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The Propagation of the Plane Elastic Wave by Wavelet-Galerkin Method

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Abstract We induced basic related formula in the applying the Wavelet-Galerkin method(WGM) to the analysis of propagation of the plane elastic wave and determined basis functions satisfying boundary conditions in the free boundary. And we obtained detail calculations on the displacements, deformations, stress fields, reflection and interference effects in propagation of the plane P-SV wave, and compared with results in the FEM.

Key words Wavelet-Galerkin method, plane P-SV wave, basis function

Introduction

The great leader Comrade **Kim Il Sung** said as follows.

"**The fundamental question in scientific research is to keep developing science and technology in the direction required by our Party and our revolution, holding fast to the Juche stand.**"("**KIM IL SUNG** WORKS" Vol. 21 P. 450)

In the literature [4] was considered the analytical method used the variable separating and Furier method which are common integration means of the partial differential equation. In the [6, 7] common analytical method on the simplest one dimensional wave problem is considered with several fundamental problems in the applying the wave phenomenon of the wavelet approximate theory. In the [5] was introduced approximate solution method of the one dimensional wave equation used wavelet method. Here, unknown displacement is represented as variable separation on the time and coordinates and Daubeches wavelet scaling function is used as basis function, and multistep calculation diagram is made by direct substituting displacement representation to wave equation.

In the [1, 2] was considered free vibration analysis of the elastic cable by Wavelet-Galerkin method.

In this paper we have introduced a method of expression of the motion equation at the yards of displacement and speed of displacement and fundamental equation of the Wavelet-Galerkin method, and determined basis functions satisfying boundary condition at the free boundary.

And on the basis of analysis algorithm numerical calculation on the displacement, deformation, stress field and reflection, interference effect in propagation of the plane P-SV wave is preceded, and validity through comparison analysis with results of finite element method is demonstrated.

1. Fundamental Equation of the Wavelet-Galerkin Method in the Displacement-Displacement Velocity Field

In the Cartesian coordinate system motion equation of the plane P-SV wave is as follows [3].

$$\left.\begin{aligned}\frac{\partial^2 \bar{u}_{\bar{x}}}{\partial \bar{t}^2} &= \frac{1}{\rho}\left(\frac{\partial \bar{\sigma}_{\bar{x}\bar{x}}}{\partial \bar{x}} + \frac{\partial \bar{\sigma}_{\bar{x}\bar{z}}}{\partial \bar{x}} + \bar{f}_{\bar{x}}\right)\\ \frac{\partial^2 \bar{u}_{\bar{z}}}{\partial \bar{t}^2} &= \frac{1}{\rho}\left(\frac{\partial \bar{\sigma}_{\bar{x}\bar{z}}}{\partial \bar{x}} + \frac{\partial \bar{\sigma}_{\bar{z}\bar{z}}}{\partial \bar{z}} + \bar{f}_{\bar{z}}\right)\end{aligned}\right\} \tag{1}$$

where $\bar{u}_{\bar{x}},\ \bar{u}_{\bar{z}}$ are displacements of particle in the medium by P-SV waves of $\bar{x},\ \bar{z}$ directions respectively, and $\bar{\sigma}_{ij}\ (i,\ j=\bar{x},\ \bar{z})$ -components of stress tensor in $(\bar{x},\ \bar{z})$, ρ -density of medium, $\bar{f}_{\bar{x}},\ \bar{f}_{\bar{z}}$ -external perturbation force of a volume.

Introducing nondimensional coordinates $x=\frac{\bar{x}}{W},\ z=\frac{\bar{z}}{H},\ t=\frac{\bar{t}}{T},\ u=\frac{\bar{u}}{W},\ w=\frac{\bar{w}}{H},\ f_x=\frac{T^2\bar{f}_x}{\rho W},$ $f_z=\frac{T^2\bar{f}_z}{\rho H}$, under the assumption that considering medium is isotropy the equations (1) can be expressed in displacement - displacement speed field as follows.

$$\left.\begin{aligned}\frac{\partial u_x}{\partial t} = \upsilon_x,\ \ \frac{\partial \upsilon_x}{\partial t} &= L_{xx}u_x + L_{xz}u_z + f_x\\ \frac{\partial u_z}{\partial t} = \upsilon_z,\ \ \frac{\partial \upsilon_z}{\partial t} &= L_{zx}u_x + L_{zz}u_z + f_z\end{aligned}\right\} \tag{2}$$

where $W,\ H$ -geometrical sizes of plane region, $L_{\alpha\beta}\ (\alpha,\ \beta=x,\ z)$ -operator being expressed as follows.

