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Evaluation of the Acoustic Radiation Impedance by Interaction between Vibration Modes on the Aperture of Cylindrical Waveguide

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Abstract We numerically analyzed and evaluated quantitatively the interactions between each vibration modes based on the principle for calculating the Rayleigh integral in case that the size of the horn aperture is reasonably larger than wavelength, and determined the mutual radiation impedances and the accuracy of planar wave approximation when sound wave is emitted through a horn or a tube.

Key words radiation impedance, cylindrical waveguide, horn-type tube, vibration mode

Introduction

The great leader Comrade **Kim Jong Il** said as follows.

"**Scientists and technicians must provide satisfactory solutions to the scientific and technical problems arising in making the national economy Juche-orientated, modern and scientifically-based.**"("**KIM JONG IL** SELECTED WORKS" Vol. 10 P. 194)

In case that acoustic wave is emitted through horn or waveguide the aperture can be considered as a plan piston radiator if the size of aperture surface is smaller than wavelength, but otherwise it can't be. Although there is an example where sound radiation from the aperture of the rectangular horn was evaluated based on the reference wave theory[1], horn aperture is mostly considered as a piston radiator because of complexity of calculation. On the other hand, the mutual radiation impedance between elements on the acoustic array has been discussed by considering vibration amplitudes distribution on the antenna elements to be uniform in general [2], but there are no examples related to one between vibration modes.

In this paper we express sound field in the horn tube as reference waves, and suggest the method for calculating the interactions between each modes based on the numerical integral and the calculation principle of Rayleigh integral to evaluate the efficiency of sound radiation from circular aperture in case that sound wave is emitted through the tube like a horn.

1. Analysis Theory

For the forward wave, n^{th} solution of cylindrical wave is as follows.

$$p_n(r, \varphi, z, t) = (A_n \cos n\varphi + B_n \sin n\varphi) \cdot J_n(\mu r) e^{i(\omega t - \sqrt{k^2 - \mu^2} Z)} \tag{1}$$

If the vibration of sound source is symmetric, only the $n = 0$ mode is presented. Thus its sound field can be expressed as follows.

$$p_0(r,z,t)=A_0J_0(\mu r)e^{i(\omega t-\sqrt{k^2-\mu^2}Z)} \tag{2}$$

Assuming that wall of guide with radius of a is hard and considering the boundary condition that normal velocity is 0,

$$u_r\,|_{r=a}=\frac{i}{\rho\omega}\frac{\partial p}{\partial r}\,|_{r=a}=0\,, \tag{3}$$

μ has discrete values $\mu_m=x_m/a$, where x_m are as follows.

$$x_m=0,\ 3.831\,7,\ 7.015\,6,\ 10.173\,5,\ 13.323\,7,\ 16.470\,6,\ 19.615\,9,\ 22.760\,1,\ 25.903\,7,\ 29.046\,8,\ \cdots$$

Considering backward wave as well as forward wave and generalizing for various μ_m, pressure caused by M vibration modes can be written as follows.

$$P=\sum_m^M J_0(\mu_m r)\left(A_m e^{-i\sqrt{k^2-\mu_m^2}Z}+R_m e^{i\sqrt{k^2-\mu_m^2}Z}\right) \tag{4}$$

Assuming that the aperture of circular tube is a symmetric planar radiator and applying the Kirchhoff's formula can yield the radiation condition on the aperture [3]. The radiation condition on the point p on the aperture is

$$P(p)=\frac{i\omega\rho}{2\pi}\iint U_n\frac{e^{-ik|\boldsymbol{r}_s-\boldsymbol{r}_p|}}{|\boldsymbol{r}_s-\boldsymbol{r}_p|}ds=-\frac{1}{2\pi}\iint\frac{\partial P}{\partial n}\frac{e^{-ik|\boldsymbol{r}_s-\boldsymbol{r}_p|}}{|\boldsymbol{r}_s-\boldsymbol{r}_p|}ds \tag{5}$$

where S is area of the aperture, $\boldsymbol{r}_s$ is position vector of integral element on the surface S and $\boldsymbol{r}_p$ is position vector of the point p. Substituting equation (4) into (5) results in the expression is

