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반파라메터최량판별규칙의 한가지 추정

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패턴인식을 비롯한 현실문제들에서는 반파라메터판별규칙에 의한 판별분석문제가 많이 제기되고있다.

선행연구[1－10]에서는 회귀분석문제로서 부분선형회귀모형과 반파라메터회귀모형의 추론문제, 파라메터판별함수에 의한 판별분석문제, 비파라메터판별함수에 의한 판별분석문제, 여러 반파라메터모형들사이의 판별분석문제, 극값판별분석과 선형판별분석모형의 일반화로서 반파라메터선형판별분석모형을 연구하였다.

론문에서는 여러 모집단에서 반파라메터최량판별규칙에 의한 판별모형을 제기하고 다차원표본자료들에 기초하여 최량판별규칙을 추정하는 문제를 취급하였다.

판별변량 $\boldsymbol{x}=(x_1, \cdots, x_p)^{\mathrm{T}}$ 에 관한 모집단 $G_r(r=\overline{1,\ m})$ 의 밀도함수와 사전확률을 각각 $f_r(\boldsymbol{x}),\ q_r\ (r=\overline{1,\ m})$ 라고 하자.

표본공간 R^p 를 공통부분이 없는 m 개의 구역 $R_1^p, \cdots, R_m^p$ 로 나누고 $\boldsymbol{x}\in R_1^p \Rightarrow \boldsymbol{x}\in G_1$, $\cdots$, $\boldsymbol{x}\in R_m^p \Rightarrow \boldsymbol{x}\in G_m$ 로 판별할 때 오판별확률

$$p=q_1\int_{\overline{R_1^p}} f_1(\boldsymbol{x})d\boldsymbol{x}+\cdots+q_m\int_{\overline{R_m^p}} f_m(\boldsymbol{x})d\boldsymbol{x}$$

가 최소로 되는 최량판별규칙을 구해보자.

$$H_r(\boldsymbol{x})=\frac{q_r f_r(\boldsymbol{x})}{\sum_{j=1}^{m} q_j f_j(\boldsymbol{x})}=P(G_r|\boldsymbol{x})\ (r=\overline{1,\ m-1}),\ H_m(\boldsymbol{x})=1-\sum_{r=1}^{m-1} H_r(\boldsymbol{x})=P(G_m|\boldsymbol{x}) \tag{1}$$

로 표시하자. 이때 모집단밀도함수와 사전확률들이 미지라고 하면 $H_r(\boldsymbol{x})\ (r=\overline{1,\ m})$ 는 미지파라메터 $\boldsymbol{\theta}=(\theta_1, \cdots, \theta_s)^{\mathrm{T}}\in\Theta$ 와 미지함수 $\boldsymbol{g}=(g_1, \cdots, g_q)^{\mathrm{T}}\in H\equiv H(R^p,\ R^q)$ 를 포함하는 반파라메터판별함수 $H_r(\boldsymbol{x})=H_r(\boldsymbol{x};\ \boldsymbol{\theta},\ \boldsymbol{g})\ \ (r=\overline{1,\ m})$ 로 된다.

반파라메터판별함수 $H_r(\boldsymbol{x})=H_r(\boldsymbol{x};\boldsymbol{\theta},\boldsymbol{g})\ (r=\overline{1,\ m})$ 에 의한 판별규칙

$$\begin{aligned}
&H_2(\boldsymbol{x};\boldsymbol{\theta},\boldsymbol{g})-H_1(\boldsymbol{x};\boldsymbol{\theta},\boldsymbol{g})<0,\ H_3(\boldsymbol{x};\boldsymbol{\theta},\boldsymbol{g})-H_1(\boldsymbol{x};\boldsymbol{\theta},\boldsymbol{g})<0,\ \cdots,\\
&\qquad\qquad \cdots,\ H_m(\boldsymbol{x};\boldsymbol{\theta},\boldsymbol{g})-H_1(\boldsymbol{x};\boldsymbol{\theta},\boldsymbol{g})<0\Rightarrow \boldsymbol{x}\in G_1\\
&H_2(\boldsymbol{x};\boldsymbol{\theta},\boldsymbol{g})-H_1(\boldsymbol{x};\boldsymbol{\theta},\boldsymbol{g})\ge 0,\ H_3(\boldsymbol{x};\boldsymbol{\theta},\boldsymbol{g})-H_2(\boldsymbol{x};\boldsymbol{\theta},\boldsymbol{g})<0,\cdots,\\
&\qquad\qquad \cdots,\ H_m(\boldsymbol{x};\boldsymbol{\theta},\boldsymbol{g})-H_2(\boldsymbol{x};\boldsymbol{\theta},\boldsymbol{g})<0\Rightarrow \boldsymbol{x}\in G_2\\
&\qquad\qquad \vdots\\
&H_m(\boldsymbol{x};\boldsymbol{\theta},\boldsymbol{g})-H_1(\boldsymbol{x};\boldsymbol{\theta},\boldsymbol{g})\ge 0,\ H_m(\boldsymbol{x};\boldsymbol{\theta},\boldsymbol{g})-H_2(\boldsymbol{x};\boldsymbol{\theta},\boldsymbol{g})\ge 0,\cdots,\\
&\qquad\qquad \cdots,\ H_m(\boldsymbol{x};\boldsymbol{\theta},\boldsymbol{g})-H_{m-1}(\boldsymbol{x};\boldsymbol{\theta},\boldsymbol{g})\ge 0\Rightarrow \boldsymbol{x}\in G_m
\end{aligned} \tag{2}$$

