Does the Size of the Signal Space Matter?

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Example

- An investor is contemplating whether or not to invest in a company, which might be good or bad.
- She thinks $Pr(G) = Pr(B) = 0.5$
- $U(\text{INVEST}|G) = U(\text{NOT INVEST}|B) = 1$
  $U(\text{INVEST}|B) = U(\text{NOT INVEST}|G) = 0$

Example

Advisor	Possible Signals	$Pr(G\|signal)$	Signal Space Size
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- *Who is a better advisor?*
  - Both advisors' (ex-ante) signal accuracies are the same.

$$\begin{aligned} EU_A &= EU(\text{INVEST}|\text{"invest"})Pr(\text{"invest"}) \\ &\quad + EU(\text{NOT INVEST}|\text{"not invest"})Pr(\text{"not invest"}) \\ &= 0.7 = EU_B. \end{aligned}$$

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  - Does the size of the signal space affect the preference for the signal?

Motivation

- People pay for the information (financial advice, medical tests) for decision-making.
- They pay more when the information is more accurate.
- Do people care about the "format" of the information?
  - Do they value the size of the signal space even when this does not affect the signal accuracy?
  - No theoretical/empirical studies of signal transmission have considered the separate effects of signal space size.
- If they do, what is the rationale behind it?

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- People might not care.
  - Most of the theories
- So far, there has been no elicitation of the preference for the size of the signal space.

Overview

- This paper provides the first empirical evidence that the size of the signal space matters in information acquisition.
  - When subjects purchase informative signals about the result of compound lotteries, the demand for the signal increases as the size of the signal space increases, even though the signal accuracies are fixed.
- However, when the signal is free, subjects no longer prefer a larger signal space.
- Experimental results suggest what is a better design to deliver information.
- Leading theoretical frameworks cannot explain the experimental findings.

Contributions

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2. The context of information design (Kamenica and Gentzkow, 2011).
   - Without loss of generality, most theoretical studies of information design have restricted the sender's signal to be "straightforward."
   - However, the experimental findings of this paper suggest that the receiver might prefer the environment where the signal space is larger than the action space.

Study 1

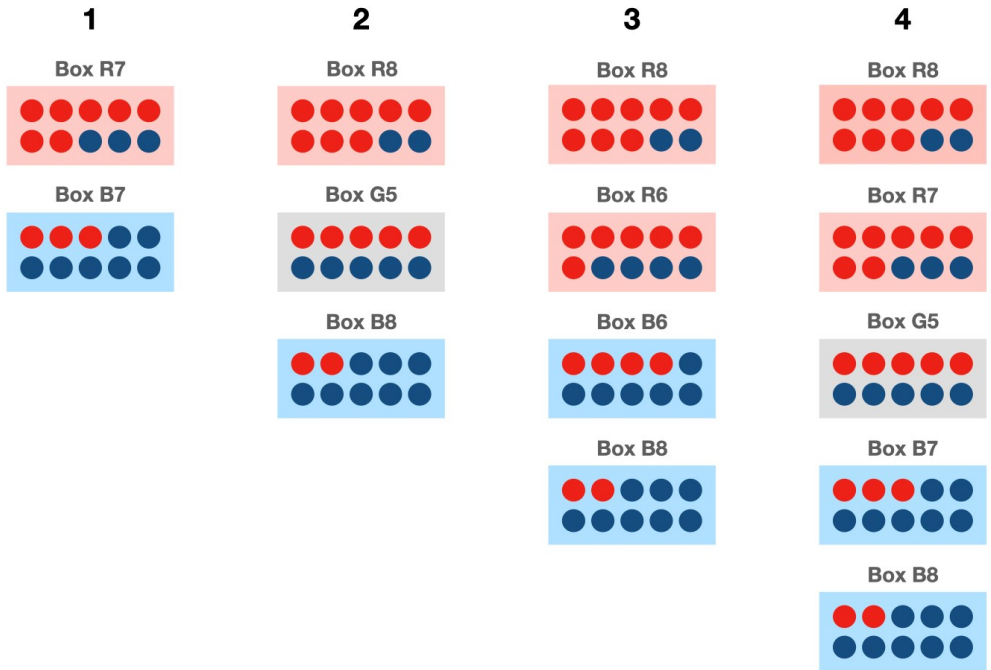
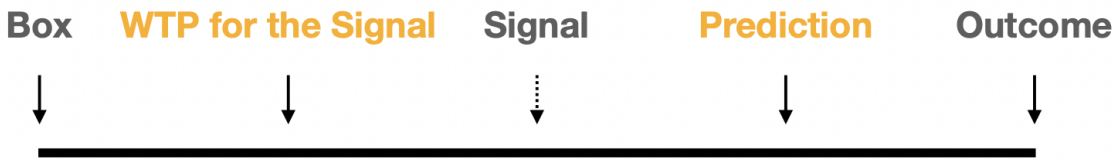


Figure: Four Lotteries

- In each lottery, the computer draws a ball through two stages.
  - Stage 1: One of the boxes is randomly selected.
  - Stage 2: A ball is drawn from the selected box.

