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A Method to Determinate the Scaleless Interval of Fractal based on Wavelet Transformation

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Abstract Fractal dimension is being widely used as a powerful tool for signal analysis in digital signal processing fields.

It is important to estimate correctly fractal dimension exhibiting the feature of signal in digital signal processing with fractal. So we have to set up good fractal measure and select correctly the scaleless interval in the fractal curve.

In the paper a method for automatically selecting scaleless interval of fractal curve by wavelet transformation is proposed.

Key words fractal dimension, scaleless interval, wavelet transformation, 3-D spline

Introduction

Fractal theory was firstly proposed by Mandelbrot in 80's of 20^{th} century. It is a non-linear science and has already influenced to the various fields [3, 4].

The fractal of real world is in some layers and it has fractal property in statistical meaning. So scale rule is materialize in certain ranges and these ranges are called scaleless interval [1, 8].

It is very important to decide the scaleless interval correctly. Identifying the scaleless interval clearly is to find a segmentation which has the widest linear range in double log coordinate system.

Correlation coefficient test method, three broken line method, self adaptation standard error methods are suggested for detecting the scaleless interval correctly [10, 12].

These methods do not identify regular bending correctly, and have to take possible combination of segments to raise the reliability of result, but at this time, there are a lot of faults in taking the combination if scale range is too large.

At the same time, to find the position of bending interval in data curve is very important to raise the accuracy of scaleless interval detection using statistical regression. But in previous researches [5－7, 12], it doesn't make clearly, which linear interval of any segmentation is regarded as scaleless interval, if there are many broken line of data point.

But if scaleless interval is selected manually, it can detect the bending of data point curve more quickly and clearly.

On the other hands, wavelet analysis's multi scale resolution ability and focus feature is completely similar to visual feature of person, so if wavelet analysis is used, scaleless interval may be found quickly and correctly, and at the same time self-adaptively [2].

This paper proposes a method to distinguish exactly, the scaleless interval utilizing the wavelet transformation modulus maximum line and wavelet transformation multi scale differential property.

1. Problem Establishment

The most important feature of fractal object is self-similarity and it is expressed with a change of correlation integral curve to straight line in double log coordinate system.

But fractal object in nature has absolute self-similarity structure not on infinite layers, but on finite layers [9].

This means that the correlation integral curve has several statistical regression straight line segmentations in double log coordinate system. Decision of the scaleless interval of scale range which corresponds to the longest segment among these statistical regression straight line segmentations is the main problem in fractal application.

In [9, 11], it is considered that fractal scaleless interval may be automatically identified based on wavelet modulus maximum curve method, but this method not only has large operation burden but also can't be applied in detection of the statistical regression straight line interval.

In this paper, to select the scaleless interval decision quickly and raise the accuracy, smooth the points on double log coordinate system and then it is proved that the longest straight line interval using the high speed wavelet transformation which moment order is 2 in its smoothing(or interpolation) spline can be found.

2. Smoothing Nature 3D Spline

Suppose the function value $y_i = f(x_i)(i = 0, 1, \cdots, N)$ on nodes in the scale interval $\Delta : a = x_0 < x_1 < \cdots < x_N = b$ on $[a, b]$ in the log-log coordinates is given by y_i^0 with some error.

Then, s(x) that minimizes a functional

$$J(f) = \int_a^b | f''(x) |^2 \, dx + \sum_{i=0}^{N} \rho_i^{-1} (y_i - y_i^0)^2$$

on the set $W_2^2[a, b]$ is a 3D spline function in the set, which is called smoothing 3D spline, where $W_2^2[a, b]$ has absolute continuous first derivative and second derivative. Second derivative is a function set that belong to $L_2[a, b]$.

While, when we set the smoothing 3D spline $s(x)$ to $M_i = S''(x_i), M_{i+1} = S''(x_{i+1})$, the expression on each interval $[x_i, x_{i+1}]$ is as follows.

$$s(x) = (1-t)y_i + ty_{i+1} - \frac{h_i^2}{6}[t(2-t)M_i + t(1+t)M_{i+1} - t^2(2-t)M_i - t^2(1+t)M_{i+1}] \qquad (1)$$

where $h_i = x_{i+1} - x_i$, $t = (x - x_i)/h_i$.

