Functions

3.27. Let $W = \{a, b, c, d\}$. Decide whether each set of ordered pairs is a function from W into W.

(a) $\{(b, a), \quad (c, d), \quad (d, a), \quad (c, d) \quad (a, d)\}$
(b) $\{(d, d), \quad (c, a), \quad (a, b), \quad (d, b)\}$
(c) $\{(a, b), \quad (b, b), \quad (c, d), \quad (d, b)\}$
(d) $\{(a, a), \quad (b, a), \quad (a, b), \quad (c, d)\}$

(a) ✓ (b) ✗ no b in the domain

(c) ✓ (d) ✗

OR use arrow diagram

One-to-one, onto and invertible

3.5. Let the functions $f: A \to B$, $g: B \to C$, $h: C \to D$ be defined by Fig. 3-9. Determine if each function is: (*a*) onto, (*b*) one-to-one, (*c*) invertible.

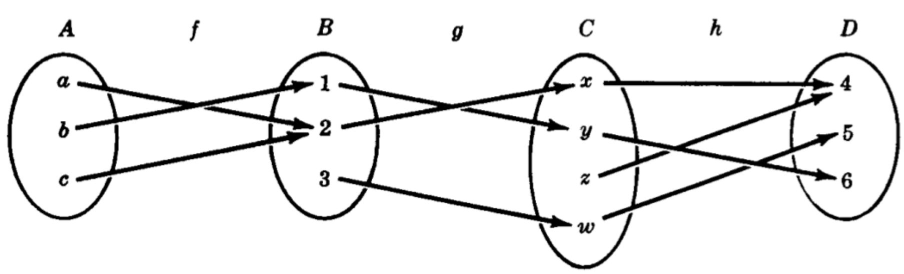

Fig. 3-9

3.7. Consider functions $f: A \to B$ and $g: B \to C$. Prove the following:

(a) If f and g are one-to-one, then the composition function $g \circ f$ is one-to-one.

(b) If f and g are onto functions, then $g \circ f$ is an onto function.

(a) Let $(a, b) \in f$ and $(b, c) \in g$,

then $(a, c) \in g \circ f$.

Suppose $g \circ f(d) = c$, since g is one-to-one only $g(b) = c$, thus $f(d) = b$. We also know that f is one-to-one, it means $a = d$. Therefore $g \circ f$ is one-to-one. $\square$

(b) Suppose c is an arbitrary element in C. Since g is onto, we can always find a b such that $g(b) = c$. Similarly, since f is onto, we also can always find an a such that $f(a) = b$. Therefore $g \circ f(a) = c$. Note that we choose c arbitrarily, thus $\forall\, c \in C$ is the image of some element of A, $g \circ f$ is onto. $\square$

Modular Arithmetic and Logarithms

$k \pmod{M}$

$k = Mq + r$ where $0 \leq r < M$

What if k is negative?

$-35 \pmod{11}$

$-240 \pmod{42}$

$f(x) = a^x$

$g(x) = \log_a x$

$2^0, 2^1, 2^2, 2^3, 2^4, 2^5, 2^6, \cdots, 2^{10}$

$\log_2 32 =$ ______

$3^0, 3^1, 3^2, \cdots$

$\log_3 27 =$ ______