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A Reserves Calculation Method by Disjunctive Kriging

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Abstract Disjunctive kriging has been known as one of non-linear geostatistics which is more efficient than linear geostatistics and the non-linear estimator estimated by that is more correct than the linear estimator [3].
In this paper, we suggested a method for calculating reserves using the Gaussian disjunctive kriging, which is the most common type of disjunctive kriging.
In order to calculate reserves more accurately and more conveniently, we set the size of estimate block at the reserves calculating area in consideration of the influence range given from the calculation result of experimental variogram.
We used only blocks of which estimates given by progressing the disjunctive kriging estimation are larger than the lowest industrial standard value on every block for calculating reserves.
This method improved the reserves calculation by the common kriging is the one that can calculate the recoverable reserves more accurately using the disjunctive kriging.
Key words disjunctive kriging, reserves

Introduction

In the geological and environmental application fields, they set a goal to get the accurate estimation from them by progressing probability estimate for the useful minerals or contaminants to exceed thresholds value given at unsampled places [2].

Disjunctive kriging(DK) has been known as the one of non-linear geostatistics which is more efficient than linear geostatistics and the non-linear estimators given by this method is more correct than linear estimators.

One researcher suggested a method that can estimate the spatial variety features of the ore body by outputting the estimate values of the ore body as a plane figure using the geostatistic analysis of tool of the ArcGIS [6].

Otherwise, some researchers studied the geostatistical theory and 3D geo-simulation technology which designed and developed the reserves calculation program combined with them, and then verified the performance and the estimation efficiency of this program [7].

In recent years, they achieved some successes to estimate the pollution level of the interested area by Disjunctive Kriging in the environmental science field [4] and some researchers verified the advantage and the accuracy of the estimating method of probability density function by the Hermite polynomials expansion and studied the coding principles and the system for disjunctive krigng estimation by using the Matlab software [5].

When there is a standard value to define the industrial condition of the mining for the grade of the useful component or thickness and depositing cost of ore and etc., the probability estimation of whether the case is greater or lower than the standard value the interested area has the most important practical significance in calculating the reserves correctly, dividing the mining blocks larger than the measures and estimate values, and in progressing the design for mining and optional mining.

In this paper, we suggested a method for calculating the reserves by using the Disjunctive kriging so that we can raise the correctness of the reserve calculation; we applied this method to some deposit to verify the correctness and effectiveness.

1. Reserves Calculation Method by the Disjunctive Kriging

1.1. Principle and estimation method of disjunctive kriging

Disjunctive kriging may take several forms, the most common of which is Gaussian disjunctive kriging[4].

The first task is to transform the actual distribution, which may have almost any form, to a standard normal one, $Y(x)$, such that

$$Z(x)=\phi[Y(x)] \tag{1}$$

The Hermite polynomials can be written as follows.

$$H_k(y)=-\frac{1}{\sqrt{k}}yH_{k-1}(y)-\sqrt{\frac{k-1}{k}}H_{k-2}(y) \tag{2}$$

Any function of $Y(x)$ can be represented as the sum of this Hermite polynomials.

$$Z(x)=\phi[Y(x)]=\sum_{k=0}^{\infty}\phi_k H_k\{Y(x)\} \tag{3}$$

By kriging them separately the estimates have only to be summed to give the disjunctive kriging estimator.

$$\hat{Z}^{\mathrm{DK}}(x)=\phi_0+\phi_1\hat{H}_1^k\{Y(x)\}+\ \phi_2\hat{H}_2^k\{Y(x)\}+\cdots \tag{4}$$

If we have N points in the neighborhood of x_0 where we want an estimate, we calculate by following equation, and then we insert them into equation (4).

$$\hat{H}_k^k\{Y(x_0)\}=\sum_{i=1}^{n}\lambda_{ik}H_k\{Y(x_i)\} \tag{5}$$

The λ_{ik} are the kriging weights, which are found by solving the equations for simple kriging because we can assume the mean is known

$$\hat{Z}^{\mathrm{DK}}(x_0)=\phi[\hat{Y}(x_0)]=\phi_0+\phi_1[\hat{H}_1^k\{Y(x_0)\}]+\phi_2[\hat{H}_2^k\{Y(x_0)\}]+\cdots \tag{6}$$

$\hat{Z}^{\mathrm{DK}}(x_0)$, which is the kriging estimator at x_0, can be estimated by (6).

