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Systematic Design Method of Disturbance Observer based on Pseudo Loop Factorization

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Abstract This paper presents a design method for Q-filter based on H_∞ norm optimization. First, the cost function for optimization is defined by considering performance and structive restrictions such as relative order and internal model order of the Q-filter. Then, it is transformed to a pseudo loop shaping problem, and then being facterized so as to overcome the structive restrictions so that the original design problem can be solved according to the solving algorithm of standard H_∞ control problem.
Key words high-performance robust motion control, disturbance observer, standard H_∞ control problem, low-pass filter, freedom control system.

Introduction

The great leader Comrade **Kim Il Sung** said as follows.

"Developing the electronics and automation industries is an important task in modernizing the national economy."("**KIM IL SUNG** WORKS" Vol. 35 P. 311)

The disturbance observer (DOB) has widely been applied to high performance robust motion control because of its high disturbance rejection ability and simple structure[1, 2]. Since the system's performances highly depends on the Q-filter in the DOB, design of the filter has been essential in controller design and analysis, which has attracted much more effort in academic society.[3－8] Especially, it was recognized that the higher order's Q-filter can provide the better performances of rejecting disturbance and noise, thus, Butterworth, Chebyshev or binomial coefficient typed high order low-pass filters(LPFs) have been commonly used [5]. However, such typical LPFs were originally developed with the purpose of effective signal reproduction, whereas the mission of Q-filter in DOB is to reject disturbance and sensor noise, keeping robustness against parameter variations. Thus they are not suitable to obtain optimal performances in control system.

On the other hands, it was shown that H_∞ norms of sensitivity functions of disturbance compensation system can sufficiently reflect the performance of attenuating disturbance and noise as well as the robust stability against model perturbations[6－8]. To systematically design a Q-filter that optimizes its norm cost function is, however, difficult due to structural restrictions such as relative order condition. Thus, most of the design algorithms with H_∞ norm cost functions employed numerical computation methods such as linear matrix inequality [7, 8].

In this paper, we present a analytic design method for DOB based on H_∞ norm optimization.

1. Design Problem of DOB

DOB based control system of 2-DOF structure is depicted in Fig. 1 and the equivalent form in Fig. 2, where u, d, y, $\hat{d}$, ξ are command input, external disturbance, output, disturbance estimate, and sensor noise, respectively. $P(s)$ and $P_n(s)$ represent the plant to be controlled and a nominal plant model respectively. $Q(s)$ is low-pass filter.

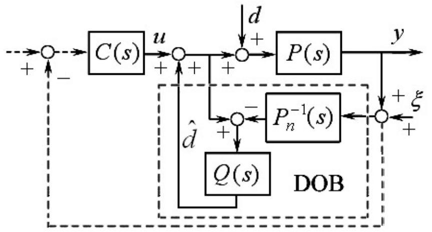

Fig. 1. DOB based control system

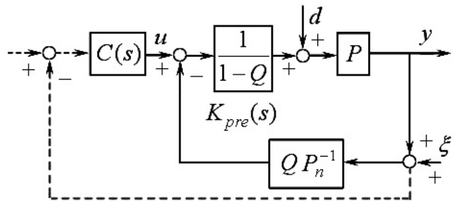

Fig. 2. Equivalent form of the control system

There exist two types of disturbance, external disturbance acting on the plant and internal disturbance due to modeling error in the control system of Fig. 1, which disturbances are considered as equivalent disturbance.

DOB estimates the equivalent disturbance, and feeds back it as a cancellation signal, as shown in Fig. 1. A low-pass filter is introduced to avoid realization of non-proper inverse plant model and filter the sensor noise. Hence, the design of DOB is reduced to the design of the low-pass filter $Q(s)$.

