ECS 20: Discrete Mathematics for Computer Science

Winter 2021

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Outline

- Counting Review
- Examples of Combinatorial Proof (More in handout)
- Encryption: Modular functions in CS (See handout, if time allows)
- Recursion Recap (if time permits)

Counting Review

Road map and problems worth mentioning:

1. Factorial notation and binominal coefficients:
   5.1(c), 5.4(c), 5.35(erratum)
2. Counting principles (sum rule, product rule or combined):
3. Inclusion-exclusion principle:
4. Permutations:
   5.12(b), 5.44(c)
5. Combinations:
   5.16(c)(erratum)
6. Pigeonhole principle:
   5.19(a)

Combinatorial Proof

Question: How do we expand $(x+y)^n$?

Example: The coefficient of xy^2 in $(x+y)^3$:

$$
\begin{aligned}
(x+y)^3 &= \underbrace{(x+y)}\underbrace{(x+y)}\underbrace{(x+y)} && (1) \\
&= xxx + xxy + xyx + \underline{xyy} + yxx + \underline{yxy} + \underline{yyx} + yyy && (2) \\
&= x^3 + 3x^2y + \underline{3xy^2} + y^3 && (3)
\end{aligned}
$$

At Step (2), choose either x or y from three $x+y$ in (1), respectively.

The coefficient of xy^2 is equal to the number of terms of xyy, namely where we have 1 x and 2 y.

We can extend the same reasoning to Binomial theorem:

$$(x+y)^n = \sum_{j=0}^{n} C(n,j)x^{n-j}y^j$$

Recursion Recap

1. What is recurrence relation?
2. Linear first order homogeneous recurrence with constant coefficient
   Method: substitution (direct iterative method)
   Exercises: Homework Problem 1
3. Linear second order homogeneous recurrence with constant coefficient
   Method: find the root(s) by characteristic equation and plug into formula
   Exercises: Homework Problem 2
4. Linear non-homogeneous recurrence
   Method: "Educated guess" (particular solution) plus the solution to the associated homogeneous recurrence
   Exercises: Homework Problem 3 - 5