에서 오판별확률

$$p = q_1 \int_{R_1^p} f_1(\boldsymbol{x})d\boldsymbol{x} + \cdots + q_m \int_{R_m^p} f_m(\boldsymbol{x})d\boldsymbol{x}$$

$$R_1^p = \{\boldsymbol{x} \mid H_2(\boldsymbol{x}; \boldsymbol{\theta}, \boldsymbol{g}) - H_1(\boldsymbol{x}; \boldsymbol{\theta}, \boldsymbol{g}) < 0,\ H_3(\boldsymbol{x}; \boldsymbol{\theta}, \boldsymbol{g}) - H_1(\boldsymbol{x}; \boldsymbol{\theta}, \boldsymbol{g}) < 0,\ \cdots,\ \cdots,\ H_m(\boldsymbol{x}; \boldsymbol{\theta}, \boldsymbol{g}) - H_1(\boldsymbol{x}; \boldsymbol{\theta}, \boldsymbol{g}) < 0\}$$

$$R_2^p = \{\boldsymbol{x} \mid H_2(\boldsymbol{x}; \boldsymbol{\theta}, \boldsymbol{g}) - H_1(\boldsymbol{x}; \boldsymbol{\theta}, \boldsymbol{g}) < 0,\ H_3(\boldsymbol{x}; \boldsymbol{\theta}, \boldsymbol{g}) - H_2(\boldsymbol{x}; \boldsymbol{\theta}, \boldsymbol{g}) < 0,\ \cdots,\ \cdots,\ H_m(\boldsymbol{x}; \boldsymbol{\theta}, \boldsymbol{g}) - H_2(\boldsymbol{x}; \boldsymbol{\theta}, \boldsymbol{g}) < 0\}$$

$$\vdots$$

$$R_m^p = \{\boldsymbol{x} \mid H_m(\boldsymbol{x}; \boldsymbol{\theta}, \boldsymbol{g}) - H_1(\boldsymbol{x}; \boldsymbol{\theta}, \boldsymbol{g}) < 0,\ H_m(\boldsymbol{x}; \boldsymbol{\theta}, \boldsymbol{g}) - H_1(\boldsymbol{x}; \boldsymbol{\theta}, \boldsymbol{g}) < 0,\ \cdots,\ \cdots,\ H_m(\boldsymbol{x}; \boldsymbol{\theta}, \boldsymbol{g}) - H_{m-1}(\boldsymbol{x}; \boldsymbol{\theta}, \boldsymbol{g}) < 0\}$$

이 최소로 되는 $\boldsymbol{\theta}^*$, $\boldsymbol{g}^*$에 의한 판별규칙을 반파라메터최량판별규칙, $\boldsymbol{\theta}^*$, $\boldsymbol{g}^*$에 의한 반파라메터판별함수를 반파라메터최량판별함수라고 부른다.

론문에서는 다차원표본자료 $\boldsymbol{x}_1, \cdots, \boldsymbol{x}_n$에 기초하여 반파라메터최량판별규칙의 미지파라메터 $\boldsymbol{\theta}^*$와 미지함수 $\boldsymbol{g}^*$를 추정하는 한가지 방법을 취급하였다.

기 본 결 과

다차원표본자료 $\boldsymbol{x}_1, \cdots, \boldsymbol{x}_n$에 기초하여 반파라메터최량판별규칙의 미지파라메터 $\boldsymbol{\theta}^*$와 미지함수 $\boldsymbol{g}^*$를 추정하기 위하여

$$E\boldsymbol{p}_k = \boldsymbol{P}_k = (P_{1k}, \cdots, P_{m-1k})^{\mathrm{T}} = (H_1(\boldsymbol{x}_k; \boldsymbol{\theta}^*, \boldsymbol{g}^*), \cdots, H_{m-1}(\boldsymbol{x}_k; \boldsymbol{\theta}^*, \boldsymbol{g}^*))^{\mathrm{T}} = \boldsymbol{H}(\boldsymbol{x}_k; \boldsymbol{\theta}^*, \boldsymbol{g}^*)$$

$$\mathrm{Var}\,\boldsymbol{p}_k = \begin{pmatrix} P_{1k}Q_{1k} & -P_{1k}P_{2k} & \cdots & -P_{1k}P_{m-1k} \\ -P_{2k}P_{1k} & P_{2k}Q_{2k} & \cdots & -P_{2k}P_{m-1k} \\ & & \vdots & \\ -P_{m-1k}P_{1k} & -P_{m-1k}P_{2k} & \cdots & P_{m-1k}Q_{m-1k} \end{pmatrix} \quad (k = \overline{1,\ m})$$