$$\begin{aligned}&\sum_m^M J_0(\mu_m r)\left(A_m e^{-i\sqrt{k^2-\mu_m^2}Z_N}+R_m e^{i\sqrt{k^2-\mu_m^2}Z_N}\right)=\\&=-\frac{1}{2\pi}\iint_S\sum_n^M\left[-i\sqrt{k^2-\mu_n^2}\left(A_n e^{-i\sqrt{k^2-\mu_n^2}Z_N}-R_n e^{i\sqrt{k^2-\mu_n^2}Z_N}\right)\right]J_0(\mu_m r)\frac{e^{-ik|\boldsymbol{r}_s-\boldsymbol{r}_p|}}{|\boldsymbol{r}_s-\boldsymbol{r}_p|}ds\end{aligned} \tag{6}$$

where Z_N is coordinate of the aperture. Multiplying mode function into both sides, integrating the expression over the surface and making use of the orthogonality of the modal functions yields,

$$\begin{aligned}&\left(A_m e^{-i\sqrt{k^2-\mu_m^2}Z_N}+R_m e^{i\sqrt{k^2-\mu_m^2}Z_N}\right)a^2J_0^2(\mu_m a)/2=\sum_n\frac{i}{2\pi}\sqrt{k^2-\mu_n^2}\left(A_n^{(N)}e^{-i\sqrt{k^2-\mu_n^2}Z_N}-\right.\\&\left.-R_n^{(N)}e^{i\sqrt{k^2-\mu_n^2}Z_N}\right)\cdot\iint J_0(\mu_m r)\left(\iint J_0(\mu_n r)\frac{e^{-ik|\boldsymbol{r}_s-\boldsymbol{r}_p|}}{|\boldsymbol{r}_s-\boldsymbol{r}_p|}ds\right)ds'\end{aligned} \tag{7}$$

Quadruple integral in the right hand side of the expression (7) means radiation impedance. Principle of calculation of Rayleigh integral can be applied in order to avoid the long time problem in calculating quadruple integral.

Extracting the terms which means radiation impedance we can obtain,

$$F(m,n)=\iint J_0(\mu_m r)\left(\iint J_0(\mu_n r)\frac{e^{-ik|\boldsymbol{r}_s-\boldsymbol{r}_p|}}{|\boldsymbol{r}_s-\boldsymbol{r}_p|}ds\right)ds' \tag{8}$$

This integral represents the mutual radiation impedance caused by interaction between sound waves of m^{th} and n^{th} vibration modes. The fact that integral surface is a circle and distribution of sound pressure is axial symmetrical can leads to the equivalence of surface elements, ds and ds', in the integrals. Thus pressure on a certain area element, ds', on the radiation surface consists of summation of pressures from the vibrations of another element areas, ds. Owing to the symmetry, the distribution of sound pressure is symmetrical with respect to the centre, and therefore the sound pressure on the element area, ds', away from the centre of radiation surface is related only to the distance b.

Each surface elements in expression (8) are coupled with another two times, once as a radiation element and then as receiving element. The consuming time for numerical integral can be reduced by calculating the integral with only once coupling as in the Rayleigh calculation[5] and doubling it.

Symmetry of the integrand with respect to the surface element in equation (8) leads,

$$F(m, n) = 2 \cdot \int_0^a J_0(\mu_m r) \cdot I'(r) \cdot r dr \tag{9}$$

$$I'(b) = \int_{-\pi/2}^{\pi/2} d\theta \int_0^{2b\cos\theta} J_0\left(\mu_n \sqrt{r_1^2 - 2r_1 b\cos\theta + b^2}\right) e^{-ikr_1} dr_1 \tag{10}$$

In calculation of equation (10), we carried out the interval transformation as shown in Fig. 1, integrating only on the inner area of element ds relative to element area ds' away from center.

