Intrinsic Preference for High Cardinality in Informative Signals

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Abstract

In the context of information acquisition, cardinality, the dimension of the signal space, represents the number of possible signals the receiver can receive. This paper provides the first experimental evidence that information receivers consider not only the signal's informativeness but also its cardinality. When purchasing informative signals, subjects prefer signals with higher cardinalities to lower ones, even if the ex-ante signal accuracies are the same. The results also reveal a pattern of (aggregate) preference reversal: when the signal was given, the preference for high cardinality vanished. Preference reversal suggests that the preference for high cardinality is intrinsic, for instance, curiosity. The experimental findings have two main practical implications. First, information providers can make their services look more attractive by simply increasing the size of the signal space. Second, the experimental findings of this paper challenge the commonly used assumption in the signaling environment in information economics, which is that signal space is equal to the action space. No known theoretical framework, including the expected utility model, recursive smooth ambiguity model, rank-dependent utility model, and prospect theory model, can explain the experimental findings.

1 Introduction

Signal transmission is an essential part of the literature on game theory, where abundant research has been conducted theoretically and empirically. However, the role of the size of the signal space in the literature has not been sufficiently discussed yet. When discussing the desirable size of the signal space, theorists assume that, without loss of generality, the signal space equals the action space. They proved that this size of the signal space is enough to find the equilibrium; therefore, a larger signal space is unnecessary. This assumption has been taken for granted for decades, but it could be criticized if the receiver prefers to receive a signal defined on a larger space. In the context of information acquisition, cardinality, the dimension of the signal space, represents the number of possible signals the receiver can receive. This paper investigates whether people have an intrinsic preference for either high or low cardinality of signal, independent of its informativeness.

Consider the example of an investor contemplating whether or not to invest in a company. State $\theta \in \{\theta_G, \theta_B\}$ represents the type of the company, and θ_G and θ_B stand for a good company and a bad company each. The investor does not know whether the company is good or bad but only knows $Pr(\theta_G) = Pr(\theta_B) = 0.5$. She wants to invest only if the company is good. Specifically, she gets utility 1 for investing good company and not investing bad company,[1] and utility 0 for investing bad company and not investing good company.

To reduce uncertainty about the investment decision, she is considering hiring a financial advisor with more knowledge of the company. There are two advisors she is considering: Advisor A and Advisor B. They provide informative signals to the investor. Advisor A will send the investor one of the two signals with equal probability: "invest" or "not invest." If his signal is "invest," the probability that the company is good is 70% ($Pr(\theta_G|\text{"invest"}) = 0.7$). If his signal is "not invest," the

[1]Note that she will be happy not to invest in a bad company as to invest in a good company, considering the opportunity cost.

probability that the company is good is 30% ($Pr(\theta_G|$"*not invest*"$) = 0.3$). Since the number of possible signals from Advisor A is 2, the cardinality of his signal is 2. Note that Advisor A is the commonly assumed sender by theorists: the action space (invest or not) is equal to the signal space.

On the other hand, Advisor B has a larger signal space. Advisor B will send the investor one of the five signals with equal probability: "strongly recommend," "recommend," "no opinion," "not recommend," "never recommend." The probability that the company is good when each signal is received is 0.8, 0.7, 0.5, 0.3, and 0.2. Since the number of possible signals from Advisor B is 5, the cardinality of his signal is 5.

Note that the ex-ante informativeness of signals from the two advisors is both 70%.[2] Hence, if the informativeness of the signal is the only factor the investor cares about, she will be indifferent between hiring Advisor A or B. However, if she also prefers high or low cardinality of signals, she might prefer one advisor to another. This paper provides the first empirical evidence that cardinality matters in information acquisition. In Study 1, subjects are asked to bet on four lotteries. Before betting, they have a chance to purchase an informative signal for each lottery. Each signal has the same ex-ante accuracy but different cardinality. The result shows the willingness to pay for the signal increases as cardinality increases, even if its informativeness does not change. It implies that people prefer Advisor B to Advisor A. Also, this preference for high cardinality drives overpayment to Advisor B. When hiring Advisor B, the investor's total earnings are lower than when hiring Advisor A.

Another interesting result I have found is a pattern of preference reversal. Lichtenstein and Slovic (1971) initially documented inconsistency between revealed preferences and prices for paired lotteries. After the initial report, the preference reversal phenomenon has been observed in numerous experimental studies for a few decades (Grether and Plott, 1979; Pommerehne et al., 1982; Tversky and Thaler, 1990). When

[2] See Appendix A.1.1 for detailed calculations.

subjects are asked to choose between a safer (high winning probability but small prize) or riskier lottery (low winning probability but high prize) with nearly the same expected values, most subjects prefer playing a safer one. However, when asked to state the minimum prices to sell each lottery when they own it, most subjects put a higher price on the riskier lottery.

A similar pattern was observed in this paper. Study 2 measured the values of four lotteries in Study 1. But in this case, there is no signal purchasing stage because the signal is given. The result reveals that, surprisingly, the preference for high cardinality vanishes.[3] It implies that if the investor hired both Advisors A and B, she no longer prefers Advisor B to Advisor A: she is indifferent now.

Among the explanations for the experimental findings, intrinsic motivation from curiosity provides the most plausible interpretation. Curiosity suggests intrinsic motivation for knowledge could not necessarily be related to the value of that it. That implies that the receiver could have a more intrinsic inclination for achieving a specific type of information than another, even if the desire disappears after having all the information. A more detailed discussion about explanations of the experimental results will be provided later in Section 5.

Does a smaller or larger signal space improve decision-making? In some environments, limiting the size of the signal space might restrict the optimal outcome. For example, in most of the standard sender-receiver literature, low cardinality of signal space might lead to inefficient outcomes (Crawford and Sobel, 1982; Heumann, 2020). In this case, a larger signal space allows better decision-making. However, in the experimental design of this paper, cardinality is independent of the efficiency of the outcomes: the informativeness of each signal is the same. Therefore, there is no behavioral or theoretical reason to predict high cardinality in signals.

[3]While this is technically a "preference reversal," I view this phenomenon thematically differently from previous literature. Tversky et al. (1990) suggests that the primary reason for preference reversal is the failure of procedure invariance, especially the overpricing of high-payoff, low-probability bets. However, the experiment of this study shows curiosity could be a reason for the systemic preference reversal. A detailed discussion will be in the remaining of this paper.

On the other hand, a decision-maker might prefer a simpler environment - a smaller size of signal space - if signals are too complicated to understand. For example, a worker might want to receive direct instructions on whether or not to proceed with the current project rather than receive abstract signals from the boss and interpret the intent. This preference could be related to complexity aversion, which illustrates a preference for simpler lotteries over complex ones, even though the expected values are the same (Huck and Weizsäcker, 1999; Sonsino et al., 2002; Halevy, 2007; Moffatt et al., 2015). However, the experimental results of this paper did not find evidence for complexity aversion.

The empirical findings of this paper have some practical implications. The first implication is for information providers. Information providers, such as financial advisors, medical test providers, or film critics, can make their services look more attractive by simply increasing the size of the signal space. For example, the result of this paper suggests that users are more attracted to a 5-star rating system than a binary suggestion, even if the two systems are equally informative. Hence, if a service provider switches the recommendation system from a binary suggestion to a 5-star rating, demand for the service will increase, even without improving the informativeness of the system.

Another implication is the theoretical aspect of signal environments. Without loss of generality, most signaling environments of information economics assume signal space is equal to the action space. For example, the Bayesian persuasion model of Kamenica and Gentzkow (2011) assumes a signal is "straightforward," implying each signal is a form of recommended action. A good example is Advisor A in the investor case. In this environment, the cardinality of the signal cannot be larger than the size of the action space. Theorists elegantly proved that equilibrium could be achieved with this size of the signal space, implying that a larger signal space is not necessarily needed. However, the experimental result of this paper challenges the assumption: the receiver might be happier if the signal space is larger than the action space. In this case, the equilibrium achieved by assuming a small size of the signal space could be questioned.

Section 3 provides theoretical predictions from various models, but no model can explain the preference for high cardinality. The expected utility model predicts the same value for each signal. The recursive smooth ambiguity model of Klibanoff et al. (2005), Rank-dependent utility model, and Prospect theory suggest different values for different signals but do not predict the systemic preference for high cardinality. Also, none of these models predict preference reversal.

