ECS 20: Discrete Mathematics for Computer Science

Winter 2021

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Outline

- Midterm 3 Recap
- Counting Review
- Examples of Combinatorial Proof (More in handout)
- Encryption: Modular functions in CS (See handout, if time allows)

Counting Review

Road map and problems worth mentioning:

1. Factorial notation and binominal coefficients:
   5.1(c), 5.4 (c), 5.35 (erratum)
2. Counting principles (sum rule, product rule or combined):
3. Inclusion-exclusion principle:
4. Permutations:
   5.12 (b), 5.44 (c)
5. Combinations:
   5.16 (c) (erratum)
6. Pigeonhole principle:
   5.19 (a)

Combinatorial Proof

Question: How do we expand $(x+y)^n$?

Example: The coefficient of xy^2 in $(x+y)^3$:

$$(x+y)^3 = \underbrace{(x+y)}\underbrace{(x+y)}\underbrace{(x+y)} \tag{1}$$

$$= xxx + xxy + xyx + \underline{xyy} + yxx + \underline{yxy} + \underline{yyx} + yyy \tag{2}$$

$$= x^3 + 3x^2y + \underline{3xy^2} + y^3 \tag{3}$$

At Step (2), choose either x or y from three $x+y$, respectively.

The coefficient of xy^2 is equal to the number of terms of xyy, namely where we have 1 x and 2 y.

We can extend the same reasoning to Binomial theorem:

$$(x+y)^n = \sum_{j=0}^{n} C(n,j)x^{n-j}y^j$$