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The Analysis of Stratum Structure based on a Coupling Model of Void Damage and Crack

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Abstract We considered mechanical modeling of complicated stratum structure with damage including voids, defects and cracks in the framework of continuum damage mechanics.
We represented voids and defects in material using damage tensor based on concept of effective stress and modeled cracks on the basis of exact crack theory. According to this modelling we determined stiffness characteristic quantity of stratum material and analyzed stress and displacement of stratum structure with cracks and void damage using ANSYS on the basis of it.

Key words opened crack, closed crack, void, damage

Introduction

Generally, stratum structures consist of complicated medium where simultaneously exist various discontinuities like various kinds of micro and macro voids (cavities), microcracks, joints and stratifications.

There are several methods to estimate strain states of discontinuous bedrocks, which can be divided into stochastic and deterministic method mainly. In the stochastic methods it gives the distribution characteristics of the discontinuous face in probability based on test data for the discontinuity of the rock and is approximating the discontinuous rock to the continuity which is equivalent to it. These are the crack theory and the estimation method of deformity on the basis of crack tensor [1]. This method is helpful to determine macroscopic state of rock, but not suitable to determine stress state in local domain of rock.

On the other hand, in the deterministic methods there is a method to model the discontinuity of rock as real geometrical form and FEM using gap element belongs to it [2]. But the disadvantage of FEM using gap elements is to increase rapidly the volume of calculation in large number of cracks.

Besides in the continuum damage mechanics it represents cracks, defects and voids only by concept of damage and the damage problem is solved on the basis of the concept of the Kachanov effective stress [3－5]. But in the strict meaning the crack problem yields the correct results by using the crack theory, which differs from the effective stress theory.

In literature [6] the Helmholtz free energy is presented by using the constants of damage tensor and stiffness characteristic quantity declined as damage even with the crack closure effect (MK model) are given. The shortcoming of this model is that material constants η_1,

η_2, η_3, η_4 should be determined for determination of model.

There were suggested the various models representing the some damage behaviors [7, 8] beside MK model. On the basis of the quasibrittle model in literature [7] a plane 2D problem was analyzed by using ABAQUS program. In the scope of this model were considered the damage problems of the ice layer material and the concrete layer material.

In this paper, we considered the mechanical modelling of the complicated stratum material with the damage including voids, defects and cracks in the framework of the continuum damage mechanics. We represented the voids and defects within material using the damage tensor based on the concept of the effective stress and modeled the cracks on the basis of exact crack theory. According to this modelling, we determined the stiffness characteristic quantities of the stratum material and analyzed stresses and displacements of the stratum structure with cracks and void damage using ANSYS.

1. The Coupling Model of the Continuous Damage and the Crack

Let us represent the voids, defects etc. in the structure in the framework of the continuum damage mechanics by using the damage tensor introducing concept of the effective stress based on the hypothesis of the strain equivalency and modelling cracks by crack theory (Fig. 1).

We suppose that the size of an element is so enough big that it involves the voids and defects.

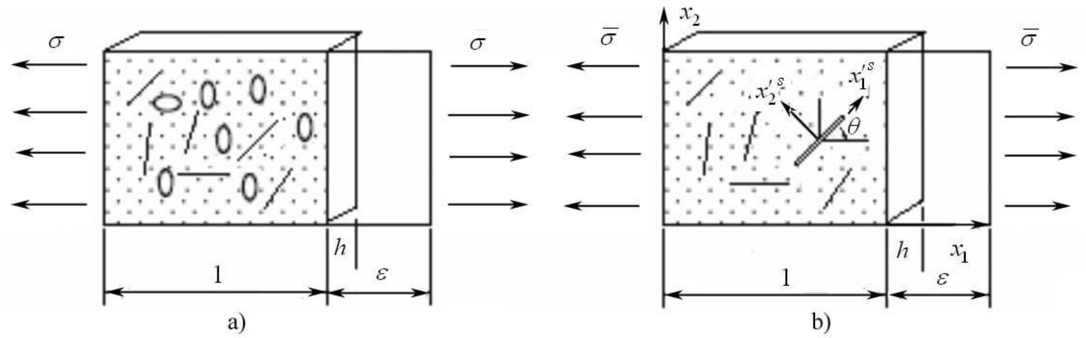

Fig. 1. Concept of the effective stress and hypothesis of the strain equivalency

a) physical space, b) effective space

Set U is the strain energy density of the elastic body, V is the supplement energy density.