Receiving an informative signal and playing a simple lottery based on the signal's information can be perceived as a two-stage lottery. In this environment, the preference for high cardinality in signals could be interpreted as a violation of the reduction of compound lottery (ROCL). If a decision-maker can reduce compound lottery, there is no reason to pay more to signal with higher cardinality under the same informativeness. Halevy (2007) revealed ambiguity neutrality and reduction of compound lotteries are tightly associated. To examine the correlation, I measured ambiguity attitudes after the information acquisition questions. The result suggests that ambiguity neutrality is not related to the preference for high cardinality.

This paper proceeds as follows. Section 2 describes the experimental design and procedure. Section 3 provides theoretical predictions of the results from various models. Section 4 reveals results and Section 5 concludes.

2 Experimental Design

Participants were assigned to one of two experiments: Study 1 or Study 2. Each study consists of two parts: Part 1 measures the value of signals (Study 1) or lotteries (Study 2) under isomorphic environments, and part 2 measured ambiguity attitudes by Ellsberg (1961) questions.

2.1 Part 1: The Value of Signals/Lotteries

There are four lotteries in this part. Each lottery contains several boxes, each containing ten balls, either red or blue. In each lottery, the computer randomly selects one of the boxes with an equal probability of $\frac{1}{4}$. Then, it draws a ball from the selected box. The task for subjects is to predict which ball will be drawn: a red ball or a blue ball. If their prediction is correct, they receive 100 points, where each point is equal to 0.01 USD. Figure 1 illustrates the four lotteries.

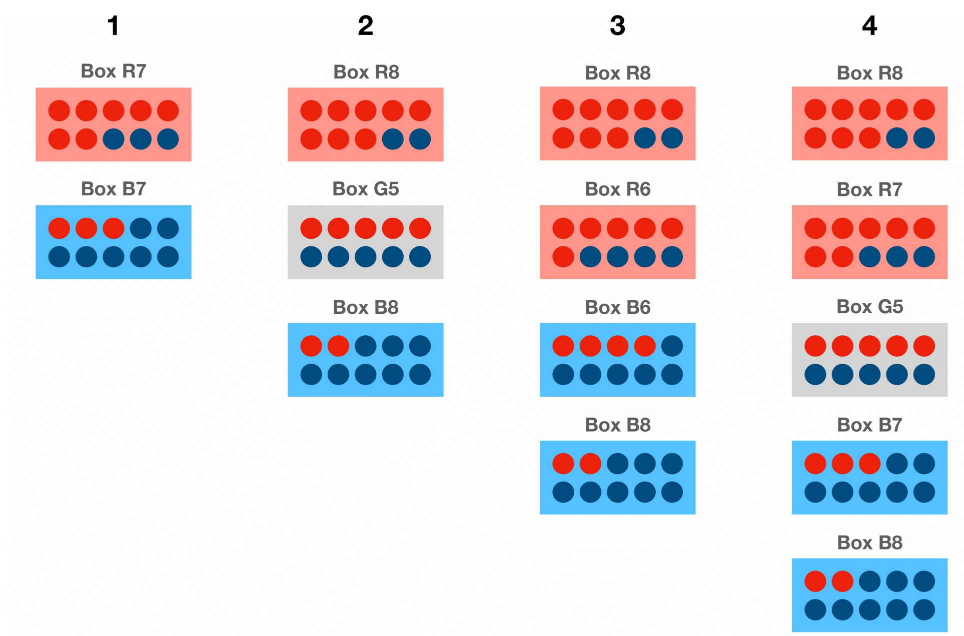

Figure 1: Four lotteries

Each box is denoted by a form of Box Xn, where $X \in \{R, B, G\}$ and $n \in \{5, 6, 7, 8, 9, 10\}$. X and n each represent the majority of balls the box contains and their number. For example, Box R7 has more red balls than blue balls, and the number of red balls is 7.[4]

[4]In the actual experiment, boxes are represented as Box R, Box B, Box G, Box RR (in case there

In the first study, the information about which box is selected is unknown. However, subjects can "buy" the information. They are endowed with 100 points, and before the prediction, there is a chance to purchase a costly signal: it will tell which box is selected. The signal increases the winning probability, but it requires some cost whether the buyer wins or loses.

For example, in lottery 2, there are three boxes: Box R8, Box G5, and Box B8. Suppose Box R8 is randomly selected. Without the signal, subjects do not know which box is chosen. Their winning probability is 50% whether they bet on a red or blue ball because there is a total of 15 red balls and 15 blue balls in lottery 2. If they buy the signal, they know that Box R8 is selected and the ball will be drawn from Box R8. The signal "Box R8" increases the odds of winning to 80% because Box R8 contains 8 red and 2 blue balls.

One of the key features of the design is that the numbers of red and blue balls in each lottery are always the same. It implies that the winning probability is 50% for all lotteries without the signal. Another essential feature is that the ex-ante signal accuracy of each lottery is the same. If subjects purchase the signal, the winning probability increases to 70% for all four lotteries. The only difference between them is the number of boxes, representing cardinality.

To investigate whether the preference for cardinality still exists after purchasing signals, Study 2 measures the value of four lotteries when the signal is given. There is no stage for purchasing the signal in Study 2 because the signal is given for free. Subjects can observe which box is selected before predicting the ball's color. They win 100 points if the predictions are correct. Except for that difference, the information structure of Study 2 is isomorphic to Study 1. I measured subjects' willingness to pay for playing each lottery.

To measure the willingness to pay for signal and lottery, the Becker-DeGroot-Marschak (BDM) mechanism (Becker et al. (1964)) was used. In Study 1, subjects

are two Box R in one lottery), and Box BB (in case there are two Box B in one lottery). The reason not to put numbers on the boxes is to provide an environment where subjects rely more on intuition.

submit the values of the signals, which are the maximum points they can pay for each lottery. After submitting values for four lotteries, one of them is randomly selected. Then, a random number is generated between 1 and 100. The random number represents the price for the signal for the selected question in the selected lottery. If a subject's submitted value in the selected lottery is greater than the price, she can see the signal and pays the price. However, if the submitted value in the selected lottery is equal to or lower than the price, she does not receive the signal and pays nothing. After the signal is revealed or not revealed, subjects predict the color of the ball. Figure 2 shows the questions in the BDM.

Q#	**Option A**	Choices	**Option B**
1	Buying a Signal for 1 point		Not Buying a Signal
2	Buying a Signal for 2 points		Not Buying a Signal
3	Buying a Signal for 3 points		Not Buying a Signal
4	Buying a Signal for 4 points		Not Buying a Signal
⋮	⋮	⋮	⋮
97	Buying a Signal for 97 points		Not Buying a Signal
98	Buying a Signal for 98 points		Not Buying a Signal
99	Buying a Signal for 99 points		Not Buying a Signal
100	Buying a Signal for 100 points		Not Buying a Signal

Figure 2: The BDM mechanism in Study 1

The procedure of the BDM in Study 2 is similar to in Study 1 (See figure 3). Before playing the lotteries, I ask subjects the maximum number of points they are willing to play each lottery. After submitting four values for four lotteries, one of the lotteries is randomly selected. Then, a random number between 1 and 100, representing a substitute prize, is generated. If the submitted value in the selected lottery is greater than the prize, a subject plays the lottery. Otherwise, she receives the prize without playing the lottery. If they play the lottery, they see which box is selected and predict the color of the ball from the selected box.

Q#	Option A	Choices	Option B
1	Playing the lottery		Receiving 1 point
2	Playing the lottery		Receiving 2 points
3	Playing the lottery		Receiving 3 points
4	Playing the lottery		Receiving 4 points
⋮	⋮	⋮	⋮
97	Playing the lottery		Receiving 97 points
98	Playing the lottery		Receiving 98 points
99	Playing the lottery		Receiving 99 points
100	Playing the lottery		Receiving 100 points

Figure 3: The BDM mechanism in Study 2

The major issue of the BDM mechanism is its difficulty and the biased results in some environments.[5] To minimize the confusion, subjects were asked to submit their maximum willingness to pay for the signal instead of deciding between Option A and B 100 times. Also, before subjects submit their actual values, an example was illustrated of how the mechanism works when a specific value is submitted.

