A Study on the Composition of a New Family of Bandlimited Interpolating Wavelet Functions Basis and the Application

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Supreme Leader **Kim Jong Un** said as follows.

"This is the age of science and technology, and its level of scientific and technological development is a criterion for judging the overall strength and status of a country."

Abstract

We proposed the mathematical methodology of the composing of a new family of band-limited interpolating wavelet functions with the property of regularity, interpolating, band-limited, symmetry, decay that are useful to nonlinear approximation and we formulated a new family of band-limited interpolating wavelet functions with the form of analysis and we demonstrated the properties of our wavelet functions composed.

And we developed new solvers of KDV equation by our wavelet functions and we get the approximation solution with any high precision according to the order of our wavelet functions and we demonstrated the advantage of our wavelet function. Our wavelet formulas exposed the advantage of the quality in advanced engineering applications.

In the next issue we will propose another new our wavelet functions formula as serials that exposed the advantages in practical quantum electronics, nanomaterial design, analysis for the financial series and the development of the electronic product of the new generation.

1. Introduction

We studied the basis theory of wavelet that is a very important in mathematics recently and their application to approximation analysis.

In the middle of 1980's, the earlier wavelet function is born by J. Morlet and the basic theory on it is formed by Y. Meyer. [2]

After that J. Stromberg composed the orthonormal wavelet basis. [2]

Y. Meyer and S. Mallat proposed MRA (Multi Resolution Analysis) and therefore proposed a systematic method for composing orthogonal wavelet. [2, 3, 4, 5, 6]

I.Daubechies composed the wavelet basis with compacted support and therefore the theory of composing wavelet basis is advanced forward. [1]

After that C. K. Chui composed spline wavelet and by G.Beylkin, R.Coifman, asymmetric wavelet is composed. [8, 12, 13, 14, 27]

And Y. Meyer, R.Coifman, M.V.Wickerhauser proposed wavelet packets and adapted wavelet. [10, 11]

After that various form of wavelet functions developed such as biorthogonal wavelet, semiorthogonal wavelet, multiwavelet, interpolating wavelet and therefore the composing theory of wavelet is expanded and they are applied widely. [7, 28-42]

In the whole, the theory of composing wavelet function has tended toward two sides. [1-9, 12-14, 16-20, 23-26]

Firstly, the problem is the composition of wavelet functions by which we can achieve the efficiency and the low complexity of the computation and the wonderful approximation of strong nonlinear object with the form of the analysis. Especially the user keenly needed the formularization of the wavelet basis function with the form of analysis.

Secondly the problem is the methodology of the unified composition of the wavelet filter so that the user can select the optimal wavelet depends on the object.

In this paper we proposed the methodology of the composition of the useful wavelet basis function with the property of the bandlimited, good normality, the interpolating, symmetry and good degree of the decay which satisfy the principle of the uncertainty.

At next series we will propose the methodology of the unified composition of the wavelet filter so that the user can select the optimal wavelet depends on the object. Here wavelet function is get with the form of numerical function.

The property of the interpolating wavelet is that the computation of the wavelet coefficient is easy by the value of the sampling and therefore the precision is high and the complexity of the computation is low. The interpolating wavelet is very important and then applied to various fields widely.

The advantages are as following.

Firstly the digital sampling space is represented directly by the MRA space by the interpolating wavelet. Secondly the interpolating wavelet expansion coefficients of the function are gotten by the linear combination of the sampling value, not by the traditional inner product. Thirdly the wavelet analysis of the function is precise and therefore the computational complexity is reduced largely.

At side of the approximation theory, by the interpolating wavelet we can get more stable and precise approximation schemas and they can represent the boundary condition of the partial differential equation. At this reason because of the advantages of the computational practice the interpolating wavelet functions are used widely.

In the prior paper, there are some interpolating wavelet functions [17-20, 40], but their forms of the composition are complex and are not the form of the analysis and have not the good normality. And they have not the property of the bandlimited that the Fourier transformation of the wavelet function has compacted support. And Shannon's wavelet has the property of the bandlimited but has the property of the low decay and then has some defects at the practice of the computation. Especially the precision of the nonlinear approximation is fall down by it. [15, 22]

The beginning of the interpolating wavelet is Shannon's interpolating wavelet and then D.L.Donoho intensified the study and proposed the composition of the interpolating wavelet by convolution of the some wavelet functions. At the next, Deslauriers[18,19] and Dubuc[20] proposed Deslauriers-Dubuc's interpolating wavelet by Daubechies's wavelet and then Schauder'wavelet by Haar'wavelet and Battle-Lemarie's wavelet are proposed.

