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Analysis of Nonlinear Vibration Response of Rotor Machine Working with Partial Impact and Friction

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Abstract We established the equation of motion by use of nonlinear function of impacting force in connection with the deformation of impacted surface in misalignment rotor and studied nonlinear dynamic characteristics of rotor machine accompanied by impact and friction.

Key words partial impact and friction, rotational machine, misalignment, nonlinear dynamic characteristics

Introduction

The great leader Comrade **Kim Il Sung** said.

"**We should actively develop the major areas of basic sciences such as mathematics, physics, chemistry and biology so as to raise the national standard of science and technology and find more effective solutions to the scientific and technical problems that arise in the different branches of the national economy.**"("**KIM IL SUNG** WORKS" Vol. 35 P. 313)

Among the all kinds of faults of rotational machine, impact and friction are very important because these should make trouble in stationary production and raise serious accident.

So these must be discovered in time. There are many studies in connection with the vibration analysis of rotational machine working with partial impact and friction [1－3].

But in the previous studies investigators used the linear function of impacting force in connection with the deformation of impacted surface in alignment rotor model, while they studied dynamic characteristics of rotor machine accompanied by impact and friction.

In this paper we will consider the nonlinear function of impacting force in connection with the deformation of impacted surface in misalignment rotor model, while we study dynamic characteristics of rotor machine accompanied by impact and friction.

1. Differential Equations of Motion

The mechanical model of considering system is like as Fig. 1. Here $k_1/2$ is stiffness coefficient of bearing, k is stiffness coefficient of shaft, c is damping coefficient of shaft, m is mass of rotor, k_c is stiffness coefficient of stator, d_c is void between rotor and stator in moving state, ω_0 is angle velocity of rotor and e is the distance of eccentricity.

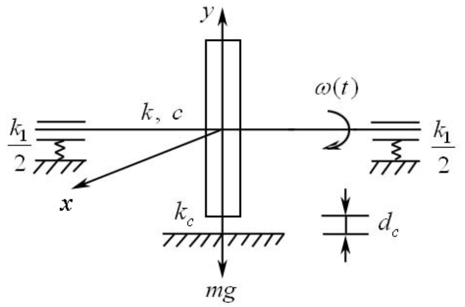

Fig. 1. Mechanical model of system

The moving differential equations of rotor in considering coordinate system are as follows:

$$\left.\begin{aligned} m\ddot{x} + c\dot{x} + \frac{kk_1 x}{k+k_1} &= me\omega_0^2 \cos\omega_0 t + F_x \\ m\ddot{y} + c\dot{y} + \frac{kk_1 y}{k+k_1} &= me\omega_0^2 \sin\omega_0 t - mg + F_y \end{aligned}\right\} \quad (1)$$

where F_x and F_y are the components of frictional force in direction of x, y axes respectively.

The relationships between stator and rotor in stop state are like as Fig. 2.

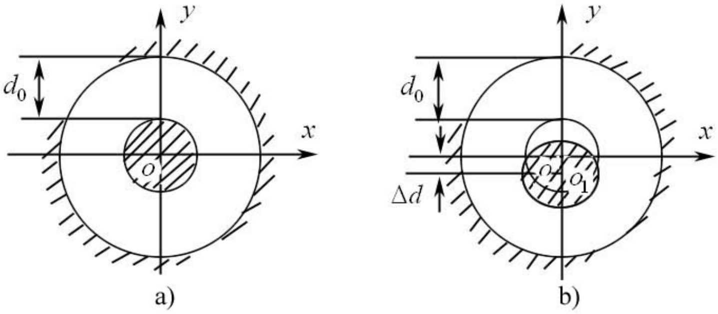

Fig. 2. Relationships between stator and rotor in stop state
a) in case of $\Delta d = 0$, b) in case of $\Delta d \neq 0$

On the other hand, the possible cases of being able to raise partial impact in the moving state in case of misalignment between rotor and stator in stop state are like as Fig. 3.

