Journal of **Kim Il Sung** University Vol. 3 No. 2
(Natural Science) Juche103(2014)

A Method for Determining Alternative of Fuzzy Multi-Criteria Group Decision Making based on Relationship Analysis between Criteria

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Abstract We presented a solution for determining the optimal alternative based on the relationship analysis between criteria in the group decision-making problem with different forms of information preference of decision-makers on criteria.
In particular, we studied the problems that consider more than one criterion in the case that the preference information of the decision-makers are stated by fuzzy numbers, which is known as fuzzy multiple criteria group decision making.
Key words fuzzy multiple criteria decision making, criteria, criteria weight

Introduction

The great leader Comrade **Kim Jong Il** said as follows.

"**Scientists and technicians should intensify their scientific research for the scientific and technical solutions to the problems arising in making the national economy scientifically-based, and thus put the production and management activities of all branches on a new scientific base.**"("**KIM JONG IL** SELECTED WORKS" Vol. 10 P. 195)

Determining optimal alternative in group decision making is very importance in management.

Up to now, there are many researches for the group decision making that consider more than one criteria, the multiple objective decision making for optimizing more than one objective and the fuzzy decision making that information are stated by fuzzy number.

However, the methods for determining optimal alternative in group decision making are not well developed until now, because each deferent evaluation method has some specific condition for its use and some limitations

Some researchers [1, 2] paid their attention to group decision-making problems, in which the decision-makers provide their preference information on alternative using different preference forms.

Meanwhile the researches on single-objective optimization model to determine the attribute weights from the interval decision matrix and criteria weight intervals made advance [10].

There are methods that get the optimal alternative by synthesizing conclusions of decision-makers when information of criteria weight is given partially and the degree of satisfaction is expressed by triangle fuzzy number[3－5].

They proposed method for determining the preference of alternative by Borda's method based on preference of interval number when the information of the decision maker on evaluation alternative is stated by interval number [12].

Literature [11] proposed a new method for solution of fuzzy multiple criteria decision making by using S, H and G functions based on fuzzy set theory.

In general, there are benefit attributes and cost attributes in multiple-attribute decision-making problems, and the “dimension” of different attributes may be different.

Literatures [8] proposed a method that determine the optimal attribute based on satisfactory matrix $\Pi = (\mu_{ij})_{m \times n}$ which is mapped from D according to the criteria type of criteria(the benefit attribute, the cost attribute)

The most general method is the arithmetic mean method for integrated decision matrix that add weight to decision matrix given by the decision-makers [13].

Literatures [6, 7] present the group decision making method that decreases the disagreement of decision-makers by fuzzy programming when the required information is given as a supplementary decision matrix.

Literature [9] divided decision-making problem into several subproblems when composition of the group decision-makers was very complex and its scale was very large and presented a method that determined the solution of decision-making problem based on the solution of every subproblem.

Literature [10] presented an optimization algorithm of the group decision making problem based on fuzzy preference information that is presented as linguistic variable.

The shortcomings of preceding methods for fuzzy multi-criteria group decision-making are as follows.

First, some methods were proposed based on relationship analysis between criteria in fuzzy multi-criteria group decision-making problem, but these methods were limited to the case only that the degree of satisfaction of alternative was given as real number, and could not be applied to many practical problems.

Second, there are methods for determining criteria weight satisfying required information of decision-makers, but required information of decision-makers are represented by linguistic variable. These methods can not express the requirement of decision-makers more detail.

So we present a method for determining the optimal alternative based on relationship analysis between criteria in the group decision-making problem with different forms of preference information of decision-makers on criteria and with the degree of satisfaction represented by the triangle fuzzy number.

1. The Method for Determining the Weights of Criteria

1.1. The type of information expression of decision-makers on criteria weights

In the group decision-making problem, the decision-maker can represent weight information of criteria as one of the following three different expression forms.

Type of utility values

$$\widetilde{U}=\{\widetilde{u}_1,\ \widetilde{u}_2,\cdots,\ \widetilde{u}_n\}$$

where $\widetilde{u}_j=(ul_j,\ um_j,\ uu_j)$ indicates utility interval of criteria weights c_j, at that time

$$0\le ul_j\le um_j\le uu_j\le 1\ \ (j=\overline{1,\ n}),$$ i. e. $ul_j,\ um_j,\ uu_j$

are infimum, median, supremum of weight of criteria c_j.

