

Determination on the Distribution of Temperature and Velocity of Nitrogen Plasma Flow by Lattice Boltzmann Method

Pae Song Gil

Abstract Because the most of plasma flows are turbulent in spraying, the distribution of temperature and velocity of plasma turbulent flow are determined correctly to increase the quality of spraying.

In previous studies [1–8], to study the plasma turbulent flow by using the lattice Boltzmann method, turbulent viscosity μ_t was assumed being linear relation with density ρ , and thus turbulent characteristics could not be accounted correctly.

We combined the lattice Boltzmann method with $k-\omega$ two equation turbulent models to obtain the distribution of temperature and velocity of nitrogen arc plasma flow.

Key words plasma, lattice Boltzmann method, $k-\omega$ model

1. Theoretical Consideration

The great leader Comrade **Kim Il Sung** said as follows.

“Efforts should be made to develop cell engineering, gene engineering, superhigh-pressure physics and ultracryogenics, exploit atomic, solar and other new sources of energy and study closely lasers and plasma so that these can be used extensively in the national economy.”(“KIM IL SUNG WORKS” Vol. 35 P.313)

Plasma flow used in plasma spraying is usually turbulent flow, because its temperature and velocity is very high.

The lattice Boltzmann equation is as follows [3].

$$\begin{aligned} f_i(\mathbf{x} + \mathbf{c}_i dt, t + dt) - f_i(\mathbf{x}, t) &= -\frac{dt}{\tau_f + 0.5dt} (f_i - f_i^e) \\ g_i(\mathbf{x} + \mathbf{c}_i dt, t + dt) - g_i(\mathbf{x}, t) &= -\frac{dt}{\tau_g + 0.5dt} (g_i - g_i^e) - \frac{dt\tau_g}{\tau_g + 0.5dt} z_i f_i \end{aligned} \quad (1)$$

where τ_f, τ_g are the relaxation times and f_i, g_i are density distribution function and density distribution function of inner energy and f_i^e, g_i^e are the equilibrium distribution function. z_i represents the effects of viscosity heating.

$$z_i = \frac{[\mathbf{c}_i - \mathbf{u}(\mathbf{x}, t)] \cdot [\mathbf{u}(\mathbf{x} + \mathbf{c}_i dt, t + dt) - \mathbf{u}(\mathbf{x}, t)]}{dt} \quad (2)$$

The two-dimensional square lattice with the nine speeds (D_2Q_9) is used (Fig. 1).

$$\mathbf{c}_0 = 0, \mathbf{c}_i = \left[\cos\left(\frac{i-1}{4}\pi\right), \sin\left(\frac{i-1}{4}\pi\right) \right] c, (i=1, 2, \dots, 8) \quad (3)$$

where $c^2 = 3RT$ and R , T are the gas constant and temperature.

The equilibrium distribution functions in lattice model are as follows.

$$f_i^e = w_i \rho \left[1 + \frac{3\mathbf{c}_i \cdot \mathbf{u}}{c^2} + \frac{9(\mathbf{c}_i \cdot \mathbf{u})^2}{2c^4} - \frac{3(u^2 + v^2)}{2c^2} \right], \quad (i=0, 1, \dots, 8) \quad (4)$$

$$g_0^e = -w_i \left[\frac{3\rho e(u^2 + v^2)}{2c^2} \right] \quad (5)$$

$$g_i^e = w_i \rho e \left[1.5 + \frac{3\mathbf{c}_i \cdot \mathbf{u}}{c^2} + \frac{9(\mathbf{c}_i \cdot \mathbf{u})^2}{2c^4} - \frac{3(u^2 + v^2)}{2c^2} \right], \quad (i=1, \dots, 4) \quad (6)$$

$$g_i^e = w_i \rho \left[3 + \frac{6\mathbf{c}_i \cdot \mathbf{u}}{c^2} + \frac{9(\mathbf{c}_i \cdot \mathbf{u})^2}{2c^4} - \frac{3(u^2 + v^2)}{2c^2} \right], \quad (i=5, \dots, 8) \quad (7)$$

where $\mathbf{u} = (u, v)$, $\rho e = \rho RT$. The weights coefficient of distribution function is

$$w_0 = \frac{4}{9}, \quad w_i = \begin{cases} \frac{1}{9}, & (i=1, 2, 3, 4) \\ \frac{1}{36}, & (i=5, 6, 7, 8) \end{cases}. \quad (8)$$

The density, velocity and temperature can be calculated by density distribution function and distribution function of inner energy.

$$\begin{aligned} \rho &= \sum_i f_i, \\ \rho \mathbf{u} &= \sum_i \mathbf{c}_i f_i, \quad \mathbf{q} = \left(\sum_i \mathbf{c}_i g_i \rho e \mathbf{u} - \frac{dt}{2} \sum_i \mathbf{c}_i f_i z_i \right) \frac{\tau_g}{\tau_g + 0.5dt}, \\ \rho e &= \sum_i g_i - \frac{dt}{2} \sum_i f_i z_i \end{aligned} \quad (9)$$

The kinetic viscosity and thermal diffusivity are given by $\nu_{eff} = \nu + \nu_t$, $\chi = 2\tau_g R\bar{T}$.

In this paper, the standard $k-\omega$ model is used to determine the turbulent viscosity. $k-\omega$ model is as follows.

$$\nu_t = \frac{k}{\omega} \quad (10)$$

where k is turbulent energy, ω —specific dissipation.

