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# Improvement of a Loose Stability Theorem of TS Fuzzy System based on Moving Rate

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**Abstract** A risk stability theorem of fuzzy control system was proposed and proved with product fuzzy reasoning method on the continuous and discrete fuzzy systems. But a loose stability theorem of fuzzy control system was not proposed. In this paper we propose and prove of a loose stability theorem of the continuous and discrete TS fuzzy system based on moving rate.

**Key words** fuzzy reasoning, moving rate, continuous TS fuzzy system, discrete TS fuzzy system, loose stability condition

## Introduction

The great leader Comrade **Kim Il Sung** said as follows.

"**Now it is very important and extremely urgent to accelerate the development of our science and technology**."("**KIM IL SUNG** WORKS" Vol. 34 P. 148)

Fuzzy system is the information processing system based on fuzzy set theory such as fuzzy modeling identification system, fuzzy diagnosis system, fuzzy predicative system, fuzzy retrieval system, fuzzy control system and so on [7－9].

Stability problem is an important problem of the practical works. Previous research [7, 8] proposed stability problem of fuzzy systems based on degree of matching-type fuzzy reasoning. In the previous papers [1, 2, 12], is presented a disadvantage of the fuzzy reasoning methods based on composition rules of fuzzy reasoning and then proposed compensation principle of the fuzzy reasoning.

The characteristic improvement of fuzzy control [3, 9], fuzzy predicative modeling method [5, 6], an improvement of an reaning ability in the fuzzy neural network [9, 10], application in DDos attack detection [2, 11], research of a new recursive fuzzy reasoning method by move rate of membership function and it's application [3] and so on, application researches were presented based on a new concept of move rate.

In the previous research [1] is proposed and proved a risk stability condition of fuzzy system. But a loose stability theorem of fuzzy control system was not proposed. In this paper we propose and prove of a loose stability theorem of the continuous and discrete TS fuzzy system based on moving rate.

## 1. Stability Condition of the TS Fuzzy System

According to parallel distributed compensation principle [4] output of a TS fuzzy controller is as follows.

$$u(t)=\sum_{i=1}^{r}\omega_i(t)F_ix(t)\Big/\sum_{i=1}^{r}\omega_i(t) \tag{1}$$

Design of TS fuzzy controller is referred to solve parameter $F_i$ of formula (1). Then the whole model of TS fuzzy system can be representing on the continuous and discrete fuzzy systems.

$$\dot{x}(t)=\frac{\sum_{i=1}^{r}\sum_{j=1}^{r}\omega_i(t)\omega_j(t)\{A_i+B_iF_j\}x(t)}{\sum_{i=1}^{r}\sum_{j=1}^{r}\omega_i(t)\omega_j(t)} \tag{2}$$

$$x(k+1)=\frac{\sum_{i=1}^{r}\sum_{j=1}^{r}\omega_i(k)\omega_j(k)\{A_i+B_iF_j\}x(k)}{\sum_{i=1}^{r}\sum_{j=1}^{r}\omega_i(k)\omega_j(k)} \tag{3}$$

In equation (3), $\omega_i$, $\omega_j$ is degree of matching of fuzzy rules by compositional rules of fuzzy reasoning[5, 6].

The continuous fuzzy systems is globally asymptotically stable if there exists a common positive definite matrix $P$ such that formula (4) for $\forall i,\ j\in\{1,\ 2,\ \cdots,\ r\}$

$$\{A_i+B_iF_j\}^T P+P\{A_i+B_iF_i\}<0 \tag{4}$$

The discrete fuzzy systems is globally asymptotically stable if there exists a common positive definite matrix $P$ such that formula (5) for $\forall i,\ j\in\{1,\ 2,\ \cdots,\ r\}$.

$$\{A_i+B_iF_j\}^T P\{A_i+B_iF_j\}-P<0 \tag{5}$$

For known $A_i$, $B_i$, stability of the TS fuzzy control system is garrented using parameter of controller $F_j$ which satisfy formula (4) or (5).