The estimation variance can calculate by followings.

The kriging variance of $\hat{H}_k\{Y(x)\}$ is

$$\delta_k^2(x_0)=1-\sum_{i=1}^{n}\lambda_{ik}\rho^k(x_i-x_0) \tag{7}$$

And the disjunctive kriging variance of $\hat{f}[Y(x_0)]$ is

$$\delta_{\mathrm{DK}}^2(x_0)=\sum_{i=1}^{\infty} f_k^2\delta_k^2(x_0) \tag{8}$$

When Hermite polynomials have been estimated at a target points, we can estimate the conditional probability that the true value there exceeds the critical value, Z_c.

The indicator can be expressed by the cumulative distribution and the Hermite polynomials.

$$\Omega[Y(x)\le Y_c]=G(y_c)+\sum_{k=1}^{\infty}\frac{1}{\sqrt{k}}H_{k-1}(y_c)g(y_c)H_k\{Y(x)\} \tag{9}$$

And the estimate for Disjunctive kriging can be estimated by follows.

$$\hat{\Omega}^{DK}[y(x_0)\le y_c]=G(y_c)+\sum_{k=1}^{L}\frac{1}{\sqrt{k}}H_{k-1}(y_c)g(y_c)\hat{H}_k^k\{Y(x_0)\} \tag{10}$$

Conversely, hyper probability can be calculated as follows.

$$\hat{\Omega}^{DK}[Z(x_0)>Z_c]=\hat{\Omega}^{DK}[y(x_0)>y_c]=1-G(y_c)-\sum_{k=1}^{L}\frac{1}{\sqrt{k}}H_{k-1}(y_c)g(y_c)\hat{H}_k^k\{Y(x_0)\} \tag{11}$$

This can be estimated the probability to exceed the lowest industrial standard value such as lowest industrial standard grade, the lowest industrial standard thickness in the unsampled points of the study area.

Through calculation process like this, we can progress disjunctive kriging estimation and calculate the reserves by using estimator given from them.

1.2. Determination of the reserves calculation index by disjunctive kriging

In order to calculate the reserves correctly, we must determine main index such as area, thickness, gravity and grade for the calculation of reserves. [1]

The method for determination of the reserves calculation index by disjunctive kriging is as followings.

① Based on the definite investigation of the geological formation and prospecting state in the deposit, it should correctly select main index to calculate the reserves in every ore bodies, levels and regions.

Reduced scale of the geological map used in reserves calculation should be usually 1∶5 000 or 1∶2 000, 1∶500, rarely 1∶10 000.

② It evaluates the characteristic of statistical distribution of the prospecting data.

It synthesizes the basic data to estimate, then, major regionalized variance should be selected as the index used in reserves calculation. It should estimate the statistical characteristic for the regionalized variance and transform to normal distribution by examination of the distribution characteristic of the measured data.

③ It should perform disjunctive kriging estimation for the reserves calculation index, major regionalized variance.

It should perform normalization for the measured data in sample location such as well and tunnel.

The normalization of standard normal distribution type for the measured data should be normally performed as follows.

As inserting to the following equation the arithmetic mean (μ), and variance (δ), data measured at sample point, it transforms corresponding $Y(x)$ to every measured point.

$$y = \frac{z - \mu}{\delta} \tag{12}$$

Hermite transformation can be calculated by Hermite polynomials using equation (2), then performed variogram estimation for them and determined the suitable theoretical variogram model. The order of Hermite polynomials should be determined by empirical method, which calculates form 5 to 7.

As it can perform the simple and ordinary kriging estimation on every Hermite polynomials by using equation (5), the Hermite coefficients should be calculated as well as polynomials for estimation of .the disjunctive kriging.

