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Propagation of Acoustical Wave in Infinite Cylindrical Solid Bar Surrounded by Porous Media Saturated with Fluid

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Abstract We established the propagation equation of acoustical wave in media with the solid-porous media cylindrical boundary condition and proceeded with a solution, and put the boundary condition of solid-porous media cylindrical. And we derived the dispersion equation and progressed a numerical calculation and a interpretation of nonlinear equation.

Key words porous media, infinite cylinder

Introduction

The great leader Comrade **Kim Il Sung** said as follows.

"**Long-term research should be conducted with a view to opening up new scientific fields and introducing the latest developments in science and technology widely in the national economy.**" ("**KIM IL SUNG** WORKS" Vol. 35 P. 313)

In the past, there has been many research works on propagation of sound in a fluid well surrounding with porous media and in a free-end solid bar, while a little of literatures on sound propagation in solid bar surrounding by porous media is available [1－5].

Today, many applications such as logging by solid bar, ultrasonic boring require theoretical studies on sound propagation in solid bar significantly.

1. Wave Equation and Solution in Solid Bar and Porous Media

We assume that solid bar is homogenous axisymmetric one, porous media is the isotropic one of which porosity is ϕ, and that linear dimension of porosity is much smaller than a wavelength.

Displacement of wave[1, 2] is $\boldsymbol{u} = \nabla\varphi + \nabla\times\nabla\times\psi\boldsymbol{e}_z$, when it oscillates harmonically, wave equation in solid bar may be expressed as

$$\nabla^2\varphi + k_p^2\varphi = 0 \tag{1}$$

$$\nabla^2\psi + k_p^2\psi = 0 \tag{2}$$

where $k_p{}^2 = \dfrac{\omega^2}{c_p{}^2}$, $c_p{}^2 = \dfrac{\lambda+2\mu}{\rho}$, $k_s{}^2 = \dfrac{\omega^2}{c_s{}^2}$, $c_s{}^2 = \dfrac{\mu}{\rho}$ and λ, μ are Lame's constants and ρ is the density of bar, ∇ is operator in cylindrical coordinate system, ω is frequency of wave.

In homogeneous and axial symmetrical bar the solutions of wave equation are as follows.

$$\varphi = AI_0(\overline{k_r}r)e^{ik_0z} \tag{3}$$

$$\psi = BI_0(\overline{k'_r}r)e^{ik_0z} \tag{4}$$

where $\overline{k_r}^2 = k_0{}^2 - k_p{}^2$, $\overline{k'_r}^2 = k_0{}^2 - k_s{}^2$, $k_0{}^2 = \dfrac{\omega^2}{c^2}$, c are velocity of acoustical wave in axial direction.

Equation of motion in porous media saturated as fluid is as follows [3].

$$-N[\nabla\times[\nabla\times\boldsymbol{u}]] + (A+2N)\nabla(\nabla\cdot\boldsymbol{u}) + \overline{Q}\nabla(\nabla\cdot\ddot{\boldsymbol{U}}) = \rho_{11}\ddot{\boldsymbol{u}} + \rho_{12}\ddot{\boldsymbol{U}} + b(\dot{\boldsymbol{u}} - \dot{\boldsymbol{U}}) \tag{5}$$

$$Q\nabla(\nabla\cdot\boldsymbol{u}) + \overline{R}\nabla(\nabla\cdot\boldsymbol{U}) = \rho_{12}\ddot{\boldsymbol{u}} + \rho_{22}\ddot{\boldsymbol{U}} - b(\dot{\boldsymbol{u}} - \dot{\boldsymbol{U}}) \tag{6}$$

where $\boldsymbol{U}$ and $\boldsymbol{u}$ are displacement vector in solid and fluid respectively, ρ_{ij} and b are combination mass and viscous coefficient respectively, which are written as

$$b(\omega) = \beta^2 H_1(\omega), \quad \rho_{22}(\omega) = \frac{\beta^2 H_2(\omega)}{\omega}, \quad \rho_{12}(\omega) = \beta\rho_f - \rho_{22}(\omega),$$

$$\rho_{11}(\omega) = (1-\beta)\rho_s - \rho_{12}(\omega), \quad H_1(\omega) + iH_2(\omega) = \frac{1}{K(\omega)}$$

where $K(\omega)$ is Darcy coefficient , is expressed as follows;

$$k(\omega) = \frac{i\beta}{\omega\rho_f} J_2\left[\frac{ia\left(\dfrac{i\omega\rho_f}{\eta}\right)^{1/2}}{J_0\left[ia\left(\dfrac{i\omega\rho_f}{\eta}\right)^{1/2}\right]}\right],$$

