# ECS 20: Discrete Mathematics for Computer Science

Winter 2021

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Week 9, March 1

# Outline: Midterm 4 Prep

- Recursion Recap and more examples
- Counting FAQ

# Recursion Recap

1. What is recurrence relation?
2. Simple recurrences, e.g. linear first order homogeneous recurrence with constant coefficient and/or constant term
   **Method:** substitution (direct iterative method)
   **Exercises:** Homework Problem 1
3. Linear second order homogeneous recurrence with constant coefficient
   **Method:** find the root(s) by characteristic equation and plug into formula
   **Exercises:** Homework Problem 2
4. Linear non-homogeneous recurrence
   **Method:** "Educated guess" (particular solution) plus the solution to the associated homogeneous recurrence
   **Exercises:** Homework Problem 3 - 5

# Example 1

Find all the solutions of $a_n = 2a_{n-1} + 3$ with $a_0 = 2$.

$b_1$: 1st term in geometric series

$m$: # of terms

Type : simple recurrence

How : substitution

$$a_n = 2a_{n-1} + 3 \quad (1)$$

$$= 2(2a_{n-2} + 3) + 3 \quad (2)$$

$$= 2^2 a_{n-2} + 2\cdot 3 + 3 \quad (3)$$

$$= 2^2(2a_{n-3} + 3) + 2\cdot 3 + 3 \quad (4)$$

Pattern 1:

$3 + n - 3 = n$

$$= 2^3 a_{n-3} + 2^2\cdot 3 + 2^1\cdot 3 + 2^0\cdot 3 \quad (5)$$

Pattern 2: quotient $q = 2$

$$= \cdots$$

$$= 2^n a_0 + \frac{b_1(q^m - 1)}{q - 1}$$

geometric series

$2^{n-1}\cdot 3, 2^{n-2}\cdot 3, \ldots, 2^0\cdot 3$

$\boxed{n}$

# Example 2

$= 2 \cdot 2 + \frac{3(2^{\ }-1)}{2-1}$ $\quad 0, \dots n-1$

$= 2^n \cdot 2 + 3(2^n - 1) = 5 \cdot 2^n - 3$ $\quad \#: n$

Find all the solutions of $a_n = 4a_{n-1} - 4a_{n-2}$ with $a_0 = 1, a_1 = 2$.

2nd + Home RR

$a_n - 4a_{n-1} + 4a_{n-2} = 0$

$r^2 - 4r + 4 = 0$ $*$

1. factrization
2. root finding formula

$(r-2)^2 = 0$ $\quad r_1 = r_2 = 2$

Theorem B

$a_n = (\alpha + \beta n) 2^n$

$a_n = 2^n$

$r = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$ar^2 + br + c = 0$

# Example 3

Find all the solutions of $a_n = 2a_{n-1} + \underset{=}{\overset{f(n)}{n^2}}$ with $\boxed{a_0 = 2}$

(*Hint:* $a_n^{(p)} = An^2 + Bn + C$)

Non-homo RR

$a_n = a_n^{(h)} + a_n^{(p)}$

$a_n^{(h)}$ $a_n = 2a_{n-1}$ by substitution

$= 2^n \underline{\underline{a_0}} = 2^n \alpha$

not 2

$An^2 + Bn + C = 2(A(n-1)^2 + B(n-1) + C) + n^2$

$= 2(A(n^2 - 2n + 1) + Bn - B + C) + n^2$

$= 2An^2 - 4An + 2A + 2Bn - 2B + 2C + n^2$

$= \underline{(2A+1)}n^2 + (2B - 4A)n + 2A - 2B + 2C$

$$\begin{cases} A = 2A + 1 \\ B = 2B - 4A \\ C = 2A - 2B + 2C \end{cases} \longrightarrow \begin{cases} A = -1 \\ B = -4 \\ C = -6 \end{cases}$$

# Midterm 4 Prep: Counting

$$a_n^{(h)} + a_n^{(p)} = -n^2 - 4n - 6 + 2^n \cdot \alpha$$

$n=0$ $\quad a_0 = 2 = -0 - 0 - 6 + 2^0 \cdot \alpha$

**Road map and problems worth mentioning:**

$2 = -6 + \alpha \qquad \alpha = 8$

$$\boxed{a_n = -n^2 - 4n - 6 + 8 \cdot 2^n}$$

1. Factorial notation and binomial coefficients:
   5.1(c), 5.4(c), 5.35(erratum)
2. Counting principles (sum rule, product rule or combined).
3. Inclusion-exclusion principle:
4. Permutations:
   5.12(b), 5.44(c)
5. Combinations:
   5.16(c)(erratum)
6. Pigeonhole principle:
   5.19(a)