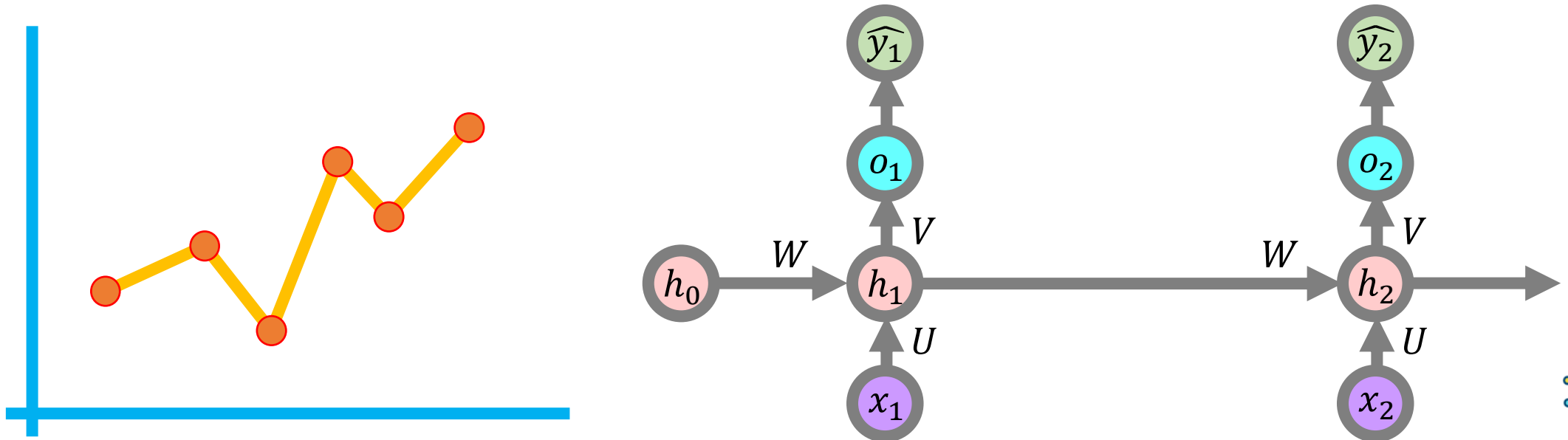


Deep Learning 101

RNN (순환신경망)

Recurrent Neural Network



안녕하세요 신박AI입니다



오늘은 RNN에 대해 같이 알아보는 시간을
가져보도록 하겠습니다

RNN은 Recurrent Neural Network의
약자로

시계열 데이터와 같은 연속적인 정보를 처리
할 때 많이 사용되는 신경망입니다

RNN은 기본적으로 과거의 정보를 ‘기억’
하면서 새로운 정보를 처리하는 것입니다

예를들면,

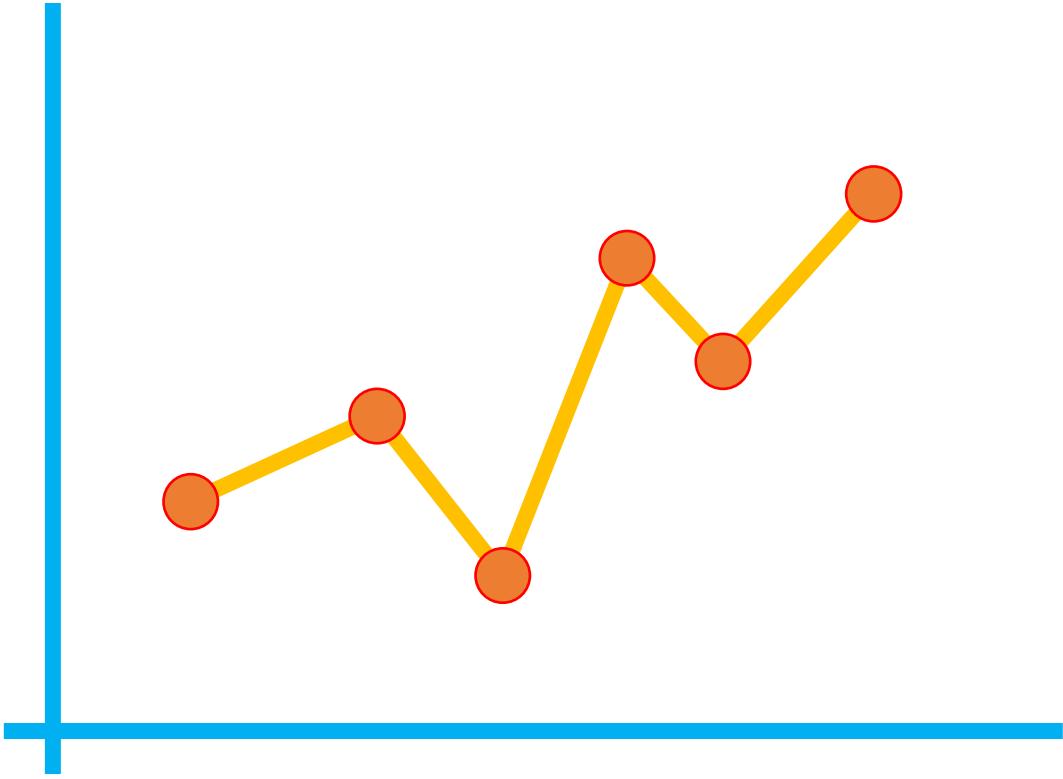
예를들면,



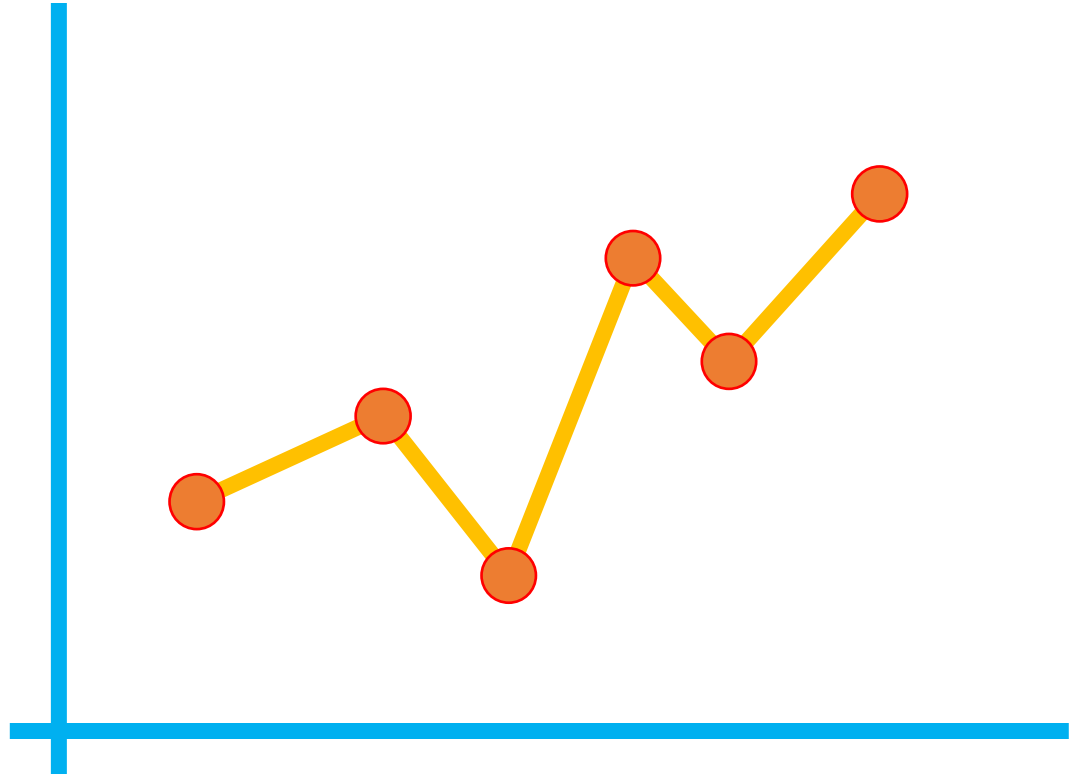
예를들면,



예를들면, 주식가격의 변동이나

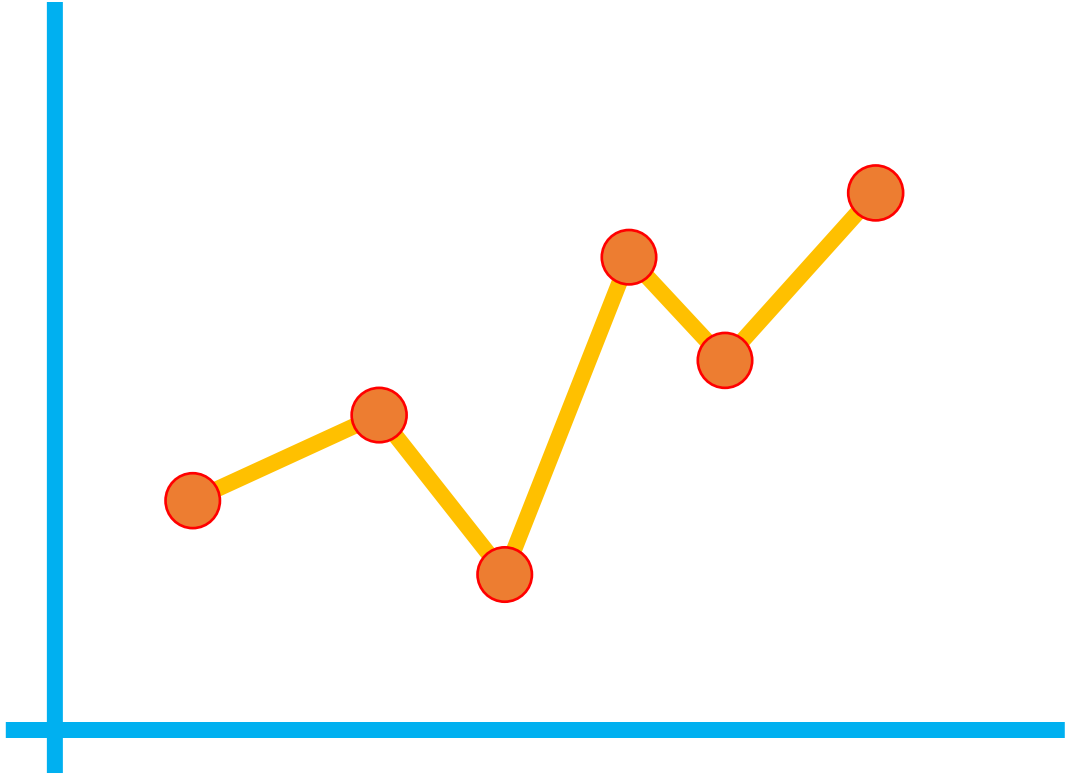


예를들면, 주식가격의 변동이나, 문장 내의 단어들처럼,



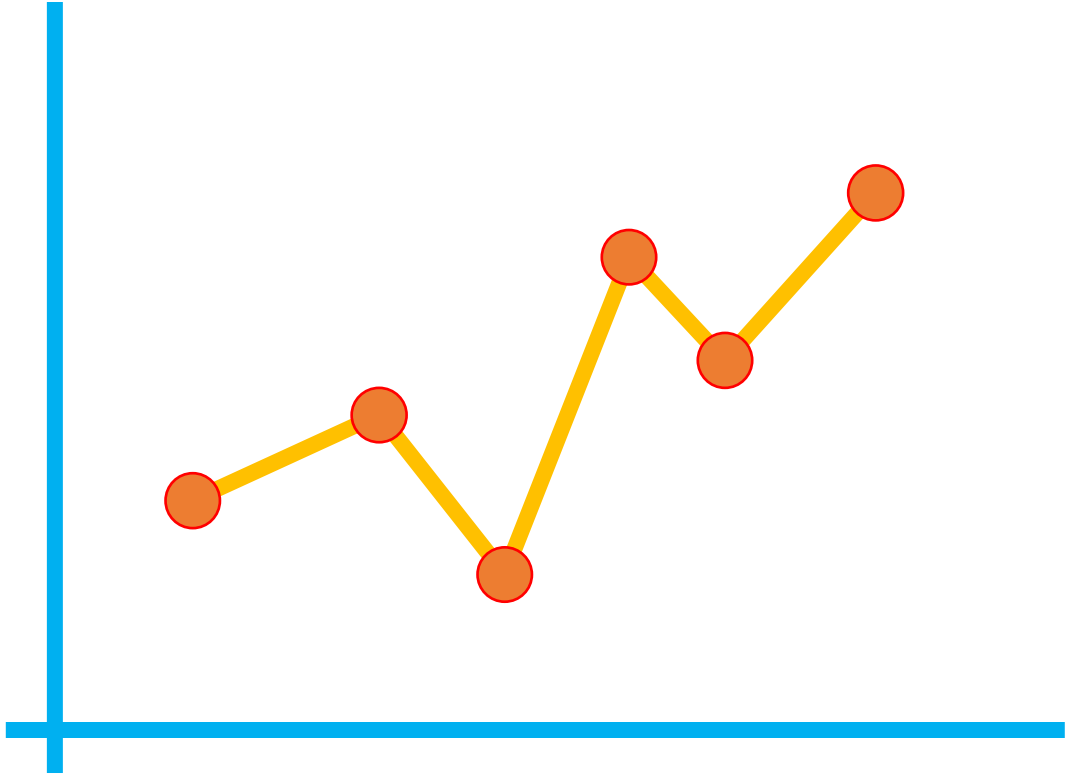
죽는 날까지

예를들면, 주식가격의 변동이나, 문장 내의 단어들처럼,



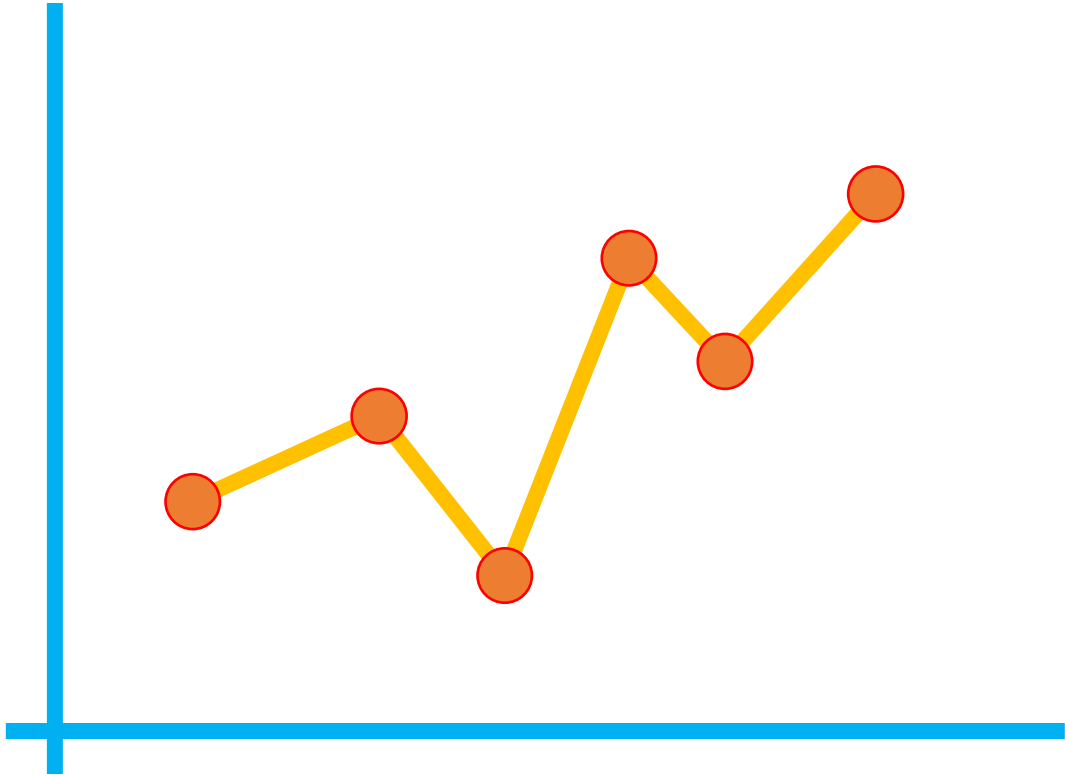
죽는 날까지 하늘을

예를들면, 주식가격의 변동이나, 문장 내의 단어들처럼,



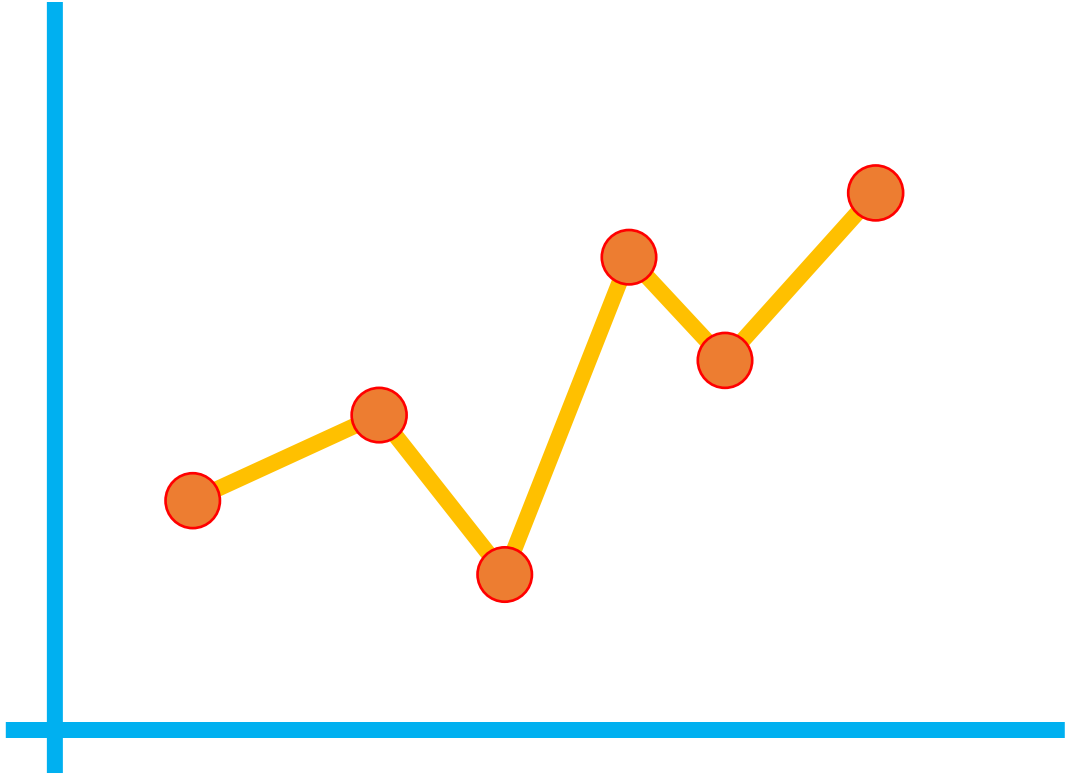
죽는 날까지 하늘을 우러러

예를들면, 주식가격의 변동이나, 문장 내의 단어들처럼,



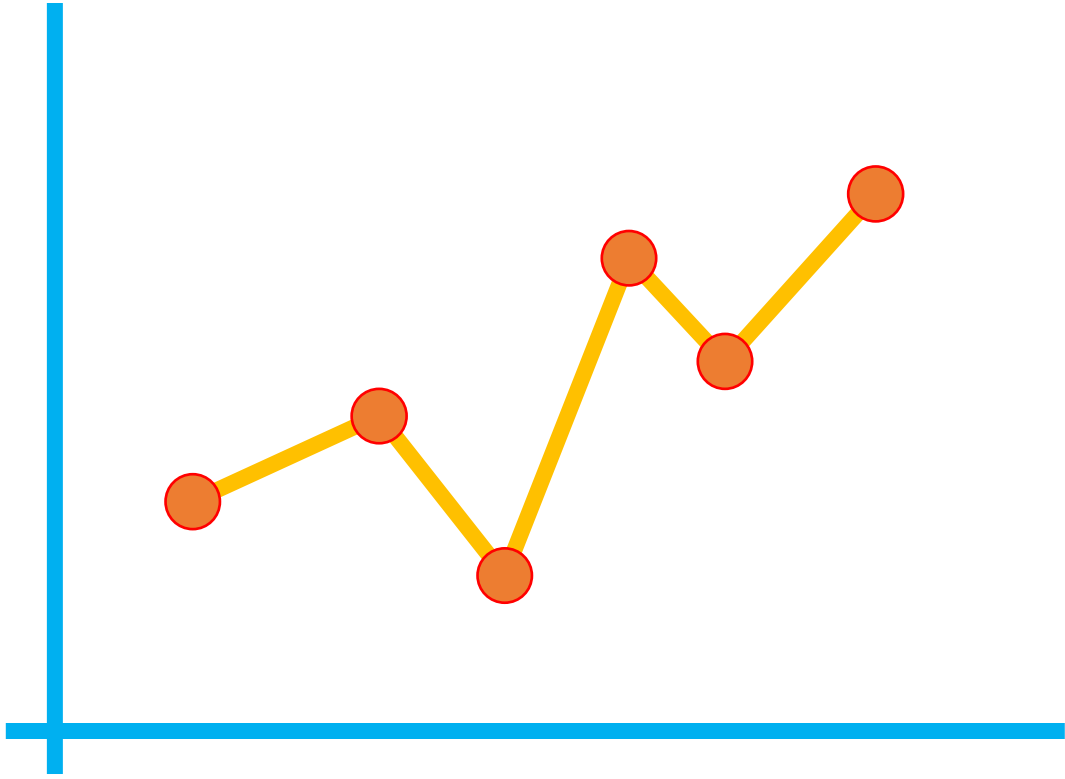
죽는 날까지 하늘을 우러러
한

예를들면, 주식가격의 변동이나, 문장 내의 단어들처럼,



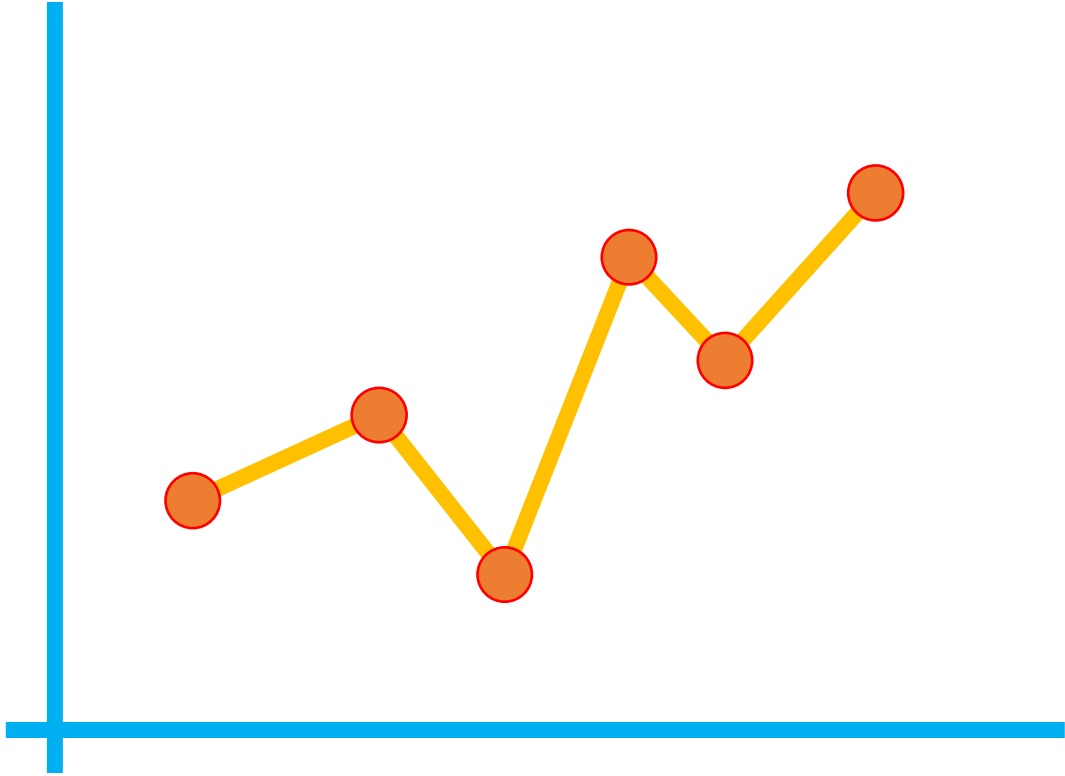
죽는 날까지 하늘을 우러러
한 점

예를들면, 주식가격의 변동이나, 문장 내의 단어들처럼,



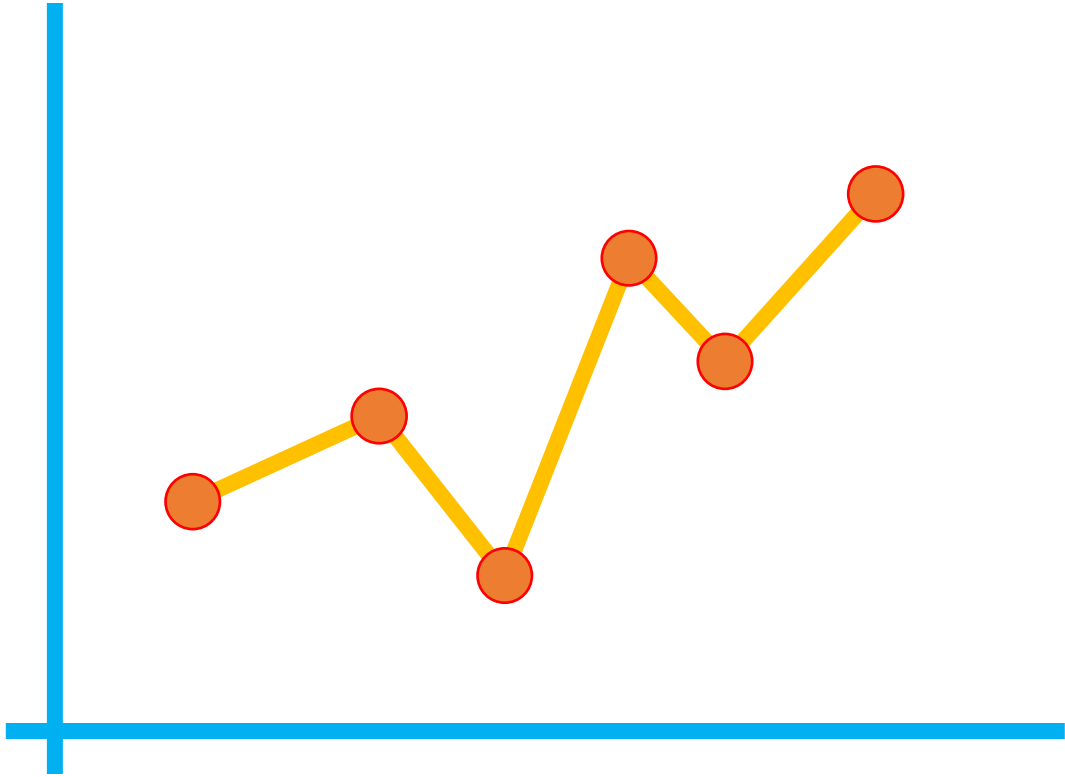
죽는 날까지 하늘을 우러러
한 점 부끄럼이

예를들면, 주식가격의 변동이나, 문장 내의 단어들처럼,



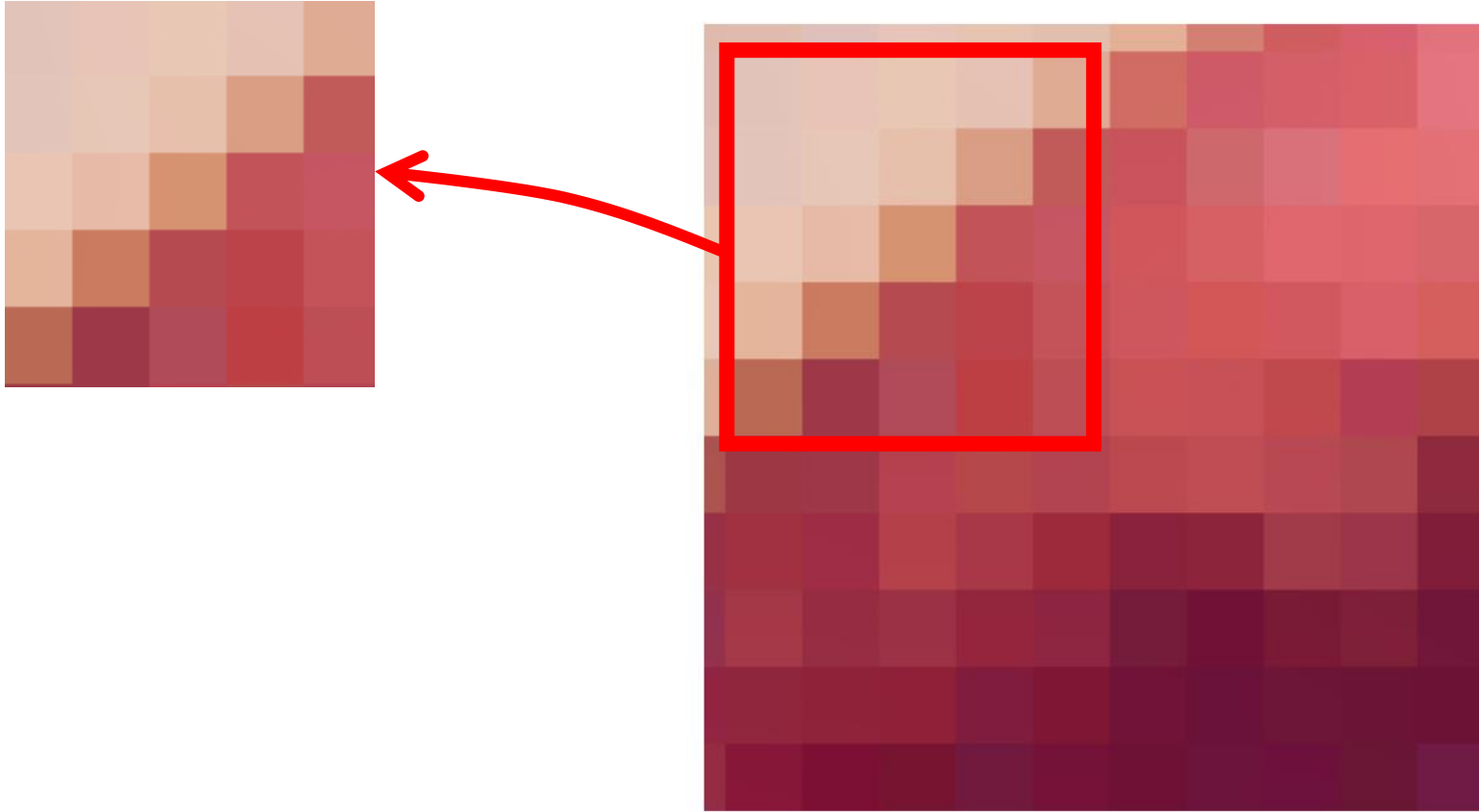
죽는 날까지 하늘을 우러러
한 점 부끄럼이 없기를

예를들면, 주식가격의 변동이나, 문장 내의 단어들처럼, 순서가 중요한 데이터들을 효과적으로 처리할 수 있습니다

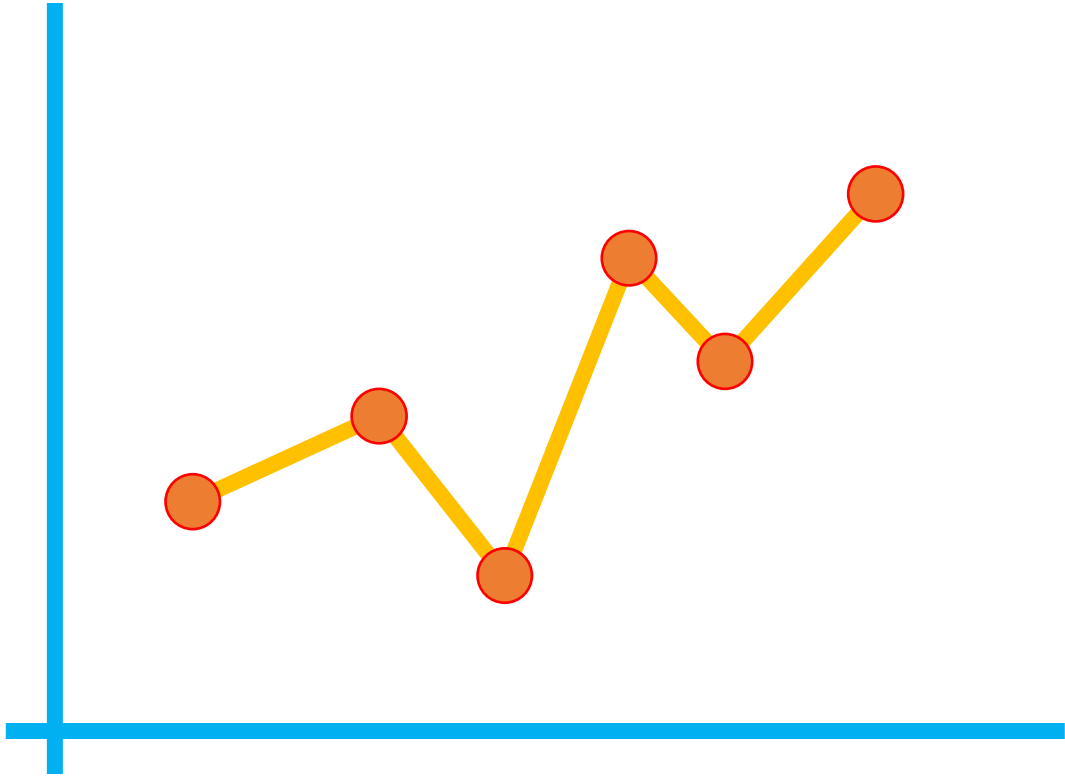


죽는 날까지 하늘을 우러러
한 점 부끄럼이 없기를

CNN이 이미지 데이터의 공간적 특징을 추출하여 학습한다면,

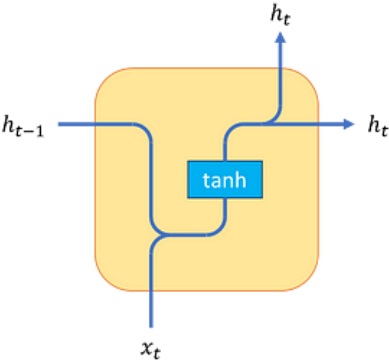


RNN은 시계열 데이터의 시간적 특징을 추출하여 학습한다고 보시면 좋을 것 같습니다

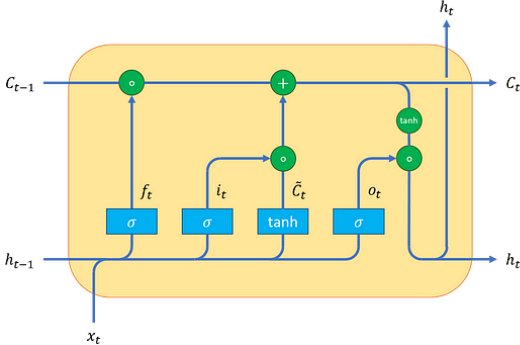


죽는 날까지 하늘을 우러러
한 점 부끄럼이 없기를

뿐만 아니라 RNN은 LSTM, GRU등을 거쳐



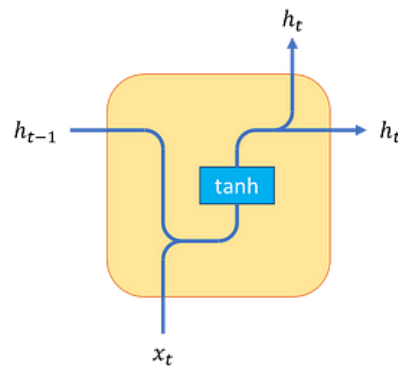
RNN



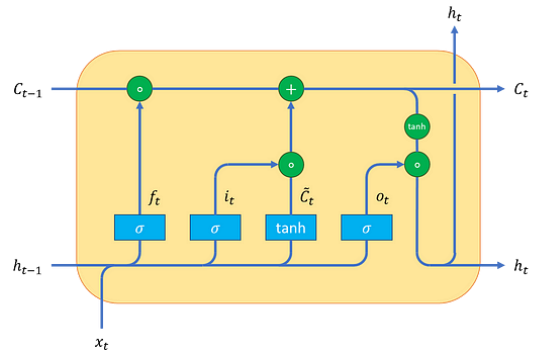
LSTM



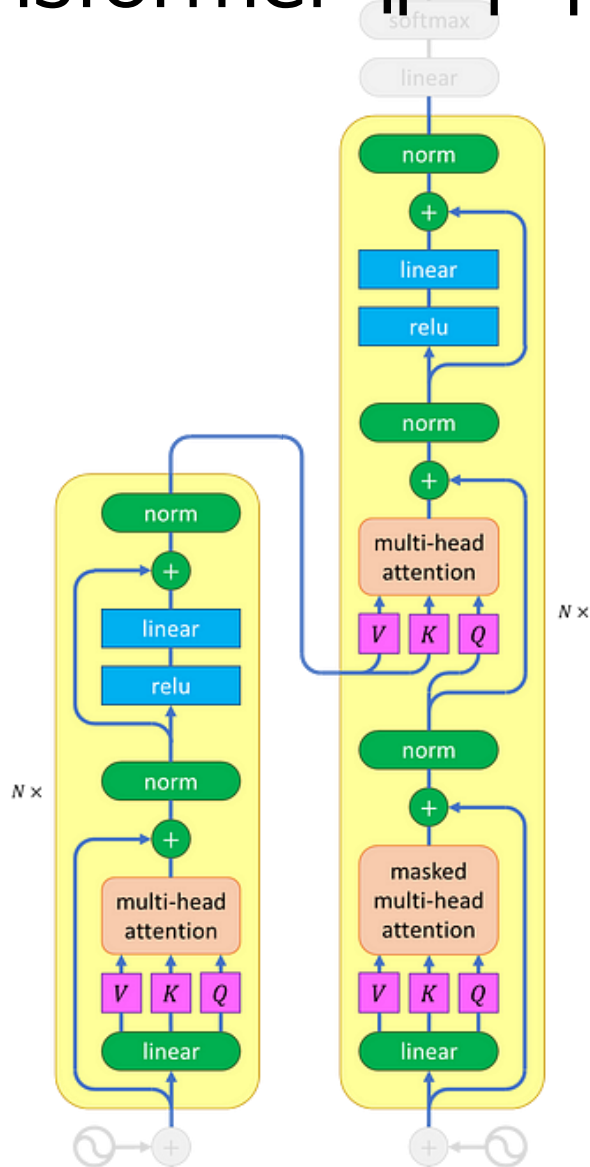
오늘날 ChatGPT의 할아버지격이라 할 수 있는 Transformer에 까지 발전하게 됩니다



RNN



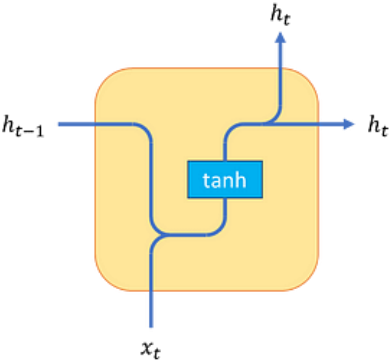
LSTM



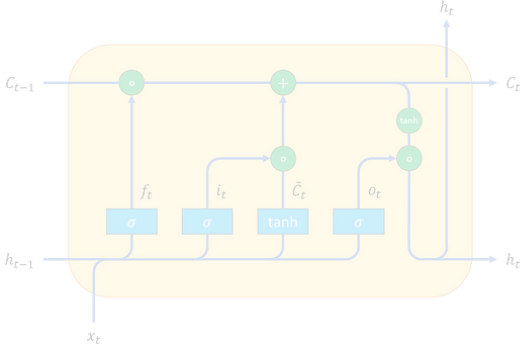
Transformer



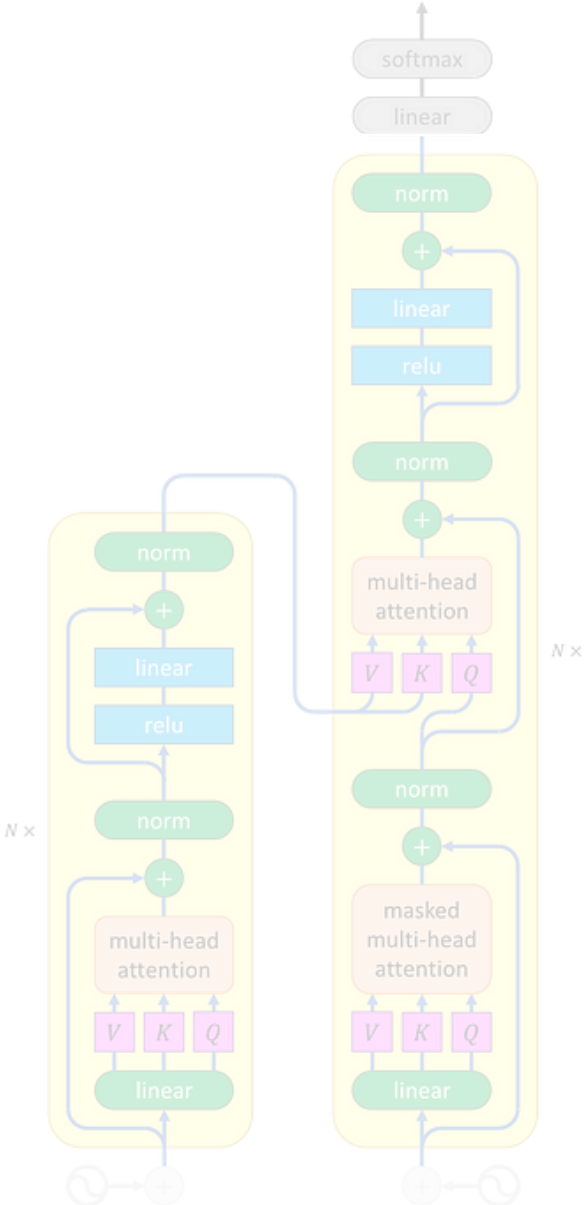
그래서 오늘은 이 RNN의 구조에 대해 소개해드리고



RNN



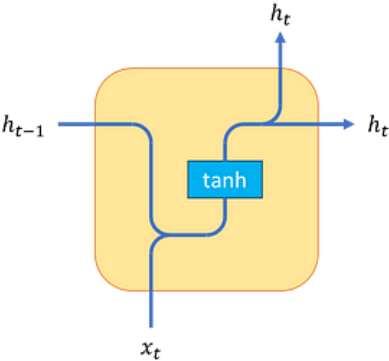
LSTM



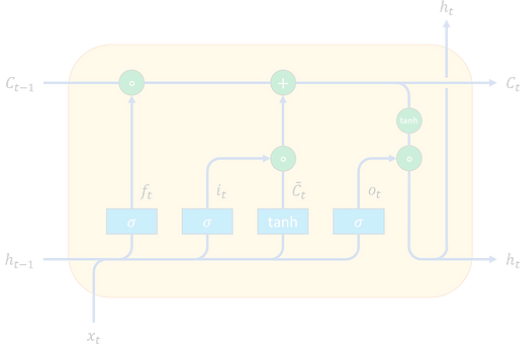
Transformer



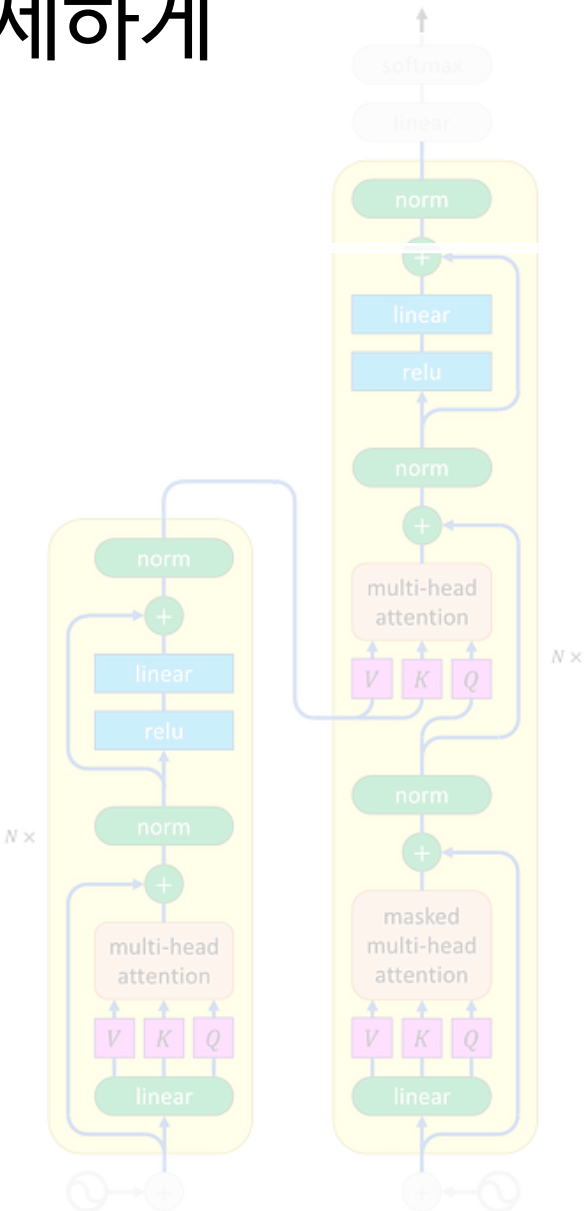
RNN이 시계열 정보를 학습하는 알고리즘에 대해 자세하게 소개해드리고자 합니다



RNN



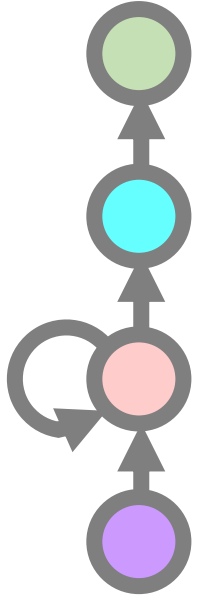
LSTM



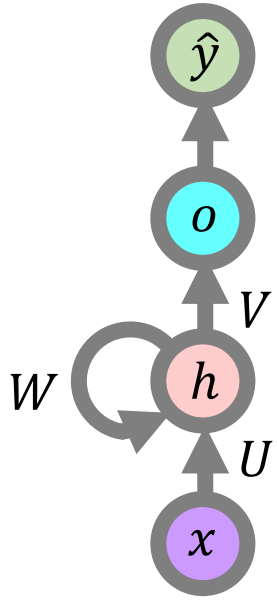
Transformer



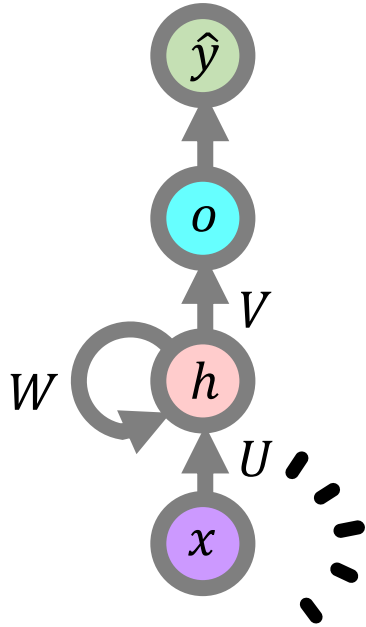
RNN의 구조는 생각보다 간단합니다



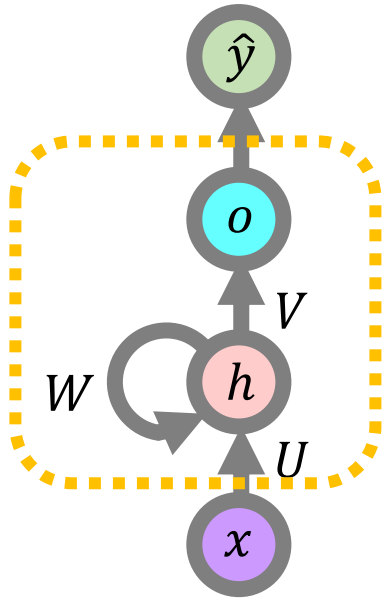
RNN의 구조는 생각보다 간단합니다



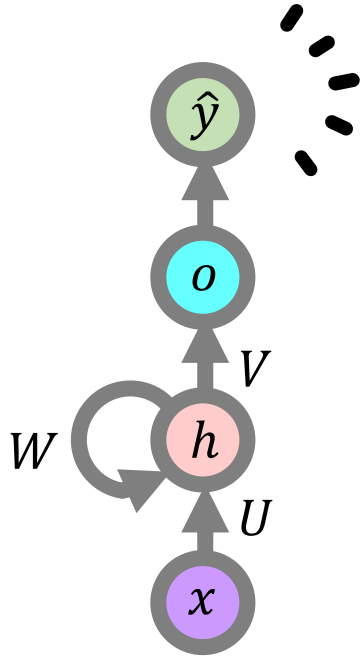
일단 RNN이 하는 일은, 입력벡터 x 를 받아서



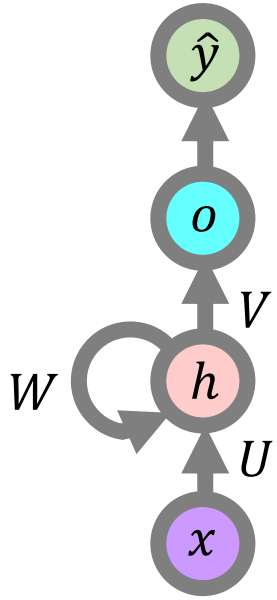
내부적인 연산을 통해서



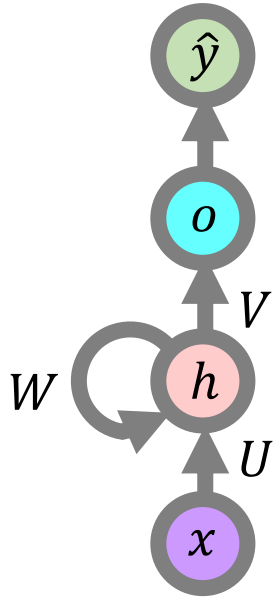
출력벡터인 \hat{y} 를 출력하는 것입니다



이것이 RNN의 순전파 feedforward과정입니다



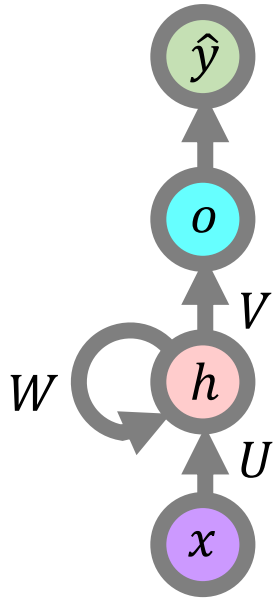
입력벡터 x 와 출력벡터 \hat{y} 의 종류는 다양합니다



그것이 음절 단위의 입력이든,

오늘 밤에도 별이 바람에 스치운다.

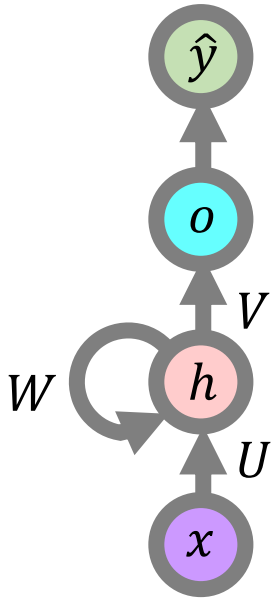
‘오’ ‘늘’ ‘밤’ ‘에’ ‘도’ ‘별’ ‘이’ ‘바’ ‘람’ ‘에’ ‘스’ ‘치’ ‘운’ ‘다’



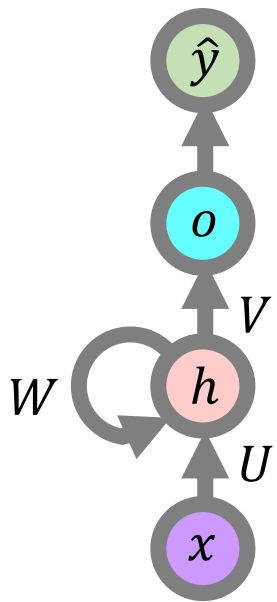
아니면 글자 (word) 단위의 입력이든,

오늘 밤에도 별이 바람에 스치운다.