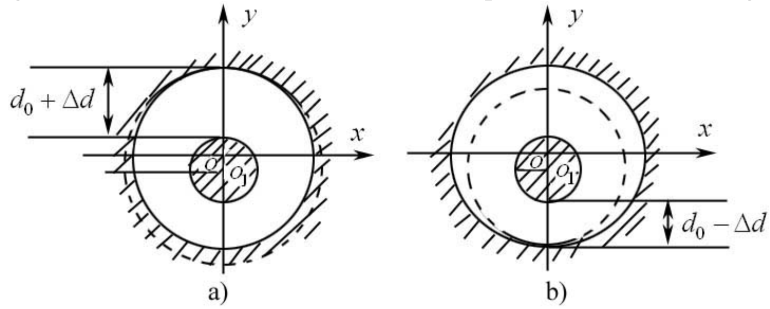

Fig. 3. The possible cases of being able to raise partial impact in moving state of rotor which has misalignment between rotor and stator
a) in case that $d = d_0 + \Delta d$, b) in case that $d = d_0 - \Delta d$

As we can see from this figure, when the partial impact is being raised by misalignment the impact angle (the angle between impacted surface and impacting direction) is changed according to the displacement of rotor. Thus, in this case we can't regard that the value of exponent n in the expression between impacting force and displacement of impacted surface $F = k_c(d - d_c)^n$ is always 1. In this case the value of exponent n is changed in the interval between the value 1 and the value 3/2.

When denoting that f is the coefficient of friction between rotor and stator, d_0 is void of design, Δd is the eccentricity distance between centers of stator and rotor, then the components of frictional force in direction of x, y axes F_x, F_y are expressed as follows.

$$\begin{Bmatrix} F_x \\ F_y \end{Bmatrix} = H_1(d-d_c)\begin{Bmatrix} f_x \\ f_y \end{Bmatrix} \tag{2}$$

$$\begin{Bmatrix} f_x \\ f_y \end{Bmatrix} = -\frac{(d-d_c)k_c}{d}\begin{bmatrix} 1 & -f \\ f & 1 \end{bmatrix}\begin{Bmatrix} x \\ y \end{Bmatrix}, \quad H_1(d-d_c)=\begin{cases} 1, & d \ge d_c \\ 0, & d < d_c \end{cases} \tag{3}$$

$$d=\sqrt{x^2+y^2}\ , \ \ d_c = d_0 + \Delta d \sin\psi\ , \ \ \psi = \arctan(y/x)\,. \tag{4}$$

And the value of exponent n is denoted like $n=-\alpha/\pi+1.5$ by impacting angle $\alpha = |\psi-\theta| = |\arctan(y/x)-\arctan(\dot{y}/\dot{x})|$ (Fig. 4).

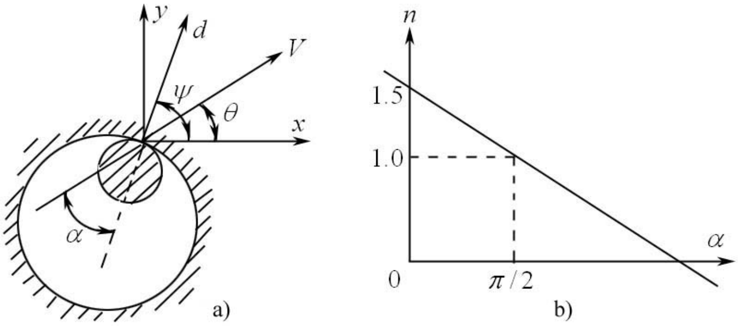

Fig. 4. The impacting angle between stator and rotor (a) and the relationship between impacting angle and exponent (b)

In Fig. 4 V is the velocity vector to radius direction of rotor.

After substituting expressions (2−4) to equation (1), we obtain motion differential equations of system as follows.

$$\left.\begin{aligned} m\ddot{x}+c\dot{x}+kx+\left[\frac{(d-d_c)^n}{d}k_c\cdot x-\frac{(d-d_c)^n}{d}f\cdot k_c\cdot y\right]\cdot H_1(d-d_c) &= me\omega_0^2\cos\omega_0 t \\ m\ddot{y}+c\dot{y}+ky+\left[\frac{(d-d_c)^n}{d}f\cdot k_c\cdot x+\frac{(d-d_c)^n}{d}k_c\cdot y\right]H_1(d-d_c) &= me\omega_0^2\sin\omega_0 t - mg \end{aligned}\right\} \tag{5}$$

The equations (5) are the motion equations of rotor with mass unbalance when the partial impact and friction takes place by misalignment between rotor and stator.

2. The Analysis of Nonlinear Response Characteristics

Using the equations obtained above we have analyzed the nonlinear response characteristics of blower of which critical rotational velocity is $\omega_{\text{crit}} = 450\,\text{rad/s}$.

The values of parameters of objects used in calculation are as follows.

$m = 100\,\text{kg}$, $l = 1\,\text{m}$, $D = 8\,\text{cm}$, $k = 2.7\times10^{11}$ N/m, $c = 1.5\times10^{2}$ Ns/m, $k_1 = 2.3\times10^{11}$ N/m.