Furthermore, even if the result is upward or downward biased, the biased result does not impair the primary purpose of the BDM mechanism, which is to compare preferences between signals and between lotteries, not to elicit their exact values.

There are two hypotheses to test in this experimental design. Study 1 measures the willingness to pay for the signal. If informativeness of signals only matters, the WTP for signals for four lotteries should be the same because ex-ante signal accuracies are the same. When S_i indicates the signal for lottery i,

$$S_1 \sim S_2 \sim S_3 \sim S_4. \tag{1}$$

Hypothesis 1. *Subjects are indifferent between signals for four lotteries.*

[5] See Noussair et al. (2004) and Cason and Plott (2014) for discussions about the biased results of the BDM.

Suppose a subject values the signal for lottery i is more than the signal for lottery j. If the demand for the signal is extrinsic, she will also value lottery i more than lottery j if the signal is given without cost. Therefore, when $L_i^{signal}(c)$ represents a lottery i with the signal with the cost c, Study 2 measures $L_i^{signal}(0)$ for four lotteries. Then, the following hypothesis holds.

Hypothesis 2. *$S_i > S_j$ implies $L_i^{signal}(0) > L_j^{signal}(0)$, for $i \neq j$.*

To avoid subjects focusing only on cardinality, lotteries were presented in order of 1-3-2-4 ($S_1 - S_3 - S_2 - S_4$, and $L_1^{signal}(0) - L_3^{signal}(0) - L_2^{signal}(0) - L_4^{signal}(0)$).

2.2 Part 2: Ellsberg Questions

After the elicitation of the value of signals, subjects' ambiguity attitudes were measured by two questions from Ellsberg (1961). Ambiguity attitude is closely related to two-stage lotteries, especially to the ability to reduction of compound lotteries (Halevy, 2007; Seo, 2009). Halevy (2007) shows the strong association between ambiguity neutrality and reduction of compound lotteries (ROCL). Since preference for high/low cardinality can be interpreted as a failure to reduce compound lotteries, this task helps to understand how ambiguity attitude is related to the preference for cardinality.

The following statement describes the task.

> Consider there is a bag containing 90 ping-pong balls. 30 balls are blue, and the remaining 60 balls are either red or yellow in unknown proportions. The computer will draw a ball from the bag. The balls are well mixed so that each ball is as likely to be drawn as any other. You will bet on the color that will be drawn from the bag.

Subjects are asked to choose their preferred options between A & B and between C & D. Table 1 illustrates the four options.

If a subject prefers A to B and D to C, there is no formulation of subjective probability that can rationalize the preference. This preference is interpreted to be a consequence of ambiguity aversion.

Options	
Option A	receiving 100 points if a blue ball is drawn.
Option B	receiving 100 points if a red ball is drawn.
Option C	receiving 100 points if a blue or yellow ball is drawn.
Option D	receiving 100 points if a red or yellow ball is drawn.

Table 1: Ellsberg questions

After rewards from Part 1 and Part 2 are determined, one of the parts is randomly selected. Subjects will get the points in the selected part. Each point is transferred to 0.01 USD.

2.3 Procedural Details

467 subjects participated in experiments through Prolific, an online platform for recruiting research participants.[6] 179 and 158 subjects participated in studies 1 and 2 each. Also, another 130 subjects participated in a robustness study, which will be discussed later. On average, subjects spent 10 minutes and earned $3.32, including $2.2 of a base payment.

2.4 Robustness Study

In addition to the two studies, additional study was implemented to investigate the robustness of the results. Since the winning probability of each option in both studies was all the same, which is 70% (with signals in Study 1), it is not apparent subjects understand the information structures. Especially, the signaling process of Study 1 could be complicated for subjects. The additional study provides evidence of whether subjects understood the procedure correctly.

[6] Gupta et al. (2021) shows Prolific can be a reliable source of high-quality data. About a subject pool about Prolific, see Palan and Schitter (2018). In both studies, only US subjects participated.

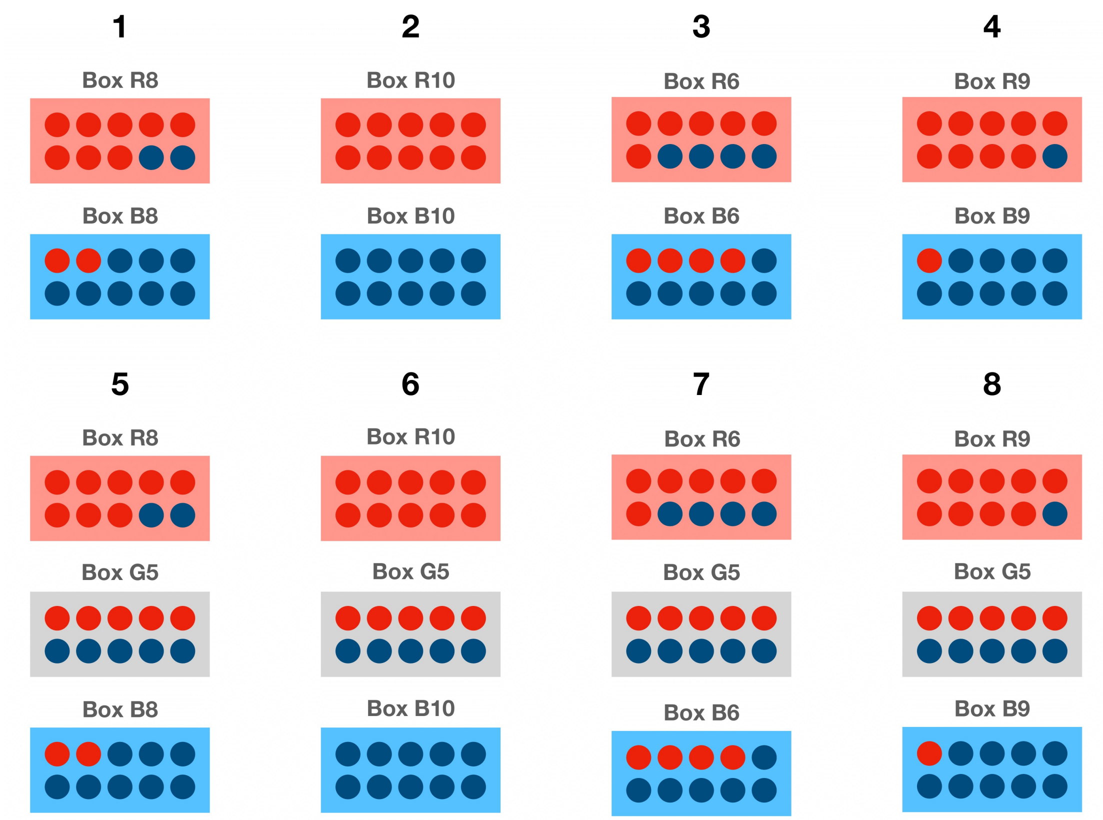

Figure 4: Lotteries in the robustness study

The procedure of this study is identical to part 1 in Study 1: subjects were asked to value signals in uncertain lotteries. To investigate subjects' understanding, values of signals for eight lotteries were measured. The signal of each lottery provides a different winning probability. If they understood this information acquisition framework correctly, they would be more willing to pay for a signal with a higher winning probability. Figure 4 illustrates lotteries of this study.

Table 2: Summary of lotteries in the robustness study

Questions	Cardinality	Winning Prob With a Signal	EU's Prediction
1	2	0.80	30
2	2	1.00	50
3	2	0.60	10
4	2	0.90	40
5	3	0.70	20
6	3	0.83	33.3
7	3	0.57	6.7
8	3	0.77	26.7

Table 2 summarizes the details of the lotteries. Lotteries 1-4 have two boxes, Box Rn and Box Bn, and lotteries 5-8 have three boxes, Box Rn, Box Gn, and Box Bn, where $n \in \{5, 6, 7, 8, 9, 10\}$. The winning probability with the signal of each lottery is described in the third column. The fourth column shows the theoretical prediction of the expected utility theory. If subjects understand the information framework of the signaling process, their demands for the signals will be in line with theoretical predictions.