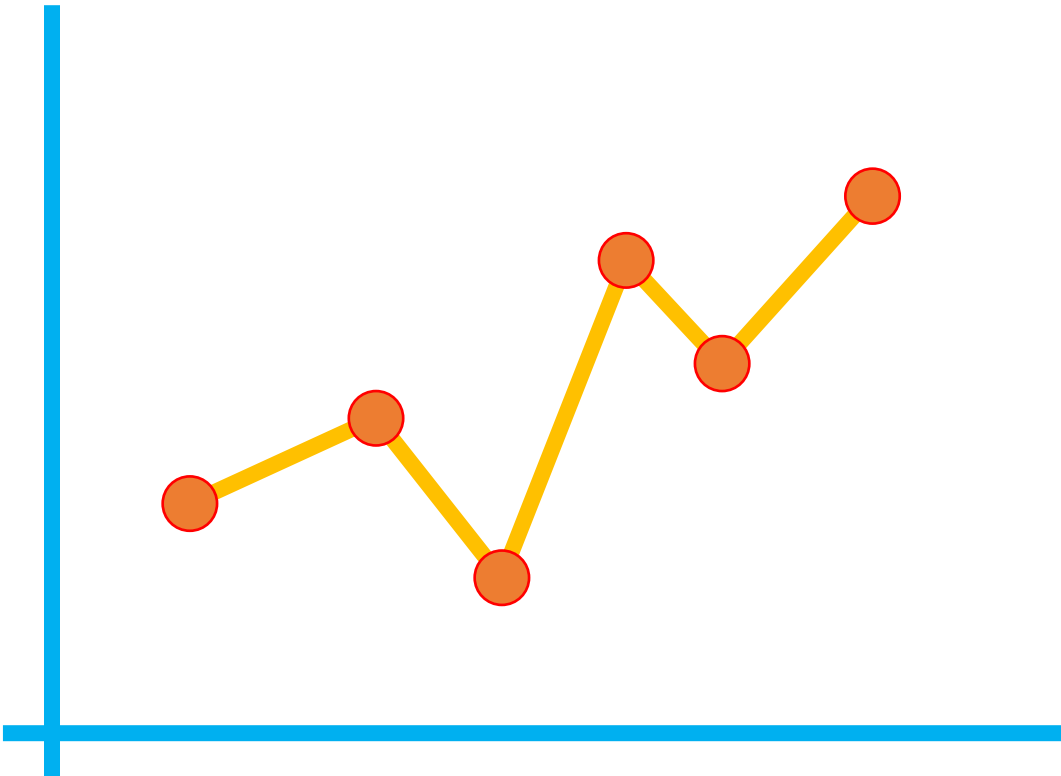
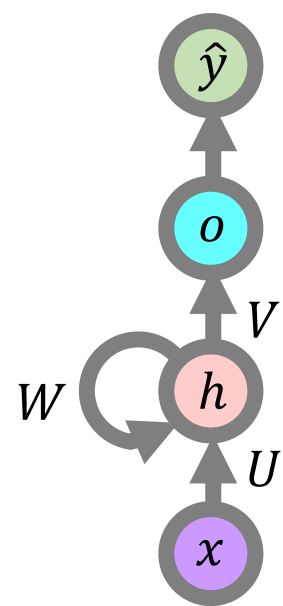
‘오늘’ ‘밤에도’ ‘별이’ ‘바람에’ ‘스치운다’



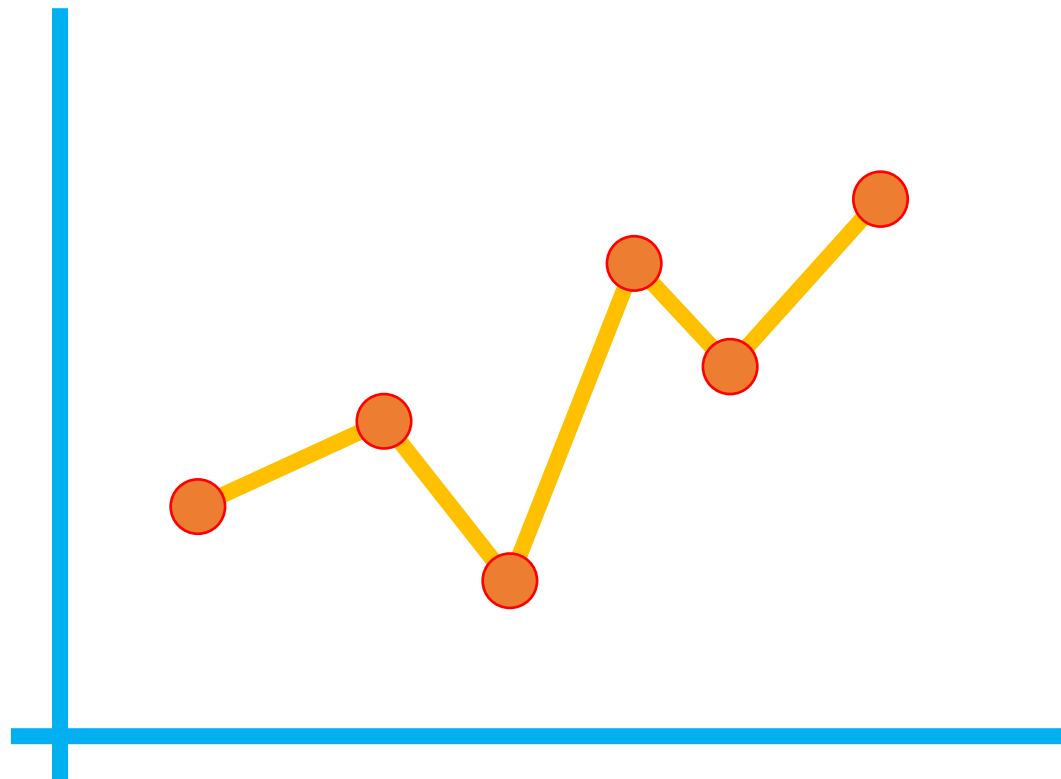
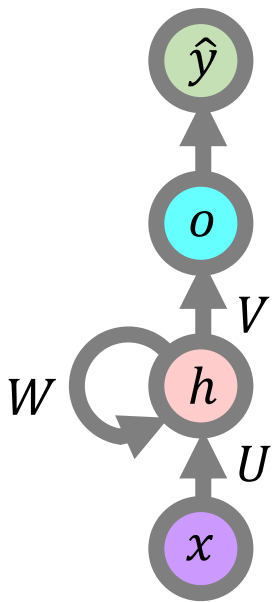
아니면 악보의 음표든



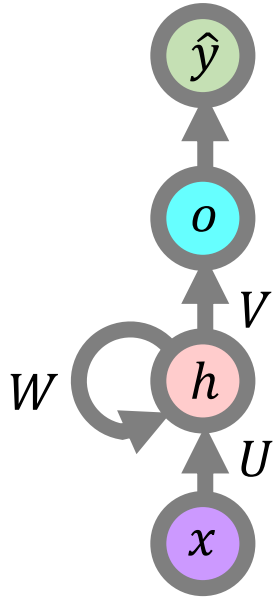
아니면 주식의 가격 변화든



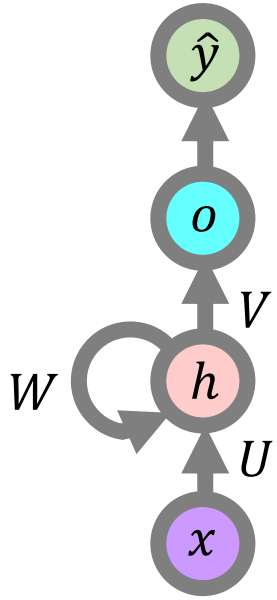
그것을 적절히 처리해서 시계열 데이터로 쓸수만 있다면 다 가능합니다



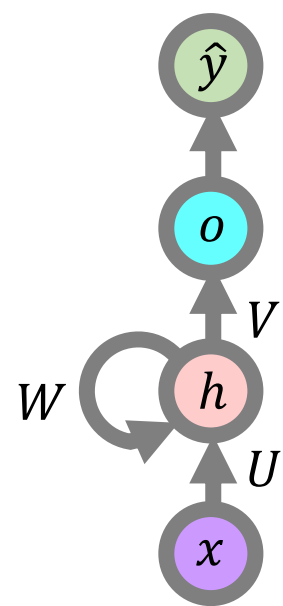
그러면 시계열 데이터를 처리한다 것은 어떤 유익이 있을까요?



예를들어 이 RNN은 영어를 한국어로 번역하는 모델이라 가정해봅시다

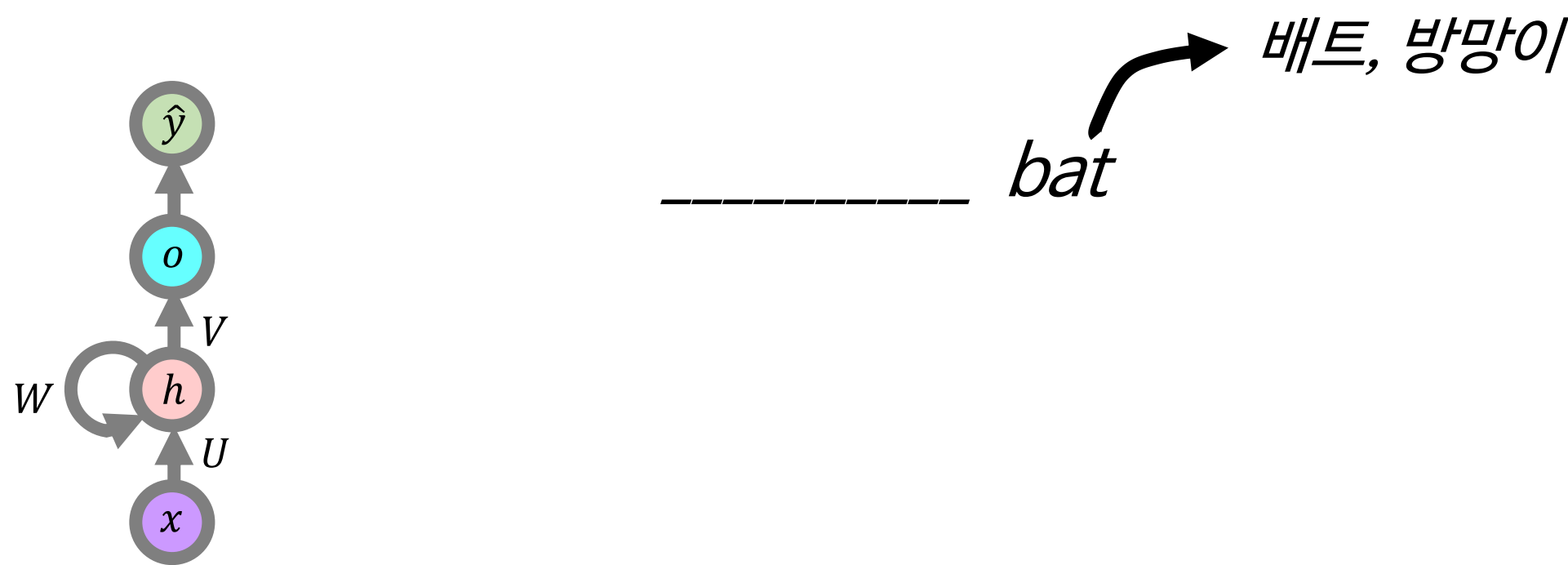


보통 bat라는 단어를 보면

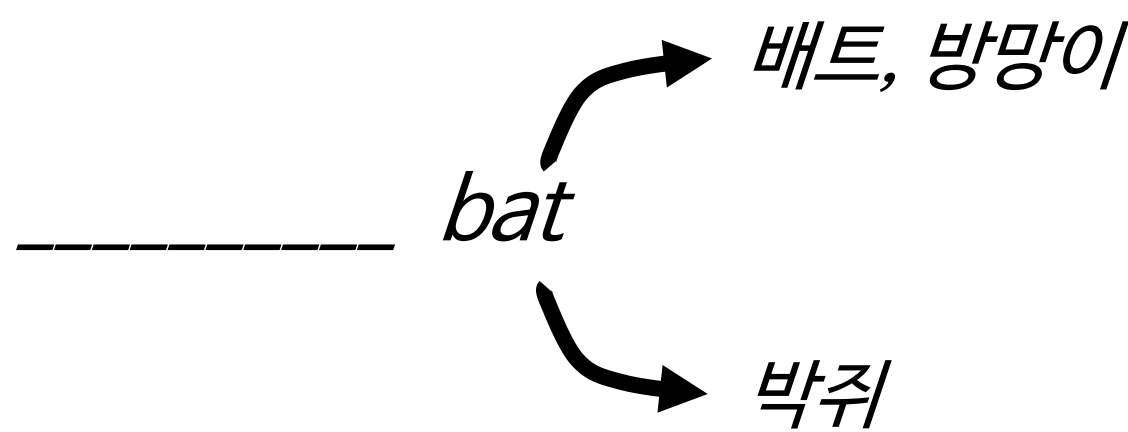
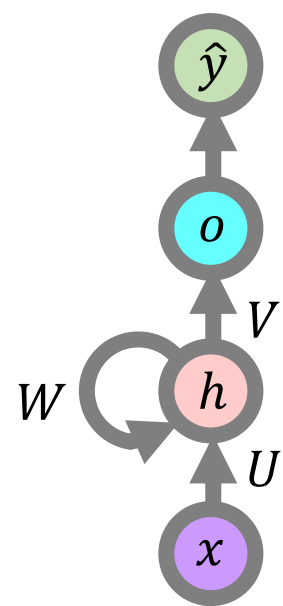


_____ *bat*

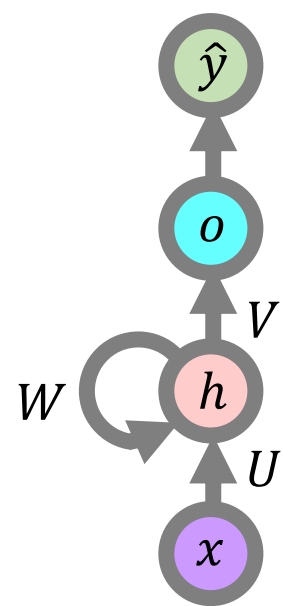
보통 bat라는 단어를 보면 배트나 방망이로 번역이 되기도 하고



박쥐로도 번역이 됩니다

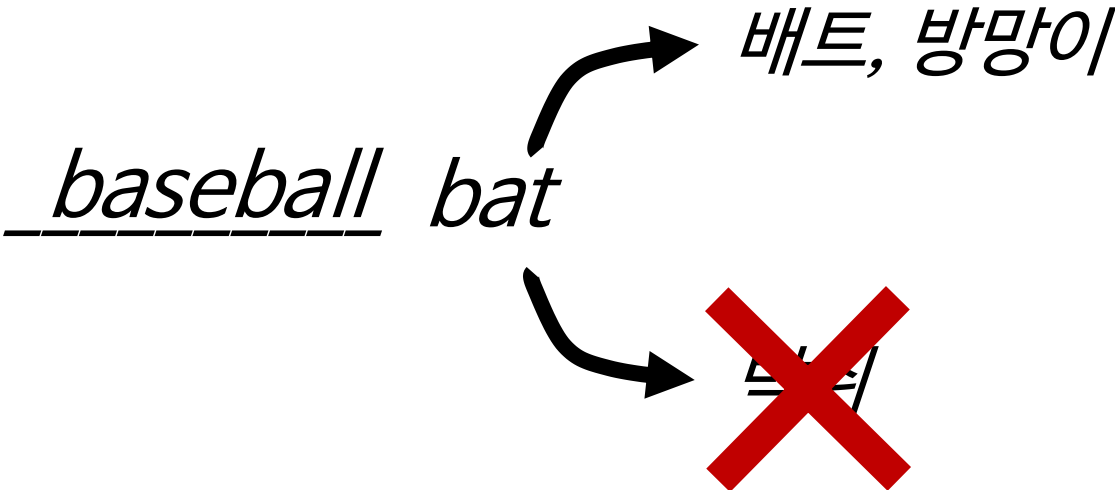
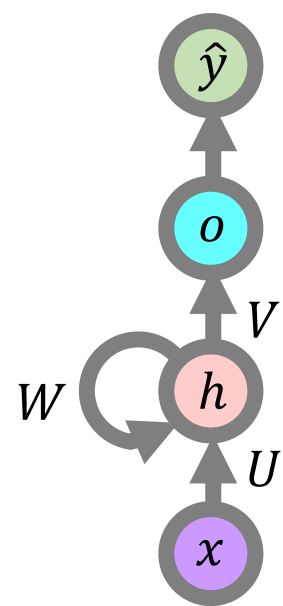


그러나 앞에 단어가 baseball일 경우

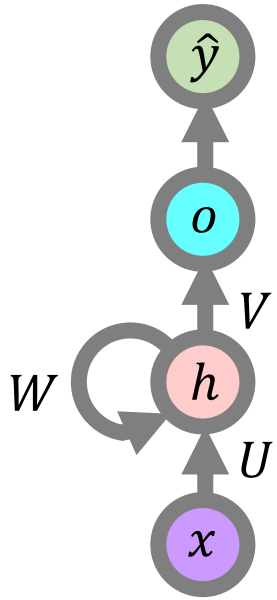


baseball bat 배트, 방망이
 박쥐

높은 확률로 bat는 배트로 번역이 됩니다

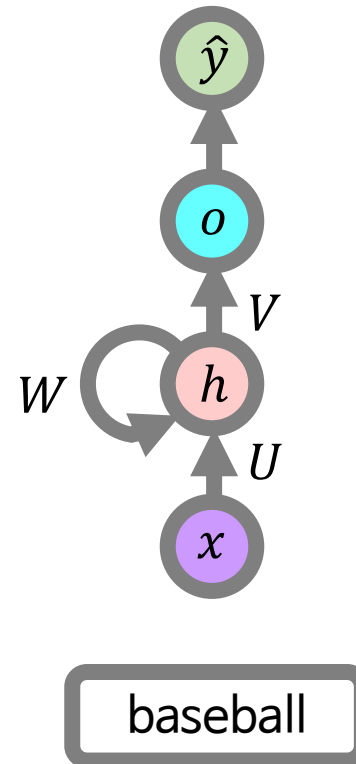
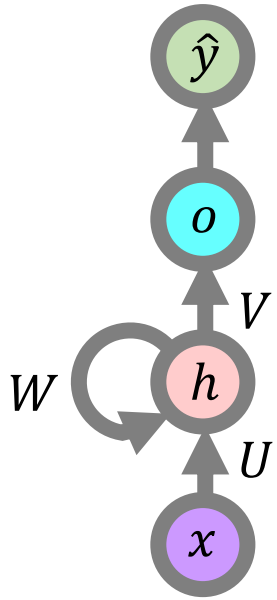


RNN은 이와같은 뇌의 능력, 기억 memory를 모방하여

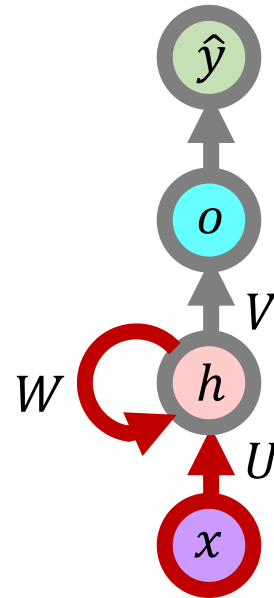
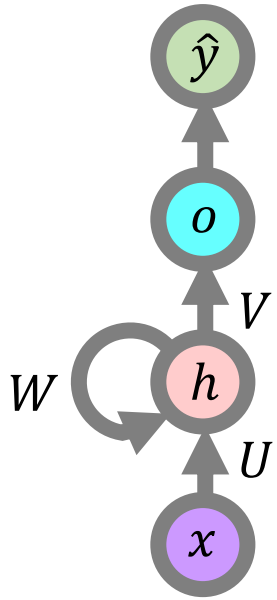


baseball

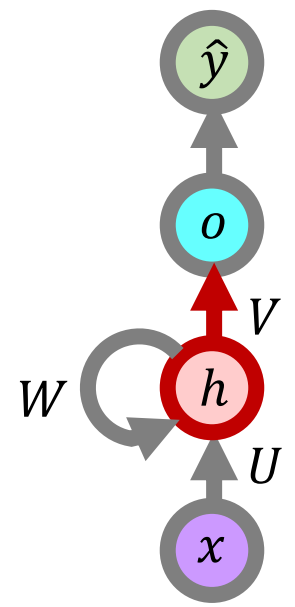
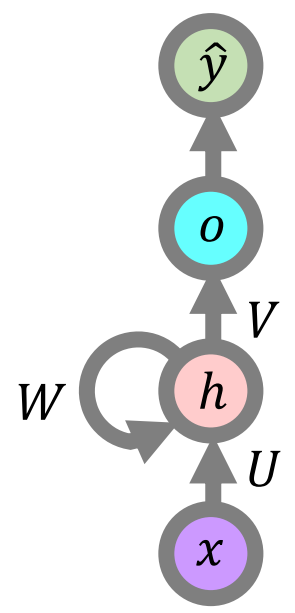
Baseball을 번역할 때,



Baseball을 번역할 때,

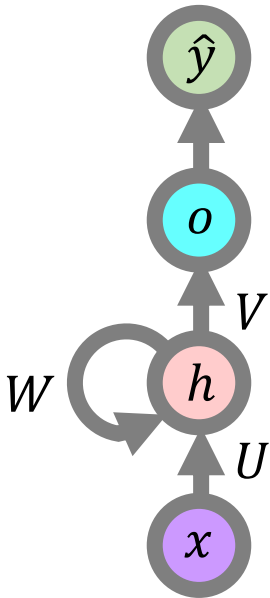
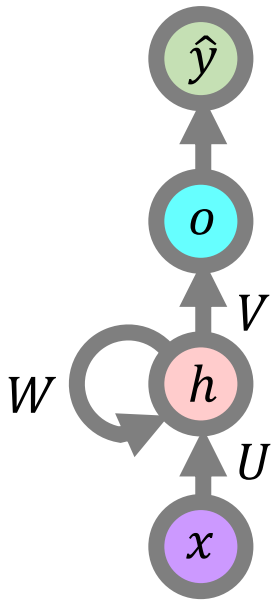


이 내부상태 h 가 baseball을 처리한 값들로 세팅이 됩니다

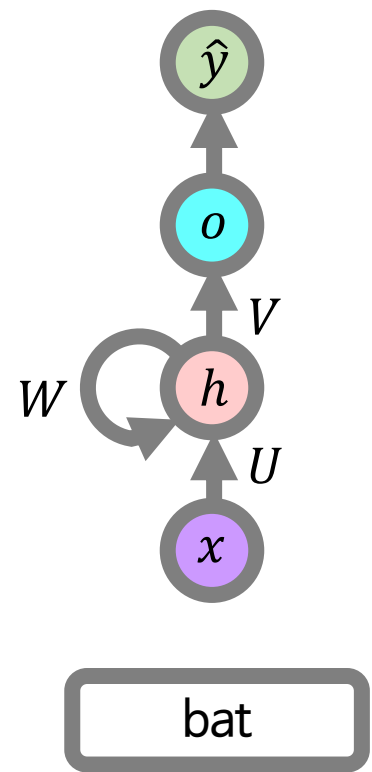
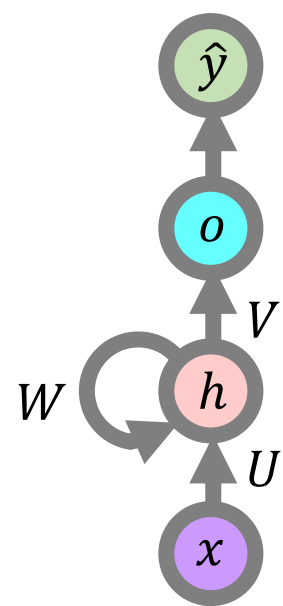


이런 내부 연산을 통해 'baseball'을 '야구'로 번역하고

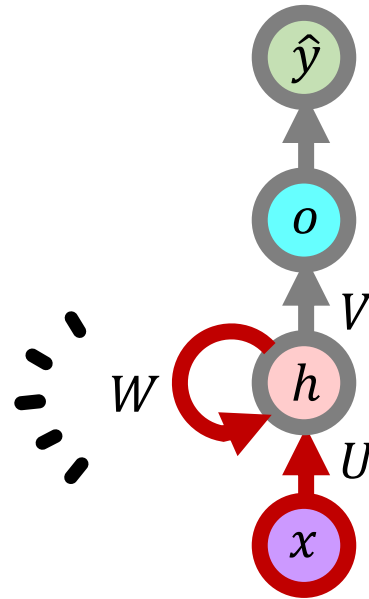
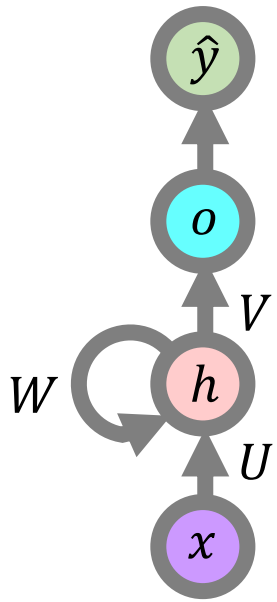
야구



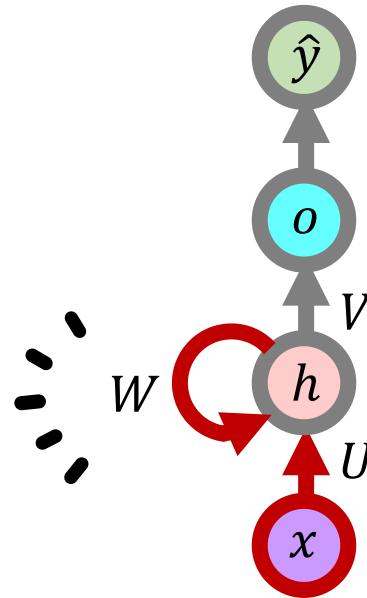
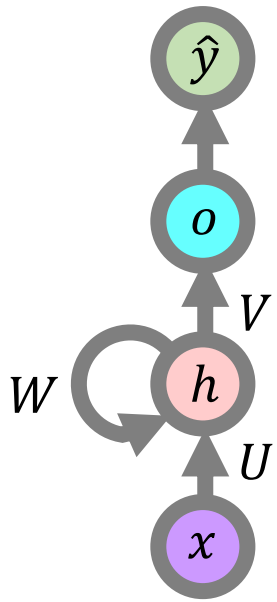
다시 'bat'를 번역할 때



앞서 'baseball'을 번역할 때 생성된 내부상태 (hidden state)가



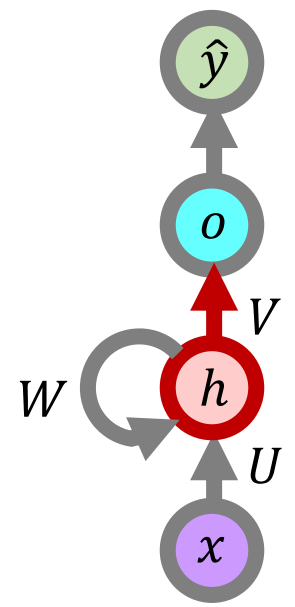
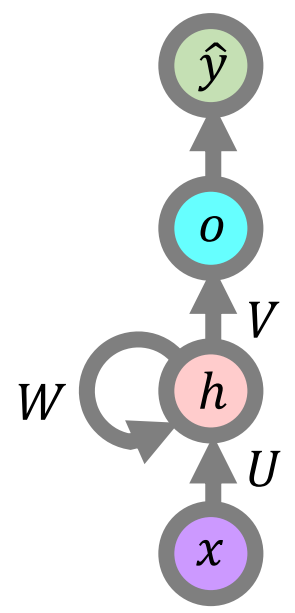
다시 'bat'를 번역할 때 영향을 끼친다는 점이 RNN의 핵심입니다



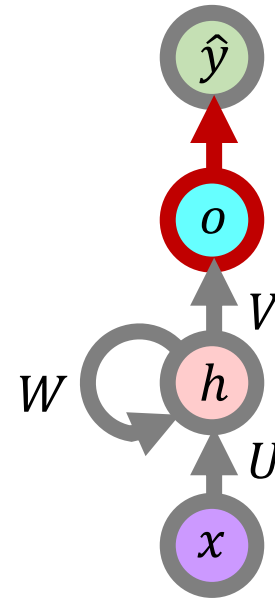
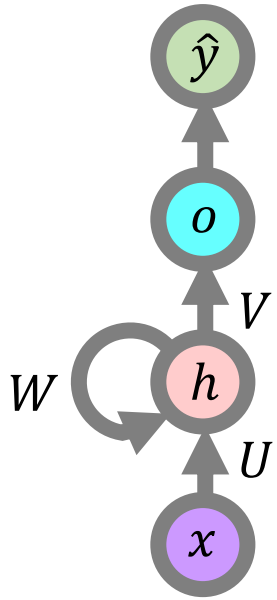
그리고 baseball과 bat를 합쳐놓은 형태의 내부상태 h 로 업데이트되고,



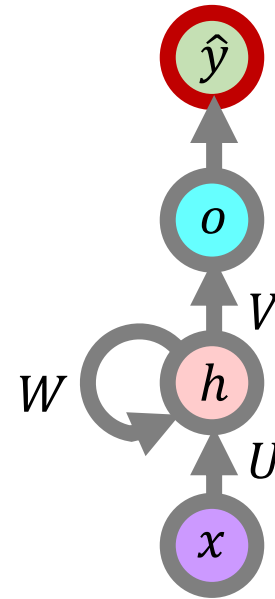
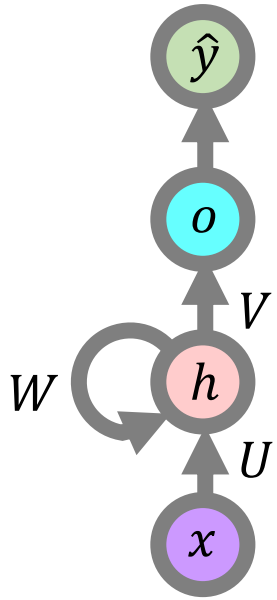
그리하여 'bat'는 박쥐로 번역될 확률보다, 방망이 혹은 배트로 번역될 확률이 높아지게 됩니다



그리하여 'bat'는 박쥐로 번역될 확률보다, 방망이 혹은 배트로 번역될 확률이 높아지게 됩니다

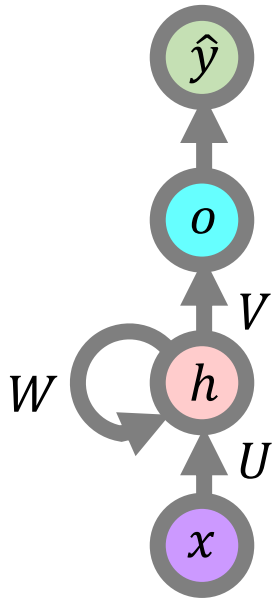
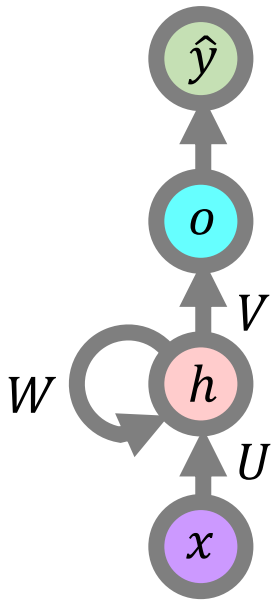


그리하여 'bat'는 박쥐로 번역될 확률보다, 방망이 혹은 배트로 번역될 확률이 높아지게 됩니다



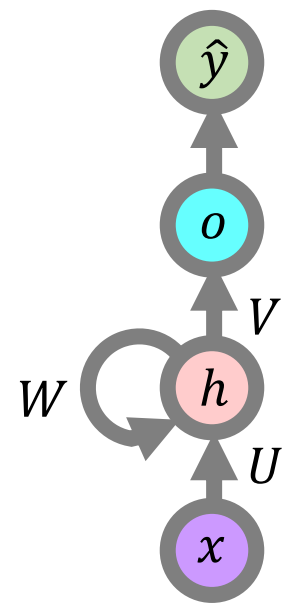
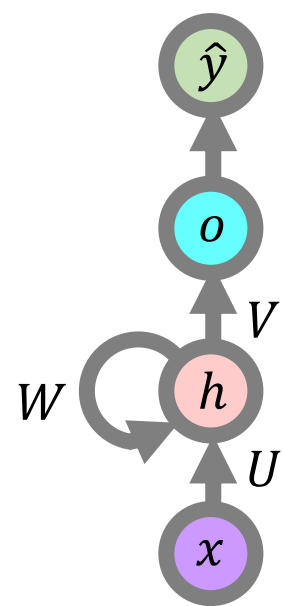
그리하여 'bat'는 박쥐로 번역될 확률보다, 방망이 혹은 배트로 번역될 확률이 높아지게 됩니다

배트



그럼 이제부터는 숫자를 넣어서 어떻게 모델안에서 순전파와 역전파가 일어나는지 알아보도록 하겠습니다

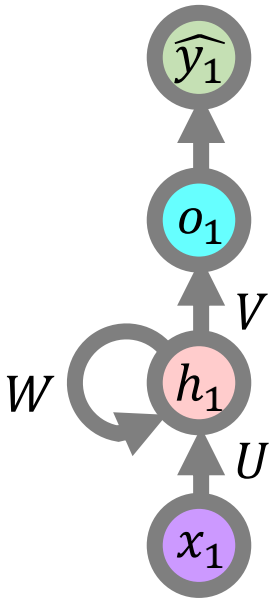
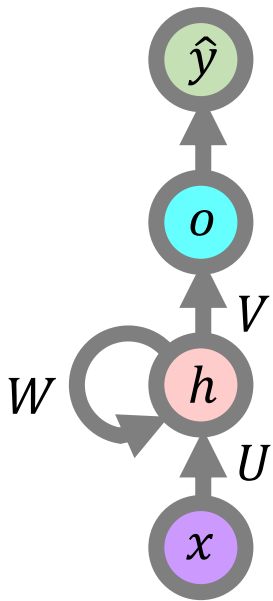
배트



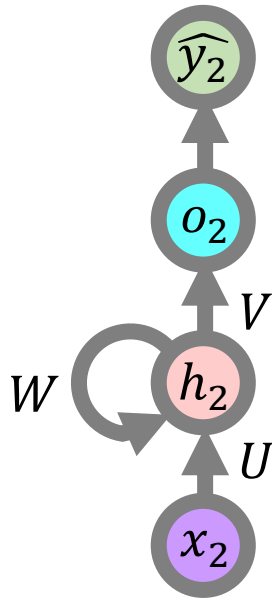
연산의 시간적 과정을 공간적으로 표현하기 위해 다음과 같이 RNN을 옆으로 풀어서 그려보도록 하겠습니다

야구

배트



baseball

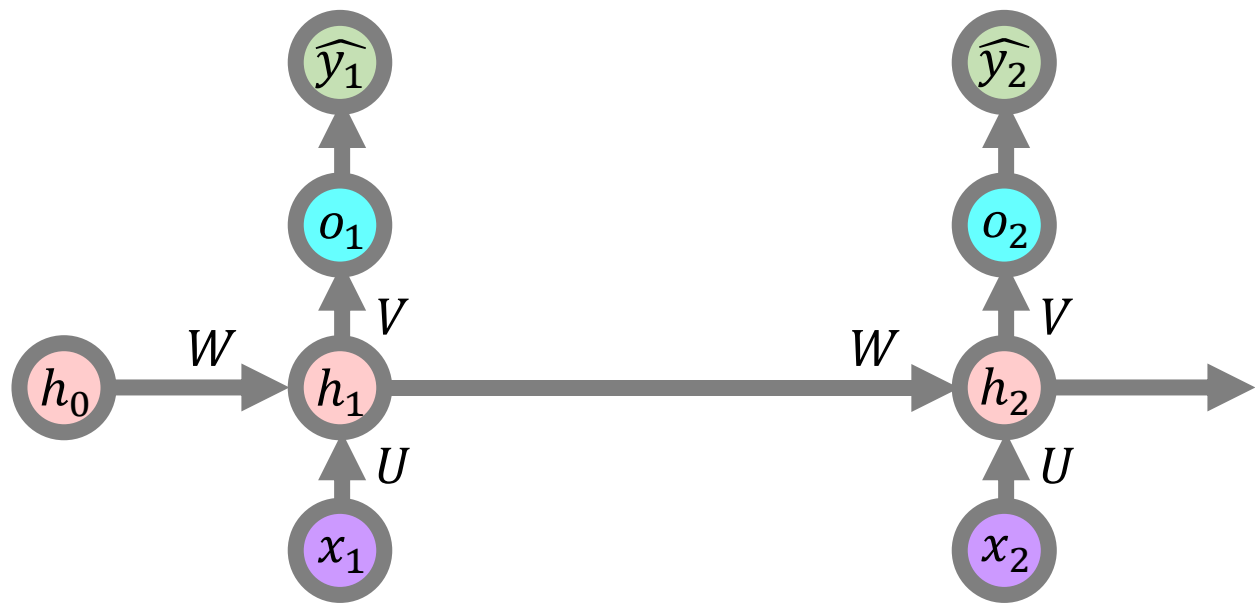
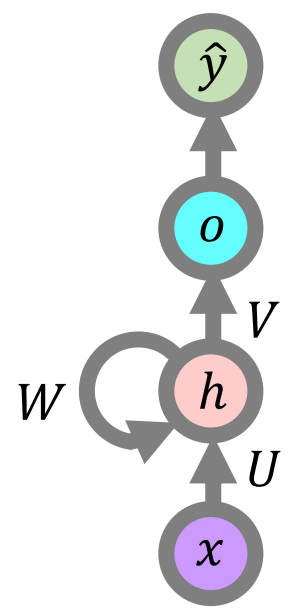


bat

그리고 h 의 시간적 업데이트를 더 잘 이해하기 위해 화살표를 다음과 같이 수정하도록 하겠습니다

야구

배트



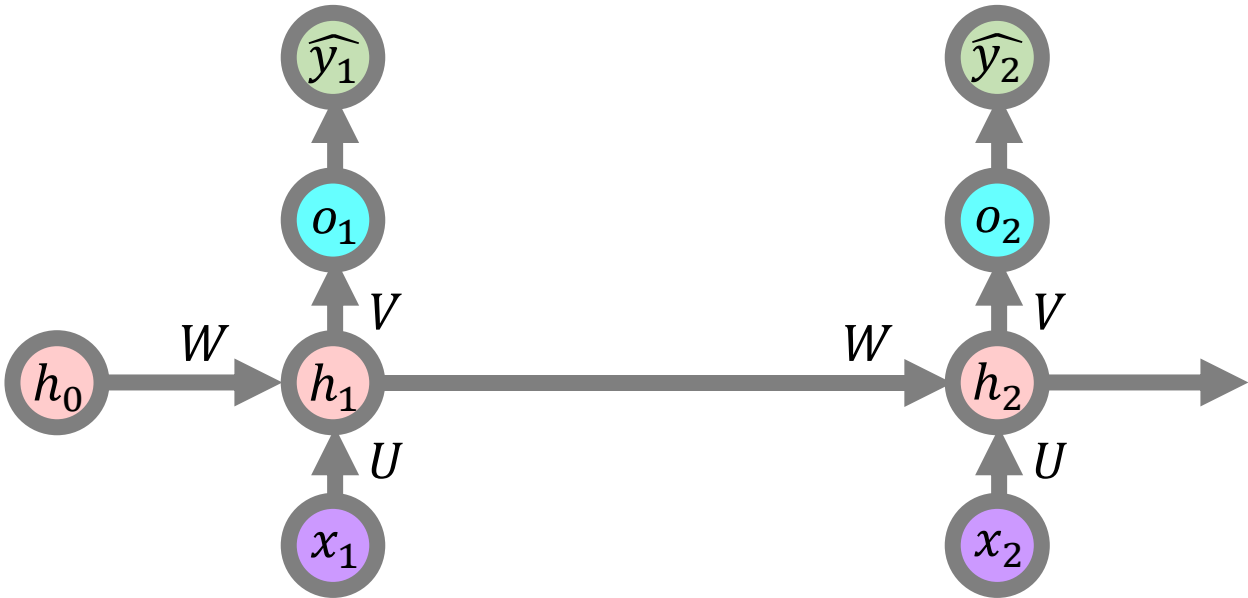
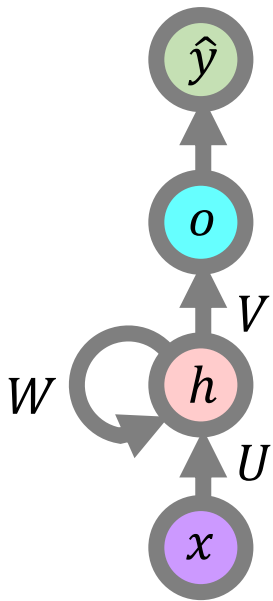
baseball

bat

자 그러면 이제 입력값부터 살펴보도록 하겠습니다

야구

배트



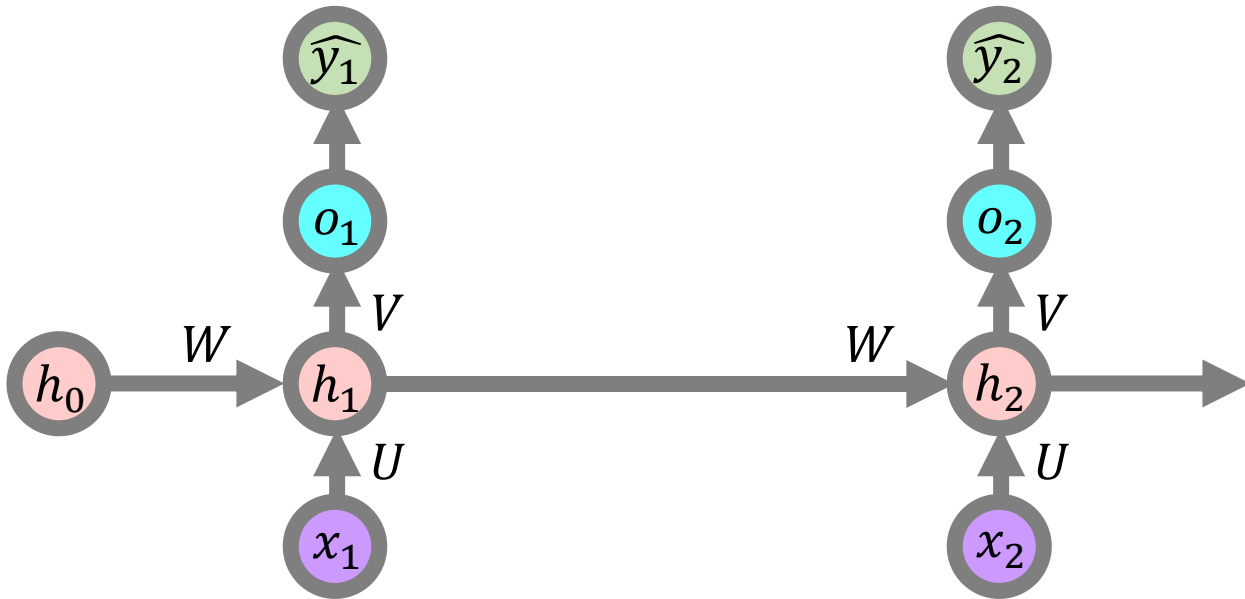
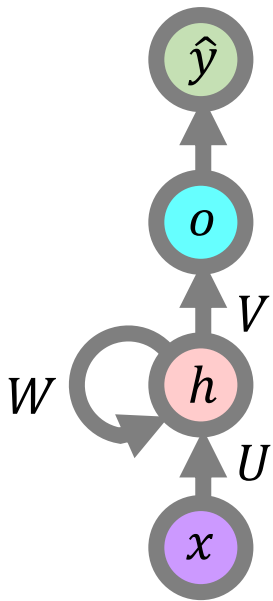
baseball

bat

설명을 간단하게 하기 위해 입력값은 one-hot vector로 하도록 하겠습니다

야구

배트



baseball

bat

one-hot vector는 모든 벡터의 값이 0이고 오직 하나의 값만 1인 벡터를 말합니다

to	$= [1, 0, 0, 0, 0, 0, 0, 0, 0, \dots, 0]$
be	$= [0, 1, 0, 0, 0, 0, 0, 0, 0, \dots, 0]$
or	$= [0, 0, 1, 0, 0, 0, 0, 0, 0, \dots, 0]$
not	$= [0, 0, 0, 1, 0, 0, 0, 0, 0, \dots, 0]$
to	$= [1, 0, 0, 0, 0, 0, 0, 0, 0, \dots, 0]$
be	$= [0, 1, 0, 0, 0, 0, 0, 0, 0, \dots, 0]$
that	$= [0, 0, 0, 0, 1, 0, 0, 0, 0, \dots, 0]$
is	$= [0, 0, 0, 0, 0, 1, 0, 0, 0, \dots, 0]$
the	$= [0, 0, 0, 0, 0, 0, 1, 0, 0, \dots, 0]$
question	$= [0, 0, 0, 0, 0, 0, 0, 1, 0, \dots, 0]$

보시는 것 처럼 같은 단어는 같은 벡터입니다

to	$= [1, 0, 0, 0, 0, 0, 0, 0, 0, \dots, 0]$
be	$= [0, 1, 0, 0, 0, 0, 0, 0, 0, \dots, 0]$
or	$= [0, 0, 1, 0, 0, 0, 0, 0, 0, \dots, 0]$
not	$= [0, 0, 0, 1, 0, 0, 0, 0, 0, \dots, 0]$
to	$= [1, 0, 0, 0, 0, 0, 0, 0, 0, \dots, 0]$
be	$= [0, 1, 0, 0, 0, 0, 0, 0, 0, \dots, 0]$
that	$= [0, 0, 0, 0, 1, 0, 0, 0, 0, \dots, 0]$
is	$= [0, 0, 0, 0, 0, 1, 0, 0, 0, \dots, 0]$
the	$= [0, 0, 0, 0, 0, 0, 1, 0, 0, \dots, 0]$
question	$= [0, 0, 0, 0, 0, 0, 0, 1, 0, \dots, 0]$

그리고 다른 단어는 서로 다른 벡터로 이루어져 있습니다

to $= [1, 0, 0, 0, 0, 0, 0, 0, 0, 0, \dots, 0]$

be $= [0, 1, 0, 0, 0, 0, 0, 0, 0, 0, \dots, 0]$

or $= [0, 0, 1, 0, 0, 0, 0, 0, 0, 0, \dots, 0]$

not $= [0, 0, 0, 1, 0, 0, 0, 0, 0, 0, \dots, 0]$

to $= [1, 0, 0, 0, 0, 0, 0, 0, 0, 0, \dots, 0]$

be $= [0, 1, 0, 0, 0, 0, 0, 0, 0, 0, \dots, 0]$

that $= [0, 0, 0, 0, 1, 0, 0, 0, 0, 0, \dots, 0]$

is $= [0, 0, 0, 0, 0, 1, 0, 0, 0, 0, \dots, 0]$

the $= [0, 0, 0, 0, 0, 0, 1, 0, 0, 0, \dots, 0]$

question $= [0, 0, 0, 0, 0, 0, 0, 1, 0, 0, \dots, 0]$

그래서 이론적으로는 벡터의 길이N이 충분히 길다면, 세상의 모든 단어
들에 대해 벡터를 각각 1:1 대응시킬 수도 있습니다

	$\overbrace{\hspace{10em}}^N$
to	$= [1, 0, 0, 0, 0, 0, 0, 0, 0, \dots, 0]$
be	$= [0, 1, 0, 0, 0, 0, 0, 0, 0, \dots, 0]$
or	$= [0, 0, 1, 0, 0, 0, 0, 0, 0, \dots, 0]$
not	$= [0, 0, 0, 1, 0, 0, 0, 0, 0, \dots, 0]$
to	$= [1, 0, 0, 0, 0, 0, 0, 0, 0, \dots, 0]$
be	$= [0, 1, 0, 0, 0, 0, 0, 0, 0, \dots, 0]$
that	$= [0, 0, 0, 0, 1, 0, 0, 0, 0, \dots, 0]$
is	$= [0, 0, 0, 0, 0, 1, 0, 0, 0, \dots, 0]$
the	$= [0, 0, 0, 0, 0, 0, 1, 0, 0, \dots, 0]$
question	$= [0, 0, 0, 0, 0, 0, 0, 1, 0, \dots, 0]$

이 영상에서는 편의상 $n=3$ 인 두개의 one-hot vector 사전을 사용하도록 하겠습니다

영어사전

baseball $= [1, 0, 0]$

bat $= [0, 0, 1]$

한국어사전

야구 $= [0, 1, 0]$

배트 $= [1, 0, 0]$

그리고 입력값도 [baseball, bat]라고 하는 입력 시퀀스, 즉 두 개의 vector를 순차적으로 세로 벡터 형태로 넣도록 하겠습니다

영어사전

baseball = [1, 0, 0]

bat = [0, 0, 1]

한국어사전

야구 = [0, 1, 0]

배트 = [1, 0, 0]

입력: $\begin{bmatrix} \text{baseball} \\ \text{bat} \end{bmatrix} = \begin{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} & \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \end{bmatrix}$

출력: $\begin{bmatrix} \text{야구} & \text{배트} \end{bmatrix} = \begin{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} & \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \end{bmatrix}$

입력시퀀스를 다 넣으면 순전파가 완료가 되는것입니다

영어사전

baseball = [1, 0, 0]

bat = [0, 0, 1]

한국어사전

야구 = [0, 1, 0]

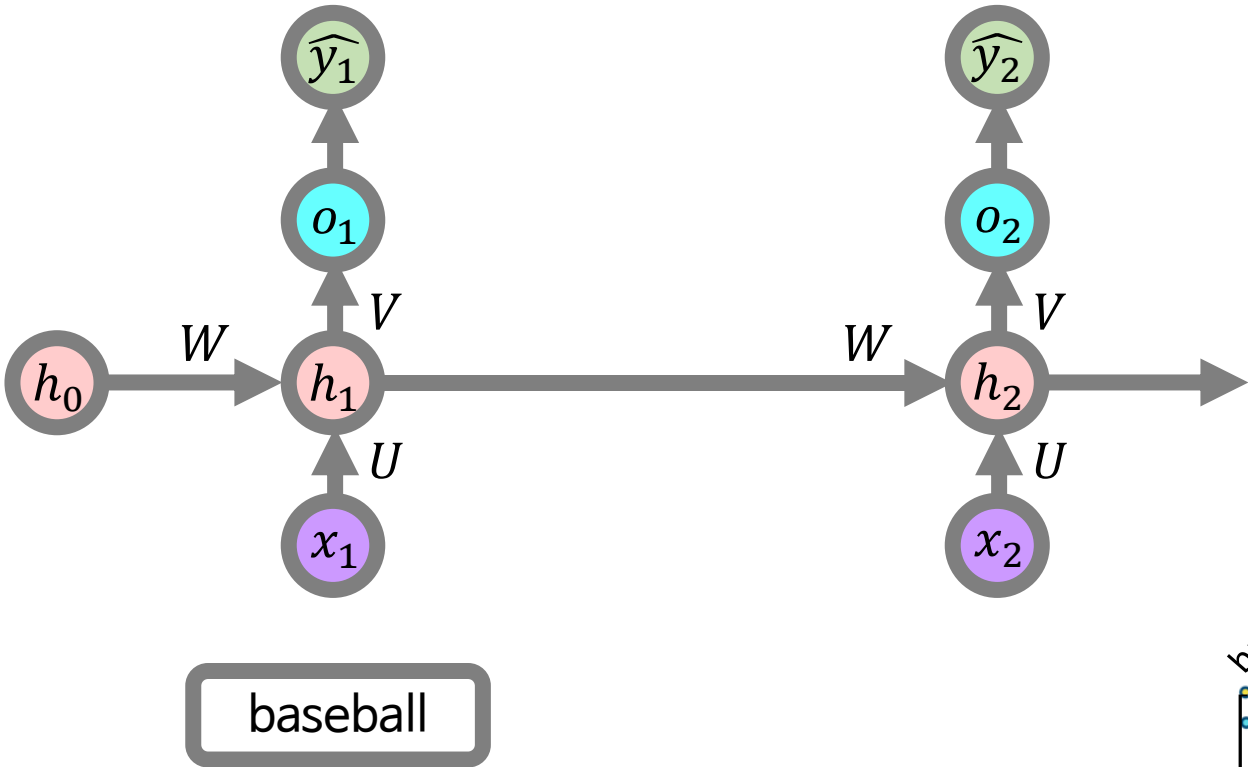
배트 = [1, 0, 0]

입력: $\begin{bmatrix} \begin{matrix} \text{baseball} \\ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \end{matrix} & \begin{matrix} \text{bat} \\ \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \end{matrix} \end{bmatrix}$, 출력: $\begin{bmatrix} \begin{matrix} \text{야구} \\ \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \end{matrix} & \begin{matrix} \text{배트} \\ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \end{matrix} \end{bmatrix}$

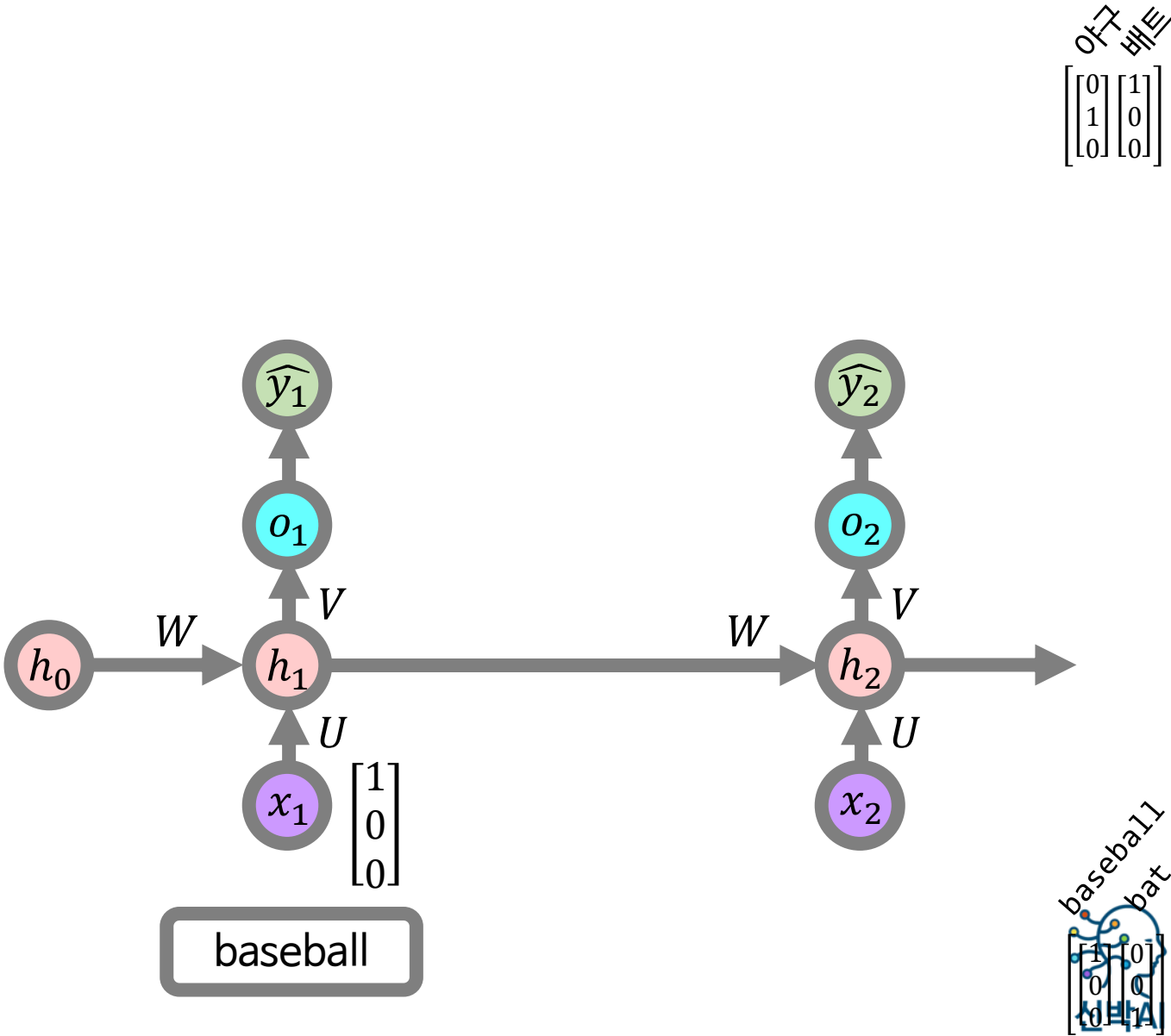
그럼 먼저 baseball의 one-hot vector를 넣도록 하겠습니다

