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Application of SWAN Model in the Design Wave Analysis

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Abstract We considered the analysis method of design wave field using the design wind on the ocean that there are not obvious observation data.

In case of the lack of the observation data on the ocean, we computes the design wave computes using the weather process which caused the largest wave historically or the design wind.

We determined the design wind on the ocean using NCEP data and analyzed the design wave with SWAN model.

SWAN model was simulated by the combination of Lax-Wenderoff and semi-emplicit schema.

Key words SWAN model, design wave, design wind, probability analysis

Introduction

It is very important to consider scientifically the design wave in building and operating the oceanic structures including seawall. Especially the building of the oil boring, the fishing and the military activity on the ocean are strengthened more and more, so the destruction of structures by wave and ships disaster are increasing more and more on the ocean. Generally, because of the observation data dissatisfied on the ocean, the design wave is calculated by wave field with dangerous weather process caused the highest wave or by using the design wind. We determined the design wind on the ocean using NCEP data, and then simulated the design wave field with SWAN model from it. SWAN(simulating wave near shore) model as the third generation wave numerical model is developed to simulate wave in near shore.

1. The Design Wind to Calculate the Marine Design Wave

We used Gumbel distribution function and Weibull distribution function, these functions are widely used in the estimation of design wind with return period up to 100 years [1].

1.1. Parameters in theoretical distribution function of extreme value

Gumbel distribution function and Weibull distribution function are as follows.

$$F(x) = 1 - \exp\left[-\left(\frac{x-a}{b}\right)^c\right] \tag{1}$$

$$F(x) = P\{x_{\max} < x\} = \exp\left[-\exp\left(\frac{x-a}{b}\right)^c\right] \tag{2}$$

where a, b, c is distribution parameters respectively.

Generally, distribution parameter is obtained by minimum square method in base of the transform of regression equation, but this method has shortcomings that only function transformed into linear has minimum error. However Gauss-Newton method solves directly without transformation process, so using this method we estimated the parameters of three variable distribution functions.

We determined with Gauss-Newton method the parameter of Weibull distribution function on the basis of annual maximum wind velocity series in 1980－2008 in our sea and used "Kolmogorove" method for its suitability.

The results which determine the suitability of theoretical distribution functions using the maximum annual wind data in 1976～2001 are as table 1.

Table 1. Suitability of theoretical distribution functions

Point	Gumbel				Weibull			
	Linear minimum square		Gauss-Newton		Linear minimum square		Gauss-Newton	
	D_n	std	D_n	std	D_n	std	D_n	std
"ㅇ"	0.08	0.04	0.05	0.03	0.06	0.03	0.05	0.03
"ㅂ"	0.14	0.06	0.11	0.06	0.17	0.07	0.11	0.06
"ㅁ"	0.09	0.04	0.07	0.04	0.10	0.04	0.07	0.04
"ㅈ"	0.10	0.04	0.08	0.04	0.09	0.04	0.09	0.04

D_n －maximum error(m/s), std－mean square error(m/s)

In case of significant level $\alpha = 0.05$, "Kolmogorove" critical value is equal to 0.275, $D_n < D_0$, so accepted the hypothesis that theoretical distribution function is equal to empirical distribution function.

As seen in table 1, maximum error is little 0.02 compared with Linear minimum square method. The error in case of Gauss-Newton method is little at nine points of Korean East Sea and Korean West Sea, so we used this method to determine the parameters of theoretical distribution functions.

1.2. Processing of wind data

The observation wind data on the ocean is very poor, thus we determined the mean maximum wind velocity during six hours with NCEP data in 1980－2008.

If two maximum wind velocity continued in observation wind data of six-hour intervals are W_1, W_2, then the mean maximum wind velocity during six hours for calculation of a tidal wave is as follows [1－4].

$$W = 0.3W_2 + 0.7W_1 , \quad W_2 > W_1 \tag{3}$$

$$W = 0.2W_2 + 0.8W_1 , \quad W_2 < W_1 \tag{4}$$

2. Numerical solution of SWAN model from combination schema

2.1. Combination schema

In SWAN the action density is as follows.

$$N = E/\sigma \tag{5}$$

where $E = f(\sigma, \theta, x, y, t)$ is wave energy spectrum, $\sigma = \sqrt{gk\tanh(kd)}$ is relative frequency, k is wave number, d is depth.