3 Theoretical Predictions

Let L_i^{prior} denote lottery i without the signal. Also, $L_i^{signal}(c)$ is lottery i with the signal with the cost c. Suppose an individual's willingness to pay for the signal for lottery i is greater than or equal to the signal for lottery j: $S_i \geq S_j$. The values of S_i and S_j are determined where

$$L_i^{signal}(S_i) = L_i^{prior}, \tag{2}$$

$$L_j^{signal}(S_j) = L_j^{prior}. \tag{3}$$

Since $L_i^{prior} = L_j^{prior}$,

$$L_i^{signal}(S_i) = L_j^{signal}(S_j). \tag{4}$$

For simplicity of notation, denote $L_i(x)$ instead of $L_i^{signal}(x)$ from now on. Note that $L_i(x)$ is a decreasing function of x. Hence, under the equation 4,

$$S_i \geq S_j \implies L_i(c) \geq L_j(c), \tag{5}$$

where $0 < c < \max S_i, \forall i \in \{1, 2, 3, 4\}$. For example, suppose a subject's willingness to pay for signal 1 (the signal in lottery 1) is 20, and signal 2 (the signal in lottery 2) is 30. Then, she will be happier to purchase signal 2 for the price of 15 than to purchase signal 1 for the price of 15. For calculation simplicity, I will compare $L_i(c)$ and $L_j(c)$ the comparison between S_i and S_j is needed.

Studies 1 and 2 elicited values of S_i for lottery i and $L_i(0)$ each. The remaining part of this section describes how different theories under uncertainty predict the two values in different lotteries.

3.1 Expected Utility

The expected utility of lottery i is given by

$$U_{EU}(L_i) = \sum_{s \in S} p(s)u(s). \tag{6}$$

The expected utility indicates that decision-makers are only interested in the expected values of lotteries but indifferent to the uncertainty resolution process. They do not care whether the lottery is a simple, compound, or a mean-preserving spread of the other lottery. Therefore, according to the expected utility model, subjects are indifferent between signals for lotteries, as well as between values of lotteries after receiving signals.

$$S_1 \sim S_2 \sim S_3 \sim S_4, \tag{7}$$
$$L_1(0) \sim L_2(0) \sim L_3(0) \sim L_4(0).$$

3.2 Recursive Smooth Ambiguity Utility

The recursive smooth ambiguity model of Klibanoff et al. (2005) (KMM, hereafter) suggests a theoretical utility model involving a second-order belief. KMM assumes that the decision-makers have a subjective expected utility on the space of second-order compound lotteries. For each f, there exists a second-order belief μ such that

$$U_{KMM}(f) = \sum_{\Delta(S)} \phi\Big(\sum_{s\in S} p(s)u(f(s))\Big)\mu(p), \tag{8}$$

where μ is a second-order subject belief, Δ is the set of possible first-order objective lotteries, and ϕ is a monotone function evaluating expected utility associated with first-order beliefs.

For example, when purchasing a signal for L_1, there are two possible outcomes in the first stage (second-order): R7 or B7. In the second stage (first-order), the expected utility is $0.7u(100-c)+0.3u(-c)$ for both cases. Therefore, the evaluation of L_1 is given by

$$\begin{aligned} U_{KMM}(L_1(c)) &= \frac{1}{2}\phi(0.7u(100-c)+0.3u(-c)) + \frac{1}{2}\phi(0.7u(100-c)+0.3u(-c)) \\ &= \phi(0.7u(100-c)+0.3u(-c)) \end{aligned}$$

Similarly, the values of lotteries are evaluated as

$$
\begin{aligned}
U_{KMM}(L_2(c)) &= \frac{2}{3}\phi(0.8u(100-c)+0.2u(-c)) + \frac{1}{3}\phi(0.5u(100-c)+0.5u(-c)),\\
U_{KMM}(L_3(c)) &= \frac{1}{2}\phi(0.8u(100-c)+0.2u(-c)) + \frac{1}{2}\phi(0.6u(100-c)+0.4u(-c)),\\
U_{KMM}(L_4(c)) &= \frac{2}{5}\phi(0.8u(100-c)+0.2u(-c)) + \frac{2}{5}\phi(0.7u(100-c)+0.3u(-c))\\
&\quad + \frac{1}{5}\phi(0.5u(100-c)+0.5u(-c)).
\end{aligned}
$$

When μ is subjective, KMM explained ambiguity aversion by concaveness of ϕ: If lottery Y is a mean-preserving spread of lottery X, then individuals prefer X to Y because of their second-order subjective probability (μ). Since $L_3(c)$ is a mean preserving spread of $L_1(c)$, decision maker prefer $L_1(c)$ to $L_3(c)$:

For easier computation, let's define $U(\alpha)$ as,

$$U(\alpha) \equiv \alpha u(100-c) + (1-\alpha)u(-c).$$

Then,

$$
\begin{aligned}
U_{KMM}(L_1(c)) &= \phi(0.7u(100-c)+0.3u(-c))\\
&= \phi(U(0.7))\\
&\geq \frac{1}{2}\phi(U(0.8)) + \frac{1}{2}\phi(U(0.6))\\
&= U_{KMM}(L_3(c)).
\end{aligned}
$$

A few more steps of calculations (See Appendix A.1 for details) show the following preferences hold.

$$
\begin{aligned}
S_1 \succeq &S_3 \succeq S_2, &(9)\\
S_1 \succeq &S_4 \succeq S_2.
\end{aligned}
$$

Since $L_i(0)$ is a specific form of $L_i(c)$, the preference among $L_i(0)$ does not change. Hence, regardless of the ambiguity attitude, KMM does not predict preference reversal.

$$\begin{aligned} L_1(0) &\succeq L_3(0) \succeq L_2(0), \\ L_1(0) &\succeq L_4(0) \succeq L_2(0). \end{aligned} \tag{10}$$

According to KMM's prediction, ambiguity averse agents most prefer the signal and the lottery with the lowest cardinality. When ϕ is convex, implying ambiguity seeking, the opposite inequality holds.

$$\begin{aligned} S_2 &\succeq S_3 \succeq S_1, \\ S_2 &\succeq S_4 \succeq S_1, \end{aligned} \tag{11}$$

$$\begin{aligned} L_2(0) &\succeq L_3(0) \succeq L_1(0), \\ L_2(0) &\succeq L_4(0) \succeq L_1(0). \end{aligned} \tag{12}$$

3.3 Simluational Predictions from Other Models

3.3.1 Rank-Dependent Utility

Rank-dependent utility (RDU) model suggested a probability weighting approach based on the order of rank of the outcomes (Quiggin, 1982; Segal, 1987, 1990). According to the RDU model, the utility of a lottery paying x_i with probability p_i is described as

$$U_{RDU}(x_1, p_1; x_2, p_2; \cdots; x_n, p_n) = u(x_1) + \sum_{i=2}^{n}[u(x_i) - u(x_{i-1})]f(\sum_{j=i}^{n} p_j), \tag{13}$$

where $x_1 \leq x_2 \leq x_3 \cdots \leq x_n$, $f : [0,1] \to [0,1]$, $f(0) = 0$ and $f(1) = 1$. For the simple lottery that gives 100 with probability p and 0 with probability $1-p$,

$$U(100, p; 0, 1-p) = u(100)f(p). \tag{14}$$

Suppose its certainty equivalent be $CE(p)$, then

$$CE(p) = CE(100, p; 0, 1-p) = u^{-1}(u(100)f(p)). \tag{15}$$

Hence,

$$U_{RDU}(L_1(0)) = u(CE(0.7)) = u(100)f(0.7).$$

Similarly, values of four lotteries with signals are calculated as

$$\begin{aligned}
U_{RDU}(L_1(0)) &= u(100)f(0.7), \\
U_{RDU}(L_2(0)) &= u(100)f(0.5) + [u(100)(f(0.8) - f(0.5))]f(\frac{2}{3}), \\
U_{RDU}(L_3(0)) &= u(100)f(0.6) + [u(100)(f(0.8) - f(0.6))]f(\frac{1}{2}), \\
U_{RDU}(L_4(0)) &= u(100)f(0.5) + [u(100)(f(0.7) - f(0.5))]f(\frac{4}{5}) \\
&\quad + [u(100)(f(0.8) - f(0.7))]f(\frac{2}{5}).
\end{aligned}$$

Preferences between lotteries vary depending on the functional form of $f(p)$. Table 3 illustrates simulational predictions of the RDU model based on different concave functions.