야구 배트

$$\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

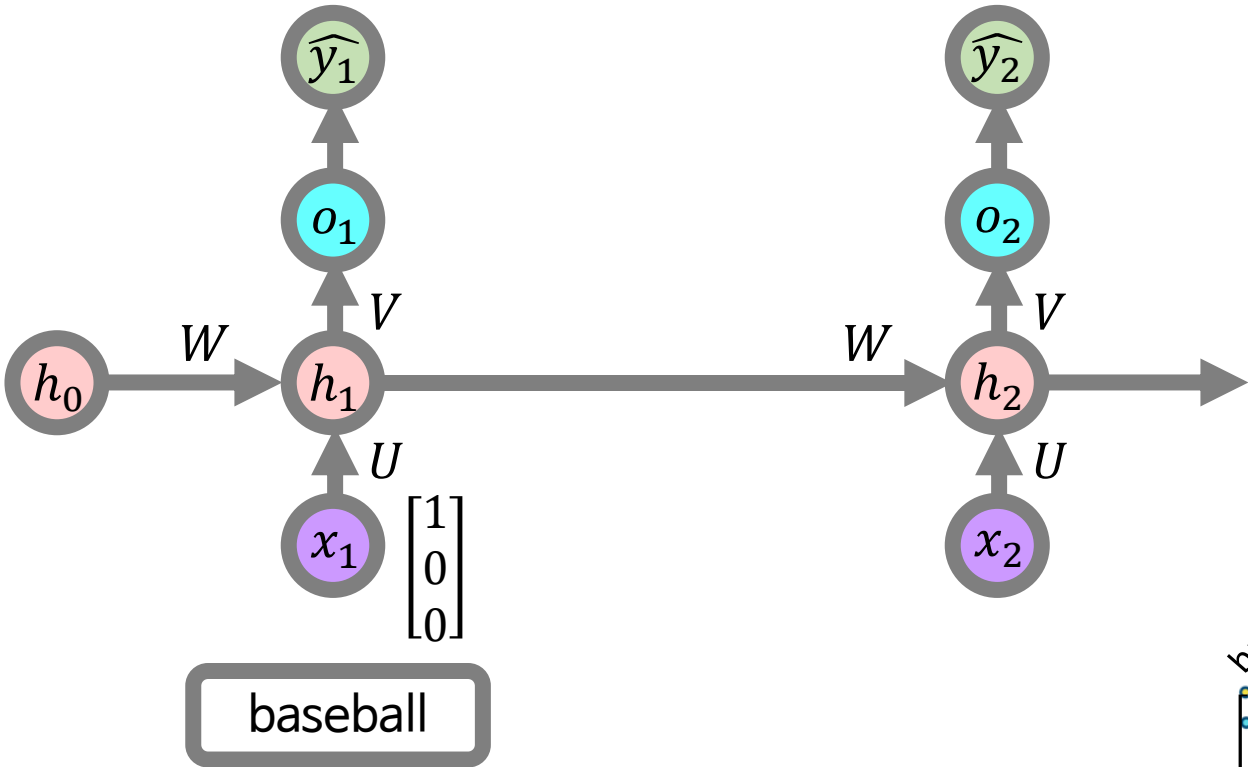


그럼 먼저 baseball의 one-hot vector를 넣도록 하겠습니다



내부상태 h 를 계산하는 공식은 다음과 같습니다

$$h_1 = \tanh(W h_0 + U x_1)$$



야구
배트

0	1
1	0
0	0

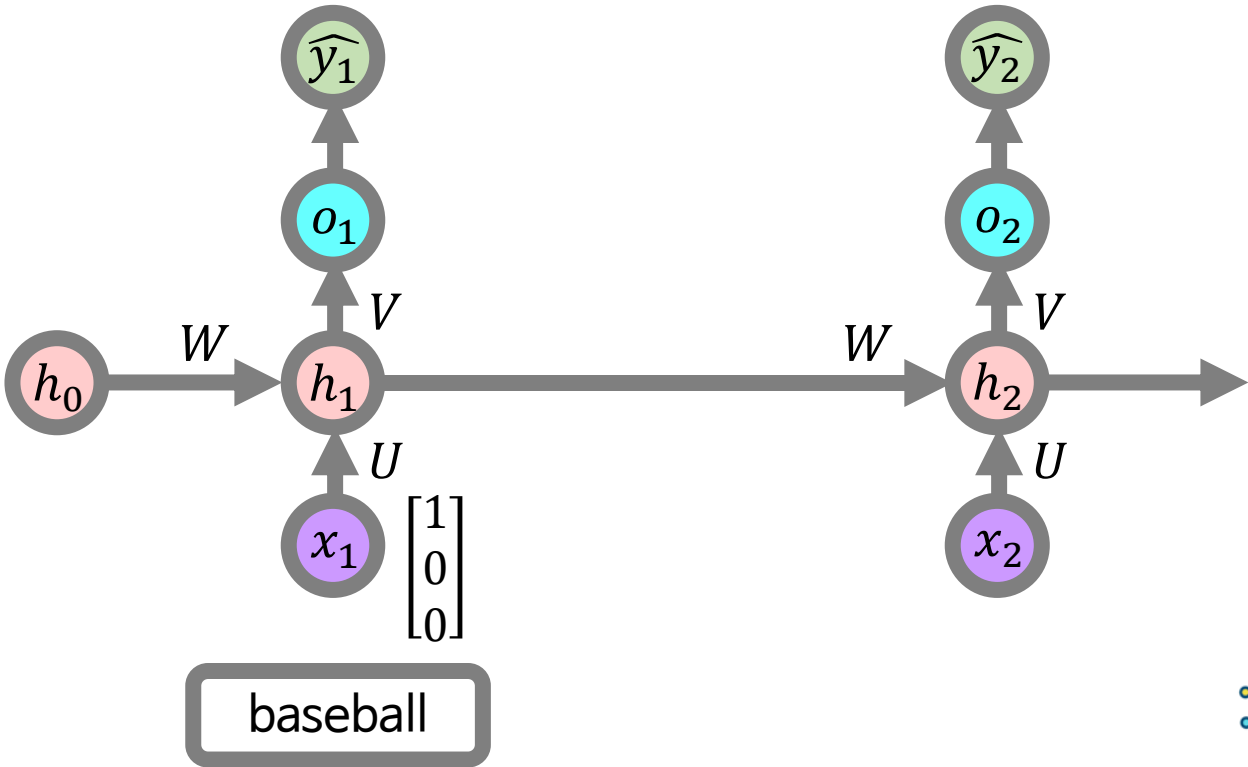
baseball
bat

1	0
0	0
0	1

완전한 공식은 여기에 편향 b를 넣어주어야 하는데

야구
배트
 $\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

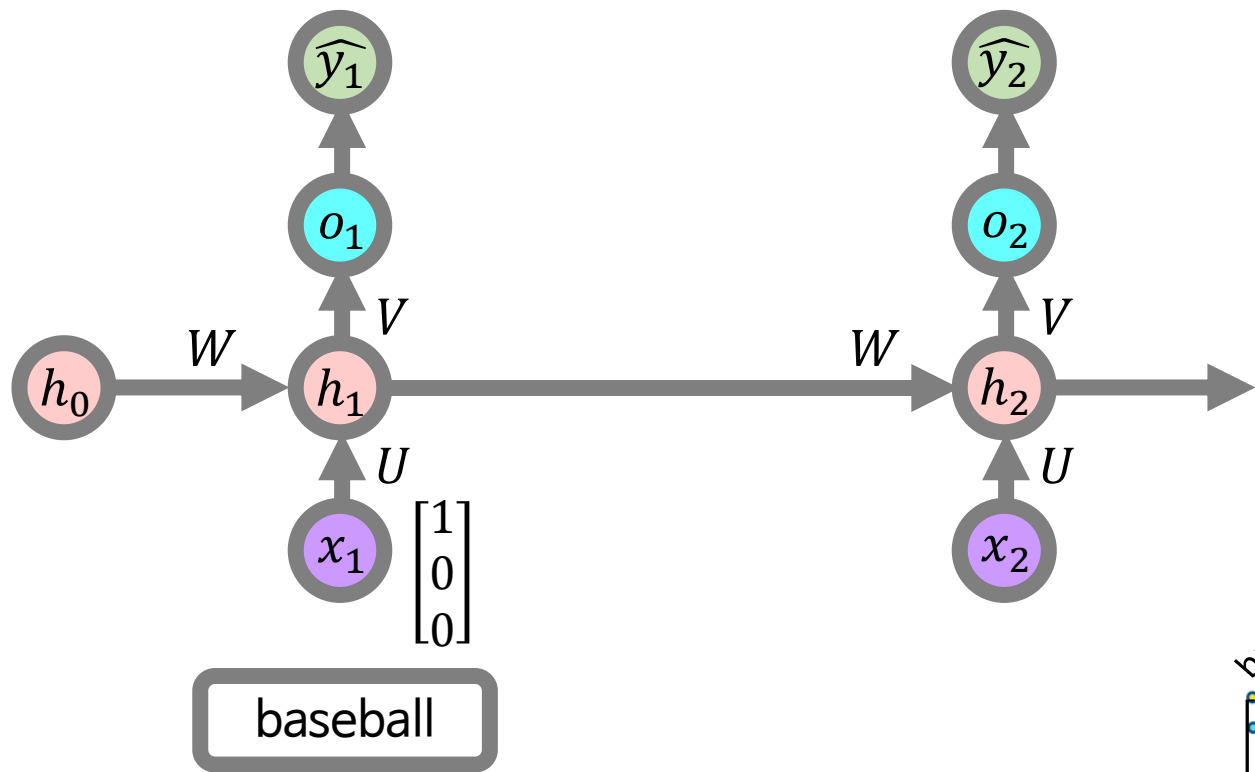
$$h_1 = \tanh(b + Wh_0 + Ux_1)$$



이 영상에서는 계산의 편의를 위해 생략하도록 하겠습니다

야구 배트
 $\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$ $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

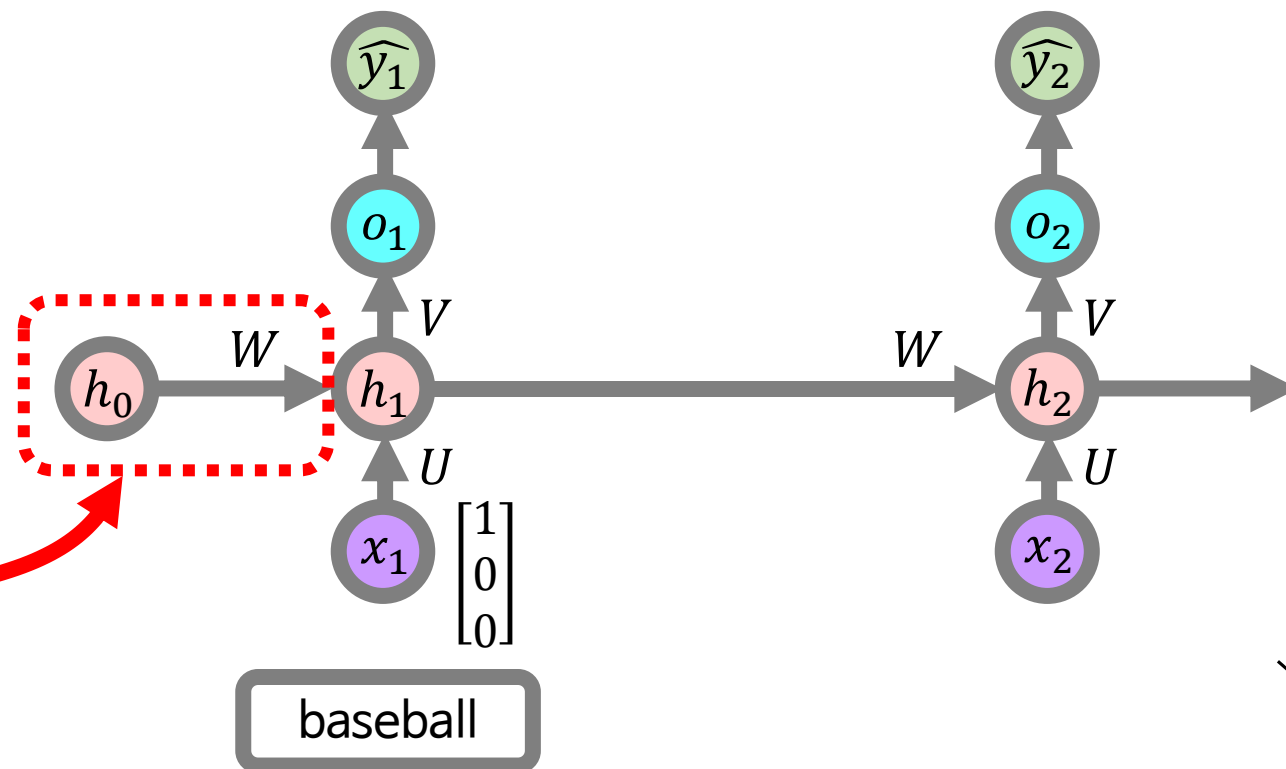
$$h_1 = \tanh(W h_0 + U x_1)$$



baseball bat
 $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ $\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

Wh_0 는 이 부분을 뜻하고

$$h_1 = \tanh(W h_0 + U x_1)$$



야구
배트

0	1
1	0
0	0

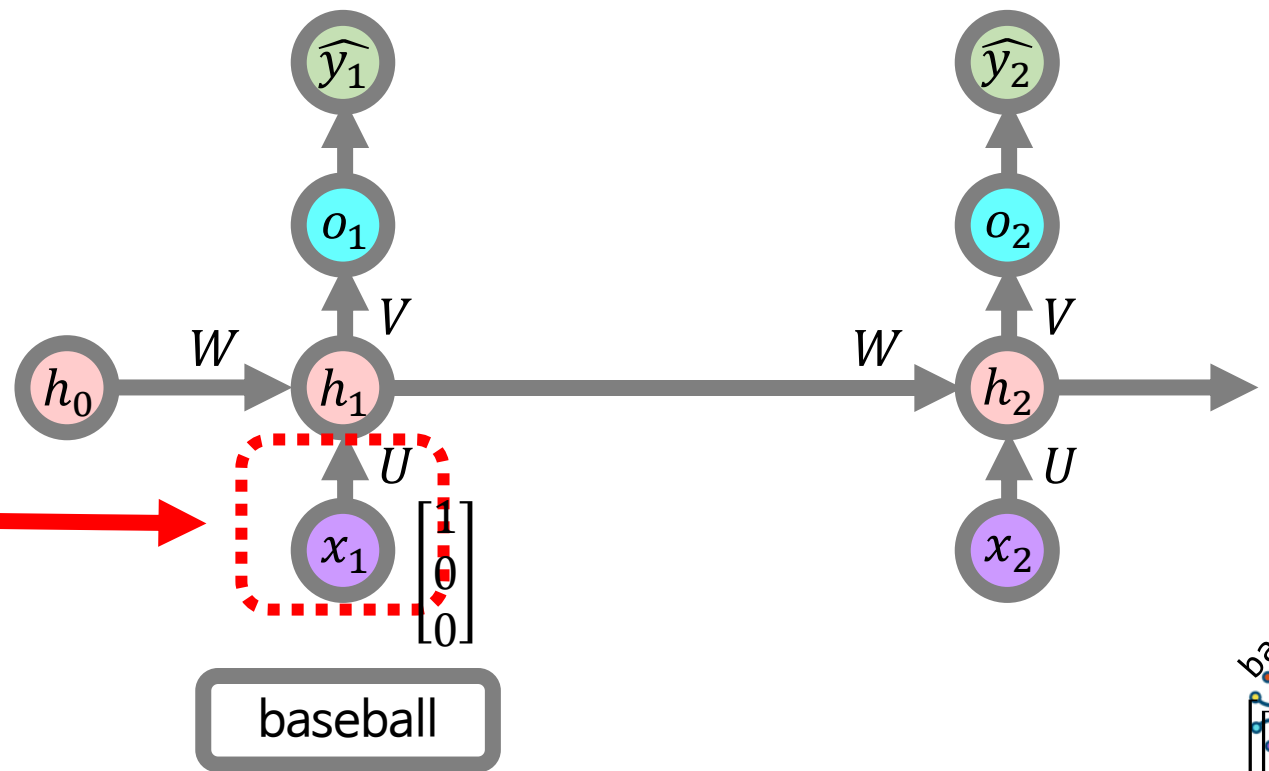
baseball
bat

1	0
0	0
0	1

Ux_1 는 이 부분을 뜻합니다

야구
배트
 $\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$ $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

$$h_1 = \tanh(W h_0 + U x_1)$$

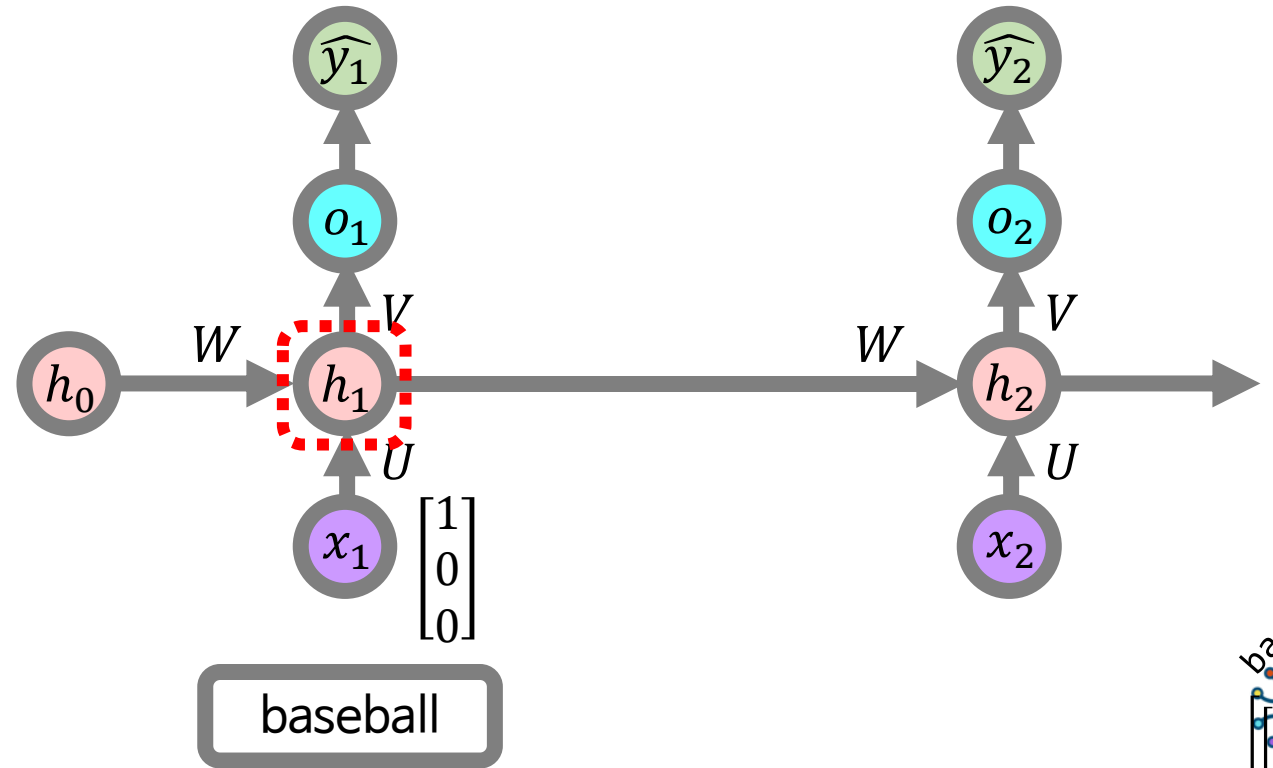


baseball
bat
 $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

tanh 는 RNN에서 쓰는 활성화 함수입니다

야구
배트
 $\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$ $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

$$h_1 = \tanh(W h_0 + U x_1)$$

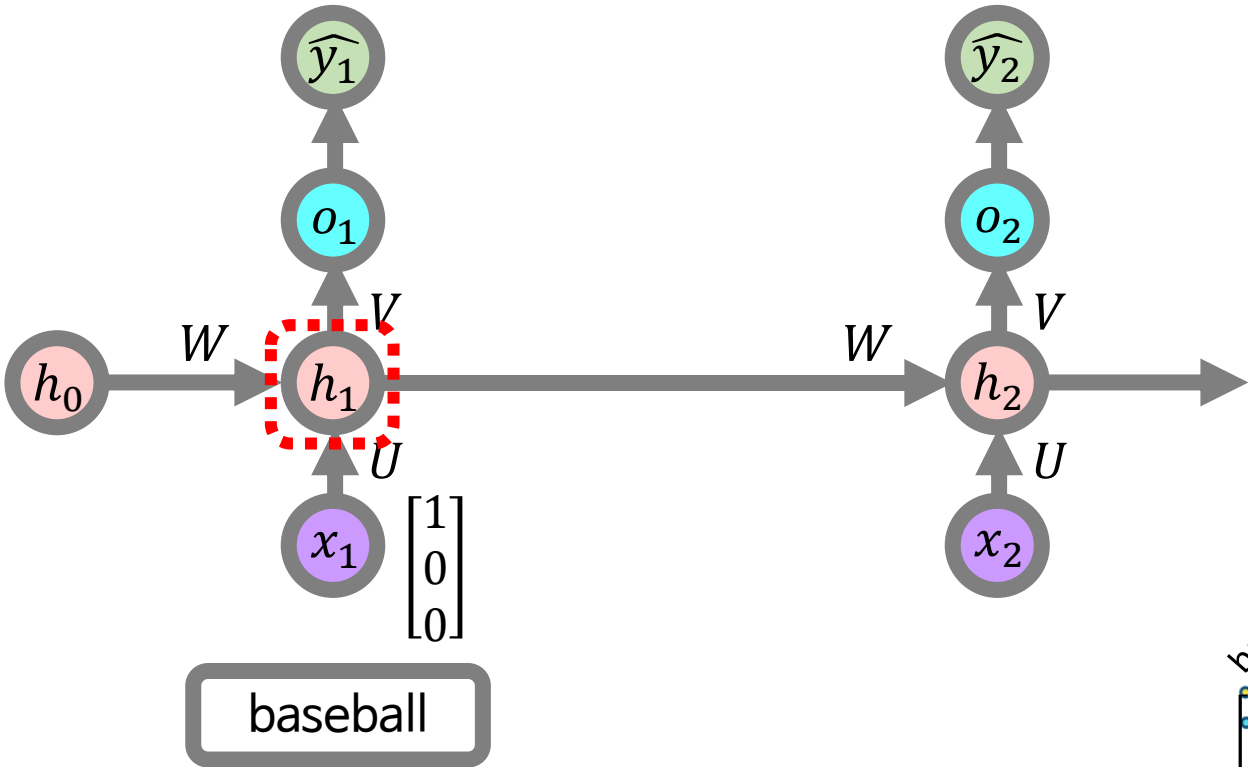


baseball
bat
 $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ $\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

자 그러면 숫자를 넣어서 계산해보도록 하겠습니다

야구
배트
 $\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$
 $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

$$h_1 = \tanh(W h_0 + U x_1)$$



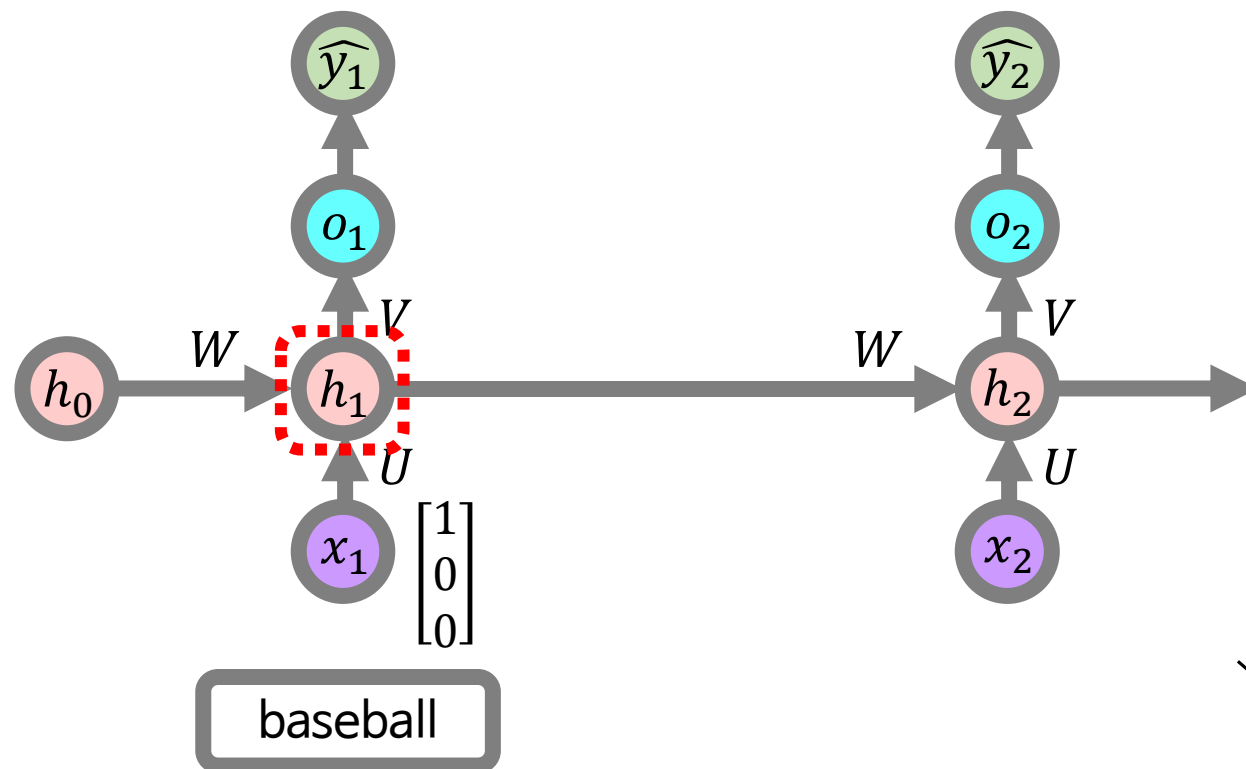
초기값으로 필요한 변수들을 다음과 같이 설정하도록 하겠습니다

$$W = \begin{bmatrix} -0.011 & 0.13 \\ -0.123 & 0.014 \end{bmatrix} \quad V = \begin{bmatrix} -0.141 & 0.038 \\ 0.056 & -0.105 \\ -0.132 & 0.14 \end{bmatrix}$$

$$U = \begin{bmatrix} 0.094 & -0.02 & 0.135 \\ 0.135 & -0.069 & -0.009 \end{bmatrix}$$

$$h_0 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$h_1 = \tanh(W h_0 + U x_1)$$



야구
배트

$$\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

baseball
bat

$$\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

자 그러면 숫자를 넣어서 계산해보도록 하겠습니다

$$W = \begin{bmatrix} -0.011 & 0.13 \\ -0.123 & 0.014 \end{bmatrix} \quad V = \begin{bmatrix} -0.141 & 0.038 \\ 0.056 & -0.105 \\ -0.132 & 0.14 \end{bmatrix}$$

$$U = \begin{bmatrix} 0.094 & -0.02 & 0.135 \\ 0.135 & -0.069 & -0.009 \end{bmatrix}$$

$$h_0 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

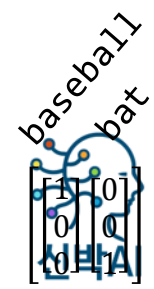
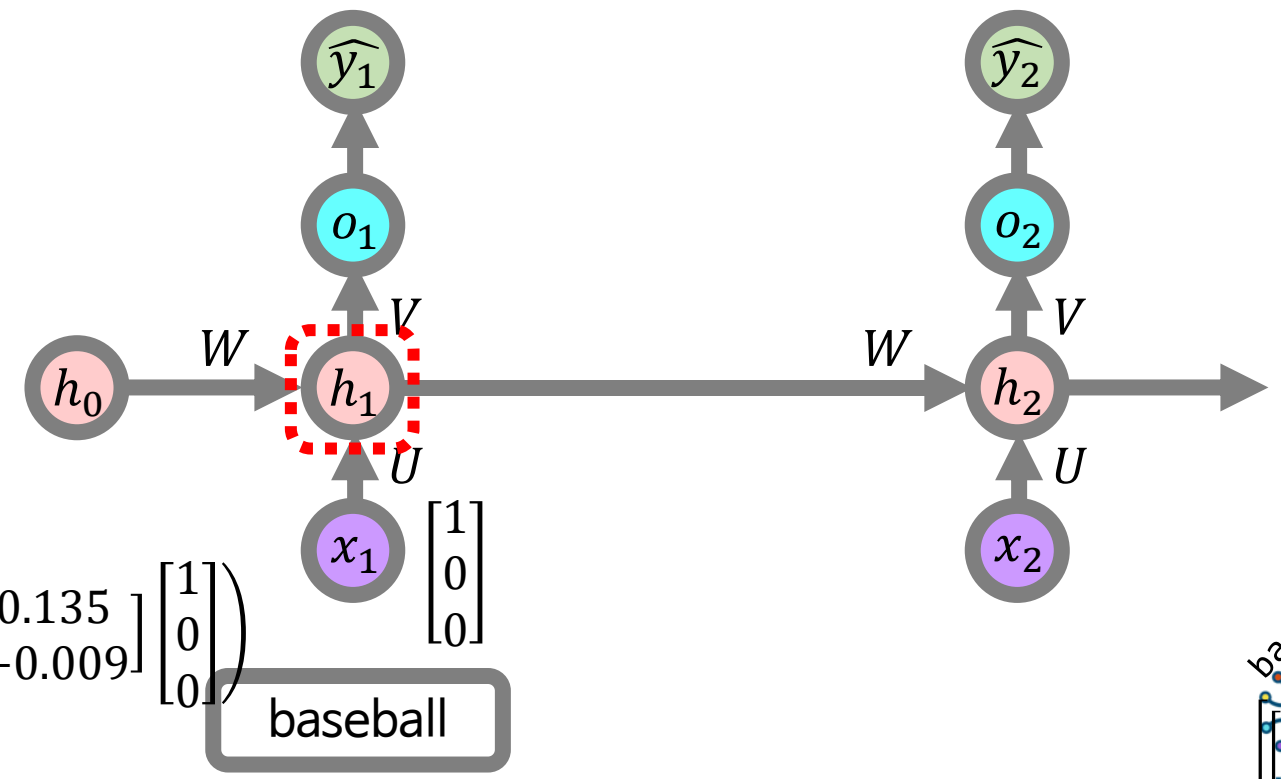
야구 배트

$$\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$h_1 = \tanh(W h_0 + U x_1)$$

$$= \tanh \left(\begin{bmatrix} -0.011 & 0.13 \\ -0.123 & 0.014 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0.094 & -0.02 & 0.135 \\ 0.135 & -0.069 & -0.009 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \right)$$

baseball



자 그러면 숫자를 넣어서 계산해보도록 하겠습니다

$$W = \begin{bmatrix} -0.011 & 0.13 \\ -0.123 & 0.014 \end{bmatrix} \quad V = \begin{bmatrix} -0.141 & 0.038 \\ 0.056 & -0.105 \\ -0.132 & 0.14 \end{bmatrix}$$

$$U = \begin{bmatrix} 0.094 & -0.02 & 0.135 \\ 0.135 & -0.069 & -0.009 \end{bmatrix}$$

$$h_0 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

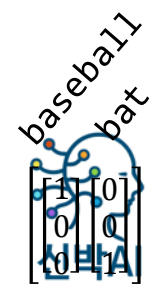
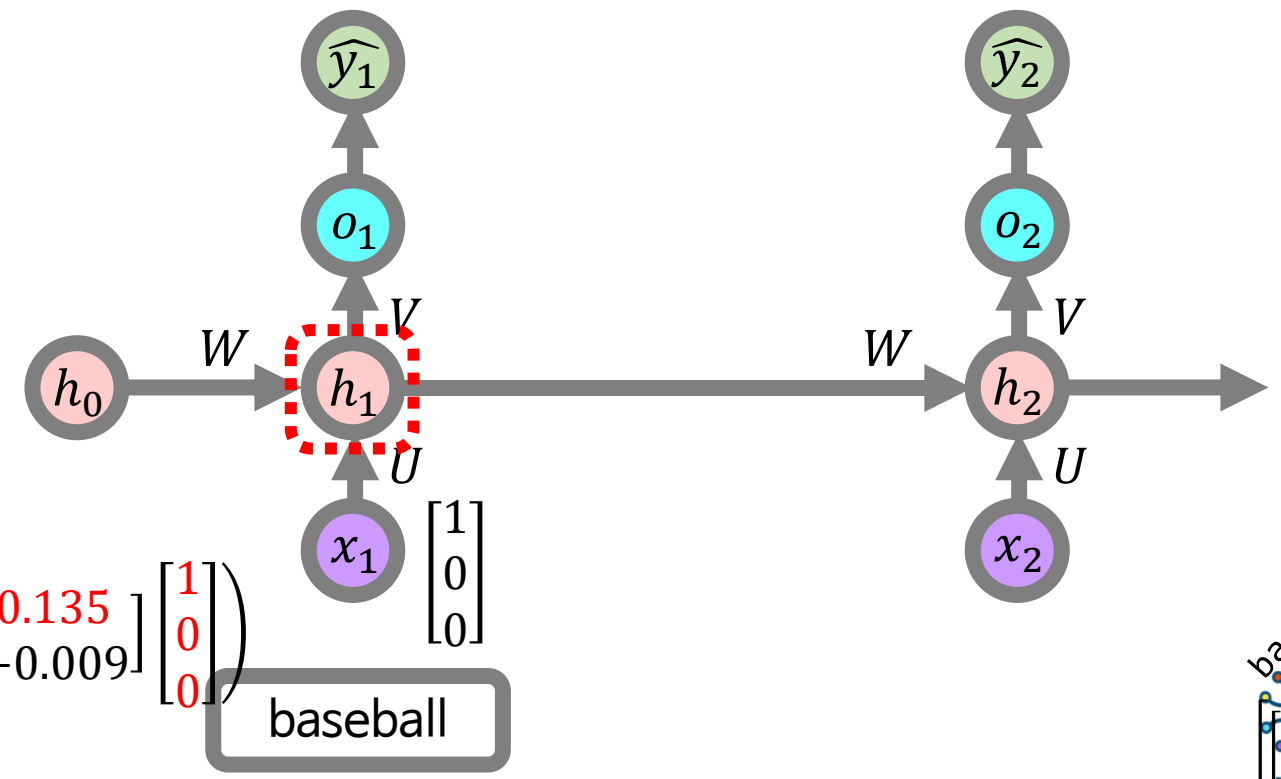
야구 배트

$$\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$h_1 = \tanh(W h_0 + U x_1)$$

$$= \tanh \left(\begin{bmatrix} -0.011 & 0.13 \\ -0.123 & 0.014 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0.094 & -0.02 & 0.135 \\ 0.135 & -0.069 & -0.009 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \right)$$

baseball



자 그러면 숫자를 넣어서 계산해보도록 하겠습니다

$$W = \begin{bmatrix} -0.011 & 0.13 \\ -0.123 & 0.014 \end{bmatrix} \quad V = \begin{bmatrix} -0.141 & 0.038 \\ 0.056 & -0.105 \\ -0.132 & 0.14 \end{bmatrix}$$

$$U = \begin{bmatrix} 0.094 & -0.02 & 0.135 \\ 0.135 & -0.069 & -0.009 \end{bmatrix}$$

$$h_0 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

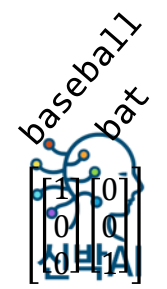
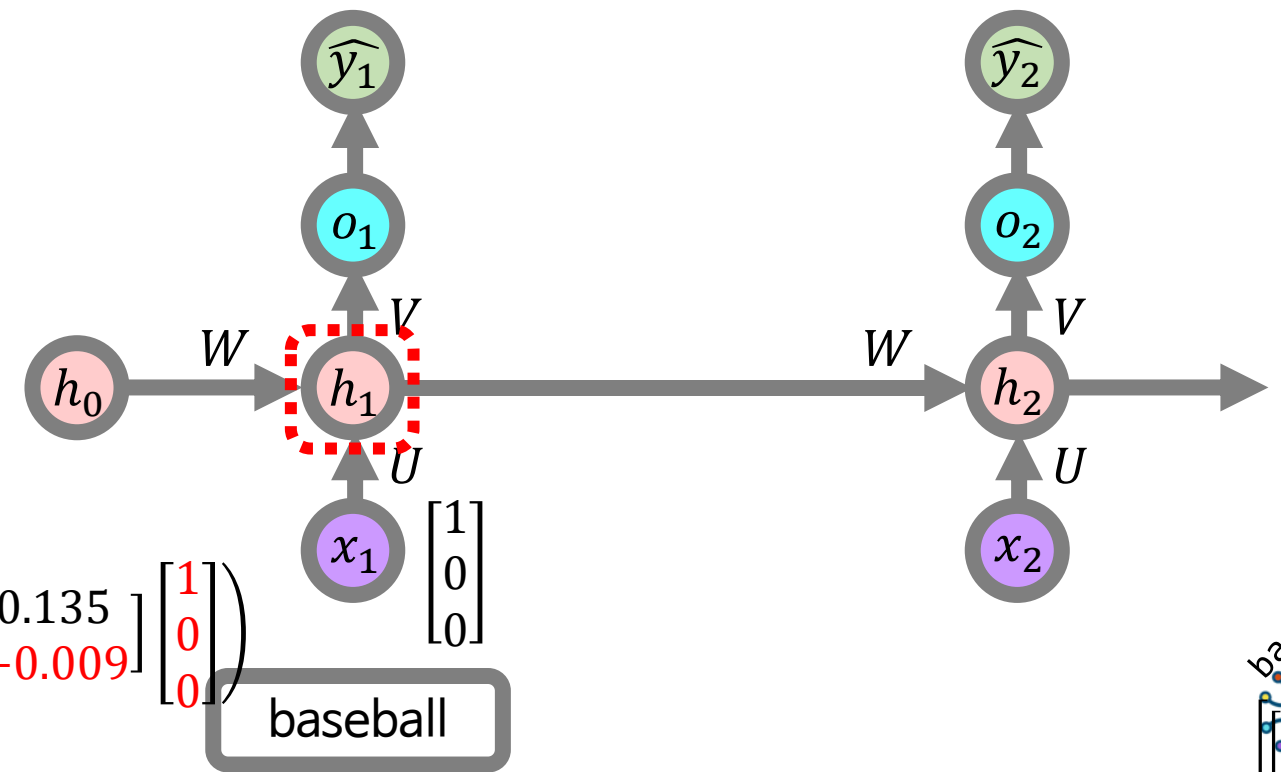
야구 배트

$$\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$h_1 = \tanh(W h_0 + U x_1)$$

$$= \tanh \left(\begin{bmatrix} -0.011 & 0.13 \\ -0.123 & 0.014 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0.094 & -0.02 & 0.135 \\ 0.135 & -0.069 & -0.009 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \right)$$

baseball



자 그러면 숫자를 넣어서 계산해보도록 하겠습니다

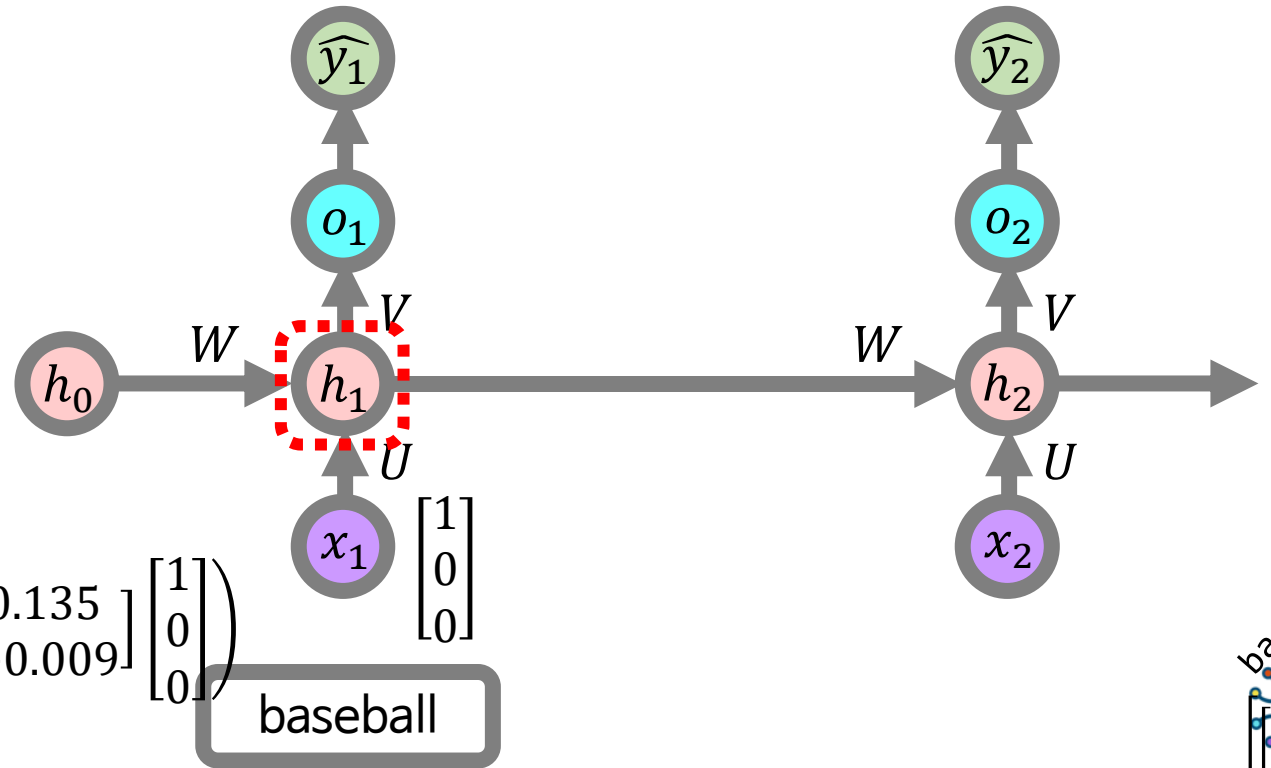
$$W = \begin{bmatrix} -0.011 & 0.13 \\ -0.123 & 0.014 \end{bmatrix} \quad V = \begin{bmatrix} -0.141 & 0.038 \\ 0.056 & -0.105 \\ -0.132 & 0.14 \end{bmatrix}$$

$$U = \begin{bmatrix} 0.094 & -0.02 & 0.135 \\ 0.135 & -0.069 & -0.009 \end{bmatrix}$$

$$h_0 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{matrix} \text{야구} & \text{베이스} \\ \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} & \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \end{matrix}$$

$$\begin{aligned} h_1 &= \tanh(W h_0 + U x_1) \\ &= \tanh\left(\begin{bmatrix} -0.011 & 0.13 \\ -0.123 & 0.014 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0.094 & -0.02 & 0.135 \\ 0.135 & -0.069 & -0.009 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}\right) \\ &= \tanh\left(\begin{bmatrix} 0.094 \\ 0.135 \end{bmatrix}\right) \end{aligned}$$



자 그러면 숫자를 넣어서 계산해보도록 하겠습니다

$$W = \begin{bmatrix} -0.011 & 0.13 \\ -0.123 & 0.014 \end{bmatrix} \quad V = \begin{bmatrix} -0.141 & 0.038 \\ 0.056 & -0.105 \\ -0.132 & 0.14 \end{bmatrix}$$

$$U = \begin{bmatrix} 0.094 & -0.02 & 0.135 \\ 0.135 & -0.069 & -0.009 \end{bmatrix}$$

$$h_0 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

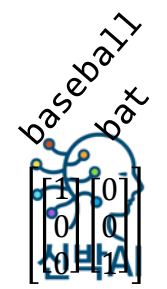
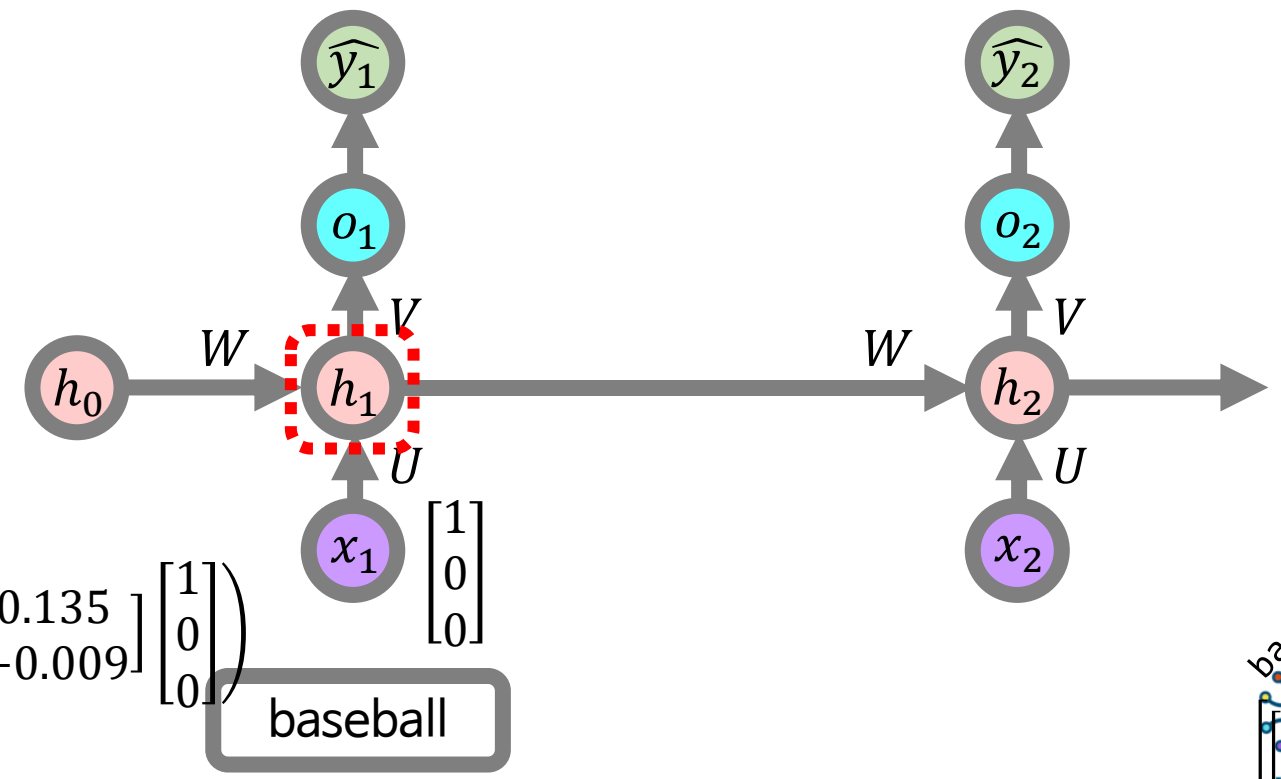
야구 배트

$$\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$h_1 = \tanh(W h_0 + U x_1)$$

$$= \tanh \left(\begin{bmatrix} -0.011 & 0.13 \\ -0.123 & 0.014 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0.094 & -0.02 & 0.135 \\ 0.135 & -0.069 & -0.009 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \right)$$

$$= \tanh \left(\begin{bmatrix} 0.094 \\ 0.135 \end{bmatrix} \right) = \begin{bmatrix} 0.094 \\ 0.134 \end{bmatrix}$$



그러면, 새로운 내부상태 h_1 은 다음과 같이 계산이 되었습니다

$$W = \begin{bmatrix} -0.011 & 0.13 \\ -0.123 & 0.014 \end{bmatrix} \quad V = \begin{bmatrix} -0.141 & 0.038 \\ 0.056 & -0.105 \\ -0.132 & 0.14 \end{bmatrix}$$

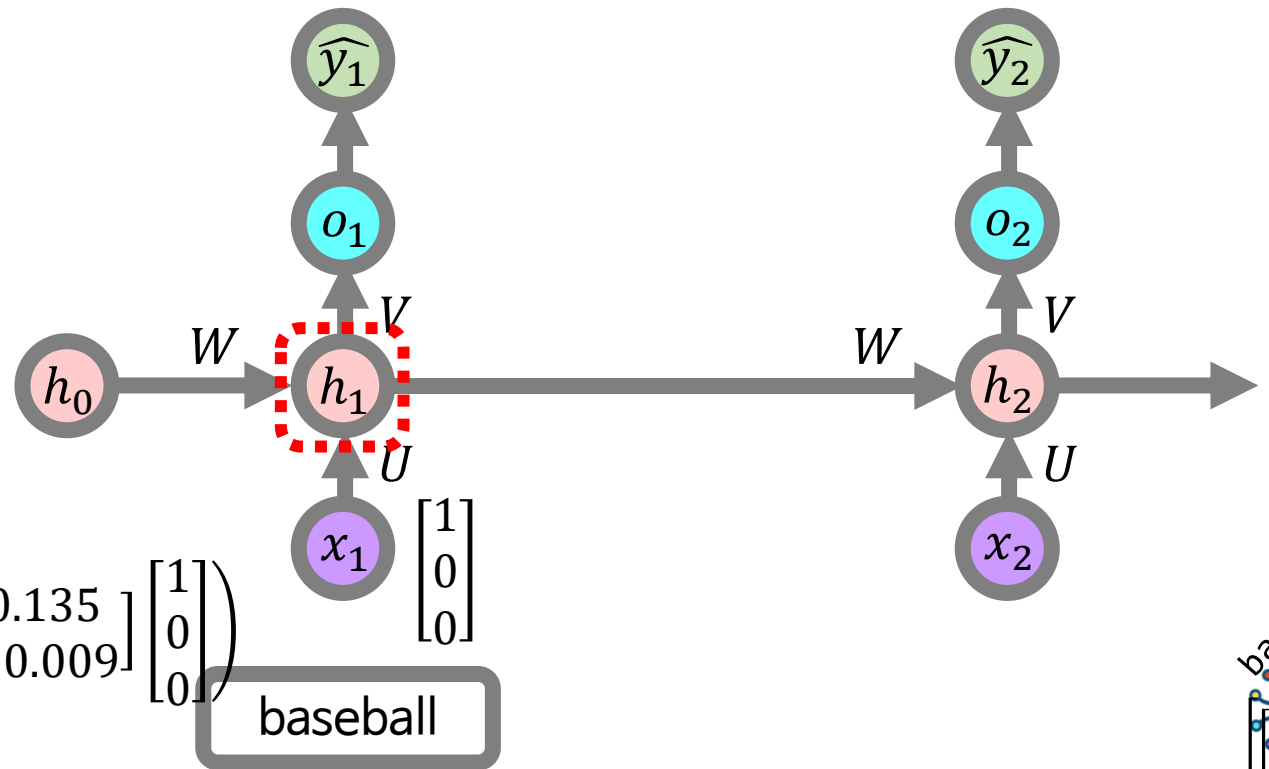
$$U = \begin{bmatrix} 0.094 & -0.02 & 0.135 \\ 0.135 & -0.069 & -0.009 \end{bmatrix}$$

$$h_0 = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad h_1 = \begin{bmatrix} 0.094 \\ 0.134 \end{bmatrix}$$

야구 배트

$$\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{aligned} h_1 &= \tanh(W h_0 + U x_1) \\ &= \tanh \left(\begin{bmatrix} -0.011 & 0.13 \\ -0.123 & 0.014 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0.094 & -0.02 & 0.135 \\ 0.135 & -0.069 & -0.009 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \right) \\ &= \tanh \left(\begin{bmatrix} 0.094 \\ 0.135 \end{bmatrix} \right) = \begin{bmatrix} 0.094 \\ 0.134 \end{bmatrix} \end{aligned}$$



그다음, o_1 은 다음과 같습니다

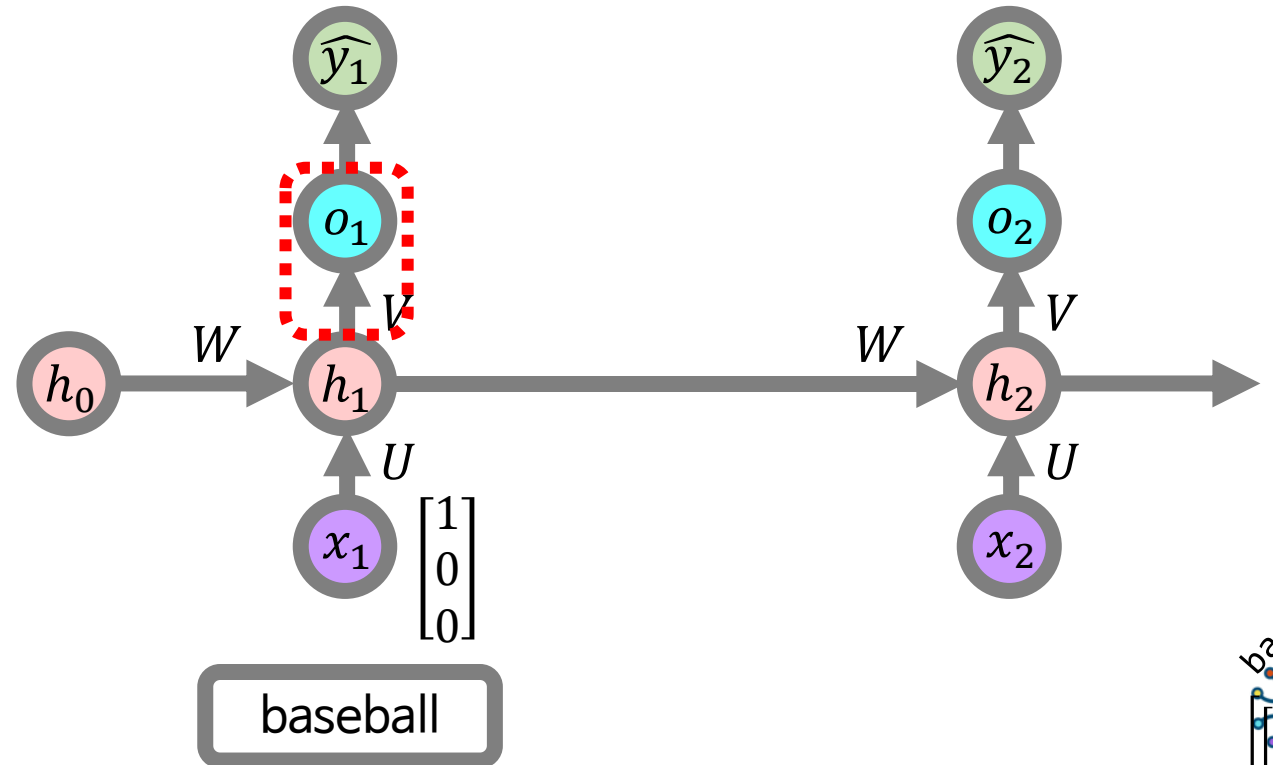
$$W = \begin{bmatrix} -0.011 & 0.13 \\ -0.123 & 0.014 \end{bmatrix} \quad V = \begin{bmatrix} -0.141 & 0.038 \\ 0.056 & -0.105 \\ -0.132 & 0.14 \end{bmatrix}$$

$$U = \begin{bmatrix} 0.094 & -0.02 & 0.135 \\ 0.135 & -0.069 & -0.009 \end{bmatrix}$$

$$h_0 = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad h_1 = \begin{bmatrix} 0.094 \\ 0.134 \end{bmatrix}$$

$$o_1 = Vh_1$$

$$h_1 = \tanh(W h_0 + U x_1)$$



야구
배트
 $\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$



그다음, o_1 은 다음과 같습니다

$$W = \begin{bmatrix} -0.011 & 0.13 \\ -0.123 & 0.014 \end{bmatrix} \quad V = \begin{bmatrix} -0.141 & 0.038 \\ 0.056 & -0.105 \\ -0.132 & 0.14 \end{bmatrix}$$

$$U = \begin{bmatrix} 0.094 & -0.02 & 0.135 \\ 0.135 & -0.069 & -0.009 \end{bmatrix}$$