ρ_s and ρ_f are densities of solid and fluid, a characteristic radius of porous, η dynamic viscous coefficient of saturated fluid, J_n Bessel function of the first kind of order n and β is porosity. A, N, $\overline{Q}$ and $\overline{R}$ are elastic constants related with Lamels constants, which are written as;

$$A = \frac{(1-\beta)\left(1-\beta-\dfrac{K_m}{K_s}\right)K_s + \dfrac{K_s}{K_f}K_m}{1-\beta-\dfrac{K_m}{K_s}+\beta\dfrac{K_s}{K_f}} - \frac{2}{3}N,$$

$$\overline{Q} = \frac{\left(1-\beta-\dfrac{K_m}{K_s}\right)\beta K_s}{1-\beta-\dfrac{K_m}{K_s}+\beta\dfrac{K_s}{K_f}},$$

$$\overline{R} = \frac{\beta^2 - K_s}{1-\beta-\dfrac{K_m}{K_s}+\beta\dfrac{K_s}{K_f}},$$

$$N = \mu_m.$$

where K_s, K_m, K_f are skeleton of solid and porous medium, and bulk modulus of fluid respectively, μ_m is shear modulus of skeleton of porous solid, which are expressed as

$K_m = (1-\beta)\cdot\rho_s\cdot\left(V_{l_1}^2 - \frac{4V_{s_1}^2}{3}\right)$, $\mu_m = (1-\beta)\cdot\rho_s\cdot\upsilon_{s_1}^2$, $K_f = \rho_f\cdot V_{l_2}^2$. V_{l_1}, V_{s_1}, V_{l_2} are velocities of longitudinal wave, shear wave in solid and longitudinal wave in fluid. The displacement vectors of porous media in solid and fluid can be expressed as;

$$\boldsymbol{u} = \sum_{j=1}^{2}\nabla\varphi_j + \nabla\times\psi_1\boldsymbol{e}_\varphi \tag{7}$$

$$\boldsymbol{U} = \sum_{j=1}^{2}\xi_j(\omega)\nabla\varphi_j + \chi(\omega)\nabla\times\psi_1\boldsymbol{e}_\varphi \tag{8}$$

where $\xi_j(\omega) = -\frac{1}{V_{lj}{}^2(\omega)}\frac{\overline{P}\overline{R}-\overline{Q}^2}{\overline{Q}\gamma_{22}(\omega)-\overline{R}\gamma_{12}(\omega)} + \frac{\overline{R}\gamma_{11}(\omega)-\overline{Q}\gamma_{12}(\omega)}{\overline{Q}\gamma_{22}(\omega)-\overline{R}\gamma_{12}(\omega)}$, $\chi(\omega) = -\frac{\gamma_{12}(\omega)}{\gamma_{22}(\omega)}$, ,

$\gamma_{ii}(\omega) = \rho_{ii}(\omega) - \frac{ib(\omega)}{\omega}$, $\gamma_{12}(\omega) = \rho_{12} + \frac{ib(\omega)}{\omega}$, $p = A + 2N$.

When acoustical field oscillates harmonically, we can introduce three Helmholtz's equations as following.

$$\nabla^2\varphi_1 + k_{p_1}^2\varphi_1 = 0 \tag{9}$$

$$\nabla^2\varphi_2 + k_{p_2}^2\varphi_2 = 0 \tag{10}$$

$$\nabla^2(\psi_1 e_\varphi) + k_{s_1}^2(\psi_1 e_\varphi) = 0 \tag{11}$$

where $k_{p_1}^2 = \frac{\omega^2}{V_{l_1}^2}$, $k_{p_2}^2 = \frac{\omega^2}{V_{l_2}^2}$, $k_{s_1}^2 = \frac{\omega^2}{V_{s_1}^2}$, V_{l_1}, V_{l_2} are velocities of longitudinal wave, V_{s_1} is velocity for rotational wave. Taking account of boundary condition, solutions of above equation are

$$\varphi_1 = A_1K_0(k_{r_1}R)e^{ik_0z}$$

$$\varphi_2 = A_2K_0(k_{r_2}R)e^{ik_0z}, \quad \psi_1 = B_1K_1(k_r'R)e^{ik_0z}$$

where $k_{r_1}^2 = k_0^2 - k_{p_1}^2$, $k_{r_2}^2 = k_0^2 - k_{p_2}^2$, $k_r'^2 = k_0^2 - k_{s_1}^2$.

