Introduction to Machine Learning

Lecture 12: Convolutional Neural Network (CNN)

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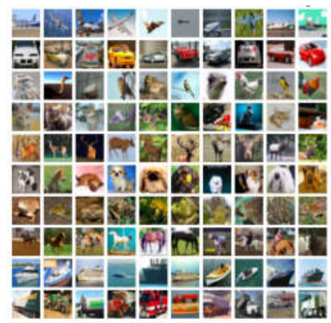
Contents

Introduction

Image Classification



- Identifying objects from various scenes is a easy task for human
- However, it is difficult for human to describe (precisely) how he/she can do it

https://medium.com/@tifa2up/image-classification-using-deep-neural-networks-a-beginner-friendly-approach-using-tensorflow-94b0a090ccd4

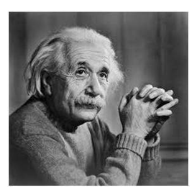
Features

≈  + ········ + Relative location

Features

- How to extract discriminative features ?
  - Hand-crafted features: HOG, SIFT, SURF etc
  - Learned features: hidden states of DNN

Hand-crafted Features – Sobel Operator



-1	0	1
-2	0	2
-1	0	1

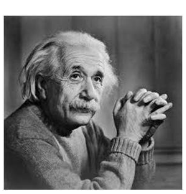
Vertical Mask

-1	-2	-1
0	0	0
1	2	1

Horizontal Mask

https://www.tutorialspoint.com/dip/sobel_operator.htm

Hand-crafted Features – Robinson Compass Mask



2	1	0
1	0	-1
0	-1	-2

South West Direction Mask

-2	-1	0
-1	0	1
0	1	2

North East Direction Mask

https://www.tutorialspoint.com/dip/robinson_compass_mask.htm

Hand-crafted Features – HOG

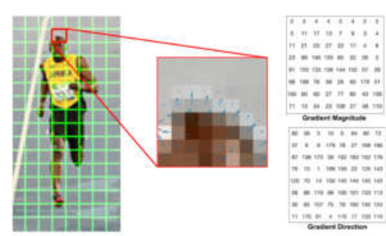
- Histograms of Oriented Gradients

$$\begin{bmatrix} -1 & 0 & 1 \end{bmatrix}$$

Vertical derivative

$$\begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$

Horizontal derivative

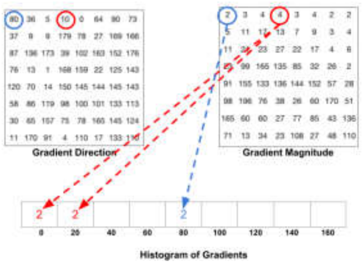


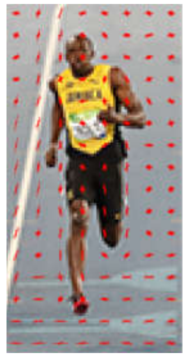
Center : The RGB patch and gradients represented using arrows. Right : The gradients in the same patch represented as numbers

https://www.learnopencv.com/histogram-of-oriented-gradients/
Dalal, Navneet, Triggs, et al. Histograms of Oriented Gradients for Human Detection. CVPR, 2005.

Hand-crafted Features – HOG

- Histograms of Oriented Gradients

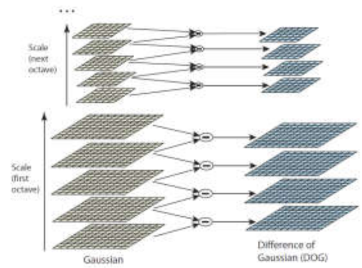




https://www.learnopencv.com/histogram-of-oriented-gradients/
Dalal, Navneet, Triggs, et al. Histograms of Oriented Gradients for Human Detection. CVPR, 2005.

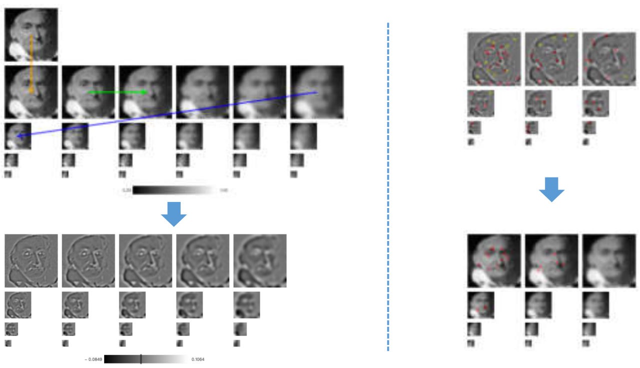
Hand-crafted Features – SIFT

- Scale-Invariant Feature Transform



Lowe D G. Distinctive Image Features from Scale-Invariant Keypoints[C]// International Journal of Computer Vision. 2004:91-110.
http://weitz.de/sift/

Hand-crafted Features – SIFT



http://weitz.de/sift/

Hand-crafted Features vs Learned Features

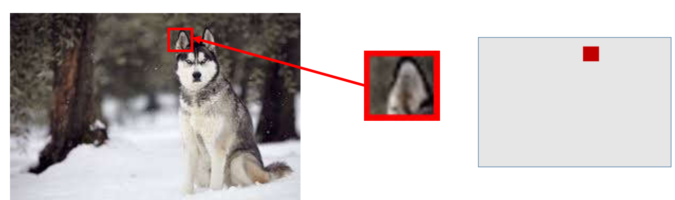
- Hand-crafted Features
  - Challenging to design
  - Require expert domain knowledge
  - Not flexible
- Learned Features
  - Automatically learned by machines
  - No need of expert domain knowledge
  - Data-driven

Learned Features

Relative location

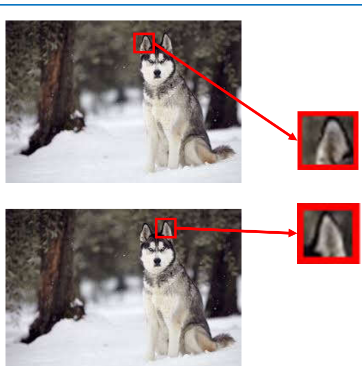
- We can design a new neural network
  - It can detect different patterns
  - It can detect similar patterns in different regions
  - It can roughly preserve the spatial information

Detect different patterns

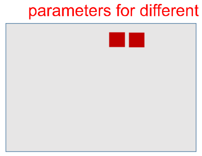


- Most patterns are much smaller in view of the whole image
  - A filter does not have to see the whole image to discover the pattern
    - One filter connects to small region with less parameters at a time

