Does the Size of the Signal Space Matter?*

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Abstract

This paper provides the first experimental evidence that information receivers consider the size of the signal space, representing the number of possible signals. In an environment of betting on the outcomes of compound lotteries, laboratory experiments measured the values of signals for the outcome (Study 1) and the values of compound lotteries (Study 2) in varying sizes of the signal space. The results show that the size of the signal space is positively correlated with the value of the signals but not the value of the compound lotteries. The preference for larger signal space suggests the key assumption in the information design literature, which is the signal space is equal to the action space, might not always hold. Leading theoretical frameworks cannot explain these experimental findings.

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1 Introduction

Signal transmission is an essential part of the literature on game theory, where a vast amount of theoretical and empirical research has been conducted. However, the desirable size of the signal space in the literature has often been overlooked. In the context of information acquisition, the size of the signal space denotes the number of possible signals. When discussing the size of the signal space, in many cases, theorists assume that the signal space equals the action space. They have shown that assuming an equivalence between the signal space and the action space is sufficient to find the equilibrium; therefore, a larger signal space is unnecessary. This assumption has been taken for granted for decades, but its validity could be limited if the receiver prefers a larger signal space. This paper investigates whether a preference exists for the size of the signal space, independent of the signal accuracy.

Consider the example of an investor contemplating whether or not to invest in a company. State $\theta \in \{G, B\}$ represents the type of the company, where G and B stand for a good company and a bad company respectively. The investor does not know whether the company is good or bad, but she thinks the probability that the company is good is 0.5. She wants to invest only if the company is good. Without loss of generality, suppose she gets utility 1 for investing in the good company or for not investing bad company, and utility 0 for investing in the bad company or for not investing in the good company.[1]

To reduce uncertainty about the investment decision, she is considering hiring a financial advisor with more knowledge of the company. There are two advisors she is considering: Advisor A and Advisor B. They both provide informative signals to the investor. Advisor A will send the investor one of thse two signals with equal probability: "invest" or "not invest." If his signal is "invest," the probability that the company is good is 70% ($Pr(G|\text{“}invest\text{”}) = 0.7$). If his signal is "not invest," the

[1] Note that she will be as happy not to invest in a bad company as to invest in a good company, considering the opportunity cost.

probability that the company is good is 30% ($Pr(G|\text{“}\textit{not invest}\text{”}) = 0.3$). Since the number of possible signals sent from Advisor A is 2, the size of his signal space is 2. Note that Advisor A is the kind of sender commonly assumed by theorists: the action space (invest or not) is equal to the signal space.

On the other hand, Advisor B has a larger signal space. Advisor B will send the investor one of these five signals with equal probability: “must invest,” “invest,” “no opinion,” “not invest,” or “never invest.” The respective probabilities that the company is good when each signal is sent are 0.8, 0.7, 0.5, 0.3, and 0.2. The size of his signal space is 5.

The advisor’s signal accuracy is defined by “winning” probability when receiving the signal from the advisor, which is consistent with the expected utility conditional on the signal. For example, if the investor receives and follows the signal from Advisor A, her winning probability is 0.7 whether she receives “invest” or “not invest.” Hence, Advisor A’s signal accuracy is 0.7. In the same way, Advisor B’s signal accuracy is also 0.7.[2] If the signal accuracy is the only factor the investor cares about, she will be indifferent between Advisors A and B. However, if she also cares about the size of the signal space, she will prefer one advisor to another.

This paper provides the first empirical evidence that the size of the signal space matters in information acquisition. In Study 1, subjects were asked to bet on four lotteries. Before betting, they had a chance to purchase a signal for each lottery. The signal for each lottery had the same accuracy but had a different signal space size, like Advisor A and Advisor B in the previous example. The results show the participants’ willingness to pay for the signal increased as the size of the signal space increased, even though signal accuracies did not change. This implies that people prefer Advisor B to Advisor A. Furthermore, the preference for larger signal space drove overpayment to Advisor B. When hiring Advisor B, the investor’s total earnings were lower than when hiring Advisor A.

[2] See Appendix A.1.1 for detailed calculations.

However, in a separate experimental study, I observed a pattern of preference reversal.[3] Study 2 measured the values of the four lotteries in Study 1. But in this case, there was no signal purchasing stage because the price for the signal was free. The result reveals that, surprisingly, subjects no longer preferred the larger signal space; they were indifferent to the size of the signal space.[4] If subjects valued signals with larger signal space when purchasing the signal, they should have also appreciated the larger signal space when the signal was free; but they did not.

Receiving a signal and playing a simple lottery based on the signal's information can be perceived as a two-stage lottery. In this environment, the preference for a larger signal space could be interpreted as a violation of the reduction of compound lottery axiom (ROCL). When a decision-maker can reduce compound lottery, there is no reason to pay more to a signal with a larger space under the same signal accuracy. Halevy (2007) revealed that ambiguity neutrality and reduction of compound lotteries are tightly associated. If his findings can be applied to this environment, the preference for a larger signal space should be correlated with ambiguity neutrality. However, the results of this paper did not find the correlation.

Among the possible explanations for the experimental findings, curiosity is the most plausible interpretation. Curiosity indicates an intrinsic motivation for seeking knowledge that might not have instrumental value. That implies that when purchasing signals, a receiver could have a more intrinsic inclination for having more possible signals, even if the size of the signal space does not matter after buying it.

[3] Lichtenstein and Slovic (1971) initially documented systemic inconsistencies between preferences of paired lotteries and their prices. Since the initial report, the preference reversal phenomenon has been observed in numerous experimental studies for a few decades (Grether and Plott, 1979; Pommerehne et al., 1982; Tversky and Thaler, 1990). When subjects are asked to choose between a safer lottery (high winning probability but small prize) and a riskier lottery (low winning probability but high prize) with nearly the same expected values, most subjects prefer to play the safer one. However, when asked to state the minimum prices to sell each lottery, most subjects put higher prices on the riskier lottery.

[4] While this is technically a "preference reversal," I view this phenomenon thematically differently from previous literature. Tversky et al. (1990) suggested that the primary reason for preference reversal is the failure of procedure invariance, especially the overpricing of high-payoff, low-probability bets. In this paper, however, the demand for greater knowledge without instrumental value suggests curiosity motivates it.

A more detailed discussion of potential explanations for the experimental results will be provided later in Section 5.

Does a smaller or larger signal space enable better decision-making? In some environments, limiting the size of the signal space might restrict the optimal outcome. For example, in most of the standard sender-receiver literature, a small size of signal space might lead to inefficient outcomes (Crawford and Sobel, 1982; Heumann, 2020). In this case, a larger signal space allows better decision-making. However, in the experimental design of this paper, the size of the signal space is independent of the efficiency of the outcomes: the signal accuracy of each signal is the same. Therefore, there is no behavioral or theoretical reason to prefer a larger signal space.

On the other hand, a decision-maker might prefer a simpler environment—a smaller signal space—if the signals are too complicated to understand. For example, a worker might want to receive direct instructions on what to do rather than receive abstract signals from the boss and interpret her intent. This preference could be related to complexity aversion, which illustrates a preference for simpler lotteries over complex ones, even though the expected values are the same (Huck and Weizsäcker, 1999; Sonsino et al., 2002; Halevy, 2007; Moffatt et al., 2015). However, the experimental results of this paper did not find evidence for complexity aversion.

This paper has two main contributions. First, the empirical findings of this paper suggest how to deliver information from the view of information providers. Information providers, such as financial advisors, medical test providers, or film critics, can make their services look more attractive by simply increasing the size of the signal space. For example, the result of this paper suggests that users are more attracted to a 5-star rating system than a binary suggestion, even if the two systems are equally accurate. Hence, if a service provider switches its recommendation system from a binary suggestion to a 5-star rating, demand for the service will increase, even without improving the system's accuracy. This implication is aligned with the experimental findings of Jin et al. (2022), called complex disclosure, suggesting senders get more benefits from using complex reports than from using easier ones.

