Rabi flopping in non-magic lattice, spatially averaged

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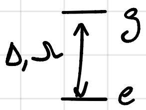
Assume two-level system subject to differential Stark shift



$$\hbar\omega_{eg} = \hbar\omega_{eg}^0 + U_e - U_g = \hbar\omega_{eg}^0 + \frac{1}{2\varepsilon_0 c}(\alpha_e - \alpha_g)\, I(\vec{x})$$

$$\hbar\Delta(\vec{x}) \equiv \hbar\omega - \hbar\omega_{eg}(\vec{x})$$

Assume that we look at a spatially-averaged response to the driving field, then

$$\langle P_e \rangle = \left\langle \frac{\Omega^2}{\Omega^2 + \Delta^2(\vec{x})} \sin^2\left(\sqrt{\Omega^2+\Delta^2(\vec{x})}\,\frac{t}{2}\right)\right\rangle$$ ✓

If we also take the time-average at long times

$$\langle P_e \rangle \longrightarrow \frac{1}{2}\left\langle \frac{\Omega^2}{\Omega^2+\Delta^2(\vec{x})}\right\rangle = \frac{1}{2}\left\langle \left[1 + \left(\frac{\Delta(\vec{x})}{\Omega}\right)^2\right]^{-1}\right\rangle = \frac{1}{2V}\int_V d^3x \, \frac{1}{1+(\Delta(\vec{x})/\Omega)^2}$$

We then approximate around point of interest $\vec{x}_0$ where the detuning is zero and neglect everything above first order

$$\Delta(\vec{x}) \simeq \underbrace{\Delta_0}_{=0} + \vec{J}\cdot(\vec{x}-\vec{x}_0)$$

Then assuming that I does not vary along z, we can reduce to 2D integral

$$\langle P_e \rangle \simeq \frac{1}{2A}\int_A d^2x \left[1 + \frac{1}{\Omega^2}(\vec{J}\cdot(\vec{x}-\vec{x}_0))^2\right]^{-1} \quad \Big| \ \vec{u} = \vec{x}-\vec{x}_0$$

$$= \frac{1}{2}\left(\frac{d}{2}\right)^{-2} \int_{-d/2}^{d/2} du_x \int_{-d/2}^{d/2} du_y \left[1 + \frac{1}{\Omega^2}(J_x u_x + J_y u_y)^2\right]^{-1}$$

Which is of the form $(a \equiv J_x/\Omega\,,\ b \equiv J_y/\Omega)$

$$\equiv \int_{-d/2}^{d/2} dx \int_{-d/2}^{d/2} dy \left[1 + (ax+by)^2\right]^{-1}$$

$$\overset{\text{mathematica}}{=} \frac{1}{ab}\left\{ (b-a)d\, \mathrm{atan}\left(\frac{(a-b)d}{2}\right) + (a+b)d\, \mathrm{atan}\left(\frac{(a+b)d}{2}\right) - 2\, \mathrm{atanh}\left(\frac{2abd^2}{4+(a^2+b^2)d^2}\right)\right\}$$

Which gives us the long-time, pixel-averaged excited state fraction depending on the local Stark shift gradient.