The behavior of the DOB loop in Fig. 1 can be analyzed by looking at the transfer functions from u, d and ξ to the output y.

$$\begin{aligned} y &= \frac{P_n P}{P_n + (P - P_n)Q} u - \frac{PQ}{P_n + (P - P_n)Q} \xi - \frac{P_n P(1-Q)}{P_n + (P - P_n)Q} d = \\ &= G_{yu}(s)u - G_{y\xi}(s)\xi - G_{yd}(s)d \end{aligned} \tag{1}$$

Assume that the nominal model of the plant is correct (i.e., $P(s) = P_n(s)$). Then (1) is simplified as

$$y = P_n(s)u - Q(s)\xi - P_n(s)(1 - Q(s))d \tag{2}$$

Note that $Q(s)$ should go to zero in order to reject noise ξ as much as possible, whereas $Q_C(s) = 1 - Q(s)$ should also be small in order to attenuate disturbance d, implying that $Q(s)$ should be 1. These two conditions are conflicting. In control applications, disturbances normally happen at low frequencies, whereas sensor noise takes effects at high frequencies. This suggests that $Q(s)$ should be a low-pass filter, and can be designed by frequency weighted minimization.

Remainder of the paper discuses an analytic design method of DOB satisfying the design conditions and restrictions.

2. Criteria Function for Designing DOB

In order to define criteria function for optimal design of DOB, it should be resolved how reflecting the restrictions such as robust stability and relative order condition and the optimality such as sensitivity performances which is related with disturbance and noise attenuation.

2.1. Robust stability

From (2), it follows that in order to guarantee the closed-loop stability in the condition of $P(s) = P_n(s)$, $Q(s)$ should be stable, that is nominal stability The system also should hold the stability in the condition of plant model variation, i.e., robust stability.

Suppose that the model uncertainty can be treated as following multiplicative perturbation

$$P(s) = P_n(s)(I + \Delta(s)) \tag{3}$$

where $\Delta(s)$ is model variation and assumed to be stable.

Disturbance observer loop is robustly stable by small gain theorem if $Q(s)$ satisfies

$$\bar{\sigma}(\Delta(j\omega)Q(j\omega)) < 1, \quad 0 \le \omega \le \infty \ .$$

But detailed $\Delta(s)$ is unknown, thus using a stable upper function $W_T(s)$, above robust stability condition is transformed as

$$\|W_T(s)Q(s)\|_\infty < 1 . \tag{4}$$

This is one of restrictions in the design of the low-pass filter $Q(s)$.

2.2. Order and relative order conditions

Disturbance observer consists of the inverse nominal plant model $P_n^{-1}(s)$ that is normally difficult to realize since it is non-proper. From Fig. 2, it is clear that $Q(s)$ should have a relative order larger than or equal to the relative order of the nominal model $P_n(s)$ in order to enable practical implementation of the DOB, and this is also a critical structural restriction to the filter's design process.

2.3. Internal model order condition

In order to attenuate perfectly a certain disturbance, the controller $K_{pre}(s)$ or $Q(s)$ should have the corresponding form. If the disturbance $d(t) = p_q t^q + p_{q-1} t^{q-1} + \cdots + p_0$, to be perfectly rejected for example, from the internal model principle [9], it follows that the controller $K_{pre}(s)$ should include q+1 integrating actions $1/s^{q+1}$. Thus, q is an important design requirement of the low-pass filter $Q(s)$.

2.4. Sensitivity analysis

From Fig. 2, the sensitivity function $S_{DOB}(s)$ and complementary sensitivity function $T_{DOB}(s)$ of the DOB inner loop are easily derived [10]

$$S_{DOB} = \frac{P_n(1-Q)}{P_n + Q(P - P_n)}, \quad T_{DOB} = \frac{PQ}{P_n + Q(P - P_n)} \tag{5}$$

This results in $T_{DOB}(s) = Q(s)$ and $S_{DOB}(s) = Q_C(s)$ for nominal condition $P(s) = P_n(s)$.