Another contribution involves the theoretical aspect of the context of information design (Kamenica and Gentzkow, 2011). Without loss of generality, most theoretical studies of information design have assumed that the signal is "straightforward," implying that each signal is a form of recommended action, such as Advisor A in the investor case. In this case, the signal space is equal to the action space. When finding equilibrium, they rely on this assumption to simplify the design of the signal structure. However, the experimental findings of this paper suggest that the receiver might prefer the environment where the signal space is larger than the action space.

Section 3 provides theoretical predictions from various models, but none of them can explain the preference for larger signal space. The expected utility model predicts the same value for each signal. The recursive smooth ambiguity model of Klibanoff et al. (2005), the rank-dependent utility model (Quiggin, 1982), and prospect theory (Kahneman and Tversky, 1979, 1992) suggest different values for different signals, but they do not predict the systemic preference for the size of the signal space. Also, none of these models predict preference reversal.

This paper proceeds as follows. Section 2 describes the experimental design and procedure. Section 3 provides theoretical predictions of the results from various models. Section 4 reveals experimental results and Section 5 concludes.

2 Experimental Design

Participants were assigned to one of two studies: Study 1 or Study 2. Each study consists of two parts: part 1 measured the value of signals (Study 1) or lotteries (Study 2) under isomorphic environments, and part 2 measured ambiguity attitudes by Ellsberg (1961) questions.

2.1 Part 1: The Value of Signals/Lotteries

There are four lotteries in Part 1. Each lottery contains several boxes, with each box containing ten balls, either red or blue. In each lottery, the computer draws a ball in two stages. In the first stage, the computer randomly selects one of the boxes with an equal probability. In the second stage, the computer randomly draws a ball from the selected box. Between the first and the second stages, subjects predict the color of the ball which will be drawn. If their prediction is correct, they receive 100 points, where each point is equal to 0.01 USD. Figure 1 illustrates the four lotteries.[5]

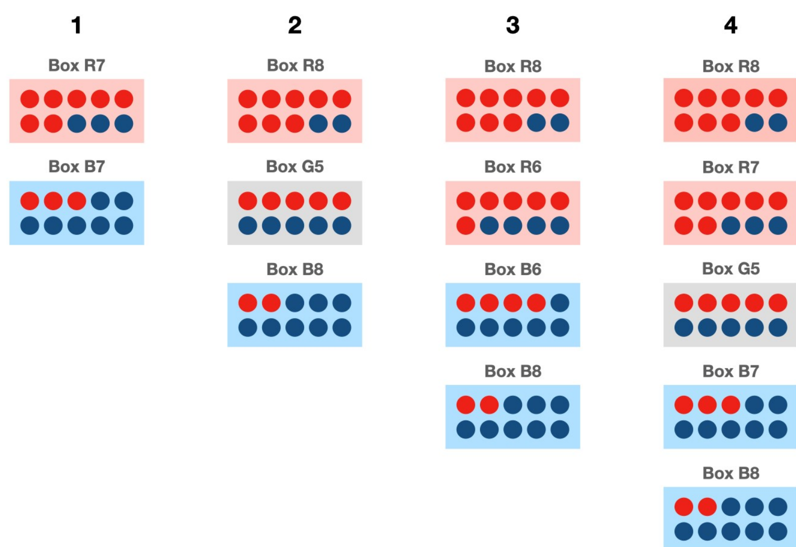

Figure 1: Four lotteries

Each box is denoted by Box Xn, where $X \in \{R, B, G\}$ and $n \in \{5, 6, 7, 8, 9\}$.[6] X and n represent the majority color of the balls in the box and the number of balls in the box, respectively. For example, Box R7 has more red balls than blue balls, and

[5]To avoid the possibility of cognitive load, the maximum size of the signal space is 5.

[6]Ambuehl and Li (2018) elicited the demand for informative signals and found that people significantly prefer information that might yield certainty. Therefore, to avoid the certainty effect, I exclude the box of $n = 10$.

the number of red balls is 7.[7]

In Study 1, subjects do not know which box was selected. However, before the prediction, subjects have a chance to "buy" a costly signal with their 100 endowment points. If they purchase a signal, the computer will tell them which box is selected. That signal increases their probability of winning but requires some cost, whether they win or lose.

For example, in lottery 2, there are three boxes: Box R8, Box G5, and Box B8. Suppose Box R8 is randomly selected. Without the signal, subjects do not know which box was chosen. Their winning probability is 50% whether they bet on a red or blue ball because there is a total of 15 red balls and 15 blue balls in lottery 2. If they buy the signal, they learn that Box R8 was selected and the ball will be drawn from Box R8. The signal "Box R8" increases the odds of winning to 80% because Box R8 contains 8 red and 2 blue balls.

One of the key features of this experiment is that each lottery always has 50% red balls and 50% blue balls. This implies that the prior, the winning probability without the signal, is 50% for all lotteries. Another essential feature is that the signal accuracy for each lottery is the same. If subjects purchase the signal, the winning probability increases to 70% for all four lotteries. The only difference between them is the number of boxes, representing the possible number of signals.

Study 2 measures the values of the four lotteries when the signals are free: before predicting the ball's color, subjects can observe which box is selected without the signal purchasing process. Note that the information structures of both studies are isomorphic. Hence, if a subject has a preference over the size of the signal space in Study 1, she will also have the same preference in Study 2. To quantify the values of the lotteries, I measured the subjects' willingness to pay to play each lottery. Figure 2 shows the timeline of both studies.

[7] In the actual experiment, boxes are represented as Box R, Box B, Box G, Box RR (if there is more than one Box R in the same lottery), and Box BB (if there is more than one Box B in the same lottery). Numerical labels are not used to provide an environment where subjects rely more on intuition.

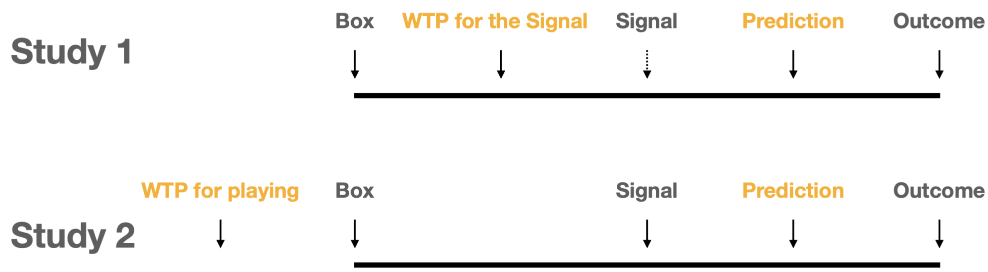

Figure 2: Timeline of both studies

To measure the willingness to pay, the Becker-DeGroot-Marschak (BDM) mechanism (Becker et al., 1964) was used. In Study 1, subjects submit the values of the signals, which are the maximum points they can pay for each lottery. After submitting values for signals for all four lotteries, one of them is randomly selected. Then, a random number is generated between 1 and 100. The random number represents the price for the signal for the selected question in the selected lottery. If a subject's submitted value in the selected lottery is greater than the price, she can see the signal and pays the price. However, if the submitted value in the selected lottery is equal to or lower than the price, she does not receive the signal and pays nothing. After the signal is revealed or not revealed, subjects predict the color of the ball. Figure 3 shows the questions in the BDM.