And Aldroubi, Unser and Saito, Beylkin proposed the interpolating wavelet depend on the Donoho's method of the convolution. Else there are Devore-Popov's wavelet, Lemarie-Malgouyres's wavelet and Sickd's wavelet. These wavelet functions have the complicated form of the composition and have not analytic representation and the good normality. After that two dimension of Devore-Jawerth-Lucier's wavelet is proposed.

These interpolating wavelet functions have not the property of the bandlimited that Fourier transformation of the wavelet has compacted support. And Shannon's wavelet has the bandlimited but property of the bad decay and therefore has some defects at the practice of the computation of that the computational complexity is increase and the precision is low and the rate of the compression is low.

2. The composition of the bandlimited interpolating wavelet with the property of the normality and the character

We proposed a method of the composing new bandlimited interpolating wavelet functions with the property of the normality and we demonstrate the character. We get formula of new wavelet functions with the property of the normality, the symmetry, the interpolating, the bandlimited and the property of the bad decay. And we demonstrate those characters.

2.1 Composition

Let continuous function $\varphi(x) \in L^2(\mathbf{R})$ satisfy following conditions.

(i) $\varphi(k) = \begin{cases} 1, k = 0 \\ 0, k \in Z \end{cases}$

(ii) $\varphi(x) = \sum_k \varphi(\frac{k}{2})\varphi(2x-k),$

$\{\varphi_{jk} | \varphi_{jk}(x) = 2^{j/2}\varphi(2^j x - k)\}_{k\in z}$: Riesz basis

(iii) $\exists R \in \mathbf{R} > 0 : \varphi(x)$ is Hölder continuous with degree of R

(iv) $|\varphi^{(m)}(x)| \le c_l(1+|x|)^{-l}, \quad x \in \mathbf{R}, \; l > 2, \; 0 \le m \le [R]$

If $V_j := \{ f \mid f = \sum_k \beta_{jk}\varphi_{jk} \}$ then the space sequence $\{V_j\}$ satisfy $V_j \subset V_{j+1}$.

When $\beta_{jk} = f(\frac{k}{2^j})$, $p_j f = \sum_k \beta_{jk}\varphi_{jk}$, the following theorem is stated.

[Theorem 2.1](D.L.Donoho) Let $f \in c_0(\mathbf{R})$, let $\varphi(x)$ satisfy condition (i)-(iv). Then

$$\left\| f - p_j f \right\|_\infty \to 0 (j \to \infty)$$

Next let us think B-spline.

$$N_1(x) = \chi_{[0,1]} = \begin{cases} 1, & x \in [0,1) \\ 0, & else \end{cases}$$

$$N_m(x) = N_{m-1} * N_1 = \int_{-\infty}^{+\infty} N_{m-1}(x-t)N_1(t)dt = \int_0^1 N_{m-1}(x-t)dt, \quad m \in N, \quad m \ge 2$$

At this the next expression is stated.

$$N_m(x) = \frac{x}{m-1}N_{m-1}(x) + \frac{m-x}{m-1}N_{m-1}(x-1)$$

Next we define next function.

$$h\mathrm{m}(x) := \frac{m}{2\varepsilon} N_m\left(\frac{m}{2\varepsilon}x + \frac{m}{2}\right), m = 1,2,\cdots, \varepsilon : 0 < \varepsilon \le \frac{\pi}{3} : \text{real number}$$

And we define a function $\varphi(x)$ of that the Fourier formation $\hat{\varphi}(\omega)$ is defined as following.

$$\hat{\varphi}(\omega) := \int_{\omega-\pi}^{\omega+\pi} h_{\mathrm{m}}(x)dx, \omega \in \mathbf{R}$$

Then they satisfy next characters.

[Lemma 2.1] $\hat{\varphi}(\omega)$ have next characters.

(i) $\hat{\varphi}(\omega) \geq 0, \hat{\varphi}(-\omega) = \hat{\varphi}(\omega)$

(ii) $\operatorname{supp} \hat{\varphi} = [-\pi - \varepsilon, \pi + \varepsilon], \hat{\varphi}(\omega):$ continuous function

(iii) $\hat{\varphi}(\omega) = 1, \omega \in [-\pi + \varepsilon, \pi - \varepsilon]$

(proof) The function $h_m(x)$ has the following characters by the definition and property of the B-spline.