로 되는 기준우연벡토르

$$\boldsymbol{p}_k = \begin{pmatrix} p_{1k} \\ \vdots \\ p_{m-1k} \end{pmatrix}, \quad p_{rk} = \begin{cases} 1 & , \ \boldsymbol{x}_k \in G_r \\ 0 & , \ \boldsymbol{x}_k \notin G_r \end{cases} \quad (r = \overline{1,\ m-1})$$

를 생각하면

$$\boldsymbol{p}_k = \boldsymbol{H}(\boldsymbol{x}_k; \boldsymbol{\theta}^*, \boldsymbol{g}^*) + \boldsymbol{\varepsilon}_k \tag{3}$$

$$E\,\boldsymbol{\varepsilon}_k = \boldsymbol{0},\ \ \mathrm{Var}\,\boldsymbol{\varepsilon}_k = \begin{pmatrix} P_{1k}Q_{1k} & -P_{1k}P_{2k} & \cdots & -P_{1k}P_{m-1k} \\ -P_{2k}P_{1k} & P_{2k}Q_{2k} & \cdots & -P_{2k}P_{m-1k} \\ & & \vdots & \\ -P_{m-1k}P_{1k} & -P_{m-1k}P_{2k} & \cdots & P_{m-1k}Q_{m-1k} \end{pmatrix}$$

로 쓸수 있다. 여기서 $Q_{rk} = 1 - P_{rk}$ $(r = \overline{1,\ m-1};\ k = \overline{1,\ n})$이다.

이때 $\boldsymbol{H}(\boldsymbol{x}; \boldsymbol{\theta}^*, \boldsymbol{g}^*)$에서 $\boldsymbol{x}$를 고정하면 $H_r(\boldsymbol{x}; \boldsymbol{\theta}^*, \boldsymbol{g}^*)\,(r = \overline{1,\ m-1})$은 $\boldsymbol{\alpha}^* = (\boldsymbol{\theta}^{*\mathrm{T}}, \boldsymbol{g}^{*\mathrm{T}})^{\mathrm{T}} \in \Theta \times H = R^{s+q}$에서 정의된 함수로 볼수 있다. 여기서는 $H_r(\boldsymbol{x}; \boldsymbol{\theta}^*, \boldsymbol{g}^*)\,(r = \overline{1,\ m-1})$이 $\boldsymbol{\alpha}^*$에

관하여 미분가능한 경우를 취급한다.

R^{s+q}에서 정의된 함수 $H_r(\boldsymbol{x};\boldsymbol{\theta}^*,\boldsymbol{g}^*)\ (r=\overline{1,\ m-1})$을 $\boldsymbol{\alpha}_0=(\boldsymbol{\theta}_0^{\mathrm{T}},\ \boldsymbol{g}_0^{\mathrm{T}})^{\mathrm{T}}$에서 테일러전개하면

$$H_r(\boldsymbol{x};\boldsymbol{\alpha}^*)=H_r(\boldsymbol{x};\boldsymbol{\alpha}_0)+\mathrm{grad}H_r(\boldsymbol{x};\boldsymbol{\alpha}_0)(\boldsymbol{\alpha}^*-\boldsymbol{\alpha}_0)+o_r(\|\boldsymbol{\alpha}^*-\boldsymbol{\alpha}_0\|)\ \ (r=\overline{1,\ m-1})$$

이다. 여기서 $U_r(\boldsymbol{x})=U_r(\boldsymbol{x};\boldsymbol{\alpha}^*)=o_r(\|\boldsymbol{\alpha}^*-\boldsymbol{\alpha}_0\|)\ (r=\overline{1,\ m-1})$으로 표시하고 간단히

$$\boldsymbol{H}(\boldsymbol{x};\boldsymbol{\alpha}^*)=\boldsymbol{H}(\boldsymbol{x};\boldsymbol{\alpha}_0)+\boldsymbol{H}'_{\boldsymbol{\alpha}^*}(\boldsymbol{x};\boldsymbol{\alpha}_0)(\boldsymbol{\alpha}^*-\boldsymbol{\alpha}_0)+\boldsymbol{o}(\|\boldsymbol{\alpha}^*-\boldsymbol{\alpha}_0\|)$$

$$\boldsymbol{o}(\|\boldsymbol{\alpha}^*-\boldsymbol{\alpha}_0\|)\equiv(U_1(x),\cdots,U_{m-1}(x))^{\mathrm{T}}=\boldsymbol{U}(x):\ \text{미지함수}$$