Table 3: Theoretical predictions by RDU

$f(p)$	Preferences between S_i	Preferences between $L_i(0)$
$p^{0.1}$	$S_2 \succ S_4 \succ S_3 \succ S_1$	$L_2(0) \succ L_4(0) \succ L_3(0) \succ L_1(0)$
$p^{0.5}$	$S_2 \succ S_4 \succ S_3 \succ S_1$	$L_2(0) \succ L_4(0) \succ L_3(0) \succ L_1(0)$
$p^{0.8}$	$S_2 \succ S_4 \succ S_3 \succ S_1$	$L_2(0) \succ L_4(0) \succ L_3(0) \succ L_1(0)$
p	$S_1 \sim S_2 \sim S_3 \sim S_4$	$L_1(0) \sim L_2(0) \sim L_3(0) \sim L_4(0)$
$log(p)$	$S_1 \succ S_3 \succ S_2 \succ S_4$	$L_1(0) \succ L_3(0) \succ L_2(0) \succ L_4(0)$
$ln(p)$	$S_1 \succ S_3 \succ S_2 \succ S_4$	$L_1(0) \succ L_3(0) \succ L_2(0) \succ L_4(0)$

Simulation results show that the RDU models with various functional forms of $f(p)$ do not predict the preference for high cardinality or preference reversal.

3.3.2 Prospect Theory

The first version of prospect theory was formulated by Kahneman and Tversky (1979), providing evidence of a systemic violation of the expected utility theory, and presenting an alternative theoretical model to explain the violation. Kahneman and Tversky (1992) (KT, henceforth) presented an extension of the original model, cumulative prospect theory, which adopted rank-dependence in probability weighting.

According to the cumulative prospect theory (CPT), the utility of a lottery paying x_i with probability p_i is described as

$$U_{CPT}(x_m, p_m; x_{m+1}, p_{m+1}; \cdots ; x_0, p_0; \cdots ; x_n, p_n) = \sum_{i=-m}^{n} \pi_i v(x_i). \tag{16}$$

where $v(\cdot)$ is a value function, which is an increasing function with $v(0) = 0$, and π is decision weight. KT defined the value function as follow.

$$v(x) = \begin{cases} x^{\alpha} & \text{if } x \geq 0, \\ -\lambda(-x)^{\beta} & \text{if } x < 0, \end{cases} \tag{17}$$

where λ is a loss aversion parameter.

Decision weights π are defined by:

$$\begin{aligned}
\pi_n^+ &= w^+(p_n), \\
\pi_{-m}^- &= w^+(p_{-m}), \\
\pi_i^+ &= w^+(p_i + ... + p_n) - w^+(p_{i+1} + ... + p_n),\ 0 \leq i \leq n-1, \\
\pi_i^- &= w^-(p_{-m} + ... + p_i) - w^-(p_m + ... + p_{i-1}),\ 1-m \leq i \leq 0,
\end{aligned} \tag{18}$$

where w^+ and w^- are the following functions.

$$w^+(p) = \frac{p^\gamma}{(p^\gamma + (1-p)^\gamma)^{1/\gamma}},\ w^-(p) = \frac{p^\delta}{(p^\delta + (1-p)^\delta)^{1/\delta}}. \tag{19}$$

To predict the preferences for S_i and $L_i(0)$ by CPT, I used the values of parameters from KT. They estimated the values from experimental data.

Table 4: Values of parameters from KT

Parameter	Meaning	Value
α	power for gains	0.88
β	power for losses	0.88
λ	loss aversion	2.25
γ	probability weighting parameter for gains	0.61
δ	probability weighting parameter for losses	0.69

Also, I assume the cost of the signal is 20, which is the theoretically expected value. Hence, preference between S_i is from simulational results from $L_i(20)$. With these values of the parameters, CPT predicts the following preferences:[7]

$$\begin{gathered}
S_1 \succ S_3 \succ S_4 \succ S_2, \\
L_1(0) \succ L_3(0) \succ L_4(0) \succ L_2(0).
\end{gathered} \tag{20}$$

[7] According to the meta-analysis from Brown et al. (2022), the mean of the loss aversion coefficient λ from numerous empirical estimates is 1.97. I found simulational results with $\lambda = 1.97$ do not change the preference between lotteries.

Summarizing the theoretical predictions, some theoretical models predict different values of signals, but no model shows the preference for high cardinality ($S_1 \succ S_2 \succ S_3 \succ S_4$).

Prediction 1. *Preference for high cardinality does not exist.*

Also, no model predicts different preferences between S_i and $L_i(0)$.

Prediction 2. *Preference reversal does not exist.*

To summarizes, no theoretical models predict preference for high cardinality or preference reversal.

4 Results

4.1 Preference (Reversal) for High Cardinality

Table 5: The willingness to pay for S_i and $L_i(0)$ with different cardinalities

		Study 1		Study 2	
Lottery	Cardinality	S_i	Number	$L_i(0)$	Number
1	2	23.6	179	52.9	158
2	3	24.9	179	48.9	158
3	4	25.8	179	51.0	158
4	5	29.8	179	52.7	158

Table 5 shows the value for each signal (S_i) and each lottery given the signal ($L_i(0)$) in points. For S_i, the theoretical predictions from the expected utility maximizer are 20 points for each lottery. Hence, overall, the demand for signals is greater than the theoretical predictions. The most notable feature of the willingness to pay for the signal is the preference for high cardinality: The demand for the signal increases as the cardinality increases.

Interestingly, in Study 2, evidence of the preference reversal is observed. The preference for high cardinality vanishes when subjects already have the signal. Kolmogorov–Smirnov test and Fisher's exact test verify that there is no significant difference among the distributions of lotteries.[8]

I did not find evidence for complexity aversion in lottery choice. According to Sonsino et al. (2002), the complexity of a lottery is measured as the product of the number of rows and the number of columns. Hence, in our environment, the number of boxes in the lottery indicates the complexity of the lottery. The results show when the signal is given, the number of boxes did not affect the values of playing lotteries.

To examine relationships formally, I consider OLS regressions of the form:

$$y_{in} = \beta_0 + \beta_1 c_i + \beta_2 AmbNeutral_n + \epsilon_{in} \tag{21}$$

$y_{i.n}$ is the value of S_i or $L_i(0)$ by individual n, c_i is the cardinality of lottery i, and $AmbNeutral$ is a dummy variable whether individual n is ambiguity neutral or not.

Table 6: Determinants of the demand for signals and lotteries

	Dependent variable: S_i			Dependent variable: $L_i(0)$		
	(1)	(2)	(3)	(4)	(5)	(6)
Cardinality	1.934***	1.934***	1.934***	0.160	0.160	0.160
	(0.3953)	(0.3955)	(0.3953)	(0.5021)	(0.5025)	(0.5021)
Ambiguity Neutrality		0.355			−2.307	
		(2.9456)			(2.5809)	
Constant	19.284***	19.116***	19.284***	50.817***	52.044***	50.817***
	(1.8731)	(2.1920)	(1.3834)	(2.1789)	(2.6221)	(1.7574)
Subject fixed effect	No	No	Yes	No	No	Yes
Observations	716	716	716	632	632	632
R-squared	0.010	0.010	0.046	0.000	0.003	0.000
F-test p-value	0.0000	0.0000	0.0000	0.7502	0.7504	0.7502

Notes: Robust standard errors clustered by subject in parentheses. Column (2) and (5) cannot include subject fixed effect because the ambiguity attitude is measured at the subject level. $^{***}p < 0.01$, $^{**}\ p < 0.05$, $^{*}p < 0.1$.

[8] P-values of Kolmogorov–Smirnov and fisher's exact test are 1.000 and 1.000 for the difference between lottery 1 and others.

The first three columns in table 6 show that cardinality significantly affects the value of signals. (F-test p-values are 0.0000 for these columns.) When the size of the signal space increase, the willingness to pay for the signal also increases.

Result 1. ***Preference for High Cardinality:*** *When purchasing informative signals, subjects are willing to pay more for the signal with high cardinality.*

Result 1 rejects Hypothesis 1. Also, columns (4)-(6) show that cardinality no longer affects the value of lotteries when the signal is given. (F-test p-values are 0.7502, 0.7504, and 0.7502 for each column.) This result rejects Hypothesis 2.