(i) $h_m(x) \in L(\mathbf{R}), \quad h_m \geq 0, \quad m = 1, \quad 2, \quad \cdots$

(ii) $\int_{-\infty}^{+\infty} h_m(x)dx = 1, \quad h_m(x)$: the even function, $m = 1, 2, \cdots$

(iii) $\operatorname{supp} \mathrm{h_m} = [-\varepsilon, \varepsilon]$

Therefore the result is proved by the composition of $\hat{\varphi}(\omega)$.

(end)

2.2 Property

[**Theorem 2.2**] $\varphi(x)$ defined upper is scaling function with the property of the symmetry, the interpolating and the bandlimited.

The wavelet function that the scaling function corresponding is $\varphi(x)$, $\operatorname{supp} \hat{\varphi} = [-\pi - \varepsilon, \pi + \varepsilon],$ is as following.

$$\psi(x) = 2\varphi(2x - 1) - \varphi\left(x - \frac{1}{2}\right)$$

And let $\varphi_m(x)$ be $\varphi(x)$ corresponding to $N_m(x)$, then it has property of the decay as following.

$$\varphi_m(x) = 0\left(\frac{1}{|x|^{m-1}}\right)$$

(proof) $\operatorname{supp} \hat{\varphi} = [-\pi - \varepsilon, \pi + \varepsilon]$ is stated by Lemma 2.1.

At $0 \leq \omega \leq 2\pi$, $\sum_k \hat{\varphi}(\omega + 2k\pi) = \hat{\varphi}(\omega) + \hat{\varphi}(\omega + 2\pi) + \hat{\varphi}(\omega - 2\pi)$

At $0 \leq \omega \leq \pi - \varepsilon$, $\sum_k \hat{\varphi}(\omega + 2k\pi) = \hat{\varphi}(\omega)$

On the other hand, by Lemma 2.1 $\hat{\varphi}(\omega)=1$ at $0\le\omega\le\pi-\varepsilon$

At $\pi-\varepsilon\le\omega\le\pi+\varepsilon$, $\hat{\varphi}(\omega)=\int_{\omega-\pi}^{\varepsilon} h_{\mathrm{m}}(x)\ \ dx$

$\hat{\varphi}(\omega-2\pi)=\int_{-\varepsilon}^{\omega-\pi} h_m(x)dx, \hat{\varphi}(\omega+2\pi)=0$. Therefore the following expression is stated.

$$\sum_k \hat{\varphi}(\omega+2k\pi)=\hat{\varphi}(\omega)+\hat{\varphi}(\omega-2\pi)=\int_{\omega-\pi}^{\varepsilon} h_m(x)dx+\int_{-\varepsilon}^{\omega-\pi} h_m(x)dx=\int_{-\varepsilon}^{\varepsilon} h_m(x)dx=1$$

And when the condition $h_{00}, h_{10},\ \ h_{20},\cdots,h_{N0},0,\cdots 0$ is satisfied, the following expression is stated.

$$\sum_k \hat{\varphi}(\omega+2k\pi)=\hat{\varphi}(\omega-2\pi)=\int_{-\varepsilon}^{\varepsilon} h_m(x)dx=1$$

Therefore $\varphi(x)$ satisfy $\varphi(n)=\delta_{n0}$ and has the property of the interpolating.

Let the definition $H\left(\frac{\omega}{2}\right):=\sum_k \hat{\varphi}(\omega+4k\pi)$ be set. Then when the condition $|\omega|\ge\pi+\varepsilon$ is satisfied, the following expression is stated.

$$H\left(\frac{\omega}{2}\right)=0,\ \ \hat{\varphi}(\omega)=0, \hat{\varphi}(\omega)=H\left(\frac{\omega}{2}\right)\hat{\varphi}\left(\frac{\omega}{2}\right)$$

And when the condition $|\omega|\le\pi+\varepsilon$ is satisfied, the following expression is stated.

$$\hat{\varphi}\left(\frac{\omega}{2}\right)=1, H\left(\frac{\omega}{2}\right)=\hat{\varphi}(\omega),\ \ \hat{\varphi}(\omega)=H\left(\frac{\omega}{2}\right)\hat{\varphi}\left(\frac{\omega}{2}\right).$$

Next Let the definition $\Phi(\omega):=\sum_k|\hat{\varphi}(\omega+2k\pi)|^2$ be set. Then $\Phi(\omega)$ is 2π-periodic even function.