## 2. Loose Stability Condition of the Discrete and Continuous Fuzzy System

Next, let's consider a loose stability condition.

Varying formula (2), (3) is derivated formula (6), (10). In this case applying the fuzzy reasoning method based on moving rate [2, 3, 4], a new two theorems are as follows.

**Theorem 1** Loose stability condition of the continuous fuzzy system based on moving rate

The continuous fuzzy system of formula [6] is as follows.

$$\dot{x}(t)=\frac{\sum_{i=j}^{r}d_i(t)d_i(t)\{A_i+B_iF_i\}x(t)}{\sum_{i=1}^{r}\sum_{j=1}^{r}d_i(t)d_j(t)}+\frac{\sum_{i<j}^{r}2d_i(t)d_j(t)\frac{\{A_i+B_iF_j\}+\{A_j+B_jF_i\}}{2}x(t)}{\sum_{i=1}^{r}\sum_{j=1}^{r}d_i(t)d_j(t)} \tag{6}$$

In formula [6], $d_i$, $d_j$ are moving rate based on the principle of compensation for fuzzy reasoning and $r$ is the number of fuzzy rules.

The continuous fuzzy system is globally asymptotically stable if there exists a common positive definite matrix $P$ which satisfy formula (7), (8).

$$\{A_i + B_iF_i\}^T P + P\{A_i + B_iF_i\} < 0 \tag{7}$$

$$G_{ij}{}^T P + PG_{ij} < 0 \quad , \quad i < j \tag{8}$$

$$G_{ij} = \frac{\{A_i + B_iF_j\} + \{A_j + B_jF_i\}}{2} \tag{9}$$

**Proof** Let's consider a critical function (Lyapunov function) which satisfy following 4 conditions.

① $V(0) = 0$ ,

② $V[x(t)] > 0,\ \ if\ \ x(t) = 0$ ,

③ $V[x(t)] \to \infty,\ \ if\ \ \|x(t)\| \to \infty$ ,

④ $V[x(t)] = x(t)^T Px(t)$

Let's consider variation of the critical function.

$$\dot{V}[x(t)] = \dot{x}(t)^T Px(t) + x(t)^T P\dot{x}(t) =$$

$$= x(t)^T \sum_{i=j}^{r} d_i'(t)d_i'(t)\{A_i + B_iF_i\}^T Px(t) + x(t)^T P\sum_{i=j}^{r} d_i'(t)d_i'(t)\{A_i + B_iF_i\}x(t) +$$

$$+ x(t)^T \sum_{i<j}^{r} 2d_i'(t)d_j'(t)\frac{\{A_i + B_iF_j\}^T + \{A_j + B_jF_i\}^T}{2} Px(t) +$$

$$+ x(t)^T P\sum_{i<j}^{r} 2d_i'(t)d_j'(t)\frac{\{A_i + B_iF_j\} + \{A_j + B_jF_i\}}{2} x(t) =$$

$$= x(t)^T \sum_{i=j}^{r} d_i'(t)d_i'(t)[\{A_i + B_iF_i\}^T P + P\{A_i + B_iF_i\}]x(t) +$$

$$+ x(t)^T \sum_{i<j}^{r} 2d_i'(t)d_j'(t)[G_{ij}{}^T P + PG_{ij}]x(t) < 0$$

Since $\dot{V}[x(t)] < 0$ , globally asymptotical stability of the continuous TS fuzzy system based on moving rate is guaranteed. (proof end)

Repairing formula (3), the discrete fuzzy control system based on moving rate can be represented to formula (10).

$$x(k+1) = \frac{\sum_{i=j}^{r} d_i(k)d_i(k)\{A_i + B_iF_i\}x(k)}{\sum_{i=1}^{r}\sum_{j=1}^{r} d_i(k)d_j(k)} + \frac{\sum_{i<j}^{r} 2d_i(k)d_j(k)\frac{\{A_i + B_iF_j\} + \{A_j + B_jF_i\}}{2}x(k)}{\sum_{i=1}^{r}\sum_{j=1}^{r} d_i(k)d_j(k)} \tag{10}$$

In formula [10], as same as theorem 1, $d_i$ , $d_j$ are moving rate based on the principle of compensation for fuzzy reasoning based on [3, 4, 11－14] and $r$ is the number of fuzzy rules.