In case of small-scale the SWAN is as follows for Cartesian coordinates (4).

$$\frac{\partial N}{\partial t} + \frac{\partial C_{gx} N}{\partial x} + \frac{\partial C_{gy} N}{\partial y} + \frac{\partial C_{\theta} N}{\partial \theta} + \frac{\partial C_{\sigma} N}{\partial \sigma} = \frac{S}{\sigma} \tag{6}$$

where $N = f(\sigma, \theta, x, y, t)$ is action density, θ is propagation direction of wave.

At ambient depth the group wave velocity in x, y direction is as follows in current .

$$\left.\begin{aligned} C_{gx} &= \frac{dx}{dt} = \frac{1}{2}\left[1 + \frac{2kd}{\sinh(2kd)}\right]\frac{\sigma k_x}{k^2} + U_x \\ C_{gy} &= \frac{dy}{dt} = \frac{1}{2}\left[1 + \frac{2kd}{\sinh(2kd)}\right]\frac{\sigma k_y}{k^2} + U_y \end{aligned}\right\} \tag{7}$$

Transform term of refraction and frequency due to current is as follows.

$$C_{\theta} = -\frac{C_g}{C}\frac{\partial C}{\partial m} - \frac{\partial U_n}{\partial m} \tag{8}$$

$$C_{\sigma} = \frac{\partial \sigma}{\partial d}\left(\frac{\partial d}{\partial t} + U\frac{\partial d}{\partial s}\right) - C_g \vec{k} \cdot \frac{\partial \bar{U}}{\partial n} \tag{9}$$

For large scale, SWAN is as follows in spherical coordinate.

Alternating $N(x, y, t, \sigma, \theta)$ with $N(\varphi, \lambda, t, \sigma, \theta)$

$$\frac{\partial N}{\partial t} + (\cos\varphi)^{-1}\frac{\partial}{\partial \varphi}\dot{\varphi}\cos\varphi N + \frac{\partial}{\partial \lambda}\dot{\lambda} N + \frac{\partial}{\partial \theta}\dot{\theta} N + \frac{\partial}{\partial \sigma}\dot{\sigma} N = \frac{S}{\sigma} \tag{10}$$

where

$$\dot{\varphi} = \left\{\frac{1}{2}\left[1 + \frac{2kd}{\sinh(2kd)}\right]\frac{\sigma k \sin\theta}{k^2} + U_{\varphi}\right\} R^{-1},$$

$$\dot{\lambda} = \left\{\frac{1}{2}\left[1 + \frac{2kd}{\sinh(2kd)}\right]\frac{\sigma k \sin\theta}{k^2} + U_{\lambda}\right\} (R\cos\varphi)^{-1},$$

$$\dot{\sigma} = C_{\sigma},$$

$$\dot{\theta} = -\frac{1}{K}\left[\frac{\partial \sigma}{\partial d}\frac{\partial d}{\partial m} + \bar{k}\frac{\partial \overline{U}}{\partial m}\right] - C_s \cos\theta \tan\varphi R^{-1},$$

R is radius of the earth.

In SWAN the source function is sum of wind input, nonlinear interaction, breaking wave, bottom friction terms.

$$S = S_{in} + S_{nl} + S_{br} + S_{bo} \tag{11}$$

2.1.1. Wind input function

S_{in} is it Miles formulated as

$$S_{in} = \alpha + \beta E(\sigma, \theta) \tag{12}$$

Linear growth effect is it Cavaleri(1981) formulated as

$$\alpha = \begin{cases} \alpha_1 U_a^2 \cos(\theta - \psi), & f = f_{\max}, \quad |\theta - \psi| < 90^\circ \\ 0, & \text{others} \end{cases} \tag{13}$$

where $\alpha_1 = \alpha_0 / (2\pi f_{\max})$.