At the arbitrary ω from the definition of $\hat{\varphi}(\omega)$, $\Phi(\omega)$ has maximum two items which are not zero and is bounded upper.

And $\hat{\varphi}(\omega)$ is bounded down when $[-\pi, \ \pi] \in \omega$.

Therefore $\{\varphi(x-n)\}_{n\in z}$ is Riesz basis.

On the other hand,

$$e^{-i(\omega/2)}\hat{\psi}(\omega) = \hat{\varphi}\left(\frac{\omega}{2}\right)\hat{\varphi}(\omega-2\pi)+\hat{\varphi}(\omega+2\pi) =$$

$$= \hat{\varphi}\left(\frac{\omega}{2}\right)(\hat{\varphi}(\omega-2\pi)+\hat{\varphi}(\omega+2\pi)+\hat{\varphi}(\omega)-\hat{\varphi}(\omega)) =$$

$$= \hat{\varphi}\left(\frac{\omega}{2}\right)(1-\hat{\varphi}(\omega)) = \hat{\varphi}\left(\frac{\omega}{2}\right)-\hat{\varphi}(\omega)$$

Let the definition $\psi\left(x+\frac{1}{2}\right) = 2\varphi(2x)-\varphi(x)$ be set.

Then $\psi(x) = 2\varphi(2x-1)-\varphi\left(x-\frac{1}{2}\right)$.

$\hat{\varphi}_m(\omega) \in c^{m-1}$ is stated. From $\varphi(x) = \int_{-\infty}^{+\infty}\hat{\varphi}(\omega)e^{i\omega x}d\omega = \int_{-\pi-\varepsilon}^{\pi+\varepsilon}\hat{\varphi}(\omega)e^{i\omega x}d\omega$,

$\varphi_m(x) = 0\left(\frac{1}{|x|^{m-1}}\right)$ is stated. (end)

[Theorem 2.3] When $m \geq 2$, using the $\varphi(x)$ composed upper, any function $f \in c_0(\boldsymbol{R})$,

$\left\| f - P_j f \right\|_\infty \to 0 (j \to \infty)$

is stated. Here $p_j f = \sum_k \beta_{jk}\varphi_{jk}$

Proof (abbreviation)

[Theorem 2.4] When $m \geq 2$, the $\varphi(x)$ composed upper satisfy the result of the theorem 2.2 for $f \in c_0(\boldsymbol{R})$.

Proof (abbreviation).

(Example 1) when $m = 2$, at real ε that satisfies $0 < \varepsilon \leq \frac{\pi}{3}$, new scaling function and wavelet function proposed upper are as following.(Fig.1, Fig.2)

$$\varphi(x)=\frac{1}{\pi}\cdot\frac{(-\sin(\pi-\varepsilon)x+2\sin\pi x-\sin(\pi+\varepsilon)x)}{x^3\varepsilon^2},x\neq 0$$

