Homework 1 Review

1.26 Which of the following sets are equal?

$$A = \{x \mid x^2 - 4x + 3 = 0\},\quad C = \{x \mid x \in \mathbf{N}, x < 3\},\quad E = \{1, 2\},\quad G = \{3, 1\},$$
$$B = \{x \mid x^2 - 3x + 2 = 0\},\quad D = \{x \mid x \in \mathbf{N}, x \text{ is odd}, x < 5\},\quad F = \{1, 2, 1\},\quad H = \{1, 1, 3\}.$$

$$A = \{x \mid x^2 - 4x + 3 = 0\}$$
$$= \{x \mid (x-1)(x-3) = 0\}$$
$$= \{1, 3\}$$

Qudratic Equation in One Variable

$ax^2 + bx + c = 0$

① By factorization

② By formula

$$B = \{x \mid x^2 - 3x + 2 = 0\}$$
$$= \{x \mid (x-1)(x-2) = 0\}$$
$$= \{1, 2\}$$

$$C = \{x \mid x \in \mathbb{N},\ x < 3\}$$
$$= \{1, 2\}$$

$$D = \{x \mid x \in \mathbb{N},\ x \text{ is odd},\ x < 5\}$$
$$= \{1, 3\}$$

$A = D = G = H \qquad B = C = E = F$

1.8 Prove Theorem 1.4. The following are equivalent: $A \subseteq B$, $A \cap B = A$, $A \cup B = B$.

Suppose $A \subseteq B$ and let $x \in A$. Then $x \in B$, hence $x \in A \cap B$ and $A \subseteq A \cap B$. By Theorem 1.3, $(A \cap B) \subseteq A$. Therefore $A \cap B = A$. On the other hand, suppose $A \cap B = A$ and let $x \in A$. Then $x \in (A \cap B)$; hence $x \in A$ and $x \in B$. Therefore, $A \subseteq B$. Both results show that $A \subseteq B$ is equivalent to $A \cap B = A$.

Suppose again that $A \subseteq B$. Let $x \in (A \cup B)$. Then $x \in A$ or $x \in B$. If $x \in A$, then $x \in B$ because $A \subseteq B$. In either case, $x \in B$. Therefore $A \cup B \subseteq B$. By Theorem 1.3, $B \subseteq A \cup B$. Therefore $A \cup B = B$. Now suppose $A \cup B = B$ and let $x \in A$. Then $x \in A \cup B$ by definition of the union of sets. Hence $x \in B = A \cup B$. Therefore $A \subseteq B$. Both results show that $A \subseteq B$ is equivalent to $A \cup B = B$.

Thus $A \subseteq B$, $A \cup B = A$ and $A \cup B = B$ are equivalent.

① $\underbrace{A \subseteq B}_{p}$ iff / if and only if / equivalent / $\Leftrightarrow$ $\underbrace{A \cap B = A}_{q}$ $p \Leftrightarrow q$

② $A \subseteq B \Leftrightarrow \underbrace{A \cup B = B}_{m}$ $p \Leftrightarrow m$

③ $A \cap B = A \Leftrightarrow A \cup B = B$ $q \Leftrightarrow m$

$q \to p \to m \Rightarrow q \Rightarrow m$

$m \to p \to q \Rightarrow m \Rightarrow q$

$q \Leftrightarrow m$

1.31 See solutions posted on Canvas

1.12 Prove: $(A \cup B)\backslash(A \cap B) = (A\backslash B) \cup (B\backslash A)$. (Thus either one may be used to define $A \oplus B$.)

Using $X\backslash Y = X \cap Y^C$ and the laws in Table 1.1, including DeMorgan's Law, we obtain:

$$\begin{aligned}(A \cup B)\backslash(A \cap B) &= (A \cup B) \cap (A \cap B)^C = (A \cup B) \cap (A^C \cup B^C)\\ &= (A \cup A^C) \cup (A \cap B^C) \cup (B \cap A^C) \cup (B \cap B^C)\\ &= \emptyset \cup (A \cap B^C) \cup (B \cap A^C) \cup \emptyset\\ &= (A \cap B^C) \cup (B \cap A^C) = (A\backslash B) \cup (B\backslash A)\end{aligned}$$