$$L_{xx}=V_1^2\frac{\partial^2}{\partial x^2}+V_2^2\frac{\partial^2}{\partial z^2},\ \ L_{xz}=V_3^2\frac{\partial^2}{\partial x\partial z},\ \ L_{zx}=V_4^2\frac{\partial^2}{\partial x\partial z},\ \ L_{zz}=V_5^2\frac{\partial^2}{\partial x^2}+V_6^2\frac{\partial^2}{\partial z^2} \tag{3}$$

where

$$V_1^2=\frac{\lambda+2\mu}{\rho}\cdot\frac{T^2}{W^2},\ \ V_2^2=\frac{\mu}{\rho}\cdot\frac{T^2}{H^2},\ \ V_3^2=\frac{\lambda+\mu}{\rho}\cdot\frac{T^2}{W^2},$$

$$V_4^2=\frac{\lambda+\mu}{\rho}\cdot\frac{T^2}{H^2},\ \ V_5^2=\frac{\mu}{\rho}\cdot\frac{T^2}{W^2},\ \ V_6^2=\frac{\lambda+2\mu}{\rho}\cdot\frac{T^2}{H^2}$$

are nondimensional quantities connected with propagation speed of the wave respectively.

Equation (2) can be written as follows;

$$\partial_t U = LU + F \tag{4}$$

where $U=\begin{Bmatrix}u_x\\ \upsilon_x\\ u_z\\ \upsilon_z\end{Bmatrix},\ \ L=\begin{pmatrix}0&1&0&0\\ L_{xx}&0&L_{xz}&0\\ 0&0&0&1\\ L_{zx}&0&L_{zz}&0\end{pmatrix},\ F=\begin{Bmatrix}0\\ f_x\\ 0\\ f_z\end{Bmatrix}.$

Solution of the equation (4) with initial condition $U(x,\ t_0)=U_0(x)$, $0\le x\le 1$ may be expressed as follows.

$$U(x,\ t)=e^{(t-t_0)L}U_0(x)+\int_{t_0}^{t}e^{(t-\tau)L}F(x,\ \tau)d\tau \qquad (5)$$

From this the equation (5) is expressed following repeat calculation scheme.

$$U_n=e^{\delta tL}U_{n-1}+\delta t\beta_0F_{n-1} \qquad (6)$$

where δt is time interval as $\delta t=t_1-t_0=t_2-t_1=\cdots=t_n-t_{n-1}$, U_n -value of $U(x,\ t)$ in time point $t_n=t_0+n\delta t$, F_{n-1} -impact force vector having constant value between t_{n-1} and t_n and $\beta_0=(e^{\delta tL}-1)(\delta tL)^{-1}$.

Neglecting high order infinitesimal over 4^{th} degree and describing as matrix is follows.

$$\{U_{n+1}\}=[L_u]\{U_n\}+\delta t[L_f]\{F_n\} \qquad (7)$$

where $[L_u]$, $[L_f]$ are function matrixes of L_{xx}, L_{xz}, L_{zx}, L_{zz} .

We assume that displacement, displacement speed in direction x, z is described by equal basis functions in the function space V_m .

$$U(x,\ z,\ t)=\sum_{k=1}^{p}\sum_{l=1}^{p}U_{k,\ l}(t)\varphi_{m,\ k-n1}(x)\varphi_{m,\ l-n1}(z),\ \ n1=2N-1,\ \ p=2^m+2N-2 \qquad (8)$$

where N -vanishing moment degree, m - scaling number.

Let`s apply the wavelet-Galerkin method to equation (7), then

$$\iint_{D^2}[\{U_{n+1}\}-[L_u]\{U_n\}-\delta t[L_f]\{F_n\}]\varphi_{m,\ k-n1}(x)\varphi_{m,\ l-n1}(z)dxdz=0 . \qquad (9)$$

where $D^2=[0,\ 1]\times[0,\ 1]$ -considering region propagating elastic wave.

In equation(9) integration is made as approximate calculation scheme by using numerical integration formula.

In the calculation process follow relation formula are used.

$$\int_0^1\varphi_{m,\ l-n1}(x)\varphi_{m,\ k-n1}(x)dx=\begin{cases}0:k\ne l\\1:k=l\end{cases},\quad \int_0^1\frac{\partial^n}{\partial x^n}\delta(x-x_0)\varphi_{m,\ k-n1}(x)dx=(-1)^n\varphi^{(n)}{}_{m,\ k-n1}(x_0)$$

2. Determination of the Basic Function Satisfying Free Boundary Condition

Solution form of the x, z and y direction by plane P-SV wave may be expressed by using two dimensional wavelet scaling function.