Simultaneous equation to decide the M_i of the smoothing 3D spline has the following structure.

$$\begin{cases} a_0 M_0 + b_0 M_1 + c_0 M_2 = g_0 \\ b_0 M_0 + a_1 M_1 + b_1 M_2 + c_1 M_3 = g_1 \\ c_{i-2} M_{i-2} + b_{i-1} M_{i-1} + a_i M_i + b_i M_{i+1} + c_i M_{i+2} = g_i,\ i = 2,\ \cdots,\ N-2 \\ c_{N-3} M_{N-3} + b_{N-2} M_{N-2} + a_{N-1} M_{N-1} + b_{N-1} M_N = g_{N-1} \\ c_{N-2} M_{N-2} + b_{N-1} M_{N-1} + a_N M_N = g_N \end{cases} \tag{2}$$

The coefficients of this equations are decided by following formula.

$$\begin{cases} a_i = \frac{1}{3}(h_{i-1} + h_i) + \frac{1}{h_{i-1}^2}\rho_{i-1} + \left(\frac{1}{h_{i-1}} + \frac{1}{h_i}\right)^2 \rho_i + \frac{1}{h_i^2}\rho_{i+1},\ i = 1,\ 2,\ \cdots,\ N-1 \\ b_i = \frac{1}{6}h_i - \frac{1}{h_i}\left[\left(\frac{1}{h_{i-1}} + \frac{1}{h_i}\right)\rho_i + \left(\frac{1}{h_i} + \frac{1}{h_{i+1}}\right)^2 \rho_{i+1}\right],\ i = 1,\ 2,\ \cdots,\ N-2 \\ c_i = \frac{1}{h_i h_{i+1}}\rho_{i+1},\ i = 1,\ 2,\ \cdots,\ N-3 \\ g_i = \frac{y_{i+1}^0 - y_i^0}{h_i} - \frac{y_i^0 - y_{i-1}^0}{h_{i-1}},\ i = 1,\ 2,\ \cdots,\ N-1 \end{cases} \tag{3}$$

If the smoothing 3D spline satisfies $S''(a) = S''(b) = 0$, for the smoothing nature 3D spline following expression is hold

$$a_0 = a_N = 1,\quad b_0 = c_0 = c_{N-2} = b_{N-1} = g_0 = g_N = 0. \tag{4}$$

From equation (2), M_i is calculated, and then y_i can be given by following equation.

$$y_i = y_i^0 - \rho_i D_i \tag{5}$$

where

$$D_0 = \frac{1}{h_0}(M_1 - M_0) \tag{6}$$

$$D_i = \frac{1}{h_i}(M_{i+1} - M_i) - \frac{1}{h_{i-1}}(M_i - M_{i-1}),\quad i = 1,\ \cdots,\ N-1 \tag{7}$$

$$D_N = -\frac{1}{h_{N-1}}(M_N - M_{N-1}) \tag{8}$$

In constituting the smoothing spline, it is important how to select the weight ρ_i. If all $\rho_i = 0$, then $y_i = y_i^0,\ i = 0,\ \cdots,\ N$, and the smoothing spline becomes to the interpolation spline. From this, to get y_i^0 more exactly on the net node, the corresponding weight factor ρ_i should be selected smaller.

3. Scaleless Interval Determination Algorithm

Let the smoothing 3D spline $s(x)$ is made in the interval $[a,\ b]$ and divide interval $[a,\ b]$ by $step = (b-a)/N$. Then represent the values of every node by $s_i,\ i = 0,\ 1,\ \cdots,\ N$.

$$h_k = \int_R \frac{1}{2}\phi\left(\frac{x}{2}\right)\overline{\phi(x-k)}dx$$

scale function ϕ that generates Db' wavelet of moment order 2 is as follows.

$$h_{-1} = \frac{1+\sqrt{3}}{4\sqrt{2}},\ h_0 = \frac{3+\sqrt{3}}{4\sqrt{2}},\ h_1 = \frac{3-\sqrt{3}}{4\sqrt{2}},\ h_2 = \frac{1-\sqrt{3}}{4\sqrt{2}},\ \text{other}\ \ h_k = 0$$

