Journal of **Kim Il Sung** University Vol. 3 No. 1
(Natural Science) Juche103(2014)

Algorithm of Configuration Control of Redundant Manipulators under the Constructional Joint Limit

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Abstract We consider the configuration control of kinetically redundant robot manipulators. We proposed the algorithm for joint limit avoidance(JLA) using configuration control approach which makes redundant manipulator possible to carry out not only the main task but also the joint limit avoidance.

Key words kinetically redundant manipulators, configuration control, joint limit avoidance

Introduction

The great leader Comrade **Kim Il Sung** said as follows.

"Today scientists and technicians are faced with a very important task. They must forge ahead more energetically with scientific research work in order to make a great contribution to raising the scientific and technological level of the country to a higher stage and developing the national economy at a rapid pace."("**KIM IL SUNG** WORKS" Vol. 37 P. 359)

Particular attention has been devoted to the study of the redundant robot manipulators, because these are utilized in performing tasks that require dexterity comparable to that of the human arm.

There are some drawbacks associated with the pseudo-inverse method [2]. By this method extra degrees of the redundant manipulators are not applied to the user-defined additional tasks.

The other alternative is presented in the extended Jacobian method [3] where the dimension of the additional task space is equal to the degree of the redundancy. In this method the dimension of the additional task should be equal to the degree of redundancy and it is not applicable for the additional tasks such as obstacle avoidance that are not active for all time.

We proposed the algorithm for JLA using configuration control approach [1] which makes redundant manipulator possible to carry out not only the main task but also the JLA.

1. Configuration Control Method

A manipulator is said to be redundant when it has a number of degrees of mobility which is greater than the number of variables that are necessary to describe a given task.

Let us denote the dimension of the joint space n, and the dimension of the task space m. Then a manipulator is kinetically redundant when $n > m$, and $r = n - m$ is the degree of redundancy.

The forward kinetic function is defined as

$$\boldsymbol{x} = \boldsymbol{f}(\boldsymbol{q}) \tag{1}$$

where $\boldsymbol{q}$ is the $(n \times 1)$ vector of the joint space, and $\boldsymbol{x}$ is the $(m \times 1)$ vector of the task space.

The differential kinetics equation of manipulator is given by

$$\dot{\boldsymbol{x}} = \boldsymbol{J}_e \dot{\boldsymbol{q}} \tag{2}$$

where $\boldsymbol{J}_e$ is the $(m \times n)$ Jacobian, and it can be viewed as a linear mapping from $\boldsymbol{R}^n$ into $\boldsymbol{R}^m$; the vector $\dot{\boldsymbol{q}} \in \boldsymbol{R}^n$ is mapped into $\dot{\boldsymbol{X}} \in \boldsymbol{R}^m$.

Two fundamental subspaces associated with a linear transformation are null space and range space(Fig. 1).

The null space denoted $\boldsymbol{N}(\boldsymbol{J_e})$ is the subspace of $\boldsymbol{R}^n$ defined by

$$\boldsymbol{N}(\boldsymbol{J_e}) = \{\dot{\boldsymbol{q}} \in \boldsymbol{R}^n \mid \boldsymbol{J}_e \dot{\boldsymbol{q}} = 0\} \tag{3}$$

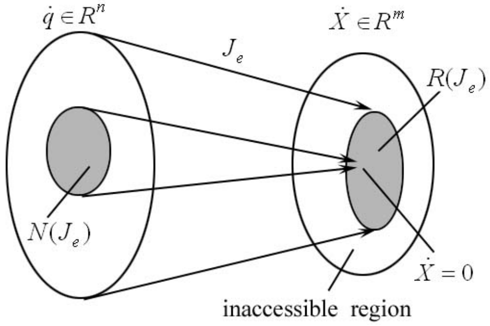

Fig. 1. Geometric representation of null space and range of J_e

The range denoted $\boldsymbol{R}(\boldsymbol{J_e})$ is a subspace of $\boldsymbol{R}^m$ defined by

$$\boldsymbol{R}(\boldsymbol{J_e}) = \{\boldsymbol{J}_e \dot{\boldsymbol{q}} \mid \dot{\boldsymbol{q}} \in \boldsymbol{R}^n\} \tag{4}$$

Equation (3) underlies the mathematical basis for redundant manipulator.