In this case it has following formula.

$$U = \sigma_{ij}\varepsilon_{ij} - V \tag{1}$$

Stress tensor σ_{ij} is expressed by damage tensor ϕ_{pj} as follows [3].

$$\sigma_{ij} = \phi_{pj}K_{ipkl}\varepsilon_{kl} \tag{2}$$

where K_{ipkl} -elastic stiffness matrix, $\phi_{pj} = \delta_{pj} - D_{pj}$, δ_{pj} -Kronecker delta, D_{pj} -damage tensor (if it is $D_{pj} = D \equiv \xi$, it is isotropic scalar damage parameter). If we coincide the global coordinates system with principal damage coordinates system, ϕ_{pj} is represented the principal value of the damage tensor $\phi_i = 1 - D_i \quad (i = 1, 2, 3)$.

If we regard the expression (2), the formula (1) is as follows.

$$U = \phi_{pj} K_{ipkl} \varepsilon_{kl} \varepsilon_{ij} - V \tag{3}$$

In the formulas (1), (3) the supplement energy V is as follows.

$$V = \frac{1}{2} \phi_{pj} K_{ipkl} \varepsilon_{kl} \varepsilon_{ij} + \Delta V_c \tag{4}$$

where ΔV_c the supplement energy by crack.

The mechanical behavior of the crack is connected with the stress states acting on the crack face.

If in the plane problems we define that tension stress is plus, we can think the following three cases according to the values of stresses σ'_{ij} in the local (crack) coordinates system $(x_1'^s, x_2'^s)$.

① $|\sigma'_{21}| > \mu |\sigma'_{22}|, \quad \sigma'_{22} \le 0$

② $|\sigma'_{21}| \le \mu |\sigma'_{22}|, \quad \sigma'_{22} \le 0$

③ $\sigma'_{22} > 0$

where μ is friction coefficient.

The case ① is the case where the friction sliding arises between the crack faces, the case ② is the case where the crack faces are restricted by friction force and elastic body behaves like no crack, the case ③ corresponds to the open crack.

1.1. The coupling model of the continuous damage and the opened crack

For the opened crack, the supplement energy by crack is as follows.

$$\Delta V_c = \sum_{s=1}^{m} \frac{2Ah}{V} \int_0^{a^s} \left[{K_{\mathrm{I}}^s}^2 + {K_{\mathrm{II}}^s}^2 \right] da^s \tag{5}$$

where s-the number of cracks, m-a number of cracks in the element, a^s-half of the crack length, h-thickness, for the case of the plane strain $A = (1 - \nu^2)/E$, for the case of the plane stress $A = 1/E$, $V = Sh$, S-area.

$$K_{\mathrm{I}}^s = F_{\mathrm{I}}^s \sigma_{22}^s \sqrt{\pi a^s} \ : \text{ First stress intensity factor,} \tag{6}$$

$$K_{\mathrm{II}}^s = F_{\mathrm{II}}^s \sigma_{21}^s \sqrt{\pi a^s} \ : \text{ Second stress intensity factor.} \tag{7}$$

F_{I}^s, F_{II}^s -the correlation coefficients of the s-crack with the neighborhood cracks and $0 \le F_{\mathrm{I}}^s \le 1$, $0 \le F_{\mathrm{II}}^s \le 1$.(If we don't account relationship, $F_{\mathrm{I}}^s = 1$, $F_{\mathrm{II}}^s = 1$.)

For the case of the plane stress we substitute

$$\sigma_{22}^s = \frac{E}{1 - \nu^2} (\varepsilon_{22}^s + \nu \varepsilon_{11}^s), \quad \sigma_{21}^s = 2G \varepsilon_{21}^s \tag{8}$$

in (6), (7) respectively and the obtained results substitute in (5), and we have

$$U=\frac{1}{2}\phi_{pj}K_{ipkl}\varepsilon_{kl}\varepsilon_{ij}-\sum_{s=1}^{m}\frac{2Ah}{V}\int_{0}^{a^s}\left[F_{\mathrm{I}}^{s\,2}\left(\frac{E}{1-\nu^2}\right)^2(\varepsilon_{22}^s+\nu\varepsilon_{11}^s)^2\pi a^s+F_{\mathrm{II}}^{s\,2}4G^2\varepsilon_{21}^{s\;2}\pi a^s\right]da^s \,. \quad (9)$$

In case for the plane strain problems, in the formulas (8), (9) $E\to E/(1-\nu^2)$, $\nu\to\nu/(1-\nu)$.