The sensitivity and complementary sensitivity functions $S_{clo}(s)$ and $T_{clo}(s)$ of the whole closed-loop system in Fig. 2 is [7].

$$S_{clo} = \frac{Q_C}{1 + P_n C} = \frac{S_{DOB}}{1 + P_n C}, \qquad T_{clo} = \frac{P_n C + Q}{1 + P_n C} = \frac{P_n C + T_{DOB}}{1 + P_n C} \tag{6}$$

From (5) and (6), it is shown that in order to achieve disturbance suppression, robustness against model uncertainties, and noise rejection, it is desirable to reduce $Q(s)$ and $Q_C(s)$ as much as possible. But since sum of them is 1, it is impossible to reduce them simultaneously over all frequencies. Therefore, it is desirable to keep the low frequency response of $Q(s)$ close to 1. Similarly, for sensor noise rejection, the magnitude of the high frequency response of $Q(s)$ should be selected to be small as possible. It concludes that $Q_C(s)$ should be minimal in low frequencies (control band), whereas $Q(s)$ should be minimal in high frequencies (model uncertainty band and noise band).

On the basis of above mentioned design restrictions and performances, we can now define the cost function of DOB by the H_∞ norm as

$$\max\gamma,\ \min_{\substack{Q(s)\in\Omega_k \\ Q(s)\in RH\infty}} \left\| \begin{bmatrix} \gamma\cdot W_C(s)\cdot(1-Q(s)) \\ W_Q(s)\cdot Q(s) \end{bmatrix} \right\|_\infty < 1 \tag{7}$$

where

$$\Omega_k = \{F(s)\,|\,F(s) = N(s)/D(s), \deg(D(s)) - \deg(N(s)) \geq k\} \tag{8}$$

is the set of transfer functions that satisfies the relative order condition *k*. Weighting function $W_C(s)$ reflects the frequency band of disturbance to be rejected and its inverse represents the desired sensitivity at low frequencies. In the same way, $W_Q(s)$ reflects frequency band of sensor noise and the upper bound of model uncertainties, and its inverse represents the desired complementary sensitivity at high frequencies. The order of $Q(s)$ is determined by *k* as well as the order of $W_C(s)$ and $W_Q(s)$.

(7) is a weighted mixed sensitivity problem in which the trade off between two conflicting minimizations of $Q(s)$ and $1-Q(s)$ is resolved by minimizing each in respective frequency domain indicated by $W_C(s)$ and $W_Q(s)$.

By maximizing γ from theorem of the cost function, the optimal $Q(s)$ can be obtained, which is robust to model uncertainty and has the best performance of disturbance and noise rejection. However, this H_∞ norm optimization problem is difficult to solve analitically due to restrictions such as relative order. In the next section, a method for solving problem (7) in the framework of standard H_∞ control problem is discussed.

3. Q-Filter Optimization

Standard H_∞ control method provides generalized design tools for a variety of control problems in which the design requirements are described by H_∞ norm constraint, and for the standard problem the systematic solving algorithms such as DGKF method were developed [11]. However, problem (7) can not be solved by the standard solving algorithm due to extra restrictions that are unacceptable in the framework of the standard scheme. This section presents a method of transforming the problem (7) to standard problem without restrictions on structure and orders.

3.1. Transforming into standard H_∞ problem

Define a scalar pseudo loop function $\tilde{L}(s)$ relatied with $Q(s)$ as

$$\tilde{L}(s) := Q(s)(1-Q(s))^{-1} \tag{9}$$

Then the sensitivity and complementary sensitivity functions of the control system are

$$Q(s)=\widetilde{L}(s)\left(1+\widetilde{L}(s)\right)^{-1},\ Q_C(s)=1-Q(s)=\left(1+\widetilde{L}(s)\right)^{-1} \tag{10}$$

$\widetilde{L}(s)$ can be regarded as the open loop transfer function of a pseudo loop system (it, in fact, is equivalent to the open loop transfer function of the inner loop in Fig. 2 under the model match condition). It is clear that $\widetilde{L}(s)\in\Omega_k$ if and only if its stable closed-loop transfer function $Q(s)$ satisfies $Q(s)\in\Omega_k$ i.e., the relative order of $\widetilde{L}(s)$ is the same as $Q(s)$.

