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Joint Inversion of Self-Potential (SP) and Magnetic Anomalies due to 2D Dike

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The earlier research papers[3] dealt with only theoretical relationship between magnetic anomaly and SP anomaly, and did not propose joint inversion. This paper newly proposed joint inversion of magnetic anomaly and SP anomaly due to 2D dike and analyzed its properties through model experiment.

1. Joint Inversion method of Magnetic and Self-Potential (SP) Anomalies due to 2D dike

When one body generates both SP and magnetic anomalies, let's suppose that that magnetic body can be represented by m_t parameters and SP polarization body by m_s parameters and m_r among m_t and m_s is repeated.

When expressing the parameter of magnetic body as $P^{(t)}$ and parameter of SP polarization body as $P^{(s)}$, the parameters of magnetic and SP polarization body can be expressed as $\boldsymbol{P}=\boldsymbol{P}^{(t)}\bigcup\boldsymbol{P}^{(s)}=\{P_1, P_2, \cdots, P_m\}$. That is

$$\begin{array}{l} \boldsymbol{P}^{(t)}=\{P_1^{(t)},\ P_2^{(t)},\ \cdots,\ P_{m_0+1}^{(t)},\ \cdots\ ,\ P_{m_t}^{(t)},\ 0,\ \cdots,\ 0\} \\ \underline{+\boldsymbol{P}^{(s)}=\{0,\ \cdots,\ 0,\ \cdots,\ 0,\ P_1^{(s)},\ P_2^{(s)},\ \cdots,\ P_{m_e}^{(s)},\cdots,P_{m_S}^{(s)}\}} \\ \boldsymbol{P}=\{P_1,\ P_2,\cdots,\ P_{m_0+1},\cdots,P_{m_e},\cdots,P_m\} \end{array} \tag{1}$$

where $m=m_t+m_s-m_r$ is number of magnetic and SP polarization body, m_0 number of zero elements among parameters of SP polarization and m_e number of parameter of magnetic body, and the relationship of $m_r=m_e-m_0$ is established.

Problem of finding parameter $\boldsymbol{P}$ of magnetic and SP polarization body from measurements of full magnetic anomaly and SP anomaly results in problem finding model parameter $\boldsymbol{P}$ minimizing the following object function:

$$\varPhi(\boldsymbol{P})=\|\boldsymbol{W}^{(s)}[U_{측}-U(\boldsymbol{P})]\|^2+\|\boldsymbol{W}^{(t)}[\Delta T_{측}-\Delta T(\boldsymbol{P})]\|^2+\alpha\|\boldsymbol{W}^{(P)}(\boldsymbol{P}-\boldsymbol{P}_0)\|^2 \tag{2}$$

where, $U(\boldsymbol{P})$ and $\Delta T(\boldsymbol{P})$ are theoretical SP anomaly and magnetic anomaly respectively corresponding to model parameters, $\boldsymbol{W}^{(s)}$ and $\boldsymbol{W}^{(t)}$ SP and magnetic weight matrixes respectively given by $\boldsymbol{W}_{jj}^{(s)}=C^{(s)}/U_{측}$ and $\boldsymbol{W}_{ii}^{(t)}=C^{(t)}/\Delta T_{측}$, $C^{(t)}$ and $C^{(s)}$ covariance matrixes for magnetic anomaly and SP anomaly respectively and in case these are not known we take these as unit.

Expanding formulas of SP and magnetic anomalies to Tyler series around initial approximate vector P_0, truncating high order terms for parameter perturbation and minimizing object function $\Phi(\boldsymbol{P})$ with respect to $\Delta\boldsymbol{P}_k$, following equation is obtained:

$$[\boldsymbol{A}_k^T\boldsymbol{W}^{(s)T}\boldsymbol{W}^{(s)}\boldsymbol{A}_k+\boldsymbol{B}_k^T\boldsymbol{W}^{(t)T}\boldsymbol{W}^{(t)}\boldsymbol{B}_k+\alpha_k\boldsymbol{I}]\Delta\boldsymbol{P}_k=A_k^T\boldsymbol{W}^{(s)T}\boldsymbol{W}^{(s)}\boldsymbol{d}_k^{(s)}+\boldsymbol{B}_k^T\boldsymbol{W}^{(t)T}\boldsymbol{W}^{(t)}\boldsymbol{d}_k^{(t)} \quad (3)$$

where, subscripts s and t in brackets express SP anomaly and magnetic anomaly respectively. And subscript k is iterative number, α control parameter, $\Delta\boldsymbol{P}_k$ parameter perturbation, I unit matrix, $\boldsymbol{A}$ and $\boldsymbol{B}$ matrixes of sensitivities of SP anomaly and magnetic anomaly respectively, elements of which are as follows:

$$a_{ji}=\begin{cases}0, & j=\overline{1,\, m_0}\\ \partial U_i(\boldsymbol{P})/\partial\boldsymbol{P}_j\,, & j=\overline{m_0+1,\, m}\end{cases} \quad (4)$$

$$b_{ji}=\begin{cases}\partial\Delta T_i(\boldsymbol{P})/\partial\boldsymbol{P}_j\,, & j=\overline{1,\, m_e}\\ 0\quad, & j=\overline{m_e+1,\, m}\end{cases} \quad (5)$$

After solving equation (3) and finding parameter perturbation $\Delta\boldsymbol{P}_k$, the next step solution is calculated by

$$\boldsymbol{P}_{k+1}=\boldsymbol{P}_k+\Delta\boldsymbol{P}_k \quad (6)$$

This iteration is repeated until object function $\Phi(\boldsymbol{P})$ converges to given error limit.