On the other hand if the direction cosines are $d_{ij}^s=\cos(x_i,\ x_j'^s)$ $(i,\ j=1,\ 2)$ between the coordinates system axes of the global coordinates system $(x_1,\ x_2)$ and the crack coordinates system, we have

$$\varepsilon_{ij}^s=d_{ki}^s\,d_{lj}^s\,\varepsilon_{kl}\,. \quad (10)$$

Therefore substituting formula (10) in (9), we obtain

$$\begin{aligned}U=\frac{1}{2}\phi_{pj}K_{ipkl}\varepsilon_{kl}\varepsilon_{ij}-\sum_{s=1}^{m}\frac{\pi Ah}{V}\Bigg[&F_{\mathrm{I}}^{s\,2}\left(\frac{E}{1-\nu^2}\right)^2(d_{i2}^s\,d_{k2}^s\,\varepsilon_{ik}d_{j2}^s\,d_{l2}^s\,\varepsilon_{jl}+\\&+2\nu\,d_{i1}^s\,d_{k1}^s\,\varepsilon_{ik}d_{j2}^s\,d_{l2}^s\,\varepsilon_{jl}+\nu^2d_{i1}^s\,d_{k1}^s\,\varepsilon_{ik}d_{j1}^s\,d_{l1}^s\,\varepsilon_{jl})+\\&+4F_{\mathrm{II}}^{s\,2}G^2d_{i2}^s\,d_{k1}^s\,\varepsilon_{ik}d_{j2}^s\,d_{l1}^s\,\varepsilon_{jl}\Bigg]\;a^{s\,2}\qquad(i,\ j,\ k,\ l,\ p=1,\ 2)\end{aligned} \quad (11)$$

If we now calculate by $\sigma_{11}=\partial U/\partial\varepsilon_{11}$, $\sigma_{22}=\partial U/\partial\varepsilon_{22}$, $\sigma_{12}=\partial U/\partial\varepsilon_{12}$, we have the following stress-strain relation.

$$\begin{Bmatrix}\sigma_{11}\\ \sigma_{22}\\ \sigma_{12}\end{Bmatrix}=\left(\begin{bmatrix}K_{11}&K_{12}&K_{13}\\K_{21}&K_{22}&K_{23}\\K_{31}&K_{32}&K_{33}\end{bmatrix}+\begin{bmatrix}\Delta K_{11}&\Delta K_{12}&\Delta K_{13}\\ \Delta K_{21}&\Delta K_{22}&\Delta K_{23}\\ \Delta K_{31}&\Delta K_{32}&\Delta K_{33}\end{bmatrix}\right)\begin{Bmatrix}\varepsilon_{11}\\ \varepsilon_{22}\\ \varepsilon_{12}\end{Bmatrix} \quad (12)$$

where K_{ij} and ΔK_{ij} are the elastic stiffness matrixes considered the effects of damage and crack respectively, for the isotropic materials in the principal damage coordinates system are determined as follows.

$$\begin{aligned}&K_{11}=\frac{E(1-\nu)(1-D_1)}{(1+\nu)(1-2\nu)},\quad K_{12}=K_{21}=\frac{E\nu(2-D_1-D_2)}{(1+\nu)(1-2\nu)},\quad K_{13}=K_{31}=0\\&K_{22}=\frac{E(1-\nu)(1-D_2)}{(1+\nu)(1-2\nu)},\quad K_{23}=K_{32}=0,\ \ K_{33}=\frac{G(2-D_1-D_2)}{2}\end{aligned} \quad (13)$$

$$\Delta K_{11}=\sum_{s=1}^{m}\frac{\pi Ah}{V}\left[F_{\mathrm{I}}^{s\,2}\left(\frac{E}{1-\nu^2}\right)^2(d_{12}^{s\;4}+2\nu d_{11}^{s\;2}d_{12}^{s\;2}+\nu^2d_{11}^{s\;4})+4F_{\mathrm{II}}^{s\,2}G^2d_{11}^{s\;2}d_{12}^{s\;2}\right]a^{s\,2}$$

$$\begin{aligned}\Delta K_{12}=\Delta K_{21}=\sum_{s=1}^{m}\frac{\pi Ah}{V}\Bigg[&F_{\mathrm{I}}^{s\,2}\left(\frac{E}{1-\nu^2}\right)^2(d_{12}^{s\;2}d_{22}^{s\;2}+\nu d_{12}^{s\;2}d_{21}^{s\;2}+\nu d_{11}^{s\;2}d_{22}^{s\;2}+\\&+\nu^2d_{11}^{s\;2}d_{21}^{s\;2})+4F_{\mathrm{II}}^{s\,2}G^2d_{11}^sd_{12}^sd_{21}^sd_{22}^s\Bigg]a^{s\,2}\end{aligned}$$