Detect the similar patterns in different regions



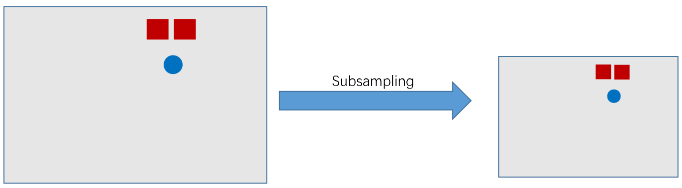
- The similar patterns appear in different regions
  - We can use one filter to detect similar patterns
    - One filter uses the same set of parameters for different regions



The spatial information

- For many vision tasks, the detected spatial information can be redundant

Roughly preserve the spatial information



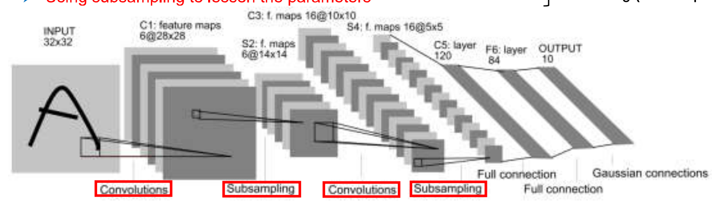
- The relative location of patterns will be preserved after subsampling
  - Using subsampling to lessen the parameters

Convolutional Neural Networks (CNN)

- Try to construct a new neural network
  - One filter connects to small region with less parameters at a time
  - One filter uses the same set of parameters for different regions
  - Using subsampling to lessen the parameters

Convolution

Pooling (Subsampling)

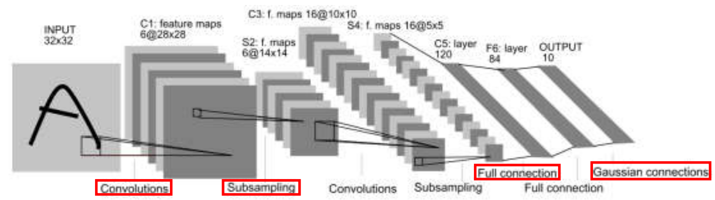


Y. Lecun, L. Bottou, Y. Bengio, and P. Haffner, "Gradient-based learning applied to document recognition," Proc. IEEE, vol. 86, no. 11, pp. 2278–2324, Feb. 1998.

Network Layers of CNN

How does CNN work

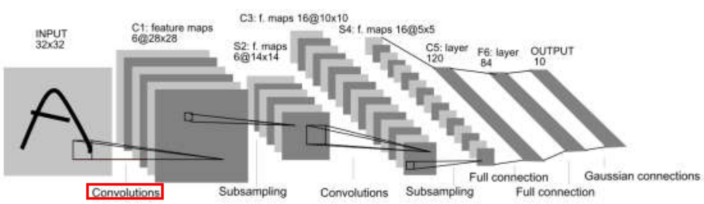
- A simple but important CNN – LeNet 5



- Given an input image, how to predict its label?

How does CNN work

- A simple but important CNN – LeNet 5



CNN – Convolution Layer

- **Input: Image**

1	1	1	1	1	1
0	0	0	0	1	0
0	1	1	1	0	0
0	0	1	0	0	0
0	1	0	0	0	0
1	0	0	0	0	0

6×6 image

- **Filters**

Filter 1

1	1	1
-1	1	-1
1	-1	-1

Filter 2

1	-1	-1
-1	1	-1
-1	-1	1

Those are parameters to be learned!

CNN – Convolution Layer

1	1	1	1	1	1
0	0	0	0	1	0
0	1	1	1	0	0
0	0	1	0	0	0
0	1	0	0	0	0
1	0	0	0	0	0

6×6 image

Filter 1

1	1	1
-1	1	-1
1	-1	-1

- One filter Connects to small region with less parameters at a time

$$1\times1+1\times1+1\times1$$
$$+0\times(-1)+0\times1+0\times(-1)$$
$$+0\times1+1\times(-1)+1\times(-1)$$

1

CNN – Convolution Layer

1	1	1	1	1	1
0	0	0	0	1	0
0	1	1	1	0	0
0	0	1	0	0	0
0	1	0	0	0	0
1	0	0	0	0	0

6×6 image

Stride = 1

Filter 1

1	1	1
-1	1	-1
1	-1	-1

- One filter uses the same set of parameters for different regions

1 2

-1

CNN – Convolution Layer

1	1	1	1	1	1
0	0	0	0	1	0
0	1	1	1	0	0
0	0	1	0	0	0
0	1	0	0	0	0
1	0	0	0	0	0

6×6 image

Stride = 3

Filter 1

1	1	1
-1	1	-1
1	-1	-1

1	5
3	

- One filter uses the same set of parameters for different regions

CNN – Convolution Layer

1	1	1	1	1	1
0	0	0	0	1	0
0	1	1	1	0	0
0	0	1	0	0	0
0	1	0	0	0	0
1	0	0	0	0	0

6×6 image

Stride = 1

1	1	1
-1	1	-1
1	-1	-1

Filter 1

1	2	2	5
-1	-2	2	0
0	5	1	1
3	0	1	0

CNN – Convolution Layer

1	1	1	1	1	1
0	0	0	0	1	0
0	1	1	1	0	0
0	0	1	0	0	0
0	1	0	0	0	0
1	0	0	0	0	0

6×6 image

Stride = 1

1	1	1
-1	1	-1
1	-1	-1

Filter 1

1	2	2	$\boxed{5}$
-1	-2	2	0
0	$\boxed{5}$	1	1
3	0	1	0

CNN – Convolution Layer

1	1	1	1	1	1
0	0	0	0	1	0
0	1	1	1	0	0
0	0	1	0	0	0
0	1	0	0	0	0
1	0	0	0	0	0

6×6 image

Stride = 1

1	1	1
-1	1	-1
1	-1	-1

Filter 1

1	2	2	5
-1	-2	2	0
0	5	1	1
3	0	1	0

CNN – Convolution Layer

1	1	1	1	1	1
0	0	0	0	1	0
0	1	1	1	0	0
0	0	1	0	0	0
0	1	0	0	0	0
1	0	0	0	0	0

6×6 image

Stride = 1

1	-1	-1
-1	1	-1
-1	-1	1

Filter 2

-1	-2	-4	-1
1	[illegible]	[illegible]	-2
-4	-1	-1	1
-1	-2	1	0

Feature Maps

2×4×4 Images

Ignore the bias and activation function!

CNN – Convolution Layer

1. How to deal with the color images?

2. How many parameters in the convolution layer?

3. What is the size of the feature maps?

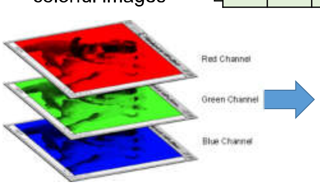
CNN – Convolution Layer

- How to deal with the color images?

