Tutorial 01

[Solutions to the tasks and some additional explanations]

Task 1

Gordon Moore was one of the co-founders of Intel and he was quite accurate in his prediction in 1965 of how the density of components in integrated circuits would evolve. He revised the forecast in 1975. Higher density would also mean that higher speed and more processing power are possible.

A. At the end of 4 and a half years, processors would be (1 to 2, the 2 to 4, then 4 to 8) 8 times faster. So, the project to increase the speed of processors by 6 times is not worth investing.

B. (i) A new faster algorithm running on the current technology requires- 100000/2 = 50,000 hours for a solution. (ii) Waiting for 3 years, the processor power will rise by 4 times. So, with the new technology and the slower algorithm, the performance is sped up by 4 times i.e. it takes 100000/4 = 25000 hours. Taking consideration of the wait time of 3 years (=3 X 365 X 24 = 26280 hours), the solution will be available in 25000 + 26180 = 51280 hours.

Task 2

In general, each positive whole number can easily be used as the base for a numbering system. For a base b, this means that each place (each digit) in a number would be a number out of 0,1,2,....,b-1. For base 10 we have 0,1,2,3,4,5,6,7,8,9, for base 2 just 0,1 and for base 16 we have 0,1,2,3,4,5,6,7,8,9,A,B,C,D,E,F.

Now, we count the digits from right to left, starting with 0 and the value of each digit is the number at this digit multiplied by the base value to the power of the step, i.e. the number we came up for this digit by counting from right to left.

3.a Calculate the conversions from the base 16 number $311AF9_{16}$ to base 10.

5	4	3	2	1	0	← Step
3	1	1	A(=10)	F(=15)	9	← Digits

(i) Slow Method:

Number (Base 16)/ (Base 10	Step	Step Value	Calculation in Base 10	Value in Base 10
$3_{16}/3_{10}$	16^5	1048576_{10}	1048576 X 3	3145728
$1_{16}/1_{10}$	16^4	65536_{10}	65536 X 1	65536
$1_{16}/1_{10}$	16^3	4096_{10}	4096 X 1	4096
$A_{16}/10_{10}$	16^2	256_{10}	256 X 10	2560
$F_{16}/15_{10}$	16^1	16_{10}	16 X 15	240
$9_{16}/9_{10}$	16^0	1_{10}	1 X 9	9
Converted number in base 10 = 3218169				

(ii) Efficient Method:

Action	Place (Base 16)/(Base 10)	Result (Base 10 Value)
Multiply by 16	$3_{16}/3_{10}$	16 X 3 = 48
Add next place	$1_{16}/1_{10}$	48 + 1 = 49
Multiply by 16		49 X 16 = 784
Add next place	$1_{16}/1_{10}$	784 + 1 = 785
Multiply by 16		785 X 16 = 12560
Add next place	$A_{16}/10_{10}$	12560 + 10 = 12570
Multiply by 16		12570 X 16 = 201120
Add next place	$F_{16}/15_{10}$	201120 +15 = 201135
Multiply by 16		201135 X 16 = 3218160
Add final place	$9_{16}/9_{10}$	3218160 + 9 = 3218169
Converted Number (Base 10)		= 3218169

3.b Convert the base 2 number 10010100101_2 to base 10 using the slow method.

10	9	8	7	6	5	4	3	2	1	0	← Step
1	0	0	1	0	1	0	0	1	0	1	← Digits

Number (Base 2)	Step	Step Value	Calculation in Base 10	Value in Base 10
1	2^{10}	1024_{10}	1024 X 1	1024
0	2^{9}	512_{10}	512 X 0	0
0	2^{8}	256_{10}	256 X 0	0
1	2^{7}	128_{10}	128 X 1	128
0	2^{6}	64_{10}	64 X 0	0
1	2^{5}	32_{10}	32 X 1	32
0	2^{4}	16_{10}	16 X 0	0
0	2^{3}	8_{10}	8 X 0	0
1	2^{2}	4_{10}	4 X 1	4
0	2^{1}	2_{10}	2 X 0	0
1	2^{0}	1_{10}	1 X 1	1
Converted number in base 10 = 1189				

3.c Convert the hexadecimal (base 16) number FF452FACD to binary without the use of addition, subtraction, multiplication, or division.

In a hexadecimal number, each digit exactly represents 4 digits in the binary number. Thus, conversion is just replacing each digit by the correct 4 digits.

Convert every digit into its corresponding binary value:

F_{16} = 1111

F_{16} = 1111

4_{16} = 0100

5_{16} = 0101

2_{16} = 0010

F_{16} = 1111

A_{16} = 1010

C_{16} = 1100

D_{16} = 1101

So, $FF452FACD_{16}$ = $1111\ 1111\ 0100\ 0101\ 0010\ 1111\ 1010\ 1100\ 1101_2$