Magnetic anomaly $\Delta T(x)$ and SP anomaly $U(x)$ over the 2D dike as seen in Fig are expressed respectively [1, 2].

$$\Delta T(x)=\frac{\mu_0 M_t\sin\alpha}{2\pi}\cdot\frac{\sin I}{\sin i_S}\left\{\cos(\alpha-2i_S)\ln\frac{r_B\cdot r_C}{r_A\cdot r_D}-\sin(\alpha-2i_S)[\varphi_A-\varphi_B-\varphi_C+\varphi_D]\right\} \quad (7)$$

$$\begin{aligned}U(x)=M_s\{&(h+l\sin\alpha)(\varphi_C-\varphi_D)+(x-x_0-l\cos\alpha-b)\cdot\ln r_C-\\&-(x-x_0-l\cos\alpha+b)\cdot\ln r_D-h(\varphi_B-\varphi_A)-(x-x_0-b)\ln r_B+(x-x_0+b)\ln r_A\}\end{aligned} \quad (8)$$

where x is horizontal position of measurement point, M_s dipole moment of SP polarization body, h and x_0 depth and horizontal position of the dike respectively, b half thickness of the dike, α inclination angle of dike, μ_0 absolute permittivity of vacuum, I the inclination of the earth's magnetic field, i_s effective magnetic inclination, M_m magnetic moment of the magnetic dike, r_A, r_B, r_C, r_D distances between measurement point M and points A, B, C and D respectively, φ_A, φ_B, φ_C, and φ_D angles between vertical line going down from measurement point M and sight lines looking points A, B, C and D (Fig.).

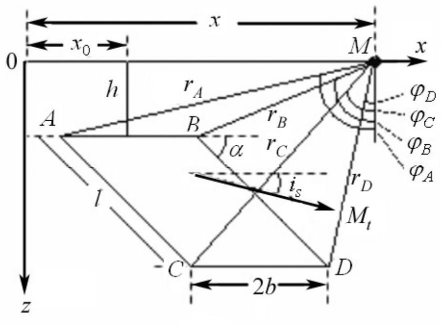

Fig. 2D dike model

So we must determine parameter of 2D dike as the following:

$$\boldsymbol{P}=\{M_t,\, h,\, x_0,\, b,\, l,\, \alpha,\, M_s\}$$

2. Properties of Joint Inversion

We estimated the properties of the proposed method through model experiment.

In model experiments, interval between measurement points was 5m and the numbers of those were 30 respectively and therefore total numbers of those (sum of SP measurements and magnetic measurements) were 60. To simulate field environment, we added random noises corresponding 5% of anomaly value in each measurement points to theoretical SP and magnetic anomalies, and iteration was finished when relative error was 5%.

The results of joint inversion, inversion of only SP anomaly and inversion of only magnetic anomaly are shown in the following table. Here, parameters of 2D dike and its initial approximation are selected so that those are the same as each other for joint inversion and individual inversions.

Table. Results of joint inversion and individual anomaly inversion

Parameter	True value	Initial approximation	Joint inversion	Magnetic inversion	SP inversion
$M_t/(\times 10^{-9}\mathrm{A\cdot m^{-1}})$	5	10	5.1	−11.5(divergence)	−
h/m	20	5	19.6	64.7(divergence)	22.4
x_0/m	75	30	74.7	−148.9(divergence)	75.9
$2b$	5	2	4.7	−9.7(divergence)	3.0
$\alpha/(^\circ)$	30	10	31.8	−5(divergence)	24.5
l/m	60	30	59.7	111.4(divergence)	60.2
$M_s/(\times 10^{-3}\mathrm{V\cdot m^2})$	5	10	5.1	−	10.0
Model relative error /%			2.7	(divergence)	28.64

As seen in the table, inversion using only magnetic anomaly could not find correct model parameter and diverged, whereas inversion using only SP anomaly found model parameter with model relative error of 28.64%. But joint inversion found comparatively correct model parameter with model relative error of 2.7%.

Conclusion

We established the method of joint inversion of magnetic and self-spontaneous anomalies over 2D dike and estimated its properties. We have verified throughout model experiment that our joint inversion has wide radius of convergence and little polyvalence than individual inversions of magnetic anomaly or self-spontaneous anomaly. Our joint inversion can be effectively used in simultaneously interpreting magnetic and self-spontaneous anomalies measured over 2D dike and clarifying the parameters.

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