Type of preference relation

$$\widetilde{P}=(\widetilde{p}_{ij})_{n\times n}$$

where $\widetilde{p}_{ij}=(pl_{ij},\ pm_{ij},\ pu_{ij})$ indicates the preference degree of the criteria c_i over c_j and is the required information on p_{ij} that is described by the following formula

$$p_{ij}=0.5(w_i-w_j+1),$$

where pl_{ij}, pm_{ij}, pu_{ij} are infimum, median, supremum of p_{ij} and w_i, w_j are the weight of criteria c_i, c_j.

From the definition, the following condition is satisfied

$$0\le bl_{ij}\le bm_{ij}\le bu_{ij}\le 1,\ \ bl_{ij}bu_{ji}=bm_{ij}bm_{ji}=bu_{ij}bl_{ji}\ge 1,\ bl_{ii}=bm_{ii}=bu_{ii}=0.5,$$

$$pl_{ii}=pm_{ii}=pu_{ii}=0.5\ \ (i,\ j=\overline{1,\ n})$$

Type of multiplicative preference relation

$$\widetilde{B}=(\widetilde{b}_{ij})_{n\times n}:$$

where $\widetilde{b}_{ij}=(bl_{ij},\ bm_{ij},\ bu_{ij})$ indicates the multiplicative preference degree of the criteria c_i over c_j and is the required information on b_{ij} that is described by the following formula

$$b_{ij}=\frac{w_i}{w_j}$$

where bl_{ij}, bm_{ij}, bu_{ij} are infimum, median, supremum of b_{ij}, w_i, w_j is the weight of criteria c_i, c_j.

From the definition, the following condition is satisfied.

$$0\le pl_{ij}\le pm_{ij}\le pu_{ij}\le 1,\ \ pl_{ij}+pu_{ji}=pm_{ij}+pm_{ji}=pu_{ij}+pl_{ji}=1,\ \ pl_{ii}=pm_{ii}=pu_{ii}=0.5$$

$$(i,\ j=\overline{1,\ n})$$

Above three expression forms are most general forms that can represent criteria weight information.

1.2. The method for determining weights of criteria

In the group decision-making problem, let $w_1,\ w_2,\cdots,\ w_n$ be the weight vector of criteria to be determined. If this weight satisfies the requirement of all decision-maker that is represented three forms, the following condition must be satisfied for $i,\ j=\overline{1,\ n}$

$$ul_j\le w_j\le uu_j \tag{1}$$

$$(w_j-um_j)^2=em_j\le\varepsilon_{j1} \tag{2}$$

$$pl_{ij} \le 0.5(w_i - w_j + 1) \le pu_{ij} \tag{3}$$

$$(0.5(w_i - w_j + 1) - pm_{ij})^2 = em_{ij} \le \varepsilon_{ij2} \tag{4}$$

$$bl_{ij} \le \frac{w_i}{w_j} \le bu_{ij} \tag{5}$$

$$\left(\frac{w_i}{w_j} - bm_{ij}\right)^2 = em_{ij} \le \varepsilon_{ij3} \tag{6}$$

where ε_{j1}, ε_{ij2}, ε_{ij3} are nonnegative real numbers that is small sufficiently.

Generally, it is difficult to satisfy all requirements of the decision-makers.

Therefore, we must determine the reasonable criteria weight that relax the contradiction of information as much as possible and satisfy requirements of the decision-makers as much as possible.

For the multiple-criteria group decision-making problems with t decision-makers under study, without loss of generality, we suppose that the decision-makers ($k = 1, 2, \cdots, t_1$) provide their preference information by means of the utility values, the decision-makers ($k = t_1 + 1, \cdots, t_2$) provide their preference information by means of the preference relation and the decision-makers ($k = t_2 + 1, \cdots, t$) provide their preference information by means of the multiplicative preference relation.