**Theorem 2** Loose stability condition of the discrete fuzzy system based on moving rate

The discrete TS fuzzy system is globally asymptotically stable if there exists a common positive definite matrix $P$ which satisfy formula (11), (12) for $\forall i,\ j \in \{1,\ 2,\ \cdots,\ r\}$ .

$$\{A_i + B_iF_i\}^T P\{A_i + B_iF_i\} - P < 0 \tag{11}$$

$$G_{ij}^T PG_{ij} - P < 0, \quad i < j \tag{12}$$

In formula [12], $G_{ij}$ is equal to formula (9).

**Proof** Let's consider a critical function (a discrete Lyapunov function) which satisfy following 4 conditions.

① $V(0)=0$,

② $V[x(k)]>0,\ if\ \ x(k)=0$,

③ $V[x(k)]\to\infty,\ if\ \ \|x(k)\|\to\infty$,

④ $V[x(k)]=x(k)^T Px(k)$

Let's consider variation of the critical function.

$\Delta V[x(k)]=V[x(k+1)]-V[x(k)]=$

$$=\left[\frac{\sum_{i=j}^{r} d_i(k)d_i(k)\{A_i+B_iF_i\}x(k)}{\sum_{i=1}^{r}\sum_{j=1}^{r} d_i(k)d_j(k)}+\frac{\sum_{i<j}^{r} 2d_i(k)d_j(k)\frac{\{A_i+B_iF_j\}+\{A_j+B_jF_i\}}{2}x(k)}{\sum_{i=1}^{r}\sum_{j=1}^{r} d_i(k)d_j(k)}\right]^T\cdot P\cdot$$

$$\cdot\left[\frac{\sum_{i=j}^{r} d_i(k)d_i(k)\{A_i+B_iF_i\}x(k)}{\sum_{i=1}^{r}\sum_{j=1}^{r} d_i(k)d_j(k)}+\frac{\sum_{i<j}^{r} 2d_i(k)d_j(k)\frac{\{A_i+B_iF_j\}+\{A_j+B_jF_i\}}{2}x(k)}{\sum_{i=1}^{r}\sum_{j=1}^{r} d_i(k)d_j(k)}\right]-$$

$$-x(k)^T Px(k)=x(k)^T\left[\sum_{i=j}^{r} d_i(k)d_i(k)M_{ii}+\sum_{i<j}^{r} 2d_i(k)d_j(k)G_{ij}\right]^T\cdot P\cdot$$

$$\cdot\left[\sum_{i=j}^{r} d_i(k)d_i(k)M_{ii}+\sum_{i<j}^{r} 2d_i(k)d_j(k)G_{ij}\right]x(k)-\left[\sum_{i=1}^{r}\sum_{j=1}^{r} d_i(k)d_j(k)\right]x(k)^T Px(k)$$

For simple, let's $d_i=d_i(k),\ M_{ij}=\{A_i+B_iF_j\}$,

$$\Delta V[x(k)]=x(k)^T\left[\sum_{i=j}^{r}\sum_{k=l}^{r} d_i^2d_k^2M_{ii}^TPM_{kk}+\sum_{i=j}^{r}\sum_{k<l}^{r} 2d_i^2d_kd_lM_{ii}^TPG_{kl}+\sum_{i<j}^{r}\sum_{k=l}^{r} 2d_id_jd_k^2G_{ij}^TPM_{kk}+\right.$$