2. Solid-Porous Media Cylindrical Boundary Condition

Cylindrical boundary condition of solid-porous media is as follows.

$$\begin{aligned}
&(u_r)^{\mathrm{I}}\big|_{r=a} = (u_r)^{\mathrm{II}}\big|_{r=a}\\
&(u_z)^{\mathrm{I}}\big|_{r=a} = (u_z)^{\mathrm{II}}\big|_{r=a}\\
&(\sigma_{rr})^{\mathrm{I}}\big|_{r=a} = (\sigma_{rr})^{\mathrm{II}}\big|_{r=a} + \sigma\,\big|_{r=a}\\
&(\sigma_{rz})^{\mathrm{I}}\big|_{r=a} = (\sigma_{rz})^{\mathrm{II}}\big|_{r=a}\\
&(U_r)^{\mathrm{II}}\big|_{r=a} = (u_r)\big|_{r=a}
\end{aligned} \tag{12}$$

where superscripts Ⅰ, Ⅱ stands for solid and porous media, u_r, u_z are displacements of solid in r and z direction, σ_{rr}, σ_{rz} are stress components, U_r is displacement of r direction in porous fluid.

3. Dispersion Equation

We calculate the components of displacement and stress from solutions of equations in bar and porous media such as

$$(u_r)^{\mathrm{I}} = A\overline{k_r} I_1(k_r R) - Bik_0 \overline{k_r'} I_1(\overline{k_r'} R), \quad (u_z)^{\mathrm{I}} = -Aik_0 I_0(\overline{k_r} R) - B\overline{k_r'}^2 I_0(\overline{k_r'} R),$$

$$(\sigma_{rr})^{\mathrm{I}} = A\left\{\left[(\lambda+2\mu)\overline{k_r}^2 - \lambda k_0^2\right] I_0(\overline{k_r} R) - 2\mu \frac{\overline{k_r}}{R} I_1(\overline{k_r} R)\right\} +$$

$$+ B\left[-2\mu i k_0 \overline{k_r}'^2 I_0(\overline{k_r}' R) + 2\mu \frac{ik_0 \overline{k_r}'}{R} I_1(\overline{k_r}' R)\right],$$

$$(u_r)^{\mathrm{II}} = -A_1 k_{r_1} K_1(k_{r_1} R) - A_2 k_{r_2} K_1(k_{r_2} R) - B_1 ik_0 K_1(k_r' R),$$

$$(U_r)^{\mathrm{II}} = -A_1 \xi_1 k_{r_1} K_1(k_{r_1} R) - A_2 \xi_2 k_{r_2} K_1(k_{r_2} R) - B_1 \chi ik_0 K_1(k_r' R),$$

$$(u_z)^{\mathrm{II}} = A_1 ik_0 K_0(k_{r_1} R) + A_2 ik_0 K_0(k_{r_2} R) - B_1 k_r' K_0(k_r' R),$$

$$(\sigma_{rr})^{\mathrm{II}} = A_1\left[2Nk_{r_1}^2 K_0(k_{r_1} R) + \frac{2Nk_{r_1}}{R} K_1(k_{r_1} R) + (A+Q\xi_1)(k_{r_1}^2 - K_0{}^2) K_0(k_{r_1} R)\right] +$$

$$+ A_2\left[2Nk_{r_2}^2 K_0(k_{r_2} R) + \frac{2Nk_{r_2}}{R} K_1(k_{r_2} R) + (A+Q\xi_2)(k_{r_2}^2 - K_0{}^2) K_0(k_{r_2} R)\right] +$$