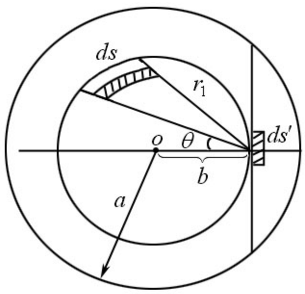


Fig. 1. Relationship between surface integral elements on the aperture

2. Analysis Result

In case that there are several kinds of wave modes the expression (8), which means mutual radiation impedance between individual vibration modes on the horn aperture, is evaluated using the numerical integral method. Self-radiation resistance and self-radiation reactance are compared with radiation impedance of a circular piston radiator as shown in Fig. 2.

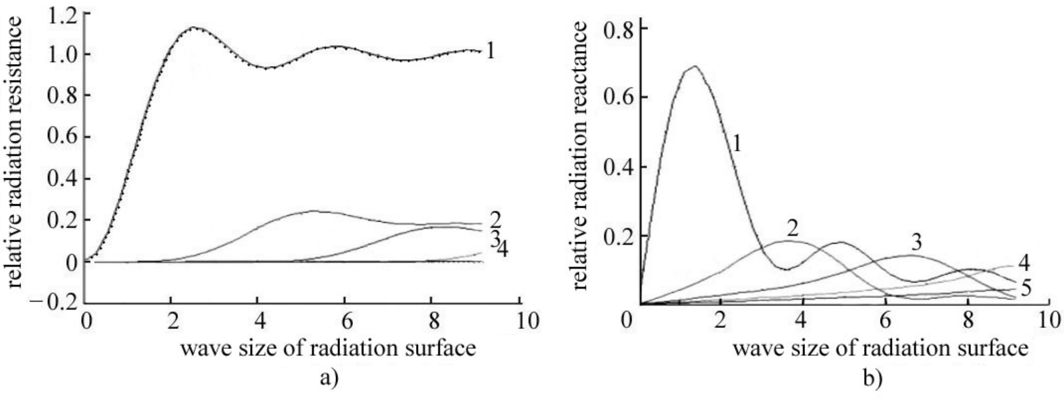


Fig. 2. The self radiation impedance for several vibration modes

a) radiation resistance, b) radiation reactance, 1−5 are the case that the order of vibration mode is from 1 to 5.

From the Fig. it can be seen that the self-radiation impedance of first vibration mode, the planar wave mode, is completely coincident with the radiation impedance of circular piston radiator in general and that the simulation is agreed with the theory.

As shown in Fig. 2, the radiation impedance on the circular aperture caused by each vibration mode can be characterized as bellows:

First, the effective radiation impedances for higher vibration modes have the same behavior as for planar wave mode, which increases with wave size of radiator surface, reaches a certain value and then becomes stable, but its peak is presented for the higher wave size and its magnitude is smaller than the one for planar mode.

Second, the reactances for higher vibration modes increase with wave size of radiator surface and then decrease to 0, and its peak is also presented for the higher wave size and its magnitude is smaller than the one for planar mode. This means that the radiation impedance of higher vibration modes can be neglected when the wave size of radiator surface is small, i.e., when the absolute size of radiator surface is smaller than wavelength, otherwise it can’t be.

Table 1 shows ratios between self-radiation resistance for planar plan wave vibration mode in each vibration mode.

Table 1. The ratios of self radiation impedance for various vibration modes relative to plane wave vibration mode (%)

Mode	1	2	3	4	5
Resistance	100	21.5	14.8	3.7	0.2
Reactance	100	26.6	20.3	16.7	6.4

As seen in table 1, the resistances of second and third modes are 21.5% and 14.8% as compared with first one respectively, and for higher than 5^{th} mode it is smaller than 0.2%.

For higher frequency each modes vibrate in maximum and thus the additional factor of radiation impedance enhances acoustic power. But its role is relatively small comparing with planar radiator.

Fig. 3 and 4 show the mutual radiation impedances between self-radiation impedances and high order vibration mode for first and second vibration modes.

Radiation impedances in Fig. 2－4 are expressed as normalized values relative to the one of infinite tube.