$$+\sum_{i<j}^{r}\sum_{k<l}^{r} 4d_id_jd_kd_lG_{ij}^TPG_{kl}-\sum_{i=j}^{r}\sum_{k=l}^{r} d_i^2d_k^2P-\sum_{i=j}^{r}\sum_{k<l}^{r} 2d_i^2d_kd_lP-\sum_{i<j}^{r}\sum_{k=l}^{r} 2d_id_jd_k^2P-$$

$$\left.-\sum_{i<j}^{r}\sum_{k<l}^{r} 4d_id_jd_kd_lP\right]x(k)=x(k)^T\left[\sum_{i=j}^{r}\sum_{k=l\le i}^{r} d_i^2d_k^2[M_{ii}^TPM_{kk}+M_{kk}^TPM_{ii}-2P]+\right.$$

$$+\sum_{i=j}^{r}\sum_{k<l}^{r} 2d_i^2d_kd_l[M_{ii}^TPG_{kl}+G_{ij}^TPM_{kk}-2P]+$$

$$\left.+\sum_{i<j}^{r}\sum_{k<l\le j}^{r} 4d_id_jd_kd_l[G_{ij}^TPG_{kl}+G_{kl}^TPG_{ij}-2P]\right]x(k)$$

In the last formula, considering 3 terms from theorem condition and theorem [1],

$$M_{ii}^T PM_{kk} + M_{kk}^T PM_{ii} - 2P < 0, \quad M_{ii}^T PG_{kl} + G_{ij}^T PM_{kk} - 2P < 0, \quad G_{ij}^T PG_{kl} + G_{kl}^T PG_{ij} - 2P < 0$$

is satisfied.

Therefore globally asymptotical stability of the discrete TS fuzzy system based on moving rate is garrented. (proof end)

## Conclusion

We proposed and proved a new theorem about loose stability conditions in the continuous and discrete TS fuzzy system based on moving rate of the fuzzy reasoning method.

Firstly, the condition of theorem 1 and theorem 2 in this paper is loosed the condition the condition of theorem 1 and theorem 2[1].

Namely, common positive definite matrix $P$ which satisfy the condition of theorem satisfy the past condition [1]. In other hands, to solve a common positive definite matrix $P$ which satisfying the condition of theorem 1 and theorem 2 in this paper is simple than to solve a common positive definite matrix $P$ which satisfy the condition of theorem 1 and theorem 2 in the paper [1]. Therefore to apply the condition of theorem 1 and theorem 2 in this paper is effective.

Secondly, the theorem 1 and theorem 2 in this paper is a kind of robust stability condition. Because conditions of theorems guaranteed stability of fuzzy systems although there exists model error in the antecedents of fuzzy model.

## References

[1] Kwak Son Il et al.; International Symposium in Commemoration of the 65th Anniversary of the Foundation of **Kim Il Sung** University (*Mathematics, Physics & Life Science*), 130, Juche101(2012).

[2] Choe Jong Ho et al.; 2012 9th International Conference on Fuzzy Systems and Knowledge Discovery(FSKD 2012), 153, 2012.

[3] Kwak Son Il et al.; IEEE computer society(IPTC), 297, 2010.

[4] Huaguang Zhang et al.; Fuzzy Modeling and Fuzzy Control, Springer, 33, 2009.

[5] Gwak Son Il et al.; Fuzzy Optimization and Decision Making, 39, 12, 592, 2012.

[6] Gwak Son Il et al.; Journal of Intelligent & Fuzzy Systems, 45, 11, 623, 2012.

[7] Ho Chol Man et al.; IEEE Computer Society, 489, 2010.

[8] 李少远; 智能控制(第二版)[M]，机械工业出版社，25～79，2009.

[9] 许哲万 等; 计算机科学，39，12，156，2012.

[10] 许哲万 等; 东大学报，33，10，1395，2012.

[11] 许哲万 等; 计算机科学，38，11，196，2011.

[12] 许哲万 等; 计算机科学，39，11，170，2012.