

## 1. Review: Polynomials, Exponentials, Logarithms, and their Derivatives

### a. Polynomials

$p(n) = \sum_{i=0}^d a_i n^i$  is called a polynomial in  $n$  of degree  $d$  where  $a_d \neq 0$ .

A polynomial is asymptotically positive if and only if  $a_d > 0$ , and we have  $p(n) = \Theta(n^d)$ .

### b. Exponentials

$a^n$  is the basic form of an exponential, where  $a$  is called base.

$$(a^n)^m = a^{mn} = (a^m)^n.$$

For all real constants  $a$  and  $b$  such that  $a > 1$ ,

$$\lim_{n \rightarrow \infty} \frac{n^b}{a^n} = 0$$

from which we can conclude that any exponential function with a base strictly greater than 1 grows faster than any polynomial function.

### c. Logarithms

$\lg n = \log_2 n$ ;  $\ln n = \log_e n$  (natural logarithm).

For all real  $a > 0$ ,  $b > 0$ ,  $c > 0$ , and  $n$ , we have the following properties:

i.  $a = b^{\log_b a}$

ii.  $\log_c(ab) = \log_c a + \log_c b$

iii.  $\log_b a^n = n \log_b a$

iv.  $\log_b a = \frac{\log_c a}{\log_c b}$

### d. Derivatives

$$f(x) = ax^n, \quad \frac{df}{dx} = anx^{n-1}.$$

$$f(x) = a^x, \quad \frac{df}{dx} = a^x \ln a.$$

$$f(x) = \ln x, \quad \frac{df}{dx} = \frac{1}{x}, \quad x > 0; \quad f(x) = \log_a x, \quad \frac{df}{dx} = \frac{1}{x \ln a}, \quad x > 0.$$

## 2. Asymptotic Notation: $O$ , $\Omega$ , $\Theta$

**Example:** We have  $f(n) = \lg n$ ,  $g(n) = \ln n$ , what is their relation represented in asymptotic notation? There are two ways.

a. By definition

b. By using limits (L'Hopital's rule)