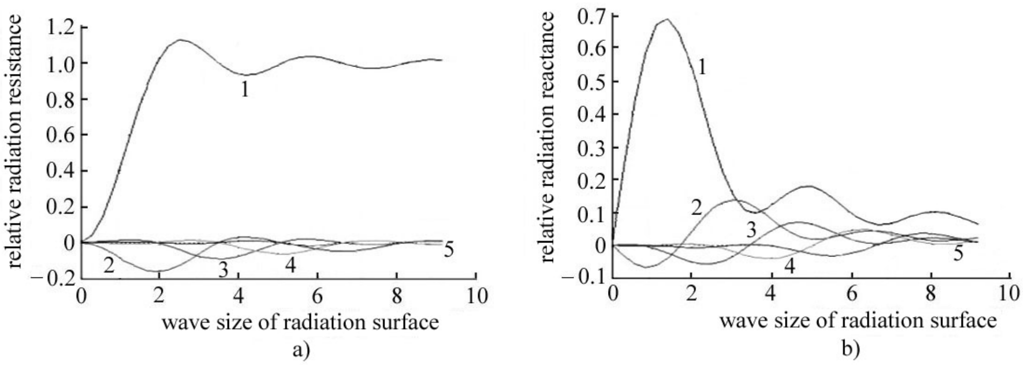
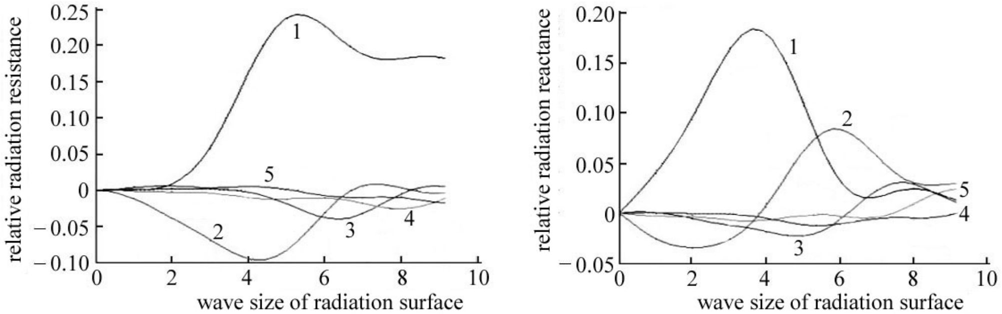


Fig. 3. Mutual radiation impedance between first mode and higher modes
a) radiation resistance, b) radiation reactance, 1－5 are the case that the order of vibration mode is from 1 to 5.

Mutual radiation impedance between several modes has characteristics different from the one previously described as below:

First, the mutual radiation impedance between various modes may be negative unlike self-radiation impedance. This means that energies of acoustic waves are transferred between several vibration modes and that effectiveness of radiation is reduced.

Second, the mutual radiation impedance between different modes is much smaller relative to the self-radiation impedance. And its ratios are 18% at most and 10% for two modes with order interval more than 2.



a) b)

Fig. 4. The mutual radiation impedance between 2nd mode and higher order modes
a) radiation resistance, b) radiation reactance, 1−5 are the case that the order of vibration mode is from 1 to 5.

Table 2 gives the ratios between self-radiation impedance of individual vibration mode and mutual radiation impedances with higher modes.

Table 2. The ratios of self radiation impedance and mutual radiation impedances with higher modes for several modes(%)

Foundational mode		Active mode						
		1	2	3	4	5	6	7
1	Resistance	100	14.3	7.9	5.4	4.2	3.4	2.1
	Reactance	100	20	10	6.8	5.2	4.0	3.3
2	Resistance		100	39.7	16.6	10.6	7.4	3.8
	Reactance		100	45.6	16.6	13.1	6.8	4.6
3	Resistance			100	50.6	18.8	8.9	3.7
	Reactance			100	51.6	16.8	6.1	1.1

Conclusion

From the results it can be concluded that uncertainty of supposing the aperture of the horn or tube as piston radiator in estimating acoustic radiation is below about 20～30 percents although there are differences according to wave size of radiation surface. Among them, uncertainty of the self radiation impedance for every vibration modes is below 27 percents, the one of mutual radiation impedance is usually below 18 percents. Particularly it is below 10 percents for the order interval more than 2.

References

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