The disjunctive kriging estimate value for every block can be calculated by using (6) and disjunctive kriging variance and the conditional probability can be calculated by using (8), (11).

1.3. Reserves calculation

We determined the size of the estimation block to consider the influence range of the theoretical variogram given during disjunctive kriging estimation process and separated the study area according to that size, and did the disjunctive kriging estimate for every block.

Then, the reserves of the blocks that its grade has greater one than the lowest industrial standard value can calculate the reserves and sum up them to calculate total reserves, so we can determine the boundary of ore bodies differently with other pre-method, using the method, we established the method to calculate the recoverable reserves for study area.

In this paper, we used only the grade and the thickness for reserves calculation of study area.

① It can calculate the estimate value of the grade and the thickness, the conditional probability for the estimate blocks through the disjunctive kriging estimate above.

First, study area can be divided into blocks with uniformed size in consideration of influence range, reduced scale of map and density of prospecting net given by variogram estimation and perform the disjunctive kriging estimation for every block.

Then, it can be got the estimation map of them by the grade and the thickness, reserves calculation index that is calculated by progressing the disjunctive kriging estimation at study area, and it can be got map of conditional probability estimated the conditional probability more than industrial standard of them in every block.

It can select the grade, thickness and conditional probability for the block more than the lowest industrial standard value, and then calculate for the block satisfied the lowest industrial standard respectively.

② It can calculate the reserves of every block with the selected estimator, then calculate the total amounts of reserves of given ore body to sum them up.

The volume (V_i) of given block ore body is calculated by the thickness (M_i) of area given by disjunctive kriging estimation for every block. (square measure: S)

$$V_i = S \times M_i \tag{13}$$

($i = \overline{1, n}$: the number of blocks)

The reserves of ore can be determined by the following expression. (d is gravity)

$$Q_i = V_i \times d \tag{14}$$

The ore reserves more than industrial standard is determined by the following expression.

$$Q_{iC} = Q_i \times \hat{\Omega}^{DK}[Z(x_i) \geq Z_c] \tag{15}$$

$\hat{\Omega}^{DK}[Z(x_i) \geq Z_c]$: the conditional probability

The amount of metals (C_{iC}) more than industrial standard can be calculated with grade (C_i) of estimated block and conditional probability, and then the metals reserves (P_{iC}) of block can be calculated with times to ore reserves of corresponding block.

$$C_{iC} = C_i \times \hat{\Omega}^{DK}[Z(x_i) \geq Z_c] \tag{16}$$

$$P_{iC} = C_{iC} \times Q_i \tag{17}$$

Through calculation for the amount of metals (C_{iC}) and the amount of ore in each block, it can calculate recoverable reserves of the study area.

$$Q_C = \sum_{i=1}^{n} Q_{iC} \tag{18}$$

$$P_C = \sum_{i=1}^{n} P_{iC} \tag{19}$$

The algorithm for this method is as Fig.