1	1	1
-1	1	-1
1	-1	-1

1	-1	-1
-1	1	-1
1	-1	-1

colorful images



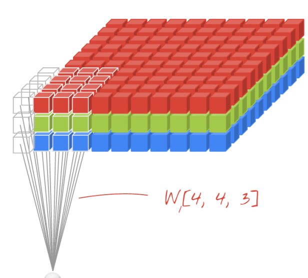
1	1	1	1	1	1
0	0	0	0	1	0
0	0	0	1	0	0
0	0	1	0	0	0
0	1	0	0	0	0
1	0	0	0	0	0

Feature maps is also a “colorful images”！

Image from: https://stackoverflow.com/questions/14113772/what-does-it-mean-to-change-the-color-channel

CNN – Convolution Layer

- How to deal with the colorful images?



https://sites.google.com/site/nttrungmtwiki/home/it/data-science---python/tensorflow/tensorflow-and-deep-learning-part-3?tmpl=%2Fsystem%2Fapp%2Ftemplates%2Fprint%2F&showPrintDialog=1

CNN – Convolution Layer

- How many parameters in the convolution layer?

1	1	1
-1	1	-1
1	-1	-1

Filter 1

$9 = 3\times3$

1	-1	-1
-1	1	-1
-1	-1	1

Filter 2

$9 = 3\times3$

……

-1	-1	-1
-1	1	-1
1	-1	1

Filter N

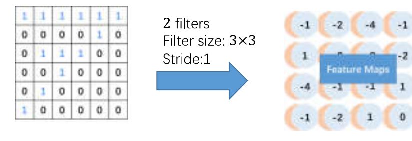
$9 = 3\times3$

There are $9N$ parameters

The power of sharing weights!

CNN – Convolution Layer

- What is the size of the feature maps?



6×6 image

2×4×4 image

Input image: $C\times H\times W$

Filters: $N\times n\times n$

Stride: s

Feature maps=? $N\times\left(\frac{H-n}{s}+1\right)\times\left(\frac{W-n}{s}+1\right)$

May not divisible!

CNN – Convolution Layer

- Padding

Stride = 2

1	1	1	1	1	1	0
0	0	0	0	1	0	0
0	1	1	1	0	0	0
0	0	1	0	0	0	
0	1	0	0	0	0	
1	0	0	0	0	0	

6×6 image

1	1	1
-1	1	-1
1	-1	-1

- Two ways
  1. Ignore the extra pixels
  2. Fill with zero

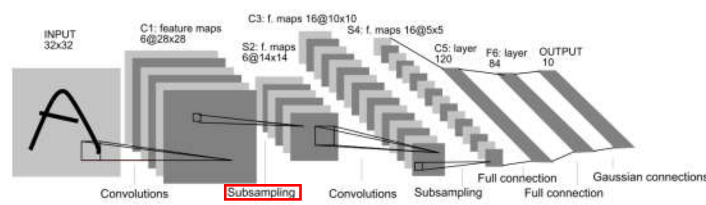
     Add P_H and P_W zeros

Feature maps = $N\times\left(\frac{H-n+P_H}{s}+1\right)\times\left(\frac{W-n+P_W}{s}+1\right)$

参考 cs231n 2017 lecture5, page62和 https://www.tensorflow.org/api_guides/python/nn#Notes_on_SAME_Convolution_Padding

CNN – Pooling Layer

- LeNet 5



CNN – Pooling Layer

- The output from the convolution layer can be huge

Input image: $1\times96\times96$

Filters: $400\times8\times8$

Stride: 1

Feature maps= $400\times(96-8+1)\times(96-8+1)$

- Output of the convolution layer

  $3168400 = 400\times(96-8+1)\times(96-8+1)$
  - Hard to train
  - Overfitting

CNN – Pooling Layer

- Max Pooling

1	1	1
-1	1	-1
1	-1	-1

Filter 1

1	2	2	5
-1	-2	2	0
0	5	1	1
3	0	1	0

Convolved Feature

2	5
5	1

Pooled Feature

- Using subsampling to lessen the parameters

CNN – Pooling Layer

- Average Pooling

1	1	1
-1	1	-1
1	-1	-1

Filter 1

1	2	2	5
-1	-2	2	0
0	5	1	1
3	0	1	0

Convolved Feature

0	2.25
2	0.75

Pooled Feature

CNN – Pooling Layer

- Overlapping Pooling

1	1	1
-1	1	-1
1	-1	-1

Filter 1

1	2	2	5
-1	-2	2	0
0	5	1	1
3	0	1	0

Convolved Feature

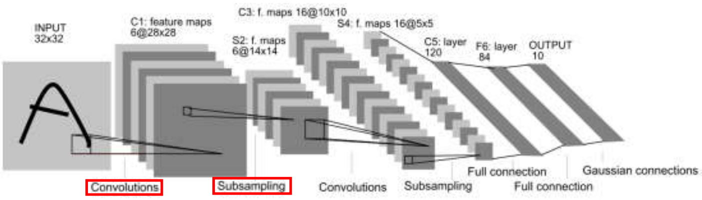
2	2	5
5	5	2
5	5	1

Pooled Feature

[1] A. Krizhevsky, I. Sutskever, and G. E. Hinton, "ImageNet Classification with Deep Convolutional Neural Networks," *NIPS*, 2012

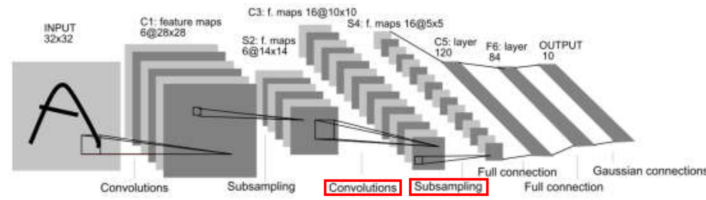
CNN - Convolution + Pooling

- LeNet 5



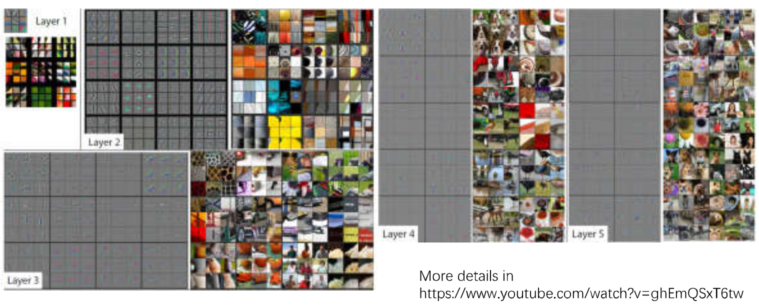
CNN - Convolution + Pooling

- LeNet 5



- Why more convolution and pooling layer?