Result 2. ***Preference Reversal:*** *When the signal is given, the preference for high cardinality vanishes.*

The results show the demand for signal increases when cardinality increases, ($S_4 \succ S_3 \succ S_2 \succ S_1$) but the value of lottery with the signal is independent with cardinality ($L_1(0) \sim L_2(0) \sim L_3(0) \sim L_4(0)$). Since no theoretical model predicts the preference for high cardinality, the result falsifies Prediction 1. Therefore, results 1-2 falsify Predictions 1-2. Also, no model except CTP predicts preference reversal. CTP predicts different preferences between S_i and $L_i(0)$, but in a wrong direction. Therefore, Predictions 1 and 2 are both falsified by the experimental results.

4.2 Ambiguity Attitudes

Table 7: Ambiguity attitudes

Ambiguity	Study 1		Study 2	
Attitude	Number	Percentage	Number	Percentage
Averse	72	40.2%	55	34.8%
Neutral	85	47.5%	84	53.2%
Seeking	22	12.3%	19	12.0%
Total	179	100.0%	158	100.0%

Table 8: The submitted values of S_i and $L_i(0)$ with different ambiguity attitudes

Attitude	S_1	S_2	S_3	S_4	$L_1(0)$	$L_2(0)$	$L_3(0)$	$L_4(0)$
Averse	22.8	23.8	24.4	29.9	55.7	49.5	47.7	55.8
Neutral	23.8	26.9	24.9	29.2	50.7	50.8	48.7	50.8
Seeking	25.5	28.7	26.5	31.4	54.2	55.8	52.9	52.2
Total	23.6	25.9	24.9	29.8	52.9	51.0	48.9	52.7
F-test p-value	0.8142				0.5988			

Table 7 and 8 each describes ambiguity attitudes of subjects, and values of S_i and $L_i(0)$ conditional on different ambiguity attitudes. It can be easily noticed that the overall patterns between the WTP for signals and lotteries are consistent even though the attitude towards ambiguity changes. The p-values of F-tests provide evidence that there is no effect of ambiguity attitude on S_i or $L_i(0)$.

The second row of table 6 verifies that ambiguity neutrality is independent of the preference for cardinality. In the previous literature from Halevy (2007), ambiguity neutrality is tightly related to the ability to reduce the compound lottery. However, since signal overbuying under higher cardinality can be interpreted as a failure of ROCL, the findings of this paper contradict his results.

Result 3. *Ambiguity neutrality does not affect the demand for signals or lotteries.*

4.3 Predictions with Signals

Table 9 illustrates subjects' prediction decisions after the signal stage. The majority of subjects followed the signal when their signal was informative (Box R or Box B). It implies that the subjects understood the information structure of the experiments. In both studies, the chi-square test and Fisher's exact test reject the null hypothesis that subjects randomly predict. (P-values are 0.000 for both studies.)

Table 9: Predictions with signals

	Predictions	Box R	Box B	Box G	No Signal
Study 1	Red	16 (94.1%)	3 (15.8%)	12 (85.7%)	85 (65.9%)
	Blue	1 (5.9%)	16 (84.2%)	2 (14.3%)	44 (34.1%)
Study 2	Red	37 (94.9%)	4 (11.1%)	8 (72.7%)	N/A
	Blue	2 (5.1%)	32 (88.9%)	3 (27.3%)	N/A
Chi-square test p-value = 0.000					

Did subjects make better decisions after receiving a high or low cardinality signal? Table 10 shows the correct decision rate with different signals. The correct decision is defined as whether the subject's prediction is consistent with the signal suggested after receiving Box R or Box B as a signal. Results show that the correct decision rate and cardinality are not correlated. (Chi-square test p-value and Fisher's exact test p-value are 0.513, 0.672 each.)

Table 10: Correct decision rate with each signal

Signal Received	S_1	S_2	S_3	S_4	Total
Correct	9 (81.8%)	3 (100.0%)	11 (84.6%)	9 (100.0%)	32 (88.9%)
Incorrect	2 (18.2%)	0 (0.0%)	2 (15.4%)	0 (0.0%)	4 (11.1%)
Total	11	3	13	9	36
Chi-square test p-value = 0.513					

4.4 Payoffs and Cardinality

This section investigates whether the preference for high cardinality hurts information buyers. Table 11 shows subjects' payoffs in points from part 1 in both studies. Overall, profits were larger in Study 1 than in Study 2 because of the 100 points endowment in Study 1. In Study 1, subjects earned the highest profit on average when they played lottery 1, the simplest lottery. In other words, they earned less profit when they played lotteries with high cardinality. However, this pattern disappears when signals are given.

Table 11: Payoffs from part 1

		Study 1		Study 2	
Lottery Selected	Cardinality	Payoff	Number	Payoff	Number
1	2	160.9	48	71.6	32
2	3	141.6	50	80.0	43
3	4	140.9	46	67.0	40
4	5	142.0	35	72.8	43
Total		146.7	179	73.1	158

Table 12 reports the regression results to clarify whether and when cardinality affects the payoffs. Columns (1) and (3) reveal the effect of cardinality on the payoffs. Results show that only Study 1 has a significant effect: Purchasing informative signals with high cardinality negatively affects payoff. Columns (2) and (4) show the effect of playing the simplest lottery (lottery 1). If a subject played more complex lotteries (lotteries 2-4), her expected payoff was 19.4 points less than when playing lottery 1. (F-test p-value is 0.0202.) The result of column (4) reveals this pattern vanishes when the signal is given.

Table 12: Determinants of the payoffs

	Dependent variable: Payoffs in Study 1		Dependent variable: Payoffs in Study 2	
	(1)	(2)	(3)	(4)
Cardinality	−6.118*		−1.168	
	(3.3800)		(2.5433)	
Simplest Lottery		19.438**		−1.788
		(8.2933)		(6.8568)
Constant	161.231***	141.458***	76.114***	73.444***
	(9.0026)	(4.3261)	(6.9543)	(3.1359)
Observations	716	716	632	632
R-squared	0.017	0.030	0.001	0.000
F-test p-value	0.0720	0.0202	0.6466	0.7946

Notes: Robust standard errors clustered by subject in parentheses.
$^{***}p < 0.01$, $^{**}\ p < 0.05$, $^{*}p < 0.1$.

Result 4. *Subjects earned less profit when purchasing signals with high cardinalities.*

The implication of Result 4 is that people overvalue informative signals when their cardinalities are high. Therefore, they submitted overpriced values for these signals, resulting in fewer earnings.

4.5 Robustness Study

Results from the robustness study show subjects understood the entire information structure, especially the informativeness of each signal. Subjects' submitted values of each lottery are consistent with the expected utility model.

Table 13: Summary of results in the robustness study

Questions	Cardinality	Winning Prob With a Signal	EU's Prediction	WTP
1	2	0.80	30	24.1
2	2	1.00	50	38.0
3	2	0.60	10	24.8
4	2	0.90	40	37.8
5	3	0.70	20	28.4
6	3	0.83	33.3	34.7
7	3	0.57	6.7	23.7
8	3	0.77	26.7	30.8

Table 13 reveals the submitted values of the willingness to pay for the signal in each lottery. What stands out in this table is that subjects valued the signals consistent with the theoretical prediction. Also, compared to the WTP for signals in lotteries 1-4, subjects overpaid signals in lotteries 5-8 because of the effect of cardinality. The chi-square test result rejects the null hypothesis that the willingness to pay for signals was randomly submitted (p-value=0.000).

Table 14: Determinants of the demand for signals

	Dependent variable: S_i			
	(1)	(2)	(3)	(4)
EU's Prediction	0.251***	0.273***	0.251***	0.273***
	(0.0646)	(0.0672)	(0.0646)	(0.0672)
Cardinality		1.505		1.505
		(1.0620)		(1.0620)
Constant	22.322***	17.969***	22.321***	17.969***
	(1.9820)	(3.7191)	(1.7506)	(3.6133)
Subject fixed effect	No	No	Yes	Yes
Observations	1040	1040	1040	1040
R-squared	0.020	0.020	0.045	0.047

Notes: Robust standard errors clustered by subject in parentheses.
$^{***}p < 0.01$, $^{**}\ p < 0.05$, $^{*}p < 0.1$.