Q#	Option A	Choices	Option B
1	Buying a Signal for 1 point		Not Buying a Signal
2	Buying a Signal for 2 points		Not Buying a Signal
3	Buying a Signal for 3 points		Not Buying a Signal
4	Buying a Signal for 4 points		Not Buying a Signal
⋮	⋮	⋮	⋮
97	Buying a Signal for 97 points		Not Buying a Signal
98	Buying a Signal for 98 points		Not Buying a Signal
99	Buying a Signal for 99 points		Not Buying a Signal
100	Buying a Signal for 100 points		Not Buying a Signal

Figure 3: The BDM mechanism in Study 1

The procedure of the BDM in Study 2 is similar to that in Study 1 (See figure 4). Before playing the lotteries, subjects are asked the maximum number of points they are willing to pay for playing each lottery. After submitting four values for four lotteries, one of the lotteries is randomly selected. Then, a random number between 1 and 100, representing a substitute prize, is generated. If the submitted value in the selected lottery is greater than the prize, a subject plays the lottery. Otherwise, she receives the prize without playing the lottery. If subjects play the lottery, they see which box is selected and predict the color of the ball from the selected box.

Q#	Option A	Choices	Option B
1	Playing the lottery		Receiving 1 point
2	Playing the lottery		Receiving 2 points
3	Playing the lottery		Receiving 3 points
4	Playing the lottery		Receiving 4 points
⋮	⋮	⋮	⋮
97	Playing the lottery		Receiving 97 points
98	Playing the lottery		Receiving 98 points
99	Playing the lottery		Receiving 99 points
100	Playing the lottery		Receiving 100 points

Figure 4: The BDM mechanism in Study 2

The major issue with the BDM mechanism is its difficulty and the biased results in some environments.[8] To minimize the confusion, subjects were asked to submit their maximum willingness to pay for the signal instead of deciding between Option A and B 100 times. Also, before subjects submit their actual values, an example was illustrated of how the mechanism works when a specific value is submitted. Furthermore, even if the result is upward or downward biased, the biased result does not impair the primary purpose of the BDM mechanism, which is to compare preferences between signals and between lotteries, not to elicit their exact values.

There are two hypotheses to test. Study 1 measures subjects' willingness to pay for the signal. If only the signal accuracy matters, the demand for signals for all four lotteries should be the same. If c_i indicates the cost subjects are willing to pay for the signal of lottery i,

$$c_1 = c_2 = c_3 = c_4. \tag{1}$$

Hypothesis 1. *The size of the signal space does not affect the demand for the signal.*

[8] See Noussair et al. (2004) and Cason and Plott (2014) for discussions about the biased results of the BDM.

If L_i denotes lottery i, let $V_i^{signal}(c)$ represent a value of L_i with the signal with the cost c. Then, Study 2 measures $V_i^{signal}(0)$ for four lotteries. Suppose a subject values the signal for lottery i is more than the signal for lottery j. Then, she will also value lottery i more than lottery j even when the signal is free: $c_i > c_j \implies V_i^{signal}(0) > V_j^{signal}(0)$. Then, the following hypothesis holds.

Hypothesis 2. *The rank among c_i is identical to the rank among $V_i(0)$.*

To avoid subjects focusing only on the size of the signal space, lotteries were presented in the order of $L_1 - L_3 - L_2 - L_4$ in both studies.

2.2 Part 2: Ellsberg Questions

After the elicitation of the value of signals, subjects' ambiguity attitudes were measured by two questions from Ellsberg (1961). Ambiguity attitude is closely related to two-stage lotteries, especially to the ability to reduce compound lotteries (Halevy, 2007; Seo, 2009). Halevy (2007) showed the strong association between ambiguity neutrality and reduction of compound lotteries. Since preference for a larger/smaller signal space can be interpreted as a failure to reduce compound lotteries, this task helps to understand how ambiguity attitude is related to a preference for the size of the signal space.

The following statement describes the task.

> Consider there is a bag containing 90 ping-pong balls. 30 balls are blue, and the remaining 60 balls are either red or yellow in unknown proportions. The computer will draw a ball from the bag. The balls are well mixed so that each ball is as likely to be drawn as any other. You will bet on the color that will be drawn from the bag.

Subjects are asked to choose their preferred options between A & B and between C & D. Table 1 illustrates the four options.

Options	
Option A	receiving 100 points if a blue ball is drawn.
Option B	receiving 100 points if a red ball is drawn.
Option C	receiving 100 points if a blue or yellow ball is drawn.
Option D	receiving 100 points if a red or yellow ball is drawn.

Table 1: Ellsberg questions

If a subject prefers A to B and D to C, there is no formulation of subjective probability that can rationalize the preference. This preference is interpreted to be a consequence of ambiguity aversion. After the rewards from Part 1 and Part 2 are determined, one of the parts is randomly selected. Subjects will get the points in the selected part. Each point is converted to 0.01 USD.

2.3 Procedural Details

467 subjects participated in experiments through Prolific, an online platform for recruiting research participants.[9] 179 and 158 subjects participated in studies 1 and 2, respectively. Also, another 130 subjects participated in a robustness study, which is discussed below. On average, subjects spent 10 minutes and earned \$3.32, including a \$2.20 base payment.

2.4 Robustness Study

In addition to the main studies, an additional study was implemented to investigate the robustness of the results. The robustness study provides evidence on whether subjects understood the procedure correctly.

[9] Gupta et al. (2021) showed Prolific can be a reliable source of high-quality data. For details on the Prolific's subject pool, see Palan and Schitter (2018). In both studies, only US subjects participated.

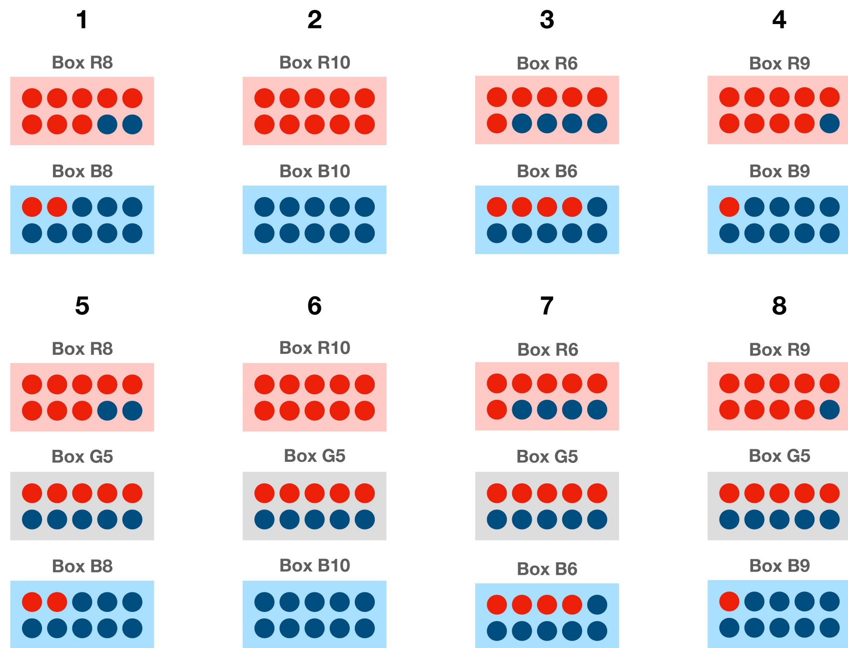

Figure 5: Lotteries in the robustness study

The procedure of this study is identical to Part 1 in Study 1: subjects were asked to value signals in uncertain lotteries. To investigate subjects' understanding, the values of signals for eight lotteries were measured. The signal of each lottery provides a different winning probability. If subjects understood this information acquisition framework correctly, they would be more willing to pay for a signal with a higher winning probability. Figure 5 illustrates the lotteries in this study.