$$\begin{aligned}\Delta K_{13}=\Delta K_{31}=\sum_{s=1}^{m}\frac{\pi Ah}{V}\Bigg[&F_{\mathrm{I}}^{s\,2}\left(\frac{E}{1-\nu^2}\right)^2(2d_{12}^{s\;3}d_{22}^s+2\nu d_{11}^sd_{12}^{s\;2}d_{21}^s+2\nu d_{11}^{s\;2}d_{12}^sd_{22}^s+\\&+2\nu^2d_{11}^{s\;3}d_{21}^s)+4F_{\mathrm{II}}^{s\,2}G^2(d_{11}^sd_{12}^{s\;2}d_{21}^s+d_{11}^{s\;2}d_{12}^sd_{22}^s)\Bigg]a^{s\,2}\end{aligned}$$

$$\Delta K_{22}=\sum_{s=1}^{m}\frac{\pi Ah}{V}\left[F_{\mathrm{I}}^{s\,2}\left(\frac{E}{1-\nu^2}\right)^2(d_{22}^{s\,4}+2\nu d_{21}^{s\,2}d_{22}^{s\,2}+\nu^2 d_{21}^{s\,4})+4F_{\mathrm{II}}^{s\,2}G^2 d_{21}^{s\,2}d_{22}^{s\,2}\right]a^{s\,2}$$

$$\Delta K_{23}=\Delta K_{32}=\sum_{s=1}^{m}\frac{\pi Ah}{V}\left[F_{\mathrm{I}}^{s\,2}\left(\frac{E}{1-\nu^2}\right)^2(2d_{12}^{s}d_{22}^{s\,3}+2\nu d_{11}^{s}d_{21}^{s}d_{22}^{s\,2}+2\nu d_{12}^{s}d_{21}^{s\,2}d_{22}^{s}+\right.$$

$$\left.+2\nu^2 d_{11}^{s}d_{21}^{s\,3})+4F_{\mathrm{II}}^{s\,2}G^2(d_{12}^{s}d_{21}^{s\,2}d_{22}^{s}+d_{11}^{s}d_{21}^{s}d_{22}^{s\,2})\right]a^{s\,2}$$

$$\Delta K_{33}=\sum_{s=1}^{m}\frac{\pi Ah}{V}\left[F_{\mathrm{I}}^{s\,2}\left(\frac{E}{1-\nu^2}\right)^2(4d_{12}^{s\,2}d_{22}^{s\,2}+8\nu d_{11}^{s}d_{12}^{s}d_{21}^{s}d_{22}^{s}+4\nu^2 d_{11}^{s\,2}d_{21}^{s\,2})+\right.$$

$$\left.+4F_{\mathrm{II}}^{s\,2}G^2(d_{11}^{s}d_{21}^{s}+d_{11}^{s}d_{22}^{s})^2\right]a^{s\,2}$$

1.2. The coupling model of the continuous damage and the closed crack

Express the stress σ'_{ij} in the local coordinates system as follows.

$$\sigma'_{ij}=\sigma_{ij}^{e}+\sigma_{ij}^{s},\ \ \sigma_{ij}^{e}=\begin{cases}\sigma'_{ij}, & ij\neq 12,\,21\\ \mu f\,|\,\sigma'_{22}, & ij=12,\,21\end{cases},\ \ \sigma_{ij}^{s}=\begin{cases}0, & ij\neq 12,\,21\\ \sigma'_{ij}-\mu f\,|\,\sigma'_{22}\,|, & ij=12,\,21\end{cases} \tag{14}$$

where f express the polarity of the sliding and $f=\mathrm{sgn}(\sigma'_{21})$.

In the expression (14) under the action of σ_{ij}^{e} the elastic body is in the state like no cracks and under the action of σ_{ij}^{s} the normal stress acting on the crack face is zero, therefore it is equivalent to opened crack. Accordingly the increment formula of the supplement energy by the sliding crack is as follows.

$$\Delta V_c=\sum_{s=1}^{m}\frac{2Ah}{V}\int_0^{a^s}K_{\mathrm{II}}^{s\,2}da$$

We should not consider the interactions between the cracks on the closed crack.