Nonlinear growth effect is it Snyder(1981) formulated as

$$\beta = \max\left\{0,\ 0.25 \frac{\rho_a}{\rho_w} 28 \frac{u_*}{c} \cos[(\theta - \psi) - 1]\right\} 2\pi f \tag{14}$$

where $c = \dfrac{g}{2\pi f}\tanh(kd)$ is wave phase speed, $u_* = \sqrt{C_D} \cdot U_{10}$ is friction speed, $C_D = (0.8 + 0.065 U_{10}) 10^{-3}$, θ is wave direction, ψ is wind direction, d is depth.

Explicit schema is easy to apply and semi-emplicit schema is possible to shorten the time of calculation, therefore we combined these schema to solve formula (6) [2, 3].

2.1.2. Explicit schema

We used Lax-Wenderoff schema for advection term in geographical space.

In case of only advection term is as follows.

$$\frac{\partial}{\partial t} N_{i,\,j,\,l,\,m}^n = -\nabla (C_g N_{i,\,j,\,l,\,m}^n) \tag{15}$$

First of all, the value of $n + 1/2$ time is as

$$N_{i+1/2,\,j,\,l,\,m}^{n+1/2} = \frac{1}{2}(N_{i+1,\,j,\,l,\,m}^n + N_{i,\,j,\,l,\,m}^n) - \frac{\Delta t}{2}\left\{\frac{C_{gi+1,\,j} N_{i+1,\,j,\,l,\,m}^n - C_{gi,\,j} N_{i,\,j,\,l,\,m}^n}{\Delta x} + \right.$$

$$\left. + \frac{C_{gi+1/2,\,j+1} N_{i+1/2,\,j+1,\,l,\,m}^n - C_{gi+1/2,\,j-1} N_{i+1/2,\,j-1,\,l,\,m}^n}{2\Delta y}\right\}$$

$$N_{i,\,j+1/2,\,l,\,m}^{n+1/2} = \frac{1}{2}(N_{i,\,j+1,\,l,\,m}^n + N_{i,\,j,\,l,\,m}^n) - \frac{\Delta t}{2}\left\{\frac{C_{gi+1,\,j+1/2} N_{i+1,\,j+1/2,\,l,\,m}^n - C_{gi-1,\,j+1/2} N_{i-1,\,j+1/2,\,l,\,m}^n}{2\Delta x} + \right.$$

$$\left. + \frac{C_{gi,\,j+1} N_{i,\,j+1,\,l,\,m}^n - C_{gi,\,j} N_{i,\,j,\,l,\,m}^n}{\Delta y}\right\}$$

In next step get $n+1$ time with $n + 1/2$ time.

$$\begin{aligned} N_{i,\,j}^{n+1} = N_{i,\,j}^n - \Delta t\{&[(1 + a_x)(C_{gi+1/2,\,j} N_{i+1/2,\,j}^{n+1/2} - C_{gi-1/2,\,j} N_{i-1/2,\,j}^{n+1/2}) + \\ &+ \frac{1}{3} a_x (C_{gi+3/2,\,j} N_{i+3/2,\,j}^{n+1/2} - C_{gi-3/2,\,j} N_{i-3/2,\,j}^{n+1/2})] / \Delta x + \\ &+ [(1 + a_y)(C_{gi,\,j+1/2} N_{i,\,j+1/2}^{n+1/2} - C_{gi,\,j-1/2} N_{i,\,j-1/2}^{n+1/2}) + \\ &+ \frac{1}{3} a_y (C_{gi,\,j+3/2} N_{i,\,j+3/2}^{n+1/2} - C_{gi,\,j-3/2} N_{i,\,j-3/2}^{n+1/2})] / \Delta y\} \end{aligned}$$

$$a_x = \frac{3}{8} - \frac{1}{8}(C_{gx} \Delta t / \Delta x)^2, \quad a_y = \frac{3}{8} - \frac{1}{8}(C_{gy} \Delta t / \Delta y)^2$$

$$C_{gx} = C_g \cos\theta, \quad C_{gy} = C_g \sin\theta$$

Refraction term apply front difference

$$N_{i,\,j,\,l,\,m}^{n+1}=N_{i,\,j,\,l,\,m}^{n}-\Delta t\begin{cases}\dfrac{(C_\theta)_{i,\,j,\,l,\,m}N_{i,\,j,\,l,\,m}^{n}-(C_\theta)_{i,\,j,\,l-1,\,m}N_{i,\,j,\,l-1,\,m}^{n}}{\Delta\theta}, & C_\theta>0\\[2ex] \dfrac{(C_\theta)_{i,\,j,\,l+1,\,m}N_{i,\,j,\,l+1,\,m}^{n}-(C_\theta)_{i,\,j,\,l,\,m}N_{i,\,j,\,l,\,m}^{n}}{\Delta\theta}, & C_\theta<0.\end{cases}$$