Table 2: Summary of lotteries in the robustness study

Questions	Signal Space Size	Signal Accuracy	Predictions
1	2	0.80	30
2	2	1.00	50
3	2	0.60	10
4	2	0.90	40
5	3	0.70	20
6	3	0.83	33.3
7	3	0.57	6.7
8	3	0.77	26.7

Table 2 summarizes the details of the lotteries. Lotteries 1-4 have two boxes, Box Rn and Box Bn, where $n \in \{5, 6, 7, 8, 9, 10\}$. Hence, the size of the signal space is 2. Also, since Lotteries 5-8 have three boxes, Box Rn, Box Gn, and Box Bn, the size of the signal space is 3 for these lotteries. The signal accuracy, which is the winning probability with the signal of each lottery, is described in the third column. The fourth column shows the theoretical prediction when the decision-maker is a risk-neutral utility maximizer. If subjects understand the information framework of the signaling process, their demands for the signals will be in line with theoretical predictions.

3 Theoretical Predictions

Let L_i^{prior} denote lottery i without the signal. Also, $L_i^{signal}(p)$ is lottery i with a signal with the cost c. Suppose an individual's willingness to pay for the signal for lottery i is greater than or equal to her willingness to pay for the signal for lottery j: $c_i \geq c_j$, where c_i denotes the elicited price for the signal for lottery i. The values of c_i and c_j are determined by

$$V_i^{signal}(c_i) = V_i^{prior}, \tag{2}$$

$$V_j^{signal}(c_j) = V_j^{prior}, \tag{3}$$

where V_i denotes the value of lottery i.

Since $V_i^{prior} = V_j^{prior}$,

$$V_i^{signal}(c_i) = V_j^{signal}(c_j). \tag{4}$$

For simplicity of notation, I denote $V_i(c)$ instead of $V_i^{signal}(c)$ from now on. Note that $L_i(x)$ is a decreasing function of x. Hence, under the equation 4,

$$c_i \geq c_j \implies V_i(c) \geq V_j(c), \tag{5}$$

where $0 \leq c \leq \max(c_i, c_j)$. For example, suppose a subject's willingness to pay for signal 1 (the signal in lottery 1) is 20, and that for signal 2 (the signal in lottery 2) is 30. She will be happier to purchase signal 2 for a price of 15 than to purchase signal 1 for a price of 15. Hence, for calculation simplicity, I will compare $V_i(c)$ and $V_j(c)$ when a comparison between c_i and c_j is needed.

Study 1 measured c_i for $i \in \{1, 2, 3, 4\}$. Also, Study 2 elicited $L_i(0)$ for $i \in \{1, 2, 3, 4\}$ because the signal is free ($c = 0$). The remaining part of this section describes how different theories under uncertainty predict the two values in different lotteries.

3.1 Expected Utility

The expected utility of lottery i is given by

$$U_{EU}(L_i) = \sum_{s \in S} p(s)u(s). \tag{6}$$

The expected utility indicates that decision-makers are only interested in the expected values of lotteries but indifferent to the uncertainty resolution process. They do not care whether one lottery is a simple, compound, or a mean-preserving spread of the other lottery. Therefore, according to the expected utility model, subjects are indifferent between signals for lotteries, as well as between values of lotteries after receiving those signals.

$$\begin{gathered} c_1 = c_2 = c_3 = c_4, \\ V_1(0) = V_2(0) = V_3(0) = V_4(0). \end{gathered} \tag{7}$$

3.2 Recursive Smooth Ambiguity Utility

The recursive smooth ambiguity model by Klibanoff et al. (2005) (KMM, hereafter) suggests a theoretical utility model involving a second-order belief. KMM assumes that the decision-makers have a subjective expected utility on the space of second-order compound lotteries. For each f, there exists a second-order belief μ such that

$$U_{KMM}(f) = \sum_{\Delta(S)} \phi\Big(\sum_{s\in S} p(s)u(f(s))\Big)\mu(p), \tag{8}$$

where μ is a second-order subject belief, Δ is the set of possible first-order objective lotteries, and ϕ is a monotone function evaluating the expected utility associated with first-order beliefs.

For example, when purchasing a signal for L_1, there are two possible outcomes in the first stage (second-order): R7 or B7. In the second stage (first-order), the expected utility is $0.7u(100-c)+0.3u(-c)$ for both cases. Therefore, the evaluation of L_1 is given by

$$\begin{aligned} U_{KMM}(L_1(c)) &= \frac{1}{2}\phi(0.7u(100-c)+0.3u(-c)) + \frac{1}{2}\phi(0.7u(100-c)+0.3u(-c)) \\ &= \phi(0.7u(100-c)+0.3u(-c)) \end{aligned}$$

Similarly, the values of lotteries are evaluated as

$$\begin{aligned} U_{KMM}(L_2(c)) &= \frac{2}{3}\phi(0.8u(100-c)+0.2u(-c)) + \frac{1}{3}\phi(0.5u(100-c)+0.5u(-c)), \\ U_{KMM}(L_3(c)) &= \frac{1}{2}\phi(0.8u(100-c)+0.2u(-c)) + \frac{1}{2}\phi(0.6u(100-c)+0.4u(-c)), \\ U_{KMM}(L_4(c)) &= \frac{2}{5}\phi(0.8u(100-c)+0.2u(-c)) + \frac{2}{5}\phi(0.7u(100-c)+0.3u(-c)) \\ &\quad + \frac{1}{5}\phi(0.5u(100-c)+0.5u(-c)). \end{aligned}$$

When μ is subjective, KMM explained ambiguity aversion by the concavity of

ϕ: if Lottery Y is a mean-preserving spread of Lottery X, then individuals prefer X to Y because of their second-order subjective probability (μ). Since $L_3(c)$ is a mean-preserving spread of $L_1(c)$, decision-makers prefer $L_1(c)$ to $L_3(c)$:

For easier computation, let's define $U(\alpha)$ as,

$$U(\alpha) \equiv \alpha u(100 - c) + (1 - \alpha)u(-c).$$

Then,

$$\begin{aligned} U_{KMM}(L_1(c)) &= \phi(0.7u(100 - c) + 0.3u(-c)) \\ &= \phi(U(0.7)) \\ &\geq \frac{1}{2}\phi(U(0.8)) + \frac{1}{2}\phi(U(0.6)) \\ &= U_{KMM}(L_3(c)). \end{aligned}$$

A few more steps of calculations (See Appendix A.1 for details) show the following preferences hold.

$$\begin{aligned} c_1 \geq & c_3 \geq c_2, \\ c_1 \geq & c_4 \geq c_2. \end{aligned} \tag{9}$$

Since $L_i(0)$ is a specific form of $L_i(c)$, the preference among $L_i(0)$ does not change. Hence, regardless of the ambiguity attitude, KMM does not predict preference reversal.

$$\begin{aligned} V_1(0) \geq & V_3(0) \geq V_2(0), \\ V_1(0) \geq & V_4(0) \geq V_2(0). \end{aligned} \tag{10}$$

When ϕ is convex, implying ambiguity seeking, the opposite inequality holds.

$$\begin{aligned} c_2 \geq & c_3 \geq c_1, && (11) \\ c_2 \geq & c_4 \geq c_1, \\ V_2(0) \geq & V_3(0) \geq V_1(0), && (12) \\ V_2(0) \geq & V_4(0) \geq V_1(0). \end{aligned}$$