In order to determine the weight vector of criteria, we establish the following linear programming problem.

$$E = \min\left(\sum_{j=1}^{n}\sum_{d=1}^{t_1} w_{J_d}(el_j^{(d)} + em_j^{(d)} + eu_j^{(d)}) + \sum_{i,j=1}^{n}\ \sum_{d=t_1+1}^{t} w_{J_d}(el_{ij}^{(d)} + em_{ij}^{(d)} + eu_{ij}^{(d)}) \right) \tag{7}$$

$$em_j^{(d)} = (w_j - um_j^{(d)})^2,\ j = \overline{1,\ n},\ d = \overline{1,\ t_1}$$

$$w_j \ge ul_j^{(d)} - el_j^{(d)},\quad j = \overline{1,\ n},\ d = \overline{1,\ t_1}$$

$$w_j \le uu_j^{(d)} + eu_j^{(d)},\quad j = \overline{1,\ n},\quad d = \overline{1, t_1}$$

$$el_j^{(d)},\ eu_j^{(d)} \ge 0,\quad j = \overline{1,\ n},\ d = \overline{1,\ t_1}$$

$$em_{ij}^{(d)} = (0.5(w_i - w_j + 1) - pm_{ij}^{(d)})^2,\ i,\ j = \overline{1,\ n},\ d = \overline{t_1 + 1,\ t_2}$$

$$0.5(w_i - w_j + 1) \ge pl_{ij}^{(d)} - el_{ij}^{(d)},\ i,\ j = \overline{1,\ n}, d = \overline{t_1 + 1,\ t_2}$$

$$0.5(w_i - w_j + 1) \le pu_{ij}^{(d)} + eu_{ij}^{(d)},\ i,\ j = \overline{1,\ n},\ d = \overline{t_1 + 1,\ t_2}$$

$$em_{ij}^{(d)} = \left(\frac{w_i}{w_j} - bm_{ij}^{(d)}\right)^2,\ i,\ j = \overline{1,\ n},\ d = \overline{t_2 + 1, t}$$

$$\frac{w_i}{w_j} \ge bl_{ij}^{(d)} - el_{ij}^{(d)},\ i,\ j = \overline{1,\ n},\ d = \overline{t_2 + 1,\ t}$$

$$\frac{w_i}{w_j} \le bu_{ij}^{(d)} + eu_{ij}^{(d)},\ i,\ j = \overline{1,\ n},\ d = \overline{t_2 + 1, t}$$

$$el_{ij}^{(d)},\ eu_{ij}^{(d)} \ge 0,\ i, j = \overline{1,\ n},\ d = \overline{t_1 + 1, t}$$

$$\sum_{j=1}^{n} w_j = 1,\ w_j \geq 0,\ \ j = \overline{1,\ n}$$

where $el_j^{(d)}$, $eu_j^{(d)}$ ($j = \overline{1,\ n},\ d = \overline{1,\ t_1}$) $el_{ij}^{(d)}$, $eu_{ij}^{(d)}$ ($i,\ j = \overline{1,\ n}$, $d = \overline{t_1 + 1,\ t}$) are the preference information deviation value of the decision-maker J_d and w_{J_d} is the weight of the decision-maker J_d .

The decision-makers can adjust the preference information with nonzero deviation values by compromising their requirement.

Then the amount of change is proportional to the deviation values and extends the range of the triangular fuzzy number.

And we establish the model again on the basis of changed preference information value and get the deviation values again. If this procedure is repeated until all of the deviation values comes to zero, the final weight values is just the optimal criteria weight vector satisfying the requirement of every decision-makers.

2. A Method for Determining the Optimal Alternative

For fuzzy multi-criteria group decision making problem, we consider a new method for selecting the appropriate alternative among feasible alternative set A in the case where d_i^j ($i = \overline{1,\ m}$, $j = \overline{1,\ n}$) is given by the triangular fuzzy number, decision matrix D is mapping to the matrix of degree of satisfaction and the weights of decision-maker is given as follows:

$$W_J = (w_{J_1},\ w_{J_2},\ \cdots,\ w_{J_t}),\ \ \sum_{d=1}^{t} w_{J_d} = 1$$

Then the matrix of degree of satisfaction of decision-maker J_d ($d = \overline{1,\ t}$) is expressed as follows:

$$\widetilde{\Pi}^{(d)} = (\mu_{ij}^{(d)}(x))_{m\times n} = \begin{pmatrix} \mu_{11}^{(d)}(x) & \mu_{12}^{(d)}(x) & \cdots & \mu_{1n}^{(d)}(x) \\ \mu_{21}^{(d)}(x) & \mu_{22}^{(d)}(x) & \cdots & \mu_{2n}^{(d)}(x) \\ \vdots & \vdots & \ddots & \vdots \\ \mu_{m1}^{(d)}(x) & \mu_{m2}^{(d)}(x) & \cdots & \mu_{mn}^{(d)}(x) \end{pmatrix},\ d = \overline{1,\ t} \tag{8}$$

where $\mu_{ij}^{(d)}(x) = (\alpha_{ij}^{(d)},\ \beta_{ij}^{(d)},\ \gamma_{ij}^{(d)})$.