$k_c = 2.7\times10^{7}$ N/m, $e = 0.005\,\text{mm}$, $f = 0.2$, $d_0 = 0.2\,\text{mm}$, $\Delta d = 0.1\,\text{mm}$, $\Delta d = 0.1\,\text{mm}$

While analyzing we used the expression of void $d_c = d_0 + \Delta d \sin\omega t$ in case of having misalignment and the expression of void $d_c = d_0$ in case of not having misalignment.

We have got response solving the motion equations (5) by MATLAB.

Fig. 5, 6 are illustrating waves of response signal when the value of exponent n is constant or variable respectively under the same calculating condition of working rotating velocity $\omega_0 = 62.8\,\text{rad/s}$.

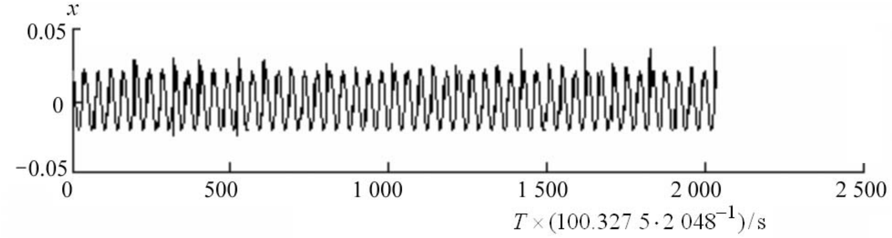

Fig. 5. The time wave of response signal when the value of exponent n is constant

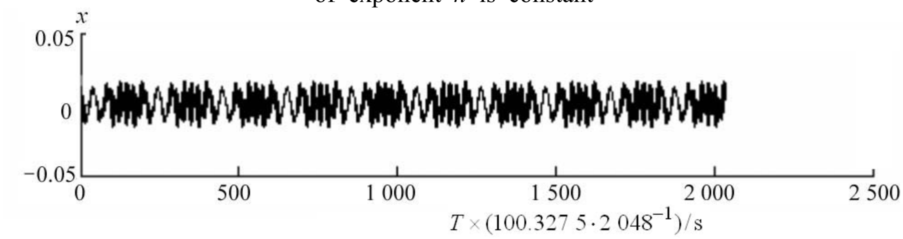

Fig. 6. The time wave of response signal when the value of exponent n is variable

From these figures we can see the fact that the response characteristics is more complex when the value of exponent n is variable.

For the consideration of the characteristics of wavelet transformation of response signal we have transformed the signals illustrated in Fig. 5, 6 into wavelet signals using MATLAB/coif5. And we have calculated the values of Ripshitz exponent and relation dimension of those wavelet signals.

The results of the analysis of calculated data are as follows.

① The result of calculation of the values of Ripshitz exponent using wavelet transformation, n is equal to 1.515 5 when the value of exponent is constant and is equal to 1.375 7 when the value of exponent is variable.

② The results of calculation of the values of singularity vector using wavelet transformation, when the value of exponent n is constant and the value of exponent is variable are like as table 1.

Table 1. The values of singularity vector of wavelet signals in case that value of exponent n is constant and variable

Number of components	In case that n = constant	In case that $n \neq$ constant	Number of components	In case that n = constant	In case that $n \neq$ constant
1	0.766 8	0.559 7	9	0.004 9	0.015 1
2	0.091 0	0.148 0	10	0.004 2	0.017 2
3	0.051 5	0.076 4	11	0.003 3	0.009 1
4	0.027 9	0.051 6	12	0.002 2	0.007 0
5	0.016 1	0.044 9	13	0.001 8	0.005 6
6	0.011 4	0.026 7	14	0.001 6	0.003 8
7	0.008 4	0.022 1	15	0.001 1	0.001 9
8	0.007 6	0.016 6	16	0.000 7	0.001 5

③ The results of calculation of the values of relationship dimension of each wavelet scale components are like as table 2.

Table 2. The values of relationship dimension of each wavelet scale components in case that value of exponent n is constant and variable

Value of scale	2	8	14	20	26	32
Value of relationship dimension ($n=1$)	0.000 603	0.000 85	0.002 1	0.002 6	0.003 2	0.003 9
Value of relationship dimension ($n \neq \mathrm{const}$)	0.000 610	0.008 10	0.014 1	0.137 0	0.006 8	0.004 3

Conclusion

We considered the response characteristics of rotor with partial impact and friction regarding the nonlinearity of impacting force and estimated the influence of the nonlinearity of impacting force on the response characteristics of rotor.

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