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Numerical Analysis Model of COD Field Considering Self-Pollution of Coastal Aquaculture Field

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Abstract In this paper, the mathematic horizontal diffusion model considering tidal current, wind current and self-pollution in an aquaculture field and half-implicit difference scheme are suggested.

The error from numerical analysis of COD is 0.1mg/L, so this result can be useful.

Key words aquaculture field, nursery, COD(chemical oxygen demand), half-implicit difference scheme

Introduction

The great leader Comrade **Kim Il Sung** said as follows.

"Our coasts and territorial waters must be protected and taken good care of, so that the surrounding seas will not be polluted."("**KIM IL SUNG** WORKS" Vol. 39 P. 352)

For a great production of non-pollution marine products, the antipollution counter plan is being studied based on investigation and analysis of spatial distribution of COD by using numerical model analyzing COD in a farm. But affection of self-pollution hasn't been much studied.

Numerical model analyzing COD is used to predict red tide and to research pollution counter plan based on assessment of self-purifying capacity of the sea.

We have made a numerical model analyzing diffusion of COD in two-dimensional field even considering self-pollution of a coastal nursery and used for establishing a counter plan of marine pollution as well as choosing of a farm.

1. Basic Equation and Boundary Condition

The sources of organic pollution in a coastal farm are mainly wastes from factories and cities, and self-pollution of farm. There horizontal distribution of COD is generally changed by advection such as tidal current, wind current, turbulent diffusion process and self-alteration through chemical and biological process and etc.

We can use following formulas [1 – 5] as 2-dimensional equation of motion and continuity equation for the tidal and wind currents to analyze horizontal advection and diffusion characteristic of pollutant in a sallow water space.

$$\frac{\partial \xi}{\partial t}+\frac{\partial}{\partial x}[(d+\xi)u]+\frac{\partial}{\partial y}[(d+\xi)v]=0 \tag{1}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-g\frac{\partial \xi}{\partial x}+\frac{\tau_{sx}}{h\rho}+\frac{\tau_{bx}}{h\rho}+fv+A_e\Delta u \tag{2}$$

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-g\frac{\partial \xi}{\partial y}+\frac{\tau_{sy}}{h\rho}+\frac{\tau_{by}}{h\rho}-fu+A_e\Delta v \tag{3}$$

where $h=d+\xi$, d—mean depth of water, τ_s —wind stress at surface, τ_b —friction stress at floor, ξ — water height, ρ — density of sea water, A_e — horizontal frictional coefficient in turbulence, f—coriolis variable, x, y—latitudinal and longitudinal coordinates.

$$\vec{\tau}_s=\rho_a r_a^2\,|\vec{w}|\,\vec{w} \tag{4}$$

$$\vec{\tau}_b=\rho r_b^2\,|\vec{v}|\,\vec{v}-\beta\vec{\tau}_s \tag{5}$$

$$r_a^2=r_b^2=2.6\times10^{-3},\ \ \beta=0.35$$

where w—wind velocity, ρ_a —air density, at coastal area $V_n=0$.

At open boundary, a formula calculating tidal height when mean spring tide appears in a water space, where regular semidiurnal tide appeared, is given.

$$\xi_{(t)}=(H_{m_2}+H_{s_2})\cos(\omega_{m_2}t-g_{m_2}) \tag{6}$$

where ω_{m_2} —angular velocity of constituent m_2, H_{m_2}, H_{s_2}, g_{m_2} —amplitude and phase angle of constituent.

Bellow shows an equation of pollutant dispersion in 2-dimensional field.

$$\frac{\partial (hc)}{\partial t}+\frac{\partial}{\partial x}(hcu)+\frac{\partial}{\partial y}(hcv)=\frac{\partial}{\partial x}\left(hk_x\frac{\partial c}{\partial x}\right)+\frac{\partial}{\partial y}\left(hk_y\frac{\partial c}{\partial y}\right)+S \tag{7}$$

where c—cardinality of pollutant (COD), k_x, k_y —dispersion coefficient, S—a load of pollutant at unit square at unit time(incensement by pollutant source and chemical also biological self-alteration).

$$S=M-KC$$

where, self-attenuation coefficient K is different with season.