$$+ B_1 2N\left[ik_0 k_r' K_0(k_r' R) + \frac{ik_0}{R} K_1(k_r' R)\right],$$

$$(\sigma)^{\mathrm{II}} = A_1(Q+R\xi_1)(k_{r_1}^2 - k_0{}^2) K_0(k_{r_1} R) + A_2(Q+R\xi_2)(k_{r_2}^2 - k_0{}^2) K_0(k_{r_2} R),$$

$$(\sigma_{rr})^{\mathrm{II}} + \sigma = A_1\left[2Nk_{r_1}^2 K_0(k_{r_1} R) + \frac{2Nk_{r_1}}{R} K_1(k_{r_1} R) + (A+Q+Q\xi_1+R)(k_{r_1}^2 - k_0^2) K_0(k_{r_1} R)\right] +$$

$$+ A_2\left[2Nk_{r_2}^2 K_0(k_{r_2} R) + \frac{2Nk_{r_2}}{R} K_1(k_{r_2} R) + (A+Q+Q\xi_2+R)(k_{r_2}^2 - k_0^2) K_0(k_{r_2} R)\right] +$$

$$+ B_1 2N\left[ik_0 k_r' K_0(k_r' R) + \frac{ik_0}{R} K_1(k_r' R)\right],$$

$$(\sigma_{rz})^{\mathrm{II}} = -A_1 2Nik_0 k_{r_1} K_1(k_{r_1} R) + A_2 2Nik_0(-k_{r_2}) K_1(k_{r_2} R) + B_1 N(k_r'^2 + k_0{}^2) K_1(k_r' R),$$

$$(P)^{\mathrm{II}} = -\frac{\sigma}{\beta} = -A_1 \frac{Q_1 + R\xi_1}{\beta}(k_{r_1}^2 - k_0{}^2) K_0(k_{r_1} R) - A_2 \frac{Q + R\xi_2}{\beta}(k_{r_2}^2 - k_0^2) K_0(k_{r_2} R).$$

Substituting the displacements and stresses into boundary condition, we can obtain matrix equation as follows.

$$\begin{pmatrix} m_{11} & m_{12} & m_{13} & m_{14} & m_{15} \\ m_{21} & m_{22} & m_{23} & m_{24} & m_{25} \\ m_{31} & m_{32} & m_{33} & m_{34} & m_{35} \\ m_{41} & m_{42} & m_{43} & m_{44} & m_{45} \\ m_{51} & m_{52} & m_{53} & m_{54} & m_{55} \end{pmatrix} \begin{pmatrix} A \\ B \\ A_1 \\ A_2 \\ B_1 \end{pmatrix} = 0 \tag{13}$$

where

$$m_{11} = \overline{kr} I_1(\overline{kr}a),$$
$$m_{21} = ik_0 I_0(\overline{kr}a),$$
$$m_{31} = [(\lambda + 2\mu)\overline{kr^2} - \lambda k_0^2] I_0(\overline{kr}a) - 2\mu \frac{\overline{kr}}{a} I_1(\overline{kr}a),$$
$$m_{41} = 2\mu i k_0 \overline{kr} I_1(\overline{kr}a),$$
$$m_{51} = 0,$$
$$m_{12} = ik_0 \overline{kr'} I_1(\overline{kr'}a),$$
$$m_{22} = -\overline{kr'^2} I_0(\overline{kr'}a),$$
$$m_{32} = 2\mu i k_0 \overline{kr'^2} I_0(\overline{kr'}a) - 2\mu \frac{ik_0 \overline{kr'}}{a} I_1(\overline{kr'}a),$$
$$m_{42} = -\mu(k_0^2 \overline{kr'} + \overline{kr'^3}) I_1(\overline{kr'}a),$$
$$m_{52} = 0,$$
$$m_{13} = kr_1 K_1(kr_1 a),$$
$$m_{23} = -ik_0 K_0(kr_1 a),$$
$$m_{33} = -2Nkr^2 K_0(kr_1 a) - \frac{2Nkr_1}{a} K_1(kr_1 a) - (A + Q + Q\xi_1 + R\xi_1)(kr_1^2 - k_0^2) K_0(kr_1 a),$$
$$m_{43} = 2Nik_0 kr_1 K_1(kr_1 a),$$
$$m_{53} = (1 - \xi_1) kr_1 K_1(kr_1 a),$$
$$m_{14} = kr_2 K_1(kr_2 a),$$
$$m_{24} = -ik_0 K_0(kr_2 a),$$
$$m_{34} = -2Nkr_2^2 = K_0(kr_2 a) - 2Nkr_2 / aK_1(kr_2 a) - (A + Q + Q\xi_2 + R\xi_2)(kr_2{}^2 - k_0{}^2) K_0(kr_2 a),$$
$$m_{44} = 2Nik_0 kr_2 K_1(kr_2 a),$$
$$m_{54} = (1 - \xi_2) kr_2 K_1(kr_2 a),$$
$$m_{15} = ik_0 K_1(kr'a),$$
$$m_{25} = kr' K_0(kr'a),$$
$$m_{35} = -2N[ik_0 kr' K_0(kr'a) + \frac{ik_0}{a} K_1(kr'a)],$$
$$m_{45} = -N(kr'^2 + k_0{}^2) K_1(kr'a),$$
$$m_{55} = (1 - \chi) i k_0 K_1(kr'a).$$