Variation term due to depth and current is same as above mentioned.

$$N_{i,\,j,\,l,\,m}^{n+1}=N_{i,\,j,\,l,\,m}^{n}-\Delta t\begin{cases}\dfrac{(C_\sigma)_{i,\,j,\,l,\,m}N_{i,\,j,\,l,\,m}^{n}-(C_\sigma)_{i,\,j,\,l,\,m-1}N_{i,\,j,\,l,\,m-1}^{n}}{\Delta\sigma}, & C_\sigma>0\\[2ex] \dfrac{(C_\sigma)_{i,\,j,\,l,\,m+1}N_{i,\,j,\,l,\,m+1}^{n}-(C_\sigma)_{i,\,j,\,l,\,m}N_{i,\,j,\,l,\,m}^{n}}{\Delta\sigma}, & C_\sigma<0.\end{cases}$$

Finally considering up to source term, total result is as

$$N_{i,\,j,\,l,\,m}^{n+1}=N_{i,\,j,\,l,\,m}^{n}+S_{i,\,j,\,l,\,m}^{n}\,. \tag{16}$$

2.1.3. Semi-emplicit schema

The numerical method by semi-emplicit schema in SWAN is as follows

From principle making emplicit schema we use $n+1$ time for preferential action term and n time for secondary action term, then semi- emplicit schema is as

$$\begin{aligned}&\frac{N_{i,\,j}^{n+1}-N_{i,\,j}^{n}}{\Delta t}+\frac{(C_{gxij}N_{i+1,\,j}-C_{gxij}N_{i-1,\,j})^n}{2\Delta x}+\frac{(C_{gyij}N_{i,\,j+1}-C_{gyij}N_{i,\,j-1})^n}{2\Delta y}+\\&+\frac{(C_{\sigma+1}-C_\sigma)N_{i,\,j}^{n+1}}{\Delta\sigma}+\frac{(C_{\theta+1}-C_\theta)N_{i,\,j}^{n+1}}{\Delta\theta}=\frac{S^n}{\sigma}.\end{aligned} \tag{17}$$

Therefore, formula (17) is written as

$$\begin{aligned}&N_{i,\,j}^{n+1}\left(1+\frac{\Delta t}{\Delta\sigma}(C_{\sigma+1}-C_\sigma)_{i,\,j,\,\theta}+\frac{\Delta t}{\Delta\theta}(C_{\theta+1}-C_\theta)_{i,\,j,\,\sigma}\right)=N_{i,\,j}^{n}+\frac{\Delta t}{\sigma}S_{i,\,j}^{n}-\\&\quad-\frac{\Delta t}{2\Delta x}C_{gxij}(N_{i+1,\,j}-N_{i-1,\,j})^n-\frac{\Delta t}{2\Delta y}C_{gyij}(N_{i,\,j+1}-C_{gyij}N_{i,\,j-1})^n\,.\end{aligned} \tag{18}$$

2.1.4. Calculating condition

We assumed that there is no wave in initial moment, thus N is zero in $t=0$.

In next time we use ahead value as initial field.

We assumed that is zero because N is unknown on almost liquid boundary, namely, $N=0$ and, or in other case assumed that the energy current through liquid boundary is zero. In solid boundary, we assumed that wave is absorbed into solid, then can assume zero condition.

Arrange of frequency is $f_{\min}=0.04\text{Hz}$, $f_{\max}=0.4\text{Hz}$ and interval of it is $\Delta f=0.01\text{Hz}$. And the interval of spectrum direction is 30°. The interval of time is to satisfy $\Delta t<\Delta x/C_g$, in which C_g is group velocity of the lowest frequency of wave.