Learned Features by CNN



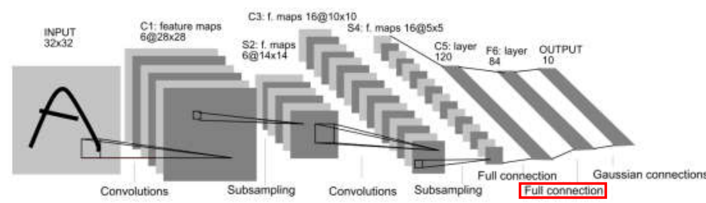
More details in
https://www.youtube.com/watch?v=ghEmQSxT6tw

- Deeper layers, more specific features

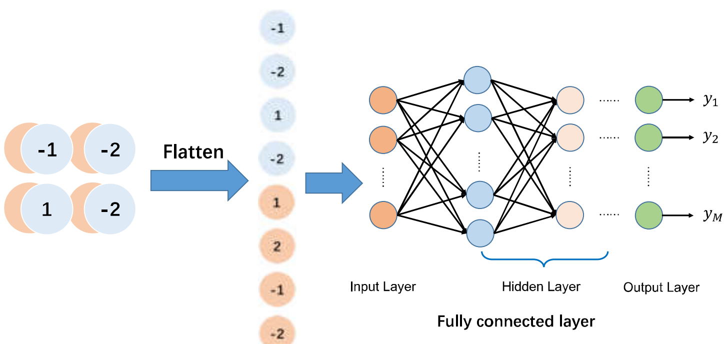
Zeiler M D, Fergus R. Visualizing and Understanding Convolutional Networks[J]. 2013, 8689:818-833.

CNN – Fully connected layer

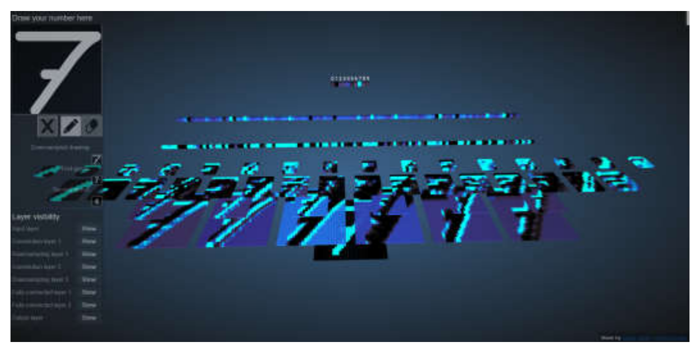
- LeNet 5



CNN – Fully connected layer



CNN – Visualization



3D convolutional network visualization http://scs.ryerson.ca/~aharley/vis/conv/

A. W. Harley, "An Interactive Node-Link Visualization of Convolutional Neural Networks," in ISVC, pages 867-877, 2015

CNN – Hyperparameters

- Convolution layers
  - Number of filters
  - Size of filters
  - Stride
- Pooling layers
  - Window size
  - Window stride
- Fully connected layers
  - Number of layers
  - Number of neurons

CNN – Traditional Methods

- Comparison

	Traditional Methods	CNN
Filters	Manually design	Learn automatically
Layers	Few	Can be quite some
Features	Low level features	From low to high level features

CNN is more powerful!

Learning a CNN

Loss functions

- Mean squared error (MSE)

$$\text{MSE} = \frac{1}{n}\sum_{i=1}^{n}(Y_i - \hat{Y}_i)^2$$

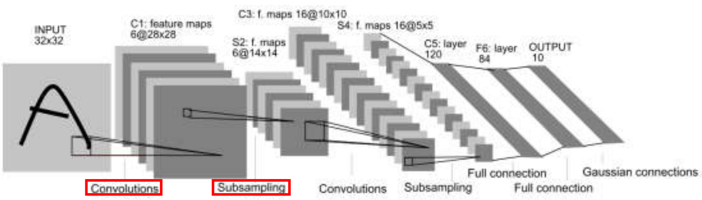
- Cross entropy loss

$$\text{Loss} = -\sum_{i=1}^{n} Y_i \log p_i$$

- User defined loss

Backpropagation

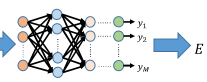
- A simple but important CNN – LeNet 5



- Back-propagation – Chain rule
- How to compute the gradients of convolution layers and pooling layers?

Backpropagation – Convolution Layer

$$\begin{array}{|c|c|c|}\hline X_{11} & X_{12} & X_{13} \\ \hline X_{21} & X_{22} & X_{23} \\ \hline X_{31} & X_{32} & X_{33} \\ \hline \end{array} + \begin{array}{|c|c|}\hline F_{11} & F_{12} \\ \hline F_{21} & F_{22} \\ \hline \end{array} \xrightarrow{f} \begin{array}{|c|c|}\hline O_{11} & O_{12} \\ \hline O_{13} & O_{14} \\ \hline \end{array} \rightarrow$$

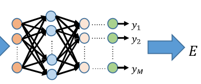


3×3 Input $\boldsymbol{X}$ 2×2 Filter $\boldsymbol{F}$ 2×2 Output $\boldsymbol{O}$

- The output of convolution operation $\boldsymbol{O} = f(\boldsymbol{F}, \boldsymbol{X})$
- The loss E
- Assume that we have already computed $\frac{\partial E}{\partial o_{ij}}$, and of course all partial derivatives of latter layers

Backpropagation – Convolution Layer

$$\begin{array}{|c|c|c|}\hline X_{11} & X_{12} & X_{13} \\ \hline X_{21} & X_{22} & X_{23} \\ \hline X_{31} & X_{32} & X_{33} \\ \hline \end{array} + \begin{array}{|c|c|}\hline F_{11} & F_{12} \\ \hline F_{21} & F_{22} \\ \hline \end{array} \xrightarrow{f} \begin{array}{|c|c|}\hline O_{11} & O_{12} \\ \hline O_{13} & O_{14} \\ \hline \end{array} \rightarrow$$