3.3 Simulational Predictions from Other Models

3.3.1 Rank-Dependent Utility

The rank-dependent utility (RDU) model suggested a probability weighting approach based on the order of rank for the outcomes (Quiggin, 1982; Segal, 1987, 1990). According to the RDU model, the utility of a lottery paying x_i with probability p_i is described as

$$U_{RDU}(x_1, p_1; x_2, p_2; \cdots; x_n, p_n) = u(x_1) + \sum_{i=2}^{n} [u(x_i) - u(x_{i-1})] f(\sum_{j=i}^{n} p_j), \tag{13}$$

where $x_1 \leq x_2 \leq x_3 \cdots \leq x_n$, $f : [0, 1] \to [0, 1]$, $f(0) = 0$ and $f(1) = 1$. For the simple lottery that gives 100 with probability p and 0 with probability $1 - p$,

$$U(100, p; 0, 1 - p) = u(100)f(p). \tag{14}$$

Suppose its certainty equivalent is $CE(p)$, then

$$CE(p) = CE(100, p; 0, 1 - p) = u^{-1}(u(100)f(p)). \tag{15}$$

Hence,

$$U_{RDU}(L_1(0)) = u(CE(0.7)) = u(100)f(0.7).$$

Similarly, values of four lotteries with signals are calculated as

$$
\begin{aligned}
U_{RDU}(L_1(0)) &= u(100)f(0.7),\\
U_{RDU}(L_2(0)) &= u(100)f(0.5) + [u(100)(f(0.8) - f(0.5))]f(\frac{2}{3}),\\
U_{RDU}(L_3(0)) &= u(100)f(0.6) + [u(100)(f(0.8) - f(0.6))]f(\frac{1}{2}),\\
U_{RDU}(L_4(0)) &= u(100)f(0.5) + [u(100)(f(0.7) - f(0.5))]f(\frac{4}{5})\\
&\quad + [u(100)(f(0.8) - f(0.7))]f(\frac{2}{5}).
\end{aligned}
$$

Preferences between lotteries vary depending on the functional form of $f(p)$. Table 3 illustrates simulational predictions of the RDU model based on different concave functions.

Table 3: Theoretical predictions by RDU

$f(p)$	Preferences between c_i	Preferences between $V_i(0)$
$p^{0.1}$	$c_2 \geq c_4 \geq c_3 \geq c_1$	$V_2(0) \geq V_4(0) \geq V_3(0) \geq V_1(0)$
$p^{0.5}$	$c_2 \geq c_4 \geq c_3 \geq c_1$	$V_2(0) \geq V_4(0) \geq V_3(0) \geq V_1(0)$
$p^{0.8}$	$c_2 \geq c_4 \geq c_3 \geq c_1$	$V_2(0) \geq V_4(0) \geq V_3(0) \geq V_1(0)$
p	$c_1 = c_2 = c_3 = c_4$	$V_1(0) = V_2(0) = V_3(0) = V_4(0)$
$log(p)$	$c_1 \geq c_3 \geq c_2 \geq c_4$	$V_1(0) \geq V_3(0) \geq V_2(0) \geq V_4(0)$
$ln(p)$	$c_1 \geq c_3 \geq c_2 \geq c_4$	$V_1(0) \geq V_3(0) \geq V_2(0) \geq V_4(0)$

Simulation results show that the RDU models with various functional forms of $f(p)$ do not predict the preference for larger signal space.

3.3.2 Prospect Theory

The first version of prospect theory was formulated by Kahneman and Tversky (1979), providing evidence of a systemic violation of the expected utility theory. The authors presented an alternative theoretical model to explain the violation. Later, Kahneman and Tversky (1992) (KT, henceforth) presented an extension of the original model, cumulative prospect theory, which adopted rank-dependence in probability weighting.

According to the cumulative prospect theory (CPT), the utility of a lottery paying x_i with probability p_i is described as

$$U_{CPT}(x_m, p_m; x_{m+1}, p_{m+1}; \cdots ; x_0, p_0; \cdots ; x_n, p_n) = \sum_{i=-m}^{n} \pi_i v(x_i), \tag{16}$$

where $v(\cdot)$ is a value function, which is an increasing function with $v(0) = 0$, and π is the decision weight. KT defined the value function as follows.

$$v(x) = \begin{cases} x^{\alpha} & \text{if } x \geq 0, \\ -\lambda(-x)^{\beta} & \text{if } x < 0, \end{cases} \tag{17}$$

where λ is a loss aversion parameter.

Decision weights π are defined by:

$$\begin{aligned} \pi_n^+ &= w^+(p_n), \\ \pi_{-m}^- &= w^+(p_{-m}), \\ \pi_i^+ &= w^+(p_i + ... + p_n) - w^+(p_{i+1} + ... + p_n),\ 0 \leq i \leq n-1, \\ \pi_i^- &= w^-(p_{-m} + ... + p_i) - w^-(p_m + ... + p_{i-1}),\ 1 - m \leq i \leq 0, \end{aligned} \tag{18}$$

where w^+ and w^- are the following functions.

$$w^+(p) = \frac{p^{\gamma}}{(p^{\gamma} + (1-p)^{\gamma})^{1/\gamma}},\ w^-(p) = \frac{p^{\delta}}{(p^{\delta} + (1-p)^{\delta})^{1/\delta}}. \tag{19}$$

To predict the preferences for c_i and $V_i(0)$ by CPT, I used the parameter values from KT. They estimated the values from experimental data.

Table 4: Values of parameters from KT

Parameter	Meaning	Value
α	power for gains	0.88
β	power for losses	0.88
λ	loss aversion	2.25[10]
γ	probability weighting parameter for gains	0.61
δ	probability weighting parameter for losses	0.69

Also, I assume the cost of the signal is 20, which is the theoretically expected value when the decision-maker is a risk-neutral expected utility maximizer. Hence, the preference between c_i is from simulational results from $L_i(20)$. With these parameter values, CPT predicts the following preferences:

$$c_1 \geq c_3 \geq c_4 \geq c_2, \tag{20}$$
$$V_1(0) \geq V_3(0) \geq V_4(0) \geq V_2(0).$$

Summarizing the theoretical predictions for the value of the signals in Study 1, no model predicts the preference for a larger signal space ($c_1 \geq c_2 \geq c_3 \geq c_4$).

Prediction 1. *Preference for a larger signal space does not exist.*

This prediction is consistent with Hypothesis 1. Also, no model predicts different preferences between c_i and $V_i(0)$, which is consistent with Hypothesis 2.

Prediction 2. *Preference reversal does not exist.*

To summarize, theoretical predictions are aligned with the hypotheses: no theoretical models predict the preference for a larger signal space or preference reversal.

[10] According to a meta-analysis by Brown et al. (2022), the mean of the loss aversion coefficient λ from numerous empirical estimates is 1.97. I found that simulational results with $\lambda = 1.97$ do not change the preference between lotteries.

4 Results

4.1 Preference for a Larger Signal Space

Table 5: Elicited values for c_i and $V_i(0)$ with different size of signal space.

		Study 1		Study 2	
Lottery	$\|S\|$	c_i	Number	$V_i(0)$	Number
1	2	23.6	179	52.9	158
2	3	24.9	179	48.9	158
3	4	25.8	179	51.0	158
4	5	29.8	179	52.7	158
Cuzick's test p-value		0.005		0.574	

The Cuzick non-parametric trend test across ordered groups reveals these differences are significant for both risk-resolution-preference and ambiguityresolution- preference categorizations.

Table 5 shows the submitted value for each signal (c_i) and each lottery given the signal ($V_i(0)$) in points. $|S|$ represents the size of the signal space. For c_i, the theoretical predictions from the risk-neutral expected utility maximizer are 20 points for each lottery. Hence, overall, the demand for signals is greater than the theoretical predictions. The most notable feature of the willingness to pay for the signal is the preference for larger signal space: the demand for the signal increases as the size of the signal space increases. However, in Study 2, the preference for larger signal space vanishes when the signal is freely accessible.

I did not find evidence for complexity aversion in lottery choice. According to Sonsino et al. (2002), a lottery's complexity is measured as the product of the number of rows and columns. Hence, in this environment, the number of boxes in the lottery indicates the complexity of the lottery. The results show that when the signal is free, the number of boxes — the size of the signal space — did not affect the values of playing the lotteries.