로 쓸수 있다. 따라서 반파라메터최량판별규칙의 $\boldsymbol{\theta}^*$, $\boldsymbol{g}^*$을 찾는 문제는 부분선형모형

$$\widetilde{\boldsymbol{p}}_k=\boldsymbol{H}'_{\boldsymbol{\theta}^*}(\boldsymbol{x}_k\ ;\boldsymbol{\alpha}_0)\cdot\boldsymbol{\theta}^*+\boldsymbol{H}'_{\boldsymbol{g}^*}(\boldsymbol{x}_k\ ;\boldsymbol{\alpha}_0)\cdot\boldsymbol{g}^*+\boldsymbol{U}+\boldsymbol{\varepsilon}_k \tag{4}$$

$$E\boldsymbol{\varepsilon}_k=\boldsymbol{0},\ \mathrm{Var}\,\boldsymbol{\varepsilon}_k=\begin{pmatrix} P_{1k}Q_{1k} & -P_{1k}P_{2k} & \cdots & -P_{1k}P_{m-1k}\\ -P_{2k}P_{1k} & P_{2k}Q_{2k} & \cdots & -P_{2k}P_{m-1k}\\ & & \vdots & \\ -P_{m-1k}P_{1k} & -P_{m-1k}P_{2k} & \cdots & P_{m-1k}Q_{m-1k}\end{pmatrix}$$

$$P_{rk}=H_r(\boldsymbol{x}_k;\boldsymbol{\theta}^*,\boldsymbol{g}^*),\ \ Q_{rk}=1-P_{rk}\ (r=\overline{1,\ m-1}),\ \ \boldsymbol{\theta}^*{}_{s\times1},\ \ \boldsymbol{g}^*{}_{q\times1}$$

$$\boldsymbol{H}'_{\boldsymbol{\theta}^*}(\boldsymbol{x}_k\ ;\boldsymbol{\alpha}_0)=\begin{pmatrix}\mathrm{grad}H_{1,\,\boldsymbol{\theta}^*}(\boldsymbol{x}_k\ ;\boldsymbol{\alpha}_0)\\ \vdots\\ \mathrm{grad}H_{m-1,\,\boldsymbol{\theta}^*}(\boldsymbol{x}_k\ ;\boldsymbol{\alpha}_0)\end{pmatrix}_{(m-1)\times s},\quad \boldsymbol{H}'_{\boldsymbol{g}^*}(\boldsymbol{x}_k\ ;\boldsymbol{\alpha}_0)=\begin{pmatrix}\mathrm{grad}H_{1,\,\boldsymbol{g}^*}(\boldsymbol{x}_k\ ;\boldsymbol{\alpha}_0)\\ \vdots\\ \mathrm{grad}H_{m-1,\,\boldsymbol{g}^*}(\boldsymbol{x}_k\ ;\boldsymbol{\alpha}_0)\end{pmatrix}_{(m-1)\times q}$$

$$\widetilde{\boldsymbol{p}}_k=\boldsymbol{p}_k-\boldsymbol{H}(\boldsymbol{x}_k;\alpha_0)+\boldsymbol{H}'_{\boldsymbol{\alpha}^*}(\boldsymbol{x}_k;\boldsymbol{\alpha}_0)\boldsymbol{\alpha}_0\,(k=\overline{1,\ n})$$

의 파라메터성분 $\boldsymbol{\theta}^*$와 비파라메터성분 $\boldsymbol{g}^*$을 찾는 문제로 되며 초기추정량 $\hat{\boldsymbol{\alpha}}_1$에 대하여 부분선형모형 (4)는

$$Y_k=X_k\cdot\boldsymbol{\theta}^*+Z_k\cdot\overline{\boldsymbol{g}}^*+\boldsymbol{\varepsilon}_k \tag{5}$$

$$Y_k=\boldsymbol{p}_k-\boldsymbol{H}(\boldsymbol{x}_k;\hat{\boldsymbol{\alpha}}_1)+\boldsymbol{H}'_{\boldsymbol{\alpha}^*}(\boldsymbol{x}_k;\hat{\boldsymbol{\alpha}}_1)\hat{\boldsymbol{\alpha}}_1,\ \ X_k=\boldsymbol{H}'_{\boldsymbol{\theta}^*}(\boldsymbol{x}_k;\hat{\boldsymbol{\alpha}}_1)$$

$$Z_k=(\boldsymbol{H}'_{\boldsymbol{g}^*}(\boldsymbol{x}_k;\hat{\boldsymbol{\alpha}}_1),I)_{(m-1)\times(q+m-1)},\ \ I=\begin{pmatrix}1&\cdots&0\\ \vdots&\ddots&\vdots\\ 0&\cdots&1\end{pmatrix}_{(m-1)\times(m-1)},\ \ \overline{\boldsymbol{g}}^*=(\boldsymbol{g}^{*\mathrm{T}},\ \boldsymbol{U}^{\mathrm{T}})^{\mathrm{T}}{}_{(q+m-1)\times1}$$

$$E\boldsymbol{\varepsilon}_k=\boldsymbol{0},\ Var\,\hat{}\ \boldsymbol{\varepsilon}_k=\begin{pmatrix} \hat{P}_{1k}\hat{Q}_{1k} & -\hat{P}_{1k}\hat{P}_{2k} & \cdots & -\hat{P}_{1k}\hat{P}_{m-1k}\\ -\hat{P}_{2k}\hat{P}_{1k} & \hat{P}_{2k}\hat{Q}_{2k} & \cdots & -\hat{P}_{2k}\hat{P}_{m-1k}\\ & & \vdots & \\ -\hat{P}_{m-1k}\hat{P}_{1k} & -\hat{P}_{m-1k}\hat{P}_{2k} & \cdots & \hat{P}_{m-1k}\hat{Q}_{m-1k}\end{pmatrix}$$

$$\hat{P}_{rk}=H_r(\boldsymbol{x}_k;\hat{\boldsymbol{\alpha}}_1),\ \ \hat{Q}_{rk}=1-\hat{P}_{rk}\ \ (r=\overline{1,\ m-1};\ k=\overline{1,\ n})$$

