ECS 20: Discrete Mathematics for Computer Science

Winter 2021

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Week 10, March 8

Outline: Midterm 4 Prep

- Recap of Asymptotic Notation (e.g. Big O)
- Graphs and Trees

Big O Recap: Frequently used functions

a. Polynomials:
   $p(n) = \sum_{i=0}^{d} a_i n^i$ is called a polynomial in n of degree d where $a_d \neq 0$.
   - $p(n) = O(n^d)$.

b. Exponentiation:
   a^n is the basic form of an exponential, where a is called base.
   - $(a^n)^m = a^{mn} = (a^m)^n$.

c. Logarithms:
   The inverse function to exponentiation, namely:
   $\log_a n = x$ exactly if $a^x = n$ and $n > 0$ and $a > 0$ and $a \neq 1$.

   Usually, base $a = 2, e, 10$ (e is the constant base ≈ 2.718 for natural logarithm)

   For all real $a > 0$, $b > 0$, $c > 0$, and n, we have:
   i. $a = b^{\log_b a}$
   ii. $\log_c(ab) = \log_c a + \log_c b$
   iii. $\log_b a^n = n \log_b a$

Big O Recap: Growth relation between functions

Frequently used functions	Order of Growth	Example
constant	$O(1)$	$f(n) = 10$
logarithm	$O(\log n)$	$f(n) = 4\log(n+2)$
linear	$O(n)$	$f(n) = 2n + 1$
linearithmic	$O(n \log n)$	$f(n) = 3n \log 4n + 1$
quadratic	$O(n^2)$	$f(n) = 2n^2 + 3n + 1$
cubic	$O(n^3)$	$f(n) = n^3 + 1$
exponential	$O(a^n), a \geq 2$	$f(n) = 3^n$

Big O Recap: Problem 2 in HW8

Show that $(x^3 + 2x)/(2x + 1)$ is $O(x^2)$.

Graphs and Trees: Terminology (1)

1. **Path**: an alternating sequence of vertices and edges.
2. **Simple Path**: a path in which all vertices are distinct.
3. **Trail**: a path in which all edges are distinct.
4. **Distance**: the length of the shortest path between u and v.
5. **Diameter**: the maximum distance between any pairs of vertices.

Graphs and Trees: Terminology (2)

Problem 8.34. Consider the graph G in Fig. 8-58. Find:

(a) degree of each vertex, and verify Handshaking Theorem;

(b) all simple paths from A to L;

(c) all **trails** (distinct edges) from B to C;

(d) $d(A, C)$, **distance** from A to C;

(e) diam(G), the **diameter** of G.

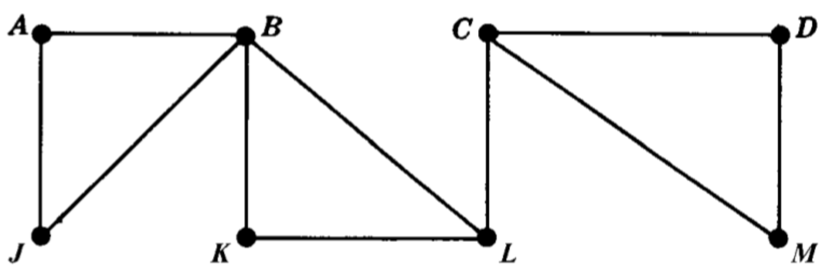

Fig. 8-58

Graphs and Trees: Graph isomorphism

To show that two simple graphs are not isomorphic, we can show that they do not share an invariant property, such as the same number of vertices, edges, degrees, etc.

Problem 8.6. Show that the six graphs in Fig. 8-39 are distinct, that is, no two of them are isomorphic.

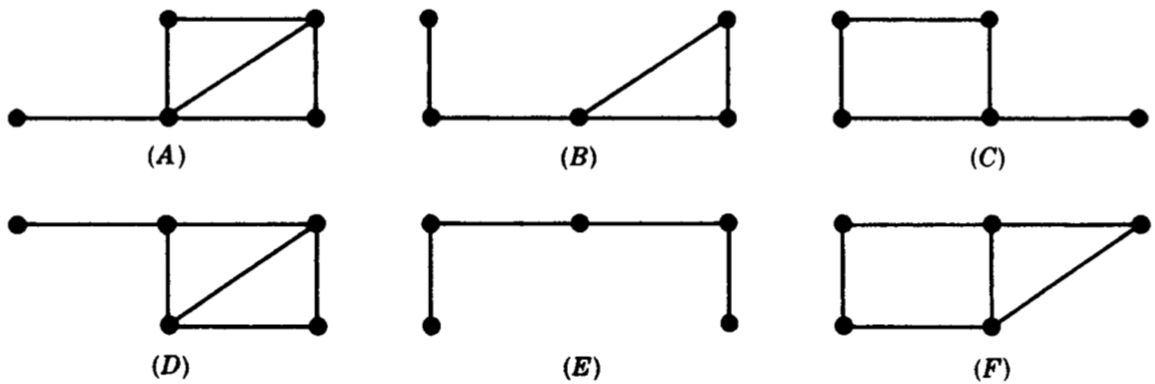

Fig. 8-39