The results from table 14 support the claim that subjects understood the whole information structure, including the meaning of the signal's informativeness. Theoretical predictions from the expected utility model are significantly related to actual submitted values. The results from the second row indicate cardinality has a positive effect on the demand for signals, but not significantly.

5 Conclusion

Economists have studied various environments with purchasing costly stochastic information. This article examines the demand for the informative signal with different levels of cardinality.

The results reveal the preference for high cardinality in the information acquisition process. Even though the signal accuracy is the same, subjects prefer a signal defined on a larger signal space than one on a small signal space. Also, a pattern of preference reversal is observed: after receiving the signal, the preference for high cardinality vanishes.

What is the behavioral reason for the desire for high cardinality? The first possible explanation is that subjects were confused and had a poor understanding of the informativeness of signals. This explanation is not plausible because the experimental design of this study allowed subjects to calculate the informativeness of each signal easily. Moreover, the result of the robustness study (see Section 4.5) rejects the hypothesis that subjects were confused about understanding the informativeness of each signal.

Another explanation is that subjects mistakenly believed higher cardinality implies higher informativeness. In many cases, the number of signals implies more information. Numerous theoretical and experimental studies have revealed the preference for frequent signals in various contexts.[9] In a similar sense, even when the possible number of signals is independent of the informativeness of the signal, people might misbelieve the correlation. However, this hypothesis cannot explain the pattern of preference reversal. If subjects believed signals with high cardinality were more informative, they should have also valued these signals even when they were given free.

The third explanation is intrinsic, i.e., curiosity. Consider an example of two famous American TV series. Breaking Bad, a crime drama TV series, used cliffhangers exceptionally well, making viewers search online to find more about the next episode or season. On the other hand, Modern Family, a family sitcom, is not mainly based on these types of narrative devices. Hence, in the view of curiosity, viewers are more likely to be curious about Breaking Bad than Modern Family before knowing the whole story. However, it does not necessarily mean that they value Breaking Bad after watching both shows: they might have enjoyed both shows equally. This viewpoint aligns with the experimental findings. Before receiving information, the receiver has a more intrinsic preference for a specific type of information than another. However,

[9] For example, in Edmond (2013)'s model of information and political regime change, the number of informative signals helps overthrow the regime. Lee and Niederle (2015) shows more signals (virtual roses) increase the success rate of the date in the internet dating market.

after having all information, the intrinsic preference no longer exists.

A contemporary view of curiosity is the intrinsically motivated drive to seek information, even when it has no instrumental value (Loewenstein, 1994; Oudeyer and Kaplan, 2007; Kidd and Hayden, 2015). Loewenstein (1994) and Golman and Loewenstein (2018) described curiosity as the information gap between what we know and what we want to know. Suppose "what we know" is defined as a measure of information, which is the probability of each outcome being realized. For example, when tossing a coin, the measure of information is 0.5 because the outcome is 50% head and 50% tail. Similarly, the measure of information of drawing a dice is 1/6. In the same way, the measure of information of Advisor A and Advisor B is 0.5 and 0.2 each. After receiving the signal, the information measures will be 1 for both. Therefore, curiosity for signals of Advisors A and B, which is the gap between the measure of information before and after the signal, is 0.5 and 0.7 each. Hence, the preference for high cardinality is consistent with the view of curiosity.

Several questions remain unanswered at present. This paper discloses the preference for high cardinality of signal when the value of cardinality is between 2 and 5. However, the result does not verify the optimal size of the signal space. Decision-makers might prefer high cardinality even when the signal space is extremely large. Or, maybe there exists the most preferred size of the signal space.

Another question is whether the results can be generalized into a non-binary action space or even a continuous one. The experimental design of this paper restricts the action space to binary. In reality, however, actions are not necessarily binary. Hence, exploring whether the results of this paper still hold in a more generalized action space is also an interesting question. I hope future studies will answer these questions.

References

Becker, G., DeGroot, M., and Marschak, J. (1964). Measuring utility by a single-response sequential method. *Behavioral Science*, 9:226–236.

Brown, A., Imai, T., Vieider, F., and Camerer, C. (2022). Meta-analysis of empirical estimates of loss-aversion. *Journal of Economic Literature, forthcoming.*

Cason, T. N. and Plott, C. R. (2014). Misconceptions and game form recognition: Challenges to theories of revealed preference and framing. *Journal of Political Economy*, 122(6):1235–1270.

Crawford, V. P. and Sobel, J. (1982). Strategic information transmission. *Econometrica*, 50(6):1431–1451.

Edmond, C. (2013). Information manipulation, coordination, and regime change. *Review of Economic Studies*, 80:1422–1458.

Ellsberg, D. (1961). Risk, ambiguity, and the savage axioms. *The Quarterly Journal of Economics*, 75(4):643–669.

Golman, R. and Loewenstein, G. (2018). Information gaps: A theory of preferences regarding the presence and absence of information. *Decision*, 5(4):143–164.

Grether, D. M. and Plott, C. R. (1979). Economic theory of choice and the preference reversal phenomenon. *American Economic Review*, 69:623–638.

Gupta, N., Rigott, L., and Wilson, A. (2021). The experimenters' dilemma: Inferential preferences over populations. *Working Paper.*

Halevy, Y. (2007). Ellsberg revisited: An experimental study. *Econometrica*, 75:503–536.

Heumann, T. (2020). On the cardinality of the message space in sender–receiver games. *Journal of Mathematical Economics*, 90:109–118.

Huck, S. and Weizsäcker, G. (1999). Risk, complexity, and deviations from expected-value maximization: Results of a lottery choice experiment. *Journal of Economic Psychology*, 20:699–715.

Kahneman, D. and Tversky, A. (1979). Prospect theory: An analysis of decision under risk. *Econometrica*, 47:263–291.

Kahneman, D. and Tversky, A. (1992). Advances in prospect theory: Cumulative representation of uncertainty. *Journal of Risk and Uncertainty*, 5:297–323.

Kamenica, E. and Gentzkow, M. (2011). Bayesian persuasion. *American Economic Review*, 101:2590–2615.

Kidd, C. and Hayden, B. Y. (2015). The psychology and neuroscience of curiosity. *Neuron*, 88:449–460.

Klibanoff, P., Marinacci, M., and Mukerji, S. (2005). A smooth model of decision making under ambiguity. *Econometrica*, 73(6):1849–1892.

Lee, S. and Niederle, M. (2015). Propose with a rose? signaling in internet dating markets. *Experimental Economics*, 18:731–755.

Lichtenstein, S. and Slovic, P. (1971). Reversals of preference between bids and choices in gambling decisions. *Journal of Experimental Psychology*, 89:46–55.

Loewenstein, G. (1994). The psychology of curiosity: a review and reinterpretation. *Psychological Bulletin*, 116:75–98.

Moffatt, P., Stefania, S., and John, Z. D. (2015). Heterogeneity in preferences towards complexity. *Journal of Risk and Uncertainty*, 51:147–170.

Noussair, C., Robin, S., and Ruffieux, B. (2004). Revealing consumers' willingness-to-pay: A comparison of the bdm mechanism and the vickrey auction. *Journal of Economic Psychology*, 25:725–741.

Oudeyer, P.-Y. and Kaplan, F. (2007). What is intrinsic motivation? a typology of computational approaches. *Front Neurorobotics*, 1:6.

Palan, S. and Schitter, C. (2018). Prolific.ac——a subject pool for online experiments. *Journal of Behavioral and Experimental Finance*, 17:22–27.

Pommerehne, W. W., Schneider, W., and Zweifel, P. (1982). Economic theory of choice and the preference reversal phenomenon: A reexamination. *American Economic Review*, 72:569–574.

Quiggin, J. (1982). A theory of anticipated utility. *Journal of Economic Behavior and Organization*, 3(4):323–343.

Segal, U. (1987). The ellsberg paradox and risk aversion: An anticipated utility approach. *International Economic Review*, 28:175–202.

Segal, U. (1990). Two-stage lotteries without the reduction axiom. *Econometrica*, 58:349–377.

Seo, K. (2009). Ambiguity and second-order belief. *Econometrica*, 77(5):1575–1605.