Corresponding high pass filter coefficients of Daubeches wavelet ψ is as follows.

$$g_{-1} = \frac{1-\sqrt{3}}{4\sqrt{2}},\ g_0 = -\frac{3-\sqrt{3}}{4\sqrt{2}},\ g_1 = \frac{3+\sqrt{3}}{4\sqrt{2}},\ g_2 = -\frac{1+\sqrt{3}}{4\sqrt{2}}$$

On the other hand, discrete input signal $a_0[n]$ in $a_0 = \{a_0[n]\}_{n\in\mathbf{Z}}$ can be regarded as weighted average of a function $f \in L_2(R)$, i.e.

$$a_0[n] = < f(\cdot),\ \phi(\cdot - n) > = \int_R f(x)\phi(x-n)dx,\ n \in \mathbf{Z}\,.$$

If

$$\phi_2(x) = 2^{-1}\phi(2^{-1}x),\ \ \bar{\phi}_2(x) = \phi_2(-x),\ \ a_1[n] = < f(\cdot),\ \varphi_2(\cdot - n) >,\ d[n] = < f(\cdot),\ \ \psi_2(\cdot - n) >$$

represent filter h with $\bar{h}[n] = h[-n]$, the following relation is given.

Theorem: Let a filter g is high pass filter of Db's wavelet of moment order 2.

Then if four components of $a_0(n-1),\ a_0(n),\ a_0(n+1),\ \ a_0(n+2)$ that are consecutive with signal a_0 are equidistant arranged points on a straight line, following expression is concluded.

$$d[n] = a_0 * \bar{g}[n] = 0$$

Proof: From expression (9) and (10)

$$d[n] = a_0 * \bar{g}[n] = a_0(n-1)g_{-1} + a_0(n)g_0 + a_0(n+1)g_1 + a_0(n+2)g_2\,.$$

While, let four points $a_0(n-1),\ a_0(n),\ a_0(n+1),\ a_0(n+2)$ are on a straight line $y = kx + c$ and gap is p.

And if $a_0(n-1) = kx_1 + c$,

$$a_0(n) = k(x_1 + p) + c,\ a_0(n+1) = k(x_1 + 2p) + c,\ a_0(n+2) = k(x_1 + 3p) + c\,.$$

Therefore,

$$\begin{aligned} d[n] &= (kx_1 + c)g_{-1} + (kx_1 + kp + c)g_0 + (kx_1 + 2kp + c)g_1 + (kx_1 + 3kp + c)g_2 \\ &= (kx_1 + c)(g_{-1} + g_0 + g_1 + g_2) + kp(g_0 + 2g_1 + 3g_2). \end{aligned}$$

where

$$g_{-1} + g_0 + g_1 + g_2 = 0\,,$$

$$g_0 + 2g_1 + 3g_2 = -\frac{3-\sqrt{3}}{4\sqrt{2}} + \frac{6+2\sqrt{3}}{4\sqrt{2}} - \frac{3+3\sqrt{3}}{4\sqrt{2}} = 0\,.$$

So $d[n] = 0$. (proof end)

As discrete points to find the scaleless fractal interval in the log-log coordinate system were given, following algorithm for finding the longest scale interval of straight line interval in the log-log coordinate system may be made.

① With smoothing(or interpolation) natural 3D spline, get $N+1$ points, $(X_i,\ Y_i)$, $i = 0,\ \cdots,\ N$ that are arrange with equidistance $H = (b-a)/N$ of scale interval.

② Calculate $d[n] = Y_{n-1}g_{-1} + Y_n g_0 + Y_{n+1}g_1 + Y_{n+2}g_2$ $\ n = 1$ to $N-2$ and represent a sequence set that consists of index n of $|d[n]| < tol$ as A. Where tol is tolerance error.

Next in A if distance of two consecutive elements is 1, replace the second element with first one, and the number of consecutive elements with first one is saved in two sequence sets, P and Q.

③ For every $P(i)$,calculate $et(i) = j-1$ from $st(i) = j$ of smallest j that $X_{P(i)} \le x(j)$ and from smallest j that $X_{P(i)+Q(i)+3} < x(j)$.