There are several criteria for evaluating the alternatives and generally, some relationships between criteria exist.

If an increase of the degree of satisfaction for one criteria cause to increase the degree of satisfaction for other criteria, then a pair of two criteria is said to be cooperative. If an increase of the degree of satisfaction for one criteria cause to decrease the degree of satisfaction for other criteria, then a pair of two criteria is said to be conflictive. If increase of the degree of satisfaction for one criterion doesn't cause the variation of the degree of satisfaction, then a pair of two criteria is said to be independent.

Definition 1 Define a triangular fuzzy number $\mu_{ijk}(x)$ as follows.

$$\mu_{ijk}(x) \overset{\Delta}{=} \mu_{ik}(x) \ominus \mu_{jk}(x)$$

where $\ominus$ denote to extended subtraction of fuzzy numbers and $\mu_{ijk}(x)$ indicate the degree of satisfaction of alternative a_i, a_j for criteria c_k.

Assume that $\mu_{ijk}(x)$ be a triangular fuzzy number $\mu_{ijk}(x) = (\alpha_{ijk}, \ \beta_{ijk}, \ \gamma_{ijk})$.

Definition 2 Cooperative degree and conflicting degree are as follows.

$$\text{Cooperative degree:} \ cp(c_k, c_l) \overset{\Delta}{=} \frac{\sum\limits_{(a_i, a_j)\in CP} (|\beta_{ijk}| + |\beta_{ijl}|)}{\sum\limits_{(a_i, a_j)\in AP} (|\beta_{ijk}| + |\beta_{ijl}|)} \tag{9}$$

$$\text{Conflicting degree:} \ cf(c_k, c_l) \overset{\Delta}{=} \frac{\sum\limits_{(a_i, a_j)\in CF} (|\beta_{ijk}| + |\beta_{ijl}|)}{\sum\limits_{(a_i, a_j)\in AP} (|\beta_{ijk}| + |\beta_{ijl}|)} \tag{10}$$

c_k, c_l: two criteria, $AP = \{(a_i, a_j) \mid \forall a_i, a_j \in A, i \neq j\}$: a set of alternative pairs

$$CP = \left\{ (a_i, a_j) \left| \left[\int\limits_{x>0} \mu_{ijk}(x) - \int\limits_{x<0} \mu_{ijk}(x) \right] \times \left[\int\limits_{x>0} \mu_{ijl}(x) - \int\limits_{x<0} \mu_{ijl}(x) \right] > 0 \right. \right\},$$

$$CF = \left\{ (a_i, a_j) \left| \left[\int\limits_{x>0} \mu_{ijk}(x) - \int\limits_{x<0} \mu_{ijk}(x) \right] \times \left[\int\limits_{x>0} \mu_{ijl}(x) - \int\limits_{x<0} \mu_{ijl}(x) \right] < 0 \right. \right\},$$

$$AP = CP \cup CF \cup IR$$

$$IR = \left\{ (a_i, a_j) \left| \left[\int\limits_{x>0} \mu_{ijk}(x) - \int\limits_{x<0} \mu_{ijk}(x) \right] \times \left[\int\limits_{x>0} \mu_{ijl}(x) - \int\limits_{x<0} \mu_{ijl}(x) \right] = 0 \right. \right\},$$

$$CP \cap CF = CF \cap IR = CP \cap IR = \phi .$$

In order to evaluate several alternatives for multi-criteria, divide overall set of criteria into two subsets based on conflicting, cooperative and irrelevant relation such that criteria belonging to each subset have maximum cooperative degree with each other.

The algorithm for dividing a set of criteria is as follows.

Step 1 Setting the initial value

$$S = \{c_2, c_3, \cdots, c_n\}, \ q = 1, \ S_1 = \{c_1\}$$

Step 2 Calculation p_j for all $c_j \in S$

$$p_j = \sum_{c_i \in S_q} (cp(c_i, c_j) - cf(c_i, c_j))$$

Step 3 Let be $p_k = \max\limits_j p_j$, $k = \arg\max\limits_j p_j$

Step 4 If $p_k > 0$, then

$$S_q = S_q + \{c_k\}, \ S = S - \{c_k\}$$

and go to step 2, or else print S_q and set as follows

$$q = q + 1,\ S_q = \{c_k\},\ S = S - \{c_k\}.$$

Step 5 If $S = \phi$, then print S_q and exit the algorithm, or else go to step 2.