For redundant manipulator, the dimension of $\boldsymbol{N}(\boldsymbol{J_e})$ is equal to $(n - m')$, where m' is the rank of the matrix $\boldsymbol{J}_e$.

If $\boldsymbol{J}_e$ has full rank, then the dimension of $\boldsymbol{N}(\boldsymbol{J_e})$ is equal to the degree of redundancy. The joint velocities belonging to $\boldsymbol{N}(\boldsymbol{J_e})$, referred to as internal joint motion and denoted by $\dot{\boldsymbol{q}}_N$, do not produce any end-effector velocity in the given manipulator posture. This shows the major advantage of redundant manipulators. In a word, the dimension of $\boldsymbol{N}(\boldsymbol{J_e})$ is equal to the degree of redundancy and the joint velocities belonging to $\boldsymbol{N}(\boldsymbol{J_e})$ can be specified without affecting the task space velocities. This gives the possibility to advantageously exploit the redundant degrees for a redundant manipulator. Therefore it is possible to achieve not only the main task for the end-effector position and orientation but also the additional tasks.

For a given $\dot{\boldsymbol{x}}$, a solution $\dot{\boldsymbol{q}}$ is selected which exactly satisfies equation (2).

Most of the methods are based on the pseudo-inverse method [2].

That is

$$\dot{\boldsymbol{q}} = \boldsymbol{J}_e^+ \boldsymbol{x} \tag{5}$$

where $\boldsymbol{J}_e^+$ denotes the pseudo-inverse of the matrix $\boldsymbol{J}_e$.

If $\boldsymbol{J}_e$ has full rank, its pseudo-inverse is given by

$$\boldsymbol{J}_e^+ = \boldsymbol{J}_e^T (\boldsymbol{J}_e \boldsymbol{J}_e^T)^{-1} \tag{6}$$

The ability of the pseudo-inverse to provide the solution in the least-squares whether equation (2) is underspecified, square, or over-specified makes it the most attractive technique in redundancy resolution. However there are some drawbacks associated with this solution.

The solution given by equation (5) does not guarantee generation of joint motions which avoid singular configurations in which $\boldsymbol{J}_e$ is no longer full ranks.

In the neighborhood of a singular, $\boldsymbol{J}_e$ has rank deficient, and there exist small velocities in the task space corresponding to the large velocities in the joint space.

Another problem with the pseudo-inverse approach is that the joint motions generated by this approach do not preserve the repeatability and cyclicity condition, i.e., a closed path in Cartesian space may not result in a closed path in joint space.

The final difficulty is that the extra degrees ($\dim(\boldsymbol{q}) > \dim(\boldsymbol{x})$) are not utilized to satisfied user-defined additional tasks.

To overcome above problems the extended Jacobian method is presented [3].

The Jacobian of the augmented task is defined by

$$\boldsymbol{J}_A = \begin{bmatrix} \boldsymbol{J}_e \\ \boldsymbol{J}_c \end{bmatrix} \tag{7}$$

where $\boldsymbol{J}_A$ is the extended Jacobian matrix, $\boldsymbol{J}_e$ and J_c being the $(m \times n)$ and $(r \times n)$ Jacobian matrices of the main and additional tasks respectively.

The velocity kinematics of the extended task are given by

$$\dot{\boldsymbol{Y}} = \begin{bmatrix} \dot{\boldsymbol{X}} \\ \dot{\boldsymbol{Z}} \end{bmatrix} = \boldsymbol{J}_A \dot{\boldsymbol{q}} \tag{8}$$

where $\dot{\boldsymbol{X}}$, $\dot{\boldsymbol{Y}}$ and $\dot{\boldsymbol{Z}}$ are the time derivations of the task velocities of the main, extended and additional tasks, $\boldsymbol{X}$, $\boldsymbol{Y}$ and $\boldsymbol{Z}$, respectively.

Equation (8) is no longer redundant, and redundancy resolution is archived by solving that equation.