로 된다. $Y_k{}^{\mathrm{T}}=\boldsymbol{\theta}^{*\mathrm{T}}X_k{}^{\mathrm{T}}+\overline{\boldsymbol{g}}^{*\mathrm{T}}Z_k{}^{\mathrm{T}}+\boldsymbol{\varepsilon}_k{}^{\mathrm{T}}$이므로 $k=1,\ \cdots,\ n$에 대하여 $\boldsymbol{Y}$를 행렬로 표시하면

$$\boldsymbol{Y}=\begin{pmatrix}Y_1^{\mathrm{T}}\\ \vdots\\ Y_n^{\mathrm{T}}\end{pmatrix}_{n\times(m-1)}=\begin{pmatrix}\boldsymbol{\theta}^{*\mathrm{T}}X_1{}^{\mathrm{T}}\\ \vdots\\ \boldsymbol{\theta}^{*\mathrm{T}}X_n{}^{\mathrm{T}}\end{pmatrix}_{n\times(m-1)}+\begin{pmatrix}\overline{\boldsymbol{g}}^{*\mathrm{T}}Z_1{}^{\mathrm{T}}\\ \vdots\\ \overline{\boldsymbol{g}}^{*\mathrm{T}}Z_n{}^{\mathrm{T}}\end{pmatrix}_{n\times(m-1)}+\begin{pmatrix}\boldsymbol{\varepsilon}_k{}^{\mathrm{T}}\\ \vdots\\ \boldsymbol{\varepsilon}_k{}^{\mathrm{T}}\end{pmatrix}_{n\times(m-1)}=\boldsymbol{FB}+\boldsymbol{LG}+\boldsymbol{E}$$

이고 여기서 $\boldsymbol{F}$, $\boldsymbol{B}$, $\boldsymbol{L}$, $\boldsymbol{G}$ 는 다음의 행렬

$$\boldsymbol{F}=\begin{pmatrix} \mathrm{grad}H_{1,\,\theta^*}(\boldsymbol{x}_1\,;\hat{\boldsymbol{\alpha}}_1),\cdots,\mathrm{grad}H_{m-1,\,\theta^*}(\boldsymbol{x}_1\,;\hat{\boldsymbol{\alpha}}_1)\\ \mathrm{grad}H_{1,\,\theta^*}(\boldsymbol{x}_2\,;\hat{\boldsymbol{\alpha}}_1),\cdots,\mathrm{grad}H_{m-1,\,\theta^*}(\boldsymbol{x}_2\,;\hat{\boldsymbol{\alpha}}_1)\\ \vdots\\ \mathrm{grad}H_{1,\,\theta^*}(\boldsymbol{x}_n\,;\hat{\boldsymbol{\alpha}}_1),\cdots,\mathrm{grad}H_{m-1,\,\theta^*}(\boldsymbol{x}_n\,;\hat{\boldsymbol{\alpha}}_1)\end{pmatrix}_{n\times s(m-1)},\ \boldsymbol{B}=\begin{pmatrix}\boldsymbol{\theta}^* & \boldsymbol{0} & \cdots & \boldsymbol{0}\\ \boldsymbol{0} & \boldsymbol{\theta}^* & \cdots & \boldsymbol{0}\\ \vdots & \vdots & \ddots & \vdots\\ \boldsymbol{0} & \boldsymbol{0} & \cdots & \boldsymbol{\theta}^*\end{pmatrix}_{s(m-1)\times(m-1)}$$

$$\boldsymbol{L}=\begin{pmatrix}(\mathrm{grad}H_{1,\,\boldsymbol{g}^*}(\boldsymbol{x}_1\,;\hat{\boldsymbol{\alpha}}_1),1,0,\cdots,0),\cdots,(\mathrm{grad}H_{m-1,\,\boldsymbol{g}^*}(\boldsymbol{x}_1\,;\hat{\boldsymbol{\alpha}}_1),0,0,\cdots,1)\\ \vdots\\ (\mathrm{grad}H_{1,\,\boldsymbol{g}^*}(\boldsymbol{x}_n\,;\hat{\boldsymbol{\alpha}}_1),1,0,\cdots,0),\cdots,(\mathrm{grad}H_{m-1,\,\boldsymbol{g}^*}(\boldsymbol{x}_n\,;\hat{\boldsymbol{\alpha}}_1),0,0,\cdots,1)\end{pmatrix}_{n\times(q+m-1)(m-1)}$$

$$\boldsymbol{G}=\begin{pmatrix}\overline{\boldsymbol{g}}^* & \boldsymbol{0} & \cdots & \boldsymbol{0}\\ \vdots & \vdots & \ddots & \vdots\\ \boldsymbol{0} & \boldsymbol{0} & \cdots & \overline{\boldsymbol{g}}^*\end{pmatrix}_{(q+m-1)(m-1)\times(m-1)}$$