This algorithm, first assigns the criterion c_1 to S_1, then removes criteria which belongs to S_2 and has maximum positive value among the other criteria of S_2, and assign to S_1. So we divide criteria into two classes that cooperate with each other.

Next the final degree of satisfaction is calculated as follows:

$$\mu^{(d)}(a_k) \overset{\Delta}{=} \sum_{c_i \in S_q^{(d)}} w_i^{(d)} \mu_{ki}^{(d)}(x) \tag{11}$$

where q satisfy following relation.

$$\sum_{c_i \in S_q^{(d)}} w_i^{(d)} = \max\left\{ \sum_{c_i \in S_1^{(d)}} w_i^{(d)},\ \sum_{c_i \in S_2^{(d)}} w_i^{(d)},\ \cdots,\ \sum_{c_i \in S_{z_d}^{(d)}} w_i^{(d)} \right\}$$

Next, we extract an intergrated final degree of satisfaction by intergrating the final degree of satisfaction of all decision-makers.

The integrated final degree of satisfaction is calculated as follows:

$$\mu(a_k) \overset{\Delta}{=} \sum_{d=1}^{t} w_{J_d} \mu^{(d)}(a_k) \tag{12}$$

where w_{J_d} is a weight of decision-maker J_d.

Then, an alternative that has maximum intergrated final degree of satisfaction is an optimal alternative.

3. Result Analysis

Suppose that a set of decision-makers is $J = \{J_1,\ J_2,\ J_3\}$, a set of alternative is $A = \{a_1,\ a_2,\ a_3,\ a_4\}$, a set of criteria is $C = \{c_1,\ c_2,\ c_3,\ c_4\}$.

The matrix of degree of satisfaction of decision-makers for criteria weight is as follows.

$$\widetilde{U}^{(1)} = \{\widetilde{u}_1^{(1)},\ \widetilde{u}_2^{(1)},\ \widetilde{u}_3^{(1)},\ \widetilde{u}_4^{(1)}\} =$$
$$= \{(0.40,\ 0.45,\ 0.50),\ (0.20,\ 0.22,\ 0.25),\ (0.15,\ 0.18,\ 0.20),\ (0.10,\ 0.13,\ 0.20)\}$$

$$\widetilde{P}^{(2)} = (\widetilde{p}_{ij}^{(2)})_{4\times 4} = \begin{pmatrix} (0.50, 0.50, 0.50) & (0.50, 0.55, 0.60) & (0.20, 0.26, 0.30) & (0.60, 0.65, 0.70) \\ (0.40, 0.45, 0.50) & (0.50, 0.50, 0.50) & (0.30, 0.34, 0.40) & (0.50, 0.60, 0.70) \\ (0.70, 0.74, 0.80) & (0.60, 0.66, 0.70) & (0.50, 0.50, 0.50) & (0.80, 0.89, 1.00) \\ (0.30, 0.35, 0.40) & (0.30, 0.40, 0.50) & (0.00, 0.11, 0.20) & (0.50, 0.50, 0.50) \end{pmatrix}$$

$$\widetilde{B}^{(3)} = (\widetilde{b}_{ij}^{(3)})_{4\times 4} = \begin{pmatrix} (1, 1, 1) & \left(\frac{13}{10}, \frac{7}{5}, \frac{3}{2}\right) & \left(\frac{3}{2}, \frac{8}{5}, \frac{8}{5}\right) & (7, 8, 9) \\ \left(\frac{2}{3}, \frac{5}{7}, \frac{10}{13}\right) & (1, 1, 1) & \left(1, \frac{9}{8}, \frac{5}{4}\right) & (5, 6, 7) \\ \left(\frac{5}{8}, \frac{5}{8}, \frac{2}{3}\right) & \left(\frac{4}{5}, \frac{8}{9}, 1\right) & (1, 1, 1) & (4, 5, 6) \\ \left(\frac{1}{9}, \frac{1}{8}, \frac{1}{7}\right) & \left(\frac{1}{7}, \frac{1}{6}, \frac{1}{5}\right) & \left(\frac{1}{6}, \frac{1}{5}, \frac{1}{4}\right) & (1, 1, 1) \end{pmatrix}$$