④ For every $P(i)$, calculate

$$dist = \sqrt{(x(et(i)) - x(st(i)))\hat{}2 + (y(et(i)) - y(st(i)))\hat{}2}$$

and find i of largest $dist$,and then regard it to maxindex.

New scaleless interval is given by $[x(st(\max index)), x(et(\max index))]$.

⑤ We determine the regression straight line by using (x_j, y_j) for $j = st(\max index):et(\max index)$ and its direction coefficient is regarded as fractal dimension to find.

4. Test Result and Analysis

In the test, we used standard database image such as cloud and rock image.

Fig. 1 shows point set with equidistance throughout autocorrelation curve of the images and equidistance relation curve of nature 3D interpolation spline.

When tolerance error is $tol = 0.02$, scaleless interval found is 2.392 0, 2.819 6 and its fractal dimension is 2.806 4.(Fig. 2)

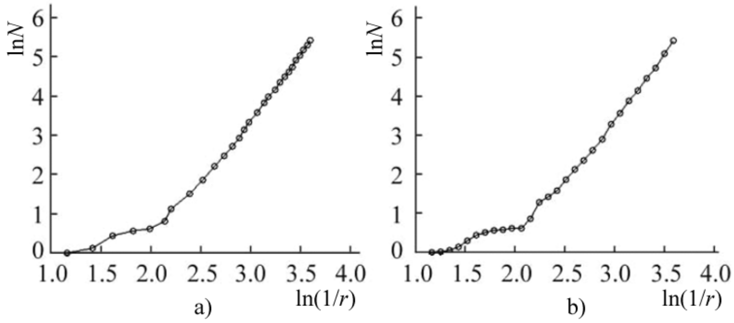

Fig. 1. Autocorrelation curve of the test image and equidistance correlation curve

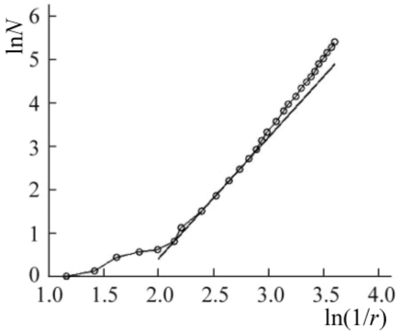

Fig. 2. Linear interval by our method

Table. Comparison of proposed method and preceding method.

	Standard FD	Paper[12]	Paper[13]	Proposed
Image 1	2.812	2.663	2.716 3	2.806 4
Image 2	2.635	2.571	2.614	2.629

Table shows that our method improved accuracy than methods calculating FD based on merely linear regression method [11] and scale transformation [12].

And method in [9] is only available for 1D fractal curve and not proper in calculation of image fractal dimension. Moreover, if several linear intervals exist, this method can’t give the accurate linear interval.

Conclusion

We proposed a method that can raise the accuracy of FD calculation to detect exactly scaleless interval based on wavelet transformation.

It is shown that the proposed method can improve the calculation accuracy, because it can detect the linear interval in fractal autocorrelation curve more correctly in calculating FD of natural image.

References

[1]. N. Sarker et al.; IEEE Transactions on Systems, 24, 115, 1994.

[2] S. Mallat; A Wavelet Tour of Signal Processing, Academic Press, 59, 2005.

[3] W. S. Chen; in Proc. IEEE Int. Conf. Acoust. Speech Signal Process, 3, 5, 1541, 2001.

[4] K. Falconer; Fractal Geometry-Mathematical Foundation and Application, John Wiley & Sons Inc, 337, 2003.

[5] W. L. Lee et al.; IEEE Trans. Med. Imag., 22, 3, 382, 2003.

[6] A. J. Lasota; Nonlinear Analysis, 64, 618, 2006.

[7] S. Novianto; Pattern Recognition Letters, 24, 365, 2003.

[8] S. Xu et al.; Pattern Recognition Letters, 27, 1942, 2006.

[9] Jian Li et al.; Pattern Recognition, 42, 2460, 2009.

[10] Mihai Ivanovici et al.; IEEE Transactions on image processing, 20, 1, 227, 2011.

[11] Jian Li et al.; ICIP-2006, 3029, 2006.

[12] 巫兆聪；测绘学报, 31, 3, 240, 2002.