Sonsino, D., Benzion, U., and Mador, G. (2002). The complexity effects on choice with uncertainty - experimental evidence. *The Economic Journal*, 112(482):936–965.

Tversky, A., Slovic, P., and Kahneman, D. (1990). The causes of preference reversal. *American Economic Review*, 80(1):204–17.

Tversky, A. and Thaler, R. H. (1990). Anomalies: preference reversals. *Journal of Economic Perspectives*, 4(2):201–211.

A Appendix

A.1 Omitted Calculations

A.1.1 Investor Example

Suppose the investor gets a signal from Advisor A. The conditional probability that the company is good is given by

$$Pr(\theta_G | \text{``\textit{invest}''}) = 0.7,$$
$$Pr(\theta_G | \text{``\textit{not invest}''}) = 0.3.$$

Let *INVEST* or *NOT INVEST* denotes the investor's action. When the signal is "invest", then the investor will invest in the company, because $Pr(\theta_G | \text{``\textit{invest}''}) = 0.7 > 0.5$. In this case, her expected utility is

$$\begin{aligned}
&u(\textit{signal}\text{=``\textit{invest}''}) \\
&= 0.7u(\textit{INVEST}, \theta_G) + 0.3u(\textit{INVEST}, \theta_B) \\
&= 0.7 * 100 + 0.3 * 0 = 70.
\end{aligned}$$

Otherwise, she will not invest because $Pr(\theta_G | \text{``\textit{not invest}''}) = 0.3 < 0.5$. Her expected utility is given by

$$\begin{aligned}
&u(\textit{signal}\text{=``\textit{not invest}''}) \\
&= 0.3u(\textit{NOT INVEST}, \theta_G) + 0.7u(\textit{NOT INVEST}, \theta_B) \\
&= 0.3 * 0 + 0.7 * 100 = 70.
\end{aligned}$$

Therefore, the expected utility when receiving Advisor A's signal is

$$0.5u(\textit{signal=“invest”}) + 0.5u(\textit{signal=“not invest”})$$
$$= 0.5 * 70 + 0.5 * 70 = 70.$$

Suppose the investor hires Advisor B. The conditional probability of the state is

$$Pr(\theta_G|\textit{“strongly recommend”}) = 0.8,$$
$$Pr(\theta_G|\textit{“recommend”}) = 0.7,$$
$$Pr(\theta_G|\textit{“no opinion”}) = 0.5,$$
$$Pr(\theta_G|\textit{“not recommend”}) = 0.3,$$
$$Pr(\theta_G|\textit{“never recommend”}) = 0.2.$$

If the signal is "strongly recommend" or "recommend," then the investor will invest because $Pr(\theta_G|\textit{“strongly recommend”})) = 0.8 > 0.5$ and $Pr(\theta_G|\textit{“recommend”})) = 0.7 > 0.5$. If the signal is "no opinion," then she is indifferent between investing or not because $Pr(\theta_G|\textit{“no opinion”}) = 0.5$. She will not invest if the signal is "not recommend" or "never recommend" because $Pr(\theta_G|\textit{“not recommend”}) = 0.3 < 0.5$ and $Pr(\theta_G|\textit{“never recommend”})) = 0.2 < 0.5$.

Hence, when Advisor B's signal is "strongly recommend", the expected utility is

$$u(\textit{signal=“strongly recommend”})$$
$$= 0.8u(INVEST, \theta_G) + 0.2u(INVEST, \theta_L)$$
$$= 0.8 * 100 + 0.2 * 0 = 80.$$

Similarly,

$$
\begin{aligned}
&u(\textit{signal}=\text{``}\textit{recommend}\text{''}) = 70\\
&u(\textit{signal}=\text{``}\textit{no opinion}\text{''}) = 50\\
&u(\textit{signal}=\text{``}\textit{not recommend}\text{''}) = 70\\
&u(\textit{signal}=\text{``}\textit{never recommend}\text{''}) = 80
\end{aligned}
$$

Hence, the expected utility of receiving a signal from Adviosr B is

$$
\begin{aligned}
&0.2u(\textit{signal}=\text{``}\textit{strongly recommend}\text{''}) + 0.2u(\textit{signal}=\text{``}\textit{recommend}\text{''}) + 0.2u(\textit{signal}=\text{``}\textit{no opinion}\text{''})\\
&+ 0.2u(\textit{signal}=\text{``}\textit{not recommend}\text{''}) + 0.2u(\textit{signal}=\text{``}\textit{never recommend}\text{''})\\
&= 0.2 * 80 + 0.2 * 70 + 0.2 * 50 + 0.2 * 70 + 0.2 * 80 = 70.
\end{aligned}
$$

A.1.2 Expected Utility

When the cost of the signal is c, the expected utility of each lottery is,

$$
\begin{aligned}
U_{EU}(L_1(c)) &= 0.7u(100-c)\\
U_{EU}(L_2(c)) &= \frac{2}{3}(0.8u(100-c) + 0.2u(-c)) + \frac{1}{3}(0.5u(100-c) + 0.5u(-c)),\\
U_{EU}(L_3(c)) &= \frac{1}{2}(0.8u(100-c) + 0.2u(-c)) + \frac{1}{2}(0.6u(100-c) + 0.6u(-c)),\\
U_{EU}(L_4(c)) &= \frac{2}{5}(0.8u(100-c) + 0.2u(-c)) + \frac{2}{5}(0.7u(100-c) + 0.3u(-c)))\\
&+ \frac{1}{5}(0.5u(100-c) + 0.5u(-c)).
\end{aligned}
$$

Similarly,

$$
\begin{aligned}
U_{EU}(L_1(0)) &= 0.7u(100) \\
U_{EU}(L_2(0)) &= \frac{2}{3}(0.8u(100)) + \frac{1}{3}(0.5u(100)), \\
U_{EU}(L_3(0)) &= \frac{1}{2}(0.8u(100)) + \frac{1}{2}(0.6u(100)), \\
U_{EU}(L_4(0)) &= \frac{2}{5}(0.8u(100)) + \frac{2}{5}(0.7u(100)) + \frac{1}{5}(0.5u(100)).
\end{aligned}
$$

A.1.3 Recursive Smooth Ambiguity Preference

$L_1(c) \succeq L_4(c)$ can be derived by the following procedure:

$$
\begin{aligned}
U_{KMM}(L_1(c)) &= \phi(U(0.7)) \\
&\geq \frac{2}{5}\phi(U(0.8)) + \frac{2}{5}\phi(U(0.7)) + \frac{1}{5}\phi(U(0.5)) \\
&= U_{KMM}(L_4(c)).
\end{aligned}
$$

Also, $L_3(c) \succeq L_2(c)$:

$$
\begin{aligned}
U_{KMM}(L_3(c)) &= \frac{1}{2}\phi(U(0.8)) + \frac{1}{2}\phi(U(0.6)) \\
&= \frac{1}{2}\phi(U(0.8)) + \frac{1}{2}\phi(\frac{1}{3}U(0.8) + \frac{2}{3}U(0.5)) \\
&\geq \frac{1}{2}\phi(U(0.8)) + \frac{1}{6}\phi(U(0.8)) + \frac{1}{3}\phi(U(0.5)) \\
&\geq \frac{2}{3}\phi(U(0.8)) + \frac{1}{3}\phi(U(0.5)) \\
&= U_{KMM}(L_2(c)),
\end{aligned}
$$

Similarly, $L_4(c) \succeq L_2(c)$:

$$\begin{aligned}
U_{KMM}(L_4(c)) &= \frac{2}{5}\phi(U(0.8)) + \frac{2}{5}\phi(U(0.7)) + \frac{1}{5}\phi(U(0.5)) \\
&= \frac{2}{5}\phi(U(0.8)) + \frac{2}{5}\phi(\frac{2}{3}U(0.8) + \frac{1}{3}U(0.5)) + \frac{1}{5}\phi(U(0.5)) \\
&\geq \frac{2}{5}\phi(U(0.8)) + \frac{4}{15}\phi(U(0.8)) + \frac{2}{15}\phi(U(0.5)) + \frac{1}{5}\phi(U(0.5)) \\
&= \frac{2}{3}\phi(U(0.8)) + \frac{1}{3}\phi(U(0.5)) \\
&= U_{KMM}(L_2(c)),
\end{aligned}$$