들이다. 따라서 식 (5)는

$$\boldsymbol{Y}=\boldsymbol{FB}+\boldsymbol{LG}+\boldsymbol{E} \tag{6}$$

$$E\boldsymbol{E}=\boldsymbol{0}_{n\times(m-1)},\quad \mathrm{Var}\boldsymbol{E}=\mathrm{Var}^{\wedge}\boldsymbol{E}=\begin{pmatrix}\mathrm{Var}^{\wedge}\boldsymbol{\varepsilon}_1 & \cdots & \boldsymbol{0}\\ \vdots & \ddots & \vdots\\ \boldsymbol{0} & \cdots & \mathrm{Var}^{\wedge}\boldsymbol{\varepsilon}_n\end{pmatrix}_{(m-1)n\times(m-1)n}$$

$$\mathrm{Var}^{\wedge}\boldsymbol{\varepsilon}_k=\begin{pmatrix}\hat{P}_{1k}\hat{Q}_{1k} & -\hat{P}_{1k}\hat{P}_{2k} & \cdots & -\hat{P}_{1k}\hat{P}_{m-1k}\\ -\hat{P}_{2k}\hat{P}_{1k} & \hat{P}_{2k}\hat{Q}_{2k} & \cdots & -\hat{P}_{2k}\hat{P}_{m-1k}\\ & & \vdots & \\ -\hat{P}_{m-1k}\hat{P}_{1k} & -\hat{P}_{m-1k}\hat{P}_{2k} & \cdots & \hat{P}_{m-1k}\hat{Q}_{m-1k}\end{pmatrix},$$

$$\hat{P}_{rk}=H_r(\boldsymbol{x}_k\,;\hat{\boldsymbol{\alpha}}_1),\quad \hat{Q}_{rk}=1-\hat{P}_{rk}\,(r=\overline{1,\ m-1};\ k=\overline{1,\ n})$$

로 된다. 모형 (6)의 미지파라메터 $\boldsymbol{\theta}^*$과 미지함수 $\boldsymbol{g}^*$을 추정하기 위하여 측면최소두제곱법과 국부선형화수법을 적용하면 다음의 결론이 나온다. 여기서 미지함수 $\boldsymbol{g}^*$은 $\boldsymbol{x}$에 관하여 미분가능하다고 가정한다.

보조정리 모형 (5)에서 오차의 분산을 등분산 즉 $\mathrm{Var}\,\boldsymbol{\varepsilon}_1=\cdots=\mathrm{Var}\,\boldsymbol{\varepsilon}_n$으로 줄 때 반파라메터최량판별규칙의 파라메터성분의 추정량은

$$\hat{\boldsymbol{\theta}}^*=\boldsymbol{e}_1^{\mathrm{T}}(\widetilde{\boldsymbol{F}}^{\mathrm{T}}\widetilde{\boldsymbol{F}})^{-1}\widetilde{\boldsymbol{F}}^{\mathrm{T}}\widetilde{\boldsymbol{Y}}$$

이며 비파라메터성분의 추정량은 $\hat{\overline{\boldsymbol{g}}}^*(\boldsymbol{x})=\hat{\boldsymbol{a}}=\boldsymbol{e}_2^{\mathrm{T}}\{\boldsymbol{D}_{\boldsymbol{x}}^{\mathrm{T}}\boldsymbol{W}_{\boldsymbol{x}}\boldsymbol{D}_{\boldsymbol{x}}\}^{-1}\boldsymbol{D}_{\boldsymbol{x}}^{\mathrm{T}}\boldsymbol{W}_{\boldsymbol{x}}(\boldsymbol{Y}-\boldsymbol{F}\hat{\boldsymbol{B}})$이다. 여기서

$$\boldsymbol{e}_1^{\mathrm{T}}=\begin{pmatrix}1&0&0&\cdots&0&0&\cdots&0\\0&1&0&\cdots&0&0&\cdots&0\\ \vdots&\vdots&\vdots&\ddots&\vdots&\vdots&\cdots&\vdots\\0&0&0&\cdots&1&0&\cdots&0\end{pmatrix}_{s\times s(m-1)},\ \boldsymbol{e}_2^{\mathrm{T}}=\begin{pmatrix}1&0&0&\cdots&0&0&\cdots&0\\0&1&0&\cdots&0&0&\cdots&0\\ \vdots&\vdots&\vdots&\ddots&\vdots&\vdots&\cdots&\vdots\\0&0&0&\cdots&1&0&\cdots&0\end{pmatrix}_{(q+m-1)\times(p+1)(q+m-1)(m-1)}$$