Considering (9), the problem (7) can be written as

$$\max\gamma,\ \min_{\substack{\widetilde{L}\in\Omega_k\\ \widetilde{L}\in\Pi}}\left\|\begin{bmatrix}\gamma\cdot W_C(s)\cdot\left(1+\widetilde{L}(s)\right)^{-1}\\ W_Q(s)\cdot\widetilde{L}(s)\left(1+\widetilde{L}(s)\right)^{-1(\ )}\end{bmatrix}\right\|_\infty<1 \tag{11}$$

where Π is the set of open loop transfer functions that make the corresponding closed-loop transfer function stable. From (11), the optimal open loop transfer function $\widetilde{L}^*(s)$ is solved and then $Q(s)$ is computed by (10). However, the problem still contains the restriction $\widetilde{L}(s)\in\Omega_k$.

To overcome this restriction, facterize $\widetilde{L}(s)$ into pseudo plant $\widetilde{P}(s)$ and pseudo controller $\widetilde{K}(s)$ as follows:

$$\widetilde{L}(s)=\widetilde{P}(s)\cdot\widetilde{K}(s) \tag{12}$$

Assumption Pseudo plant $\widetilde{P}(s)$ satisfies the following conditions:

a) $\widetilde{P}(s)$ is bounded on the open half-left plane (dose not have any unstable poles);

b) $\widetilde{P}(s)$ dose not have any zeros except infinite zeros;

c) $\widetilde{P}(s)$ satisfies $\widetilde{P}(s)\in\Omega_k$.

We can simply select $\widetilde{P}(s)=\alpha/(s+\beta)^k$ for arbitrary real number $\alpha>0$ and $\beta>0$.

Note that the pseudo plant $\widetilde{P}(s)$ is independent of real plant $P(s)$ and only preserves the relative order of $P_n(s)$. The way to factorize $\widetilde{L}(s)$ is also independent of $P(s)$ and the only restriction on $\widetilde{L}(s)$ is $\widetilde{L}(s)\in\Omega_k\cap\Pi$.

Substitute (12) into (11), then we have following optimization problem with respect to $\widetilde{K}(s)$:

$$\max\gamma,\ \min_{\widetilde{K}\in\Pi}\left\|\begin{bmatrix}\gamma\cdot W_C(s)\cdot\left(1+\widetilde{P}(s)\widetilde{K}(s)\right)^{-1}\\ W_Q(s)\cdot\widetilde{P}(s)\widetilde{K}(s)\left(1+\widetilde{P}(s)\widetilde{K}(s)\right)^{-1}\end{bmatrix}\right\|_\infty<1 \tag{13}$$

Theorem Assume that the plant $\widetilde{P}(s)$ of H_∞ norm optimization problem (13) satisfies the Assumption Let $\widetilde{K}^*(s)$ be the optimal solution of (13). The optimal loop function $\widetilde{L}^*(s)=\widetilde{P}(s)\cdot\widetilde{K}^*(s)$ is independent of the selection of $\widetilde{P}(s)$ and uniquely determined.

Proof: Let Σ_P be a set of pseudo plants that satisfy Assumption. Assume that $\widetilde{P}(s)\in\Sigma_P$ is a pseudo plant with which the corresponding optimal pseudo controller $\widetilde{K}^*(s)$ makes the norm in problem (13) minimal. Then $\widetilde{P}(s)$ has k stable poles $p_i^*\,(i=1,\cdots,k)$ and from loop shaping theory, it is clear that the optimal solution $\widetilde{K}^*(s)$ of the problem (13) includes k zeros equal to these poles as

$$\widetilde{K}^*(s)=\widetilde{K}_0(s)\cdot\prod_{i=1}^{k}(s+p_i^*)\cdot$$

Since $\widetilde{L}^*(s)=\widetilde{P}(s)\cdot\widetilde{K}^*(s)=\widetilde{K}_0(s)$, $\widetilde{K}_0(s)$ is optimal open loop function.