In order that matrix equation has a nonzero solutions, determinant of the coefficient must be zero. Thus, the following dispersion equation must be established as follows.

$$\begin{vmatrix} m_{11} & m_{12} & m_{13} & m_{14} & m_{15} \\ m_{21} & m_{22} & m_{23} & m_{24} & m_{25} \\ m_{31} & m_{32} & m_{33} & m_{34} & m_{35} \\ m_{41} & m_{42} & m_{43} & m_{44} & m_{45} \\ m_{51} & m_{52} & m_{53} & m_{54} & m_{55} \end{vmatrix} = 0$$

4. Numerical Calculation and Interpretation of Dispersion Equation

The dispersion equation is a nonlinear equation of about 70 orders, therefore the analytic solution may be impossible, so we must calculate approximately by computer. We make the numerical calculation using Newton method by MATLAB.

In case that radius of bar is $a = 0.1\text{m}$, porosity $\phi = 0.19$, viscous coefficient $\eta = 0.01$, density of bar $\rho_1 = 1\,500\text{kg}/\text{m}^3$, velocity of longitudinal wave $c_{p_1} = 2\,450\text{m}/\text{s}$, velocity of transversal wave in bar $c_{s_1} = 1\,500\text{m}/\text{s}$, density of solid $\rho_2 = 2\,650\text{kg}/\text{m}^3$, longitudinal velocity c_{p2}=3 670m/s and transversal velocity in porous media c_{s2}=2 170m/s respectively, density of porous media $\rho_f = 1\,000\text{kg}/\text{m}^3$, velocity in porous fluid c_{p3} = 1 500m/s, bulk modulus of porous solid, we obtained the dispersion curves shown in Fig.

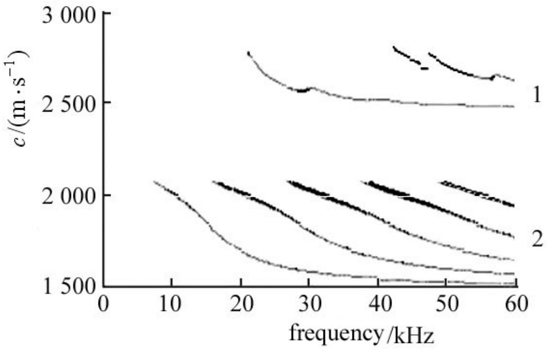

Fig. Dispersion curve of velocity according to frequency
1—longitudinal wave, 2—transversal wave

The dispersion curves have two forms, where, curves of first form approach from velocity of transversal wave in porous solid to velocity of transversal wave in bar, second forms approach from velocity of longitudinal wave in porous solid to velocity of longitudinal wave in bar. Imaginary parts of velocity of acoustical wave of second form are bigger than those of first form. Therefore a wave of second form does not exist.

Transversal reference wave modes in solid bar are corresponding to first form, while longitudinal reference wave modes to second form.

Conclusion

In this paper we established the propagation equation of acoustical wave in media with the solid-porous media cylindrical boundary condition and proceeded with a solution.

We put the solid-porous media cylindrical boundary condition and leaded the dispersion equation and progressing a numerical calculation of nonlinear equation of 70^{th} order and a interpretation, obtained a phase velocity dispersion curve.

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