Week 7 Discussion

1. Graph Basics

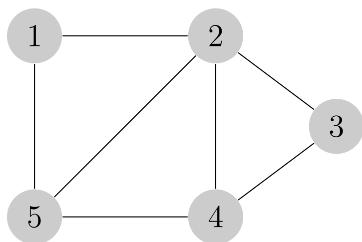


Figure 1: An undirected graph G_1 with 5 vertices and 7 edges

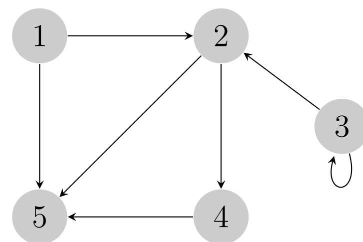
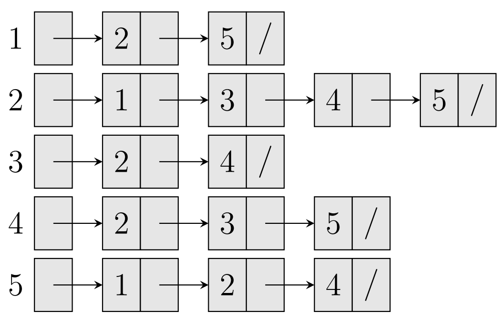


Figure 2: A directed graph G_2 with 5 vertices and 7 edges.

a. Representation: Adjacency List, Adjacency Matrix

What is the adjacency list and adjacency matrix of G_1 and G_2, respectively?



	1	2	3	4	5
1	0	1	0	0	1
2	1	0	1	1	1
3	0	1	0	1	0
4	0	1	1	0	1
5	1	1	0	1	0

Figure 3: Adjacency list and adjacency matrix representation of G_1

How about G_2? It's your turn now!

1. Adjacency matrix is of size $|V| \times |V|$ while adjacency list needs $\Theta(|V| + |E|)$ space.

2. If G is undirected, its adjacency matrix A is symmetric. Namely, $A^T = A$. Further, the main diagonal entries of A are all zeros.

3. **Self-loops**—edges from a vertex to itself—are possible in a directed graph, but are forbidden in an undirected graph.

b. Degree

1. $\sum_{u \in V}$ degree$(u) = 2|E|$, where G is an undirected graph.
2. $\sum_{u \in V}$ out-degree$(u) = \sum_{u \in V}$ in-degree$(u) = |E|$, where G is a directed graph.
3. degree(u) = out-degree(u) + in-degree(u), where $u \in V$ and G is a directed graph.
4. A vertex whose degree is 0 is **isolated**.

2. BFS

```
BFS(G, s)
// G: input graph (sorted in alphabetical/ascending order);
// s: source vertex
for each vertex u ∈ V − {s}
    d[u] = +INFTY
d[s] = 0

// create FIFO queue
Q = EMPTY
ENQUEUE(G, s)
while Q not EMPTY
    u = DEQUEUE(G)
    for each v ∈ Adj[u]
        if d[v] = +INFTY
            d[v] = d[u] + 1
            ENQUEUE(G, v)
return d
```

Let's run BFS on graph G_1!