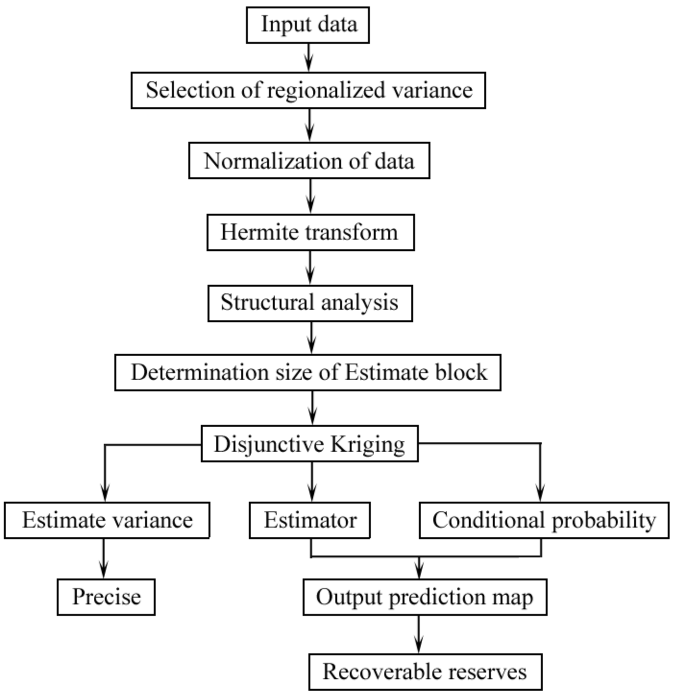

Fig. Algorithm of reserves calculation by Disjunctive kriging

2. Reserves Calculation of the Study Area and Its Estimation

2.1. Reserves calculation of the study area

We estimated the recoverable reserves for the Ponsan area of “ㅎ” deposit using the reserves calculation suggested in this paper.

We got the estimate values and the conditional probability given by disjunctive kriging estimation on the grade and the thickness of the 3rd ore body which is the main one in the deposit, and we calculated the recoverable reserves of the 3rd ore body using them.

In the paper, we calculated the reserves according to the well arrangement map of the reduced scale 1 ∶ 5 000.

The 185 wells which exist in the ore body area, range in the rectangle area between (125 000, 49 000) and (121 000, 42 500) of relative coordinates.

From the analysis data, it is known that there are Au, Ag, Pb, Zn elements in the 3rd ore body, but we synthesized only the analysis data of the Au element's content and the thickness data and used the average bulk density 3t/m^3 estimated in the ore deposit.

In this paper, we selected the grade and the thickness of the element, Au as the main regionalized variable to calculate the reserves of Au in the 3rd ore body area.

The recoverable reserves that we are going to calculate are taken from the estimation on the grade and the thickness of the ore body regardless of the industrial properties such as the hydrogeological and mining conditions of the deposit area.

First, we performed the normalization to transform the data into standard normal distribution type.

Then we calculated the Hermite transformed values according to the normalized values and got the experimental variogram from the Hermite ones at the sample points.

In the direction of strike and slope respectively, experimental variogram on 25 lag numbers do not differ largely and their tendency is relatively same.

Generally, the strike of the 3rd ore body is northwest and the direction of the slope is southwest.

We took the result that selecting the spherical model with nugget effect as the theoretical variogram model is more suitable and due to the consistence of the experimental variogram in the direction of strike and slope, we determined their parameters by the weighted polynomial regression and establish the isotropic spherical model by approximating every experimental variogram..

The parameters determined from this are as follows.

Grade: $c_0 = 1.839\ 8$, $c = 0.525\ 4$, $a = 156\ 9$

Thickness: $c_0 = 0.084\ 6$, $c = 0.641\ 8$, $a = 66$

$$\gamma_1(h) = 1.839\ 8 + 0.525\ 4 \times \mathrm{sph}(159\ 6) \tag{20}$$

$$\gamma_2(h) = 0.084\ 6 + 0.641\ 8 \times \mathrm{sph}(66) \tag{21}$$

where, $\mathrm{sph}(159\,6)$ means that spherical model with influence range of 1 596m.

So theoretical variogram model for Hermite transformed value of the grade and the thickness is equation (20), (21).

We fitted theoretical variogram above model and calculated the disjunctive kriging estimator and variance of the block for estimation, and the conditional probability of every block.

Before estimation, it is important to determine the size of the estimation block.

In the paper, we set up the square block as the estimate block, of which one side is smaller than the influence range on the basis of the empirical method of the previous theory, because of the none existence of the anisotropy in the direction of strike and slope, and estimated them by disjunctive kriging.

That is, as the influence range of the thickness is smaller than the one of grade, in consideration of the one of the thickness we set up the square block nets of 50m×50m and estimated the qualities and the thicknesses of all blocks by disjunctive kriging.

We estimated the Au grade and the vertical thickness of the ore body in every block using disjunctive kriging and calculated the average grade and thickness by log-inverse transform because of log-transform of the original data over the estimators.