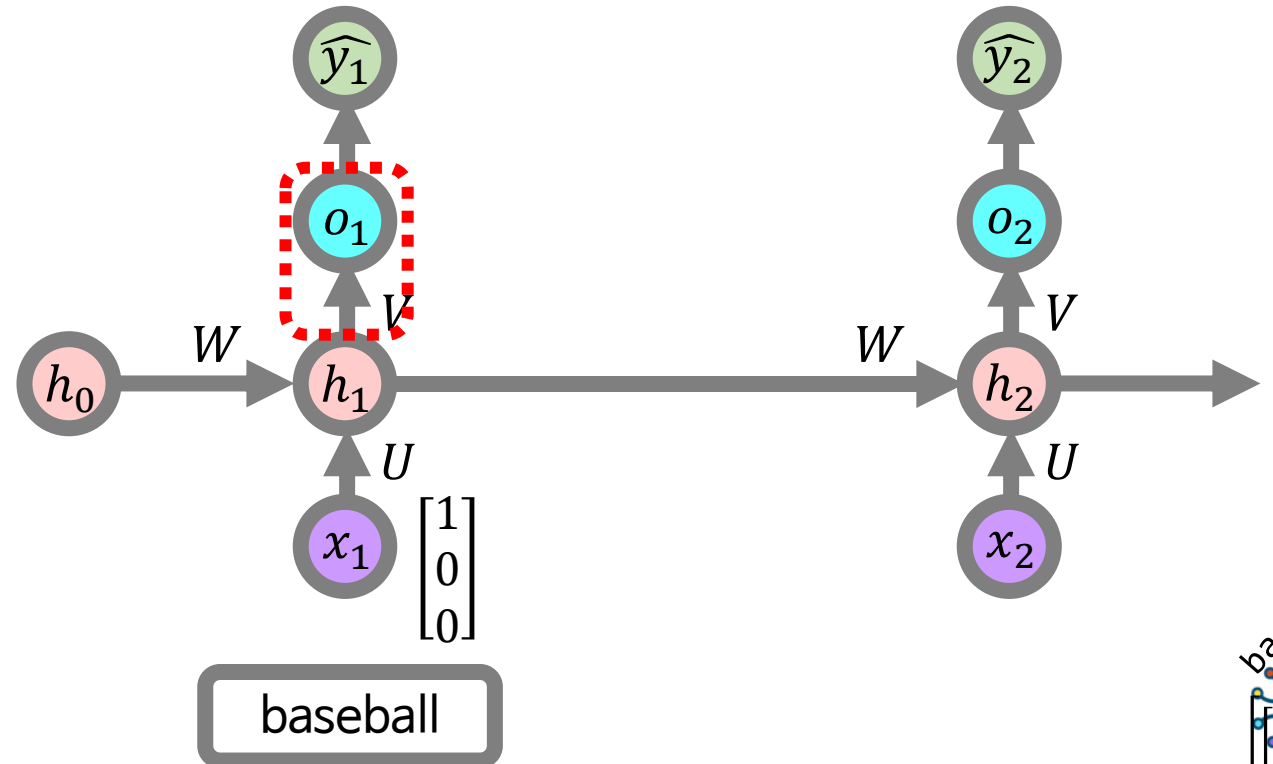
$$h_0 = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad h_1 = \begin{bmatrix} 0.094 \\ 0.134 \end{bmatrix}$$

$$o_1 = Vh_1$$

$$h_1 = \tanh(W h_0 + U x_1)$$

$$o_1 = \begin{bmatrix} -0.141 & 0.038 \\ 0.056 & -0.105 \\ -0.132 & 0.14 \end{bmatrix} \begin{bmatrix} 0.094 \\ 0.134 \end{bmatrix}$$

야구
배트
 $\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$



baseball
bat
 $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

그다음, o_1 은 다음과 같습니다

$$W = \begin{bmatrix} -0.011 & 0.13 \\ -0.123 & 0.014 \end{bmatrix} \quad V = \begin{bmatrix} -0.141 & 0.038 \\ 0.056 & -0.105 \\ -0.132 & 0.14 \end{bmatrix}$$

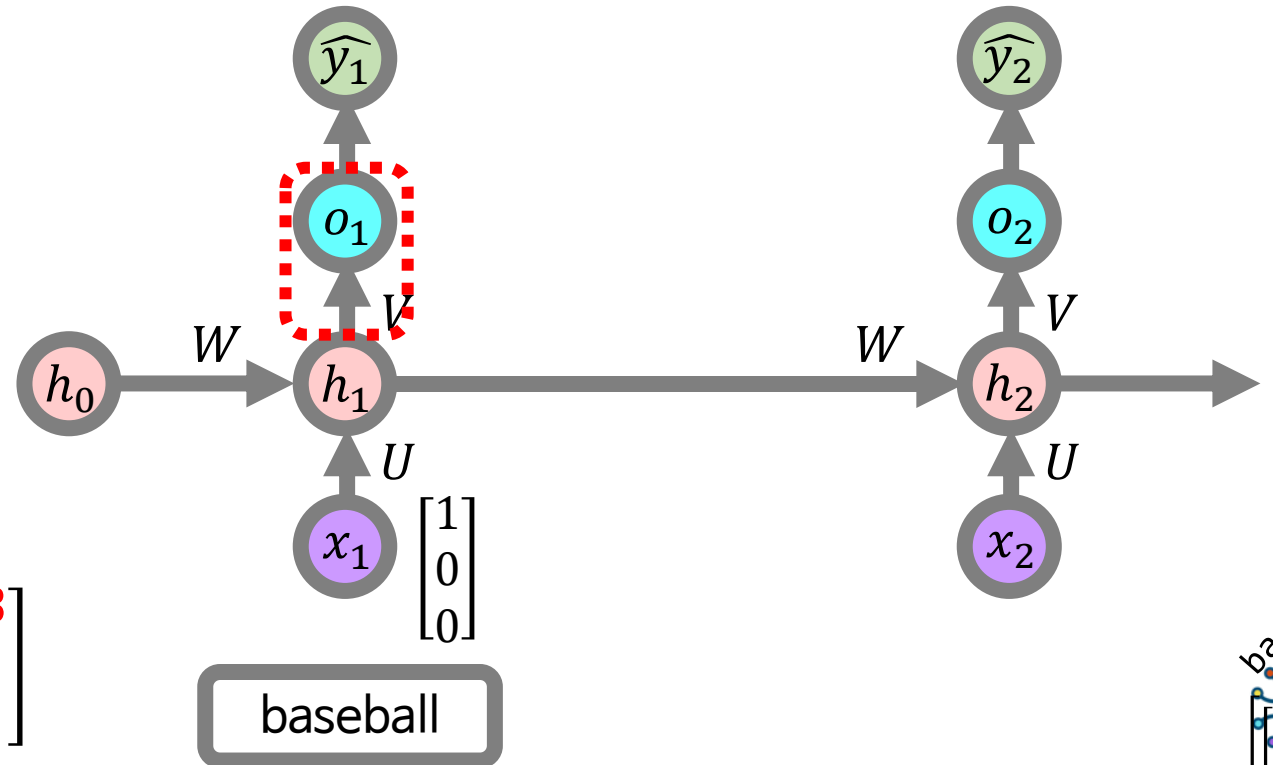
$$U = \begin{bmatrix} 0.094 & -0.02 & 0.135 \\ 0.135 & -0.069 & -0.009 \end{bmatrix}$$

$$h_0 = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad h_1 = \begin{bmatrix} 0.094 \\ 0.134 \end{bmatrix}$$

$$o_1 = Vh_1$$

$$h_1 = \tanh(W h_0 + U x_1)$$

$$o_1 = \begin{bmatrix} -0.141 & 0.038 \\ 0.056 & -0.105 \\ -0.132 & 0.14 \end{bmatrix} \begin{bmatrix} 0.094 \\ 0.134 \end{bmatrix} = \begin{bmatrix} -0.008 \end{bmatrix}$$



야구
배트
 $\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$



그다음, o_1 은 다음과 같습니다

$$W = \begin{bmatrix} -0.011 & 0.13 \\ -0.123 & 0.014 \end{bmatrix} \quad V = \begin{bmatrix} -0.141 & 0.038 \\ 0.056 & -0.105 \\ -0.132 & 0.14 \end{bmatrix}$$

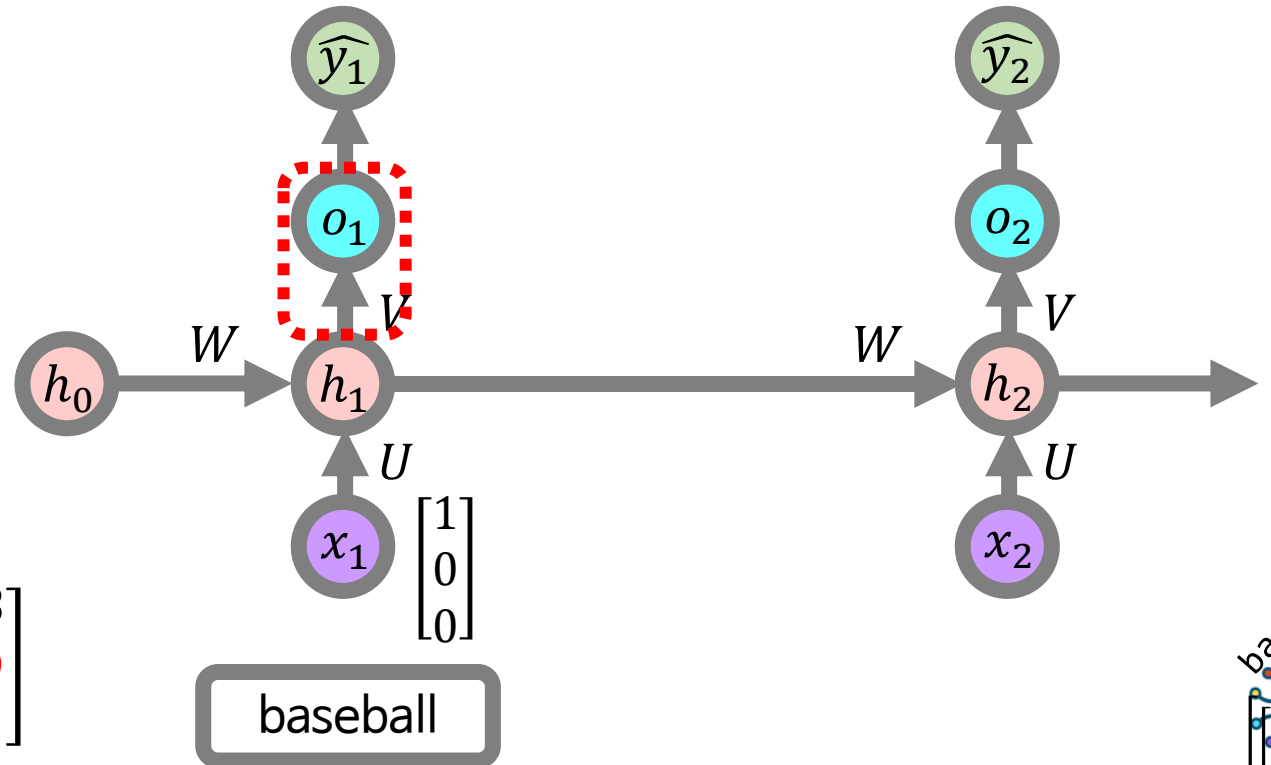
$$U = \begin{bmatrix} 0.094 & -0.02 & 0.135 \\ 0.135 & -0.069 & -0.009 \end{bmatrix}$$

$$h_0 = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad h_1 = \begin{bmatrix} 0.094 \\ 0.134 \end{bmatrix}$$

$$o_1 = Vh_1$$

$$h_1 = \tanh(W h_0 + U x_1)$$

$$o_1 = \begin{bmatrix} -0.141 & 0.038 \\ 0.056 & -0.105 \\ -0.132 & 0.14 \end{bmatrix} \begin{bmatrix} 0.094 \\ 0.134 \end{bmatrix} = \begin{bmatrix} -0.008 \\ -0.009 \end{bmatrix}$$



야구
배트

$$\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

baseball
bat

그다음, o_1 은 다음과 같습니다

$$W = \begin{bmatrix} -0.011 & 0.13 \\ -0.123 & 0.014 \end{bmatrix} \quad V = \begin{bmatrix} -0.141 & 0.038 \\ 0.056 & -0.105 \\ -0.132 & 0.14 \end{bmatrix}$$

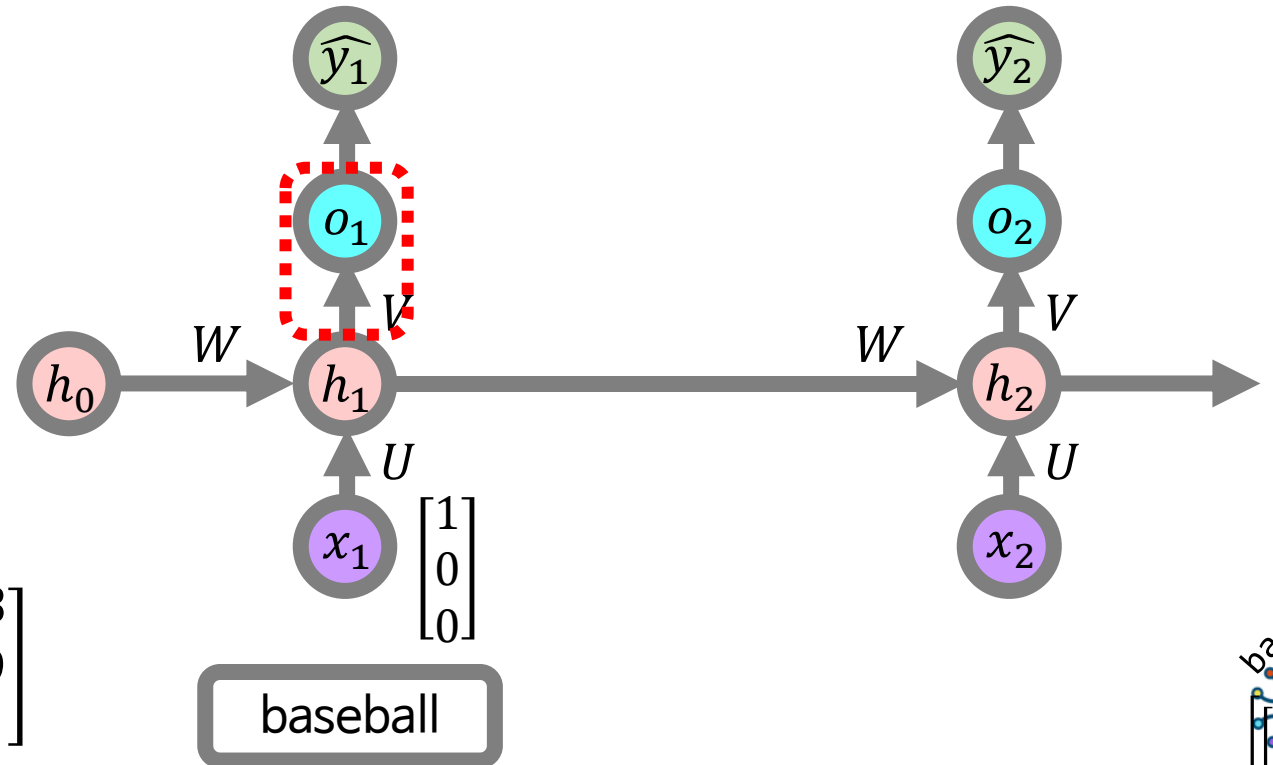
$$U = \begin{bmatrix} 0.094 & -0.02 & 0.135 \\ 0.135 & -0.069 & -0.009 \end{bmatrix}$$

$$h_0 = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad h_1 = \begin{bmatrix} 0.094 \\ 0.134 \end{bmatrix}$$

$$o_1 = Vh_1$$

$$h_1 = \tanh(W h_0 + U x_1)$$

$$o_1 = \begin{bmatrix} -0.141 & 0.038 \\ 0.056 & -0.105 \\ -0.132 & 0.14 \end{bmatrix} \begin{bmatrix} 0.094 \\ 0.134 \end{bmatrix} = \begin{bmatrix} -0.008 \\ -0.009 \\ 0.006 \end{bmatrix}$$



야구
배트
 $\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$



o_1 을 계산했습니다

$$W = \begin{bmatrix} -0.011 & 0.13 \\ -0.123 & 0.014 \end{bmatrix} \quad V = \begin{bmatrix} -0.141 & 0.038 \\ 0.056 & -0.105 \\ -0.132 & 0.14 \end{bmatrix}$$

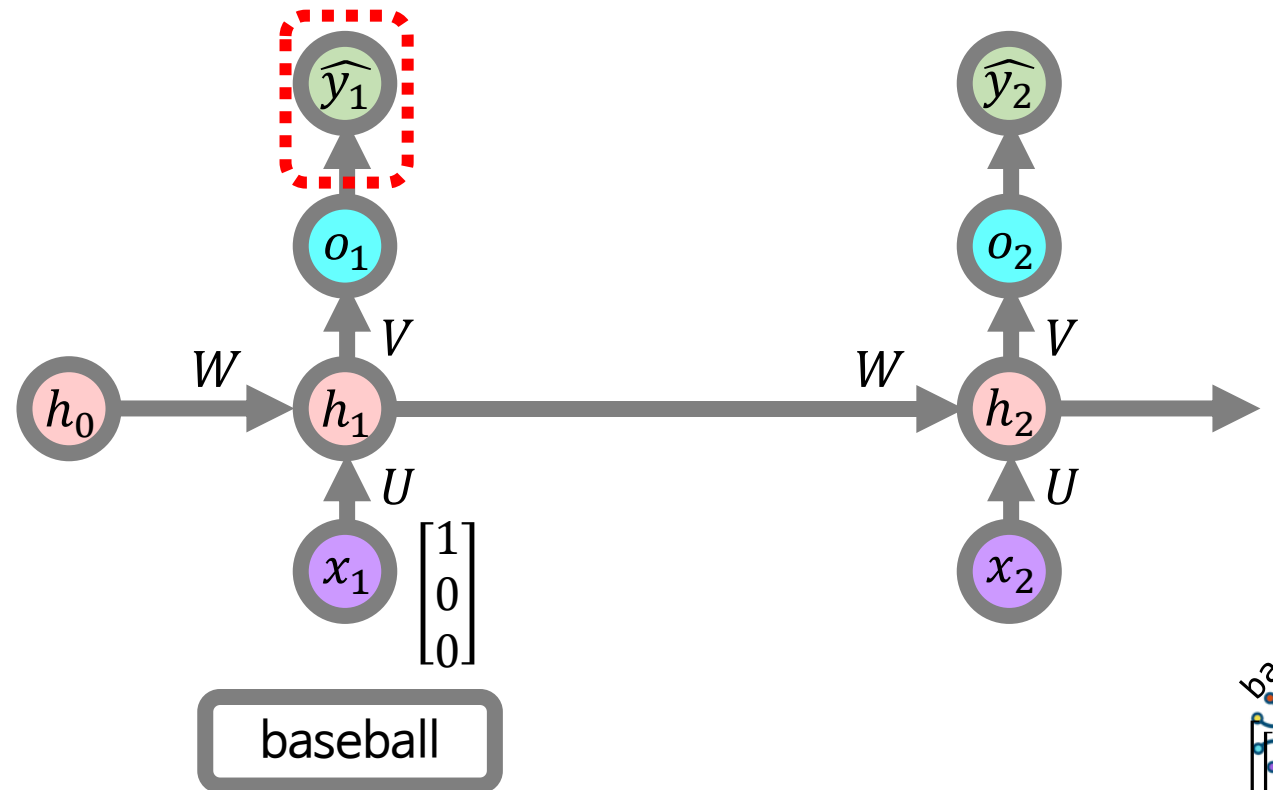
$$U = \begin{bmatrix} 0.094 & -0.02 & 0.135 \\ 0.135 & -0.069 & -0.009 \end{bmatrix}$$

$$h_0 = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad h_1 = \begin{bmatrix} 0.094 \\ 0.134 \end{bmatrix}$$

$$o_1 = \begin{bmatrix} -0.008 \\ -0.009 \\ 0.006 \end{bmatrix}$$

$$o_1 = Vh_1$$

$$h_1 = \tanh(W h_0 + U x_1)$$



그리고 마지막 \hat{y}_1 은 o_1 에 softmax 함수를 적용하면 됩니다

$$W = \begin{bmatrix} -0.011 & 0.13 \\ -0.123 & 0.014 \end{bmatrix} \quad V = \begin{bmatrix} -0.141 & 0.038 \\ 0.056 & -0.105 \\ -0.132 & 0.14 \end{bmatrix}$$

$$U = \begin{bmatrix} 0.094 & -0.02 & 0.135 \\ 0.135 & -0.069 & -0.009 \end{bmatrix}$$

$$h_0 = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad h_1 = \begin{bmatrix} 0.094 \\ 0.134 \end{bmatrix}$$

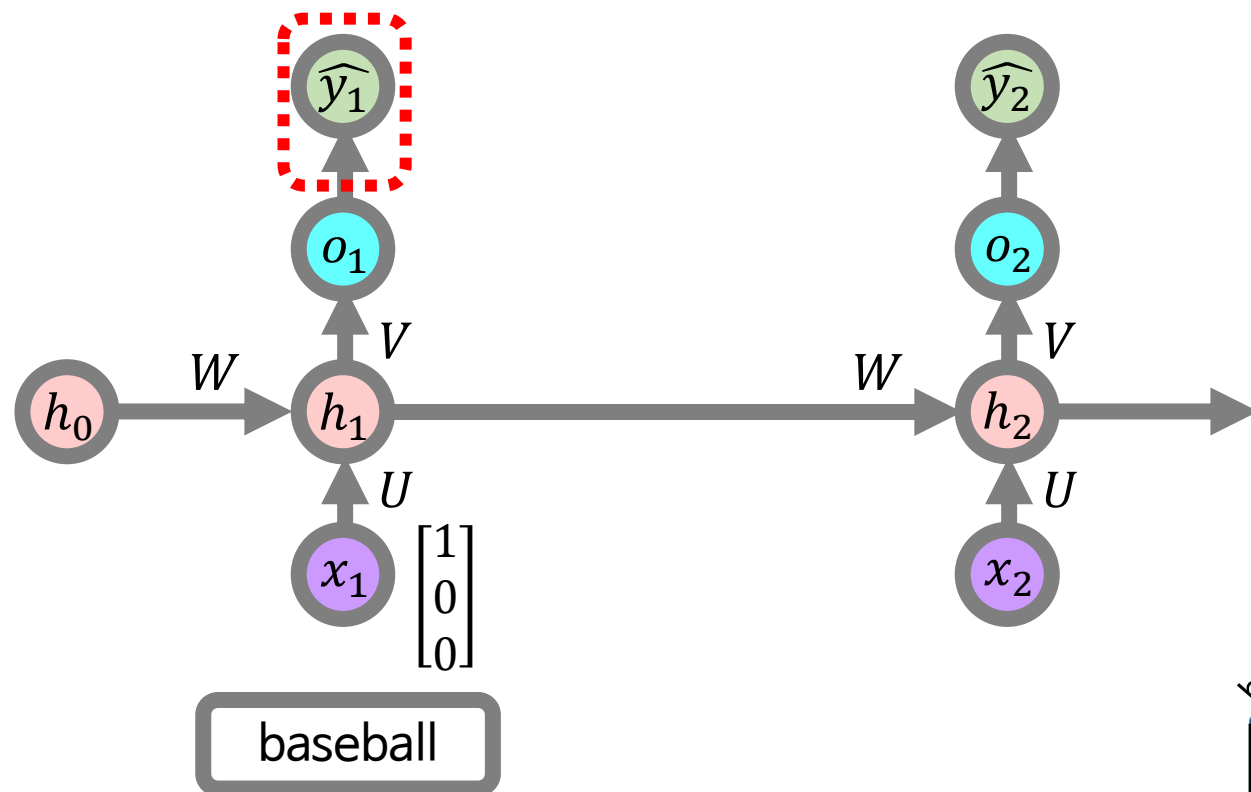
$$o_1 = \begin{bmatrix} -0.008 \\ -0.009 \\ 0.006 \end{bmatrix}$$

$$\hat{y}_1 = \text{softmax}(o_1)$$

$$o_1 = Vh_1$$

$$h_1 = \tanh(W h_0 + U x_1)$$

야구
배트
 $\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$ $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$



baseball
bat
 $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ $\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

Softmax함수는 간단하게 말씀드리면, 신경망의 raw output을 확률로 바꾸어주는 편리한 함수입니다

$$\begin{array}{ccc} o_1 & & \hat{y}_1 \\ \begin{bmatrix} -0.008 \\ -0.009 \\ 0.006 \end{bmatrix} & \xrightarrow{\quad} \frac{e^{z_i}}{\sum_{j=1}^K e^{z_j}} \xrightarrow{\quad} & \begin{bmatrix} 0.332 \\ 0.332 \\ 0.337 \end{bmatrix} \end{array}$$

o_1 의 경우라면, 다음과 같이 계산합니다

$$\begin{matrix} o_1 \\ \begin{bmatrix} -0.008 \\ -0.009 \\ 0.006 \end{bmatrix} \end{matrix} \rightarrow \frac{e^{z_i}}{\sum_{j=1}^K e^{z_j}} \rightarrow \begin{matrix} \hat{y}_1 \\ \begin{bmatrix} 0.332 \\ 0.332 \\ 0.337 \end{bmatrix} \end{matrix}$$

$$0.332 = \frac{e^{-0.008}}{e^{-0.008} + e^{-0.009} + e^{0.006}}$$

$$0.332 = \frac{e^{-0.009}}{e^{-0.008} + e^{-0.009} + e^{0.006}}$$

$$0.337 = \frac{e^{0.006}}{e^{-0.008} + e^{-0.009} + e^{0.006}}$$

그래서 이렇게 \hat{y}_1 까지 계산해보았습니다

$$W = \begin{bmatrix} -0.011 & 0.13 \\ -0.123 & 0.014 \end{bmatrix} \quad V = \begin{bmatrix} -0.141 & 0.038 \\ 0.056 & -0.105 \\ -0.132 & 0.14 \end{bmatrix}$$

$$U = \begin{bmatrix} 0.094 & -0.02 & 0.135 \\ 0.135 & -0.069 & -0.009 \end{bmatrix}$$

$$h_0 = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad h_1 = \begin{bmatrix} 0.094 \\ 0.134 \end{bmatrix}$$

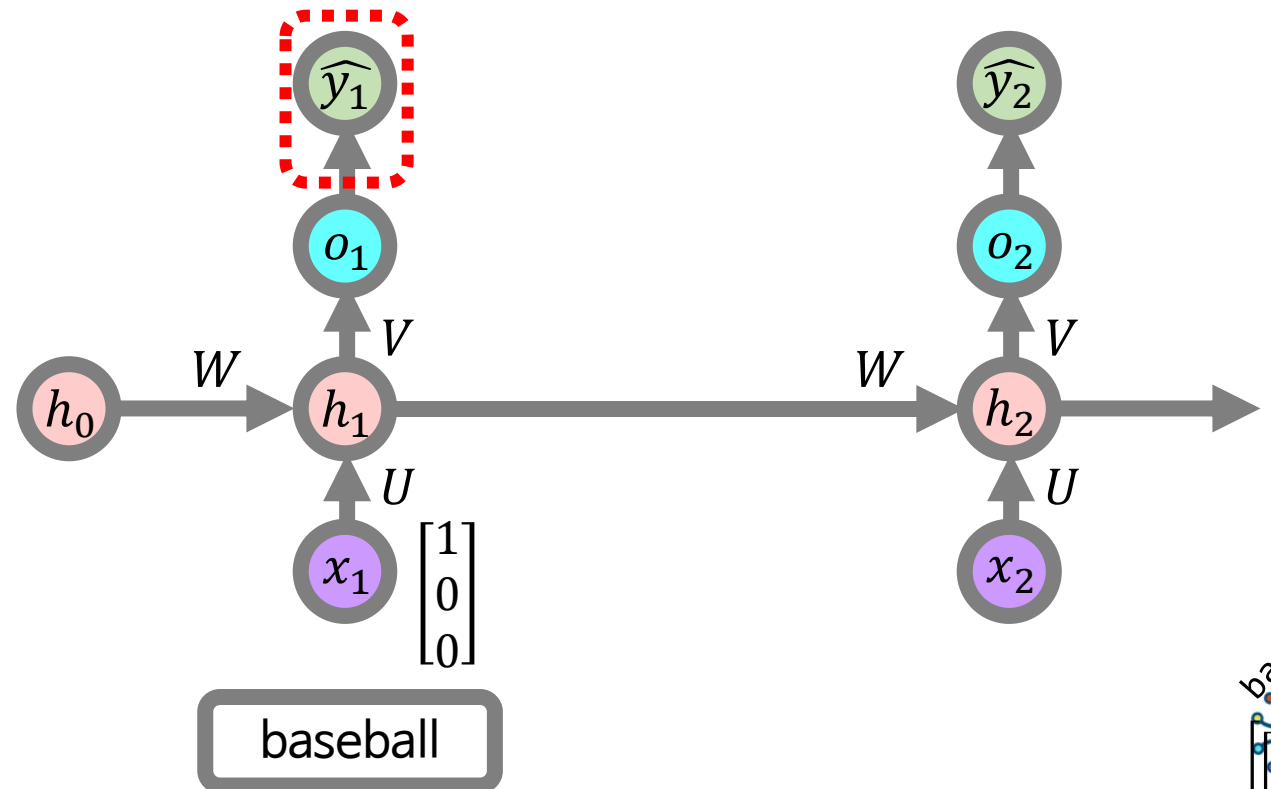
$$o_1 = \begin{bmatrix} -0.008 \\ -0.009 \\ 0.006 \end{bmatrix} \quad \hat{y}_1 = \begin{bmatrix} 0.332 \\ 0.332 \\ 0.337 \end{bmatrix}$$

$$\hat{y}_1 = \text{softmax}(o_1)$$

$$o_1 = Vh_1$$

$$h_1 = \tanh(W h_0 + U x_1)$$

야구
배트
 $\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$ $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$



그러면 \hat{y}_2 도 똑같은 방법으로 계산해볼 수 있습니다

$$W = \begin{bmatrix} -0.011 & 0.13 \\ -0.123 & 0.014 \end{bmatrix} \quad V = \begin{bmatrix} -0.141 & 0.038 \\ 0.056 & -0.105 \\ -0.132 & 0.14 \end{bmatrix}$$

$$U = \begin{bmatrix} 0.094 & -0.02 & 0.135 \\ 0.135 & -0.069 & -0.009 \end{bmatrix}$$

$$h_0 = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad h_1 = \begin{bmatrix} 0.094 \\ 0.134 \end{bmatrix} \quad h_2 = \begin{bmatrix} 0.15 \\ -0.019 \end{bmatrix}$$

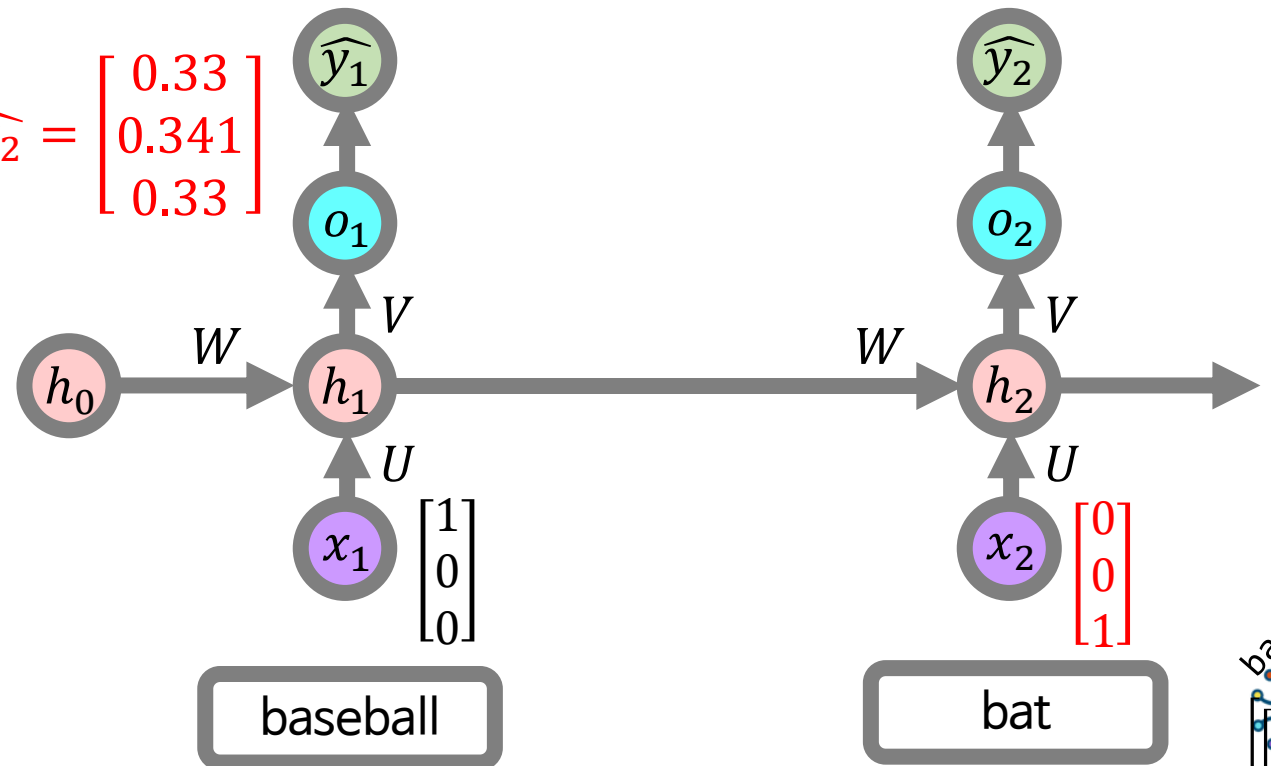
$$o_1 = \begin{bmatrix} -0.008 \\ -0.009 \\ 0.006 \end{bmatrix} \quad \hat{y}_1 = \begin{bmatrix} 0.332 \\ 0.332 \\ 0.337 \end{bmatrix} \quad o_2 = \begin{bmatrix} -0.022 \\ 0.01 \\ -0.022 \end{bmatrix} \quad \hat{y}_2 = \begin{bmatrix} 0.33 \\ 0.341 \\ 0.33 \end{bmatrix}$$

$$\hat{y}_2 = \text{softmax}(o_2)$$

$$o_2 = Vh_2$$

$$h_2 = \tanh(W h_1 + U x_2)$$

야구
배트
 $\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$ $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$



baseball
bat
 $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ $\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

RNN에서 주목해야할 것은 이전에 계산한 h_1 값이 h_2 계산에 쓰인다는 것
입니다

$$W = \begin{bmatrix} -0.011 & 0.13 \\ -0.123 & 0.014 \end{bmatrix} \quad V = \begin{bmatrix} -0.141 & 0.038 \\ 0.056 & -0.105 \\ -0.132 & 0.14 \end{bmatrix}$$

$$U = \begin{bmatrix} 0.094 & -0.02 & 0.135 \\ 0.135 & -0.069 & -0.009 \end{bmatrix}$$

$$h_0 = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad h_1 = \begin{bmatrix} 0.094 \\ 0.134 \end{bmatrix} \quad h_2 = \begin{bmatrix} 0.15 \\ -0.019 \end{bmatrix}$$

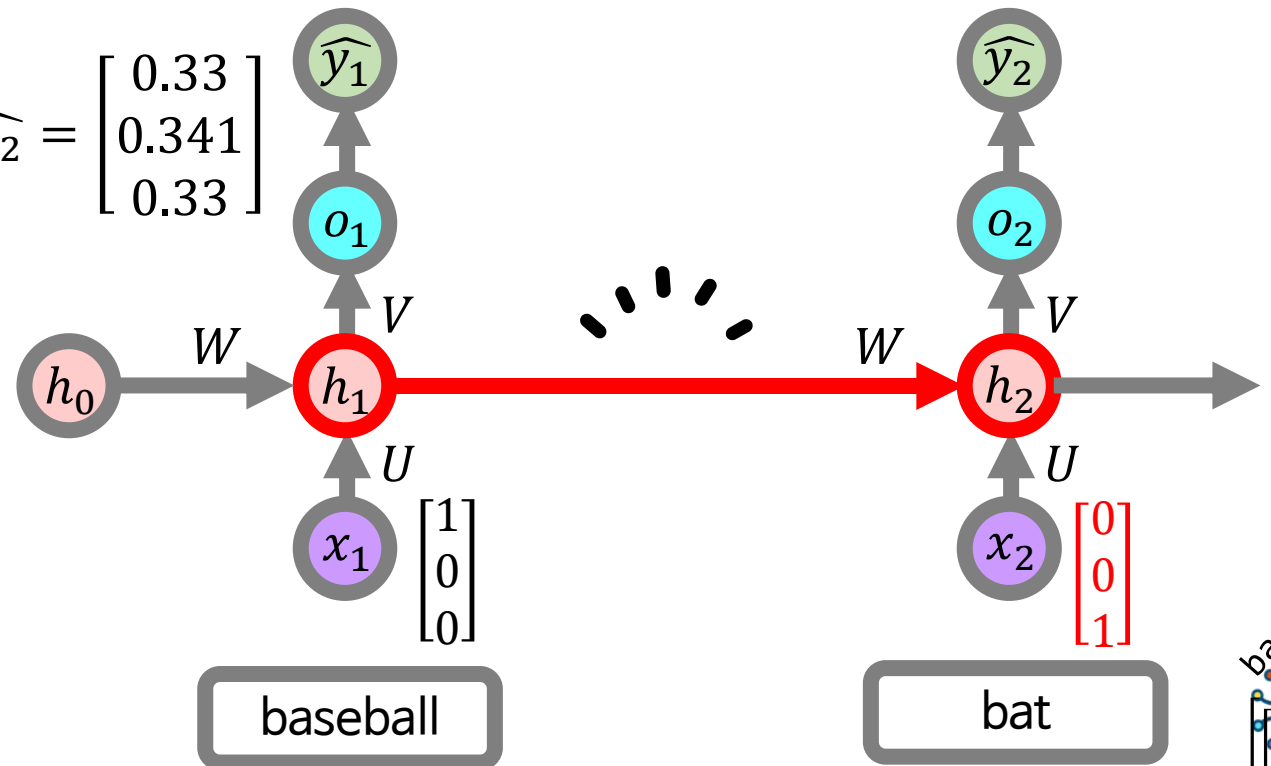
$$o_1 = \begin{bmatrix} -0.008 \\ -0.009 \\ 0.006 \end{bmatrix} \quad \hat{y}_1 = \begin{bmatrix} 0.332 \\ 0.332 \\ 0.337 \end{bmatrix} \quad o_2 = \begin{bmatrix} -0.022 \\ 0.01 \\ -0.022 \end{bmatrix} \quad \hat{y}_2 = \begin{bmatrix} 0.33 \\ 0.341 \\ 0.33 \end{bmatrix}$$

$$\hat{y}_2 = \text{softmax}(o_2)$$

$$o_2 = Vh_2$$

$$h_2 = \tanh(W h_1 + U x_2)$$

$$\begin{matrix} \text{야구} & \text{베이스} \\ \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} & \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \end{matrix}$$



자 이렇게 각 입력 시퀀스에 대한 출력값을 내어보았습니다

$$W = \begin{bmatrix} -0.011 & 0.13 \\ -0.123 & 0.014 \end{bmatrix} \quad V = \begin{bmatrix} -0.141 & 0.038 \\ 0.056 & -0.105 \\ -0.132 & 0.14 \end{bmatrix}$$

$$U = \begin{bmatrix} 0.094 & -0.02 & 0.135 \\ 0.135 & -0.069 & -0.009 \end{bmatrix}$$

$$h_0 = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad h_1 = \begin{bmatrix} 0.094 \\ 0.134 \end{bmatrix} \quad h_2 = \begin{bmatrix} 0.15 \\ -0.019 \end{bmatrix}$$

$$o_1 = \begin{bmatrix} -0.008 \\ -0.009 \\ 0.006 \end{bmatrix} \quad \hat{y}_1 = \begin{bmatrix} 0.332 \\ 0.332 \\ 0.337 \end{bmatrix} \quad o_2 = \begin{bmatrix} -0.022 \\ 0.01 \\ -0.022 \end{bmatrix} \quad \hat{y}_2 = \begin{bmatrix} 0.33 \\ 0.341 \\ 0.33 \end{bmatrix}$$

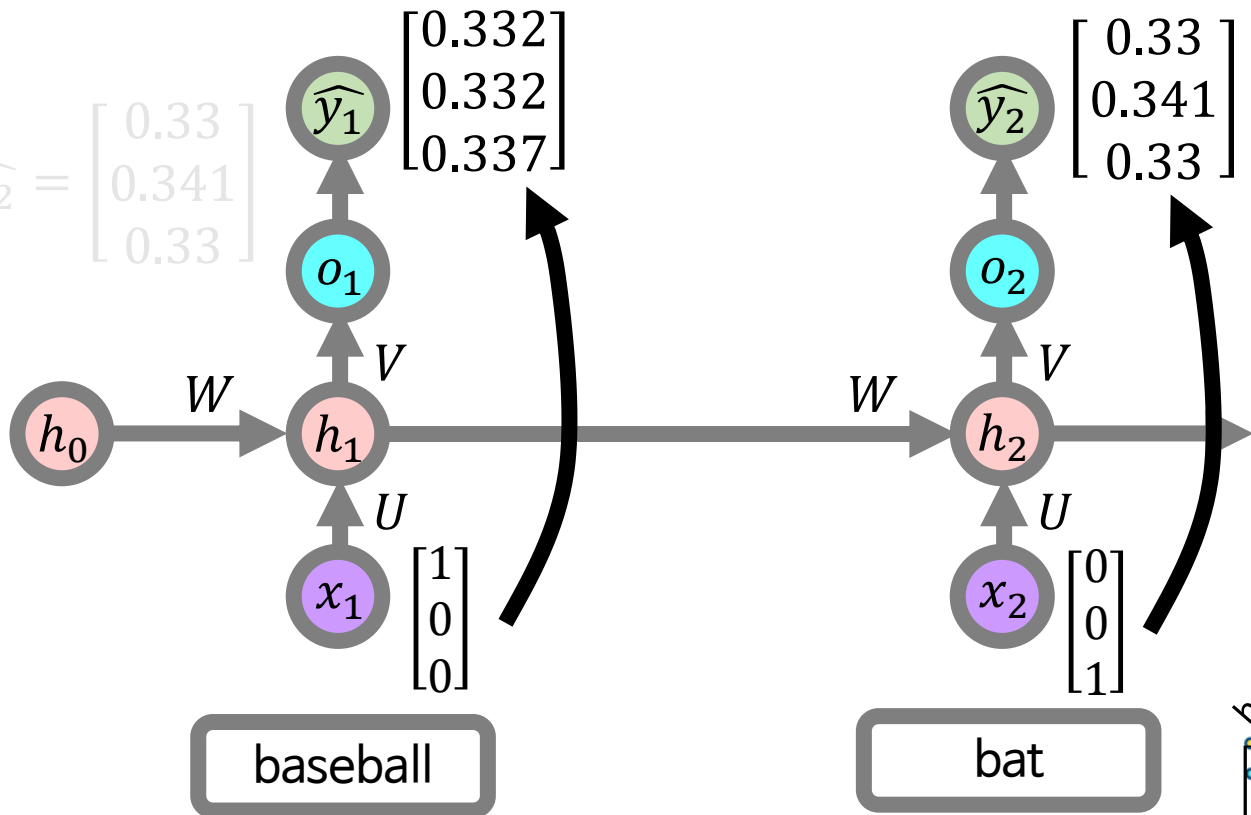
$$\hat{y}_2 = \text{softmax}(o_2)$$

$$o_2 = Vh_2$$

$$h_2 = \tanh(W h_1 + U x_2)$$

야구
배트

0	1
1	0
0	0



하지만 보시다시피, 예측값은 실제값과 차이가 커 보입니다

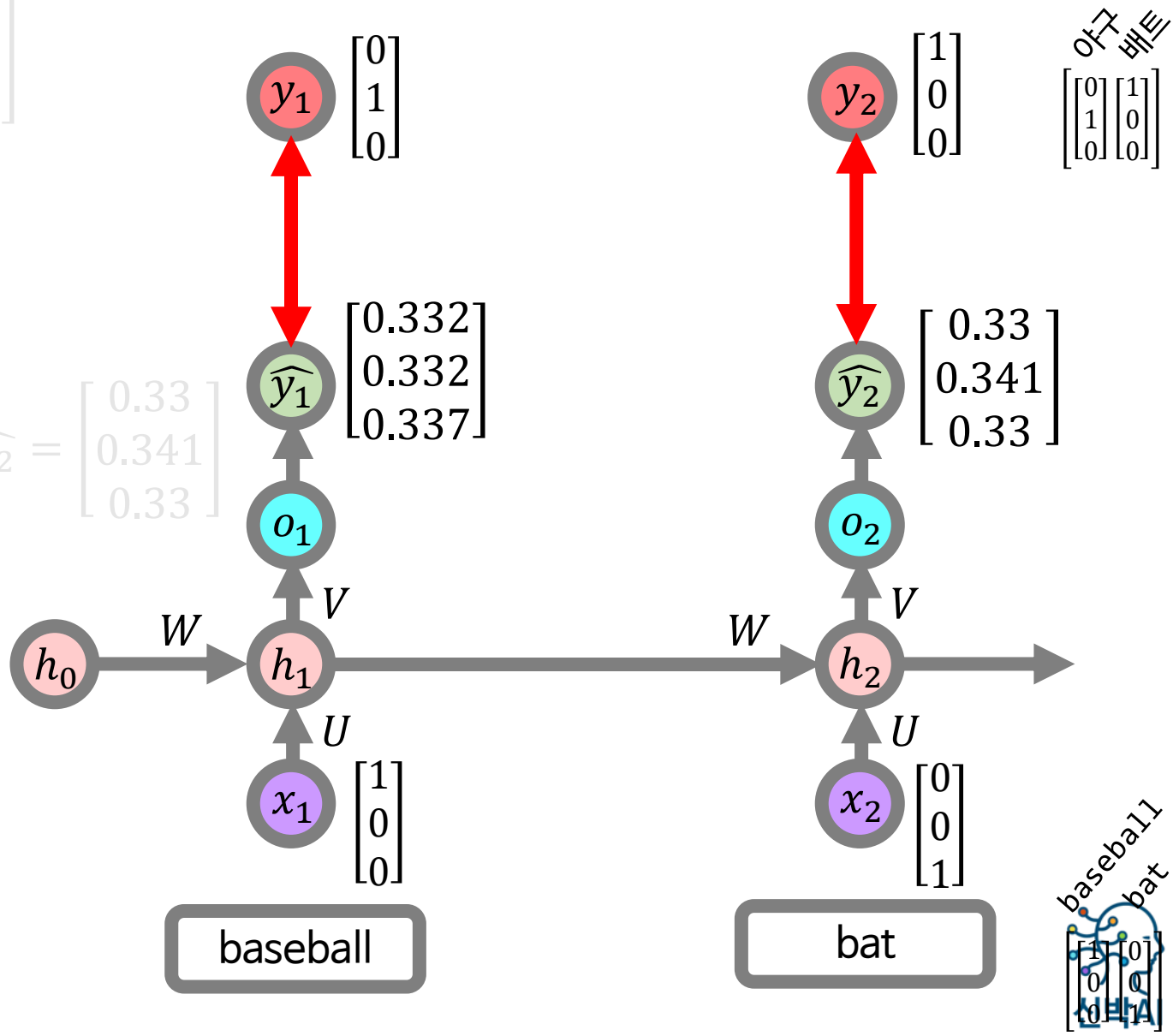
$$W = \begin{bmatrix} -0.011 & 0.13 \\ -0.123 & 0.014 \end{bmatrix} \quad V = \begin{bmatrix} -0.141 & 0.038 \\ 0.056 & -0.105 \\ -0.132 & 0.14 \end{bmatrix}$$

$$U = \begin{bmatrix} 0.094 & -0.02 & 0.135 \\ 0.135 & -0.069 & -0.009 \end{bmatrix}$$

$$h_0 = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad h_1 = \begin{bmatrix} 0.094 \\ 0.134 \end{bmatrix} \quad h_2 = \begin{bmatrix} 0.15 \\ -0.019 \end{bmatrix}$$

$$o_1 = \begin{bmatrix} -0.008 \\ -0.009 \\ 0.006 \end{bmatrix} \quad \hat{y}_1 = \begin{bmatrix} 0.332 \\ 0.332 \\ 0.337 \end{bmatrix} \quad o_2 = \begin{bmatrix} -0.022 \\ 0.01 \\ -0.022 \end{bmatrix} \quad \hat{y}_2 = \begin{bmatrix} 0.33 \\ 0.341 \\ 0.33 \end{bmatrix}$$

$$\hat{y}_2 = softmax(o_2)$$
$$o_2 = Vh_2$$
$$h_2 = tanh(W h_1 + U x_2)$$



예측값과 실제값의 차이를 우리는 손실 loss, 혹은 비용cost라 부릅니다

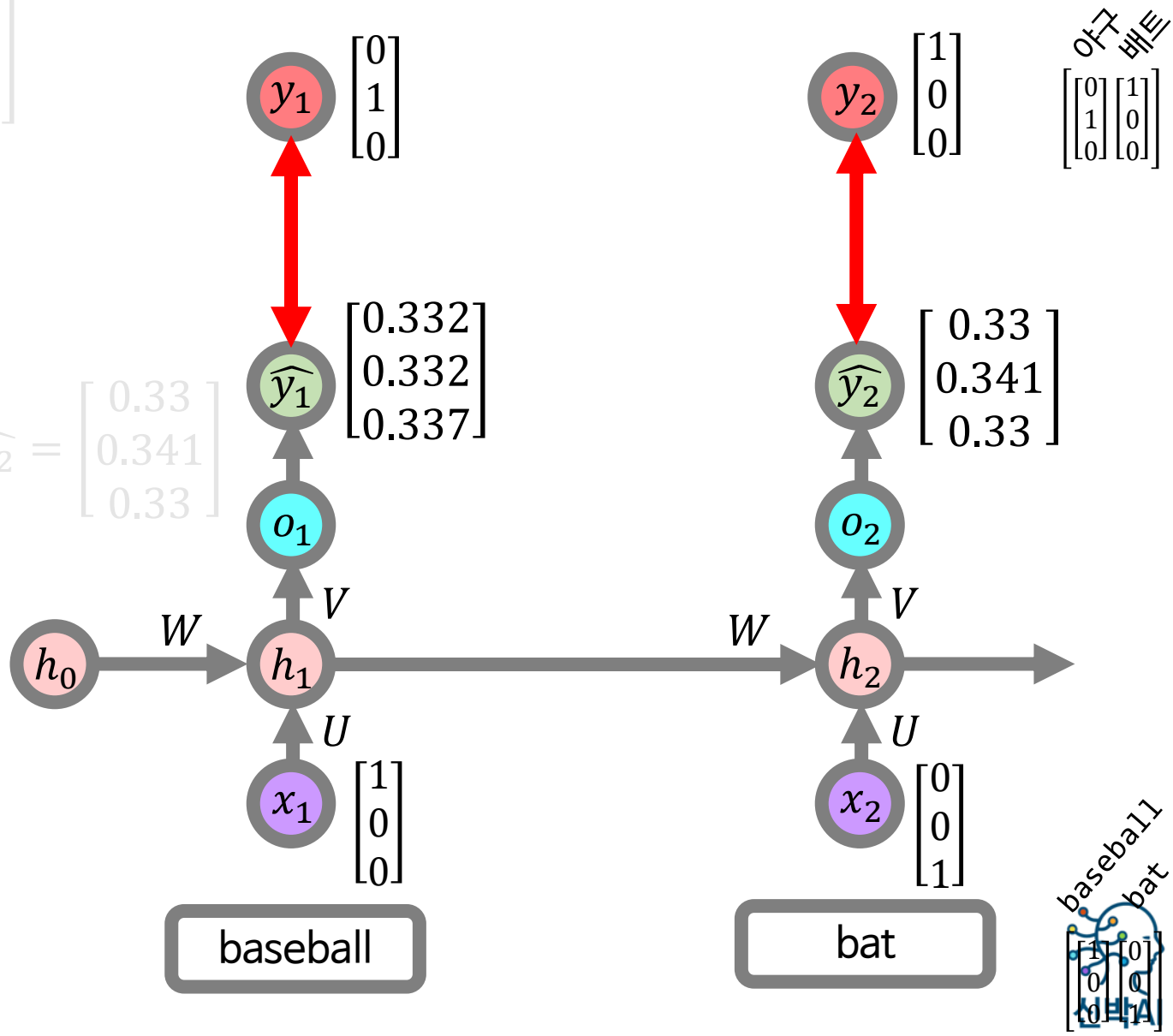
$$W = \begin{bmatrix} -0.011 & 0.13 \\ -0.123 & 0.014 \end{bmatrix} \quad V = \begin{bmatrix} -0.141 & 0.038 \\ 0.056 & -0.105 \\ -0.132 & 0.14 \end{bmatrix}$$

$$U = \begin{bmatrix} 0.094 & -0.02 & 0.135 \\ 0.135 & -0.069 & -0.009 \end{bmatrix}$$

$$h_0 = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad h_1 = \begin{bmatrix} 0.094 \\ 0.134 \end{bmatrix} \quad h_2 = \begin{bmatrix} 0.15 \\ -0.019 \end{bmatrix}$$

$$o_1 = \begin{bmatrix} -0.008 \\ -0.009 \\ 0.006 \end{bmatrix} \quad \hat{y}_1 = \begin{bmatrix} 0.332 \\ 0.332 \\ 0.337 \end{bmatrix} \quad o_2 = \begin{bmatrix} -0.022 \\ 0.01 \\ -0.022 \end{bmatrix} \quad \hat{y}_2 = \begin{bmatrix} 0.33 \\ 0.341 \\ 0.33 \end{bmatrix}$$

$$\hat{y}_2 = softmax(o_2)$$
$$o_2 = Vh_2$$
$$h_2 = tanh(W h_1 + U x_2)$$



이 loss를 구하는 여러 공식들이 있지만 여기에서는

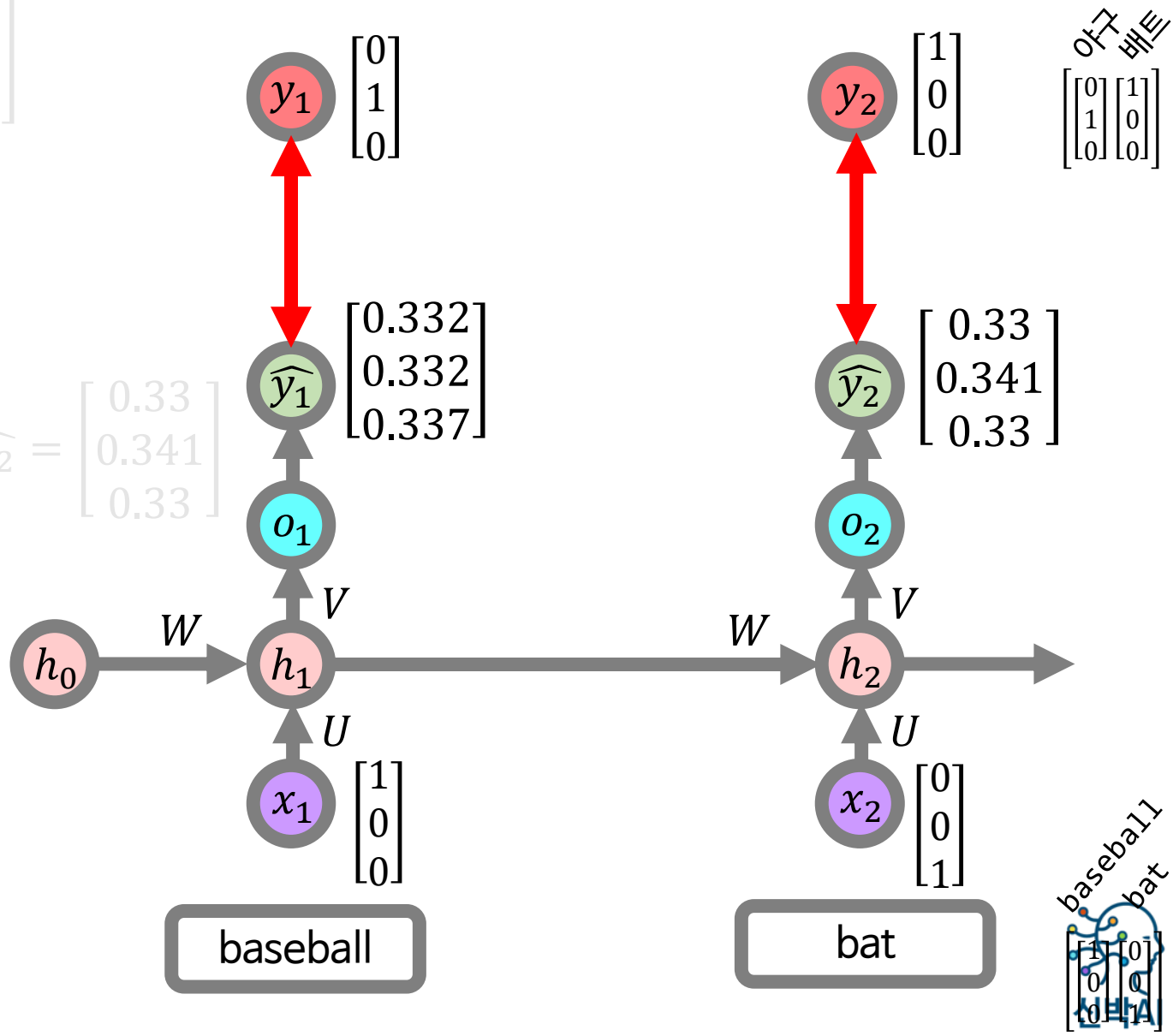
$W = \begin{bmatrix} -0.011 & 0.13 \\ -0.123 & 0.014 \end{bmatrix}$ $V = \begin{bmatrix} -0.141 & 0.038 \\ 0.056 & -0.105 \\ -0.132 & 0.14 \end{bmatrix}$