Consider any another $\widetilde{P}_1(s)\in\Sigma_P$ whose poles are $\widetilde{p}_i\,(i=1,\cdots,k)$, and $\widetilde{p}\neq p^*$ (entirely or partially different). Then, from (13), it is clear that controller

$$\widetilde{K}_1^*(s)=\widetilde{K}_0(s)\cdot\prod_{i=1}^{k}(s+\widetilde{p}_i)$$

can minimize the norm of problem (13) for $\widetilde{P}_1(s)$ and internal stability is always satisfied since unstable zero-pole cancelling between controller and plant never appear. $\widetilde{K}_0(s)$ is also a optimal open loop function for $\widetilde{P}_1(s)$. This means that selection of $\widetilde{P}(s)$ satisfying Assumption does not have any affect on minimizing the norm of the problem (13), resulting in $\widetilde{L}^*(s)=\widetilde{K}_0(s)$. (Proof end)

Factorization of $\widetilde{L}(s)$ according to (12) and Assumption ensures that the optimal solution $\widetilde{L}^*(s)$ of the problem (13) satisfies the condition $\widetilde{L}^*(s)\in\Omega_k\cap\Pi$.

Lemma 1 Let $\widetilde{K}(s)$ be an optimal solution of order n and relative order k for any $\widetilde{P}(s)\in\Sigma_P$ for (13). Then transfer function $\widetilde{L}^*(s)=\widetilde{P}(s)\cdot\widetilde{K}^*(s)$ of order n and relative order k is the optimal loop function that gives smallest norm value of (11) among other transfer functions of the same order construction.

Lemma 2 For any $\widetilde{P}(s)\in\Sigma_P$, two H_∞ norm optimization problems (11) and (13) are perfectly equivalent i.e.

$$\begin{gathered}\min_{\substack{L\in\Omega_k\\ L\in\Pi}}\left\|\begin{bmatrix}\gamma\cdot W_C(s)\cdot(1+\widetilde{L}(s))^{-1}\\ W_Q(s)\cdot\widetilde{L}(s)(1+\widetilde{L}(s))^{-1}\end{bmatrix}\right\|_\infty<1,\qquad \max\gamma\\ \min_{K\in\Pi}\left\|\begin{bmatrix}\gamma\cdot W_C(s)\cdot(1+\widetilde{P}(s)\widetilde{K}(s))^{-1}\\ W_Q(s)\cdot\widetilde{P}(s)\widetilde{K}(s)(1+\widetilde{P}(s)\widetilde{K}(s))^{-1}\end{bmatrix}\right\|_\infty<1,\quad \max\gamma\end{gathered}\tag{14}$$

and

$$\widetilde{L}^*(s)=\widetilde{P}(s)\widetilde{K}^*(s)\tag{15}$$

where $\widetilde{L}^*(s)$ and $\widetilde{K}^*(s)$ are the optimal solutions of problem (11) and (13), respectively.

The proofs of the Lemma 1 and Lemma 2 are similar to that of Theorem.

Equation (13) is a weighted mixed sensitivity problem without relative order restriction and can be solved by solving framework of standard H_∞ control problem. Above theorem and Lemmas show that the H_∞ norm optimization problem with relative order restriction can be transformed to standard H_∞ control problem (13) without order restriction by pseudo loop shaping problem, so that it can be analytically solved.

The order of $\widetilde{K}^*(s)$ does not exceed the one of the augmented plant of standard problem that consists of weighting functions $W_C(s)$ and $W_Q(s)$ and pseudo plant $\widetilde{P}(s)$. The order of $\widetilde{L}^*(s)$ is the same as that of $\widetilde{K}^*(s)$ since all the poles of $\widetilde{P}(s)$ is cancelled by some zeros of $\widetilde{K}^*(s)$. $\widetilde{L}^*(s)$ satisfies $\widetilde{L}^*(s)\in\Omega_k$ because $\widetilde{K}^*(s)$ is proper, thus $Q^*(s)\in\Omega_k$.

3.2. Design procedure of optimal Q-filter of DOB

From above discussions, Q-filter can be designed in the following procedures:

Step 1 Determine the weighting functions $W_C(s)$ and $W_Q(s)$ that reflect the design specifications such as frequency response demands and internal model order etc. (as seen later)

Step 2 Select an arbitrary $\widetilde{P}(s) \in \Sigma_P$ that reflects relative order condition k of Q-filter.

Step 3 According to the solving framework of standard H_∞ control problem, solve the optimal solution $\widetilde{K}^*(s)$ of the norm optimization problem (13).