$$\boldsymbol{D}_{\boldsymbol{x}}=\begin{pmatrix}\left(\mathrm{grad}H_{1,\,\boldsymbol{g}^*}(\boldsymbol{x}_1\,;\hat{\boldsymbol{\alpha}}_1),1,0,\cdots,0,\frac{x_1^1-x_1^0}{h_1}\mathrm{grad}H_{1,\,\boldsymbol{g}^*}(\boldsymbol{x}_1\,;\hat{\boldsymbol{\alpha}}_1),\frac{x_1^1-x_1^0}{h_1},0,\right.\\ \left.\cdots,0,\cdots,\frac{x_p^1-x_p^0}{h_p}\mathrm{grad}H_{1,\,\boldsymbol{g}^*}(\boldsymbol{x}_1\,;\hat{\boldsymbol{\alpha}}_1),\frac{x_p^1-x_p^0}{h_p},0,\cdots,0\right),\cdots,\\ \cdots,\left(\mathrm{grad}H_{m-1,\,\boldsymbol{g}^*}(\boldsymbol{x}_1\,;\hat{\boldsymbol{\alpha}}_1),0,0,\cdots,1,\frac{x_1^1-x_1^0}{h_1}\mathrm{grad}H_{m-1,\,\boldsymbol{g}^*}(\boldsymbol{x}_1\,;\hat{\boldsymbol{\alpha}}_1),0,0,\right.\\ \left.\cdots,\frac{x_1^1-x_1^0}{h_1},\cdots,\frac{x_p^1-x_p^0}{h_p}\mathrm{grad}H_{m-1,\,\boldsymbol{g}^*}(\boldsymbol{x}_1\,;\hat{\boldsymbol{\alpha}}_1),0,0,\cdots,\frac{x_p^1-x_p^0}{h_p}\right)\\ \vdots\\ \left(\mathrm{grad}H_{1,\,\boldsymbol{g}^*}(\boldsymbol{x}_n\,;\hat{\boldsymbol{\alpha}}_1),1,0,\cdots,0,\frac{x_1^n-x_1^0}{h_1}\mathrm{grad}H_{1,\,\boldsymbol{g}^*}(\boldsymbol{x}_n\,;\hat{\boldsymbol{\alpha}}_1),\frac{x_1^n-x_1^0}{h_1},0,\right.\\ \left.\cdots,0,\cdots,\frac{x_p^n-x_p^0}{h_p}\mathrm{grad}H_{1,\,\boldsymbol{g}^*}(\boldsymbol{x}_n\,;\hat{\boldsymbol{\alpha}}_1),\frac{x_p^n-x_p^0}{h_p},0,\cdots,0\right),\cdots,\\ \cdots,\left(\mathrm{grad}H_{m-1,\,\boldsymbol{g}^*}(\boldsymbol{x}_n\,;\hat{\boldsymbol{\alpha}}_1),0,0,\cdots,1,\frac{x_1^n-x_1^0}{h_1}\mathrm{grad}H_{m-1,\,\boldsymbol{g}^*}(\boldsymbol{x}_n\,;\hat{\boldsymbol{\alpha}}_1),0,0,\right.\\ \left.\cdots,\frac{x_1^n-x_1^0}{h_1},\cdots,\frac{x_p^n-x_p^0}{h_p}\mathrm{grad}H_{m-1,\,\boldsymbol{g}^*}(\boldsymbol{x}_n\,;\hat{\boldsymbol{\alpha}}_1),0,0,\cdots,\frac{x_p^n-x_p^0}{h_p}\right)\end{pmatrix}_{n\times(p+1)(q+m-1)(m-1)}$$

$$\widetilde{\boldsymbol{D}}_{\boldsymbol{x}}=\begin{pmatrix}(\mathrm{grad}H_{1,\,\boldsymbol{g}^*}(\boldsymbol{x}_1\,;\hat{\boldsymbol{\alpha}}_1),1,0,\cdots,0,0\cdot\mathrm{grad}H_{1,\,\boldsymbol{g}^*}(\boldsymbol{x}_1\,;\hat{\boldsymbol{\alpha}}_1),0,0,\cdots,0,\cdots,\\ 0\cdot\mathrm{grad}H_{1,\,\boldsymbol{g}^*}(\boldsymbol{x}_1\,;\hat{\boldsymbol{\alpha}}_1),0,0,\cdots,0),\cdots,(\mathrm{grad}H_{m-1,\,\boldsymbol{g}^*}(\boldsymbol{x}_1\,;\hat{\boldsymbol{\alpha}}_1),0,0,\cdots,1,\\ 0\cdot\mathrm{grad}H_{m-1,\,\boldsymbol{g}^*}(\boldsymbol{x}_1\,;\hat{\boldsymbol{\alpha}}_1),0,0,\cdots,0,\cdots,0\cdot\mathrm{grad}H_{m-1,\,\boldsymbol{g}^*}(\boldsymbol{x}_1\,;\hat{\boldsymbol{\alpha}}_1),0,0,\cdots,0)\\ \vdots\\ (\mathrm{grad}H_{1,\,\boldsymbol{g}^*}(\boldsymbol{x}_n\,;\hat{\boldsymbol{\alpha}}_1),1,0,\cdots,0,0\cdot\mathrm{grad}H_{1,\,\boldsymbol{g}^*}(\boldsymbol{x}_n\,;\hat{\boldsymbol{\alpha}}_1),0,0,\cdots,0,\cdots,\\ 0\cdot\mathrm{grad}H_{1,\,\boldsymbol{g}^*}(\boldsymbol{x}_n\,;\hat{\boldsymbol{\alpha}}_1),0,0,\cdots,0),\cdots,(\mathrm{grad}H_{m-1,\,\boldsymbol{g}^*}(\boldsymbol{x}_n\,;\hat{\boldsymbol{\alpha}}_1),0,0,\cdots,1,\\ 0\cdot\mathrm{grad}H_{m-1,\,\boldsymbol{g}^*}(\boldsymbol{x}_n\,;\hat{\boldsymbol{\alpha}}_1),0,0,\cdots,0,\cdots,0\cdot\mathrm{grad}H_{m-1,\,\boldsymbol{g}^*}(\boldsymbol{x}_n\,;\hat{\boldsymbol{\alpha}}_1),0,0,\cdots,0)\end{pmatrix}_{n\times(p+1)(q+m-1)(m-1)}$$

