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Neutron Transport Calculation in Hexagonal Fuel Assembly by Transmission Probability Method

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Abstract We have made the numerical calculation model to solve the neutron integral transport equation in hexagonal fuel assembly by transmission probability method, and developed computational code TPHEX. The accuracy and efficiency of this numerical method were verified from the comparison with the numerical result of MCNP5 for the standard problem.

Key words hexagonal geometry, fuel assembly, transmission probability method

Introduction

The great leader Comrade **Kim Il Sung** said as follows.

"Many power stations using atomic energy and other new sources of power should be built to increase power output radically."("**KIM IL SUNG** WORKS" Vol. 35 P. 317)

Cores of most of fast breeding reactors and VVERs consist of hexagonal fuel assembly. Neutron transport calculation in hexagonal fuel assembly is investigated less than for the rectangular. Transmission probability method, one of integral transport methods, is widely applied in physics calculation on fuel assembly of nuclear reactor. The accuracy of calculation results is considerably related with approximation applied to spatial and angular distribution of neutron flux within and surfaces of homogenized cell.

Preceding studies[1－4] assumed that the spatial distribution of neutron flux within cell is linear or quadratic, but at surfaces of cell is constant. And response matrix method was used to get expansion factors of spatial distribution equation, so consumed a considerable computational time.

In this paper the interior neutron flux and source within homogenized hexagonal cells which constitute fuel assembly are assumed as to be quadratic spatial distribution and isotropic angular distribution. At cell surfaces the simplified P_1 approximation for anisotropic angular distribution and quadratic spatial distribution are considered.

And by introducing co-ordinates of rotation transform, the expansion factors were represented with the average neutron fluxes at the inside region and boundaries of cell, so that efficiency of calculation was enhanced. Based on this model, the computer code TPHEX which calculates spatial-energy distribution and homogeneous parameters of hexagonal fuel assembly is developed.

1. Calculation Model

Hexagonal fuel assembly is divided by homogenized hexagonal cells. Neutron multi-group integral transport equation in individual cell is as follows.

$$\phi_g(r,\Omega)=\int_0^{l_0} Q_g(r',\Omega)\exp\left(-\Sigma_{tg}l/\cos\theta\right)\frac{dl}{\cos\theta}+\phi_g(r_s,\Omega)\exp\left(-\Sigma_{tg}l_0/\cos\theta\right) \tag{1}$$

Surfaces of each hexagonal cell in assembly are assigned numbers as Fig. 1 a).

For $k=1,\cdots,6$ surface of cell, new rectangular co-ordinates, $(u,\ v)_s$ provided by counterclockwise rotation transform of Cartesian co-ordinates as $(k-1)\cdot\pi/3$, are introduced(Fig. 1. b)).

For interior neutron flux and source within homogenized hexagonal cell, their spatial and angular distributions in new coordinates are represented as follows.

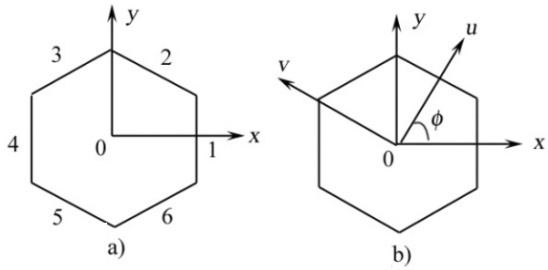

Fig. 1. Assignment of number to surfaces in hexagonal cell and rotation transformation of co-ordinates.

$$\phi_g(u,\ v,\ \Omega)=\frac{1}{4\pi}\left(\phi_0^g+\phi_u^g\frac{u}{a}+\phi_v^g\frac{v}{a}+\phi_{u^2}^g\frac{u^2}{a^2}+\phi_{v^2}^g\frac{v^2}{a^2}+\phi_{uv}^g\frac{uv}{a^2}\right) \tag{2}$$

$$Q_g(u,\ v,\ \Omega)=\frac{1}{4\pi}\left(Q_0^g+Q_u^g\frac{u}{a}+Q_v^g\frac{v}{a}+Q_{u^2}^g\frac{u^2}{a^2}+Q_{v^2}^g\frac{v^2}{a^2}+Q_{uv}^g\frac{uv}{a}\right) \tag{3}$$

where a is the length of a side in hexagonal cell.