Step 4 According to (15), obtain $\widetilde{L}^*(s)$ and then compute the optimal Q-filter $Q^*(s)$.

DGKF method [11] based on two Ricatti equation solutions was used to solve the standard problem of Step 3. Note that $Q^*(s)$ and $\widetilde{K}^*(s)$ have the same order which depends on augmented plant's order i.e., order sum of weighting functions and pseudo plant. More exactly, as suboptimal solution approaches optimal that by γ-maximization, at least one of state modes of the controller is vanished. As a result, the order of optimal solution is at least 1 order lower than the augmented plant [12].

4. Design Examples

Consider a Q-filter of order 3 and relative order 2 the form of which is most frequently employed in practical servo control systems. The order specifications of the filter design are $n=3$, $m=1$, $k=n-m=2$. Let $q=m=1$ which is the highest possible order.

Define weighting function $W_C(s)$ simply such that its frequency response is tangent of $Q_C(s)$ at $s \to 0$. Similarly, define weighting function $W_Q(s)$ such that its frequency response is tangent of $Q(s)$ at $s \to \infty$. Therefore, we can simply take weighting functions as

$$\overline{W}_C(s) = \gamma \cdot W_C(s) = \gamma/(s+\lambda)^2, \quad W_Q(s) = s^2/\alpha \tag{16}$$

where $\lambda = 0.0001$ and $\alpha = 667$ is used to specify filter's cut-off frequency. Select a pseudo plant satisfying Assumption for given relative order as

$$\widetilde{P}(s) = 1/(s+\delta)^2 \tag{17}$$

where $\delta = 3$ was arbitrarily selected by Theorem and Lemma 1.

The augmented plant of standard problem (13) and its state space representation are as

$$G_\gamma(s) = \left[\begin{array}{cc} \gamma \cdot W_C(s) & -\gamma \cdot W_C(s)\widetilde{P}(s) \\ 0 & W_Q(s)\widetilde{P}(s) \\ \hline I & -\widetilde{P}(s) \end{array}\right] \tag{18}$$

$$\left[\begin{array}{c|c|c} A & B_1 & B_2 \\ \hline C_1 & D_{11} & D_{12} \\ \hline C_2 & D_{21} & D_{22} \end{array}\right] = \left[\begin{array}{cc|c|c} A_C & -B_C C_P & B_C & 0 \\ 0 & A_P & 0 & B_P \\ \hline \gamma \cdot C_C & 0 & 0 & 0 \\ 0 & C_Q & 0 & D_Q \\ \hline 0 & C_P & I & 0 \end{array}\right] \tag{19}$$

where [A_C, B_C, C_C, 0] and [A_P, B_P, C_P, 0] are the state space representations of $W_C(s)$ and $\widetilde{P}(s)$ respectively, and [A_P, B_P, C_Q, D_Q] is the state space representations of $W_Q(s)\widetilde{P}(s)$. From (16) and

(17), it is clear that the augmented plant's order is 4, and therefore the optimal controller's order is 3 as $\gamma \to \gamma_{\max}$. We can use the function hinfopt of MATLAB Robust Control Toolbox to solve this problem and realize optimization through γ-maximization. At $\gamma_{\max} \approx 150.203$, resultant solution $\widetilde{K}_{\lambda}^{*}(s)$ and final optimal Q-filter are obtained as

$$\widetilde{K}_{\lambda}^{*}(s) = \widetilde{K}_{\lambda,0}^{*} \cdot \frac{1}{s^2} = \frac{666.7(s+8.444)(s+3)^2}{s^2(s+37.48)} \tag{20}$$

$$Q^{*}(s) = \frac{\widetilde{L}^{*}(s)}{1+\widetilde{L}^{*}(s)} = \frac{\widetilde{K}_{\lambda}^{*}(s)\widetilde{P}(s)}{1+\widetilde{K}_{\lambda}^{*}(s)\widetilde{P}(s)} = \frac{666.7s+5632}{s^3+37.48s^2+666.7s+5\,632} \tag{21}$$