However, there following drawbacks associated with this method. The dimension of the additional task should be equal to the degree of redundancy which makes the approach not applicable for additional tasks such as obstacle avoidance that are not active for all time. The extended Jacobian $\boldsymbol{J}_A$ becomes rank deficient if either of matrices $\boldsymbol{J}_e$ or $\boldsymbol{J}_c$ is singular, or there is a conflict between the main and additional tasks(which translates into linear dependence of the rows of $\boldsymbol{J}_e$ and $\boldsymbol{J}_c$). Therefore the solution of Equation (8) based on the inverse of the extended Jacobian $\boldsymbol{J}_A$ may result in instability near a singular configuration.

They proposed a singular robust and the task prioritized formulation in which the drawbacks associated with the pseudo-inverse and the extended Jacobian methods are overcome [1].

Under configuration control, the $(m \times 1)$ main task vector $\boldsymbol{x}$ is augmented $[(m+k) \times 1]$ by the $(k \times 1)$ additional task vector $\boldsymbol{z}$.

The augmented task vector is defined by $\boldsymbol{y}^T = [\boldsymbol{x}^T, \ \boldsymbol{z}^T]^T$.

The augmented differential kinematics are given by

$$\dot{\boldsymbol{y}} = \begin{bmatrix} \dot{\boldsymbol{x}} \\ \dot{\boldsymbol{z}} \end{bmatrix} = \boldsymbol{J}_A \dot{\boldsymbol{q}} \tag{9}$$

where $\boldsymbol{J}_A = \begin{bmatrix} \boldsymbol{J}_e \\ \boldsymbol{J}_c \end{bmatrix}$ is the augmented Jacobian matrix, $\boldsymbol{J}_e$ and $\boldsymbol{J}_c$ being the $(m \times n)$ and $(k \times n)$ Jacobian matrices of the main and additional respectively.

They proposed a singularity robust and task prioritized formulation using the weighted damped last-squares method at the velocity level.

The solution is given by;

$$\dot{\boldsymbol{q}} = [\boldsymbol{J}_e^T \boldsymbol{W}_e \boldsymbol{J}_e + \boldsymbol{J}_c^T \boldsymbol{W}_c \boldsymbol{J}_c + \boldsymbol{W}_\mathrm{v}]^{-1} [\boldsymbol{J}_e^T \boldsymbol{W}_e \dot{\boldsymbol{x}}^d + \boldsymbol{J}_c^T \boldsymbol{W}_c \dot{\boldsymbol{z}}^d] \tag{10}$$

which minimizes the following cost function

$$\gamma = \dot{\boldsymbol{E}}_e^T \boldsymbol{W}_e \dot{\boldsymbol{E}}_e + \dot{\boldsymbol{E}}_c^T \boldsymbol{W}_c \dot{\boldsymbol{E}}_c + \dot{\boldsymbol{q}}^T \boldsymbol{W}_\mathrm{v} \dot{\boldsymbol{q}} \tag{11}$$

where $\boldsymbol{W}_e(m \times m)$, $\boldsymbol{W}_c(k \times k)$, $\boldsymbol{W}_\mathrm{v}(n \times n)$ are diagonal positive-definite weighting matrices that assign priority between the main, additional, and singularity robustness tasks. $\dot{\boldsymbol{E}}_e = \dot{\boldsymbol{x}} - \dot{\boldsymbol{x}}^d$ and $\dot{\boldsymbol{E}}_c = \dot{\boldsymbol{z}} - \dot{\boldsymbol{z}}^d$ are the $n-$ and $k-$dimensional vectors representing the residual velocity errors of the main and additional tasks respectively. The subscript d denotes desired trajectories for the tasks. The joint velocity (10) gives a special solution that minimizes the joint velocities when $k < r$, i.e., there are not as many active tasks as the degree-of-redundancy, and the best solution in the least-squares sense when $k > r$. In all cases the presence of $\boldsymbol{W}_\mathrm{v}$ ensures the boundness of joint velocities.

2. Description of Algorithm for the Additional Task-JLA

We consider the additional task for JLA.