Table 6: Individual preferences among c_i and among $V_i(0)$.

	Study 1		Study 2	
Preference	Number	Percentage	Number	Percentage
Larger Signal Space	39	21.8%	17	10.8%
Indifferent	33	18.4%	28	17.7%
Smaller Signal Space	6	3.4%	15	9.5%
Others	101	56.4%	98	62.0%
Total	179	100.0%	158	100.0%

Table 6 illustrates the individual preferences between signals and lotteries. A larger proportion of subjects preferred the larger signal space in Study 1 ($c_4 \geq c_3 \geq c_2 \geq c_1$, but not $c_1 = c_2 = c_3 = c_4$) than in Study 2 ($V_4(0) \geq V_3(0) \geq V_2(0) \geq V_1(0)$, but not $V_1(0) = V_2(0) = V_3(0) = V_4(0)$). Also, a smaller proportion of subjects preferred the smaller signal space in Study 1 ($c_1 \geq c_2 \geq c_3 \geq c_4$, but not $c_1 = c_2 = c_3 = c_4$) than in Study 2 ($V_1(0) \geq V_2 0) \geq V_3(0) \geq V_4(0)$, but not $V_1(0) = V_2(0) = V_3(0) = V_4(0)$). There is no proportional difference between the groups who showed indifference to signal space size ($c_1 = c_2 = c_3 = c_4$ or $V_1(0) = V_2(0) = V_3(0) = V_4(0)$)

To examine these relationships formally, I consider OLS regressions of the form:

$$y_{in} = \beta_0 + \beta_1 |S|_i + \beta_2 AmbNeutral_n + \beta_3 |S|_i * AmbNeutral_n + \epsilon_{in}. \tag{21}$$

$y_{i.n}$ is the value of c_i or $V_i(0)$ by individual n, $|S|_i$ is a dummy variable indicating whether individual n is ambiguity neutral or not.

Table 7: Determinants of the demand for signals and lotteries

	Dependent variable: c_i			Dependent variable: $V_i(0)$		
	(1)	(2)	(3)	(4)	(5)	(6)
Signal Space Size	1.93***	2.03***	1.93***	0.16	0.08	0.16
	(0.40)	(0.55)	(0.40)	(0.50)	(0.73)	(0.50)
Ambiguity Neutrality		1.06			−2.85	
		(3.77)			(4.37)	
Signal Space Size ×		−0.20			0.16	
Ambiguity Neutrality		(0.79)			(1.01)	
Constant	19.28***	18.78***	19.28***	50.82***	52.34***	50.82***
	(1.87)	(2.50)	(1.38)	(2.18)	(3.20)	(1.76)
Subject fixed effect	No	No	Yes	No	No	Yes
Observations	716	716	716	632	632	632
R-squared	0.010	0.010	0.046	0.000	0.003	0.000
F-test p-value	0.0000	0.0001	0.0000	0.7502	0.8211	0.7502

Notes: Robust standard errors clustered by subject in parentheses. Columns (2) and (5) cannot include subject fixed effect because the ambiguity attitude is measured at the subject level. ***$p < 0.01$, ** $p < 0.05$, *$p < 0.1$.

The first three columns in Table 7 show that the signal space size significantly affects the value of signals. (F-test p-values < 0.0001 for these columns.) When the size of the signal space increases, the willingness to pay for the signal also increases.

Result 1. ***Preference for Larger Signal Space:*** *When purchasing signals, the willingness to pay for the signal increases as the signal space size increases.*

Result 1 rejects Hypothesis 1. Also, columns (4)-(6) show that the signal space size no longer affects the value of lotteries when the signal is free. (F-test p-values are 0.7502, 0.7504, and 0.7502 for each column.) This result rejects Hypothesis 2.

Result 2. ***Preference Reversal:*** *When the signal is free, subjects no longer prefer a larger signal space.*

Since no theoretical model predicts the preference for larger signal space, the result falsifies Prediction 1. Also, no model predicts preference reversal. Therefore, Predictions 1 and 2 are both falsified by the experimental results.

4.2 Ambiguity Attitudes

Table 8: Ambiguity attitudes

Ambiguity Attitude	Study 1 Number	Study 1 Percentage	Study 2 Number	Study 2 Percentage
Averse	72	40.2%	55	34.8%
Neutral	85	47.5%	84	53.2%
Seeking	22	12.3%	19	12.0%
Total	179	100.0%	158	100.0%

Table 9: The submitted values of c_i and $V_i(0)$ with different ambiguity attitudes

Attitude	c_1	c_2	c_3	c_4	$V_1(0)$	$V_2(0)$	$V_3(0)$	$V_4(0)$
Averse	22.8	23.8	24.4	29.9	55.7	49.5	47.7	55.8
Neutral	23.8	26.9	24.9	29.2	50.7	50.8	48.7	50.8
Seeking	25.5	28.7	26.5	31.4	54.2	55.8	52.9	52.2
Total	23.6	25.9	24.9	29.8	52.9	51.0	48.9	52.7
F-test p-value	0.8142				0.5988			

Table 8 and 9 respectively describe the ambiguity attitudes of subjects, and values of c_i and $V_i(0)$ conditional on different ambiguity attitudes. It can be easily seen that the overall patterns between the WTP for signals and lotteries are consistent even though the attitude towards ambiguity changes. The p-values of F-tests provide evidence that there is no effect of ambiguity attitude on c_i or $V_i(0)$.

The third row of Table 7 verifies that ambiguity neutrality is independent of the preference for the size of the signal space. In Halevy (2007), ambiguity neutrality is tightly related to the ability to reduce the compound lottery. However, the findings of this paper contradict these results.

Result 3. *Ambiguity neutrality is not related to the preference for the signal space size.*

4.3 Predictions with Signals

Table 10 illustrates subjects' prediction decisions after the signal stage. The majority of subjects followed the signal when their signal was informative (Box R or Box B). This implies that the subjects understood the information structure of the experiments. In both studies, the chi-square test and Fisher's exact test reject the null hypothesis that subjects randomly predicted. (p-values < 0.001 for both studies.)

Table 10: Predictions with signals

	Predictions	Box R	Box B	Box G	No Signal
Study 1	Red	16 (94.1%)	3 (15.8%)	12 (85.7%)	85 (65.9%)
	Blue	1 (5.9%)	16 (84.2%)	2 (14.3%)	44 (34.1%)
Study 2	Red	37 (94.9%)	4 (11.1%)	8 (72.7%)	N/A
	Blue	2 (5.1%)	32 (88.9%)	3 (27.3%)	N/A
Chi-square test p-value = 0.000					

Did subjects make better decisions when receiving signals from smaller or larger signal spaces? Table 11 shows the correct decision rate with different signals. The correct decision is defined as whether the subject's prediction is consistent with the signal suggested after receiving Box R or Box B as a signal. Results show that the correct decision rate and the signal space size are not correlated. (Chi-square test p-value and Fisher's exact test p-value are approximately 0.513 and 0.672, respectively).

Table 11: Correct decision rate with each signal

Signal Received	s_1	s_2	s_3	s_4	Total
Correct	9 (81.8%)	3 (100.0%)	11 (84.6%)	9 (100.0%)	32 (88.9%)
Incorrect	2 (18.2%)	0 (0.0%)	2 (15.4%)	0 (0.0%)	4 (11.1%)
Total	11	3	13	9	36
Chi-square test p-value = 0.513					

4.4 Payoffs and the Size of the Signal Space

This section investigates whether the preference for a larger signal space hurts information buyers. Table 12 shows subjects' payoffs in points from Part 1 in both studies.

Overall, profits were larger in Study 1 than in Study 2 because of the 100-point endowment in Study 1. In Study 1, subjects earned the highest profit on average when they played Lottery 1, the simplest lottery. In other words, they earned less profit when they played lotteries with larger signal space. However, this pattern disappeared when signals were free.