Let us integrate both sides of equation (2) over the inside region and surfaces of cell and rewrite. Then expansion factors are represented by the average values at the inside region and boundaries. In case of source, its expansion coefficients are written by ones of neutron flux as follows.

$$Q_\mu^g=\sum_{g'=1}^{G}\left[\Sigma_{g'\to g}+\frac{\chi_g}{k}(\nu\Sigma_f)_{g'}\right]\phi_\mu^{g'},\quad \mu=0,u,v,u^2,v^2,uv \tag{4}$$

On the other hand, spatial and angular distributions of neutron flux at each boundary of cell are respectively assumed by quadratic and simplified P_1 approximations as follows.

$$\phi_k^g(r_s,\ \Omega)=\bar{\phi}_k^g\left(C_{k_0}^g+C_{k_1}^g\frac{v}{a}+C_{k_2}^g\frac{v^2}{a^2}\right)\frac{1}{4\pi}(f_0^g+3f_1^g\cos\theta\cos\varphi) \tag{5}$$

where

$$\left.\begin{aligned} &C_{k_0}^g=(\phi_0^g+\sqrt{3}\phi_u^g/2+\phi_{u^2}^g/4)/\bar{\phi}_k^g,\quad C_{k_1}^g=\phi_v^g+\sqrt{3}\phi_{uv}^g/2/\bar{\phi}_k^g,\\ &C_{k_2}^g=\phi_{v^2}^g/\bar{\phi}_k^g,\qquad f_0^g=1,\qquad f_1^g=\bar{j}_k^g/\bar{\phi}_k^g \end{aligned}\right\} \tag{6}$$

Let us substitute the expansions (3) and (5) into equations (1), and multiply both sides of them by $(\Omega\cdot n^+)\mathrm{d}\Omega\mathrm{d}S_n$.

Then, integrating and rewriting equations, the following equation will be found.

$$J_k^{g+} = V\overline{Q}^g E_k^g + \sum_{k' \neq k} J_k^{g-} T_{k',\,k}^g, \quad k,\, k' = \overline{1,\, 6}, \quad g = \overline{1,\, G} \tag{7}$$

where the escape and transmission probabilities of neutron, E_k^g and $T_{k',\,k}^g$, are formed as follows.

$$E_k^g = \frac{1}{\overline{Q}^g}[Q_0^g E_k^g(1) + Q_0^g E_k^g(u/a) + Q_0^g E_k^g(u^2/a^2) + Q_0^g E_k^g(v^2/a^2)] \tag{8}$$

In derivation of equation (7), we considered that $E_k^g(v/a) = 0$ and $E_k^g(uv/a^2) = 0$.

$$T_{k',\,k}^g = \frac{J_k^{g-} + J_k^{g+}}{2J_k^{g-}} T_{k',\,k}^{g(0)} + \frac{3(J_k^{g-} - J_k^{g+})}{4J_k^{g-}} T_{k',\,k}^{g(1)}, \quad k,\, k' = \overline{1,\, 6},\ g = \overline{1,\, G} \tag{9}$$

$$T_{k',\,k}^{g(l')} = C_{k',\,0}^g T_{k',\,k}^{g(l')}(1) + C_{k',\,1}^g T_{k',\,k}^{g(l')}(v'/a) + C_{k',\,2}^g T_{k',\,k}^{g(l')}(v'^2/a^2), \quad l' = 1,\, 2 \tag{10}$$

Each of terms in escape and transmission probabilities is as follows.

$$E_k^g(\bullet) = \frac{1}{4\pi V} \int_{S_k} ds \int (\Omega \cdot n_k^+) d\Omega \int_0^{l_0} (\bullet) \exp(-\Sigma_t^g / \cos\theta) dl / \cos\theta \tag{11}$$

$$T_{k',\,k}^{g(l')}(\bullet) = \frac{1}{\pi a} \int_{S_{k'}} ds' \int (\bullet) \exp(-\Sigma_t^g / \cos\theta)(\Omega \cdot n_{k'}^-)^{1+l'} d\Omega \tag{12}$$

As we can see, each of terms in spatial distributions of escape and transmission probabilities at all of surfaces will be represented by identical equations since rotation transform co-ordinates are applied from the geometric symmetry of hexagonal homogeneous cell, and probabilities have the only difference in expansion factors appropriate to the number of surface.

This method is much simpler and more efficient than the previous one using response matrix[1－4].

Neutron balance equation in cell is as follows.

$$\frac{1}{V} \sum_{k=1}^{6} (J_k^{g+} - J_k^{g-}) + \Sigma_t^g \overline{\phi}^g = \overline{Q}^g \tag{13}$$

The neutron average source of g group is formed by

$$\overline{Q}^g = \sum_{g'=1}^{G} [\Sigma_{g' \to g} + \frac{\chi_g}{k} (\nu\Sigma_f)_{g'}] \overline{\phi}^{g'} \tag{14}$$

Equations (7), (13) and (14) constitute a set of main equations in transmission probability method using simplified P_1 approximation.