- How to compute $\frac{\partial E}{\partial F_{11}}$? $\xrightarrow{\text{Chain Rule}}$ $\frac{\partial E}{\partial F_{11}} = \left(\frac{\partial E}{\partial \boldsymbol{O}}\right)^T \frac{\partial \boldsymbol{O}}{\partial F_{11}}$

$$\frac{\partial E}{\partial \boldsymbol{O}} = \nabla E(O_{11}, O_{12}, O_{21}, O_{22})$$

$$= \left(\frac{\partial E}{\partial O_{11}}, \frac{\partial E}{\partial O_{12}}, \frac{\partial E}{\partial O_{21}}, \frac{\partial E}{\partial O_{22}}\right)^T$$

$$\boldsymbol{O} = f(\boldsymbol{F}, \boldsymbol{X}) = (O_{11}, O_{12}, O_{21}, O_{22})^T \triangleq \left(f_1(\boldsymbol{F}, \boldsymbol{X}), f_2(\boldsymbol{F}, \boldsymbol{X}), f_3(\boldsymbol{F}, \boldsymbol{X}), f_4(\boldsymbol{F}, \boldsymbol{X})\right)^T$$

$$\frac{\partial \boldsymbol{O}}{\partial F_{11}} = \left(\frac{\partial f_1}{\partial F_{11}}, \frac{\partial f_2}{\partial F_{11}}, \frac{\partial f_3}{\partial F_{11}}, \frac{\partial f_4}{\partial F_{11}}\right)^T$$

Backpropagation – Convolution Layer

$$\frac{\partial E}{\partial F_{11}} = \frac{\partial E}{\partial O_{11}}\frac{\partial f}{\partial F_{11}} + \frac{\partial E}{\partial O_{12}}\frac{\partial f}{\partial F_{11}} + \frac{\partial E}{\partial O_{21}}\frac{\partial f}{\partial F_{11}} + \frac{\partial E}{\partial O_{22}}\frac{\partial f}{\partial F_{11}}$$

$$\frac{\partial E}{\partial F_{12}} = \frac{\partial E}{\partial O_{11}}\frac{\partial f}{\partial F_{12}} + \frac{\partial E}{\partial O_{12}}\frac{\partial f}{\partial F_{12}} + \frac{\partial E}{\partial O_{21}}\frac{\partial f}{\partial F_{12}} + \frac{\partial E}{\partial O_{22}}\frac{\partial f}{\partial F_{12}}$$

$$\frac{\partial E}{\partial F_{21}} = \frac{\partial E}{\partial O_{11}}\frac{\partial f}{\partial F_{21}} + \frac{\partial E}{\partial O_{12}}\frac{\partial f}{\partial F_{21}} + \frac{\partial E}{\partial O_{21}}\frac{\partial f}{\partial F_{21}} + \frac{\partial E}{\partial O_{22}}\frac{\partial f}{\partial F_{21}}$$

$$\frac{\partial E}{\partial F_{22}} = \frac{\partial E}{\partial O_{11}}\frac{\partial f}{\partial F_{22}} + \frac{\partial E}{\partial O_{12}}\frac{\partial f}{\partial F_{22}} + \frac{\partial E}{\partial O_{21}}\frac{\partial f}{\partial F_{22}} + \frac{\partial E}{\partial O_{22}}\frac{\partial f}{\partial F_{22}}$$

Backpropagation – Convolution Layer

$$\boldsymbol{O} = f(\boldsymbol{F}; \boldsymbol{X}) \quad f_1(\boldsymbol{X}) = O_{11} = F_{11}X_{11} + F_{12}X_{12} + F_{21}X_{21} + F_{22}X_{22}$$

$$f_2(\boldsymbol{X}) = O_{12} = F_{11}X_{12} + F_{12}X_{13} + F_{21}X_{22} + F_{22}X_{23}$$

$$f_3(\boldsymbol{X}) = O_{21} = F_{11}X_{21} + F_{12}X_{22} + F_{21}X_{31} + F_{22}X_{32}$$

$$f_4(\boldsymbol{X}) = O_{22} = F_{11}X_{22} + F_{12}X_{23} + F_{21}X_{32} + F_{22}X_{33}$$

$$\frac{\partial f_1}{\partial F_{11}} = X_{11} \quad \frac{\partial f_2}{\partial F_{11}} = X_{12} \quad \frac{\partial f_3}{\partial F_{11}} = X_{21} \quad \frac{\partial f_4}{\partial F_{11}} = X_{22}$$

$$\vdots \quad \vdots \quad \vdots \quad \vdots$$

$$\frac{\partial f_1}{\partial F_{22}} = X_{22} \quad \frac{\partial f_2}{\partial F_{22}} = X_{23} \quad \frac{\partial f_3}{\partial F_{22}} = X_{32} \quad \frac{\partial f_4}{\partial F_{22}} = X_{33}$$

Backpropagation – Convolution Layer

$$\frac{\partial E}{\partial F_{11}} = \frac{\partial E}{\partial O_{11}}X_{11} + \frac{\partial E}{\partial O_{12}}X_{12} + \frac{\partial E}{\partial O_{21}}X_{21} + \frac{\partial E}{\partial O_{22}}X_{22}$$

$$\frac{\partial E}{\partial F_{12}} = \frac{\partial E}{\partial O_{11}}X_{12} + \frac{\partial E}{\partial O_{12}}X_{13} + \frac{\partial E}{\partial O_{21}}X_{22} + \frac{\partial E}{\partial O_{22}}X_{23}$$

$$\frac{\partial E}{\partial F_{21}} = \frac{\partial E}{\partial O_{11}}X_{21} + \frac{\partial E}{\partial O_{12}}X_{22} + \frac{\partial E}{\partial O_{21}}X_{31} + \frac{\partial E}{\partial O_{22}}X_{32}$$

$$\frac{\partial E}{\partial F_{22}} = \frac{\partial E}{\partial O_{11}}X_{22} + \frac{\partial E}{\partial O_{12}}X_{23} + \frac{\partial E}{\partial O_{21}}X_{32} + \frac{\partial E}{\partial O_{22}}X_{33}$$