$$\varphi(0)=1$$

$$\Psi(x)=2\varphi(2x-1)-\varphi\left(x-\frac{1}{2}\right)$$

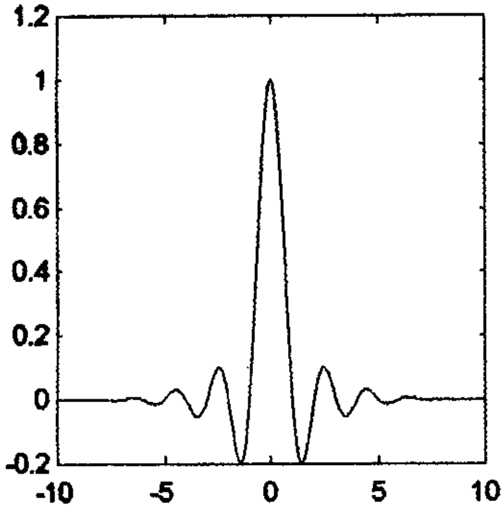

Fig.1 Scaling function

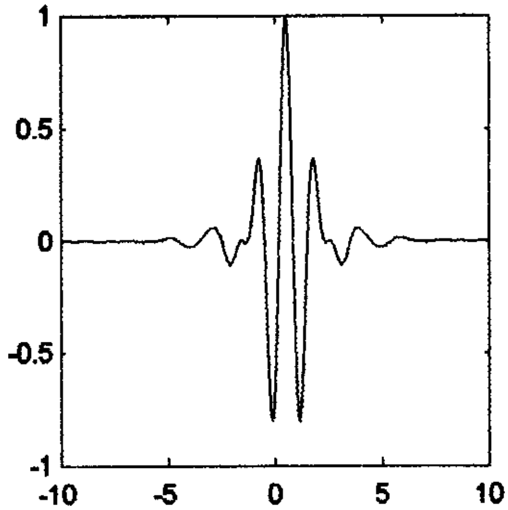

Fig. 2 Wavelet function

(Example 2) When $m=3$, at real ε that satisfies $0<\varepsilon\leq\frac{\pi}{3}$, new scaling function and wavelet function proposed upper are as following.

$$\varphi(x)=\frac{1}{\pi}\frac{27\left(-\cos(\pi-\varepsilon)x+3\cos\left(\pi-\frac{\varepsilon}{3}\right)x-3\cos\left(\pi+\frac{\varepsilon}{3}\right)x+\cos(\pi+\varepsilon)x\right)}{8x^4\varepsilon^3},x\neq 0$$

$$\varphi(0)=1$$

$$\Psi(x)=2\varphi(2x-1)-\varphi\left(x-\frac{1}{2}\right)$$

A new sampling theorem by our composed upper

[Theorem 2.5] If the support of $\hat{\varphi}_m(x),\hat{f}$ which are the Fourier transformations of our scaling function $\varphi_m(x)$ with the order m and $f(x)\in L^2(R)$ is included in $[-\pi/T,\pi/T]$, then following equation is stated.

$$f(t)=\sum_{n=-\infty}^{+\infty} f(nT)\varphi_m(t-nT)$$

Proof (abbreviation).

[Remark] Our sampling theorem is advantaged by our scaling function than the sampling theorem by Shannon. Because of the advantages for the decaying of wavelet, in the computational practice the computational complexity is lower and the precision is higher. As the order m of our wavelet is larger, the order of decaying of our wavelet is larger and then in the computational practice the computational complexity is lower and the precision is higher.

We will propose our new wavelet that has advantages in various engineering controls at the next issue.

3. Application of the wavelet basis proposed upper

We studied on new engine for a solver of KDV equation that is the mathematical model of financial series and is the important key for the explore of the new generation electronic products and we explored a solver of Schrödinger equation which is the important mathematical model of quantum mechanics with high precision and then on the based on it we contributed analysis quantum electromagnetic field.

The former solver for KDV equation has not so high precision with reason of not high degree of approximation on strength nonlinearity of equation. [40-42] Also, when using basis existing wavelet, the precision of the wavelet approximation is not high because of the limitation of the property of the wavelet functions.

We composed a new wavelet basis appropriate to the approximation of the KDV equation and then based on it, we exploit a solver with high precision. This solver has any precision according to the order of our new wavelet. This solver is very efficiency to solve the Schrödinger equation and the analysis quantum field.

Composition of a new wavelet basis function and a composition of a wavelet solver of KDV equation

In this section, we use new wavelet basis functions composed upper and then based on it, we composed a new solver of KDV equation with high precision and then we proved the convergence.

We use wavelet function with $m=2$ at formula. These are as following.

When $m=2$, at real ε with $0<\varepsilon\leq\frac{\pi}{3}$,

$$\varphi(x) = \frac{1}{\pi} \cdot \frac{(-\sin(\pi-\varepsilon)x + 2\sin \pi x - \sin(\pi+\varepsilon)x)}{x^3 \varepsilon^2}, x \neq 0$$

$$\varphi(0) = 1$$

$$\Psi(x) = 2\varphi(2x-1) - \varphi\left(x - \frac{1}{2}\right) \qquad (1)$$

A study on the solver of KDV equation has been widely because of the importance. Especially Fourier analysis method and Wavelet Analysis method are used. Typically Daubechies Wavelet, Spline-Wavelet and Wavelet packet are used. These schemas are Galerkin schema, Petrov-Galerkin schema and modified Petrov-Galerkin schema. But when using them, because of the limitation of the functions for basis, the precisions of the schemas are not high enough.

The key of lifting the precision is related to the property of basis function and therefore the key is the using and composing of the basis function with the superior property that approximating the nonlinearity more exactly.