In this case we assume that $u_x=u$ and $u_z=w$ are represented as same basis functions in the function space V_m, and solution form is expressed as follows

$$u(x,\ z,\ t)=\sum_{k=1}^{p}\sum_{l=1}^{p}u_{k,\ l}(t)\varphi_{m,\ k-n1}(x)\varphi_{m,\ l-n1}(z),\ \ w(x,\ z,\ t)=\sum_{k=1}^{p}\sum_{l=1}^{p}w_{k,\ l}(t)\varphi_{m,\ k-n1}(x)\varphi_{m,\ l-n1}(z) .$$

In the free boundary (the earth surface) boundary condition can be written as;

$$[\sigma_{i\bar{z}}]_{\bar{z}=0}=0,\ \ i=\bar{x},\ \bar{y},\ \bar{z}$$

where $\sigma_{\overline{xz}}$ and $\sigma_{\overline{zz}}$ by non-dimensions are as follows.

$$\sigma_{xz} = c_1\left(\frac{\partial u}{\partial z} + \frac{1}{c_1^2}\frac{\partial w}{\partial x}\right),\ \ \sigma_{zz} = c_2\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z}, \tag{10}$$

σ_{xz}, σ_{zz}, c_1, c_2 can be written respectively

$$\sigma_{xz} = \bar{\sigma}_{\overline{xz}}/\mu,\ \ \sigma_{zz} = \bar{\sigma}_{\overline{zz}}/(\lambda + 2\mu),\ \ c_1 = W/H,\ \ c_2 = \lambda/(\lambda + 2\mu).$$

From (9) by representing $u_{1,1}(t)$, $w_{1,1}(t)$ respectively, displacement formula of the x, z directions considering boundary condition are obtained .

$$u(x,\ z,\ t) = u_{1,2}(t)\varphi_3 + \sum_{k=2}^{p}\sum_{l=2}^{p}(u_{k,l}(t)\varphi_{m,\ k-n1}(x)\bar{\varphi}_{m,\ k-n1}(z) +$$
$$+ (w_{k,l}(t)Q_{k,l}(x,\ z=0) + w_{2,1}(t)\varphi_4 + w_{1,2}(t)\varphi_5)/c_1^2)$$

$$w(x,\ z,\ t) = w_{1,2}(t)\varphi_3 + \sum_{k=2}^{p}\sum_{l=2}^{p}(w_{k,l}(t)\varphi_{m,\ k-n1}(x)\bar{\varphi}_{m,\ k-n1}(z) +$$
$$+ c_2(u_{k,l}(t)Q_{k,l}(x,\ z=0) + u_{2,1}(t)\varphi_4 + u_{1,2}(t)\varphi_5))$$

where φ_3, φ_4, φ_5, $Q_{k,\ l}$, $\bar{\varphi}_{m,\ l-n1}(z)$ are as;

$$\varphi_3 = \varphi_{m,\ 1-n1}(x)[\varphi(n1-2) - \varphi_1\cdot\varphi(n1-1)],\ \ \varphi_4 = \frac{\varphi^2(n1-1)}{\varphi'^2(n1-1)}\varphi_{m,\ 1-n1}(x)(1-\varphi_2),\ \ \varphi_5 = 1-\varphi_1,$$

$$\bar{\varphi}_{m,\ l-n1}(z) = \begin{cases}\varphi_{m,\ l-n1}(z) - \dfrac{\varphi(n1-1)}{\varphi'(n1-1)}\varphi(n1-l) - \dfrac{c_2}{c_1^2}\dfrac{\varphi'_{m,\ k-n1}(x)}{\varphi_{m,\ 1-n1}(x)}\cdot\dfrac{\varphi^2(n1-1)}{\varphi'^2(n1-1)}\varphi(n1-l), & 1 < l < n1\\ \varphi_{m,\ l-n1}(z), & p \ge l \ge n1\end{cases},$$

$$Q_{k,\ l}(x,\ z=0) = \varphi'_{m,\ k-n1}(x)\varphi(n1-l)\varphi(n1-1)/\varphi'(n1-1).$$

3. Calculation Example

When there exist two wave source s_1, s_2 in the plane region, we consider a change characteristics of displacement fields in the reception point R of the media (Fig. 1).

First, we consider the case that shock loading with the characteristics of strong local change apply to the wave source region given by

$$\bar{f}_{\bar{x}}(\bar{x},\ \bar{z},\ \bar{t}) = \bar{f}_{\bar{z}}(\bar{x},\ \bar{z},\ \bar{t}) = \sum_{i=1}^{n}\delta(\bar{x}-\bar{x}_i)\delta(\bar{z}-\bar{z}_i)\bar{f}(\bar{t}).$$

where $\bar{f}(\bar{t})$ is a source time function and $(\bar{x}_i,\ \bar{z}_i)$ is a source position, the time history of an impulsive excitation $\bar{f}(\bar{t})$ has been taken as; $\bar{f}(\bar{t}) = K\cdot\bar{t}\cdot\exp(-\omega(\bar{t}-t_0)^2)$.