We determined that the interval of time is 30 minutes from calculating experiment.

2.2. Test result

We selected and tested 29 cases of wind over 6 hours in bay "ㅅ" of Korean West Sea and port "ㄷ" of Korean East Sea because of consideration of enough development state of wave field. The computing example is as table 2, 3.

Table 2. Calculation parameter

Division	Δt/s	Δx/m	Δy/m	$\Delta \sigma$/s	$\Delta \theta$/(°)
Semi-explicit schema	1 800	250	250	0.01	30
Explicit schema	15	250	250	0.01	30

Table 3. Test in bay "ㅅ"

No.	Wind speed /($m \cdot s^{-1}$)	Wind direction	Wind duration /h	Observation value/m	Calculation value/m	Absolute error/m	Relative error/%
1	25	ws	24	6	6.4	0.4	5.7
2	25	ws	24	7	6.8	0.2	2.8
3	15	n	24	4.5	4.2	0.3	4.2
4	12	es	24	2.2	3.2	1	14.2
5	4	s	48	2.5	1.6	0.9	12.8
6	10	es	48	2.5	3.0	0.5	7.1
7	15	s	12	2.5	2.9	0.4	5.7
8	9	es	24	3.2	2.4	0.8	11.4
9	15	e	12	3.0	3.3	0.3	4.2
10	19	ws	6	3.0	2.7	0.3	4.2
11	14	s	12	3.0	3.9	0.9	12.8
12	10	s	12	3.0	3.1	0.1	1.4
13	10	wn	24	3.5	2.5	1	14.2
14	15	wn	24	5.0	4.6	0.4	5.7
15	13	w	24	3.6	3.2	0.4	5.7

As seen in table 3, in case of semi-explicit schema, mean absolute error is 0.5m, mean relative error is 7.5%. Thus we knew that this method provides not only calculation time but accuracy in contrast with explicit schema(mean relative error 8.2%) of Korean East Sea.

Maximum absolute error from 6^{th} June 6 o'clock to 8^{th} June 0 o'clock in 1984 is 0.3m in some degree at the point "ㄱㅊ 4" of bay "ㅅ". In point "ㄷ" we calculated ten processes over 6 hours of duration among diurnal four times data in 1972 as the same method, thus mean absolute error is 0.23m, mean square error is 0.34m, mean relative error is 6.4%.

3. Analysis process of design wave used design marine wind

Analysis process of design wave used design marine wind is as follows.

① Selection of effective wind direction

We selected the most dangerous direction on structures and direction favorable to develop wave and storm surge.

② Return period selection

Return period of design wave should be 1 or 10 years in design of temporary aquaculture structures, 50 years in design of permanent aquaculture structures, 50 or 100 years according to a degree of seawall.

The optimum design is to minimum the sum of construction and repair cost, so return period of wave is to determine within 5－20 years according to a degree of structures.

③ Determination of design marine wind speed according to the return period and effective direction of wind.

④ If wave data is not, the design wave is computed by SWAN using above selected effective direction of wind and design wind speed.

In this case wave calculation contains two steps.

First of all, source term is computed in each wave grid with certain interval time. And advection term is computed by propagation schema, significant wave height and means wave period are computed and outputted at each necessary time.

The analysis process of design wave by SWAN consist of speed and direction of design marine wind of return period, duration of wind, speed and direction of current, depth data input, smoothing of depth data processing of source and advection term, Significant wave height, mean period of wave and smoothing, and output of wave field.

Based on it, we computed the design wave to guarantee the safety of marine aquaculture structure at point "ㅅ" and point "ㅎ". The example of design wave of each return period at marine aquaculture area "ㅅ" and "ㅎ" is as Table 4.

Table 4. Design wave of each return period

Calculating point (*i*, *j*)	10years	20years	50years	100years
ㅅ 25, 10	3.5	3.8	4.1	4.4
ㅎ 12, 13	4.3	4.8	5.5	6.0

Conclusion

The design wave is basic data in designing and control of marine structures such as aquaculture farm, oil tower etc.

Using above method, the computation of design wave under insufficient data series is possible, in which it is important to determine correctly the design marine wind.

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