In this section using a new band-limited interpolation wavelet that we composed, we compose the approximation schema of KDV equation with higher precision than $10^{-11} \sim 10^{-12}$.

Generalized KDV equation is as following.

$$\frac{\partial u(x,\ t)}{\partial t} + \alpha u^n(x,\ t)\frac{\partial u(x,\ t)}{\partial x} + \beta \frac{\partial^3 u(x,\ t)}{\partial x^3} = 0,\ \alpha,\ \beta : \qquad (2)$$

This is KDV equation when $n = 1$ and has Soliton solution.

$u(x,t) = 3\eta \operatorname{sec}h^2(Ax - Bt + D),\ \ \eta > 0$: Soliton solution.

$A = \frac{1}{2}(\alpha\,\eta/\beta)^{1/2},\ B = \eta\beta A \qquad D$: initial state.

Initial condition

$$u(x,0) = 3\eta \operatorname{sec} h^2 (Ax + D) \qquad (3)$$

Let $\varphi(x)$ is equation (1) , V_0 is the generating space by φ, $\{V_n\}$ is the generating multi-resolution analysis by V_0.

Wavelet-Galerkin schema of (2) is as following when $u_j \in V_j$.

For arbitrary $v \in V_j$,

$$\begin{cases} \int_{\mathbf{R}} \frac{\partial u(x,t)}{\partial t} v dx + \alpha \int_{\mathbf{R}} u_j(x,t) \frac{\partial u(x,t)}{\partial x} v dx + \beta \int_{\mathbf{R}} v \frac{\partial^3 u_j(x,t)}{\partial x^3} dx = 0 \\ u_j(0) = u_{0j} \end{cases} \qquad (4)$$

Lossless generality, let $v = \varphi_{jk} = 2^{j/2} \varphi(2^j x - k), k \in x$ is stated.

Then equation (4) can be written again as equation (5) considering $u_j(x) = \sum_k u_j(\frac{k_1}{2^j}) \varphi(2^j x - k_1)$.

$$\begin{cases} \frac{d}{dt} u_j\left(\frac{k}{2^j}\right) + \alpha \sum_{k_1,k_2} u_j\left(\frac{k_1}{2^j}\right) u_j\left(\frac{k_2}{2^j}\right) A_{k_1k_2}^{jk} + \beta \sum_k u_j\left(\frac{k}{2^j}\right) B_{kk_1}^j = 0, k \in Z \\ u_j(0) = u_{0j} \end{cases} \qquad (5)$$

$$A_{k_1k_2}^{jk} = \int_{\mathbf{R}} \varphi_{jk_1} \cdot \varphi_{jk_2} \varphi_{jk} dx, B_{kk_1}^j = \int_{\mathbf{R}} \varphi_{jk_1}^{(3)} \varphi_{jk} dx.$$

[Theorem 3.1] For arbitrary integer k and $u_j(x) \in V_j$,it exist some sequences $\{A^{jk}_{kk_1}\}_{k_1 \in z}, \{B^{j}_{kk_1}\}_{k_1 \in z}$ that the solutions of equation(4)과 (5) are equal.

Proof (abbreviation)

For the arbitrary j , we have Taylor expand $u_j(t)$ and $u_j(t+\Delta t) \approx u_j(t) + \Delta t[-\alpha c_j u_j(t) - \beta E_j u_j(t)] + 0(\Delta t^2)$, where

$$c_j : u_j \mapsto \sum_{k_1, k_2} u_j^{k_1} u_j^{k_2} A^j_{k_1 k_2}, u_j^{k_1} = u_j\left(\frac{k_1}{2^j}\right)$$

$$E_j : u_j \mapsto \sum_{k_1} u_j^{k_1} B^j_{k_1}$$

At equation (4), the iteration schema for t is same as

$$\begin{cases} u_j^0 = u_0 \\ u_j^{k+1} = u_j^k + \Delta t[-\alpha c_j u_j^k - \beta E_j u_j^k] \end{cases} \qquad (6)$$

Then the convergence theorem is stated.