We set $K = 2\cdot 10^{13}\,\mathrm{N/(m^2s)}$, $\omega = 3\cdot 10^3\,\mathrm{s^{-2}}$, $t_0 = 0.01$.

And other parameters use $\delta t = 0.04\mathrm{ms}$, $m = 8$, $N = 6$, $t = 1\mathrm{ms}$, $\rho = 2\,200\mathrm{kg/m^3}$ and $E = 1.728\cdot 10^4\mathrm{MPa}$, $\nu = 0.28$, $x_1 = z_2 = 0.25$, $x_2 = z_1 = 0.75$.

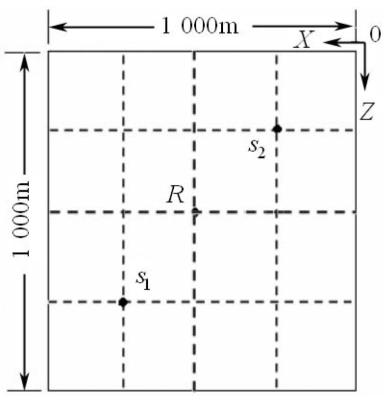

Fig. 1. Analysis model

In this calculations we treat the cases of action of the wave sources s_1 and s_2.

When only wave source s_1 exists, the longitudinal wave produced from a wave source arrives the recovers R with velocity of $\sqrt{\lambda+2\mu/\rho}=3.15(\text{km/s})$ after $t=1.122\text{s}$.

In this calculation, we have checked the vibration amplitude (greatest defection of particle from equilibrium position) from $t=1.122\text{s}$ to $t=1.116\text{s}$. Fig. 2 represents the analysis results of thesis's method and finite element method(ANSIS) and relative error is about 21%(number of element are 16 384 in the both of WGM and FEM).

Fig. 3 shows the case in which only wave source s_1 exists and the case in which both s_1 and s_2 exist.

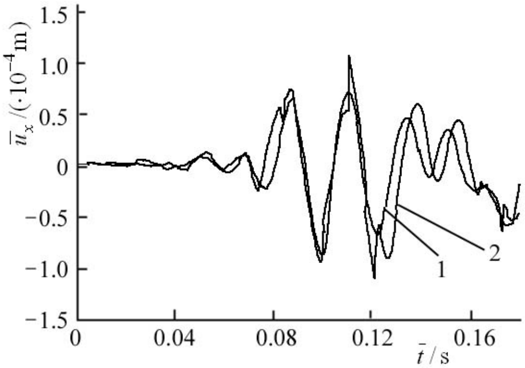

Fig. 2. Comparison curve when one wave source exists
1—WEM, 2—FEM

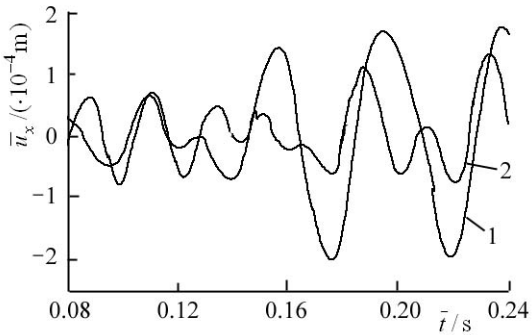

Fig. 3. Comparison of vibration curve at receive point R
1—Case that one wave source exists,
2—Case that two waves source exist

In this case amplitude of displacement increases in the place where both of compression regions and of rarity regions overlap each other, but decreases where compression region and rarity region overlap in the medium. When singular loads exist in both wave sources simultaneously, relative error of displacement values of x-direction determined by WGM with FEM in local time interval $(t=0.08\sim0.24\text{s})$ is 17% at maximum displacement point.

As seen above, the thesis's method strongly displayed the local change characteristics in the near of maximum, minimum points than the FEM.

Conclusion

We suggested a method of expressing motion equation of the plane P-SV elastic wave in displacement-displacement velocity field and derived main relation of wavelet-Galerkin method and determined basis function satisfying free boundary conditions.

From the computation results according to settled models when the loading having strong change characteristics and slow change characteristics, results of thesis's method and FEM have relative error of average 11.5, 4.15% according to two wave type respectively.

References

[1] M. Qassab et al.; Journal of Engineering Mechanics, 42, 3, 350, 2003.

[2] M. Qassab et al.; Journal of Sound and Vibration, 270, 4, 191, 2004.

[3] J. Zhong et al.; J. Int. Geophys, 136, 431, 1999.

[4] Kit Tung: Partial Differential Equations and Fourer Analysis-A short Introduction, University of Washington, 53～64, 2007.

[5] 吴爱弟 等; 天津职业技术师范大学学报, 11, 1, 2001.

[6] 谢长珍 等; 信阳师范学院学报(自然科学版), 10, 45, 1997.

[7] 袁修贵 等; 数学理论与应用, 23, 11, 2003.