$U = \begin{bmatrix} 0.094 & -0.02 & 0.135 \\ 0.135 & -0.069 & -0.009 \end{bmatrix}$

$h_0 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ $h_1 = \begin{bmatrix} 0.094 \\ 0.134 \end{bmatrix}$ $h_2 = \begin{bmatrix} 0.15 \\ -0.019 \end{bmatrix}$

$o_1 = \begin{bmatrix} -0.008 \\ -0.009 \\ 0.006 \end{bmatrix}$ $\hat{y}_1 = \begin{bmatrix} 0.332 \\ 0.332 \\ 0.337 \end{bmatrix}$ $o_2 = \begin{bmatrix} -0.022 \\ 0.01 \\ -0.022 \end{bmatrix}$ $\hat{y}_2 = \begin{bmatrix} 0.33 \\ 0.341 \\ 0.33 \end{bmatrix}$

$\hat{y}_2 = softmax(o_2)$
 $o_2 = Vh_2$
 $h_2 = tanh(W h_1 + U x_2)$



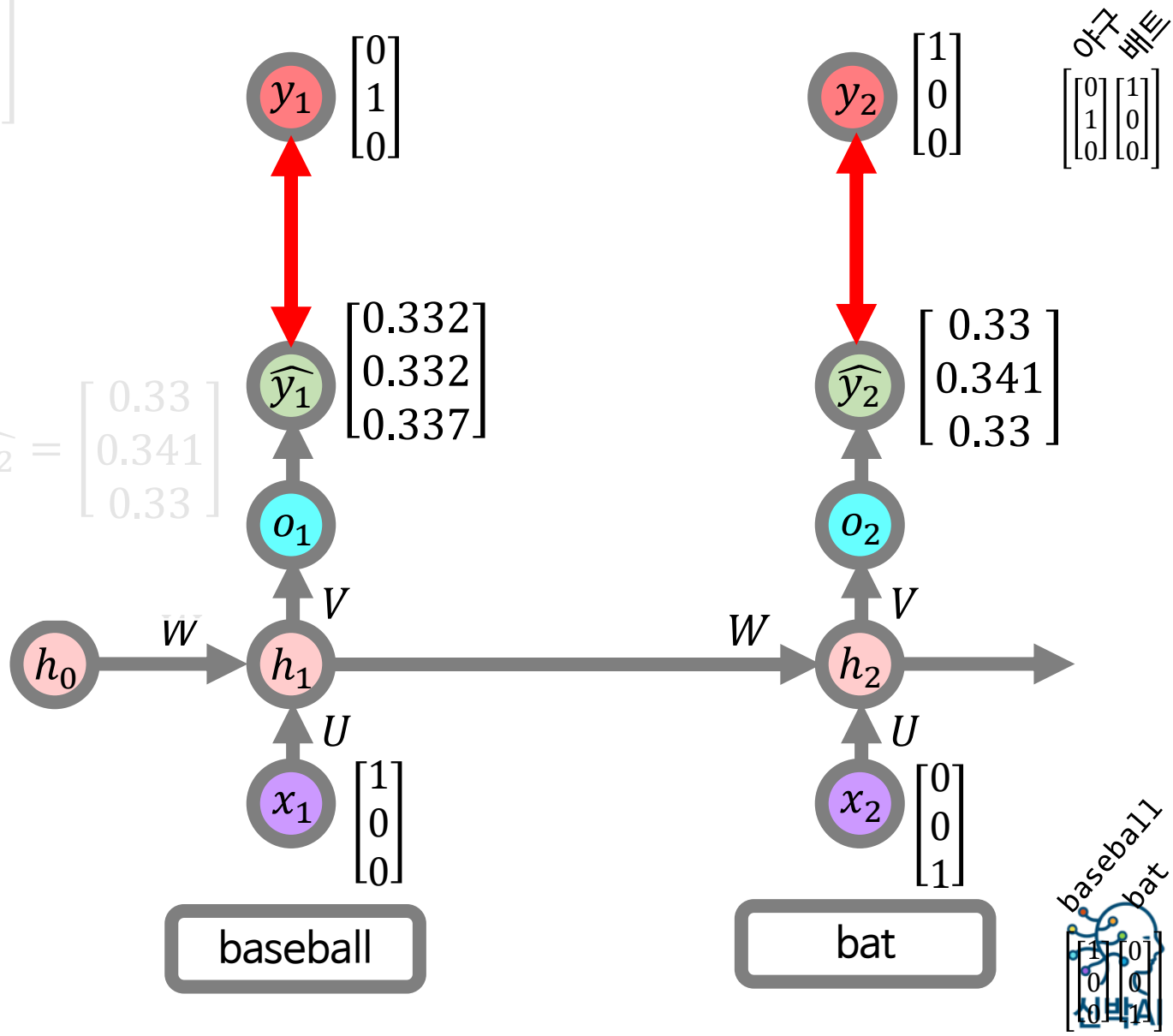
Cross-entropy라는 손실함수를 사용하도록 하겠습니다

$$W = \begin{bmatrix} -0.011 & 0.13 \\ -0.123 & 0.014 \end{bmatrix} \quad V = \begin{bmatrix} -0.141 & 0.038 \\ 0.056 & -0.105 \\ -0.132 & 0.14 \end{bmatrix}$$

$$U = \begin{bmatrix} 0.094 & -0.02 & 0.135 \\ 0.135 & -0.069 & -0.009 \end{bmatrix}$$

$$CE = - \sum_i^C t_i \log((f(s))_i)$$
$$h_0 = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad h_1 = \begin{bmatrix} 0.094 \\ 0.134 \end{bmatrix} \quad h_2 = \begin{bmatrix} 0.15 \\ 0.019 \end{bmatrix}$$
$$o_1 = \begin{bmatrix} -0.008 \\ -0.009 \\ 0.006 \end{bmatrix} \quad \hat{y}_1 = \begin{bmatrix} 0.332 \\ 0.332 \\ 0.337 \end{bmatrix} \quad o_2 = \begin{bmatrix} -0.022 \\ 0.01 \\ -0.022 \end{bmatrix} \quad \hat{y}_2 = \begin{bmatrix} 0.33 \\ 0.341 \\ 0.33 \end{bmatrix}$$

$$\hat{y}_2 = softmax(o_2)$$
$$o_2 = Vh_2$$
$$h_2 = tanh(W h_1 + U x_2)$$



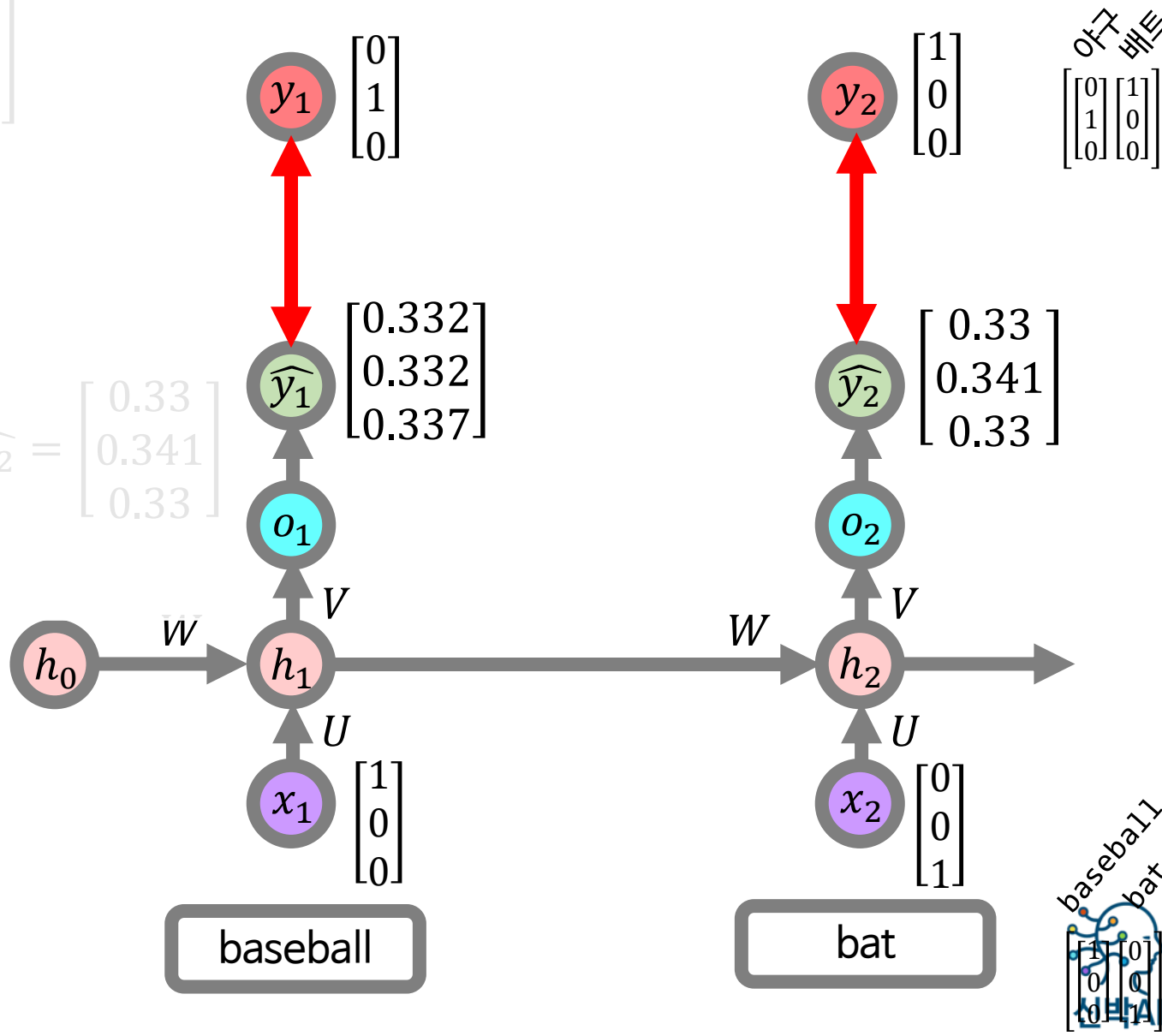
Cross-entropy를 쓰는 이유는 추후에 다룰 역전파의 계산에서

$$W = \begin{bmatrix} -0.011 & 0.13 \\ -0.123 & 0.014 \end{bmatrix} \quad V = \begin{bmatrix} -0.141 & 0.038 \\ 0.056 & -0.105 \\ -0.132 & 0.14 \end{bmatrix}$$

$$U = \begin{bmatrix} 0.094 & -0.02 & 0.135 \\ 0.135 & -0.069 & -0.009 \end{bmatrix}$$

$$CE = - \sum_i^c t_i \log((f(s)_i)$$
$$h_0 = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad h_1 = \begin{bmatrix} 0.094 \\ 0.134 \end{bmatrix} \quad h_2 = \begin{bmatrix} 0.15 \\ 0.019 \end{bmatrix}$$
$$o_1 = \begin{bmatrix} -0.008 \\ -0.009 \\ 0.006 \end{bmatrix} \quad \hat{y}_1 = \begin{bmatrix} 0.332 \\ 0.332 \\ 0.337 \end{bmatrix} \quad o_2 = \begin{bmatrix} 0.33 \\ 0.01 \\ -0.022 \end{bmatrix} \quad \hat{y}_2 = \begin{bmatrix} 0.33 \\ 0.341 \\ 0.33 \end{bmatrix}$$

$$\hat{y}_2 = softmax(o_2)$$
$$o_2 = Vh_2$$
$$h_2 = tanh(W h_1 + U x_2)$$



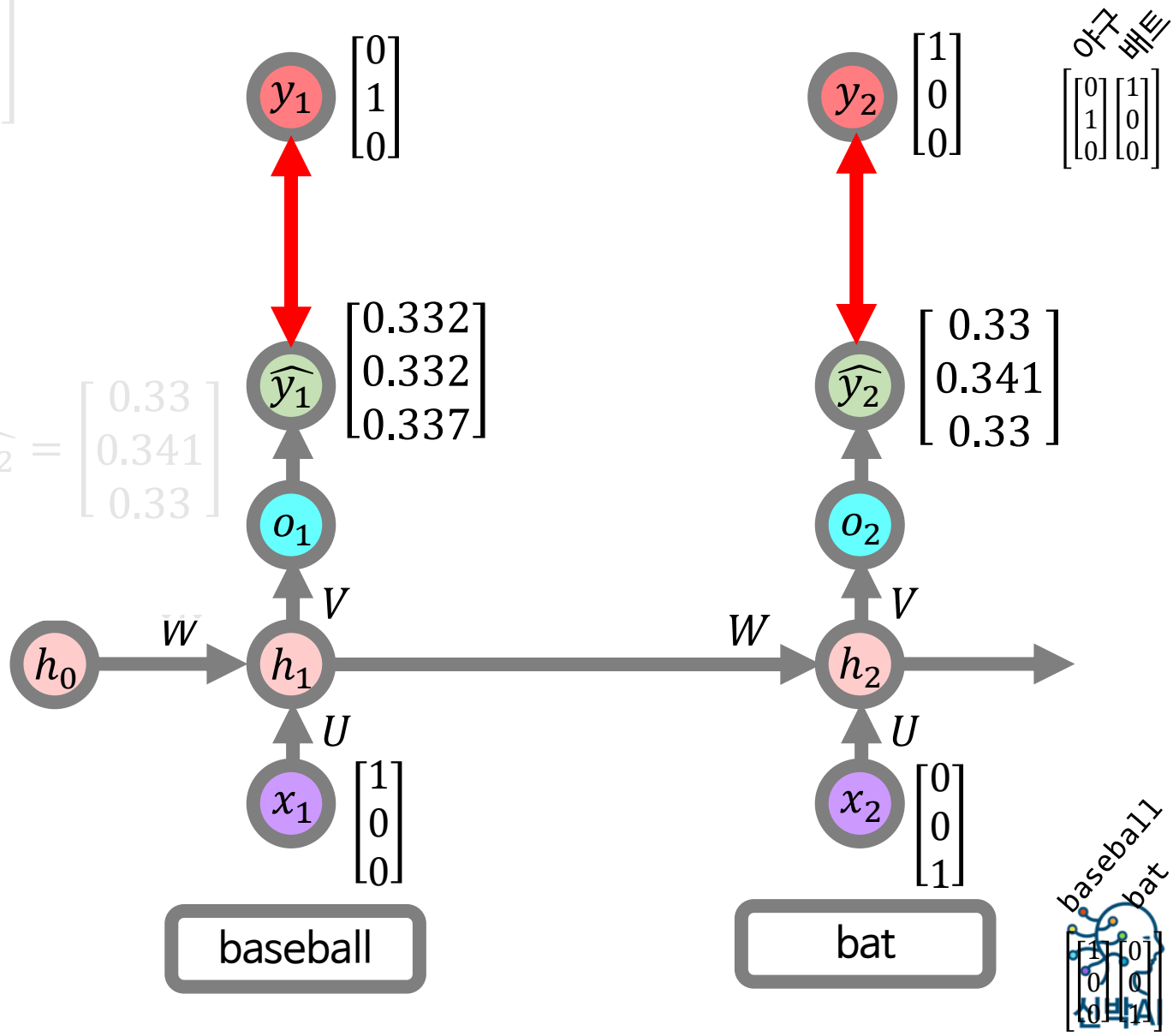
Softmax함수와 함께 쓸 때 역전파 계산이 아주 용이해지기 때문입니다

$$W = \begin{bmatrix} -0.011 & 0.13 \\ -0.123 & 0.014 \end{bmatrix} \quad V = \begin{bmatrix} -0.141 & 0.038 \\ 0.056 & -0.105 \\ -0.132 & 0.14 \end{bmatrix}$$

$$U = \begin{bmatrix} 0.094 & -0.02 & 0.135 \\ 0.135 & -0.069 & -0.009 \end{bmatrix}$$

$$CE = - \sum_i^c t_i \log((f(s)_i)$$
$$h_0 = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad h_1 = \begin{bmatrix} 0.094 \\ 0.134 \end{bmatrix} \quad h_2 = \begin{bmatrix} 0.15 \\ 0.019 \end{bmatrix}$$
$$o_1 = \begin{bmatrix} -0.008 \\ -0.009 \\ 0.006 \end{bmatrix} \quad \hat{y}_1 = \begin{bmatrix} 0.332 \\ 0.332 \\ 0.337 \end{bmatrix} \quad o_2 = \begin{bmatrix} 0.33 \\ 0.01 \\ -0.022 \end{bmatrix} \quad \hat{y}_2 = \begin{bmatrix} 0.33 \\ 0.341 \\ 0.33 \end{bmatrix}$$

$$\hat{y}_2 = softmax(o_2)$$
$$o_2 = Vh_2$$
$$h_2 = tanh(W h_1 + U x_2)$$



자 이제는 backpropagation을 통해 가중치 업데이트를 해보겠습니다

$$W = \begin{bmatrix} -0.011 & 0.13 \\ -0.123 & 0.014 \end{bmatrix} \quad V = \begin{bmatrix} -0.141 & 0.038 \\ 0.056 & -0.105 \\ -0.132 & 0.14 \end{bmatrix}$$

$$U = \begin{bmatrix} 0.094 & -0.02 & 0.135 \\ 0.135 & -0.069 & -0.009 \end{bmatrix}$$

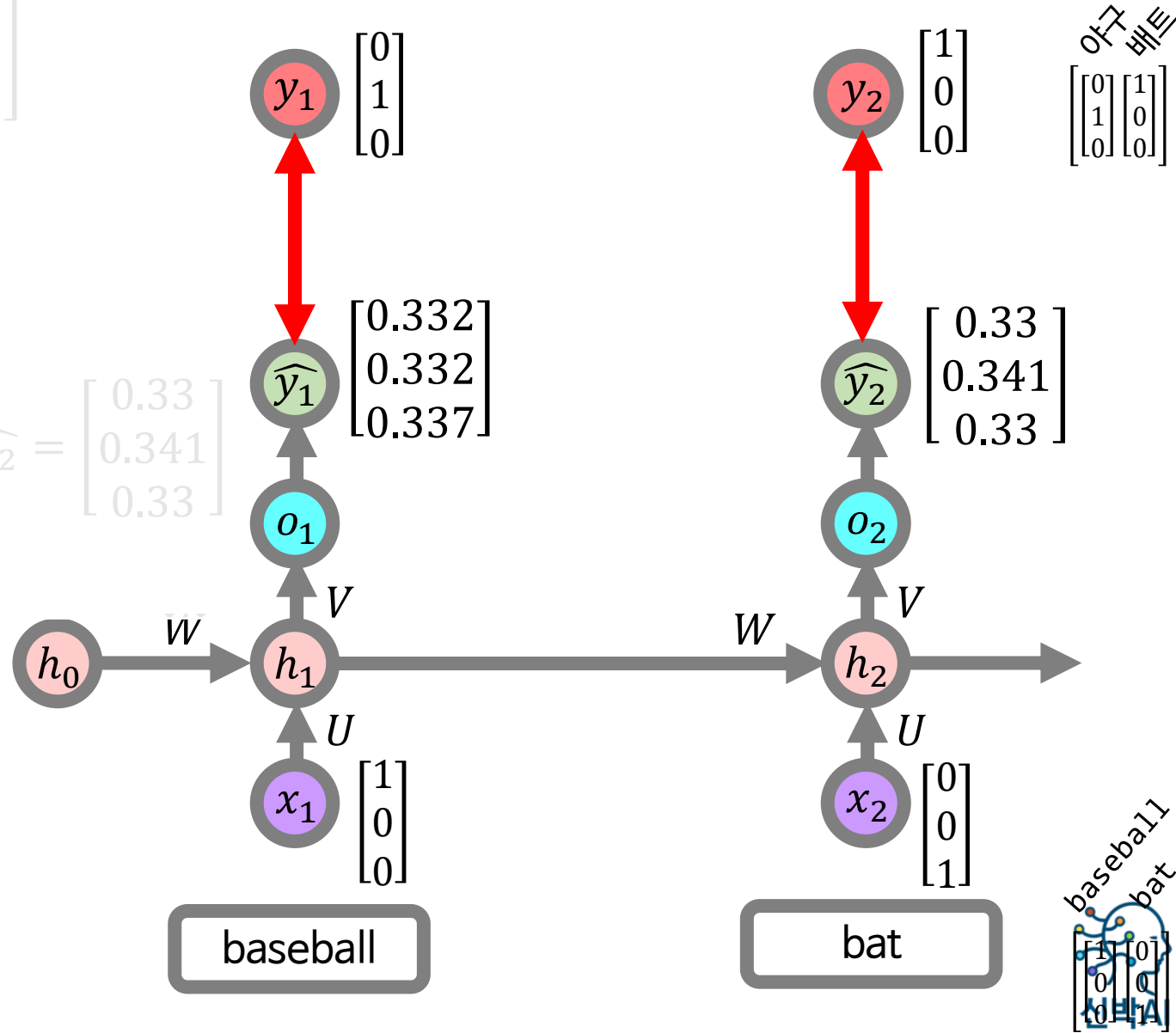
$$h_0 = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad h_1 = \begin{bmatrix} 0.094 \\ 0.134 \end{bmatrix} \quad h_2 = \begin{bmatrix} 0.15 \\ -0.019 \end{bmatrix}$$

$$o_1 = \begin{bmatrix} -0.008 \\ -0.009 \\ 0.006 \end{bmatrix} \quad \hat{y}_1 = \begin{bmatrix} 0.332 \\ 0.332 \\ 0.337 \end{bmatrix} \quad o_2 = \begin{bmatrix} -0.022 \\ 0.01 \\ -0.022 \end{bmatrix} \quad \hat{y}_2 = \begin{bmatrix} 0.33 \\ 0.341 \\ 0.33 \end{bmatrix}$$

$$\hat{y}_2 = softmax(o_2)$$

$$o_2 = Vh_2$$

$$h_2 = tanh(W h_1 + U x_2)$$



RNN에서의 backpropagation은 BPTT라고 불립니다

$$W = \begin{bmatrix} -0.011 & 0.13 \\ -0.123 & 0.014 \end{bmatrix} \quad V = \begin{bmatrix} -0.141 & 0.038 \\ 0.056 & -0.105 \\ -0.132 & 0.14 \end{bmatrix}$$

$$U = \begin{bmatrix} 0.094 & -0.02 & 0.135 \\ 0.135 & -0.069 & -0.009 \end{bmatrix}$$

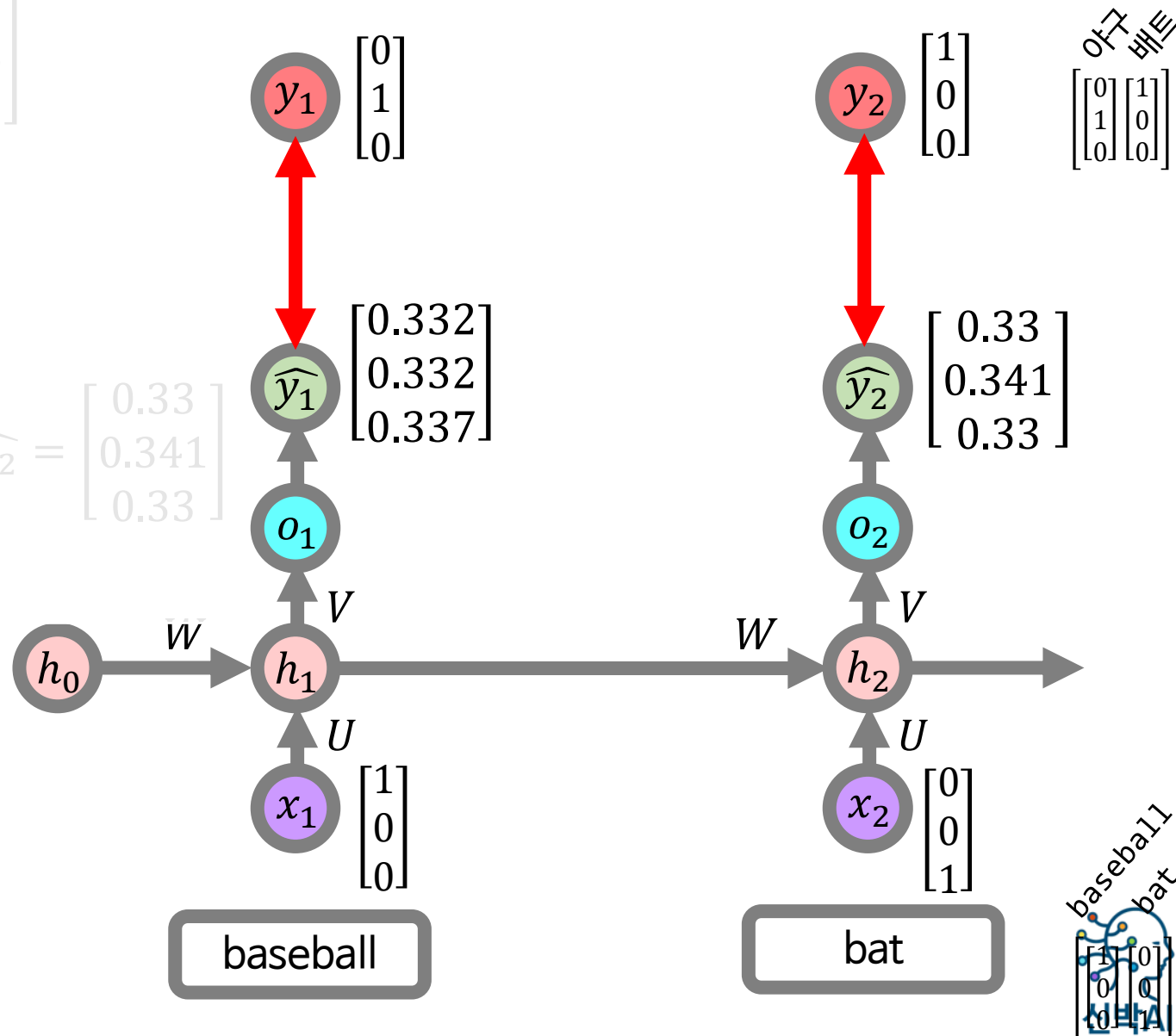
$$h_0 = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad h_1 = \begin{bmatrix} 0.094 \\ 0.134 \end{bmatrix} \quad h_2 = \begin{bmatrix} 0.15 \\ -0.019 \end{bmatrix}$$

$$o_1 = \begin{bmatrix} -0.008 \\ -0.009 \\ 0.006 \end{bmatrix} \quad \hat{y}_1 = \begin{bmatrix} 0.332 \\ 0.332 \\ 0.337 \end{bmatrix} \quad o_2 = \begin{bmatrix} -0.022 \\ 0.01 \\ -0.022 \end{bmatrix} \quad \hat{y}_2 = \begin{bmatrix} 0.33 \\ 0.341 \\ 0.33 \end{bmatrix}$$

$$\hat{y}_2 = \text{softmax}(o_2)$$

$$o_2 = Vh_2$$

$$h_2 = \tanh(W h_1 + U x_2)$$



BPTT (BackPropagation Through Time)의 약자로서

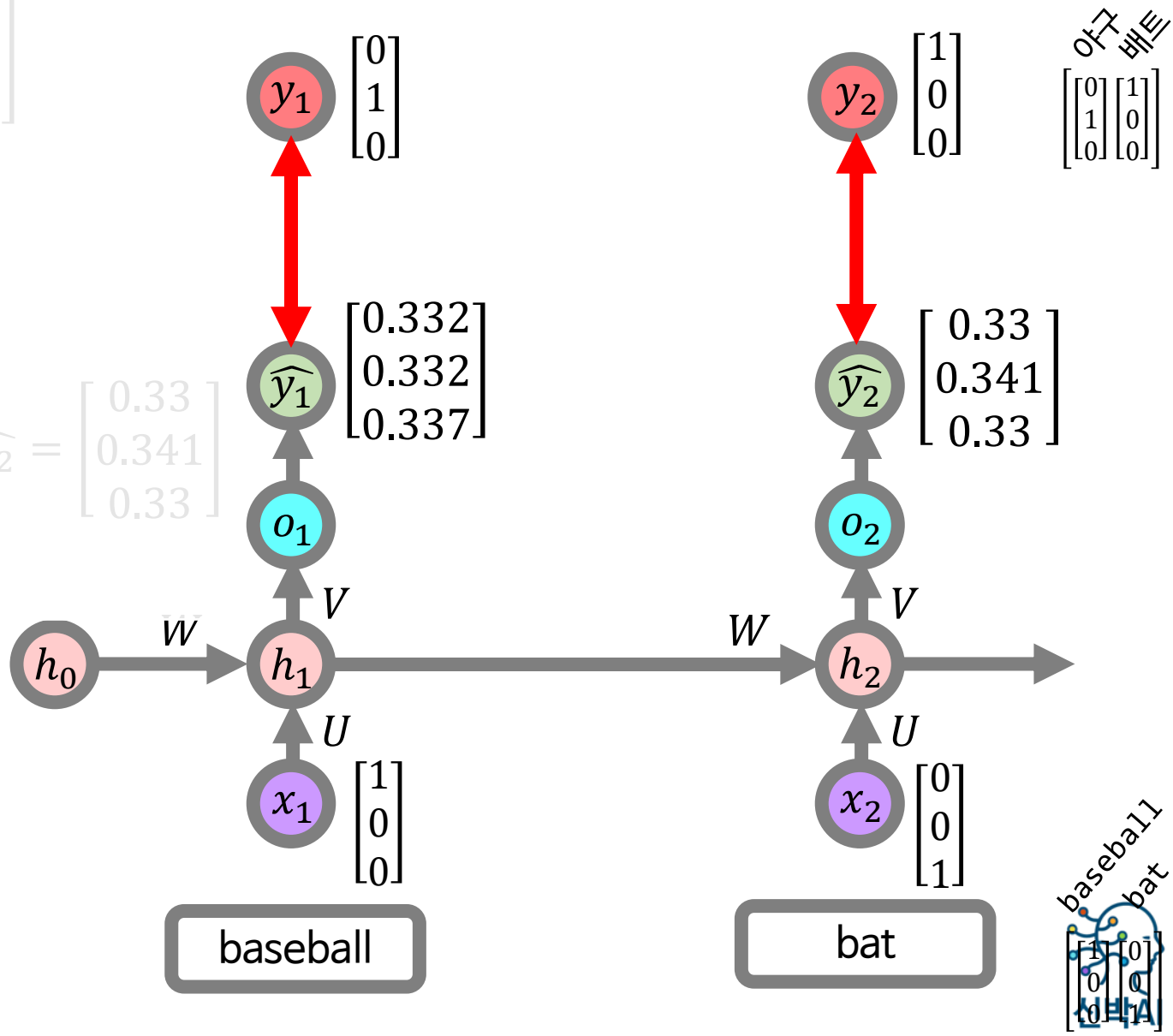
$$W = \begin{bmatrix} -0.011 & 0.13 \\ -0.123 & 0.014 \end{bmatrix} \quad V = \begin{bmatrix} -0.141 & 0.038 \\ 0.056 & -0.105 \\ -0.132 & 0.14 \end{bmatrix}$$

$$U = \begin{bmatrix} 0.094 & -0.02 & 0.135 \\ 0.135 & -0.069 & -0.009 \end{bmatrix}$$

$$h_0 = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad h_1 = \begin{bmatrix} 0.094 \\ 0.134 \end{bmatrix} \quad h_2 = \begin{bmatrix} 0.15 \\ -0.019 \end{bmatrix}$$

$$o_1 = \begin{bmatrix} -0.008 \\ -0.009 \\ 0.006 \end{bmatrix} \quad \hat{y}_1 = \begin{bmatrix} 0.332 \\ 0.332 \\ 0.337 \end{bmatrix} \quad o_2 = \begin{bmatrix} -0.022 \\ 0.01 \\ -0.022 \end{bmatrix} \quad \hat{y}_2 = \begin{bmatrix} 0.33 \\ 0.341 \\ 0.33 \end{bmatrix}$$

$$\hat{y}_2 = softmax(o_2)$$
$$o_2 = Vh_2$$
$$h_2 = tanh(W h_1 + U x_2)$$



우리말로는 시간을 통한 역전파라고 할 수 있겠습니다

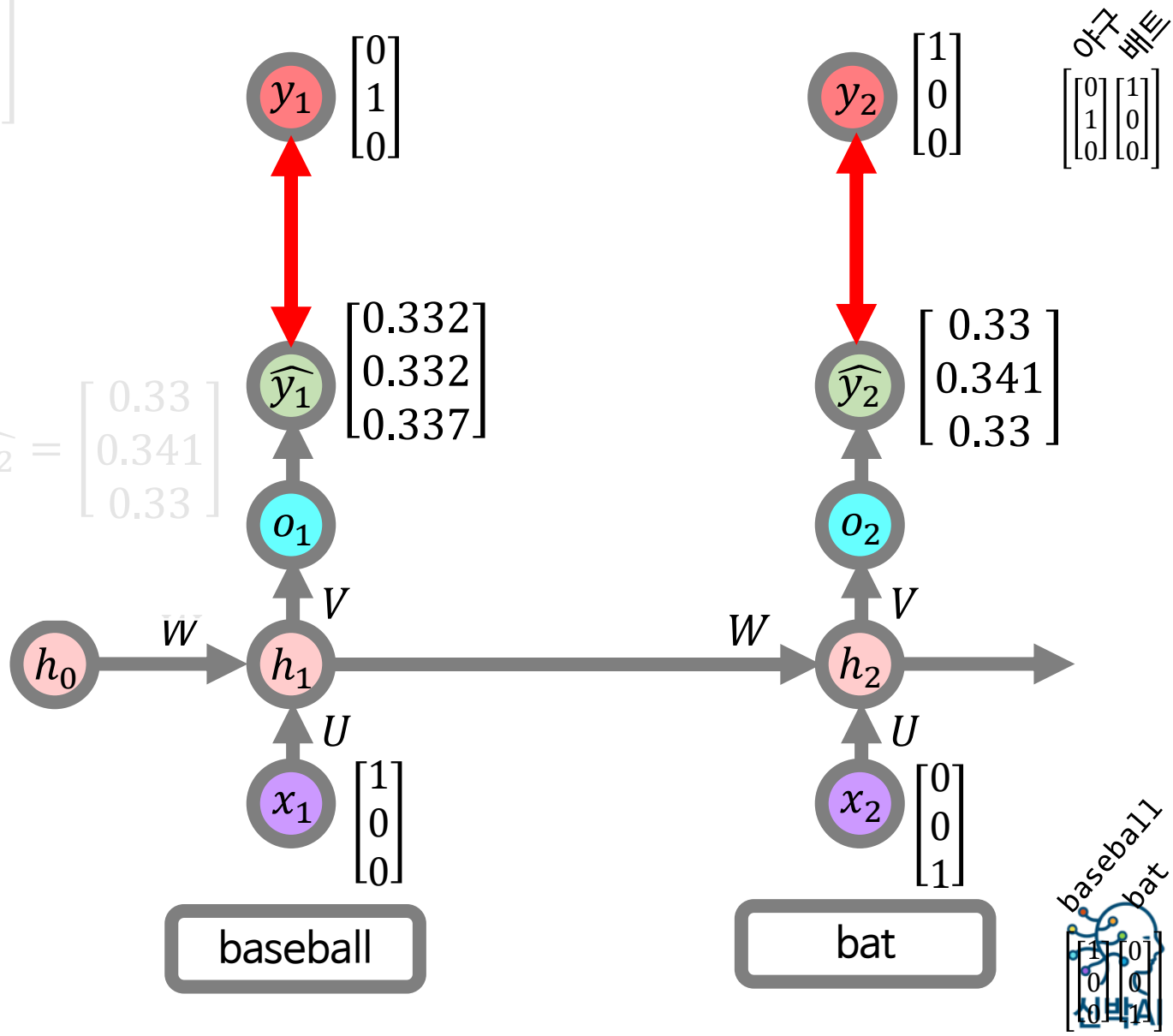
$$W = \begin{bmatrix} -0.011 & 0.13 \\ -0.123 & 0.014 \end{bmatrix} \quad V = \begin{bmatrix} -0.141 & 0.038 \\ 0.056 & -0.105 \\ -0.132 & 0.14 \end{bmatrix}$$

$$U = \begin{bmatrix} 0.094 & -0.02 & 0.135 \\ 0.135 & -0.069 & -0.009 \end{bmatrix}$$

$$h_0 = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad h_1 = \begin{bmatrix} 0.094 \\ 0.134 \end{bmatrix} \quad h_2 = \begin{bmatrix} 0.15 \\ -0.019 \end{bmatrix}$$

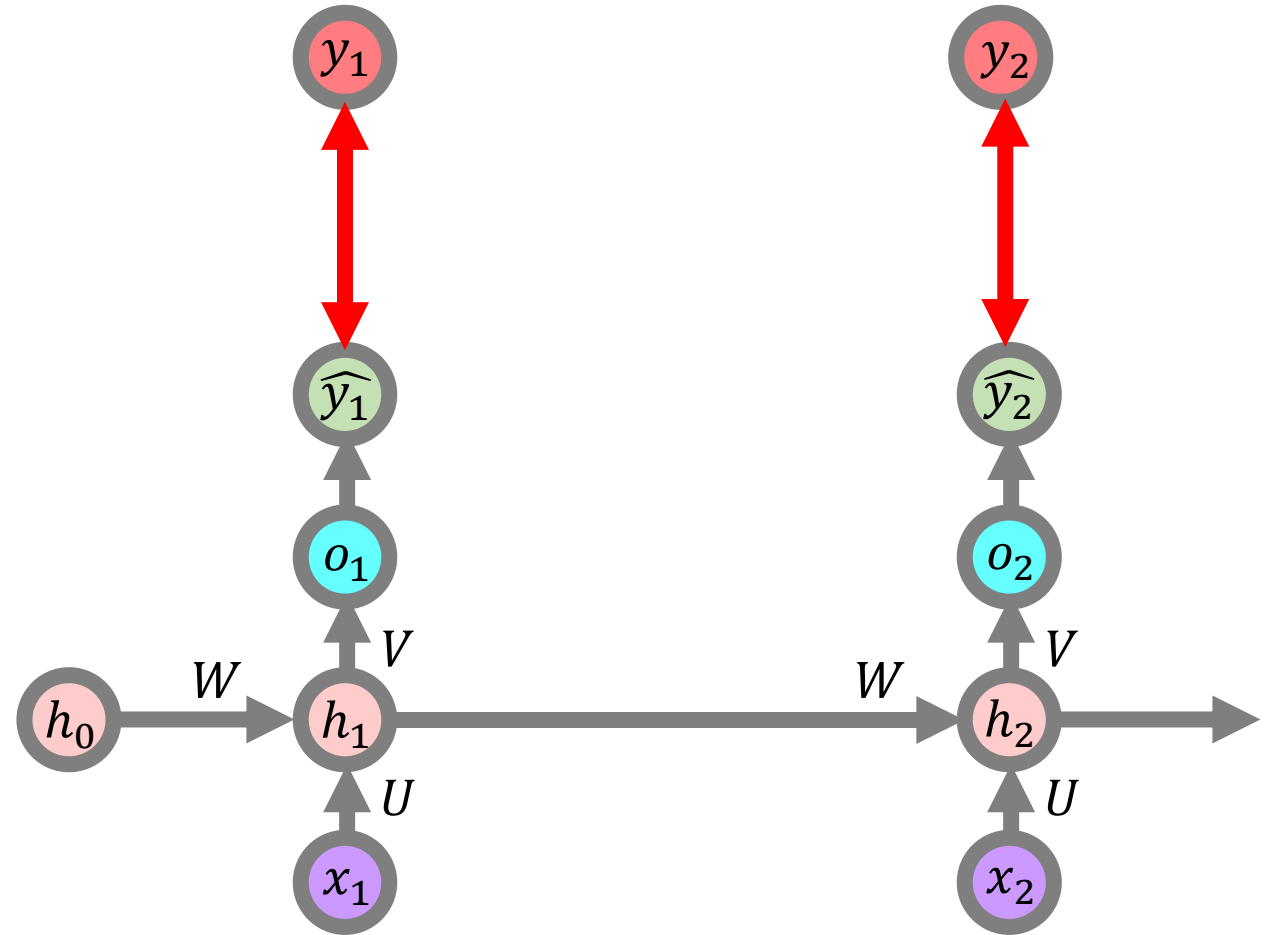
$$o_1 = \begin{bmatrix} -0.008 \\ -0.009 \\ 0.006 \end{bmatrix} \quad \hat{y}_1 = \begin{bmatrix} 0.332 \\ 0.332 \\ 0.337 \end{bmatrix} \quad o_2 = \begin{bmatrix} -0.022 \\ 0.01 \\ -0.022 \end{bmatrix} \quad \hat{y}_2 = \begin{bmatrix} 0.33 \\ 0.341 \\ 0.33 \end{bmatrix}$$