[Theorem 3.2] For arbitrary natural number $j \in \boldsymbol{N}$, let us define

$$D_j u := \sum_k u\left(\frac{k}{2^j}\right)\varphi^{(j)}(2^j x - k), t > 0 \cdot$$

Then subsequent equation is stated**.**

$$\left\| \frac{\partial^n u}{\partial x^n} - D_j u \right\|_{H^3} = 0(2^{-j(N-2)})$$

$$\left\| u^{\bar{k}} - u_j^{\bar{k}} \right\|_{H^3} = 0(\Delta t 2^{-j(N-2)})$$

Proof (abbreviation).

Using this approximation schema, we get approximation solution of KDV equation and then the error curve of it for the correct solution of KDV equation .And we comprised our approximation solution with the approximation solution by the other method. [40, 41, 42]

As the result, we demonstrated the advantages of our result. Our result is get with arbitrary precision according to the order of our wavelet.(Fig.3,Fig.4)

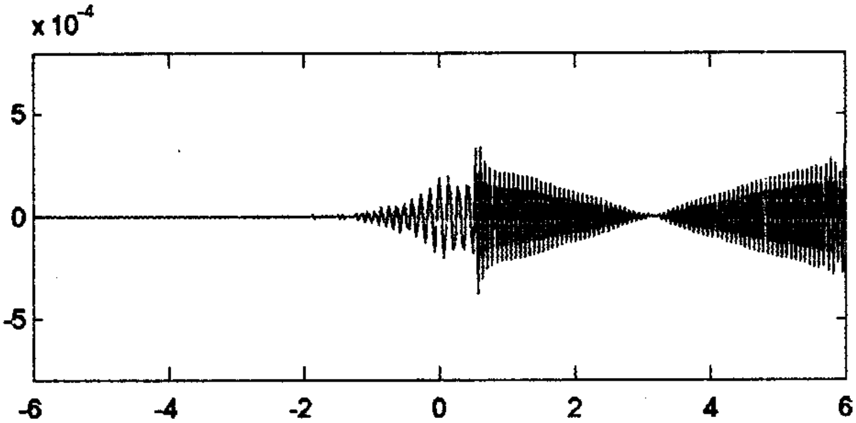

Fig.3. when $n = 2$, the error curve by using Petrov-Galerkin method

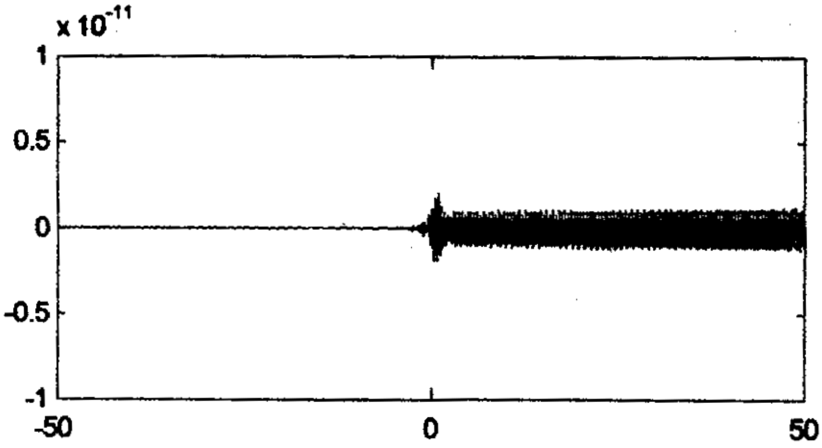

Fig.4. when $n = 2$, the error curve by using our wavelet- Galerkin method

(**Remark**) The precision of our wavelet approximation schema result is higher according to the higher order of our wavelet.

[Theorem 3.3] For any precision, there exist order m with that we use our wavelet as wavelet basis functions and then we get the approximation solution of KDV equation at the precision.

Proof (abbreviation).

4. Conclusion

We proposed the methodology of composing new wavelet functions with regularity, property of interpolating, bandlimited, symmetry, decaying and we get the formula of a family of wavelet functions as the form of analysis.

Our new wavelet functions has much of advantages at nonlinear approximation and using our new wavelet functions we proposed new solver of KDV equation with high precision and for example m=2 we get the approximation solution of KDV equation with the precision of $10^{-11} \sim 10^{-12}$. For any precision, there exist order m with that we use wavelet as wavelet basis functions and then we get the approximation solution of KDV equation at the precision. This solver is very efficient at Schrödinger equation.

We will propose wavelet that has advantages in various engineering controls at the next issue.

At next series we will propose the methodology of the unified composition of the wavelet filter so that the user can select the optimal wavelet depends on the object.

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