Timeline of Study 1

Study 1



- Correct prediction $\Longrightarrow$ 100 points (=\$1)

Key Features and Hypotheses

1. Each lottery always has 50% red balls and 50% blue balls.
   - The prior, the winning probability without the signal is 50% across for all lotteries.
2. The signal accuracy of each lottery is the same.
   - If subjects purchase the signal, the winning probability increases to 70% across all four lotteries.

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Hypothesis 1

The size of the signal space does not affect the demand for the signal.

Result of Study 1

Table: Elicited values for c_i with different size of signal space.

Lottery	$\lvert S \rvert$	c_i	Number
1	2	23.6	179
2	3	24.9	179
3	4	25.8	179
4	5	29.8	179
Cuzick's test p-value		0.005	

Result 1 (Preference for a Larger Signal Space)

When purchasing signals, the willingness to pay for the signal increases as the signal space size increases.

- Result 1 rejects Hypothesis 1.

Possible Explanations

1. Subjects do not understand the meaning of signal accuracy.

Confusion?

- Is confusion the main reason for the preference for a larger space?
- Robustness Study: same procedure as in Study 1, but the signal of each lottery has different signal accuracy.
- The results show subjects' willingness to pay for the signal increases as the signal accuracy increases.

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- Is confusion the main reason for the preference for a larger space?
- Robustness Study: same procedure as in Study 1, but the signal of each lottery has different signal accuracy.
- The results show subjects' willingness to pay for the signal increases as the signal accuracy increases.
- Subjects understood the information structure correctly. No confusion!