$$\hat{y}_2 = softmax(o_2)$$
$$o_2 = Vh_2$$
$$h_2 = tanh(W h_1 + U x_2)$$



역전파 backpropagation을 한마디로 정의하면 손실이 줄어드는 방향으로 연결가중치를 변화시켜가는 과정이라 할 수 있습니다

RNN에서는 세 개의 연결 가중치가 있습니다

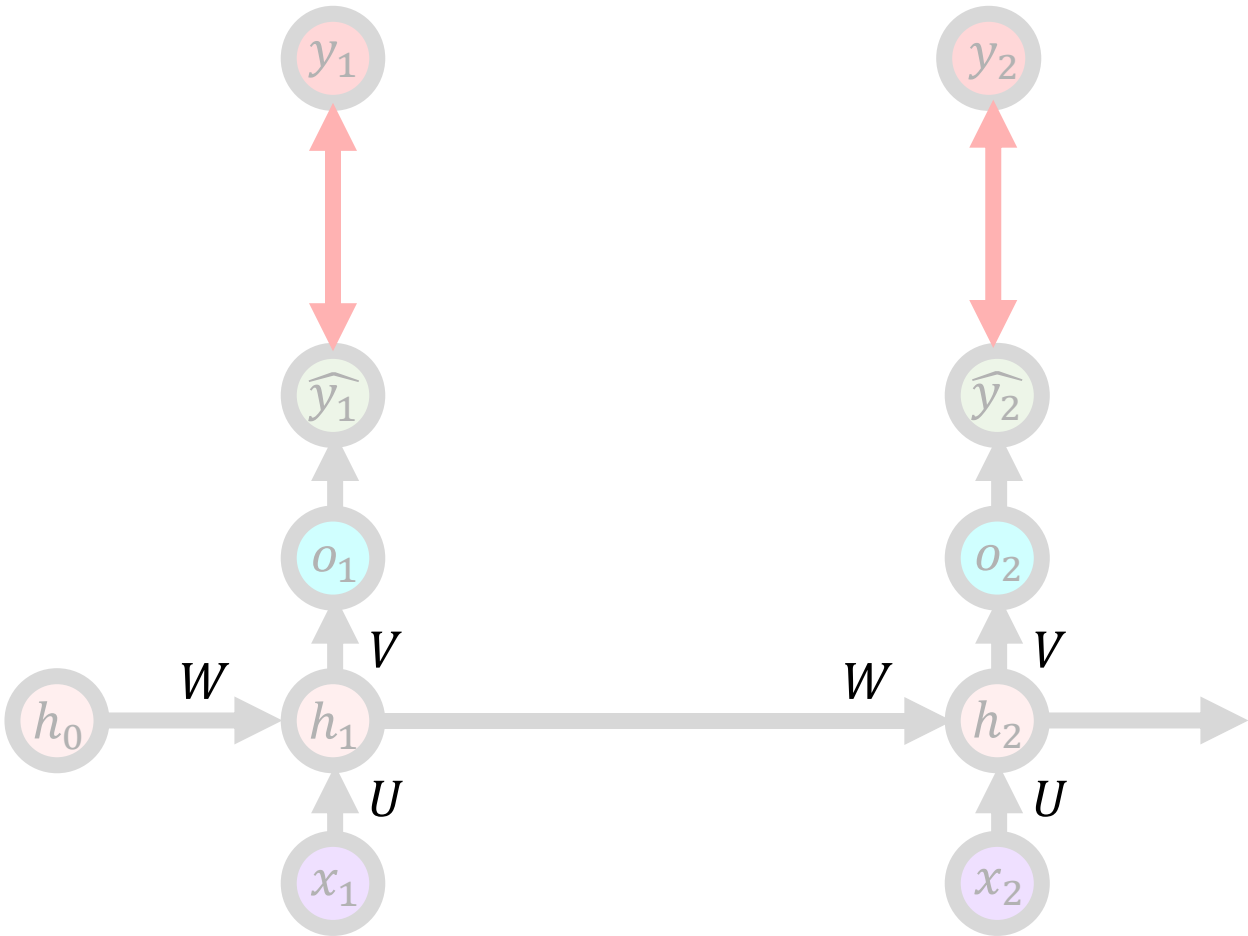


V, W, U가 바로 그것입니다

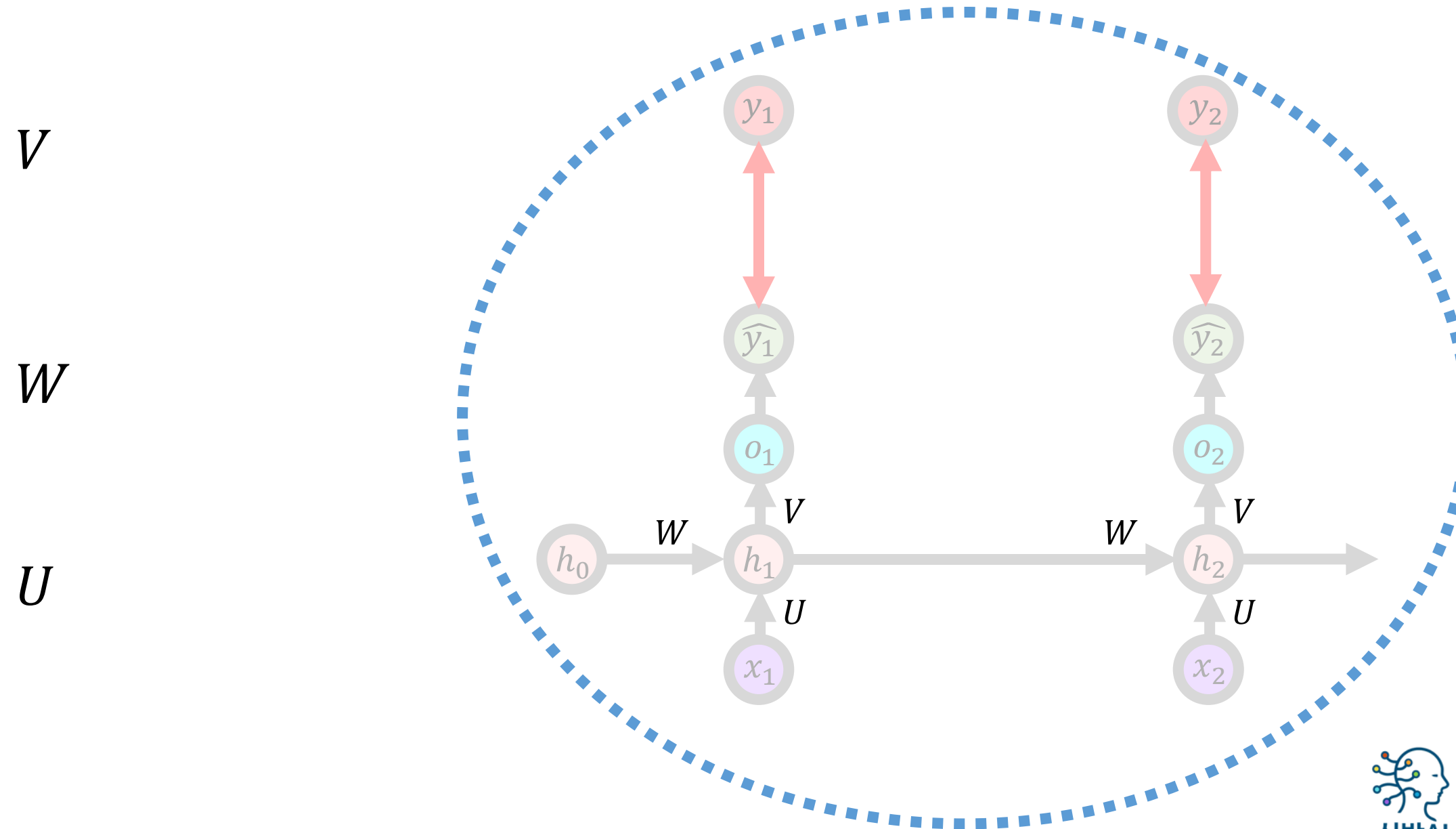
V

W

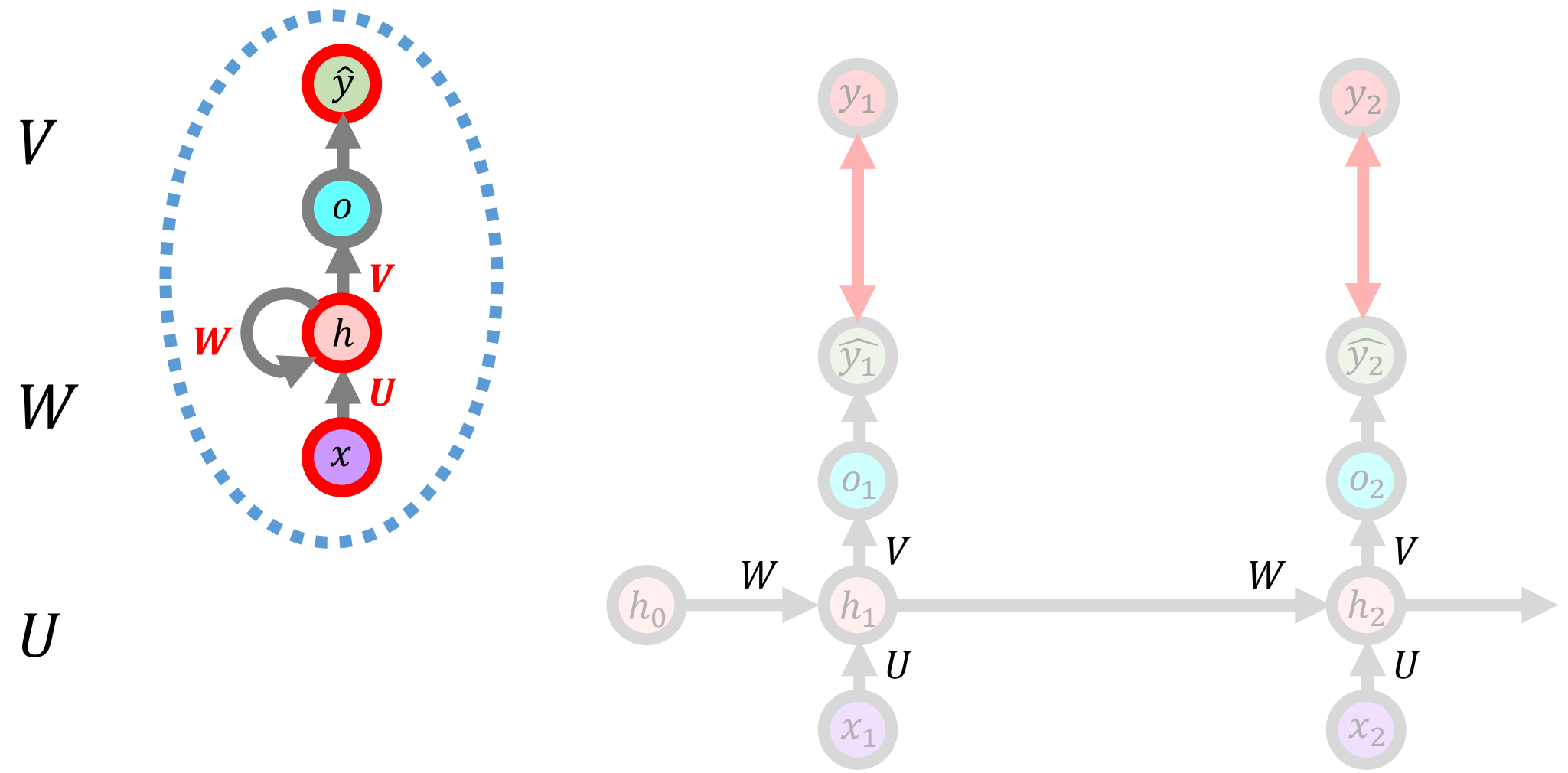
U



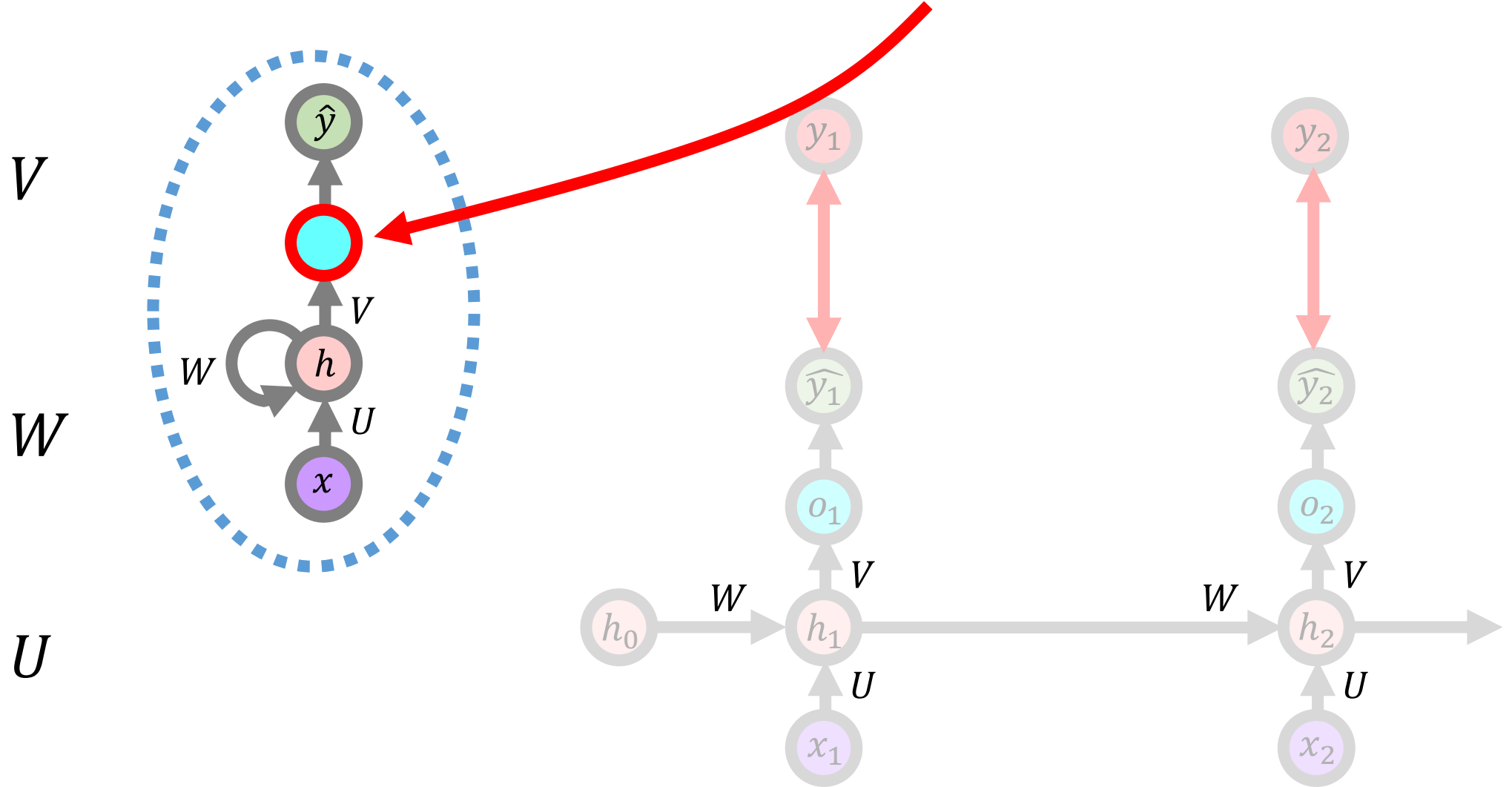
이 RNN은 시간적으로 풀어놓은 것이라 복잡해보이지만,



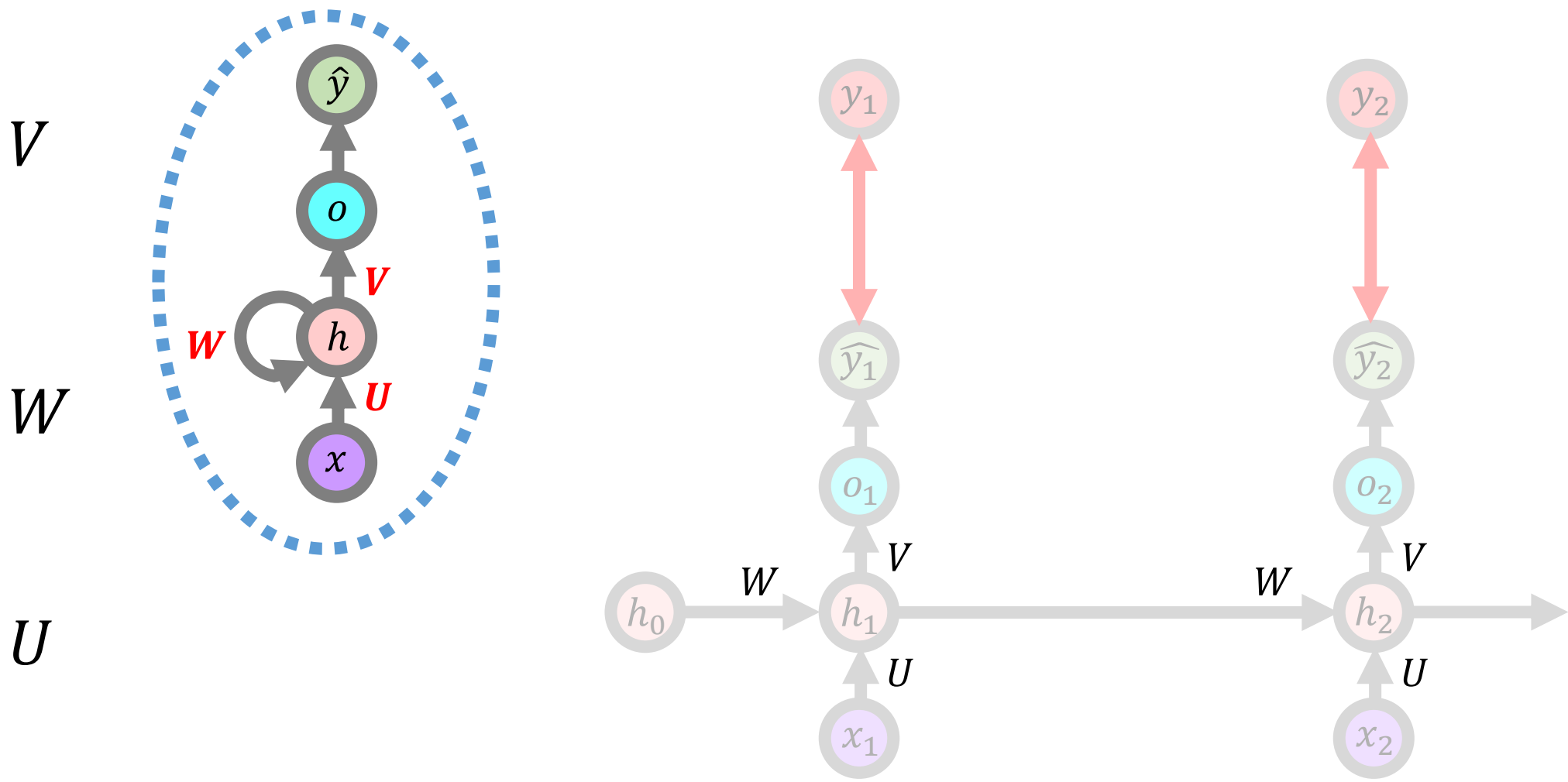
실제적으로는 3개의 노드와 3개의 연결가중치를 가진 간단한 신경망입니다



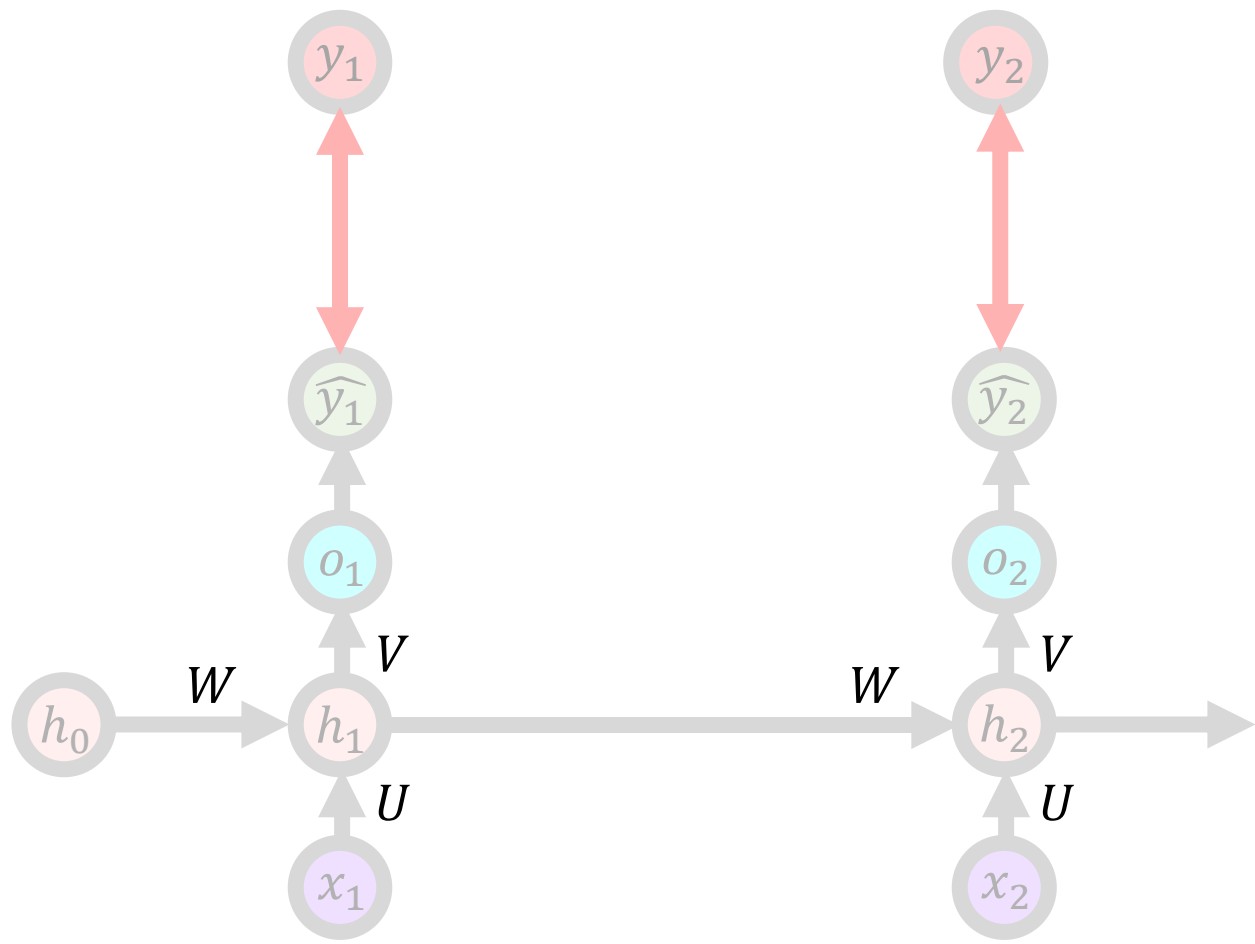
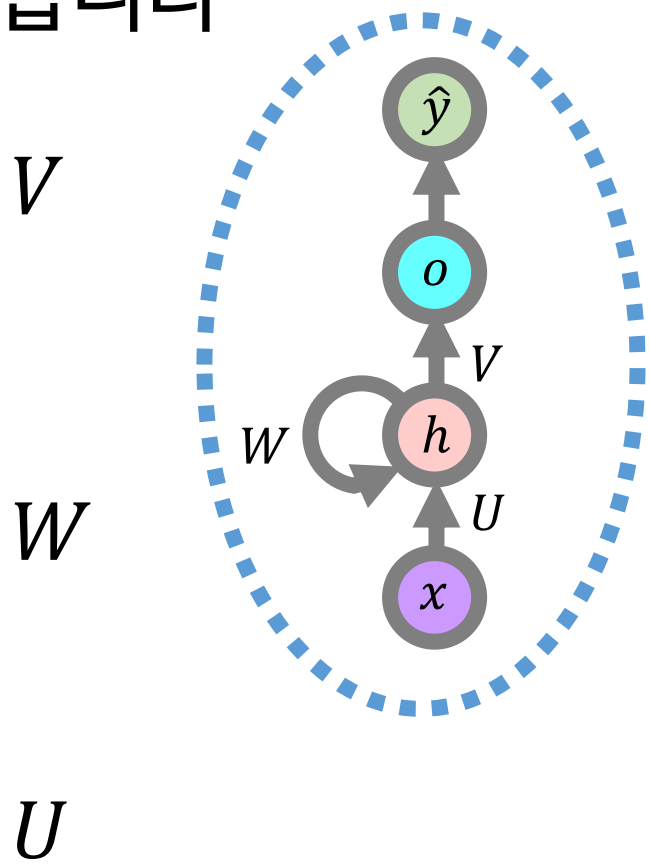
o 는 softmax계산 과정을 나타내기 위한 것이지만 실제 노드는 아닙니다



그러므로 이 세 연결가중치 V, W, U 만 업데이트 하면 되는 것입니다



가중치 업데이트는 다른 신경망 모델들과 마찬가지로 경사하강법과 역전파를 이용합니다

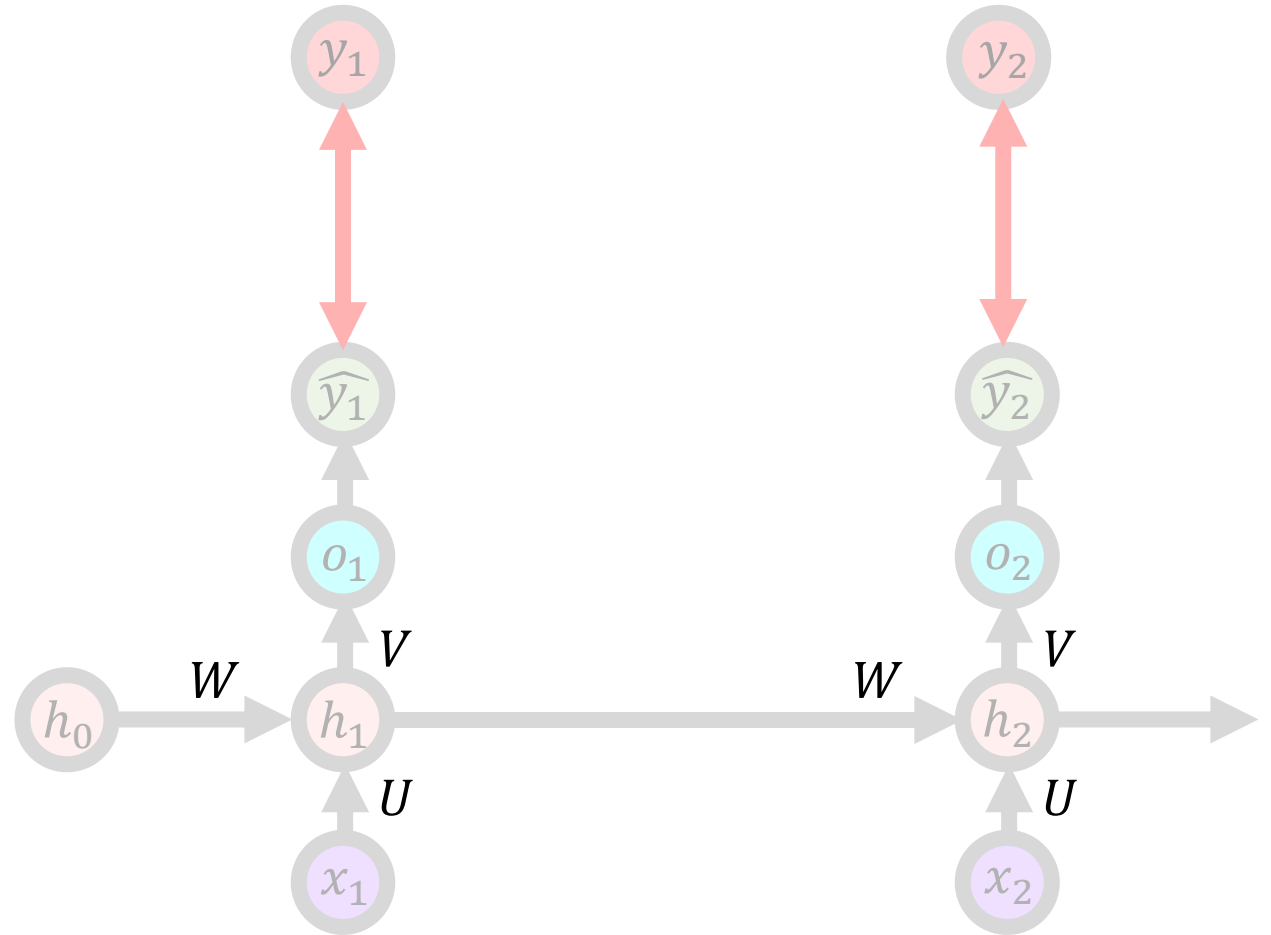


따라서 V , W , U 를 업데이트 하는 공식은 다음과 같습니다 (α 는 학습률)

$$V^* = V - \alpha \frac{\partial L}{\partial V}$$

$$W^* = W - \alpha \frac{\partial L}{\partial W}$$

$$U^* = U - \alpha \frac{\partial L}{\partial U}$$

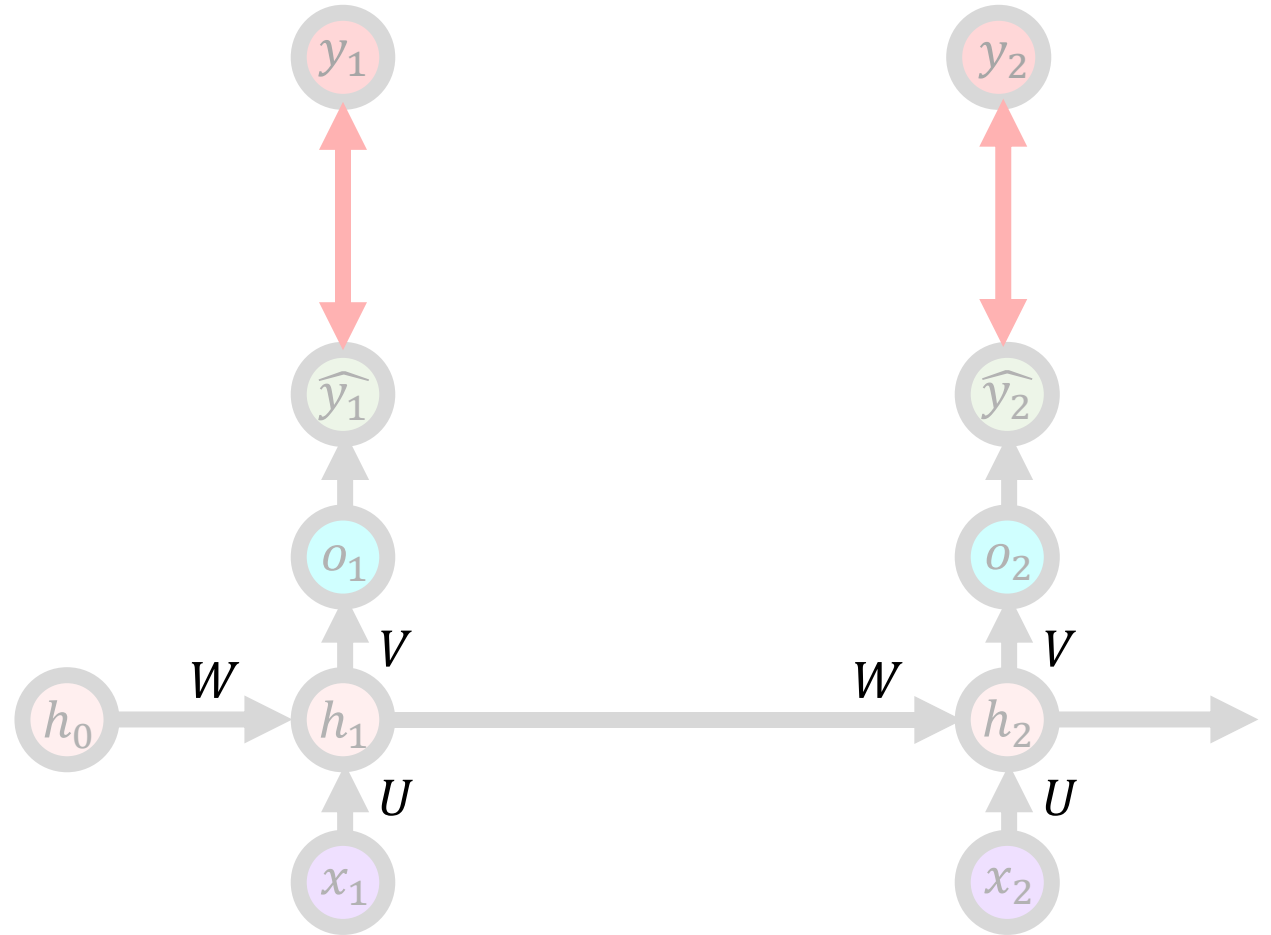


그래서 우리는 다음 기울기들만 구하면 RNN을 학습시킬 수 있습니다

$$V^* = V - \alpha \frac{\partial L}{\partial V}$$

$$W^* = W - \alpha \frac{\partial L}{\partial W}$$

$$U^* = U - \alpha \frac{\partial L}{\partial U}$$

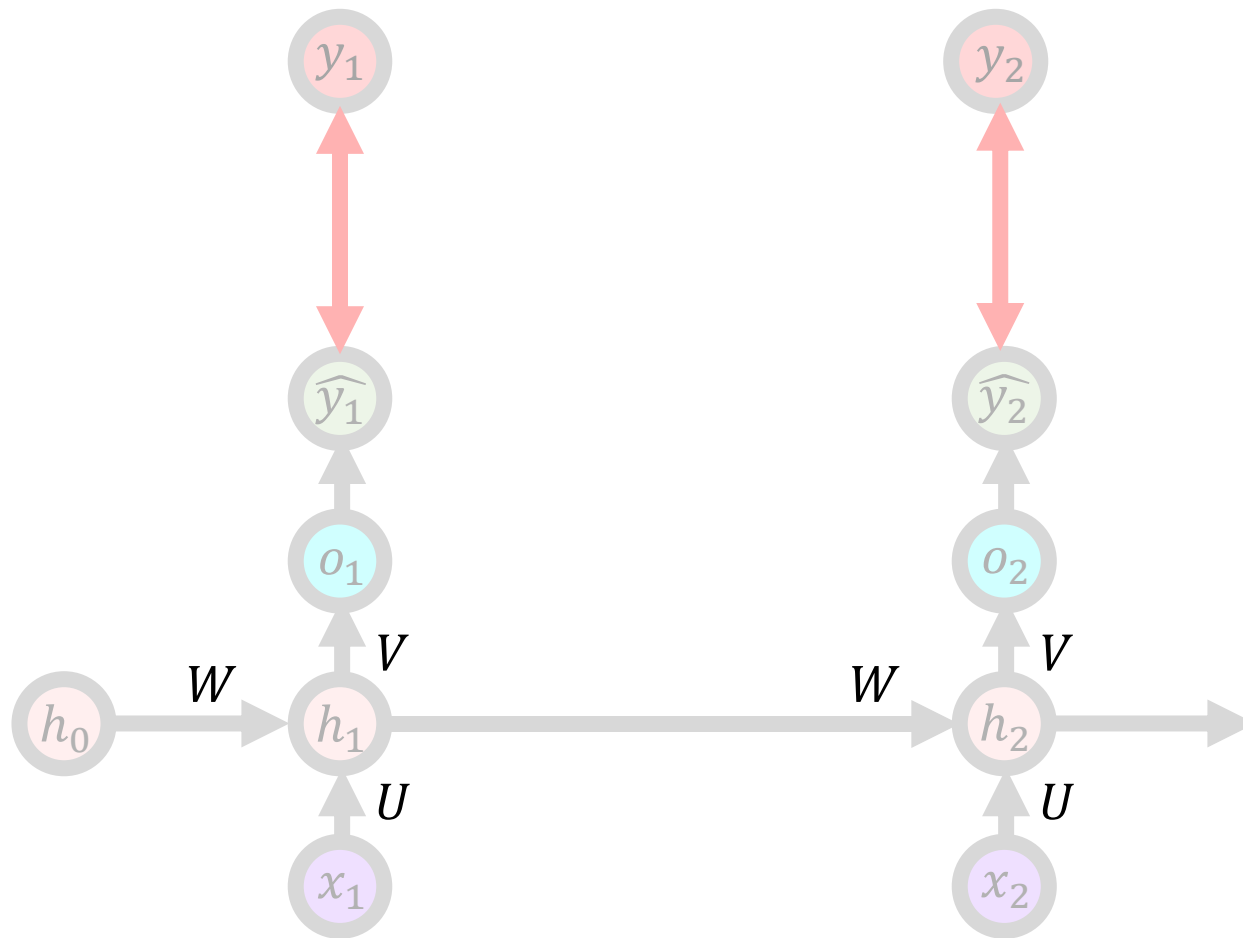


이제부터 하나씩 구해보도록 하겠습니다

$$V^* = V - \alpha \frac{\partial L}{\partial V}$$

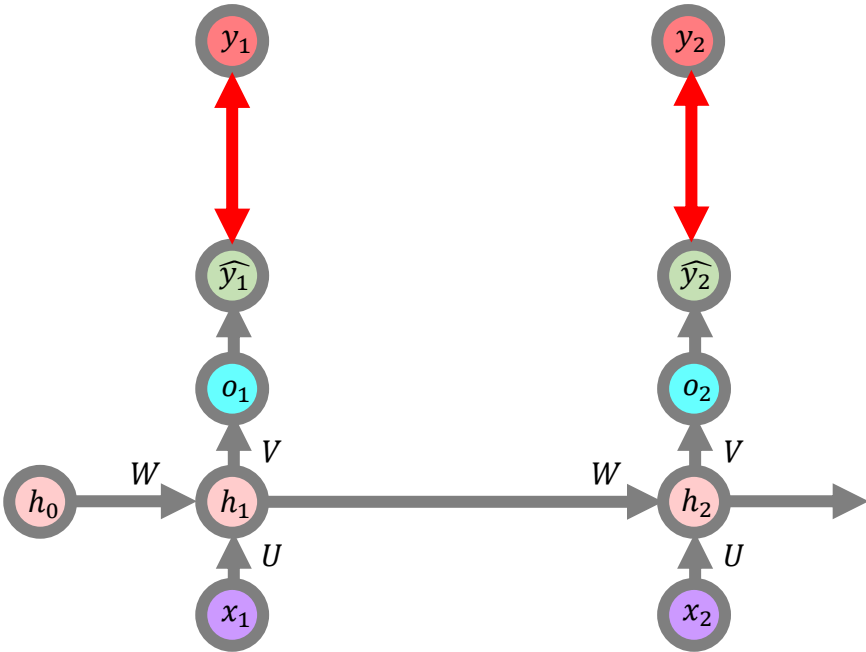
$$W^* = W - \alpha \frac{\partial L}{\partial W}$$

$$U^* = U - \alpha \frac{\partial L}{\partial U}$$



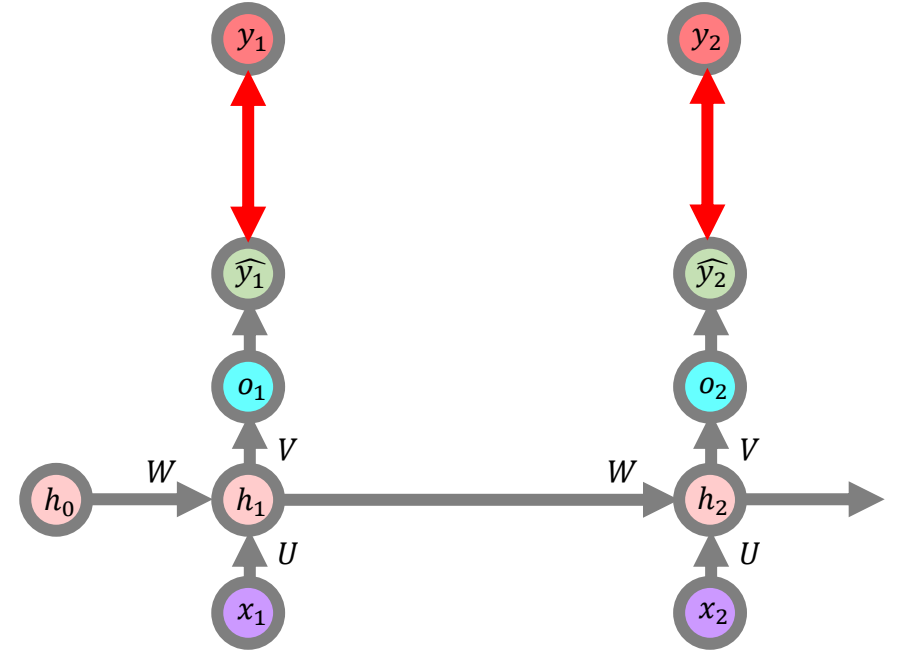
수식을 많이 써야하기 때문에 글자크기와 그림크기를 좀 줄였습니다

$$\frac{\partial L}{\partial V}$$



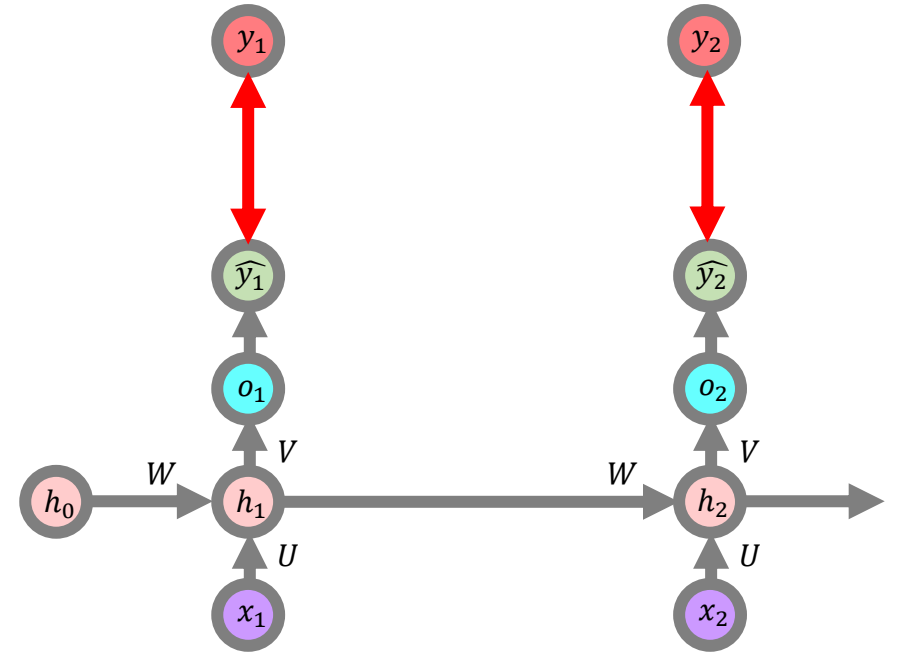
우선 $\frac{\partial L}{\partial V}$ 은 입력시퀀스 전체에 대한 기울기이므로 다음과 같이 표현할 수 있습니다

$$\frac{\partial L}{\partial V} = \frac{\partial L_1}{\partial V} + \frac{\partial L_2}{\partial V}$$



$\frac{\partial L}{\partial V}$ 의 값을 구하기 위해 우리는 연쇄법칙을 이용합니다

$$\frac{\partial L}{\partial V} = \frac{\partial L_1}{\partial V} + \frac{\partial L_2}{\partial V}$$

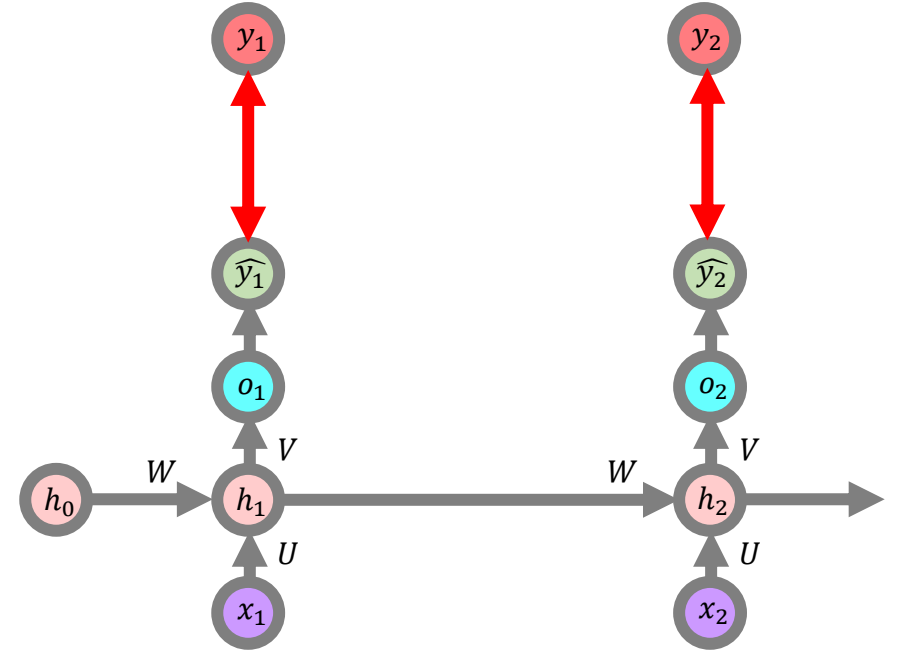


Chain rule..

연쇄법칙을 이용하면 $\frac{\partial L_1}{\partial V}$ 와 $\frac{\partial L_2}{\partial V}$ 는 다음과 같이 분해될 수 있습니다

$$\frac{\partial L}{\partial V} = \frac{\partial L_1}{\partial V} + \frac{\partial L_2}{\partial V}$$

$$\frac{\partial L}{\partial V} = \frac{\partial L_1}{\partial \hat{y}_1} \frac{\partial \hat{y}_1}{\partial o_1} \frac{\partial o_1}{\partial V} + \frac{\partial L_2}{\partial \hat{y}_2} \frac{\partial \hat{y}_2}{\partial o_2} \frac{\partial o_2}{\partial V}$$

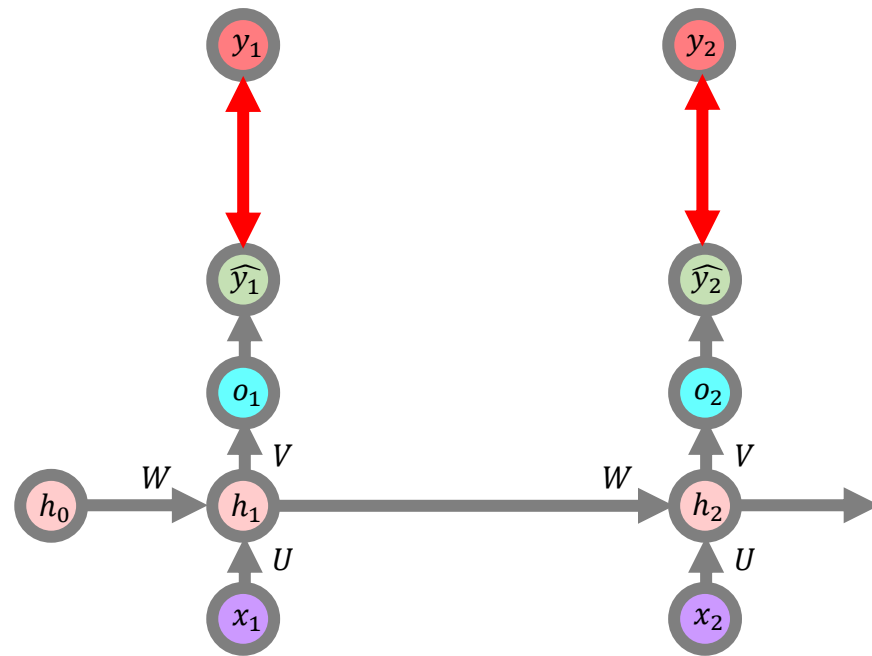


그리고 이 부분은 우리가 cross-entropy와 softmax를 쓰기 때문에 간단하게 기울기를 이렇게 계산할 수 있습니다

$$\frac{\partial L}{\partial V} = \frac{\partial L_1}{\partial V} + \frac{\partial L_2}{\partial V}$$

$$\frac{\partial L}{\partial V} = \frac{\partial L_1}{\partial \hat{y}_1} \frac{\partial \hat{y}_1}{\partial o_1} \frac{\partial o_1}{\partial V} + \frac{\partial L_2}{\partial \hat{y}_2} \frac{\partial \hat{y}_2}{\partial o_2} \frac{\partial o_2}{\partial V}$$

$$\frac{\partial L}{\partial V} = (\hat{y}_1 - y_1) \frac{\partial o_1}{\partial V} + (\hat{y}_2 - y_2) \frac{\partial o_2}{\partial V}$$

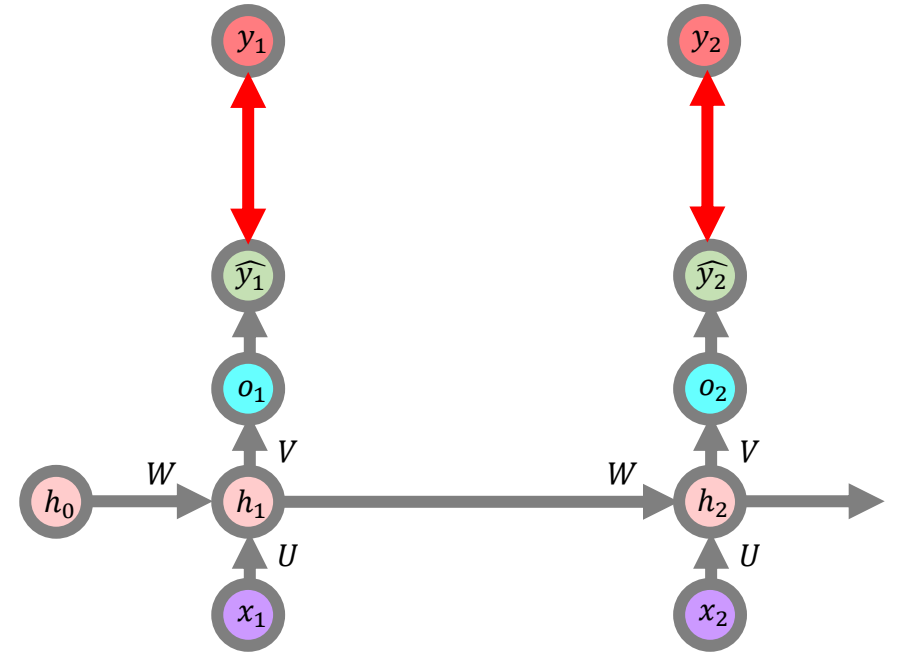


이 부분은 제가 다른 영상에서 도출과정을 보여드리려고 합니다.
 많이 기대해주세요 ;)

$$\frac{\partial L}{\partial V} = \frac{\partial L_1}{\partial V} + \frac{\partial L_2}{\partial V}$$

$$\frac{\partial L}{\partial V} = \frac{\partial L_1}{\partial \hat{y}_1} \frac{\partial \hat{y}_1}{\partial o_1} \frac{\partial o_1}{\partial V} + \frac{\partial L_2}{\partial \hat{y}_2} \frac{\partial \hat{y}_2}{\partial o_2} \frac{\partial o_2}{\partial V}$$

$$\frac{\partial L}{\partial V} = (\hat{y}_1 - y_1) \frac{\partial o_1}{\partial V} + (\hat{y}_2 - y_2) \frac{\partial o_2}{\partial V}$$

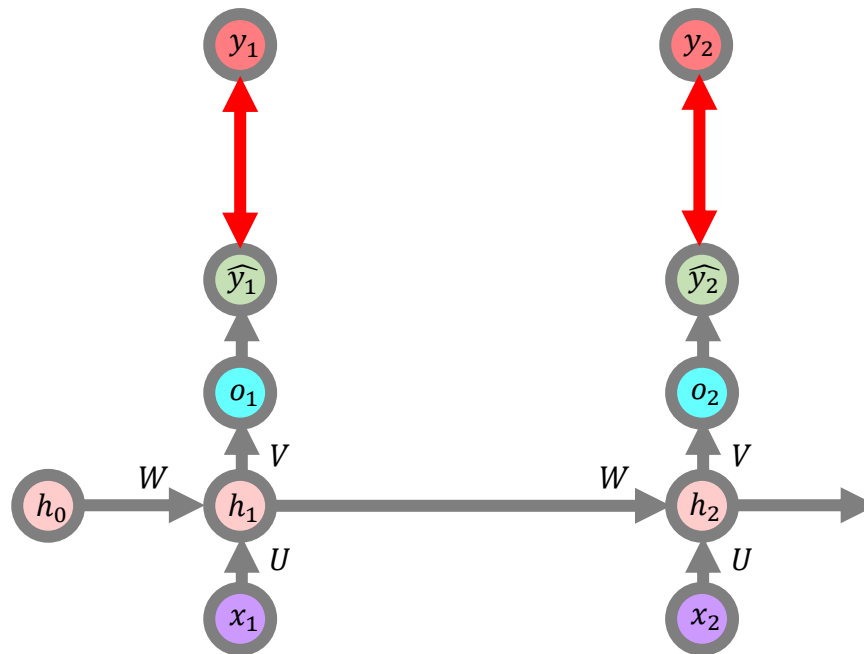


그리고 $\frac{\partial o_1}{\partial V}$ 은, 아래와 같은 공식에서 구할 수 있습니다

$$\frac{\partial L}{\partial V} = \frac{\partial L_1}{\partial V} + \frac{\partial L_2}{\partial V}$$

$$\frac{\partial L}{\partial V} = \frac{\partial L_1}{\partial \hat{y}_1} \frac{\partial \hat{y}_1}{\partial o_1} \frac{\partial o_1}{\partial V} + \frac{\partial L_2}{\partial \hat{y}_2} \frac{\partial \hat{y}_2}{\partial o_2} \frac{\partial o_2}{\partial V}$$

$$\frac{\partial L}{\partial V} = (\hat{y}_1 - y_1) \frac{\partial o_1}{\partial V} + (\hat{y}_2 - y_2) \frac{\partial o_2}{\partial V}$$



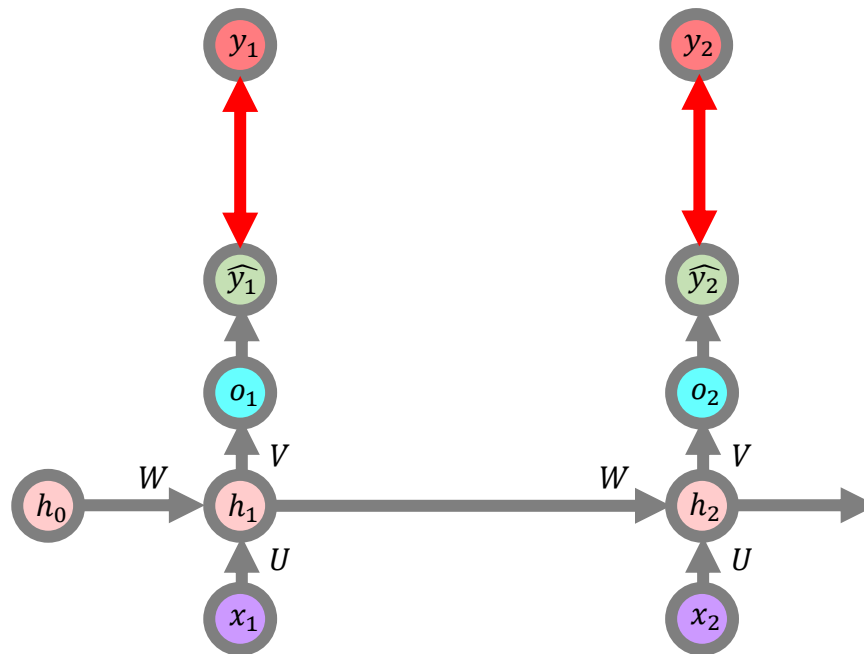
$$o_1 = Vh_1 \quad o_2 = Vh_2$$

그리고 $\frac{\partial o_1}{\partial V}$ 은, 아래와 같은 공식에서 구할 수 있습니다

$$\frac{\partial L}{\partial V} = \frac{\partial L_1}{\partial V} + \frac{\partial L_2}{\partial V}$$

$$\frac{\partial L}{\partial V} = \frac{\partial L_1}{\partial \hat{y}_1} \frac{\partial \hat{y}_1}{\partial o_1} \frac{\partial o_1}{\partial V} + \frac{\partial L_2}{\partial \hat{y}_2} \frac{\partial \hat{y}_2}{\partial o_2} \frac{\partial o_2}{\partial V}$$

$$\frac{\partial L}{\partial V} = (\hat{y}_1 - y_1) \frac{\partial o_1}{\partial V} + (\hat{y}_2 - y_2) \frac{\partial o_2}{\partial V}$$



$$o_1 = Vh_1$$

$$o_2 = Vh_2$$

$$\frac{\partial o_1}{\partial V} = h_1$$

$$\frac{\partial o_2}{\partial V} = h_2$$

그러므로 $\frac{\partial L}{\partial V}$ 는 다음과 같이 계산할 수 있습니다

$$\frac{\partial L}{\partial V} = \frac{\partial L_1}{\partial V} + \frac{\partial L_2}{\partial V}$$

$$\frac{\partial L}{\partial V} = \frac{\partial L_1}{\partial \hat{y}_1} \frac{\partial \hat{y}_1}{\partial o_1} \frac{\partial o_1}{\partial V} + \frac{\partial L_2}{\partial \hat{y}_2} \frac{\partial \hat{y}_2}{\partial o_2} \frac{\partial o_2}{\partial V}$$

$$\frac{\partial L}{\partial V} = (\hat{y}_1 - y_1) \frac{\partial o_1}{\partial V} + (\hat{y}_2 - y_2) \frac{\partial o_2}{\partial V}$$

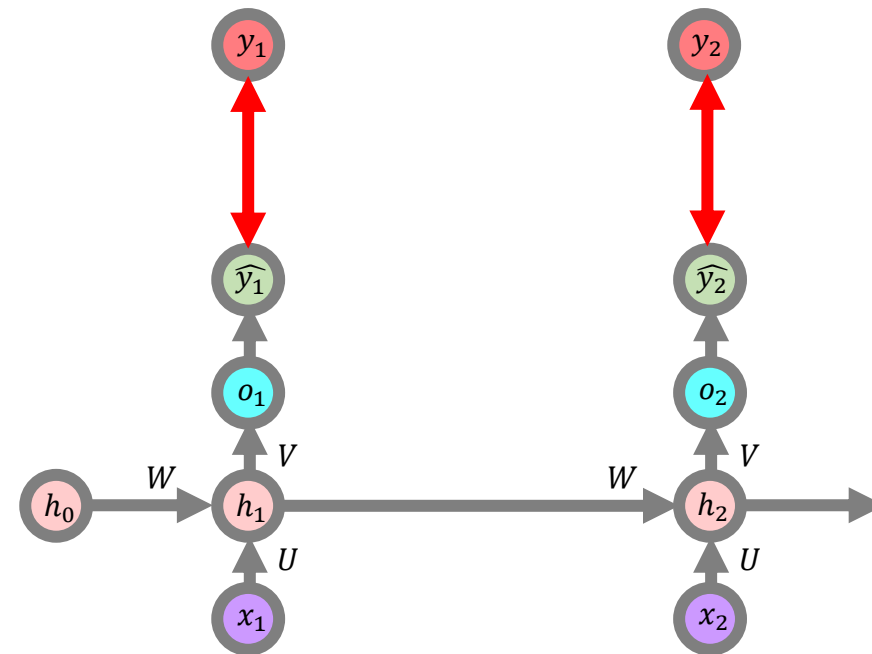
$$\frac{\partial L}{\partial V} = (\hat{y}_1 - y_1)h_1 + (\hat{y}_2 - y_2)h_2$$

$$o_1 = Vh_1$$

$$o_2 = Vh_2$$

$$\frac{\partial o_1}{\partial V} = h_1$$

$$\frac{\partial o_2}{\partial V} = h_2$$



그러므로 $\frac{\partial L}{\partial V}$ 는 다음과 같이 계산할 수 있습니다

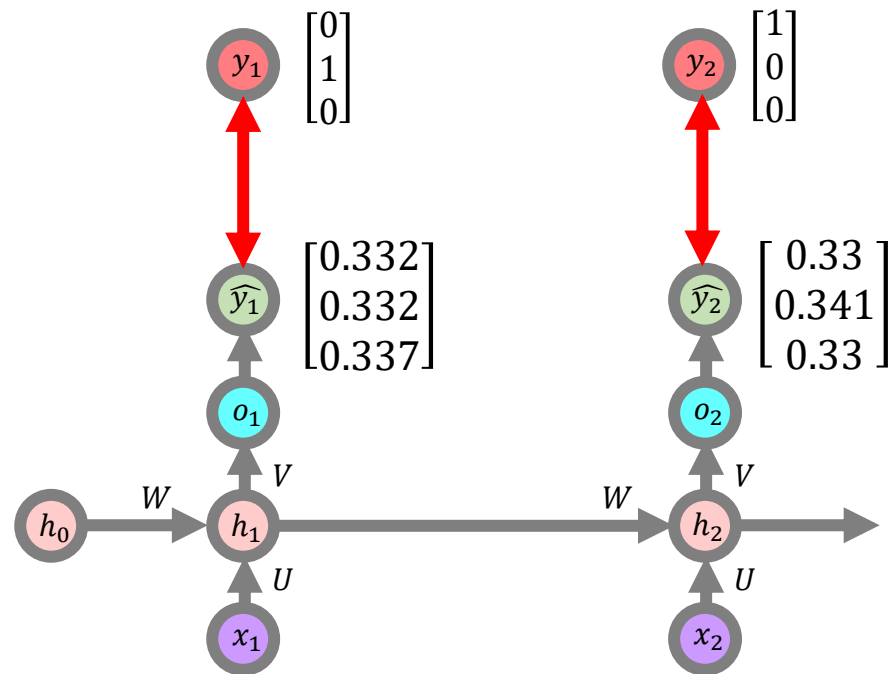
$$\frac{\partial L}{\partial V} = \frac{\partial L_1}{\partial V} + \frac{\partial L_2}{\partial V}$$

$$\frac{\partial L}{\partial V} = \frac{\partial L_1}{\partial \hat{y}_1} \frac{\partial \hat{y}_1}{\partial o_1} \frac{\partial o_1}{\partial V} + \frac{\partial L_2}{\partial \hat{y}_2} \frac{\partial \hat{y}_2}{\partial o_2} \frac{\partial o_2}{\partial V}$$

$$\frac{\partial L}{\partial V} = (\hat{y}_1 - y_1) \frac{\partial o_1}{\partial V} + (\hat{y}_2 - y_2) \frac{\partial o_2}{\partial V}$$

$$\frac{\partial L}{\partial V} = (\hat{y}_1 - y_1)h_1 + (\hat{y}_2 - y_2)h_2$$

$$\frac{\partial L}{\partial V} = \left(\begin{bmatrix} 0.332 \\ 0.332 \\ 0.337 \end{bmatrix} - \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \right) [0.094 \quad 0.134] + \left(\begin{bmatrix} 0.33 \\ 0.341 \\ 0.33 \end{bmatrix} - \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \right) [0.15 \quad -0.019]$$



그러므로 $\frac{\partial L}{\partial V}$ 는 다음과 같이 계산할 수 있습니다

$$\frac{\partial L}{\partial V} = \frac{\partial L_1}{\partial V} + \frac{\partial L_2}{\partial V}$$