$$\widetilde{\Pi}^{(1)}=(\mu_{ij}^{(1)}(x))_{4\times 4}=\begin{pmatrix}(0.0,\ 0.1,\ 0.2) & (0.4,\ 0.5,\ 0.6) & (0.1,\ 0.2,\ 0.3) & (0.8,\ 0.9,\ 1.0)\\ (0.7,\ 0.8,\ 0.9) & (0.1,\ 0.2,\ 0.3) & (0.2,\ 0.4,\ 0.5) & (0.0,\ 0.1,\ 0.2)\\ (0.5,\ 0.6,\ 0.8) & (0.7,\ 0.8,\ 0.9) & (0.6,\ 0.7,\ 1.0) & (0.0,\ 0.1,\ 0.2)\\ (0.2,\ 0.4,\ 0.5) & (0.4,\ 0.5,\ 0.6) & (0.6,\ 0.7,\ 1.0) & (0.4,\ 0.5,\ 0.6)\end{pmatrix}$$

$$\widetilde{\Pi}^{(2)}=(\mu_{ij}^{(2)}(x))_{4\times 4}=\begin{pmatrix}(0.2,\ 0.4,\ 0.5) & (0.5,\ 0.6,\ 0.8) & (0.0,\ 0.1,\ 0.2) & (0.1,\ 0.2,\ 0.3)\\ (0.4,\ 0.5,\ 0.6) & (0.7,\ 0.8,\ 0.9) & (0.6,\ 0.7,\ 1.0) & (0.4,\ 0.5,\ 0.6)\\ (0.0,\ 0.1,\ 0.2) & (0.8,\ 0.9,\ 1.0) & (0.2,\ 0.4,\ 0.5) & (0.1,\ 0.2,\ 0.3)\\ (0.2,\ 0.4,\ 0.5) & (0.7,\ 0.8,\ 0.9) & (0.0,\ 0.1,\ 0.2) & (0.2,\ 0.4,\ 0.5)\end{pmatrix}$$

$$\widetilde{\Pi}^{(3)}=(\mu_{ij}^{(3)}(x))_{4\times 4}=\begin{pmatrix}(0.0,\ 0.1,\ 0.2) & (0.4,\ 0.5,\ 0.6) & (0.5,\ 0.6,\ 0.8) & (0.6,\ 0.7,\ 1.0)\\ (0.0,\ 0.1,\ 0.2) & (0.1,\ 0.2,\ 0.3) & (0.2,\ 0.4,\ 0.5) & (0.7,\ 0.8,\ 0.9)\\ (0.2,\ 0.4,\ 0.5) & (0.5,\ 0.6,\ 0.8) & (0.6,\ 0.7,\ 1.0) & (0.8,\ 0.9,\ 1.0)\\ (0.4,\ 0.5,\ 0.6) & (0.1,\ 0.2,\ 0.3) & (0.2,\ 0.4,\ 0.5) & (0.6,\ 0.7,\ 1.0)\end{pmatrix}$$

First, determine the weight of criteria. The resultant optimal weight vector calculated by proposed method is

$$W^*=(0.405,\ 0.271,\ 0.271,\ 0.053).$$

The weight vector calculated by presented method of criteria is

$$W=(0.366,\ 0.250,\ 0.262,\ 0.122).$$

The presented weight decision method decreases the error to 43.12% than previous one.

Next, using weight vector of criteria, determine the optimal alternative.

As a result, when the weight of decisions is as follows:

$$W_J=\left(\frac{1}{3},\ \frac{1}{3},\ \frac{1}{3}\right)$$

integrated final degree of decision-makers of alternatives is as follows.

$$\mu(a_1)=(0.23, 0.30, 0.40)$$
$$\mu(a_2)=(0.26, 0.36, 0.46)$$
$$\mu(a_3)=(0.43, 0.51, 0.64)$$
$$\mu(a_4)=(0.30, 0.39, 0.49)$$

Therefore, optimal alternative is a_3, that is, third alternative is the best alternative.

The result shows that proposed method is more comfortable and appropriate method.

Conclusion

We presented a method for determining the criteria weights in the case that the preference information form of decision-makers to criteria weight are different and a method for determining optimal alternative in the case that the degree of satisfaction are stated by the triangular fuzzy number in fuzzy multi-criteria group decision-making problem.

And we illustrate the effectiveness of this method with the practical example.

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