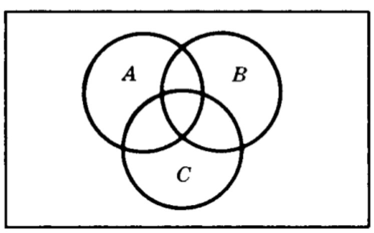
should be $\cap$ $\leftarrow$

treat $A \cup B$ as a whole

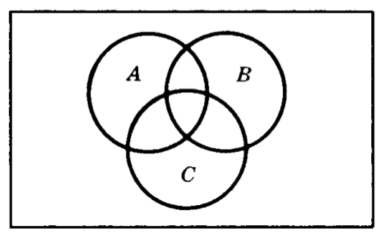
$$(A \cup B) \cap (A^c \cup B^c)$$
$$= ((A \cup B) \cap A^c) \cup ((A \cup B) \cap B^c)$$
$$= (A^c \cap (A \cup B)) \cup (B^c \cap (A \cup B))$$
$$= ((A^c \cap A) \cup (A^c \cap B)) \cup ((B^c \cap A) \cup (B^c \cap B))$$
$$= (A^c \cap B) \cup (B^c \cap A)$$
$$= (B \setminus A) \cup (A \setminus B)$$

1.34 The Venn diagram in Fig. 1-5(a) shows sets A, B, C. Shade the following sets:

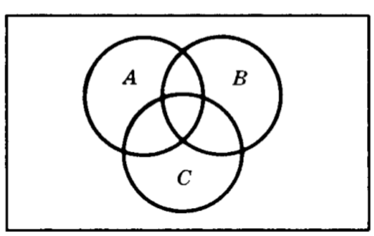
(a) $A \setminus (B \cup C)$; (b) $A^C \cap (B \cup C)$; (c) $A^C \cap (C \setminus B)$.



(a)



(a)



(a)

1.21 Let $\mathbf{N} = \{1, 2, 3, \ldots\}$ and, for each $n \in \mathbf{N}$, Let $A_n = \{n, 2n, 3n, \ldots\}$. Find:

(a) $A_3 \cap A_5$; (b) $A_4 \cap A_5$; (c) $\bigcup_{i \in Q} A_i$ where $Q = \{2, 3, 5, 7, 11, \ldots\}$ is the set of prime numbers.

(a) Those numbers which are multiples of both 3 and 5 are the multiples of 15; hence $A_3 \cap A_5 = A_{15}$.

(b) The multiples of 12 and no other numbers belong to both A_4 and A_6, hence $A_4 \cap A_6 = A_{12}$.

(c) Every positive integer except 1 is a multiple of at least one prime number; hence

should be $A_4 \cap A_6$

$$\bigcup_{i \in Q} A_i = \{2, 3, 4, \ldots\} = \mathbf{N} \backslash \{1\}$$

LCM (Least Common Multiple)

1.47 Determine whether or not each of the following is a partition of the set $\mathbf{N}$ of positive integers:

(a) $[\{n \mid n > 5\}, \{n \mid n < 5\}]$; (b) $[\{n \mid n > 6\}, \{1, 3, 5\}, \{2, 4\}]$;

(c) $[\{n \mid n^2 > 11\}, \{n \mid n^2 < 11\}]$.

(c) $n^2 > 11$

$n > \sqrt{11}$ or $n < -\sqrt{11} \rightarrow \{n \mid n \in \mathbb{Z},\ n \leq -4 \text{ or } n \geq 4\}$

$n^2 < 11$

$-\sqrt{11} < n < \sqrt{11} \longrightarrow \{n \mid -3, -2, -1, 0, 1, 2, 3\}$

Partitions

Let S be a nonempty set. A *partition* of S is a subdivision of S into nonoverlapping, nonempty subsets. Precisely, a *partition* of S is a collection $\{A_i\}$ of nonempty subsets of S such that:

(i) Each a in S belongs to one of the A_i.
(ii) The sets of $\{A_i\}$ are mutually disjoint; that is, if

$$A_j \neq A_k \quad \text{then} \quad A_j \cap A_k = \emptyset$$