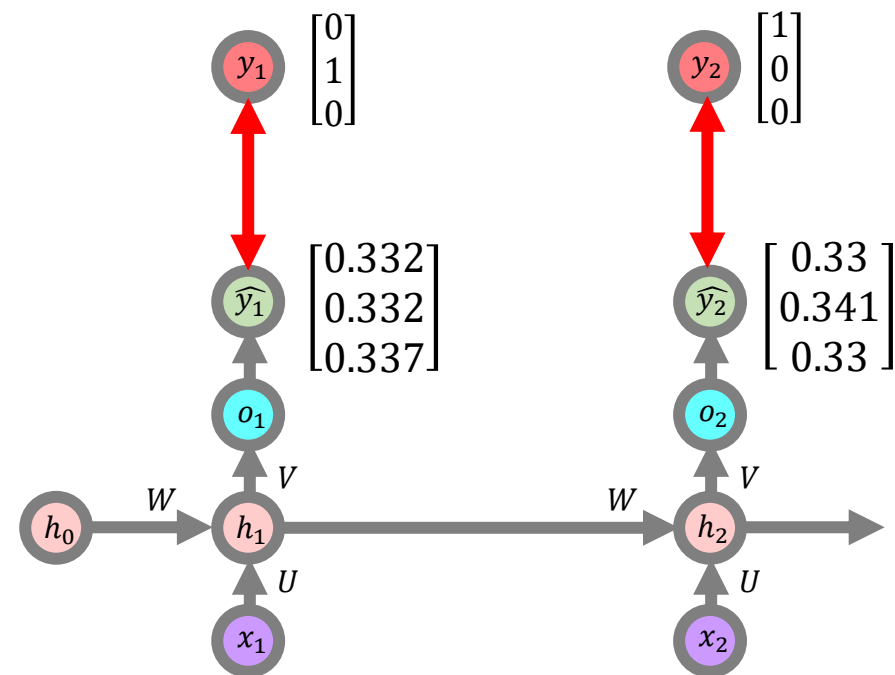
$$\frac{\partial L}{\partial V} = \frac{\partial L_1}{\partial \hat{y}_1} \frac{\partial \hat{y}_1}{\partial o_1} \frac{\partial o_1}{\partial V} + \frac{\partial L_2}{\partial \hat{y}_2} \frac{\partial \hat{y}_2}{\partial o_2} \frac{\partial o_2}{\partial V}$$

$$\frac{\partial L}{\partial V} = (\hat{y}_1 - y_1) \frac{\partial o_1}{\partial V} + (\hat{y}_2 - y_2) \frac{\partial o_2}{\partial V}$$

$$\frac{\partial L}{\partial V} = (\hat{y}_1 - y_1)h_1 + (\hat{y}_2 - y_2)h_2$$

$$\frac{\partial L}{\partial V} = \left(\begin{bmatrix} 0.332 \\ 0.332 \\ 0.337 \end{bmatrix} - \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \right) [0.094 \quad 0.134] + \left(\begin{bmatrix} 0.33 \\ 0.341 \\ 0.33 \end{bmatrix} - \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \right) [0.15 \quad -0.019]$$

$$\frac{\partial L}{\partial V} = \begin{pmatrix} 0.031 & 0.045 \\ -0.063 & -0.09 \\ 0.032 & 0.045 \end{pmatrix} + \begin{pmatrix} -0.101 & 0.012 \\ 0.051 & -0.006 \\ 0.05 & -0.006 \end{pmatrix}$$



그러므로 $\frac{\partial L}{\partial V}$ 는 다음과 같이 계산할 수 있습니다

$$\frac{\partial L}{\partial V} = \frac{\partial L_1}{\partial V} + \frac{\partial L_2}{\partial V}$$

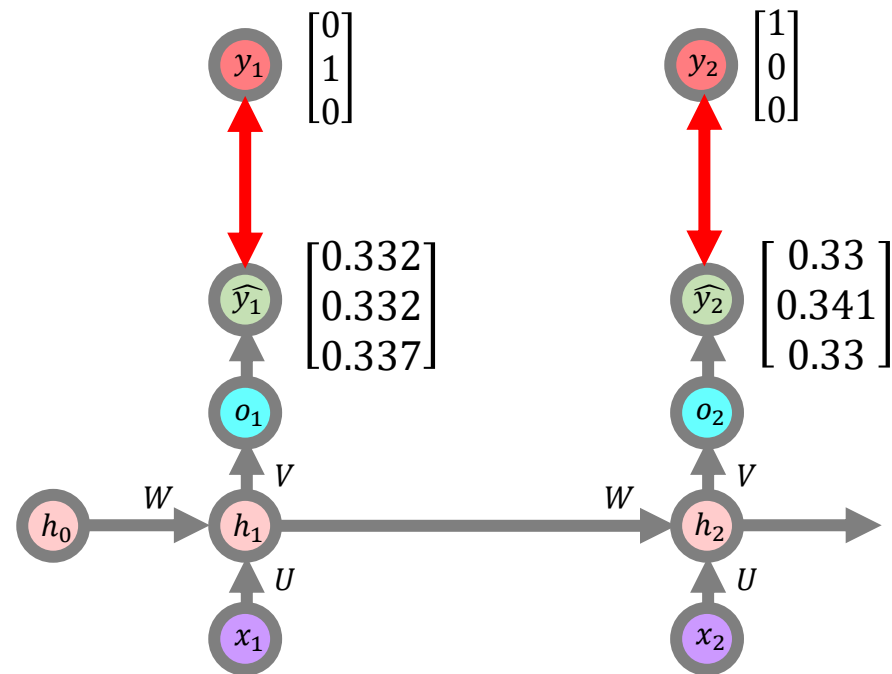
$$\frac{\partial L}{\partial V} = \frac{\partial L_1}{\partial \hat{y}_1} \frac{\partial \hat{y}_1}{\partial o_1} \frac{\partial o_1}{\partial V} + \frac{\partial L_2}{\partial \hat{y}_2} \frac{\partial \hat{y}_2}{\partial o_2} \frac{\partial o_2}{\partial V}$$

$$\frac{\partial L}{\partial V} = (\hat{y}_1 - y_1) \frac{\partial o_1}{\partial V} + (\hat{y}_2 - y_2) \frac{\partial o_2}{\partial V}$$

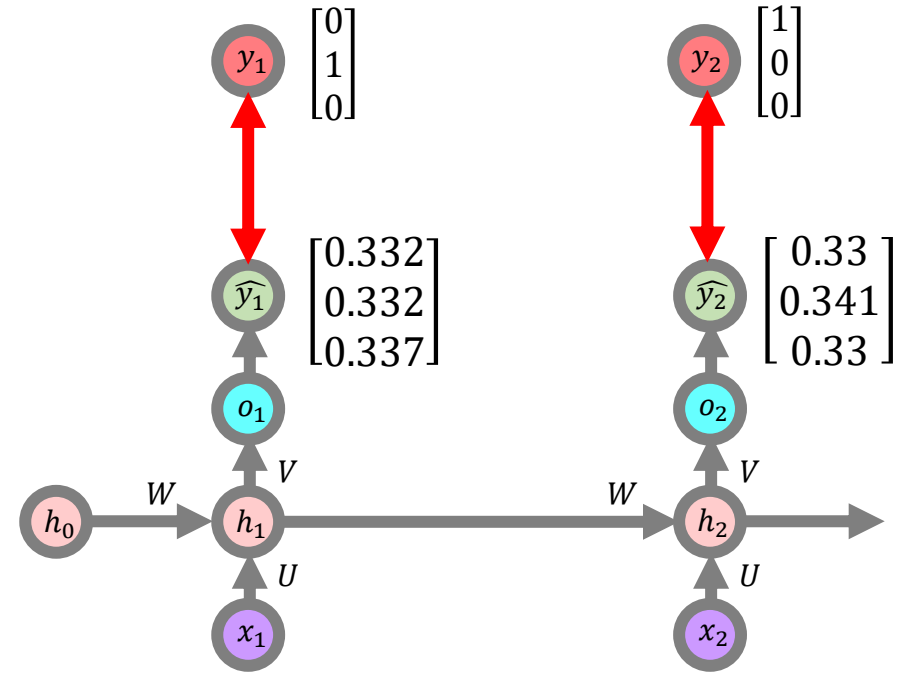
$$\frac{\partial L}{\partial V} = (\hat{y}_1 - y_1)h_1 + (\hat{y}_2 - y_2)h_2$$

$$\frac{\partial L}{\partial V} = \left(\begin{bmatrix} 0.332 \\ 0.332 \\ 0.337 \end{bmatrix} - \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \right) [0.094 \quad 0.134] + \left(\begin{bmatrix} 0.33 \\ 0.341 \\ 0.33 \end{bmatrix} - \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \right) [0.15 \quad -0.019]$$

$$\frac{\partial L}{\partial V} = \begin{pmatrix} 0.031 & 0.045 \\ -0.063 & -0.09 \\ 0.032 & 0.045 \end{pmatrix} + \begin{pmatrix} -0.101 & 0.012 \\ 0.051 & -0.006 \\ 0.05 & -0.006 \end{pmatrix} = \begin{pmatrix} -0.07 & 0.057 \\ -0.011 & -0.096 \\ 0.081 & 0.039 \end{pmatrix}$$

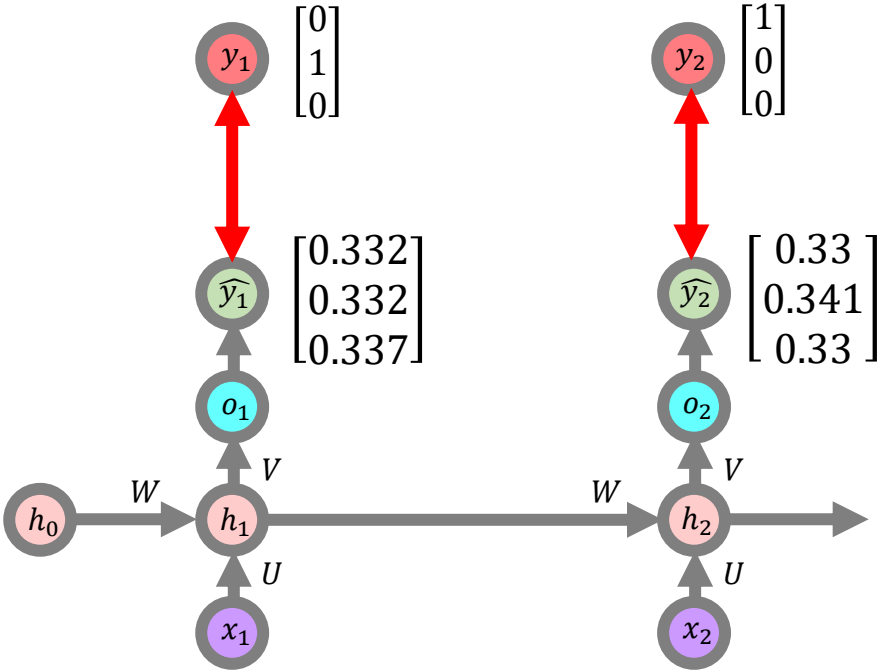


자 이제는 $\frac{\partial L}{\partial W}$ 를 계산할 차례입니다



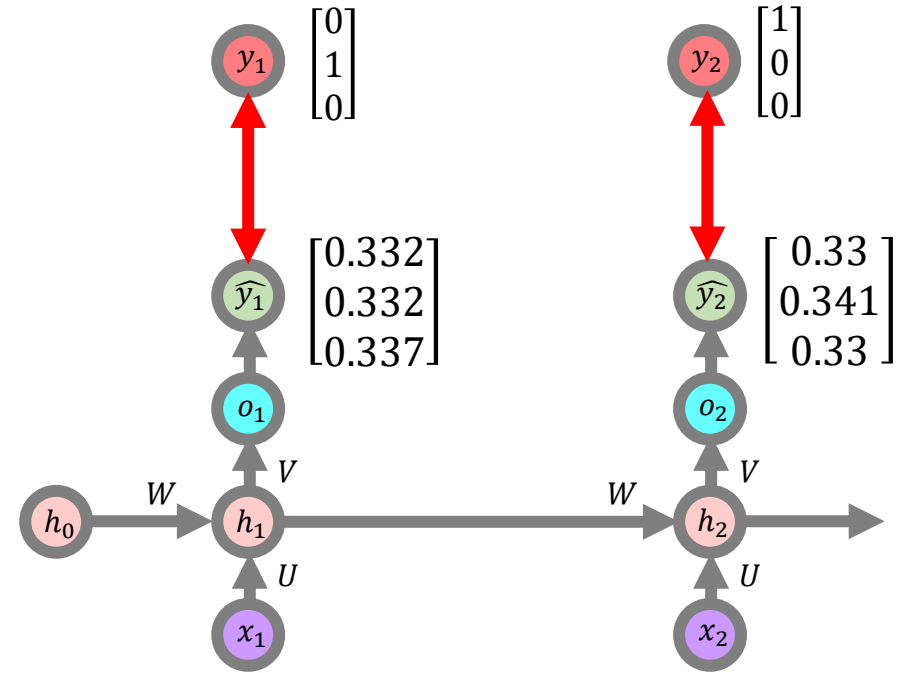
시간 순서 역순대로 먼저 $\frac{\partial L_2}{\partial W}$ 를 계산해보도록 하겠습니다

$$\frac{\partial L_2}{\partial W} =$$



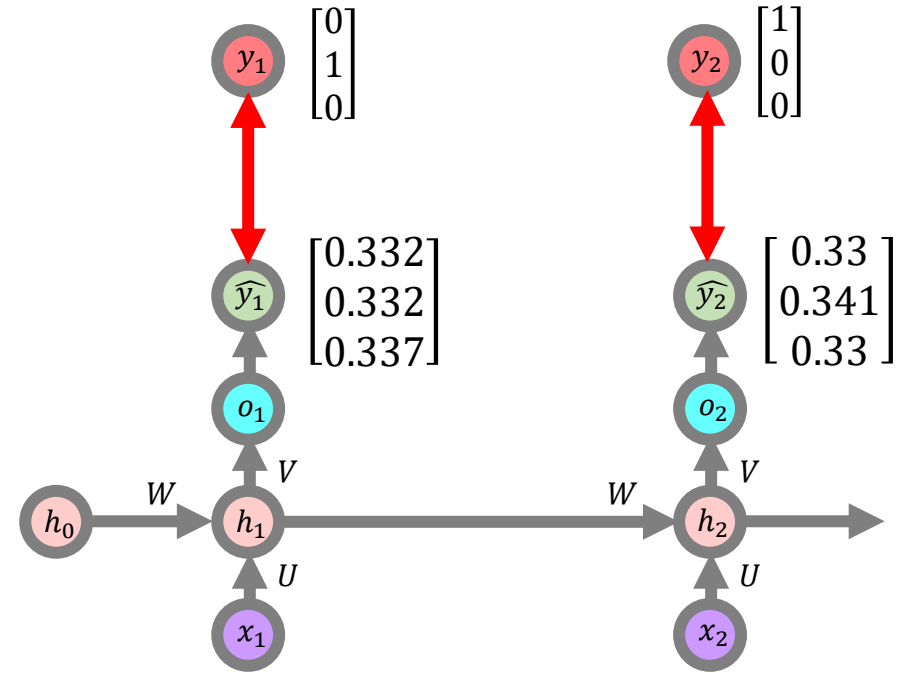
$\frac{\partial L_2}{\partial W}$ 는 체인룰에 의하여 다음과 같이 쓸수 있습니다

$$\frac{\partial L_2}{\partial W} = \frac{\partial L_2}{\partial \hat{y}_2} \frac{\partial \hat{y}_2}{\partial o_2} \frac{\partial o_2}{\partial h_2} \frac{\partial h_2}{\partial W}$$



이 식의 의미는..

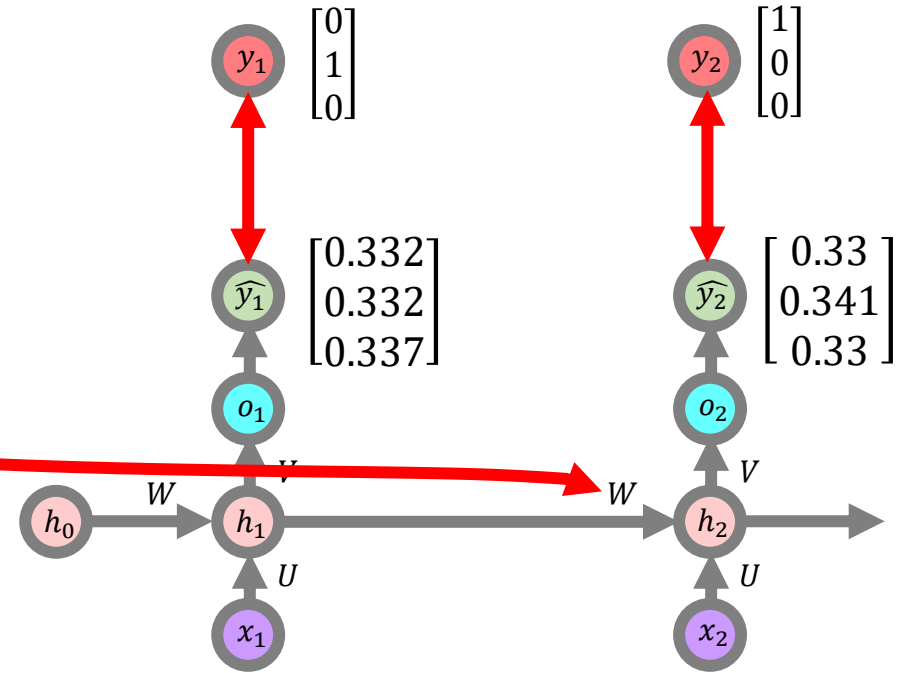
$$\frac{\partial L_2}{\partial W} = \frac{\partial L_2}{\partial \hat{y}_2} \frac{\partial \hat{y}_2}{\partial o_2} \frac{\partial o_2}{\partial h_2} \frac{\partial h_2}{\partial W}$$



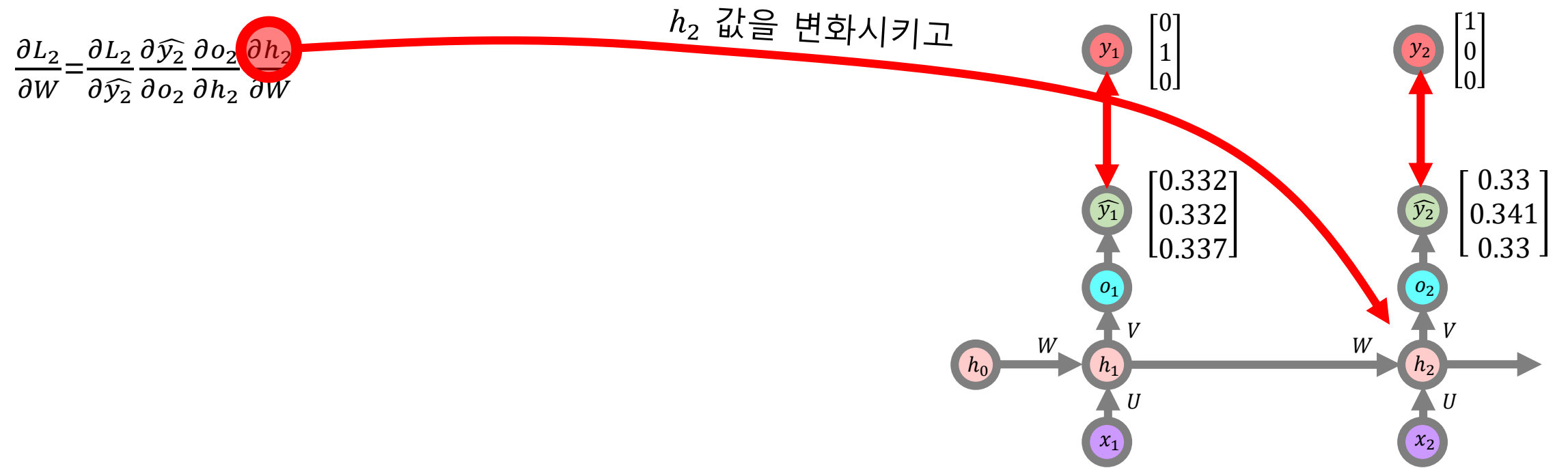
이 식의 의미는..

$$\frac{\partial L_2}{\partial W} = \frac{\partial L_2}{\partial \hat{y}_2} \frac{\partial \hat{y}_2}{\partial o_2} \frac{\partial o_2}{\partial h_2} \frac{\partial h_2}{\partial W}$$

이 가중치 W의 변화가..



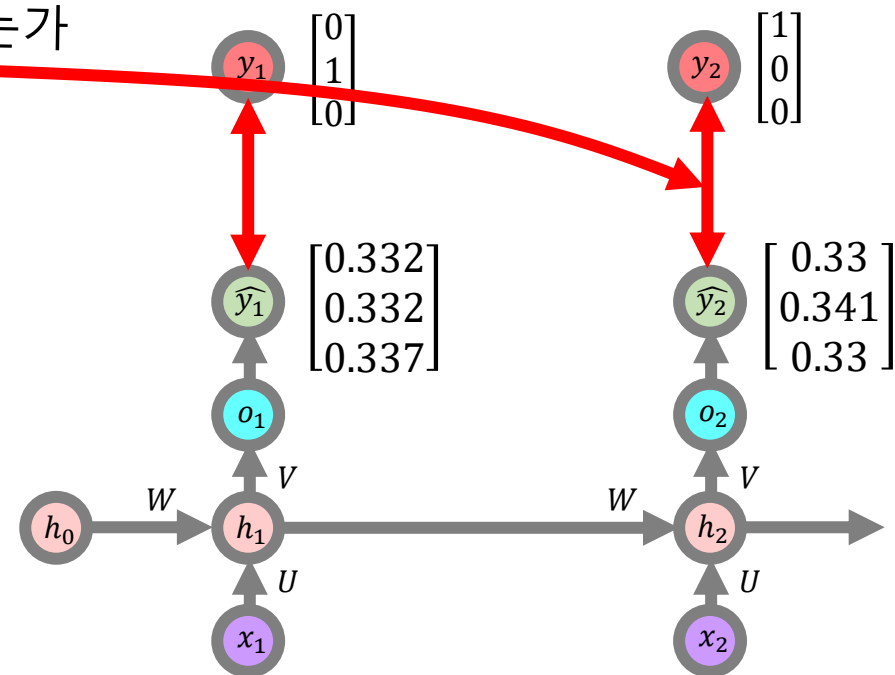
이 식의 의미는..



...에 관한 식이라고 할 수가 있습니다

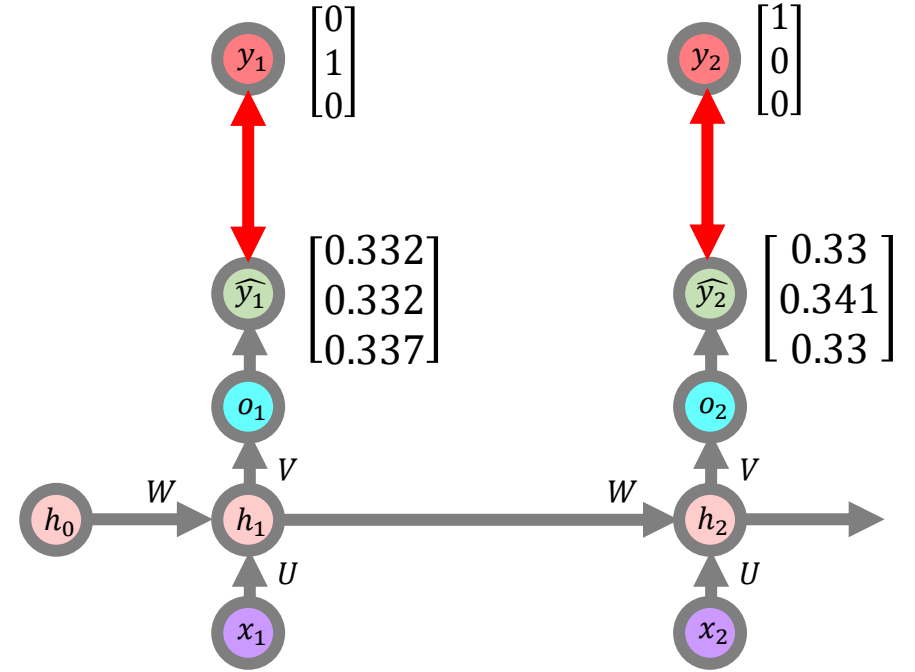
그 변화가 L_2 값을 얼마나 변화시키는지

$$\frac{\partial L_2}{\partial W} = \frac{\partial L_2}{\partial \hat{y}_2} \frac{\partial \hat{y}_2}{\partial o_2} \frac{\partial o_2}{\partial h_2} \frac{\partial h_2}{\partial W}$$



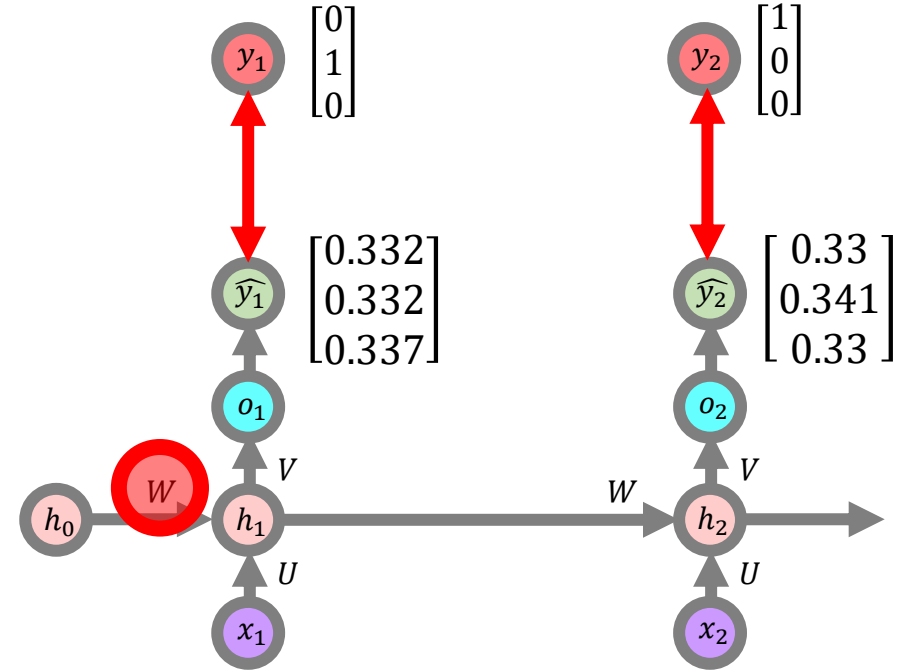
그런데 L_2 값에 변화를 주는 W 의 변화는 이것만이 아닙니다

$$\frac{\partial L_2}{\partial W} = \frac{\partial L_2}{\partial \hat{y}_2} \frac{\partial \hat{y}_2}{\partial o_2} \frac{\partial o_2}{\partial h_2} \frac{\partial h_2}{\partial W}$$



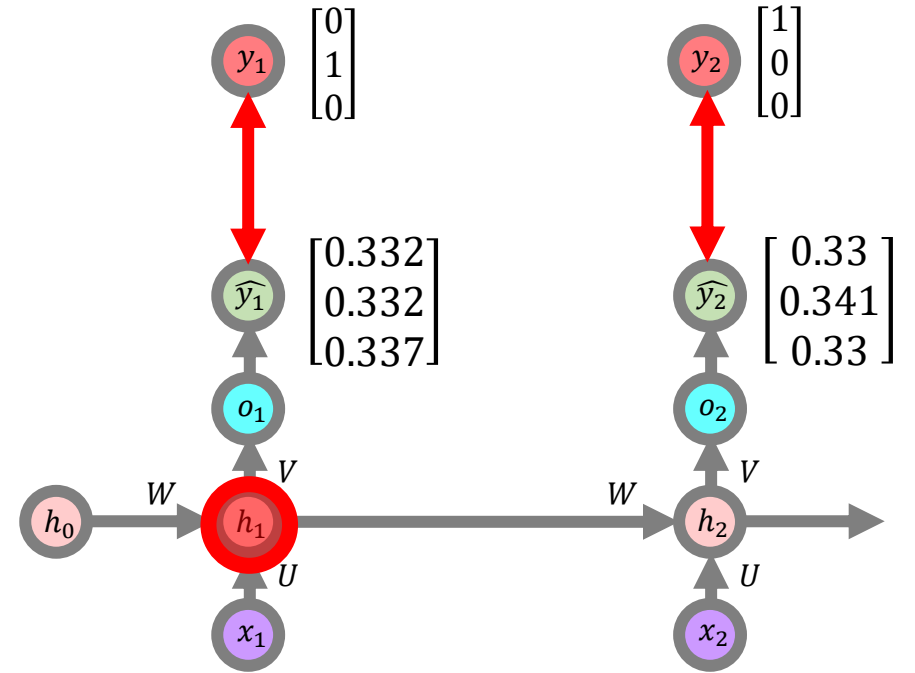
t=1에서의 W 의 변화 또한

$$\frac{\partial L_2}{\partial W} = \frac{\partial L_2}{\partial \hat{y}_2} \frac{\partial \hat{y}_2}{\partial o_2} \frac{\partial o_2}{\partial h_2} \frac{\partial h_2}{\partial W}$$



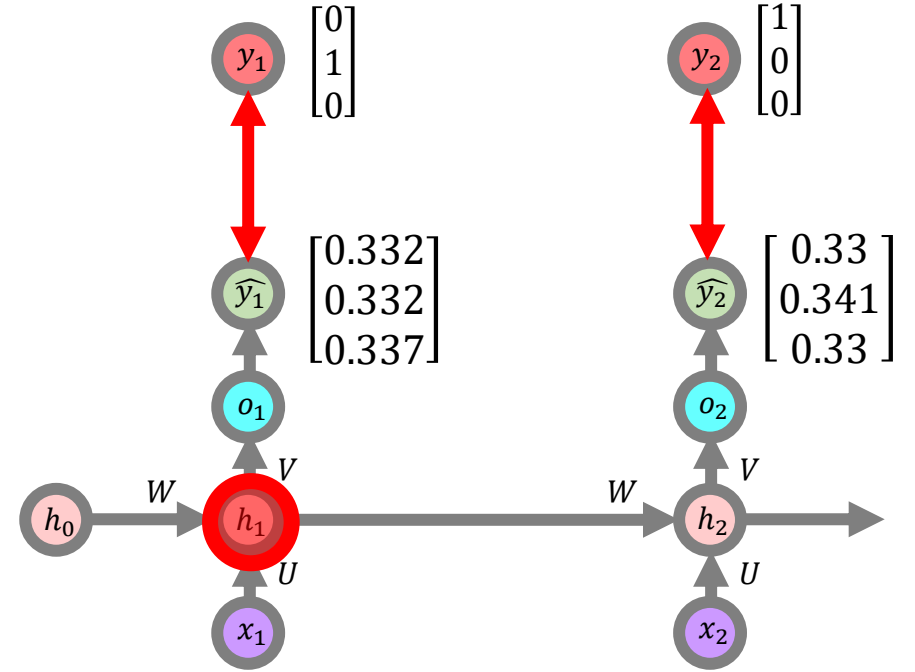
h_1 의 변화를 가져오고

$$\frac{\partial L_2}{\partial W} = \frac{\partial L_2}{\partial \widehat{y}_2} \frac{\partial \widehat{y}_2}{\partial o_2} \frac{\partial o_2}{\partial h_2} \frac{\partial h_2}{\partial W}$$



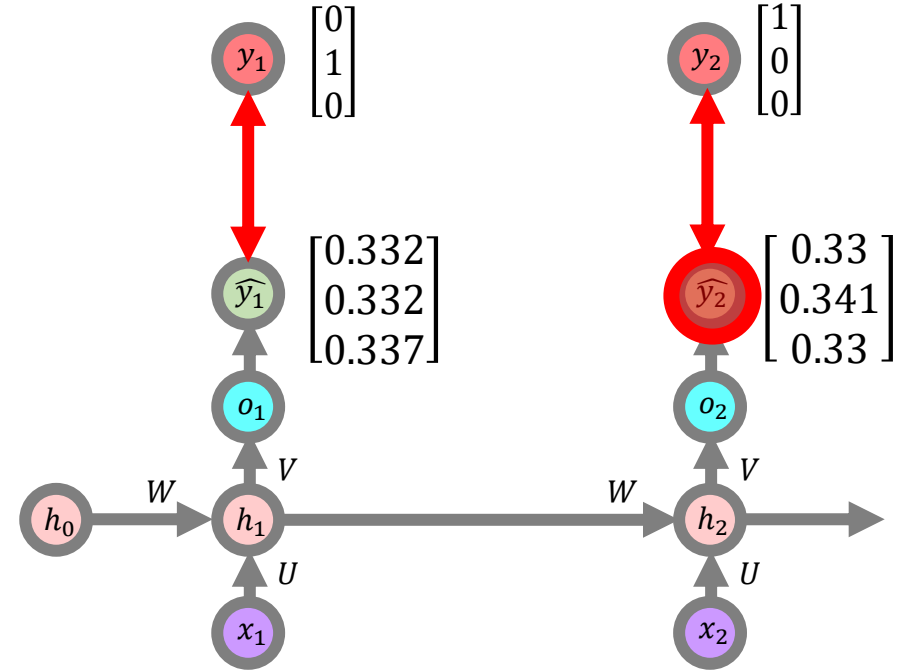
또한 h_1 의 변화는 h_2 의 변화를 낳고..

$$\frac{\partial L_2}{\partial W} = \frac{\partial L_2}{\partial \hat{y}_2} \frac{\partial \hat{y}_2}{\partial o_2} \frac{\partial o_2}{\partial h_2} \frac{\partial h_2}{\partial W}$$



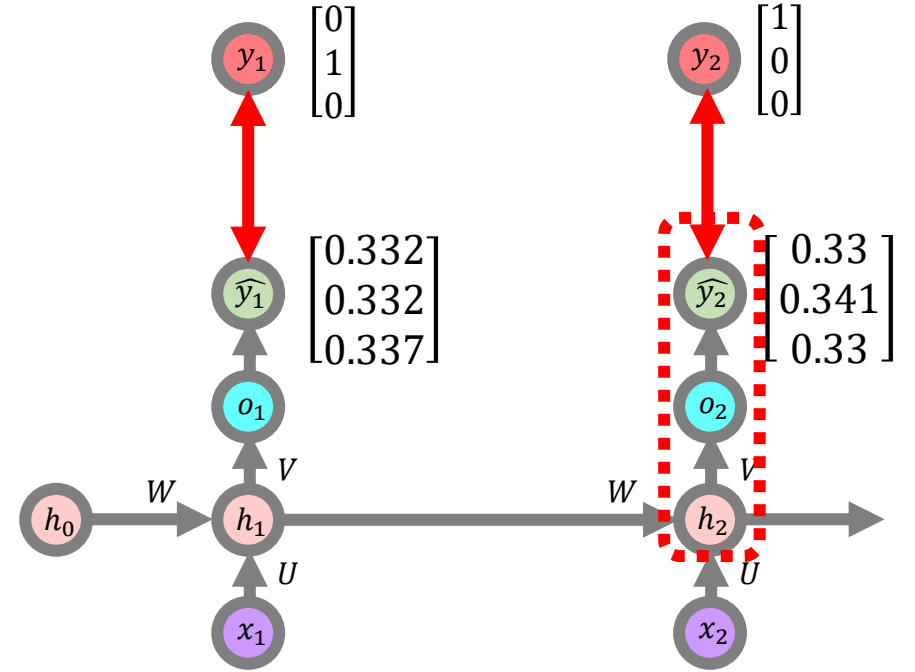
또한 h_2 의 변화는 결국 \hat{y}_2 의 변화를 가져옵니다

$$\frac{\partial L_2}{\partial W} = \frac{\partial L_2}{\partial \hat{y}_2} \frac{\partial \hat{y}_2}{\partial o_2} \frac{\partial o_2}{\partial h_2} \frac{\partial h_2}{\partial W}$$



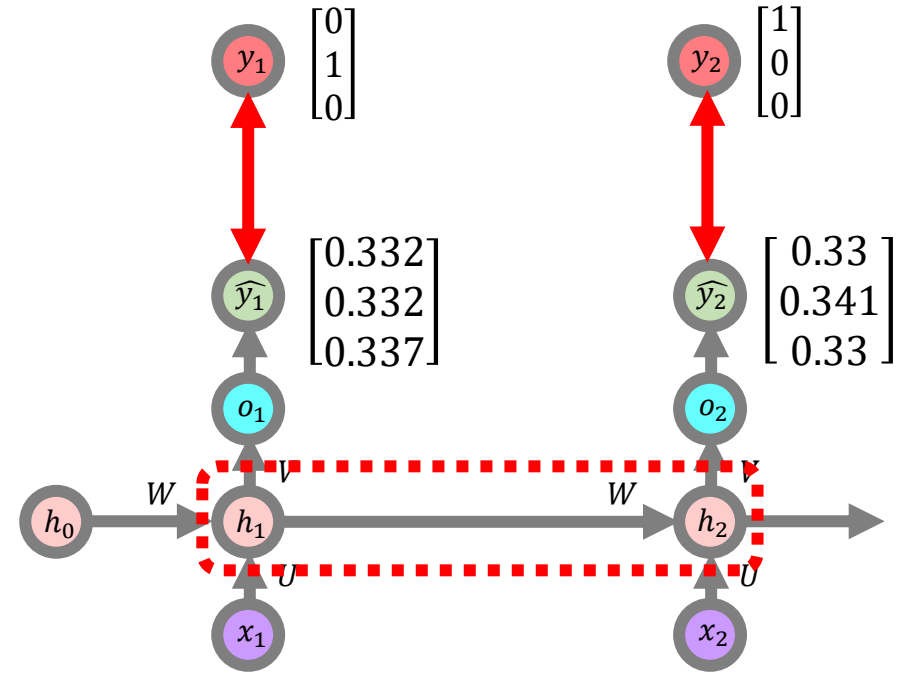
즉 정리하자면, $\frac{\partial L_2}{\partial W}$ 의 변화는 t=2에서 h_2 의 변화와

$$\frac{\partial L_2}{\partial W} = \frac{\partial L_2}{\partial \hat{y}_2} \frac{\partial \hat{y}_2}{\partial o_2} \frac{\partial o_2}{\partial h_2} \frac{\partial h_2}{\partial W}$$



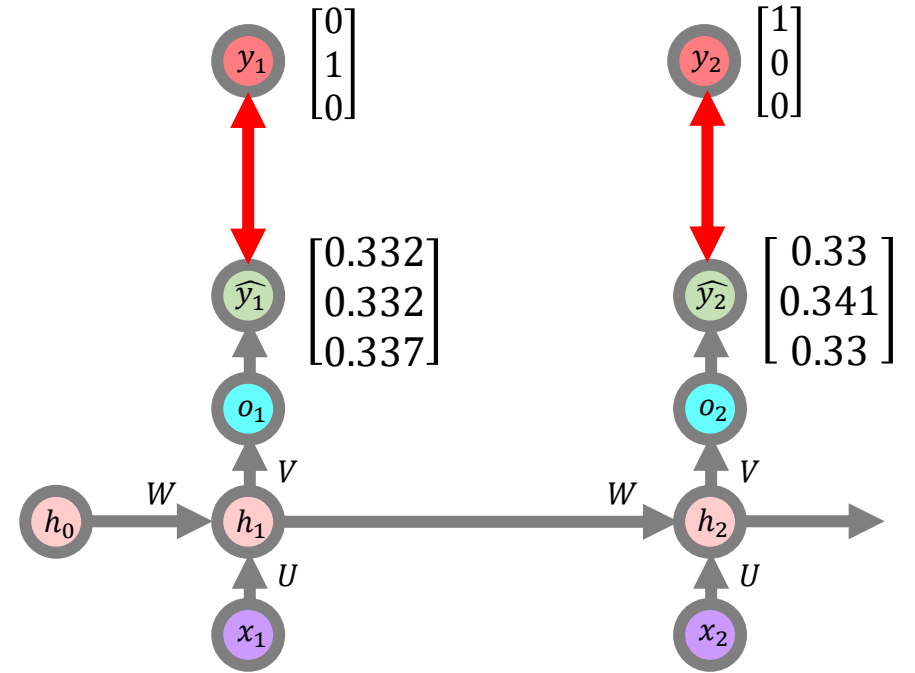
t=1에서 h_1 의 변화가 이끌어내는 h_2 의 변화가 다 같이 영향을 준다고 볼 수 있습니다

$$\frac{\partial L_2}{\partial W} = \frac{\partial L_2}{\partial \hat{y}_2} \frac{\partial \hat{y}_2}{\partial o_2} \frac{\partial o_2}{\partial h_2} \frac{\partial h_2}{\partial W}$$



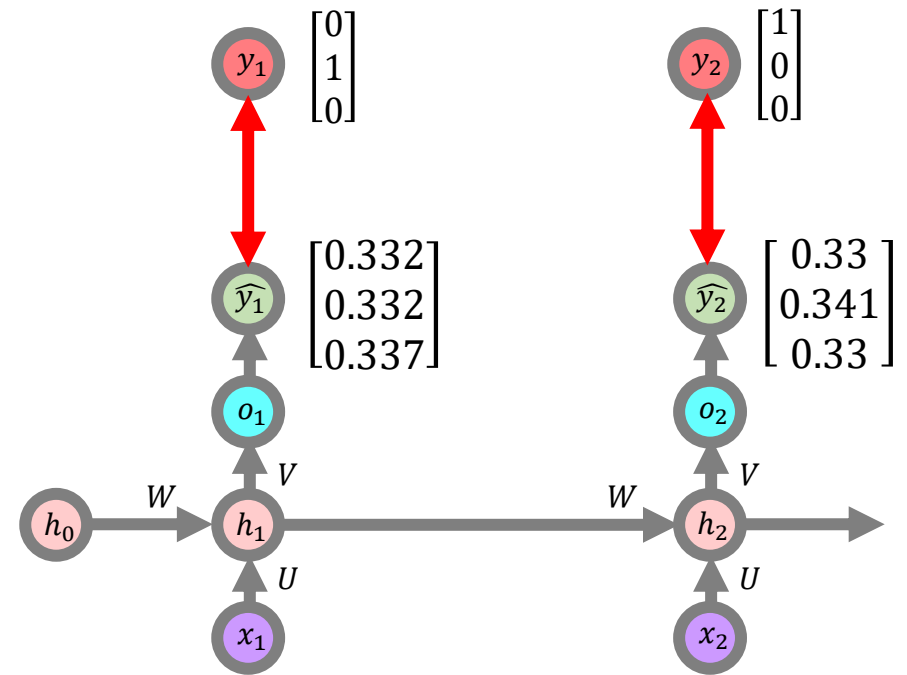
그래서 $\frac{\partial L_2}{\partial W}$ 을 더 완전한 모습으로 표현하면 다음과 같이 됩니다

$$\frac{\partial L_2}{\partial W} = \frac{\partial L_2}{\partial \hat{y}_2} \frac{\partial \hat{y}_2}{\partial o_2} \frac{\partial o_2}{\partial h_2} \frac{\partial h_2}{\partial W} + \frac{\partial L_2}{\partial \hat{y}_2} \frac{\partial \hat{y}_2}{\partial o_2} \frac{\partial o_2}{\partial h_2} \frac{\partial h_2}{\partial h_1} \frac{\partial h_1}{\partial W}$$



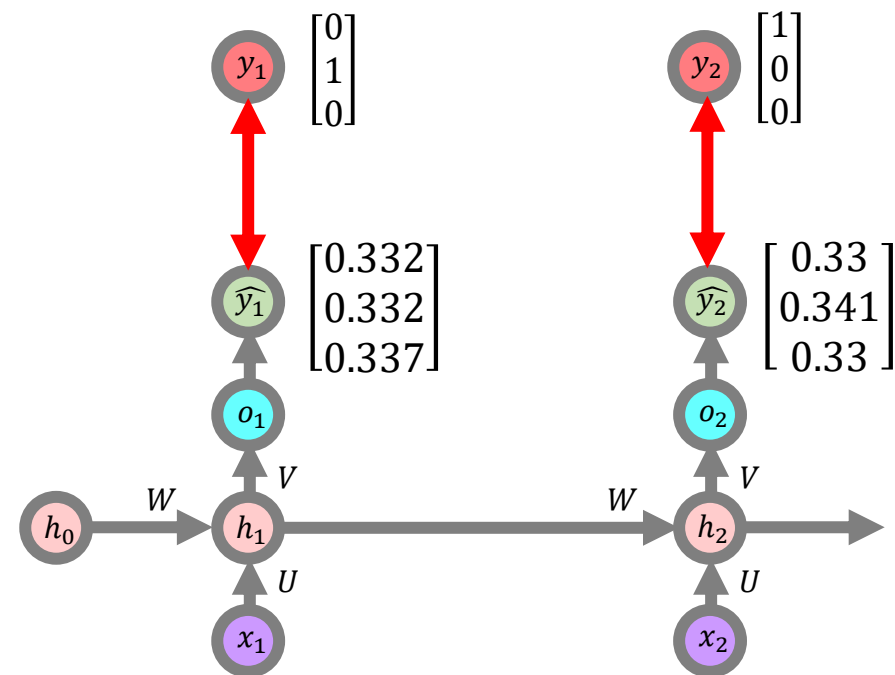
이 부분이 h_1 과 h_2 의 관계 변화를 나타내는 부분입니다

$$\frac{\partial L_2}{\partial W} = \frac{\partial L_2}{\partial \hat{y}_2} \frac{\partial \hat{y}_2}{\partial o_2} \frac{\partial o_2}{\partial h_2} \frac{\partial h_2}{\partial W} + \frac{\partial L_2}{\partial \hat{y}_2} \frac{\partial \hat{y}_2}{\partial o_2} \frac{\partial o_2}{\partial h_2} \frac{\partial h_2}{\partial h_1} \frac{\partial h_1}{\partial W}$$



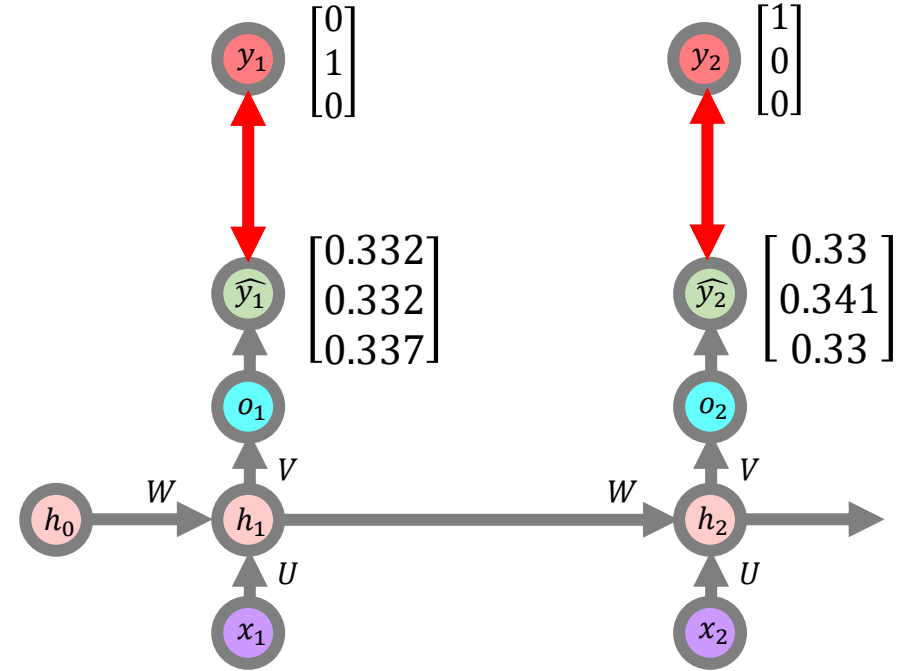
그리고 이 부분은 앞서 계산한 방법과 동일하게

$$\frac{\partial L_2}{\partial W} = \underbrace{\frac{\partial L_2}{\partial \hat{y}_2} \frac{\partial \hat{y}_2}{\partial o_2} \frac{\partial o_2}{\partial h_2} \frac{\partial h_2}{\partial W}} + \underbrace{\frac{\partial L_2}{\partial \hat{y}_2} \frac{\partial \hat{y}_2}{\partial o_2} \frac{\partial o_2}{\partial h_2} \frac{\partial h_2}{\partial h_1} \frac{\partial h_1}{\partial W}}$$



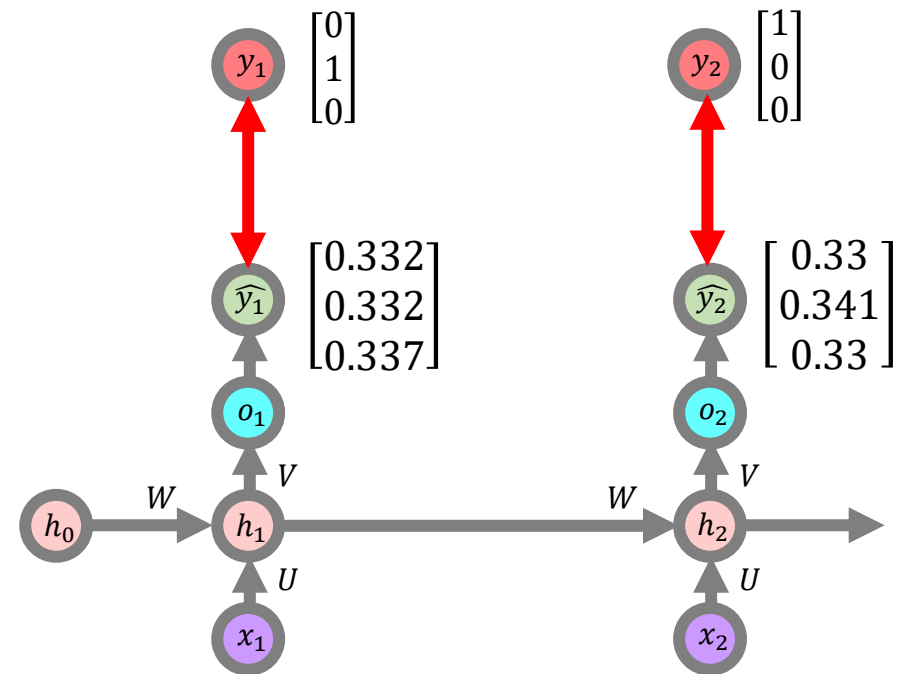
이렇게 바꿀수 있습니다

$$\begin{aligned}\frac{\partial L_2}{\partial W} &= \frac{\partial L_2}{\partial \hat{y}_2} \frac{\partial \hat{y}_2}{\partial o_2} \frac{\partial o_2}{\partial h_2} \frac{\partial h_2}{\partial W} + \frac{\partial L_2}{\partial \hat{y}_2} \frac{\partial \hat{y}_2}{\partial o_2} \frac{\partial o_2}{\partial h_2} \frac{\partial h_2}{\partial h_1} \frac{\partial h_1}{\partial W} \\ &= (\hat{y}_2 - y_2) \frac{\partial o_2}{\partial h_2} \frac{\partial h_2}{\partial W} + (\hat{y}_2 - y_2) \frac{\partial o_2}{\partial h_2} \frac{\partial h_2}{\partial h_1} \frac{\partial h_1}{\partial W}\end{aligned}$$



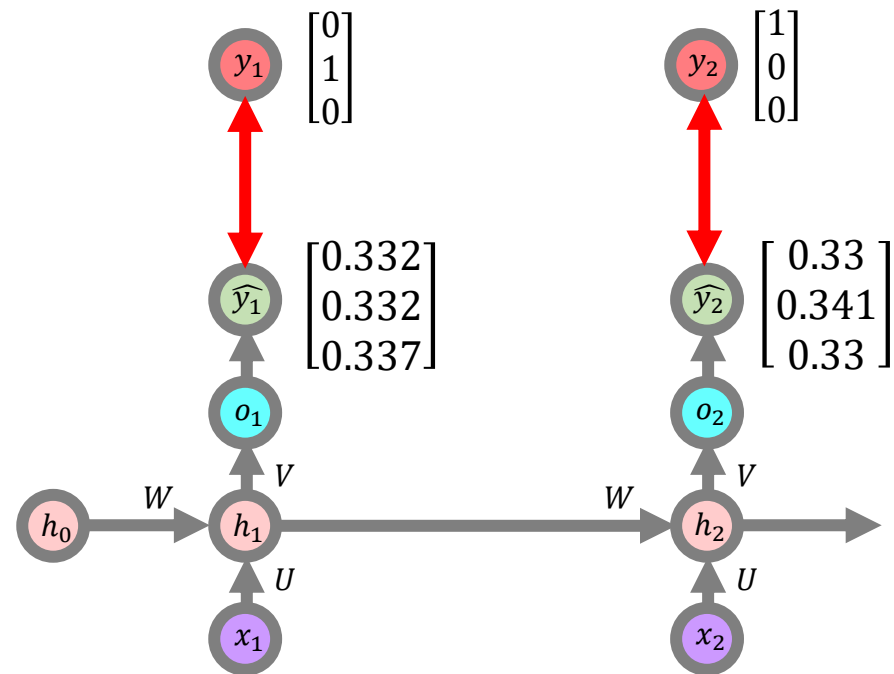
그리고 $o_2 = Vh_2$ 공식에 의해서 다음과 같이 바꿀수 있습니다

$$\begin{aligned}
 \frac{\partial L_2}{\partial W} &= \frac{\partial L_2}{\partial \hat{y}_2} \frac{\partial \hat{y}_2}{\partial o_2} \frac{\partial o_2}{\partial h_2} \frac{\partial h_2}{\partial W} + \frac{\partial L_2}{\partial \hat{y}_2} \frac{\partial \hat{y}_2}{\partial o_2} \frac{\partial o_2}{\partial h_2} \frac{\partial h_2}{\partial h_1} \frac{\partial h_1}{\partial W} \\
 &= (\hat{y}_2 - y_2) \frac{\partial o_2}{\partial h_2} \frac{\partial h_2}{\partial W} + (\hat{y}_2 - y_2) \frac{\partial o_2}{\partial h_2} \frac{\partial h_2}{\partial h_1} \frac{\partial h_1}{\partial W} \\
 &= (\hat{y}_2 - y_2) \underbrace{V}_{\frac{\partial o_2}{\partial h_2}} \frac{\partial h_2}{\partial W} + (\hat{y}_2 - y_2) \underbrace{V}_{\frac{\partial o_2}{\partial h_2}} \frac{\partial h_2}{\partial h_1} \frac{\partial h_1}{\partial W}
 \end{aligned}$$



