

# ECS 20: Discrete Mathematics for Computer Science

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# Outline

- ▶ Counting Review
- ▶ Examples of Combinatorial Proof (More in handout)
- ▶ Encryption: Modular functions in CS (See handout, if time allows)
- ▶ Recursion Recap (if time permits)

# Counting Review

## Road map and problems worth mentioning:

1. Factorial notation and binominal coefficients:  
5.1(c), 5.4(c), 5.35(erratum)
2. Counting principles (sum rule, product rule or combined):
3. Inclusion-exclusion principle:
4. Permutations:  
5.12(b), 5.44(c)
5. Combinations:  
5.16(c)(erratum)
6. Pigeonhole principle:  
5.19(a)

## Combinatorial Proof

**Question:** How do we expand  $(x + y)^n$  ?

**Example:** The coefficient of  $xy^2$  in  $(x + y)^3$ :

$$(x + y)^3 = \underbrace{(x + y)} \underbrace{(x + y)} \underbrace{(x + y)} \quad (1)$$

$$= xxx + xxy + xyx + \underline{xyy} + yxx + \underline{yxy} + \underline{yyx} + yyy \quad (2)$$

$$= x^3 + 3x^2y + \underline{3xy^2} + y^3 \quad (3)$$

At Step (2), choose either  $x$  or  $y$  from three  $x + y$  in (1), respectively.

The coefficient of  $xy^2$  is equal to the number of terms of  $xyy$ , namely where we have 1  $x$  and 2  $y$ .

We can extend the same reasoning to Binomial theorem:

$$(x + y)^n = \sum_{j=0}^n C(n, j) x^{n-j} y^j$$

# Recursion Recap

1. What is recurrence relation?
2. Simple recurrences, e.g. linear first order homogeneous recurrence with constant coefficient  
**Method:** substitution (direct iterative method)  
**Exercises:** Homework Problem 1
3. Linear second order homogeneous recurrence with constant coefficient  
**Method:** find the root(s) by characteristic equation and plug into formula  
**Exercises:** Homework Problem 2
4. Linear non-homogeneous recurrence  
**Method:** "Educated guess" (particular solution) plus the solution to the associated homogeneous recurrence  
**Exercises:** Homework Problem 3 - 5