Backpropagation – Convolution Layer

$$\begin{bmatrix} X_{11} & X_{12} & X_{13} \\ X_{21} & X_{22} & X_{23} \\ X_{31} & X_{32} & X_{33} \end{bmatrix} + \begin{bmatrix} \frac{\partial E}{\partial O_{11}} & \frac{\partial E}{\partial O_{12}} \\ \frac{\partial E}{\partial O_{21}} & \frac{\partial E}{\partial O_{22}} \end{bmatrix} \Rightarrow \begin{bmatrix} \frac{\partial E}{\partial F_{11}} & \frac{\partial E}{\partial F_{12}} \\ \frac{\partial E}{\partial F_{21}} & \frac{\partial E}{\partial F_{22}} \end{bmatrix}$$

$$\frac{\partial E}{\partial F_{11}} = \frac{\partial E}{\partial O_{11}} X_{11} + \frac{\partial E}{\partial O_{12}} X_{12} + \frac{\partial E}{\partial O_{21}} X_{21} + \frac{\partial E}{\partial O_{22}} X_{22}$$

$$\frac{\partial E}{\partial F_{12}} = \frac{\partial E}{\partial O_{11}} X_{12} + \frac{\partial E}{\partial O_{12}} X_{13} + \frac{\partial E}{\partial O_{21}} X_{22} + \frac{\partial E}{\partial O_{22}} X_{23}$$

$$\frac{\partial E}{\partial F_{21}} = \frac{\partial E}{\partial O_{11}} X_{21} + \frac{\partial E}{\partial O_{12}} X_{22} + \frac{\partial E}{\partial O_{21}} X_{31} + \frac{\partial E}{\partial O_{22}} X_{32}$$

$$\frac{\partial E}{\partial F_{22}} = \frac{\partial E}{\partial O_{11}} X_{22} + \frac{\partial E}{\partial O_{12}} X_{23} + \frac{\partial E}{\partial O_{21}} X_{32} + \frac{\partial E}{\partial O_{22}} X_{33}$$

Backpropagation – Convolution Layer

- How to compute $\frac{\partial E}{\partial X_{11}}$?

Chain Rule

$$\frac{\partial E}{\partial X_{11}} = \left(\frac{\partial E}{\partial \boldsymbol{O}}\right)^T \frac{\partial \boldsymbol{O}}{\partial X_{11}}$$

$$\frac{\partial E}{\partial \boldsymbol{O}} = \nabla E(O_{11}, O_{12}, O_{21}, O_{22}) = \left(\frac{\partial E}{\partial O_{11}}, \frac{\partial E}{\partial O_{12}}, \frac{\partial E}{\partial O_{21}}, \frac{\partial E}{\partial O_{22}}\right)^T$$

$$\boldsymbol{O} = f(\boldsymbol{F}, \boldsymbol{X}) = (O_{11}, O_{12}, O_{21}, O_{22})^T \triangleq \left(f_1(\boldsymbol{F}, \boldsymbol{X}), f_2(\boldsymbol{F}, \boldsymbol{X}), f_3(\boldsymbol{F}, \boldsymbol{X}), f_4(\boldsymbol{F}, \boldsymbol{X})\right)^T$$

$$\frac{\partial \boldsymbol{O}}{\partial X_{11}} = \left(\frac{\partial f_1}{\partial X_{11}}, \frac{\partial f_2}{\partial X_{11}}, \frac{\partial f_3}{\partial X_{11}}, \frac{\partial f_4}{\partial X_{11}}\right)^T$$

Backpropagation – Convolution Layer

$$\boldsymbol{O} = f(\boldsymbol{F}; \boldsymbol{X}) \quad \begin{aligned} f_1(\boldsymbol{X}) &= O_{11} = F_{11}X_{11} + F_{12}X_{12} + F_{21}X_{21} + F_{22}X_{22} \\ f_2(\boldsymbol{X}) &= O_{12} = F_{11}X_{12} + F_{12}X_{13} + F_{21}X_{22} + F_{22}X_{23} \\ f_3(\boldsymbol{X}) &= O_{21} = F_{11}X_{21} + F_{12}X_{22} + F_{21}X_{31} + F_{22}X_{32} \\ f_4(\boldsymbol{X}) &= O_{22} = F_{11}X_{22} + F_{12}X_{23} + F_{21}X_{32} + F_{22}X_{33} \end{aligned}$$

$$\frac{\partial E}{\partial X_{11}} = \frac{\partial E}{\partial O_{11}} F_{11} + \frac{\partial E}{\partial O_{12}} 0 + \frac{\partial E}{\partial O_{21}} 0 + \frac{\partial E}{\partial O_{22}} 0 \qquad \frac{\partial E}{\partial X_{12}} = \frac{\partial E}{\partial O_{11}} F_{12} + \frac{\partial E}{\partial O_{12}} F_{11} + \frac{\partial E}{\partial O_{21}} 0 + \frac{\partial E}{\partial O_{22}} 0 \qquad \frac{\partial E}{\partial X_{13}} = \frac{\partial E}{\partial O_{11}} 0 + \frac{\partial E}{\partial O_{12}} F_{12} + \frac{\partial E}{\partial O_{21}} 0 + \frac{\partial E}{\partial O_{22}} 0$$

$$\frac{\partial E}{\partial X_{21}} = \frac{\partial E}{\partial O_{11}} F_{21} + \frac{\partial E}{\partial O_{12}} 0 + \frac{\partial E}{\partial O_{21}} F_{11} + \frac{\partial E}{\partial O_{22}} 0 \qquad \frac{\partial E}{\partial X_{22}} = \frac{\partial E}{\partial O_{11}} F_{22} + \frac{\partial E}{\partial O_{12}} F_{21} + \frac{\partial E}{\partial O_{21}} F_{12} + \frac{\partial E}{\partial O_{22}} F_{11} \qquad \frac{\partial E}{\partial X_{23}} = \frac{\partial E}{\partial O_{11}} 0 + \frac{\partial E}{\partial O_{12}} F_{22} + \frac{\partial E}{\partial O_{21}} 0 + \frac{\partial E}{\partial O_{22}} F_{11}$$

$$\frac{\partial E}{\partial X_{31}} = \frac{\partial E}{\partial O_{11}} 0 + \frac{\partial E}{\partial O_{12}} 0 + \frac{\partial E}{\partial O_{21}} F_{21} + \frac{\partial E}{\partial O_{22}} 0 \qquad \frac{\partial E}{\partial X_{32}} = \frac{\partial E}{\partial O_{11}} 0 + \frac{\partial E}{\partial O_{12}} 0 + \frac{\partial E}{\partial O_{21}} F_{22} + \frac{\partial E}{\partial O_{22}} F_{21} \qquad \frac{\partial E}{\partial X_{33}} = \frac{\partial E}{\partial O_{11}} 0 + \frac{\partial E}{\partial O_{12}} 0 + \frac{\partial E}{\partial O_{21}} 0 + \frac{\partial E}{\partial O_{22}} F_{22}$$

Backpropagation – Convolution Layer

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & \frac{\partial E}{\partial O_{11}} & \frac{\partial E}{\partial O_{12}} & 0 \\ 0 & \frac{\partial E}{\partial O_{21}} & \frac{\partial E}{\partial O_{22}} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} F_{22} & F_{21} \\ F_{12} & F_{11} \end{bmatrix} \Rightarrow \begin{bmatrix} \frac{\partial E}{\partial X_{11}} & \frac{\partial E}{\partial X_{12}} & \frac{\partial E}{\partial X_{13}} \\ \frac{\partial E}{\partial X_{21}} & \frac{\partial E}{\partial X_{22}} & \frac{\partial E}{\partial X_{23}} \\ \frac{\partial E}{\partial X_{31}} & \frac{\partial E}{\partial X_{32}} & \frac{\partial E}{\partial X_{33}} \end{bmatrix}$$