The joint limits are presented as a set of inequality constraints. If all the computed values of the joint variables satisfy the inequalities, the redundancy can be used for other tasks. However if one or more of these inequalities are violated, the JLA secondary task should be activated. This task is defined as follows;

$$\left.\begin{aligned} z_i &= q_i \\ z_i^d &= q_{m_i} \end{aligned}\right\} \tag{12}$$

where q_{m_i} replaces either the maximum or the minimum values of the joints for $i = 1, 2, \cdots, n$, and the corresponding constraint Jacobian is defined by the equation:

$$J_{c_i} = \frac{\partial z_i}{\partial \boldsymbol{q}} = \boldsymbol{e}_i^T \tag{13}$$

where $\boldsymbol{e}_i$ is the i^{th} column of the identity matrix.

For smooth incorporation of the main and additional tasks, it is desirable to define a "buffer" region. When the inequality constraint is inactive, the corresponding weight W_{c_i} is zero, and on entering the "buffer" region increases gradually to its maximum value. The algorithm to select this weight can be given as follows:

$$\begin{aligned} &W_{c_i} = 0, \ \ q_i \le q_{i\max} - \tau \\ &W_{c_i} = W_0 (q_i - q_{i\max} + \tau)/(2\tau), \ \ q_{i\max} - \tau \le q_i \le q_{i\max} \\ &W_{c_i} = W_0 / 2, \ \ q_i > q_{i\max} \end{aligned} \tag{14}$$

where W_0 and τ are constants representing the coefficient for the weight and width of the buffer region respectively.

3. Simulations

The simulations were carried out on a three-link planar manipulator. Link lengths were $l_1 = l_2 = l_3 = 1\text{m}$, and the limits of the joints were

$$60^\circ \le q_1 \le 145^\circ,\ -60^\circ \le q_2 \le -40^\circ,\ -75^\circ \le q_3 \le -60^\circ.$$

The desired trajectory is shown in Fig. 2.

Fig. 3 shows the joint variable when the JLA provision was not activated. In this case, the third joint violates its minimum limit.

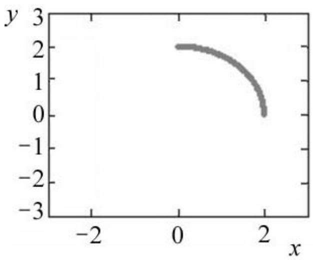

Fig. 2. The desired trajectory

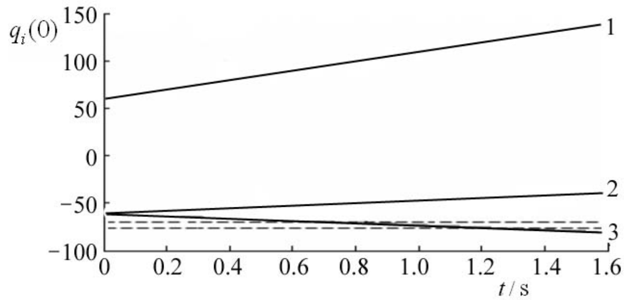

Fig. 3. The joint variable when the JLA provision was not activated

1— q_1, 2— q_2 and 3— q_∞

In the second simulation, the JLA provision based on the

$$W_0 = 100,\ W_\text{v} = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix},\ W_e = \begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix},$$

and the buffer region $\tau = 5^\circ$.

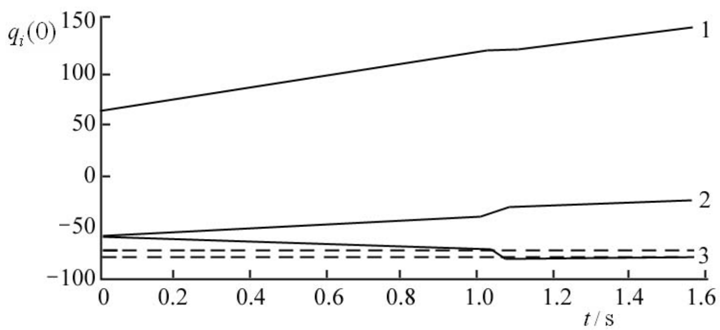

Fig. 4. The joint variable when the JLA provision was activated

1, 2 and 3 are equal to Fig. 3

Fig. 4 shows that in this case, the third joint variable does not violate its limit.

Conclusion

In this paper, we proposed the algorithm for JLA which is one of the most useful additional tasks and verified advantage throughout simulations using configuration control approach.

The configuration control is the most suitable approach in which the drawbacks associated with the pseudo-inverse and the extended Jacobian methods are overcome.

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