

ECS 20: Discrete Mathematics for Computer Science

Winter 2021

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Week 5, February 1

Outline

- ▶ Relations in Computer Science
- ▶ Propositional Logic in Computer Science
- ▶ Proof Techniques Recap

Relations in Computer Science

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Yes, we can. $R = \{(Ji\ Wang, 912345678), (Jayneel\ Vora, 923456789), (Parichaya\ Chatterji, 934567890), \dots\}$

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However, more often, we want more information about a student, for instance, undergrad/graduate, department, etc. **Can we extend relation R to n -ary?**

Relations in Computer Science

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App: Relational Databases ¹

1. Present the data to the user as relations;
2. Provide relational operators to manipulate the data in tabular form.

In practice, we organize data into one or more tables (or “relations”) of *columns and rows*, with a unique key identifying each row. Rows are also called records or tuples while columns are also called attributes or fields.

¹Wikipedia

Relations in Computer Science

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Name	ID	Kerberos	Section	Major	...	Grade
Rachel	932221234	rgreen	A01	MGT	...	A-
Monica	932221235	mgeller	A01	FST	...	A+
Phoebe	921119876	pbuffay	A03	PSC	...	B+
Joey	920001234	jtri	A03	DRA	...	A-
Chandler	913339876	cbing	A02	ECS	...	A
Ross	913339877	rgeller	A02	BIS	...	A+

Manipulate data by **SQL** (Structured Query Language), e.g. Addition, Deletion, Update and Search.

Relations in Computer Science (More in ECS-165)

Name	ID	Kerberos	Section	Major	...	Grade
Rachel	932221234	rgreen	A01	MGT	...	A-
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1. Add student Gunther (no longer on the waitlist):
`INSERT INTO ecs20_roster VALUES ('Gunther',
931234567, 'gcentral', ...);`
2. Delete Rachel's record (she drops):
`DELETE FROM ecs20_roster WHERE Name='Rachel';`
3. Update Joey's grade to A:
`UPDATE ecs20_roster SET Grade = 'A' WHERE Name
='Joey';`
4. Search for all the students in Section A03:
`SELECT * FROM ecs20_roster WHERE Section = 'A03';`

Propositional Logic in Computer Science

1. in Database Systems (Boolean Search)
2. in Programming languages
3. in Computer Architecture (Logical Circuit)
4. ...

Propositional Logic in Computer Science

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1. Search for all the students in Section A02 **AND** their grades are better than A-:

```
SELECT * FROM ecs20_roster WHERE Section = 'A02'  
AND GRADE  $\geq$  'A-';
```

2. Search for all the students in Section A02 **OR** in Section A03:

```
SELECT * FROM ecs20_roster WHERE Section = 'A02'  
OR Section = 'A03';
```

3. Search for all the students **NOT** majoring in Computer Science:

```
SELECT * FROM ecs20_roster WHERE NOT Major =  
'ECS';
```

Propositional Logic in Computer Science (More in ECS-120/140)

In many of the if conditionals and for/while loop, we might encounter propositional logic.

```
if (a >= b && b >= c) {  
    return c;  
}  
else if (a >= b && b < c) {  
    return b;  
}  
else  
    return a;
```



The Boolean satisfiability problem (abbreviated SATISFIABILITY, **SAT**) is the problem of determining if there exists an interpretation that satisfies a given Boolean formula.

$$x \geq 0 \wedge y = x + 1 \wedge (y > 2 \vee y < 1)$$

Propositional Logic in Computer Science (More in ECS-154A)

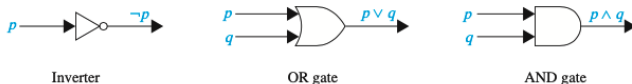


FIGURE 1 Basic logic gates.

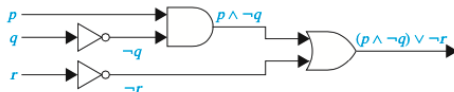


FIGURE 2 A combinational circuit.

Proof Techniques Recap

So far, we've learned:

1. Direct proof
2. Proof by contraposition
3. Proof by contradiction

New materials to come this week:

1. Equivalence proof
2. Proof by counterexample
3. **Mathematical induction**

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Example 1. Prove that if m and n are integers and mn is even, then m is even or n is even.

Since there is no obvious way of showing that m is even or n is even directly from the assumption that mn is even, we attempt a proof by contraposition.

Proof.



Proof Techniques Recap

Example 2. Prove that if $5n + 4$ is odd, then n is odd.

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Still, direct proof is not easy in the direction from a greater number ($5n + 4$) to a smaller number (n).

Proof.



Proof.