Turbulent energy equation:

$$\frac{\partial}{\partial x_j} \left[\rho u_j k - \left(\mu + \frac{\mu_t}{\sigma_k} \right) \frac{\partial k}{\partial x_j} \right] = \rho P_k - \rho \beta^* k \omega \quad (11)$$

Specific dissipation equation:

$$\frac{\partial}{\partial x_j} \left[\rho u_j \omega - \left(\mu + \frac{\mu_t}{\sigma_\omega} \right) \frac{\partial \omega}{\partial x_j} \right] = \rho \alpha \frac{\omega}{k} P_k - \rho \beta \omega^2 + \rho S_\omega \quad (12)$$

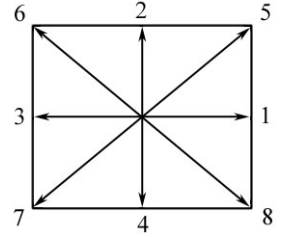


Fig. 1. D_2Q_9 lattice model

where $\alpha = \frac{5}{9}$, $\beta = \frac{3}{40}$, $\beta^* = \frac{9}{100}$, $\sigma_k = 2$, $\sigma_\omega = 2$, $S_\omega = 0$.

In this paper, we considered that the plasma flow is cylinder symmetric and thus calculated only up half-plane of plasma flow (Fig. 2).

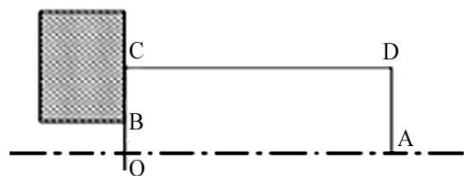


Fig. 2. Calculation domain of flow field

Calculation domain $OA \times OC = 130\text{mm} \times 15\text{mm}$ and lattice separation 2600×300 , where OB is velocity boundary, BC reflection boundary, CD, AD free boundary and OA symmetric boundary.

Boundary conditions are as follows.

Boundary OB : $u = u_0[1 - (r/R)^2]$, $v = 0$, $T = (T_0 - T_w)[1 - (r/R)^4] + T_w$, boundary BC : $u = 0, v = 0$, boundary CD : $u = 0, v = 0$.

2. Calculation Results

In usual, plasma flow is converted to turbulent flow in plasma torch when Re is over 300 [2].

In this paper, we studied the distribution of temperature and velocity of nitrogen plasma flow in Re=450 (Fig. 3, 4).

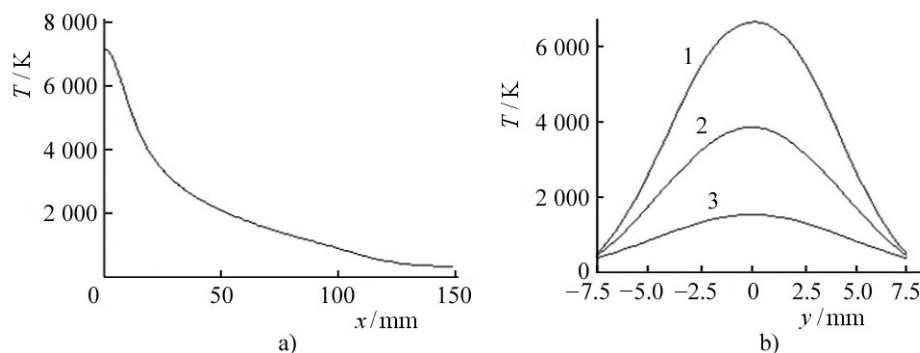


Fig. 3. The temperature distribution of nitrogen plasma flow

a) The temperature distribution along axial distance, b) The temperature distribution along radial distance; 1–3 are the case that x is 5, 20 and 70mm

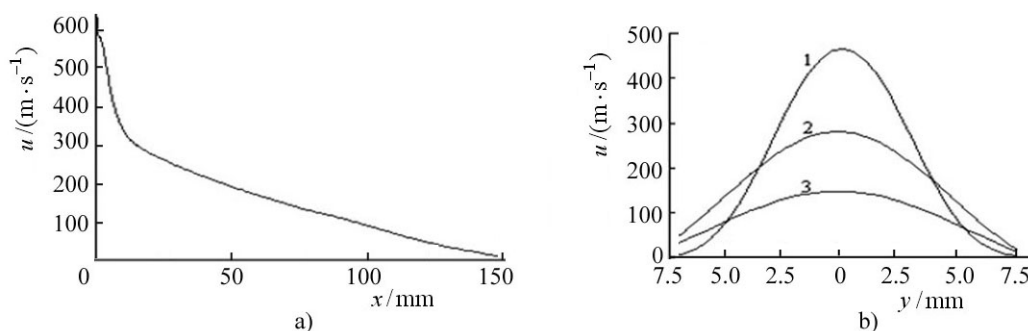


Fig. 4. The velocity distribution of nitrogen plasma flow

a) The velocity distribution along axial distance,
b) The velocity distribution along radial distance;
1–3 are equal to Fig. 3.

As shown in Fig. 3, 4, the distribution of temperature and velocity of nitrogen plasma flow decrease rapidly along axial distance. Radial distribution also decrease rapidly when far from axis and those tendency dominant when approach toward the nozzle of plasmatron.

Conclusion

The distribution of temperature and velocity of nitrogen arc plasma flow was studied by combining $k-\omega$ turbulent model with lattice Boltzmann method.

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