Robustness Study

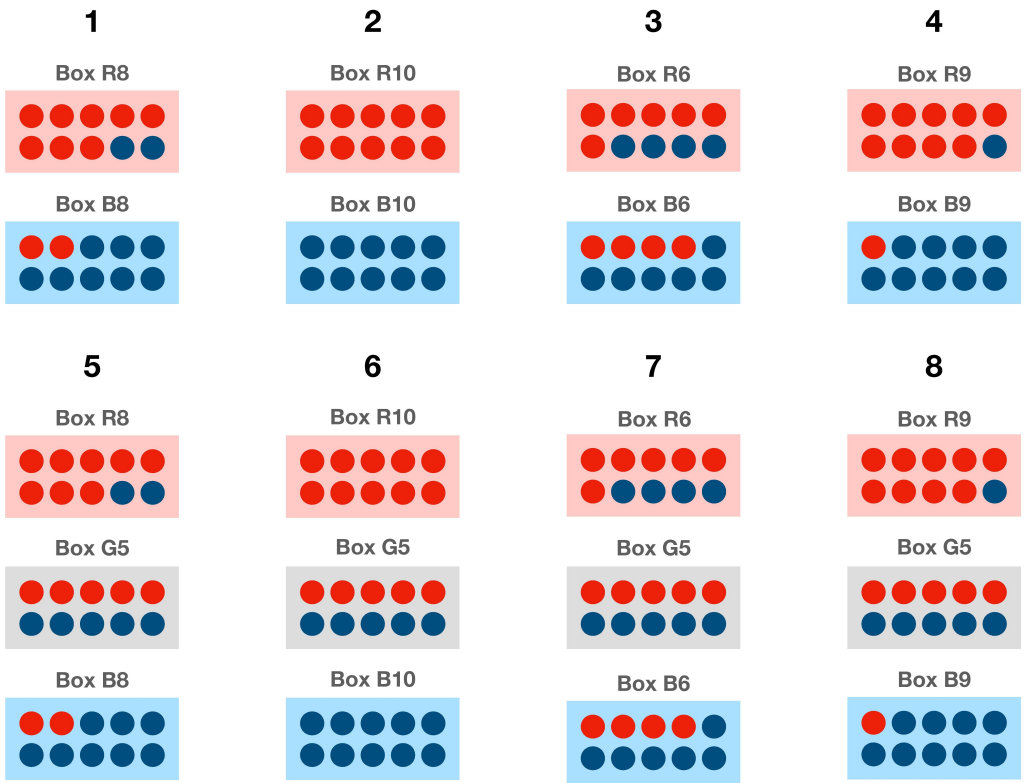


Figure: Lotteries in the Robustness Study

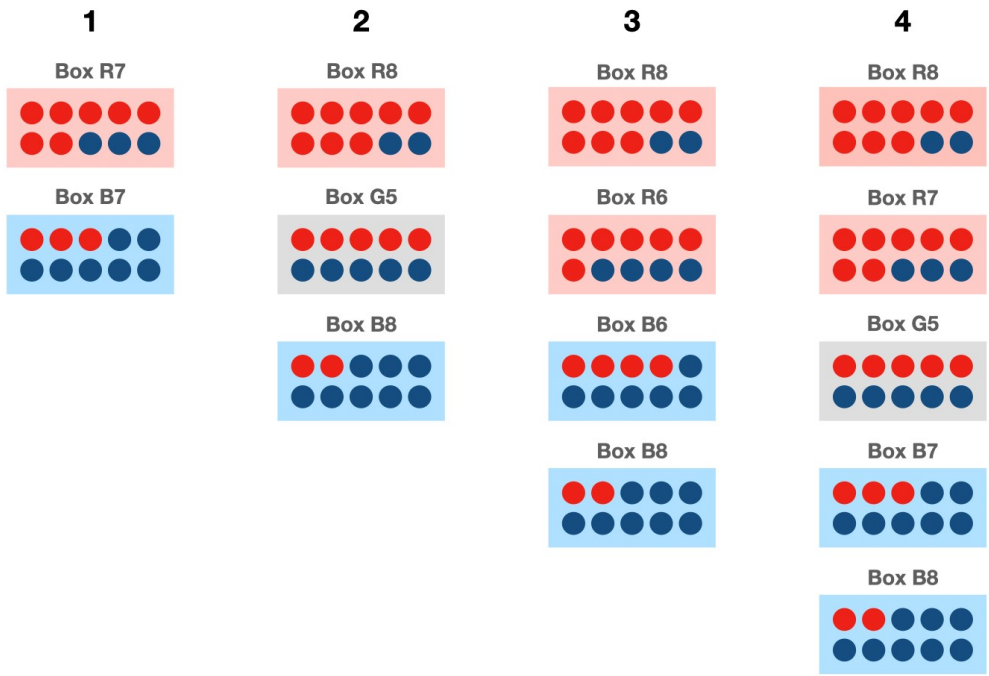
Possible Explanations

1. ~~Subjects do not understand the meaning of signal accuracy.~~
2. Subjects mistakenly believe larger signal space implies higher signal accuracy.

Misbelief on Signal Accuracy?

- Another explanation would be that subjects mistakenly believe larger signal space implies higher signal accuracy.
- Study 2 measures the values of four lotteries when the signals are free ($V_i(0)$).
- The WTP for playing each lottery is measured.
- If subjects believe larger signal space induces higher signal accuracy, the preference for larger signals should still exist.

Study 2



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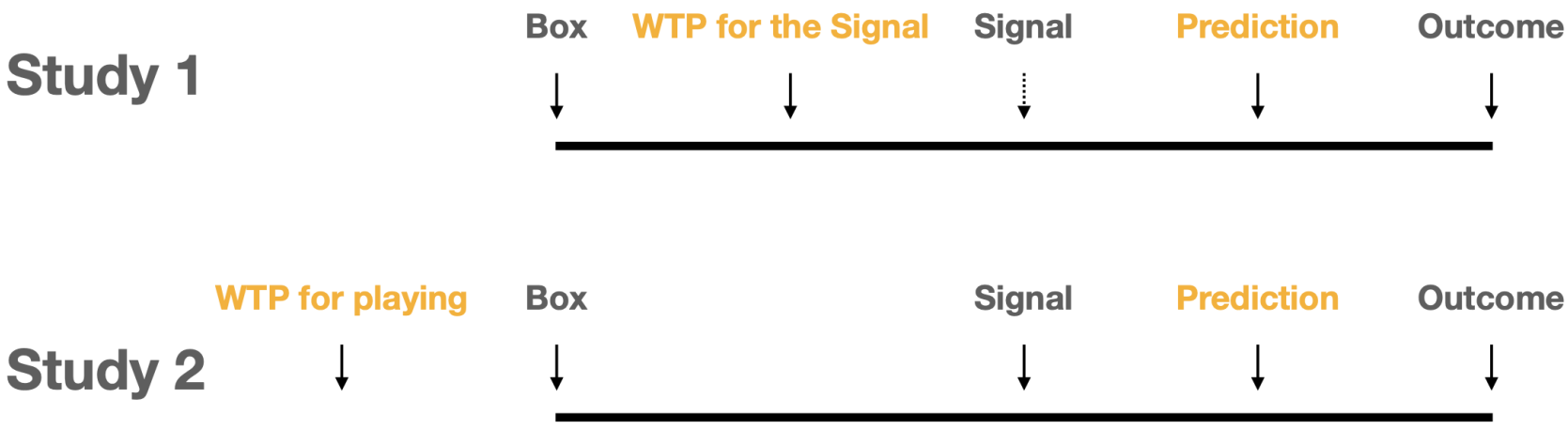
Timeline of Study 2