If we regard the formula (14), we have $\sigma_{21}^{s}=\sigma'_{21}-\mu f\sigma'_{22}=\xi_{ij}^{s}\sigma'_{ij}$, where

$$\left.\begin{aligned}\xi_{11}^{s}&=d_{11}^{s}d_{12}^{s}-\mu fd_{12}^{s\,2}\\ \xi_{12}^{s}&=d_{12}^{s}d_{21}^{s}-\mu fd_{12}^{s}d_{22}^{s}\\ \xi_{21}^{s}&=d_{22}^{s}d_{11}^{s}-\mu fd_{22}^{s}d_{12}^{s}\\ \xi_{22}^{s}&=d_{22}^{s}d_{21}^{s}-\mu fd_{22}^{s\,2}\end{aligned}\right\}.$$

By using the expressions (9), (10), from (8) we have

$$\Delta V_c=\sum_{s=1}^{m}\frac{2\pi Ah}{V}\int_0^{a^s}\sigma_{21}^{s\,2}\pi a da=\sum_{s=1}^{m}\frac{\pi Ah}{V}a^{s\,2}\xi_{ij}^{s}\xi_{pq}^{s}K_{ijkl}K_{pqrt}\varepsilon_{kl}\varepsilon_{rt},\ \ i,\,j,\,k,\,l,\,p,\,q,\,r,\,t=1,\,2\,.$$

By using the expressions (4) and (11), formula (3) is expressed as follows.

$$U=\frac{1}{2}\phi_{pj}K_{ipkl}\varepsilon_{kl}\varepsilon_{ij}-\sum_{s=1}^{m}\frac{\pi Ah}{V}a^{s\,2}\xi_{ij}^{s}\xi_{pq}^{s}K_{ijkl}K_{pqrt}\varepsilon_{kl}\varepsilon_{rt},\ \ i,\,j,\,k,\,l,\,p,\,q,\,r,\,t=1,\,2$$

The elastic stiffness matrix K_{ij} regarded the effect of damage by the expression (12) coincides with the (13) and the elastic stiffness matrix ΔK_{ij} regarded the effect of crack is determined in the principal damage coordinate system for the isotropic material and plane problem as follows.

$$\Delta K_{11}=\sum_{s=1}^{m}\frac{2\pi Ah}{V}a^{s^2}M_{11}^2,\quad \Delta K_{12}=\Delta K_{21}=\sum_{s=1}^{m}\frac{2\pi Ah}{V}a^{s^2}M_{11}M_{22},$$

$$\Delta K_{22}=\sum_{s=1}^{m}\frac{2\pi Ah}{V}a^{s^2}M_{22}^2,\quad \Delta K_{13}=\Delta K_{31}=\sum_{s=1}^{m}\frac{2\pi Ah}{V}a^{s^2}M_{11}M_{12},$$

$$\Delta K_{23}=\Delta K_{32}=\sum_{s=1}^{m}\frac{2\pi Ah}{V}a^{s^2}M_{22}M_{12},\quad \Delta K_{33}=\sum_{s=1}^{m}\frac{2\pi Ah}{V}a^{s^2}M_{12}^2$$

where $M_{kl}=\sum_{i,\ j}\xi_{ij}K_{ijkl}\quad (i,\ j,\ k,\ l=1,\ 2)$.

2. The Numerical Simulations by ANSYS

We have done the finite element simulations by using the model plane 13 of ANSYS 12.0 for the case that uniaxial distribution load with $P=0.5\text{MPa}$ in the direction x_2 (tension for the opened crack, compression for the closed crack) is acted on the plane area $3\times 3\text{m}^2$ in the total dimension (Fig. 2).

Initial data used at the simulation are $E=58\text{GPa}$, $\nu=0.23$, $\mu=0.65$, $D_1=D_2=0.2$, $F_{\text{I}}=F_{\text{II}}=1$.

In the Fig. 2－4, we have shown the displacements and the stress change characteristics following the various crack parameters.

As shown in figures, the displacement increases when the length of the crack is increasing. The maximum principal stress σ_i and the absolute value of the minimum principal stress $|\sigma_3|$ and the intensity of the stress σ_i increase according to the length of crack, and when the direction angle of crack θ increases between $0°\sim 90°$ and then decrease.

And by the calculation results we can know that the maximum, minimum principal stresses and the intensity of the stress have maximum values in the neighborhood of the crack end.