A measured value can be used as increase of COD by industrial and urban waste at sour ce item M, and the complement amount equals to RP because it is in proportioned to P (the amount of marine animals) and R was selected experientially.

Because vertical distribution of velocity in tidal current and wind one, the percent more than 80 of it, is nigh on logarithmic distribution (not when the tidal velocity is 0)

$$k_x=5.93h\sqrt{g}n\cdot d^{1/6}u\ ,\ \ k_x=5.93h\sqrt{g}n\cdot d^{1/6}v \tag{8}$$

where $n=0.34$ —Manning coefficient.

Boundary condition: there is no flux of pollutant at closed boundary, that is

$$k_n\frac{\partial c}{\partial n}=0$$

At open boundary c is constant in inflow condition; calculable boundary condition is given in outflow. If initial condition is as close as possible to real distribution, calculation is quickly progressed.

2. Numerical Analyzing Model

Forward difference quotient and symmetrical difference quotient are substituted for time derivative and spatial derivative respectively, and made by applying half-implicit difference schema (Fig. 1).

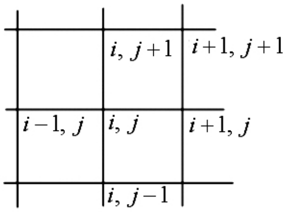

Fig. 1. Difference grid

Difference equation of formula (1) is

$$\xi_{i,\,j}^{n+1} = \xi_{i,\,j}^{n} - \Delta t\left\{\left[\frac{\partial}{\partial x}(hu)\right]_{i,\,j}^{n} + \left[\frac{\partial}{\partial y}(hv)\right]_{i,\,j}^{n}\right\}$$

$$h_{i,\,j}^{n} = \xi_{i,\,j}^{n} + d_{i,\,j}\,.$$

Difference equation of formula (2) is

$$u_{i,\,j}^{n+1} = \left\{u_{i,\,j}^{n} - \Delta t\left[v_{i,\,j}^{n}\left(\frac{\partial u}{\partial y}\right)_{i,\,j}^{n} + g\left(\frac{\partial \xi}{\partial x}\right)_{i,\,j}^{n} - \frac{(\tau_{sx})_{i,\,j}^{n}}{h_{i,\,j}^{n+1}\rho} - \frac{(\tau_{bx})_{i,\,j}^{n}}{h_{i,\,j}^{n+1}\rho} - fv_{i,\,j}^{n} - A_e A_u^n\right]\right\} \Big/ \left\{1 + \Delta t\left[\left(\frac{\partial u}{\partial x}\right)_{i,\,j}^{n} + A_e B u\right]\right\}.$$

where

$$\left(\frac{\partial u}{\partial y}\right)_{i,\,j}^{n} = \frac{u_{i,\,j+1}^{n} - u_{i,\,j-1}^{n}}{2\Delta y},\quad \left(\frac{\partial \xi}{\partial x}\right)_{i,\,j}^{n+1} = \frac{\xi_{i+1,\,j}^{n+1} - \xi_{i-1,\,j}^{n+1}}{2\Delta x},\quad \left(\frac{\partial u}{\partial x}\right)_{i,\,j}^{n} = \frac{u_{i+1,\,j}^{n} - u_{i-1,\,j}^{n}}{2\Delta x}$$

$$A_u^n = \frac{u_{i+1,\,j}^{n} + u_{i-1,\,j}^{n}}{\Delta x^2} + \frac{u_{i,\,j+1}^{n} + u_{i,\,j-1}^{n}}{\Delta y^2},\quad B = \frac{2}{\Delta x^2} + \frac{2}{\Delta y^2}\,.$$

Difference equation of formula (3) in same way is

$$v_{i,\,j}^{n+1} = \left\{v_{i,\,j}^{n} - \Delta t\left[u_{i,\,j}^{n+1}\left(\frac{\partial v}{\partial x}\right)_{i,\,j}^{n} + g\left(\frac{\partial \xi}{\partial y}\right)_{i,\,j}^{n+1} - \frac{(\tau_{sy})_{i,\,j}^{n}}{h_{i,\,j}^{n+1}\rho} - \frac{(\tau_{by})_{i,\,j}^{n}}{h_{i,\,j}^{n+1}\rho} - fu_{i,\,j}^{n+1} - A_e A_v^n\right]\right\} \Big/ \left\{1 + \Delta t\left[\left(\frac{\partial v}{\partial y}\right)_{i,\,j}^{n} + A_e B v\right]\right\}.$$