Procedures

- 467 subjects participated in experiments through Prolific.
- 179 and 158 subjects participated in studies 1 and 2 each.
- Also, another 130 subjects participated in a robustness study.
- On average, subjects spent 10 minutes and earned \$3.32, including \$2.20 of a base payment.

Key Features and Hypotheses

3. The information structures of Study 1 and Study 2 are isomorphic.

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Hypothesis 2

The rank among c_i are identical to the rank among $V_i(0)$.

Result of Study 2

Table: Elicited values for $V_i(0)$ with different size of signal space.

Lottery	$\|S\|$	$V_i(0)$	Number
1	2	52.9	158
2	3	48.9	158
3	4	51.0	158
4	5	52.7	158
Cuzick's test p-value		0.574	

Result 2 (Preference Reversal)

When the signal is free, subjects no longer prefer a larger signal space.

- Result 2 rejects Hypothesis 2.

Possible Explanations

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- When purchasing signals, individuals want to know the signal itself.
  - Instead of predicting the color of the ball, they focus on finding the "correct box."

Lottery	$\lvert S\rvert$	Prob to find the selected box without the signal
1	2	50%
2	3	33.3%
3	4	25%
4	5	20%

- When the signal space is larger, subjects pay more to uncover the uncertainty about the box.
- However, when subjects value the entire lottery, they have the ability to reduce the compound lottery: they know all lotteries are the same.

Theoretical Predictions

Model	Preferences among c_i
Expected Utility	$c_1 = c_2 = c_3 = c_4$
Recursive Smooth Ambiguity Utility	$c_1 \geq c_3 \geq c_2$ $c_1 \geq c_4 \geq c_2$
Rank-Dependent Utility	$c_2 \geq c_4 \geq c_3 \geq c_1$
Cumulative Prospect Theory	$c_1 \geq c_3 \geq c_4 \geq c_2$
Suspense and Surprise	$c_3 \geq c_4 \geq c_2 \geq c_1$

- Preference among c_i and among $V_i(0)$ are identical.
- **Prediction 1** Preference for a larger signal space does not exist.
- **Prediction 2** Preference reversal does not exist.
- Theoretical predictions are consistent with hypotheses.

Ambiguity Attitudes

- The preference for larger signal space (violation of Hypothesis 1) could be interpreted as a violation of the reduction of compound lottery axiom (ROCL).
- Halevy (2007) revealed ambiguity neutrality and reduction of compound lotteries are tightly associated.
- However, the results show the preference for larger signal space is not correlated with ambiguity neutrality.

Result 3

Ambiguity neutrality is not related to the preference for the signal space size.

Payoffs and the Size of the Signal Space

Table: Payoffs from part 1

Lottery Selected	Signal Space Size	Study 1 Payoff	Study 1 Std. Error	Study 1 Number	Study 2 Payoff	Study 2 Std. Error	Study 2 Number
1	2	160.9	7.1	48	71.6	6.2	32
2	3	141.6	7.5	50	80.0	4.5	43
3	4	140.9	7.0	46	67.0	6.2	40
4	5	142.0	8.2	35	72.8	5.4	43
	Total	146.7	3.7	179	73.1	2.8	158

Result 4

Subjects earned less profit when purchasing signals from a larger signal space.

Summary

- This article is the first to reveal the preference for a larger signal space in the information acquisition process.
- However, when the signals are free, the preference for the larger signal space vanishes.
- Leading theoretical frameworks cannot explain the experimental findings.

Thanks!

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https://sites.google.com/view/hyundamje