ECS 20: Discrete Mathematics for Computer Science

Winter 2021

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Outline

- Counting Review
- Examples of Combinatorial Proof (More in handout)
- Encryption: Modular functions in CS (See handout, if time allows)
- Recursion Recap (if time permits)

Counting Review

Road map and problems worth mentioning:

1. Factorial notation and binominal coefficients:
   5.1(c), 5.4(c), 5.35(erratum)

2. Counting principles (sum rule, product rule or combined):

3. Inclusion-exclusion principle:

4. Permutations:
   5.12(b), 5.44(c)

5. Combinations:
   5.16(c)(erratum)

6. Pigeonhole principle:
   5.19(a)

Combinatorial Proof

Question: How do we expand $(x + y)^n$?

Example: The coefficient of xy^2 in $(x + y)^3$:

$$(x + y)^3 = \underbrace{(x + y)}\underbrace{(x + y)}\underbrace{(x + y)} \tag{1}$$

$$= xxx + xxy + xyx + \underline{xyy} + yxx + \underline{yxy} + \underline{yyx} + yyy \tag{2}$$

$$= x^3 + 3x^2y + \underline{3xy^2} + y^3 \tag{3}$$

At Step (2), choose either x or y from three $x+y$ in (1), respectively.

The coefficient of xy^2 is equal to the number of terms of xyy, namely where we have 1 x and 2 y.

We can extend the same reasoning to Binomial theorem:

$$(x + y)^n = \sum_{j=0}^{n} C(n, j) x^{n-j} y^j$$

Recursion Recap

1. What is recurrence relation?

2. Simple recurrences, e.g. linear first order homogeneous recurrence with constant coefficient
   Method: substitution (direct iterative method)
   Exercises: Homework Problem 1

3. Linear second order homogeneous recurrence with constant coefficient
   Method: find the root(s) by characteristic equation and plug into formula
   Exercises: Homework Problem 2

4. Linear non-homogeneous recurrence
   Method: "Educated guess" (particular solution) plus the solution to the associated homogeneous recurrence
   Exercises: Homework Problem 3 - 5