With the results by this estimation, we calculated the Au reserves of the ore body per block according to the reserve calculating method suggested in the paper.

In the paper we selected only the blocks of which grade is larger than the lowest industrial standard, calculated and summed up the reserves of them with the disjunctive kriging estimators by them to estimate the reserves of the study area.

2.2. Comparison with previous geo-statistics methods

2.2.1. Estimation of correctness by comparing the estimation variance

In general, which estimate method is more correct, is determined by the estimation variance in relation to that one. That is, the method has less error in all blocks of the deposit is more accurate.

Comparing the estimation variance by disjunctive kriging, ordinary kriging, simple kriging and indicator kriging, we can see that the estimation variance by disjunctive kriging is less than other krigings(table 1).

Table 1. Comparison of the estimation variance by the disjunctive kriging and previous geo-statistics

Division	Disjunctive kriging	Ordinary kriging	Simple kriging	Indicator kriging
Variance of estimated values	7.936	10.310	8.276	—
Variance of the conditional probability	0.452 1	—	—	0.492 1

As you see in table 1, we can think that disjunctive kriging is a good estimate method that can make the calculating and estimating of the reserves of deposit more correctly, due to the less estimation variance than the previous geo-statistics.

2.2.2. Verification of the effectiveness by comparison with the reserves

In this paper, we set up the blocks of 50m×50m as one unit and estimated them by disjunctive kriging.

At first we calculated the ore recoverable reserves of the blocks whose grade is larger than the lowest industrial standard and summed them up to calculate the total recoverable ore reserves of the 3rd ore body in the study area.

And then we calculated the metal reserves of every block which has larger grade than the lowest industrial standard and summed them up to estimate the recoverable metal reserves.

Comparing the reserves by the disjunctive kriging with one by ordinary kriging, we can see that the later was over estimated than the earlier.

Comparing with the practical data, the later is 80.14% of the detail exploration reserves of this ore body area and the earlier are 68.79% of that (table 2).

Table 2. Comparative results of reserves

Current deposit data		Estimated recoverable reserves	
C_1-level reserves	Ratio of converting to recoverable reserves	Ordinary kriging	Disjunctive kriging
100%	71.17%	80.14%	68.79%

According to the synthesis of the prospecting data until now, recoverable reserves were about 71.17% detailed prospecting reserves.

As shown in table 2, you can see that the recoverable reserves which are calculated by the disjunctive kriging are very fit with the practical data.

Conclusion

In this paper, we suggested reserves calculation method of deposits using the disjunctive kriging, which is the one of the main methods of the geo-statistics, widely used in the mining industry, environment science and agriculture fields and applied this one to calculating the reserves of the 3rd ore body in the study area to estimate the correctness and the effectiveness of the reserve calculation by disjunctive kriging.

Especially, this method sets up the estimate block size in the reserve calculation area in consideration of the influence range taken from the results of the experimental variogram, and uses only for calculating reserves the blocks, of which the estimators are larger than the lowest industrial standard by disjunctive kriging of every block, so that it makes the reserves more accurately and effectively.

The comparison with previous geo-statistics shows that the reserves calculation suggested in this paper is more correct and effective, which is based on the disjunctive kriging.

References

[1] 배영수; 지질통계적방법에 의한 광석매장량평가, **김일성**종합대학출판사, 16～54, 1990.
[2] 김해룡; 유용광물매장량계산의 현대적방법과 그 응용, 공업출판사, 148～231, 1995.
[3] D. D. Sarrma; Geostatistics with Application in Earth Sciences, Springer, 139～150, 2009.
[4] R. Webster et al.; Geostatistics for Environmental Scientists, Second Edition, John Wiley & Sons, Ltd., 243～266, 2007.
[5] X. Emery; Computers & Geosciences, 32, 3, 965, 2006.
[6] 高峰 等; 勘探地球物理进展, 33, 1, 64, 2006.
[7] 张焱 等. 金属矿山, 41, 5, 93, 2011.