Difference equation of formula (7) is

$$\frac{h_{i,\,j}^{n+1}\cdot c_{i,\,j}^{n+1} - h_{i,\,j}^{n}\cdot c_{i,\,j}^{n}}{\Delta t} = -\frac{(hu)_{i+1,j}^{n+1}\cdot c_{i+1,j}^{n} - (hu)_{i-1,j}^{n+1}\cdot c_{i-1,j}^{n}}{2\Delta x} - \frac{(hv)_{i,j+1}^{n+1}\cdot c_{i,j+1}^{n} - (hv)_{i,j-1}^{n+1}\cdot c_{i,j-1}^{n}}{2\Delta y} +$$

$$+\left(\frac{\partial hk_x}{\partial x}\right)_{i,\,j}^{n+1}\left(\frac{\partial c}{\partial x}\right)_{i,\,j}^{n} + (hk_x)_{i,j}^{n+1}\left(\frac{\partial^2 c}{\partial x^2}\right)_{i,\,j}^{n} + \left(\frac{\partial hk_y}{\partial y}\right)_{i,\,j}^{n+1}\left(\frac{\partial c}{\partial y}\right)_{i,\,j}^{n} +$$

$$+(hk_y)_{i,\,j}^{n+1}\left(\frac{\partial^2 c}{\partial y^2}\right)_{i,\,j}^{n} + S_{i,j}^{n} = \overline{R}_{i,\,j} \Rightarrow c_{i,\,j}^{n+1} = \frac{\overline{R}_{i,\,j}\Delta t + h_{i,j}^{n} c_{i,j}^{n}}{h_{i,\,j}^{n+1}}.$$

3. Examination and Application of Numerical Analysis Model

When we tested by using value(Δt) which was determined by the stability condition of Kurant, the numerical calculable schema is stable. Not only calculable speed is able to be 5 times faster than the explicit difference schema, however calculable schema is simple. Numerical analyzed result of tidal current velocity, water height and COD is also satisfied to physical logic nature.

To examine reliability of COD, numerical analyzing model, we took comparative analysis that compare numerical result with the value of COD analyzed in the way that use potassium permanganate. Numerical analysis errors of COD in individual places are shown in table. Fig. 2 shows survey places.

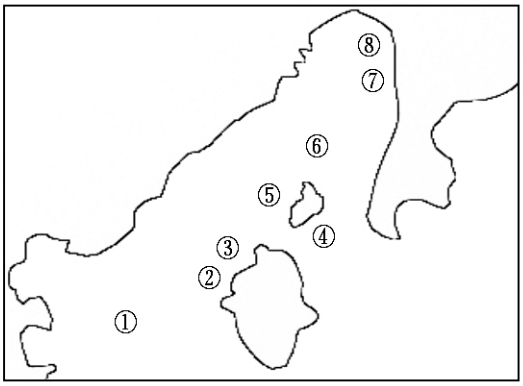

Fig. 2. Places for investigation

Table. Numerical analyzed error of COD(mg/L)

Number of place	Precedent way	New way
1	0.1	0.0
2	0.5	0.0
3	0.8	0.0
4	0.5	0.1
5	1.6	0.2
6	0.5	0.2
7	0.2	0.0
8	0.2	0.1
Mean	0.55	0.1

Mean absolute deviation of COD is 0.1mg/L. So we can understand that we can believe results sufficiently. Mean absolute error of precedent researching way that doesn't consider the self-pollution of a farm is 0.55mg/L.

We can know the field distribution of COD and its alterative characteristics and we can use this model effectively to establish a counter plan of coastal pollution like as selecting aquacultural density of a farm, determining of admissible limit of COD at a drain where discharge contaminant.

Conclusion

In the field numerical analysis model of COD, tidal current and wind current, pollutant discharge from land area and self-pollution in a farm are considered, and the mean absolute error is as small as 0.1mg/L that the result is sufficiently reliable. This model can be used for choosing location of coastal farm, selecting density of aquaculture and coastal environmental conservation.

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