Backpropagation – Pooling Layer

- Max Pooling

$$\begin{bmatrix} X^i_{11} & X^i_{12} & X^i_{13} & X^i_{14} \\ X^i_{21} & X^i_{22} & X^i_{23} & X^i_{24} \\ X^i_{31} & X^i_{32} & X^i_{33} & X^i_{34} \\ X^i_{41} & X^i_{42} & X^i_{43} & X^i_{44} \end{bmatrix} \Rightarrow \begin{bmatrix} X^o_{11} & X^o_{23} \\ X^o_{42} & X^o_{33} \end{bmatrix} \Rightarrow E$$

4×4 Input

2×2 output

- Only need to compute the gradients of the inputs

$$\frac{\partial E}{\partial X^i_{11}} = \frac{\partial E}{\partial X^o_{11}} \qquad \frac{\partial E}{\partial X^i_{12}} = 0 \qquad \frac{\partial E}{\partial X^i_{21}} = 0 \qquad \frac{\partial E}{\partial X^i_{22}} = 0$$

$$\frac{\partial E}{\partial X^i_{13}} = 0 \qquad \frac{\partial E}{\partial X^i_{14}} = 0 \qquad \frac{\partial E}{\partial X^i_{23}} = \frac{\partial E}{\partial X^o_{23}} \qquad \frac{\partial E}{\partial X^i_{24}} = 0$$

$$\vdots$$

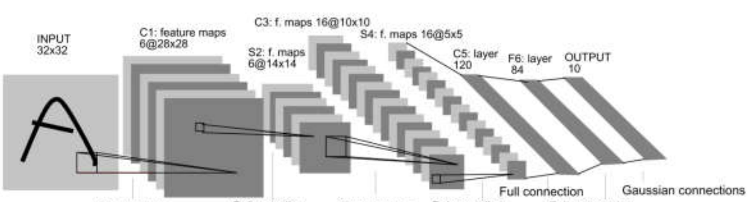
We assign the error to where it comes from

Examples of CNN Architecture

LeNet (1998)

- Gradient-based learning applied to document recognition

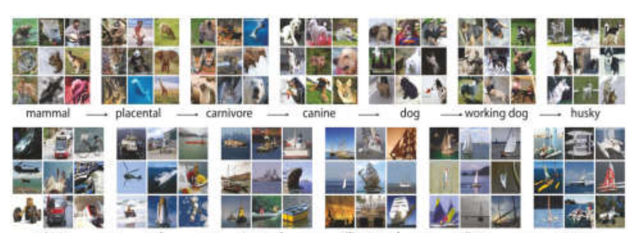
[Yann LeCun, Leon Bottou, Yoshua Bengio, and Patrick Haffner 1998]



LeNet-5 5 layers

ImageNet (Benchmark dataset)

- ImageNet
  - About 1.5×10^7 images, 2.2×10^4 categories
  - An image database organized according to the WordNet hierarchy



J Deng, W Dong, R Socher, LJ Li, K Li, L Fei-Fei. ImageNet: A large-scale hierarchical image. *CVPR*, 2009.

ISVRC

Output:
Scale
T-shirt
Steel drum
Drumstick
Mud turtle

Output:
Scale
T-shirt
Giant panda
Drumstick
Mud turtle

Classification Task

Classification + Localization Task

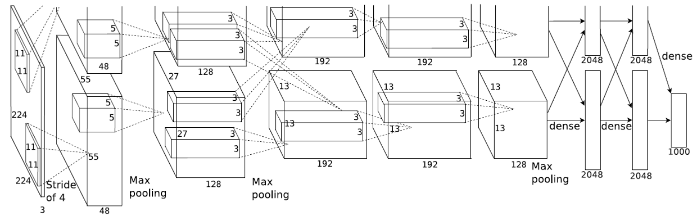
Deng et al. 2009, Russakovsky et al. 2015

Example credit: Fei-Fei Li and Jia Deng

AlexNet (2012)

- ImageNet Classification with Deep Convolutional Neural Networks

  [Alex Krizhevsky, Ilya Sutskever, and Geoffrey E. Hinton 2012]



AlexNet 8 layers Top-5 Error rate: 16.4%

VGGNet (2014)

- Very Deep Convolutional Networks for Large-Scale Image Recognition

  [Karen Simonyan, and Andrew Zisserman 2014]

ConvNet Configuration					
A	A-LRN	B	C	D	E
11 weight layers	11 weight layers	13 weight layers	16 weight layers	16 weight layers	19 weight layers
input (224 × 224 RGB image)					
conv3-64	conv3-64 **LRN**	conv3-64 **conv3-64**	conv3-64 conv3-64	conv3-64 conv3-64	conv3-64 conv3-64
maxpool					
conv3-128	conv3-128	conv3-128 **conv3-128**	conv3-128 conv3-128	conv3-128 conv3-128	conv3-128 conv3-128
maxpool					
conv3-256 conv3-256	conv3-256 conv3-256	conv3-256 conv3-256	conv3-256 conv3-256 **conv1-256**	conv3-256 conv3-256 **conv3-256**	conv3-256 conv3-256 conv3-256 **conv3-256**
maxpool					
conv3-512 conv3-512	conv3-512 conv3-512	conv3-512 conv3-512	conv3-512 conv3-512 **conv1-512**	conv3-512 conv3-512 **conv3-512**	conv3-512 conv3-512 conv3-512 **conv3-512**
maxpool					
conv3-512 conv3-512	conv3-512 conv3-512	conv3-512 conv3-512	conv3-512 conv3-512 **conv1-512**	conv3-512 conv3-512 **conv3-512**	conv3-512 conv3-512 conv3-512 **conv3-512**
maxpool					
FC-4096					
FC-4096					
FC-1000					
soft-max					

VGG-16 & VGG-19

16 layers (trainable)

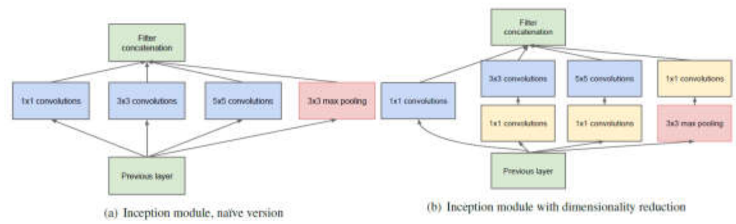
19 layers (trainable)