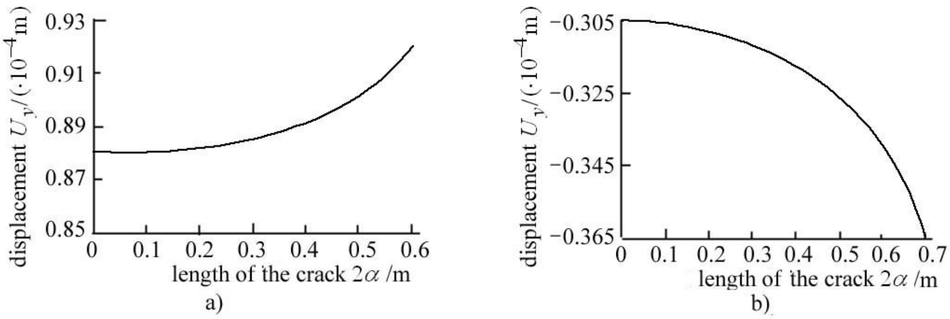

Fig. 2. Displacement according to length of the crack ($\theta=45°$)

a) opened, b) closed

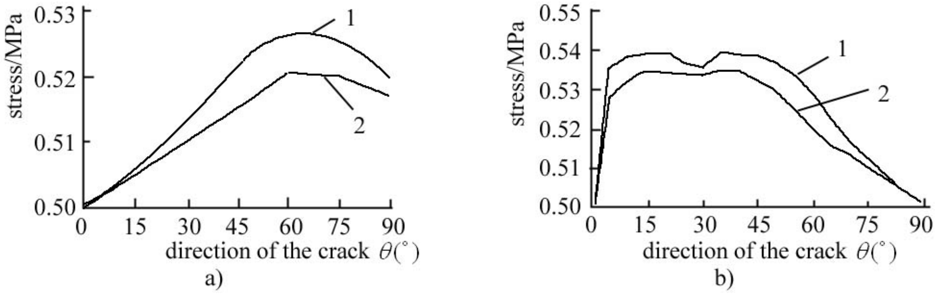

Fig. 3. Stress according to direction of the crack ($2a = 0.4\text{m}$)

a) opened, 1- σ_1 , 2- σ_i , b) closed, 1- $|\sigma_3|$, 2- σ_i

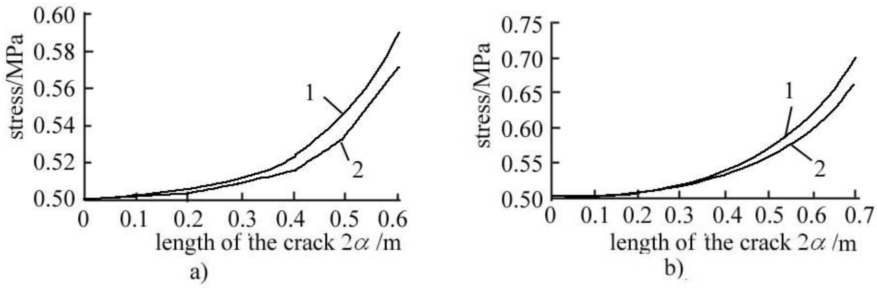

Fig. 4. Stress by length of the crack ($\theta = 45°$)

a) open, 1- σ_1 , 2- σ_i , b) closed, 1- $|\sigma_3|$, 2- σ_i

Conclusion

We considered the new mechanical modelling of the complicated stratum structure with the damage including voids, defects and cracks in the framework of the continuum damage mechanics.

We represented voids and defects etc. using damage tensor based on the concept of the effective stress and modeled the cracks on the basis of the exact crack theory. According to this modelling, we determined the stiffness characteristics quantities of the stratum material and on the basis of this analyzed the stress and the displacement of the stratum structure with the cracks and the void damage using ANSYS.

References

[1] M. Oda; Soils and Foundations, 22, 4, 96, 1982.

[2] R. E. Goodman et al.; ASCE, 94, 3, 637, 1968.

[3] G. Voyiadjis et al.; Journal of Applied Mechanics, Transactions ASME, 64, 1, 106, 1997.

[4] I. Carol Rizzi et al.; International Journal of Solids and Structures, 38, 4, 491, 2001.

[5] N. Challamel et al.; International Journal of Mechanical Sciences, 47, 3, 459, 2005.

[6] S. Murakami et al.; International Journal of Mechanical Sciences, 39, 4, 473, 1997.

[7] H. D. Dragon et al.; Archive of Applied Mechanics, 72, 6, 498, 2002.

[8] J. J. Skrzypek et al.; Engineering Fracture Mechanics, 71, 4, 681, 2004.