FIT1047 Lab 1

Topics

- Moore's law
- Conversion of numbers between different bases

Instructions

- The tasks are supposed to be done in groups and might involve considerable discussions. Some have a specific solutions, others are supposed to stipulate understanding and thinking about the concepts.
- Form groups of 3 students to work through the exercises together.
- The tutorial is based on the first lecture, but might require to search for additional information.
- We are aware that students in FIT1047 come from diverse backgrounds and also aim at different degrees. Therefore, the first questions should be the basis to discuss some of the basic concepts and terminology. Discuss them briefly in your group and if you are familiar with the concepts, please explain them to your fellow students who dont have the same background.
- If you are unsure what to do or stuck at any point, dont hesitate to ask your tutor. Tutorials and labs are a chance to get a better understanding of the topics.

Task 1: Moore's law

Moore's law is the observation that, over the history of computing hardware, the number of transistors in a dense integrated circuit has doubled approximately every two years (initially it was once per year). This observation was published in a paper in 1965 by Gordon E. Moore, one of the co-founders of Intel. In a simplified version of Moore's law let us assume that microprocessor power doubles every 18 months.

2.a Assume you have a great idea to speed up microprocessors by a factor of 6. But, you need 4 and a half years to raise money and develop the prototype. If Moore's law (in the simplified form) holds, it is worth investing in the new technology?

  - 4.5 years means we look at 54 months, this is 3 times 18 months.
  - Microprocessor power x doubles after 18 months: new power is $2x$, then after 36 months we have $2(2x)$ and after 54 months it is $2(2(2x)) = 2^3x = 8x$. Thus, after 4.5 years, improving by factor 6 is slower than other developments that achieve the improvement matching Moore's law.

2.b If a problem today takes 100,000 hours to compute, which approach would give us the solution first:

(i) Replace the algorithm with one that is twice as fast and let it run on current technology.

(ii) Wait 3 years and then use the slower algorithm on the new technology.

- The faster algorithm would need 50,000 hours. This is around 2083.3 days, which is around 5.7 years.
- After three years, we have a speed-up by factor 4. Thus, we only need 25.000 hours, half the time of the other faster algorithm. Thus, we need around 2.85 years for the computation. Plus the 3 years wait, we get 5.85 years. Thus, waiting is slower.
- Of course, waiting and then use the faster algorithm is better ...

Task 2: Conversion of numbers between different bases

There are different methods to convert between different bases. In this task you will look at two methods, one slow method and another more efficient method. The advantage of the slow method is, that it nicely shows how number systems with different bases work.

Slow method of converting numbers between bases

This methods needs one preparation step. First, one needs to convert the values for the different places and then add them up to get the converted number. In order to do this, we first need to look at the values for the different places of the number. The following table shows the base 10 values for base 3 and base 10 places 0 to 5.

Base 3			Base 10	
3^5	243_{10}		10^5	100000
3^4	81_{10}		10^4	10000
3^3	27_{10}		10^3	1000
3^2	9_{10}		10^2	100
3^1	3_{10}		10^1	10
3^0	1_{10}		10^0	1

In order to convert a number in base 3, we now compute the base 10 value for each single place of the base 3 number and then add these up. As example, the following table shows the conversion of number 120211_3 to base 10.

Number	Step	Step value	Calculation base 10	Value in base 10
1	3^5	243_{10}	243×1	243
2	3^4	81_{10}	81×2	162
0	3^3	27_{10}	27×0	0
2	3^2	9_{10}	9×2	18
1	3^1	3_{10}	3×1	3
1	3^0	1_{10}	1×1	1
Converted number base 10				427

To convert back from base 10 to base 3, we use a similar approach, but instead of multiplying with the step value we divide by the step value and continue with the remainder as shown in the following table.

Remainder	Step	Step value	Calculation base 10	Value in base 3 for this step
427	3^5	243_{10}	427/243	1
184	3^4	81_{10}	184/81	2
22	3^3	27_{10}	22/27	0
22	3^2	9_{10}	22/9	2
4	3^1	3_{10}	4/3	1
1	3^0	1_{10}	1/1	1
Concatenate top down to get number base 3				120211

This method works for all bases, but always requires to calculate with rather large intermediate results using the values for each place.

More efficient way of converting numbers between bases

For the faster method, take the leftmost place (i.e. in 120211 base 3 take the leftmost 1) multibly by the base, add the next digit, multiply by the base and continue until all numbers are used. By doing this, each place is multiplied by the correct base value for this step. The following table shows the conversion for 120211 base 3.

Action	place base 3	result (base 10 value)
Multiply by 3	1	3
Add next place	2	5
Multiply by 3		15
Add next place	0	15
Multiply by 3		45
Add next place	2	47
Multiply by 3		141
Add next place	1	142
Multiply by 3		426
Add final place	1	427
Base 10 result		427
Converted number base 10		427

Finally, converting from base 10 to base 3 requires division by three and looking at the remainders.

Division		Result	Remainder	Build base 3 number
427/3	=	142	1	Base 3 number is XXXXX1
142/3	=	47	1	Base 3 number is XXXX11
47/3	=	15	2	Base 3 number is XXX211
15/3	=	5	0	Base 3 number is XX0211
5/3	=	1	2	Base 3 number is X20211
1/3	=	0	1	Base 3 number is 120211
Converted number base 3				120211

Tasks

Concentrate on the algorithm to calculate the conversion (dont just google the result).

3.a Convert the base 16 number **123C9F** to base 10 using both methods

Slow method:

Number	Step	Step value	Calculation base 10	Value in base 10
1	16^5	1048576_{10}	1048576×1	1048576
2	16^4	65536_{10}	65536×2	131072
3	16^3	4096_{10}	4096×3	12288
C	16^2	256_{10}	256×12	3072
9	16^1	16_{10}	16×9	144
F	16^0	1_{10}	1×15	15
Converted number base 10				1195167

Faster method:

123C9F

Action	place base 3	result (base 10 value)
Multiply by 16	1	16
Add next place	2	18
Multiply by 16		288
Add next place	3	291
Multiply by 16		4656
Add next place	12	4668
Multiply by 16		74688
Add next place	9	74697
Multiply by 16		1195152
Add final place	15	1195167
Base 10 result		1195167
Converted number base 10		1195167

3.b Convert the base 2 number **1100011101** to base 10 using the *slow* method.

Number	Step	Step value	Calculation base 10	Value in base 10
1	2^9	512_{10}	512×1	512
1	2^8	256_{10}	256×1	256
0	2^7	128_{10}	128×0	0
0	2^6	64_{10}	64×0	0
0	2^5	32_{10}	32×0	0
1	2^4	16_{10}	16×1	16
1	2^3	8_{10}	8×1	8
1	2^2	4_{10}	4×1	4
0	2^1	2_{10}	2×0	0
1	2^0	1_{10}	1×1	1
Converted number base 10				797

3.c Convert the hexadecimal (base 16) number **AFC934B2D** to binary without the use of addition, substraction, multiplication, or division.

For each number just replace with the matching 4 digits of the binary representation:

A	F	C	9	3	4	B	2	D
1010	1111	1100	1001	0011	0100	1011	0010	1101