$$\widetilde{\boldsymbol{F}}=\boldsymbol{F}-\boldsymbol{SF}\,,\quad \widetilde{\boldsymbol{Y}}=\boldsymbol{Y}-\boldsymbol{SY}\,,\quad \boldsymbol{S}=\widetilde{\boldsymbol{D}}_{\boldsymbol{x}}\{\boldsymbol{D}_{\boldsymbol{x}}^{\mathrm{T}}\boldsymbol{W}_{\boldsymbol{x}}\boldsymbol{D}_{\boldsymbol{x}}\}^{-1}\boldsymbol{D}_{\boldsymbol{x}}^{\mathrm{T}}\boldsymbol{W}_{\boldsymbol{x}}(\boldsymbol{Y}-\boldsymbol{FB})$$

$\boldsymbol{W}_{\boldsymbol{x}}=\mathrm{diag}(K_{\boldsymbol{h}}(\boldsymbol{x}_1-\boldsymbol{x}_0),\cdots,K_{\boldsymbol{h}}(\boldsymbol{x}_n-\boldsymbol{x}_0))_{n\times n}$, $K_{\boldsymbol{h}}=\prod_{i=1}^{p}K(x_i/h_i)/h_i$, $\boldsymbol{x}=(x_1,\cdots,x_p)^{\mathrm{T}}$, $K(\cdot)$는 일정한 조건을 만족시키는 핵함수, $\boldsymbol{h}=(h_1,\cdots,h_p)^{\mathrm{T}}$는 평활화파라메터, $\hat{\boldsymbol{B}}=(\widetilde{\boldsymbol{F}}^{\mathrm{T}}\widetilde{\boldsymbol{F}})^{-1}\widetilde{\boldsymbol{F}}^{\mathrm{T}}\widetilde{\boldsymbol{Y}}$이다.

정리 모형 (5)에서 반파라메터최량판별규칙의 파라메터성분의 추정량은

$$\hat{\hat{\boldsymbol{\theta}}}^*=\boldsymbol{e}_3^{\mathrm{T}}((\widetilde{\boldsymbol{F}}\otimes\boldsymbol{I}_{m-1})^{\mathrm{T}}\hat{\boldsymbol{\Sigma}}^{-1}(\widetilde{\boldsymbol{F}}\otimes\boldsymbol{I}_{m-1}))^{-1}(\widetilde{\boldsymbol{F}}\otimes\boldsymbol{I}_{m-1})^{\mathrm{T}}\hat{\boldsymbol{\Sigma}}^{-1}\mathrm{Vec}(\widetilde{\boldsymbol{Y}})$$

이며 비파라메터성분의 추정량은 $\hat{\hat{g}}^*(x)=\hat{a}=e_2^{\mathrm{T}}\{D_x^{\mathrm{T}}W_xD_x\}^{-1}D_x^{\mathrm{T}}W_x\left(Y-F\hat{\hat{B}}\right)$ 이다. 여기서

$$e_3^{\mathrm{T}}=\begin{pmatrix}1&\cdots&0&0&\cdots&0&\cdots&0&\cdots&0&0&\cdots&0\\0&\cdots&0&1&\cdots&0&\cdots&0&\cdots&0&0&\cdots&0\\&\vdots&&&\vdots&&\ddots&&\vdots&&&\vdots&\\0&\cdots&0&0&\cdots&0&\cdots&1&\cdots&0&0&\cdots&0\end{pmatrix}_{s\times s(m-1)(m-1)}$$

, I_{m-1} 은 $m-1$ 차단위행렬,

$\hat{\Sigma}=\mathrm{Var}\,\hat{E}$, $\mathrm{Vec}(\hat{\hat{B}}^{\mathrm{T}})=((\tilde{F}\otimes I_{m-1})^{\mathrm{T}}\hat{\Sigma}^{-1}(\tilde{F}\otimes I_{m-1}))^{-1}(\tilde{F}\otimes I_{m-1})^{\mathrm{T}}\hat{\Sigma}^{-1}\mathrm{Vec}(\tilde{Y})$ 이다.

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An Estimation of the Semiparametric Optimal Discriminant Rule

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In this paper, using multivariate samples, $x_1, \cdots, x_n$, we estimate unknown parameter θ^* and function g^* of the semiparametric optimal discriminant rule for several populations. Here we apply profile least squares.

Key words: semiparametric discriminant rule, misclassification probability, discriminant analysis