Table 12: Payoffs from part 1

Lottery	Signal Space	Study 1			Study 2		
Selected	Size	Payoff	Std. Error	Number	Payoff	Std. Error	Number
1	2	160.9	7.1	48	71.6	6.2	32
2	3	141.6	7.5	50	80.0	4.5	43
3	4	140.9	7.0	46	67.0	6.2	40
4	5	142.0	8.2	35	72.8	5.4	43
Total		146.7	3.7	179	73.1	2.8	158

Table 13 reports the regression results to clarify whether and when signal space size affects the payoffs. Columns (1) and (3) reveal the effect of the signal space size on the payoffs. Results show that only Study 1 has a significant effect: purchasing signals from larger signal spaces negatively affected payoffs.

Columns (2) and (4) show the effect of playing the simplest lottery (Lottery 1). If a subject played more complex lotteries (Lotteries 2-4), her expected payoff was 19.4 points less than when playing Lottery 1 (F-test p-value is 0.0202). The result of Column (4) reveals that this pattern vanishes when the signal is free.

Table 13: Determinants of the payoffs

	Dependent variable: Payoffs in Study 1		Dependent variable: Payoffs in Study 2	
	(1)	(2)	(3)	(4)
Signal Space Size	−6.12*		−1.17	
	(3.38)		(2.54)	
Simplest Lottery		19.44**		−1.79
		(8.29)		(6.86)
Constant	161.23***	141.46***	76.11***	73.44***
	(9.00)	(4.33)	(6.95)	(3.14)
Observations	716	716	632	632
R-squared	0.017	0.030	0.001	0.000
F-test p-value	0.0720	0.0202	0.6466	0.7946

Notes: Robust standard errors clustered by subject in parentheses.
***$p < 0.01$, ** $p < 0.05$, *$p < 0.1$.

Result 4. *Subjects earned less profit when purchasing signals from a larger signal space.*

The implication of Result 4 is that people overvalue signals when the signal space is large. Therefore, they submitted overpriced values for these signals, resulting in lower earnings.

4.5 Robustness Study

Results from the robustness study show that subjects understood the entire information structure, especially the accuracy of each signal. Subjects' submitted values for each lottery are consistent with the expected utility model.

Table 14: Summary of results in the robustness study

Questions	Signal Space Size	Winning Prob With Signals	Predictions	WTP
1	2	0.80	30	24.1
2	2	1.00	50	38.0
3	2	0.60	10	24.8
4	2	0.90	40	37.8
5	3	0.70	20	28.4
6	3	0.83	33.3	34.7
7	3	0.57	6.7	23.7
8	3	0.77	26.7	30.8

Table 14 reveals the submitted values of the willingness to pay for the signal in each lottery. What stands out in this table is that subjects valued the signals consistent with the theoretical prediction. Also, compared to the WTP for signals in Lotteries 1-4, subjects overpaid for signals in Lotteries 5-8 because of the effect of the signal space size. The chi-square test result rejects the null thesis that the willingness to pay for signals was randomly submitted (p-value$<$ 0.001).

Table 15: Determinants of the demand for signals

	Dependent variable: c_i			
	(1)	(2)	(3)	(4)
Predictions	0.25***	0.27***	0.25***	0.27***
	(0.07)	(0.07)	(0.07)	(0.07)
Signal Space Size		1.51		1.51
		(1.06)		(1.06)
Constant	22.32***	17.97***	22.32***	17.97***
	(1.98)	(3.72)	(1.75)	(3.61)
Subject fixed effect	No	No	Yes	Yes
Observations	1040	1040	1040	1040
R-squared	0.020	0.020	0.045	0.047

Notes: Robust standard errors clustered by subject in parentheses.
***$p < 0.01$, ** $p < 0.05$, *$p < 0.1$.

The results from Table 15 support the claim that subjects understood the whole information structure, including the meaning of the signal accuracy. Theoretical predictions from the risk-neutral expected utility model are significantly related to the actual submitted values. The results from the second row indicate that the signal space size has a positive effect on the demand for signals, but not significantly positive.

5 Conclusion

Economists have studied various environments on purchasing costly stochastic information. This article is the first to examine the demand for a signal with different sizes of signal space. The results reveal a preference for a larger signal space in the information acquisition process. Even when the signal accuracy was fixed, subjects preferred to receive a signal from a larger signal space. Also, a pattern of preference reversal was observed: when the prices of the signals were free, the preference for the larger signal space vanished.

What is the behavioral reason for the desire for larger signal space? The first possible explanation is that subjects were confused and had a poor understanding of signal accuracy. This explanation is not plausible because the experimental design of this study allowed subjects to calculate the signal accuracy easily. Moreover, the result of the robustness study (see Section 4.5) rejects the argument that subjects were confused about understanding the signal accuracy.

Another explanation is that subjects mistakenly believed a larger signal space implies higher signal accuracy. In many cases, the number of signals implies more information. Numerous theoretical and experimental studies have revealed the preference for frequent signals in various contexts.[11] In a similar sense, even when the possible number of signals is independent of the signal accuracy, people might misconstrue the correlation. However, this argument cannot explain the pattern of preference reversal.

[11]For example, in Edmond (2013)'s model of information and political regime change, the number of informative signals helps overthrow the regime. Lee and Niederle (2015) showed that more signals (virtual roses) increase the success rate of the date in the internet dating market.

If subjects believed signals from larger spaces were more accurate, they should have also valued these signals even when they were free.

The third explanation is intrinsic, i.e., curiosity. Consider an example of two famous American TV series. *Breaking Bad*, a crime drama TV series, used cliffhangers exceptionally well, making viewers search online to find more about the next episode or season. On the other hand, *Modern Family*, a family sitcom, rarely used these narrative devices. Hence, viewers are more likely to be curious about *Breaking Bad* than *Modern Family* before knowing the whole story. However, that does not necessarily mean that they value *Breaking Bad* more after watching both shows: they might have enjoyed both shows equally. This viewpoint aligns with the experimental findings. Before receiving information, the receiver has a more intrinsic preference for a specific type of information than another. However, after having all information, the intrinsic preference no longer exists.

A contemporary view of curiosity defines it as the intrinsically motivated drive to seek information, even when it has no instrumental value (Loewenstein, 1994; Oudeyer and Kaplan, 2007; Kidd and Hayden, 2015). Loewenstein (1994) and Golman and Loewenstein (2018) described curiosity as the information gap between what we know and what we want to know. Suppose "what we know" is defined as a measure of information, which is the probability of each outcome being realized. For example, when tossing a coin, "what we know" is measured as 0.5 because the outcome is 50% heads and 50% tails. Similarly, the measure of "what we know" when rolling a die is 1/6. In the same way, the measures of the signals from Advisor A and Advisor B are 0.5 and 0.2 respectively.

After receiving the signal, the information measures ("what we want to know") will be 1 for both. Therefore, curiosity about the signals of Advisors A and B, which is the gap between the measure of information before and after the signal, accounts for 0.5 and 0.8 respectively. According to the view of curiosity, the investor prefers Advisor B to Advisor A, which is consistent with the experimental results.

Table 16: Information measures by curiosity

	"what we know"	"what we want to know"	curiosity
Coin	0.5	1	0.5
Dice	1/6	1	5/6
Advisor A	0.5	1	0.5
Advisor B	0.2	1	0.8

Several questions remain unanswered at present. This paper discloses a preference for a larger signal space when the signal space size is between 2 and 5. However, the result does not verify the optimal size of the signal space. Decision-makers might prefer a larger space even when the signal space is extremely large, or there may be a most preferred signal space size.