Top-5 Error rate: 7.3% (VGG-19)

GooLeNet (2014)

- Going Deeper with Convolutions

  [Karen Simonyan, Andrew Zisserman 2014]



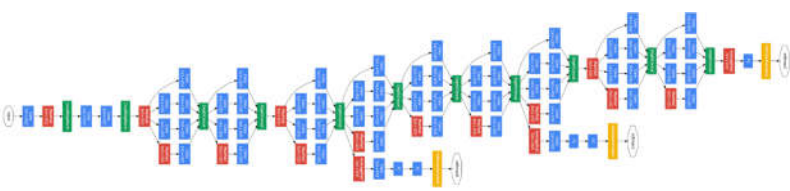
(a) Inception module, naïve version

(b) Inception module with dimensionality reduction

GooLeNet (2014)

- Going Deeper with Convolutions

  [Karen Simonyan, Andrew Zisserman 2014]



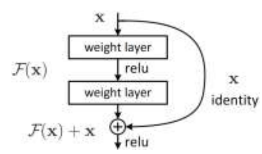
GoogLeNet

Only 3 layers??? 22 layers!!! Top-5 Error rate: 6.7%

ResNet

- Deep Residual Learning for Image Recognition

  [Kaiming He, Xiangyu Zhang, Shaoqing Ren, and Jian Sun 2015]

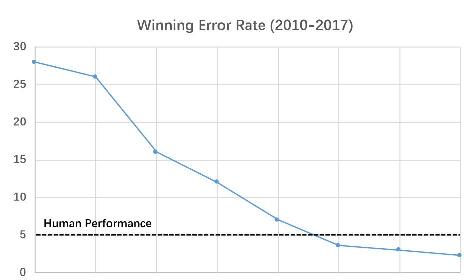


Residual learning: a building block

ResNet

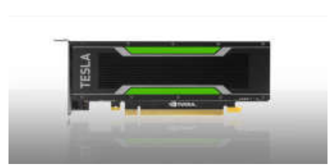
Only 2 layers??? Of course NOT!!! 152 layers!!! Top-5 Error rate: 3.57%

ResNet



Hardware

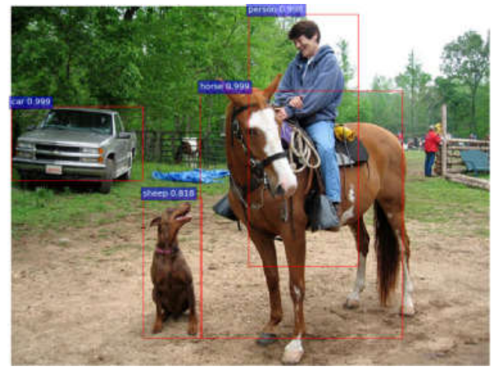
- Trend
  - Deeper than deeper
  - Need a great number of computation resources



Hardware Thanks to NVIDIA! Software

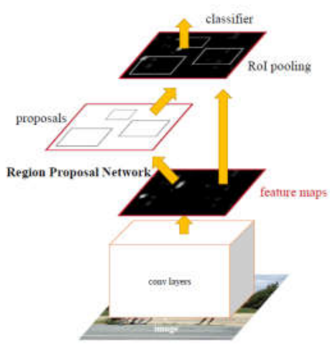
More Applications

Applications – Object Detection



Ren S, He K, Girshick R, et al. Faster R-CNN: Towards Real-Time Object Detection with Region Proposal Networks. NIPS 2015.

Applications – Object Detection

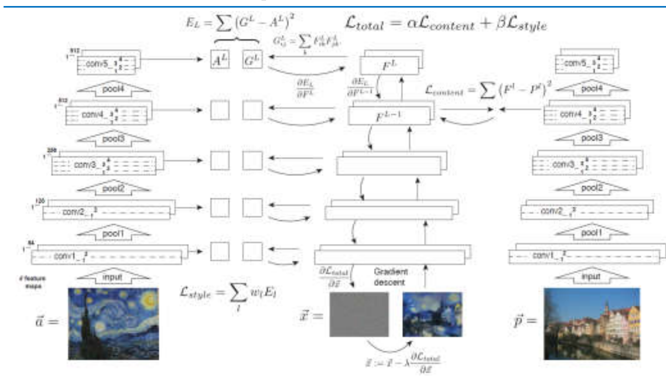


Ren S, He K, Girshick R, et al. Faster R-CNN: Towards Real-Time Object Detection with Region Proposal Networks. NIPS 2015.

Applications – Style Transfer

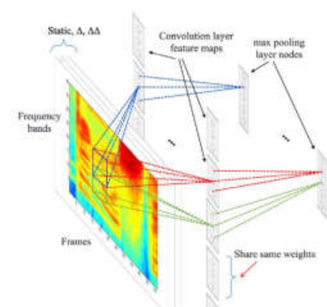
Gatys L A, Ecker A S, Bethge M. Image Style Transfer Using Convolutional Neural Networks. CVPR, 2016.

Applications – Style Transfer



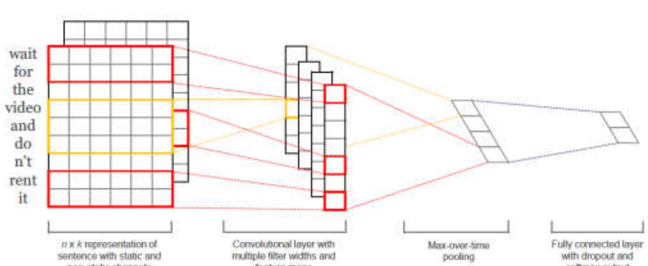
Gatys L A, Ecker A S, Bethge M. Image Style Transfer Using Convolutional Neural Networks. CVPR, 2016.

Applications – Speech Recognition



Abdel-Hamid O, Mohamed A R, Jiang H, et al. Convolutional Neural Networks for Speech Recognition[J]. IEEE/ACM Transactions on Audio Speech & Language Processing, 2014, 22(10):1533-1545.

Applications – Text Classification



Yoon Kim. Convolutional Neural Networks for Sentence Classification. EMNLP, 2014.

Materials

- Paper: Gradient-based Learning Applied to Document Recognition

  Y. Lecun, L. Bottou, Y. Bengio, and P. Haffner, 1998.

- Paper: Deep Learning

  Y. Lecun, Y. Bengio, and G. Hinton, *Nature*, 2015.

- Course: Convolutional Neural Networks for Visual Recognition

  http://cs231n.stanford.edu/

- Tool: CNN Visualization

  https://blog.keras.io/how-convolutional-neural-networks-see-the-world.html

Questions