It is clear that the result satisfies the given order and relative order demands. The frequency magnitude responses of $Q^{*}(s)$, $W_Q^{-1}(s)$, $Q_C^{*}(s)$, and $W_C^{-1}(s)$ are shown in Fig. 3 and 4 which show that the all norm restriction demands in (7) are satisfied.

$$Q_b(s) = \frac{3(\sigma s)+1}{(\sigma s)^3+3(\sigma s)^2+3(\sigma s)+1} \tag{22}$$

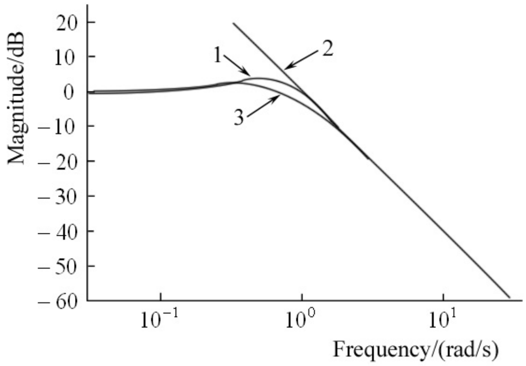

Fig. 3. Frequency magnitude responses of $Q^{*}(s)$ (1), $W_Q^{-1}(s)$ (2) and $Q_b(s)$ (3)

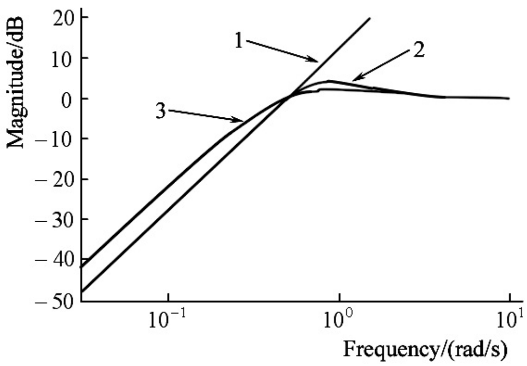

Fig. 4. Frequency magnitude responses of $W_C^{-1}(s)$ (1), $1-Q^{*}(s)$ (2), $1-Q_b(s)$ (3)

In order to compare with previous traditional DOB's Q-filter, consider a binomial coefficient model filter [5, 10].

Select the time constant σ to be $\sigma = 0.065$ so that its roll-off frequency response coincides with $Q^{*}(s)$, meaning that two noise rejection performances are equal. The high frequency responses of $Q^{*}(s)$ and $Q_b(s)$ are shown in Fig. (3). Through simple computations, we can know $|W_Q^{-1}(j\omega)| < |Q_b(j\omega)|$, $\omega \ge \omega_H = 70$ rad/s. This shows that noise rejection performance of $Q^{*}(s)$ is better than $Q_b(s)$ at this frequency band. Under this condition, we can verify $\left|1-Q^{*}(j\omega)\right| < \left|1-Q_b(j\omega)\right|$ at $\omega < \omega_L = 10$ rad/s as shown in Fig. 4 and the magnitude error at sufficiently low frequency is approximately 6 dB, showing that $Q^{*}(s)$ has the low frequency disturbances attenuation performance stronger 2.2 times than $Q_b(s)$. If we focus on noises at $\omega \ge \omega_H$ and disturbances at $\omega < \omega_L$ in design of in disturbance suppression performance get lager as the order of the observer get higher. For example, in Q-filter, $Q^{*}(s)$ is better than $Q_b(s)$ and many practical cases are similar to this case. These differences the case of order 5 and relative order 1(i.e., n = 5, m = 4), the magnitude difference between $1-Q^{*}(s)$ and $1-Q_b(s)$ reaches 22.3 dB, meaning that former's disturbance attenuation performance at low frequencies is higher 13 times than latter's.

Conclusion

In this paper, a systematic and straightforward design method of disturbance observer based on H_∞ norm optimization has been presented. The pseudo factorization of loop transfer function of DOB system makes it possible for the Q-filter design problem with structural restrictions to be transformed into standard H_∞ control problem without any restriction. The numerical example showed effectiveness and validity of the proposed method and that various design specifications can be implemented by suitable selection of weighting functions under the condition of keeping the order structure.

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