Another question is whether these results can be generalized into a non-binary action space or even a continuous one. The experimental design of this paper restricts the action space to binary. In reality, however, actions are not necessarily binary. Hence, exploring whether the results of this paper still hold in a more generalized action space is also an interesting question. I hope future studies will answer these questions.

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A Appendix

A.1 Omitted Calculations

A.1.1 Investor Example

Suppose the investor gets a signal from Advisor A. The conditional probability that the company is good is given by

$$Pr(G|\text{“}invest\text{”}) = 0.7,$$
$$Pr(G|\text{“}not\ invest\text{”}) = 0.3.$$

Let *INVEST* or *NOT INVEST* denotes the investor's action. When the signal is "invest", then the investor will invest in the company, because $Pr(G|\text{“}invest\text{”}) = 0.7 > 0.5$. In this case, her expected utility is

$$\begin{aligned} & u(signal{=}\text{“}invest\text{”}) \\ & = 0.7u(INVEST, G) + 0.3u(INVEST, B) \\ & = 0.7 * 1 + 0.3 * 0 = 0.7. \end{aligned}$$

Otherwise, she will not invest because $Pr(G|\text{“}not\ invest\text{”}) = 0.3 < 0.5$. Her expected utility is given by

$$\begin{aligned} & u(signal{=}\text{“}not\ invest\text{”}) \\ & = 0.3u(NOT\ INVEST, G) + 0.7u(NOT\ INVEST, B) \\ & = 0.3 * 0 + 0.7 * 1 = 0.7. \end{aligned}$$

Therefore, the expected utility when receiving Advisor A's signal is

$$0.5u(\textit{signal=“invest”}) + 0.5u(\textit{signal=“not invest”})$$
$$= 0.5 * 0.7 + 0.5 * 0.7 = 0.7.$$

Suppose the investor hires Advisor B. The conditional probability of the state is

$$Pr(G|\textit{“must invest”}) = 0.8,$$
$$Pr(G|\textit{“invest”}) = 0.7,$$
$$Pr(G|\textit{“no opinion”}) = 0.5,$$
$$Pr(G|\textit{“not invest”}) = 0.3,$$
$$Pr(G|\textit{“never invest”}) = 0.2.$$

If the signal is “must invest” or “invest,” then the investor will invest because $Pr(G|\textit{“must invest”})) = 0.8 > 0.5$ and $Pr(G|\textit{“invest”})) = 0.7 > 0.5$. If the signal is “no opinion,” then she is indifferent between investing or not because $Pr(G|\textit{“no opinion”}) = 0.5$. She will not invest if the signal is “not invest” or “never invest” because $Pr(G|\textit{“not invest”}) = 0.3 < 0.5$ and $Pr(G|\textit{“never invest”})) = 0.2 < 0.5$.

Hence, when Advisor B's signal is “must invest”, the expected utility is

$$u(\textit{signal=“must invest”})$$
$$= 0.8u(INVEST, G) + 0.2u(INVEST, B)$$
$$= 0.8 * 1 + 0.2 * 0 = 0.8.$$

Similarly,

$$u(\textit{signal}=\text{``}\textit{invest}\text{''}) = 0.7,$$
$$u(\textit{signal}=\text{``}\textit{no opinion}\text{''}) = 0.5,$$
$$u(\textit{signal}=\text{``}\textit{not invest}\text{''}) = 0.7,$$
$$u(\textit{signal}=\text{``}\textit{never invest}\text{''}) = 0.8.$$

Hence, the expected utility of receiving a signal from Adviosr B is

$$\begin{aligned}
&0.2u(\textit{signal}=\text{``}\textit{must invest}\text{''}) + 0.2u(\textit{signal}=\text{``}\textit{invest}\text{''}) + 0.2u(\textit{signal}=\text{``}\textit{no opinion}\text{''}) \\
&+ 0.2u(\textit{signal}=\text{``}\textit{not invest}\text{''}) + 0.2u(\textit{signal}=\text{``}\textit{never invest}\text{''}) \\
&= 0.2 * 0.8 + 0.2 * 0.7 + 0.2 * 0.5 + 0.2 * 0.7 + 0.2 * 0.8 = 0.7.
\end{aligned}$$

A.1.2 Expected Utility

When the cost of the signal is c, the expected utility of each lottery is,

$$\begin{aligned}
U_{EU}(L_1(c)) &= 0.7u(100-c) \\
U_{EU}(L_2(c)) &= \frac{2}{3}(0.8u(100-c) + 0.2u(-c)) + \frac{1}{3}(0.5u(100-c) + 0.5u(-c)), \\
U_{EU}(L_3(c)) &= \frac{1}{2}(0.8u(100-c) + 0.2u(-c)) + \frac{1}{2}(0.6u(100-c) + 0.6u(-c)), \\
U_{EU}(L_4(c)) &= \frac{2}{5}(0.8u(100-c) + 0.2u(-c)) + \frac{2}{5}(0.7u(100-c) + 0.3u(-c))) \\
&+ \frac{1}{5}(0.5u(100-c) + 0.5u(-c)).
\end{aligned}$$

Similarly,

$$\begin{aligned}
U_{EU}(L_1(0)) &= 0.7u(100) \\
U_{EU}(L_2(0)) &= \frac{2}{3}(0.8u(100)) + \frac{1}{3}(0.5u(100)), \\
U_{EU}(L_3(0)) &= \frac{1}{2}(0.8u(100)) + \frac{1}{2}(0.6u(100)), \\
U_{EU}(L_4(0)) &= \frac{2}{5}(0.8u(100)) + \frac{2}{5}(0.7u(100)) + \frac{1}{5}(0.5u(100)).
\end{aligned}$$

A.1.3 Recursive Smooth Ambiguity Preference

$V_1(c) \geq V_4(c)$ can be derived by the following procedure:

$$\begin{aligned}
U_{KMM}(L_1(c)) &= \phi(U(0.7)) \\
&\geq \frac{2}{5}\phi(U(0.8)) + \frac{2}{5}\phi(U(0.7)) + \frac{1}{5}\phi(U(0.5)) \\
&= U_{KMM}(L_4(c)).
\end{aligned}$$

Also, $V_3(c) \geq V_2(c)$:

$$\begin{aligned}
U_{KMM}(L_3(c)) &= \frac{1}{2}\phi(U(0.8)) + \frac{1}{2}\phi(U(0.6)) \\
&= \frac{1}{2}\phi(U(0.8)) + \frac{1}{2}\phi(\frac{1}{3}U(0.8) + \frac{2}{3}U(0.5)) \\
&\geq \frac{1}{2}\phi(U(0.8)) + \frac{1}{6}\phi(U(0.8)) + \frac{1}{3}\phi(U(0.5)) \\
&\geq \frac{2}{3}\phi(U(0.8)) + \frac{1}{3}\phi(U(0.5)) \\
&= U_{KMM}(L_2(c)),
\end{aligned}$$

Similarly, $V_4(c) \geq V_2(c)$:

$$\begin{aligned}
U_{KMM}(L_4(c)) &= \frac{2}{5}\phi(U(0.8)) + \frac{2}{5}\phi(U(0.7)) + \frac{1}{5}\phi(U(0.5)) \\
&= \frac{2}{5}\phi(U(0.8)) + \frac{2}{5}\phi(\frac{2}{3}U(0.8) + \frac{1}{3}U(0.5)) + \frac{1}{5}\phi(U(0.5)) \\
&\geq \frac{2}{5}\phi(U(0.8)) + \frac{4}{15}\phi(U(0.8)) + \frac{2}{15}\phi(U(0.5)) + \frac{1}{5}\phi(U(0.5)) \\
&= \frac{2}{3}\phi(U(0.8)) + \frac{1}{3}\phi(U(0.5)) \\
&= U_{KMM}(L_2(c)),
\end{aligned}$$