Boundary conditions are as follows.

Interface conditions

$$\left.\begin{aligned} J_1^{g+}(i,\, j) &= J_4^{g-}(i,\, j+1) \\ J_2^{g+}(i,\, j) &= J_5^{g-}(i+1,\, j) \\ J_3^{g+}(i,\, j) &= J_6^{g-}(i+1,\, j-1) \end{aligned}\right\} \tag{15}$$

Boundary conditions

$$J_k^{g-} = \beta(S_k) J_k^{g+}, \quad 0 \le \beta(S_k) \le 1 \tag{16}$$

The inner and outer iteration scheme is employed to solve equations (7), (13) and (14), so we can obtain multiplication factor, neutron flux distribution and power distribution, homogenized few group constants of hexagonal fuel assembly. Based on proposed model, we developed the computational code TPHEX which calculate neutron transport in hexagonal fuel assembly.

2. Numerical Results

To evaluate the accuracy of established computing system, calculation results of new method on the Benchmark problem for hexagonal assembly was compared with those obtained by Monte Carlo particle transport calculation code, MCNP5.

Configuration and main lattice parameters of Benchmark hexagonal assembly problem in [5] are shown in Fig. 2 and table 1.

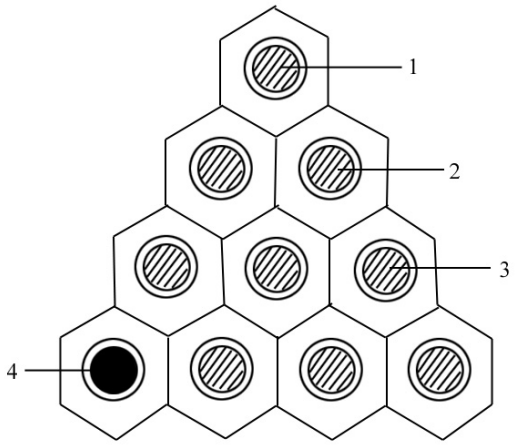

Fig. 2. 1/6 configuration of Benchmark problem for hexagonal fuel assembly
1– fuel(UO_2), 2– cladding(Zr), 3– moderator(H_2O), 4–fuel including burnable absorber(Gd)

Table 1. Main parameters of Benchmark problem for hexagonal fuel assembly

Lattice parameters	Nuclear density/($10^{24} \cdot cm^{-3}$)			
	Material	Nuclide symbol	Fuel cell	Burnable absorber cell
Cell step 12.75mm Diameter of fuel rod 7.72mm Outer diameter of clad 9.1mm Temperature Fuel rod 881K Cladding 610K Moderator(H_2O) 575K	fuel	^{235}U	$9 \cdot 10^{-4}$	$4 \cdot 10^{-4}$
		^{238}U	$2.03 \cdot 10^{-4}$	$2.08 \cdot 10^{-4}$
		^{16}O	$4.24 \cdot 10^{-4}$	$4.24 \cdot 10^{-4}$
		^{154}Gd	0	$2.79 \cdot 10^{-4}$
		^{155}Gd	0	$1.92 \cdot 10^{-4}$
		^{156}Gd	0	$2.66 \cdot 10^{-4}$
		^{157}Gd	0	$2.03 \cdot 10^{-4}$
		^{158}Gd	0	$3.23 \cdot 10^{-4}$
	clad	Zr	$4.23 \cdot 10^{-2}$	$4.23 \cdot 10^{-2}$
	moderator	H	$4.82 \cdot 10^{-2}$	$4.82 \cdot 10^{-2}$
		^{16}O	$2.41 \cdot 10^{-2}$	$2.41 \cdot 10^{-2}$

Table 2. Calculation results of k_∞ in hexagonal assembly

Code	k_∞	Relative deviation with MCNP5/%	Computing time/s
TPHEX	1.273 39	0.26	1.55
MCNP5 (skip 10, run a total of 70 cycles with 50 000 neutrons per cycle)	1.276 71±0.000 66		5 003

The results of table 2 show that the relative deviation with MCNP5 in k_∞ by TPHEX is below 0.3% and the computing time is very short. (The calculations were performed by the computer with 3GHz of clock and 512MByte of DDRAM.)

Conclusion

A method to solve neutron multigroup integral transport equation in hexagonal fuel assembly by transmission probability using spatial and angular distributions assumed respectively by quadratic and simplified P_1 approximations has been developed.

The numerical results of the Benchmark problem are compared with those of code MCNP5, and these are all in good agreement.

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