ECS 20: Discrete Mathematics for Computer Science

Winter 2021

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Week 5, February 3

Outline

- Propositional Logic in Computer Science
- Proof Techniques Recap

Propositional Logic in Computer Science

1. in Database Systems (Boolean Search)
2. in Programming languages
3. in Computer Architecture (Logical Circuit)
4. $\cdots$

Intro to Databases: An app of Relations in CS

Definition. Let $A_1, A_2, \cdots, A_n$ be sets. An *n*-ary relation on these sets is a subset of $A_1 \times A_2 \times \cdots \times A_n$.

Intro to Databases: An app of Relations in CS

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App: Relational Databases

1. Present the data to the user as relations;
2. Provide relational operators to manipulate the data in tabular form.

Example: Class roster

Name	ID	Kerberos	Section	Major	$\cdots$	Grade
Rachel	932221234	rgreen	A01	MGT	$\cdots$	A-
Monica	932221235	mgeller	A01	FST	$\cdots$	A+
Phoebe	921119876	pbuffay	A03	PSC	$\cdots$	B+
Joey	920001234	jtri	A03	DRA	$\cdots$	A-
Chandler	913339876	cbing	A02	ECS	$\cdots$	A
Ross	913339877	rgeller	A02	BIS	$\cdots$	A+

Manipulate data by **SQL** (Structured Query Language), e.g. Addition, Deletion, Update and Search.

Propositional Logic in Computer Science

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1. Search for all the students in Section A02 **AND** their grades are better than A-:
   `SELECT * FROM ecs20_roster WHERE Section = 'A02' AND GRADE ≥ 'A-';`
2. Search for all the students in Section A02 **OR** in Section A03:
   `SELECT * FROM ecs20_roster WHERE Section = 'A02' OR Section = 'A03';`
3. Search for all the students **NOT** majoring in Computer Science:
   `SELECT * FROM ecs20_roster WHERE NOT Major = 'ECS';`

Propositional Logic in Computer Science (More in ECS-120/140)

In many of if-conditionals and for/while loops, we might encounter propositional logic.

```
if (( a >= b && b >= c) || (b >= a && a >= c )) {
    return c;
}
else if (( a >= b && b < c ) || ( b >= a && a < c )) {
    return b;
}
else
    return a;
```

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```

$$\downarrow$$

The Boolean satisfiability problem (abbreviated SATISFIABILITY, **SAT**) is the problem of determining if there exists an interpretation that satisfies a given Boolean formula. [1]

$$(a \geq b \wedge b \geq c) \vee (b \geq a \wedge a \geq c)$$

[1] Wikipedia

Propositional Logic in Computer Science (More in ECS-154A)

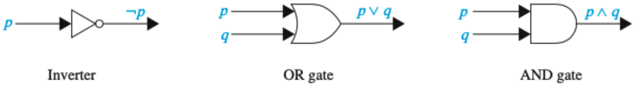

FIGURE 1 **Basic logic gates.**

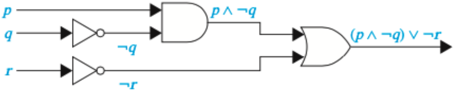

FIGURE 2 **A combinatorial circuit.**

Proof Techniques Recap

So far, we've learned:

1. Direct proof
2. Proof by contraposition
3. Proof by contradiction

New materials to come this week:

1. Equivalence proof
2. Proof by counterexample
3. **Mathematical induction**

Proof Techniques Recap

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Proof Techniques Recap

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Example 1. Prove that if m and n are integers and mn is even, then m is even or n is even.

Since there is no obvious way of showing that m is even or n is even directly from the assumption that mn is even, we attempt a proof by contraposition.

Proof. $\square$

Proof Techniques Recap

Example 2. Prove that if $5n + 4$ is odd, then n is odd.

Proof Techniques Recap

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Still, direct proof is not easy in the direction from a greater number $(5n + 4)$ to a smaller number (n).

Proof.
by Contraposition

$\square$

Proof.
by Contradiction

$\square$

Mathematical Induction: Approach

When applicable: Prove that a property $P(n)$ holds for every natural number n.

How it works:

1. Show that a property $P(n)$ holds for the base case, usually when $n = 0$ or 1.
2. Assume that $P(n)$ is true for $n = k$ where k is greater than the base case.
3. Prove that $P(n)$ is true for $n = k + 1$. In this step we will use the assumption above.
4. Then, by the principle of induction, we can conclude that $P(n)$ is true for every natural number that is greater than or equal to the base case.

Mathematical Induction: Example

Problem Statement: Prove the following statement $P(n)$ is true

$$\sum_{i=1}^{n} i^3 = (\frac{n(n+1)}{2})^2$$

Build a skeleton:

- Basis Step:
- Inductive Hypothesis:
- Inductive Step:
- Thus, by the principle of induction, we can conclude that the statement above is true for every natural number n.

Mathematical Induction: Example

$$1^3 + 2^3 + \cdots + n^3 = (\frac{n(n+1)}{2})^2$$

- Basis Step:
  Verify $P(n)$ is true when $n = 1$. $1^3 = (\frac{1 \cdot (1+1)}{2})^2$.

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- Basis Step:

  Verify $P(n)$ is true when $n = 1$. $1^3 = (\frac{1 \cdot (1+1)}{2})^2$.
- Inductive Hypothesis:

  Assume $P(n)$ is true when $n = k$. Then, we have $1^3 + 2^3 + \cdots + k^3 = (\frac{k(k+1)}{2})^2$.

Mathematical Induction: Example

$$1^3 + 2^3 + \cdots + n^3 = (\frac{n(n+1)}{2})^2$$

- Inductive Step: We need to show $P(n)$ is true when $n = k + 1$.

$$\begin{aligned}
1^3 + 2^3 + \cdots + (k+1)^3 &= 1^3 + 2^3 + \cdots + k^3 + (k+1)^3 \\
&= (\frac{k(k+1)}{2})^2 + (k+1)^3 \\
&= \frac{k^2(k+1)^2}{4} + \frac{4(k+1)^3}{4} \\
&= \frac{(k+1)^2(k^2 + 4(k+1))}{4} \\
&= \frac{(k+1)^2(k+2)^2}{4} \\
&= (\frac{(k+1)((k+1)+1)}{2})^2
\end{aligned}$$