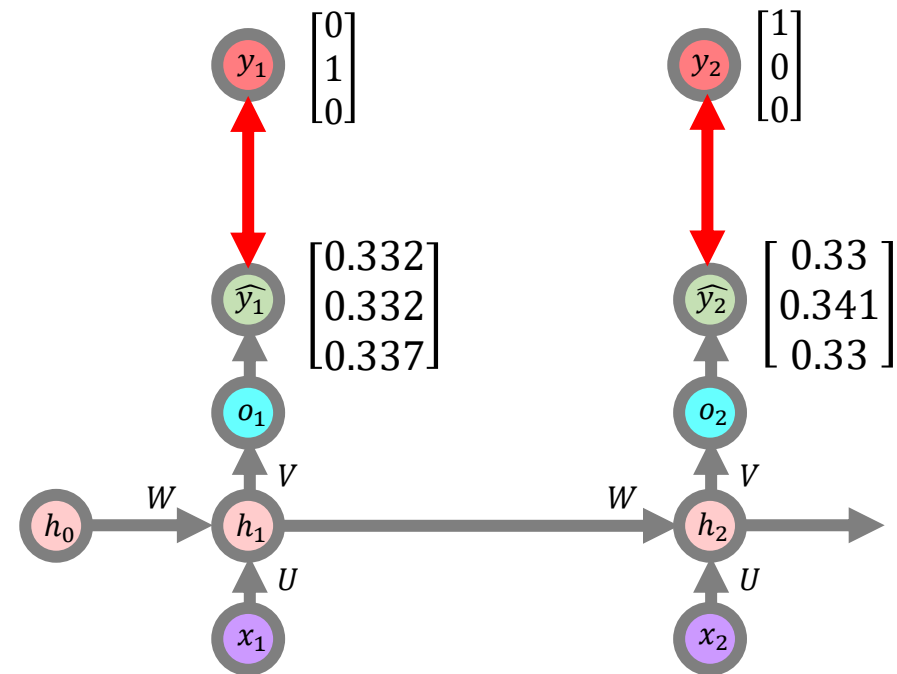
자 이젠 $\frac{\partial h_2}{\partial W}$ 을 구해야 하는데요

$$\begin{aligned}\frac{\partial L_2}{\partial W} &= \frac{\partial L_2}{\partial \hat{y}_2} \frac{\partial \hat{y}_2}{\partial o_2} \frac{\partial o_2}{\partial h_2} \frac{\partial h_2}{\partial W} + \frac{\partial L_2}{\partial \hat{y}_2} \frac{\partial \hat{y}_2}{\partial o_2} \frac{\partial o_2}{\partial h_2} \frac{\partial h_2}{\partial h_1} \frac{\partial h_1}{\partial W} \\ &= (\hat{y}_2 - y_2) \frac{\partial o_2}{\partial h_2} \frac{\partial h_2}{\partial W} + (\hat{y}_2 - y_2) \frac{\partial o_2}{\partial h_2} \frac{\partial h_2}{\partial h_1} \frac{\partial h_1}{\partial W} \\ &= (\hat{y}_2 - y_2) V \frac{\partial h_2}{\partial W} + (\hat{y}_2 - y_2) V \frac{\partial h_2}{\partial h_1} \frac{\partial h_1}{\partial W}\end{aligned}$$



앞서 본 바와 같이 h 와 W 의 관계는 다음과 같습니다

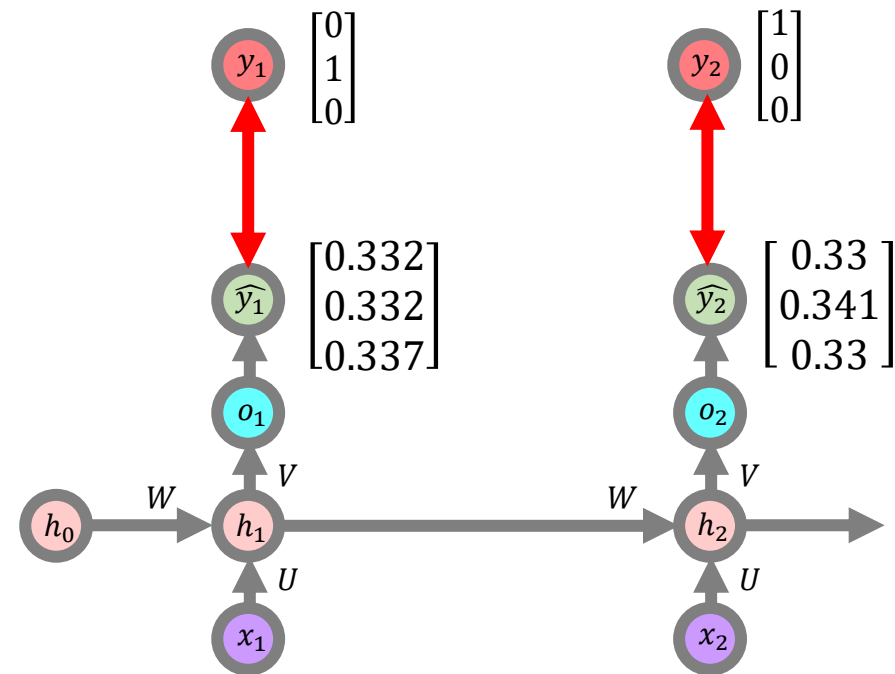
$$\begin{aligned}\frac{\partial L_2}{\partial W} &= \frac{\partial L_2}{\partial \hat{y}_2} \frac{\partial \hat{y}_2}{\partial o_2} \frac{\partial o_2}{\partial h_2} \frac{\partial h_2}{\partial W} + \frac{\partial L_2}{\partial \hat{y}_2} \frac{\partial \hat{y}_2}{\partial o_2} \frac{\partial o_2}{\partial h_2} \frac{\partial h_2}{\partial h_1} \frac{\partial h_1}{\partial W} \\ &= (\hat{y}_2 - y_2) \frac{\partial o_2}{\partial h_2} \frac{\partial h_2}{\partial W} + (\hat{y}_2 - y_2) \frac{\partial o_2}{\partial h_2} \frac{\partial h_2}{\partial h_1} \frac{\partial h_1}{\partial W} \\ &= (\hat{y}_2 - y_2) V \frac{\partial h_2}{\partial W} + (\hat{y}_2 - y_2) V \frac{\partial h_2}{\partial h_1} \frac{\partial h_1}{\partial W}\end{aligned}$$



$$h_2 = \tanh(W h_1 + U x_2)$$

그런데 뭐랄까 이 식으로는 $\frac{\partial h_2}{\partial W}$ 를 구하기 어려워 보입니다

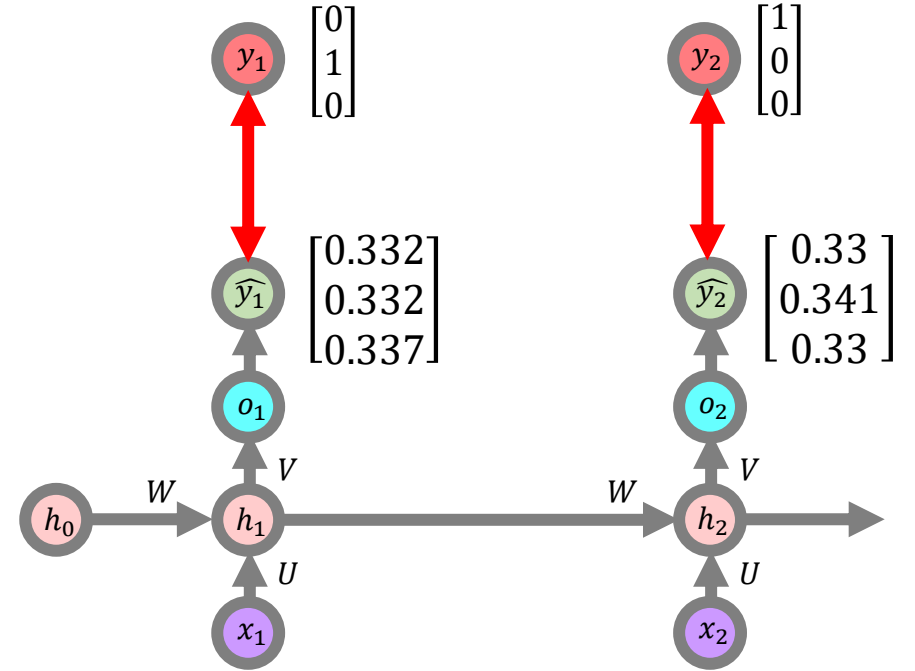
$$\begin{aligned}\frac{\partial L_2}{\partial W} &= \frac{\partial L_2}{\partial \hat{y}_2} \frac{\partial \hat{y}_2}{\partial o_2} \frac{\partial o_2}{\partial h_2} \frac{\partial h_2}{\partial W} + \frac{\partial L_2}{\partial \hat{y}_2} \frac{\partial \hat{y}_2}{\partial o_2} \frac{\partial o_2}{\partial h_2} \frac{\partial h_2}{\partial h_1} \frac{\partial h_1}{\partial W} \\ &= (\hat{y}_2 - y_2) \frac{\partial o_2}{\partial h_2} \frac{\partial h_2}{\partial W} + (\hat{y}_2 - y_2) \frac{\partial o_2}{\partial h_2} \frac{\partial h_2}{\partial h_1} \frac{\partial h_1}{\partial W} \\ &= (\hat{y}_2 - y_2) V \frac{\partial h_2}{\partial W} + (\hat{y}_2 - y_2) V \frac{\partial h_2}{\partial h_1} \frac{\partial h_1}{\partial W}\end{aligned}$$



$$h_2 = \tanh(W h_1 + U x_2)$$

그래서 다음과 같은 트릭을 사용해 보겠습니다

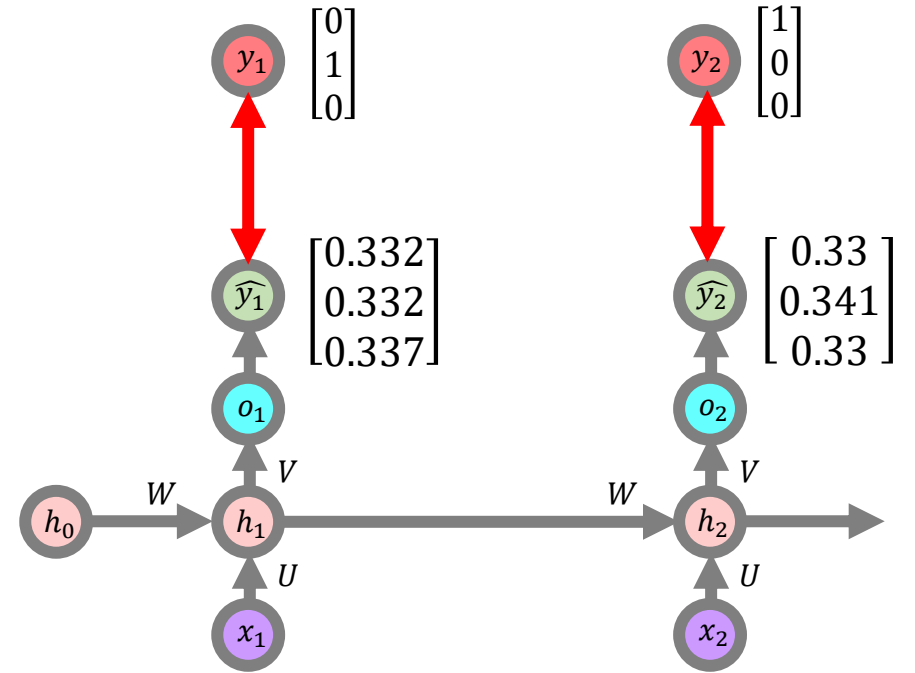
$$\begin{aligned}\frac{\partial L_2}{\partial W} &= \frac{\partial L_2}{\partial \hat{y}_2} \frac{\partial \hat{y}_2}{\partial o_2} \frac{\partial o_2}{\partial h_2} \frac{\partial h_2}{\partial W} + \frac{\partial L_2}{\partial \hat{y}_2} \frac{\partial \hat{y}_2}{\partial o_2} \frac{\partial o_2}{\partial h_2} \frac{\partial h_2}{\partial h_1} \frac{\partial h_1}{\partial W} \\ &= (\hat{y}_2 - y_2) \frac{\partial o_2}{\partial h_2} \frac{\partial h_2}{\partial W} + (\hat{y}_2 - y_2) \frac{\partial o_2}{\partial h_2} \frac{\partial h_2}{\partial h_1} \frac{\partial h_1}{\partial W} \\ &= (\hat{y}_2 - y_2) V \frac{\partial h_2}{\partial W} + (\hat{y}_2 - y_2) V \frac{\partial h_2}{\partial h_1} \frac{\partial h_1}{\partial W}\end{aligned}$$



$$h_2 = \tanh(W h_1 + U x_2)$$

이렇게 식을 두개로 나누어 보겠습니다

$$\begin{aligned}\frac{\partial L_2}{\partial W} &= \frac{\partial L_2}{\partial \hat{y}_2} \frac{\partial \hat{y}_2}{\partial o_2} \frac{\partial o_2}{\partial h_2} \frac{\partial h_2}{\partial W} + \frac{\partial L_2}{\partial \hat{y}_2} \frac{\partial \hat{y}_2}{\partial o_2} \frac{\partial o_2}{\partial h_2} \frac{\partial h_2}{\partial h_1} \frac{\partial h_1}{\partial W} \\ &= (\hat{y}_2 - y_2) \frac{\partial o_2}{\partial h_2} \frac{\partial h_2}{\partial W} + (\hat{y}_2 - y_2) \frac{\partial o_2}{\partial h_2} \frac{\partial h_2}{\partial h_1} \frac{\partial h_1}{\partial W} \\ &= (\hat{y}_2 - y_2) V \frac{\partial h_2}{\partial W} + (\hat{y}_2 - y_2) V \frac{\partial h_2}{\partial h_1} \frac{\partial h_1}{\partial W}\end{aligned}$$

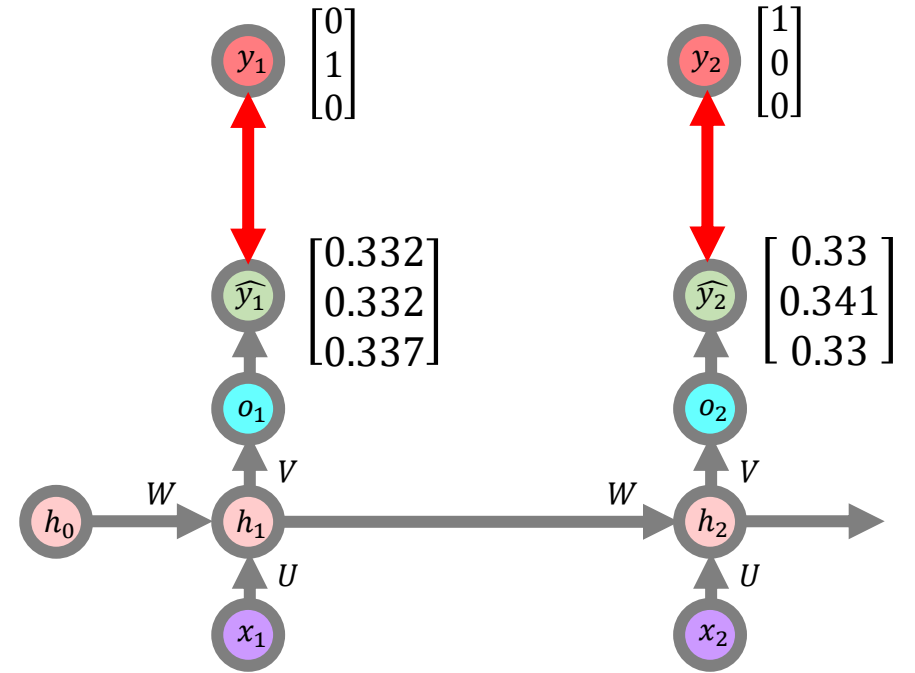


$$h_2 = \tanh(z_2)$$

$$z_2 = Wh_1 + Ux_2$$

결국 $\frac{\partial h_2}{\partial z_2}$ 은 $1 - h_2^2$ 이 됩니다

$$\begin{aligned}\frac{\partial L_2}{\partial W} &= \frac{\partial L_2}{\partial \hat{y}_2} \frac{\partial \hat{y}_2}{\partial o_2} \frac{\partial o_2}{\partial h_2} \frac{\partial h_2}{\partial W} + \frac{\partial L_2}{\partial \hat{y}_2} \frac{\partial \hat{y}_2}{\partial o_2} \frac{\partial o_2}{\partial h_2} \frac{\partial h_2}{\partial h_1} \frac{\partial h_1}{\partial W} \\ &= (\hat{y}_2 - y_2) \frac{\partial o_2}{\partial h_2} \frac{\partial h_2}{\partial W} + (\hat{y}_2 - y_2) \frac{\partial o_2}{\partial h_2} \frac{\partial h_2}{\partial h_1} \frac{\partial h_1}{\partial W} \\ &= (\hat{y}_2 - y_2) V \frac{\partial h_2}{\partial W} + (\hat{y}_2 - y_2) V \frac{\partial h_2}{\partial h_1} \frac{\partial h_1}{\partial W}\end{aligned}$$

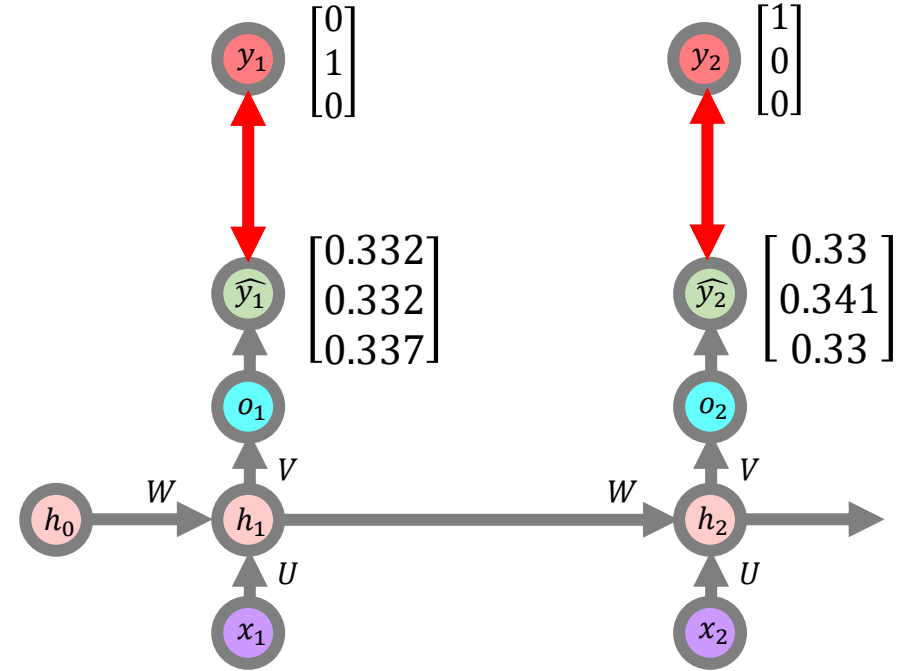


$$h_2 = \tanh(z_2) \quad \rightarrow \quad \frac{\partial h_2}{\partial z_2} = 1 - \tanh^2(z_2) = 1 - h_2^2$$

$$z_2 = W h_1 + U x_2 \quad \rightarrow \quad \frac{\partial z_2}{\partial W} = h_1$$

그러면 식을 다시 풀어서 써보겠습니다

$$\begin{aligned}
 \frac{\partial L_2}{\partial W} &= \frac{\partial L_2}{\partial \hat{y}_2} \frac{\partial \hat{y}_2}{\partial o_2} \frac{\partial o_2}{\partial h_2} \frac{\partial h_2}{\partial W} + \frac{\partial L_2}{\partial \hat{y}_2} \frac{\partial \hat{y}_2}{\partial o_2} \frac{\partial o_2}{\partial h_2} \frac{\partial h_2}{\partial h_1} \frac{\partial h_1}{\partial W} \\
 &= (\hat{y}_2 - y_2) \frac{\partial o_2}{\partial h_2} \frac{\partial h_2}{\partial W} + (\hat{y}_2 - y_2) \frac{\partial o_2}{\partial h_2} \frac{\partial h_2}{\partial h_1} \frac{\partial h_1}{\partial W} \\
 &= (\hat{y}_2 - y_2) V \frac{\partial h_2}{\partial W} + (\hat{y}_2 - y_2) V \frac{\partial h_2}{\partial h_1} \frac{\partial h_1}{\partial W} \\
 &= (\hat{y}_2 - y_2) V \frac{\partial h_2}{\partial z_2} \frac{\partial z_2}{\partial W} + (\hat{y}_2 - y_2) V \frac{\partial h_2}{\partial z_2} \frac{\partial z_2}{\partial h_1} \frac{\partial h_1}{\partial z_1} \frac{\partial z_1}{\partial W}
 \end{aligned}$$

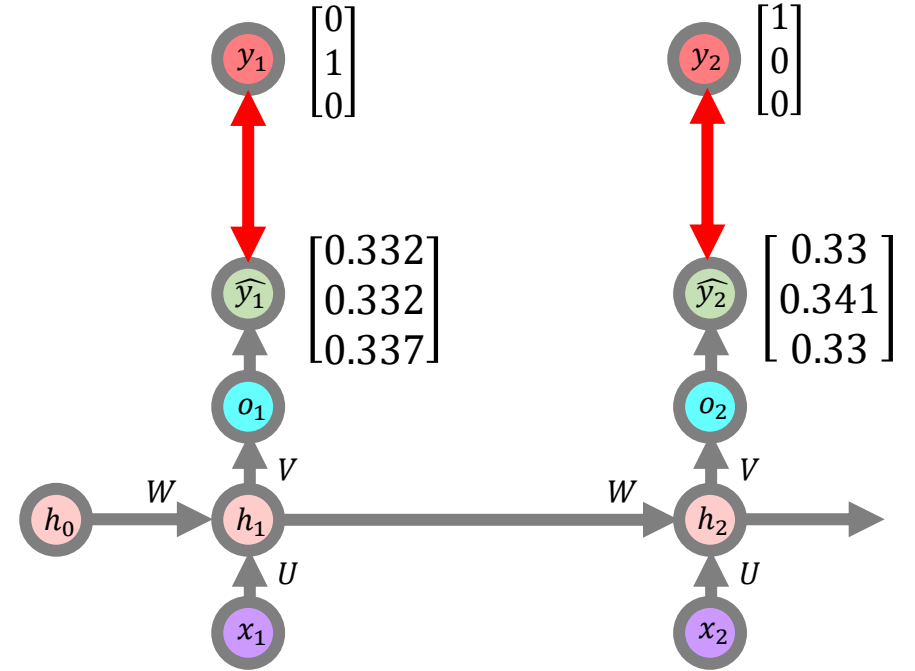


$$h_2 = \tanh(z_2) \quad \rightarrow \quad \frac{\partial h_2}{\partial z_2} = 1 - \tanh^2(z_2) = 1 - h_2^2$$

$$z_2 = W h_1 + U x_2 \quad \rightarrow \quad \frac{\partial z_2}{\partial W} = h_1$$

여기에 각각을 대입하면..

$$\begin{aligned}
 \frac{\partial L_2}{\partial W} &= \frac{\partial L_2}{\partial \hat{y}_2} \frac{\partial \hat{y}_2}{\partial o_2} \frac{\partial o_2}{\partial h_2} \frac{\partial h_2}{\partial W} + \frac{\partial L_2}{\partial \hat{y}_2} \frac{\partial \hat{y}_2}{\partial o_2} \frac{\partial o_2}{\partial h_2} \frac{\partial h_2}{\partial h_1} \frac{\partial h_1}{\partial W} \\
 &= (\hat{y}_2 - y_2) \frac{\partial o_2}{\partial h_2} \frac{\partial h_2}{\partial W} + (\hat{y}_2 - y_2) \frac{\partial o_2}{\partial h_2} \frac{\partial h_2}{\partial h_1} \frac{\partial h_1}{\partial W} \\
 &= (\hat{y}_2 - y_2) V \frac{\partial h_2}{\partial W} + (\hat{y}_2 - y_2) V \frac{\partial h_2}{\partial h_1} \frac{\partial h_1}{\partial W} \\
 &= (\hat{y}_2 - y_2) V \frac{\partial h_2}{\partial z_2} \frac{\partial z_2}{\partial W} + (\hat{y}_2 - y_2) V \frac{\partial h_2}{\partial z_2} \frac{\partial z_2}{\partial h_1} \frac{\partial h_1}{\partial z_1} \frac{\partial z_1}{\partial W}
 \end{aligned}$$

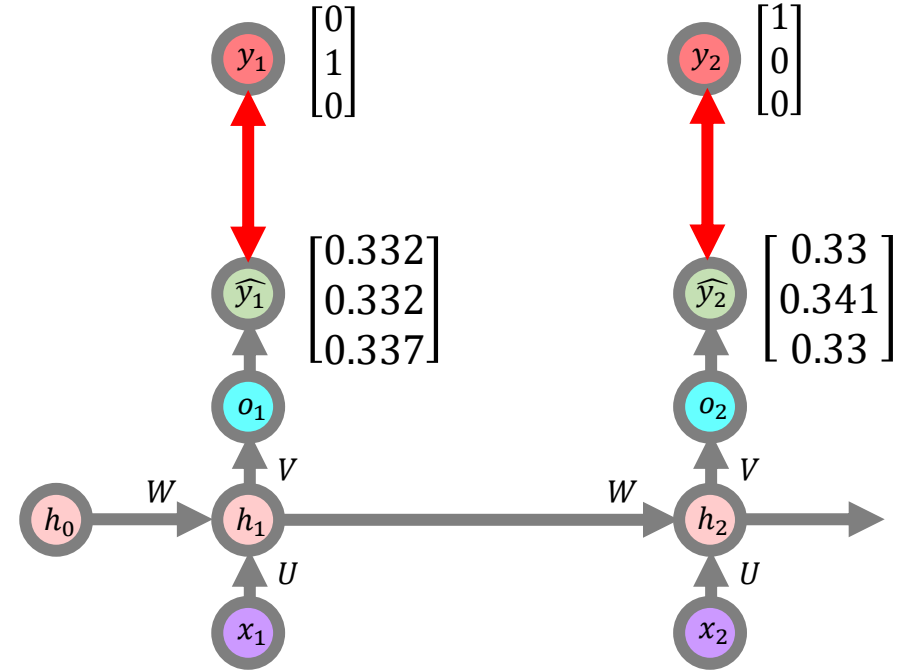


$$h_2 = \tanh(z_2) \quad \rightarrow \quad \frac{\partial h_2}{\partial z_2} = 1 - \tanh^2(z_2) = 1 - h_2^2$$

$$z_2 = W h_1 + U x_2 \quad \rightarrow \quad \frac{\partial z_2}{\partial W} = h_1$$

이렇게 되는 것을 확인하실 수 있습니다!

$$\begin{aligned}
 \frac{\partial L_2}{\partial W} &= \frac{\partial L_2}{\partial \hat{y}_2} \frac{\partial \hat{y}_2}{\partial o_2} \frac{\partial o_2}{\partial h_2} \frac{\partial h_2}{\partial W} + \frac{\partial L_2}{\partial \hat{y}_2} \frac{\partial \hat{y}_2}{\partial o_2} \frac{\partial o_2}{\partial h_2} \frac{\partial h_2}{\partial h_1} \frac{\partial h_1}{\partial W} \\
 &= (\hat{y}_2 - y_2) \frac{\partial o_2}{\partial h_2} \frac{\partial h_2}{\partial W} + (\hat{y}_2 - y_2) \frac{\partial o_2}{\partial h_2} \frac{\partial h_2}{\partial h_1} \frac{\partial h_1}{\partial W} \\
 &= (\hat{y}_2 - y_2) V \frac{\partial h_2}{\partial W} + (\hat{y}_2 - y_2) V \frac{\partial h_2}{\partial h_1} \frac{\partial h_1}{\partial W} \\
 &= (\hat{y}_2 - y_2) V \frac{\partial h_2}{\partial z_2} \frac{\partial z_2}{\partial W} + (\hat{y}_2 - y_2) V \frac{\partial h_2}{\partial z_2} \frac{\partial z_2}{\partial h_1} \frac{\partial h_1}{\partial z_1} \frac{\partial z_1}{\partial W} \\
 &= (\hat{y}_2 - y_2) V (1 - h_2^2) h_1 + (\hat{y}_2 - y_2) V \frac{\partial h_2}{\partial z_2} \frac{\partial z_2}{\partial h_1} \frac{\partial h_1}{\partial z_1} \frac{\partial z_1}{\partial W}
 \end{aligned}$$

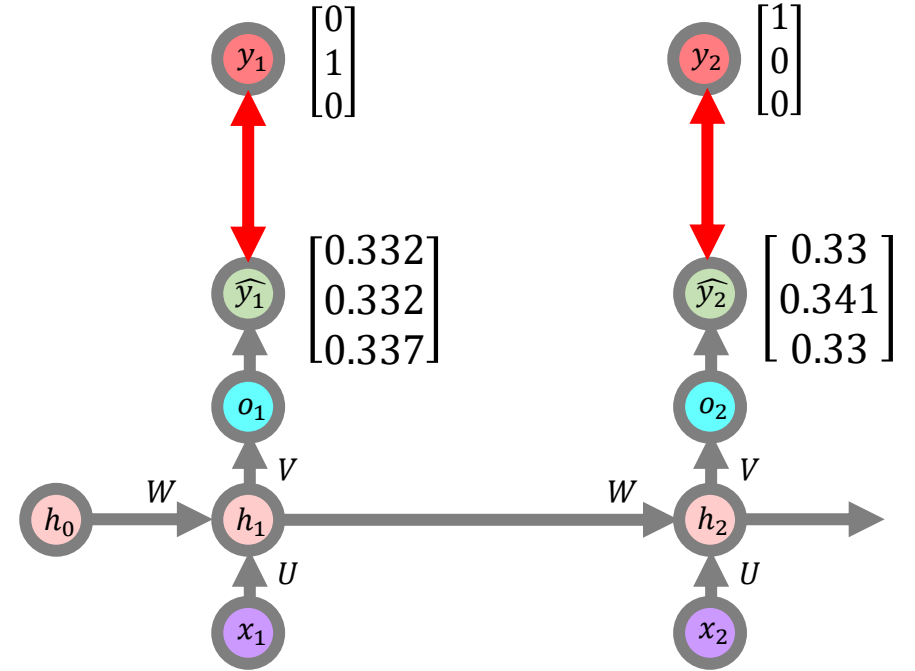


$$h_2 = \tanh(z_2) \quad \rightarrow \quad \frac{\partial h_2}{\partial z_2} = 1 - \tanh^2(z_2) = 1 - h_2^2$$

$$z_2 = W h_1 + U x_2 \quad \rightarrow \quad \frac{\partial z_2}{\partial W} = h_1$$

뿐만 아니라, $\frac{\partial h_2}{\partial z_2}$ 는 여기서도 재사용 되며,

$$\begin{aligned}
 \frac{\partial L_2}{\partial W} &= \frac{\partial L_2}{\partial \hat{y}_2} \frac{\partial \hat{y}_2}{\partial o_2} \frac{\partial o_2}{\partial h_2} \frac{\partial h_2}{\partial W} + \frac{\partial L_2}{\partial \hat{y}_2} \frac{\partial \hat{y}_2}{\partial o_2} \frac{\partial o_2}{\partial h_2} \frac{\partial h_2}{\partial h_1} \frac{\partial h_1}{\partial W} \\
 &= (\hat{y}_2 - y_2) \frac{\partial o_2}{\partial h_2} \frac{\partial h_2}{\partial W} + (\hat{y}_2 - y_2) \frac{\partial o_2}{\partial h_2} \frac{\partial h_2}{\partial h_1} \frac{\partial h_1}{\partial W} \\
 &= (\hat{y}_2 - y_2) V \frac{\partial h_2}{\partial W} + (\hat{y}_2 - y_2) V \frac{\partial h_2}{\partial h_1} \frac{\partial h_1}{\partial W} \\
 &= (\hat{y}_2 - y_2) V \frac{\partial h_2}{\partial z_2} \frac{\partial z_2}{\partial W} + (\hat{y}_2 - y_2) V \frac{\partial h_2}{\partial z_2} \frac{\partial z_2}{\partial h_1} \frac{\partial h_1}{\partial z_1} \frac{\partial z_1}{\partial W} \\
 &= (\hat{y}_2 - y_2) V (1 - h_2^2) h_1 + (\hat{y}_2 - y_2) V \frac{\partial h_2}{\partial z_2} \frac{\partial z_2}{\partial h_1} \frac{\partial h_1}{\partial z_1} \frac{\partial z_1}{\partial W}
 \end{aligned}$$

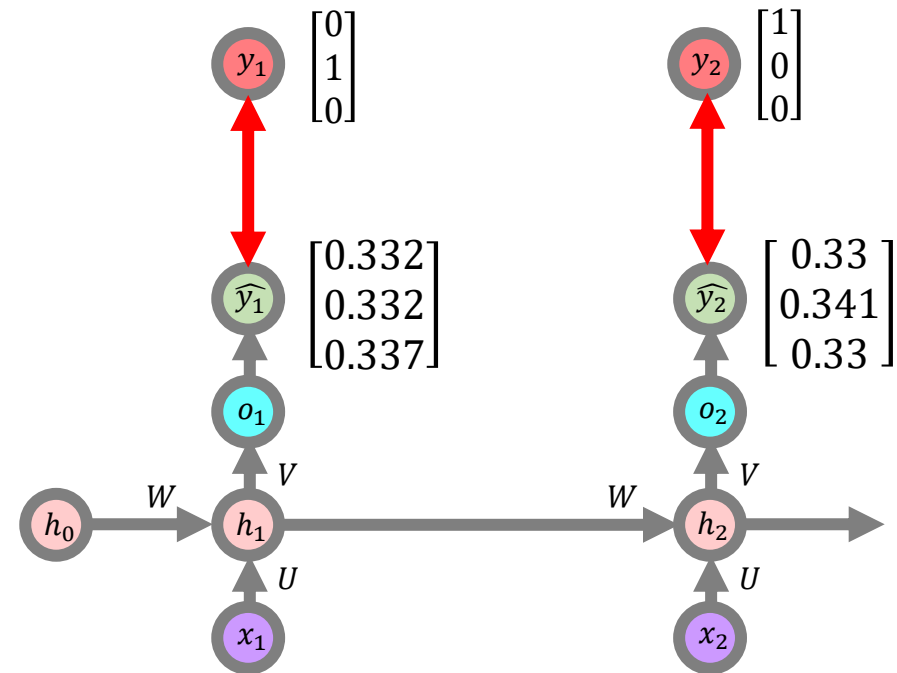


$$h_2 = \tanh(z_2) \quad \rightarrow \quad \frac{\partial h_2}{\partial z_2} = 1 - \tanh^2(z_2) = 1 - h_2^2$$

$$z_2 = W h_1 + U x_2 \quad \rightarrow \quad \frac{\partial z_2}{\partial W} = h_1$$

$\frac{\partial z_2}{\partial h_1}$ 은 여기서 도출이 가능합니다

$$\begin{aligned}
 \frac{\partial L_2}{\partial W} &= \frac{\partial L_2}{\partial \hat{y}_2} \frac{\partial \hat{y}_2}{\partial o_2} \frac{\partial o_2}{\partial h_2} \frac{\partial h_2}{\partial W} + \frac{\partial L_2}{\partial \hat{y}_2} \frac{\partial \hat{y}_2}{\partial o_2} \frac{\partial o_2}{\partial h_2} \frac{\partial h_2}{\partial h_1} \frac{\partial h_1}{\partial W} \\
 &= (\hat{y}_2 - y_2) \frac{\partial o_2}{\partial h_2} \frac{\partial h_2}{\partial W} + (\hat{y}_2 - y_2) \frac{\partial o_2}{\partial h_2} \frac{\partial h_2}{\partial h_1} \frac{\partial h_1}{\partial W} \\
 &= (\hat{y}_2 - y_2) V \frac{\partial h_2}{\partial W} + (\hat{y}_2 - y_2) V \frac{\partial h_2}{\partial h_1} \frac{\partial h_1}{\partial W} \\
 &= (\hat{y}_2 - y_2) V \frac{\partial h_2}{\partial z_2} \frac{\partial z_2}{\partial W} + (\hat{y}_2 - y_2) V \frac{\partial h_2}{\partial z_2} \frac{\partial z_2}{\partial h_1} \frac{\partial h_1}{\partial z_1} \frac{\partial z_1}{\partial W} \\
 &= (\hat{y}_2 - y_2) V (1 - h_2^2) h_1 + (\hat{y}_2 - y_2) V \frac{\partial h_2}{\partial z_2} \frac{\partial z_2}{\partial h_1} \frac{\partial h_1}{\partial z_1} \frac{\partial z_1}{\partial W}
 \end{aligned}$$

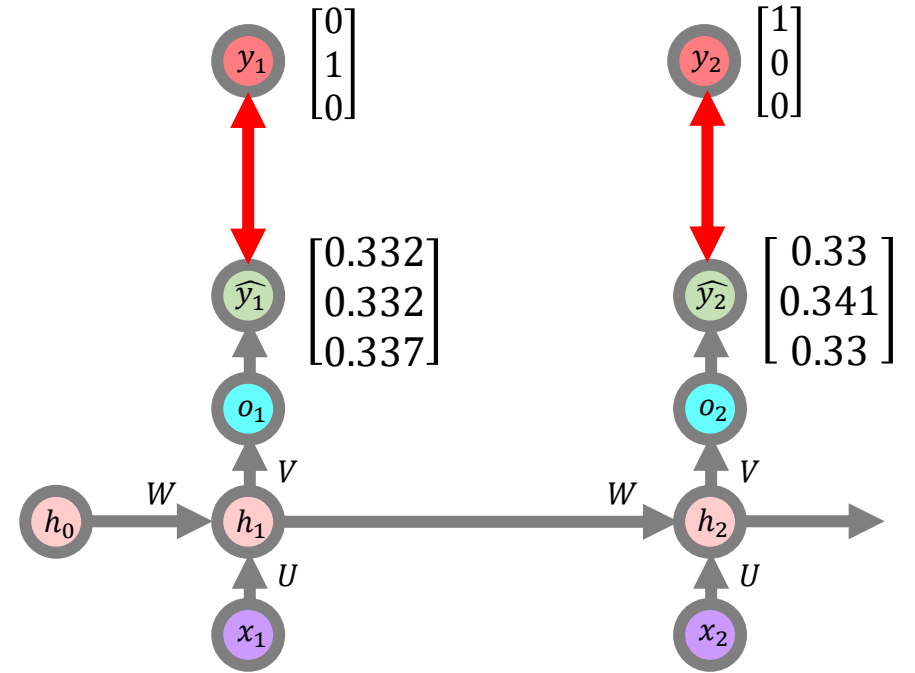


$$h_2 = \tanh(z_2) \quad \rightarrow \quad \frac{\partial h_2}{\partial z_2} = 1 - \tanh^2(z_2) = 1 - h_2^2$$

$$z_2 = W h_1 + U x_2 \quad \rightarrow \quad \frac{\partial z_2}{\partial W} = h_1 \quad \frac{\partial z_2}{\partial h_1} = W$$

계속해서 식을 대입하여 풀어나가면 다음과 같이 정리해볼 수 있습니다

$$\begin{aligned}
 \frac{\partial L_2}{\partial W} &= \frac{\partial L_2}{\partial \hat{y}_2} \frac{\partial \hat{y}_2}{\partial o_2} \frac{\partial o_2}{\partial h_2} \frac{\partial h_2}{\partial W} + \frac{\partial L_2}{\partial \hat{y}_2} \frac{\partial \hat{y}_2}{\partial o_2} \frac{\partial o_2}{\partial h_2} \frac{\partial h_2}{\partial h_1} \frac{\partial h_1}{\partial W} \\
 &= (\hat{y}_2 - y_2) \frac{\partial o_2}{\partial h_2} \frac{\partial h_2}{\partial W} + (\hat{y}_2 - y_2) \frac{\partial o_2}{\partial h_2} \frac{\partial h_2}{\partial h_1} \frac{\partial h_1}{\partial W} \\
 &= (\hat{y}_2 - y_2) V \frac{\partial h_2}{\partial W} + (\hat{y}_2 - y_2) V \frac{\partial h_2}{\partial h_1} \frac{\partial h_1}{\partial W} \\
 &= (\hat{y}_2 - y_2) V \frac{\partial h_2}{\partial z_2} \frac{\partial z_2}{\partial W} + (\hat{y}_2 - y_2) V \frac{\partial h_2}{\partial z_2} \frac{\partial z_2}{\partial h_1} \frac{\partial h_1}{\partial z_1} \frac{\partial z_1}{\partial W} \\
 &= (\hat{y}_2 - y_2) V (1 - h_2^2) h_1 + (\hat{y}_2 - y_2) V \frac{\partial h_2}{\partial z_2} \frac{\partial z_2}{\partial h_1} \frac{\partial h_1}{\partial z_1} \frac{\partial z_1}{\partial W} \\
 &= (\hat{y}_2 - y_2) V (1 - h_2^2) h_1 + (\hat{y}_2 - y_2) V (1 - h_2^2) W \frac{\partial h_1}{\partial z_1} \frac{\partial z_1}{\partial W}
 \end{aligned}$$

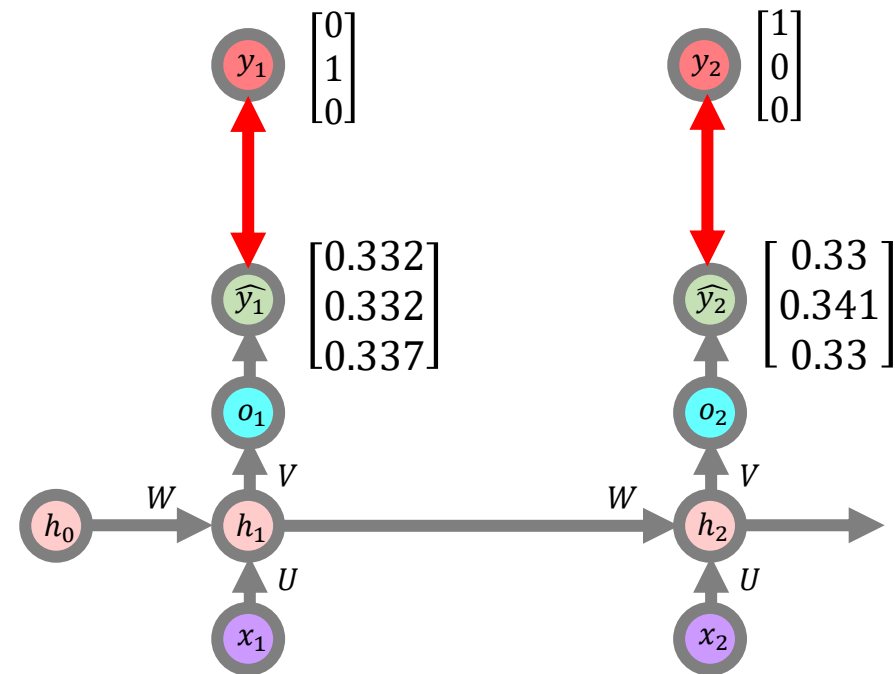


$$h_2 = \tanh(z_2) \quad \rightarrow \quad \frac{\partial h_2}{\partial z_2} = 1 - \tanh^2(z_2) = 1 - h_2^2$$

$$z_2 = W h_1 + U x_2 \quad \rightarrow \quad \frac{\partial z_2}{\partial W} = h_1 \quad \frac{\partial z_2}{\partial h_1} = W$$

그러면 이제 이 부분부터 값들을 넣어보겠습니다

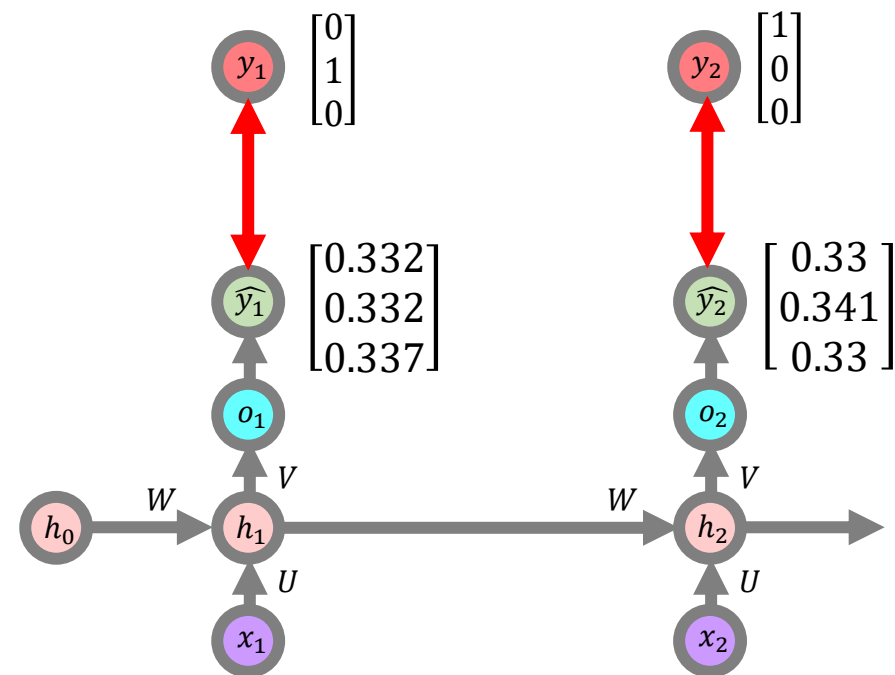
$$\begin{aligned}
 \frac{\partial L_2}{\partial W} &= \frac{\partial L_2}{\partial \hat{y}_2} \frac{\partial \hat{y}_2}{\partial o_2} \frac{\partial o_2}{\partial h_2} \frac{\partial h_2}{\partial W} + \frac{\partial L_2}{\partial \hat{y}_2} \frac{\partial \hat{y}_2}{\partial o_2} \frac{\partial o_2}{\partial h_2} \frac{\partial h_2}{\partial h_1} \frac{\partial h_1}{\partial W} \\
 &= (\hat{y}_2 - y_2) \frac{\partial o_2}{\partial h_2} \frac{\partial h_2}{\partial W} + (\hat{y}_2 - y_2) \frac{\partial o_2}{\partial h_2} \frac{\partial h_2}{\partial h_1} \frac{\partial h_1}{\partial W} \\
 &= (\hat{y}_2 - y_2) V \frac{\partial h_2}{\partial W} + (\hat{y}_2 - y_2) V \frac{\partial h_2}{\partial h_1} \frac{\partial h_1}{\partial W} \\
 &= (\hat{y}_2 - y_2) V \frac{\partial h_2}{\partial z_2} \frac{\partial z_2}{\partial W} + (\hat{y}_2 - y_2) V \frac{\partial h_2}{\partial z_2} \frac{\partial z_2}{\partial h_1} \frac{\partial h_1}{\partial z_1} \frac{\partial z_1}{\partial W} \\
 &= (\hat{y}_2 - y_2) V (1 - h_2^2) h_1 + (\hat{y}_2 - y_2) V \frac{\partial h_2}{\partial z_2} \frac{\partial z_2}{\partial h_1} \frac{\partial h_1}{\partial z_1} \frac{\partial z_1}{\partial W} \\
 &= (\hat{y}_2 - y_2) V (1 - h_2^2) h_1 + (\hat{y}_2 - y_2) V (1 - h_2^2) W \frac{\partial h_1}{\partial z_1} \frac{\partial z_1}{\partial W} \\
 &= \begin{pmatrix} -0.141 & 0.038 \\ 0.056 & -0.105 \\ -0.132 & 0.14 \end{pmatrix}^T \left(\begin{bmatrix} 0.33 \\ 0.341 \\ 0.33 \end{bmatrix} - \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \right) * (0.977 \quad 1)
 \end{aligned}$$



그러면 이렇게 계산값이 나옵니다

$$\begin{aligned}
 \frac{\partial L_2}{\partial W} &= \frac{\partial L_2}{\partial \hat{y}_2} \frac{\partial \hat{y}_2}{\partial o_2} \frac{\partial o_2}{\partial h_2} \frac{\partial h_2}{\partial W} + \frac{\partial L_2}{\partial \hat{y}_2} \frac{\partial \hat{y}_2}{\partial o_2} \frac{\partial o_2}{\partial h_2} \frac{\partial h_2}{\partial h_1} \frac{\partial h_1}{\partial W} \\
 &= (\hat{y}_2 - y_2) \frac{\partial o_2}{\partial h_2} \frac{\partial h_2}{\partial W} + (\hat{y}_2 - y_2) \frac{\partial o_2}{\partial h_2} \frac{\partial h_2}{\partial h_1} \frac{\partial h_1}{\partial W} \\
 &= (\hat{y}_2 - y_2) V \frac{\partial h_2}{\partial W} + (\hat{y}_2 - y_2) V \frac{\partial h_2}{\partial h_1} \frac{\partial h_1}{\partial W} \\
 &= (\hat{y}_2 - y_2) V \frac{\partial h_2}{\partial z_2} \frac{\partial z_2}{\partial W} + (\hat{y}_2 - y_2) V \frac{\partial h_2}{\partial z_2} \frac{\partial z_2}{\partial h_1} \frac{\partial h_1}{\partial z_1} \frac{\partial z_1}{\partial W} \\
 &= (\hat{y}_2 - y_2) V (1 - h_2^2) h_1 + (\hat{y}_2 - y_2) V \frac{\partial h_2}{\partial z_2} \frac{\partial z_2}{\partial h_1} \frac{\partial h_1}{\partial z_1} \frac{\partial z_1}{\partial W} \\
 &= (\hat{y}_2 - y_2) V (1 - h_2^2) h_1 + (\hat{y}_2 - y_2) V (1 - h_2^2) W \frac{\partial h_1}{\partial z_1} \frac{\partial z_1}{\partial W}
 \end{aligned}$$

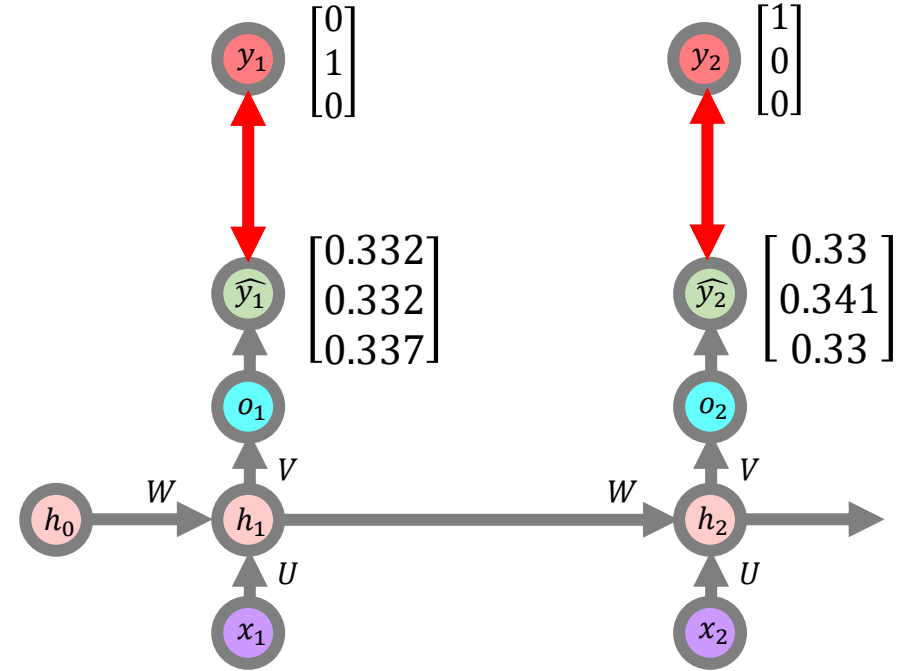
$$\begin{pmatrix} 0.068 \\ -0.015 \end{pmatrix}$$



이 값은 두 곳에 공통적으로 사용할 수 있습니다

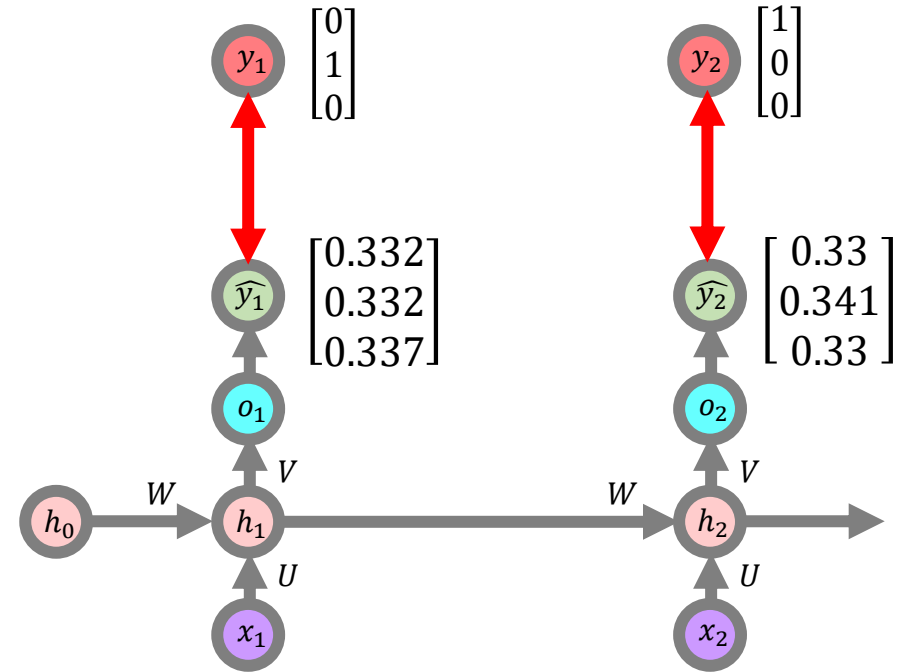
$$\begin{aligned}
 \frac{\partial L_2}{\partial W} &= \frac{\partial L_2}{\partial \hat{y}_2} \frac{\partial \hat{y}_2}{\partial o_2} \frac{\partial o_2}{\partial h_2} \frac{\partial h_2}{\partial W} + \frac{\partial L_2}{\partial \hat{y}_2} \frac{\partial \hat{y}_2}{\partial o_2} \frac{\partial o_2}{\partial h_2} \frac{\partial h_2}{\partial h_1} \frac{\partial h_1}{\partial W} \\
 &= (\hat{y}_2 - y_2) \frac{\partial o_2}{\partial h_2} \frac{\partial h_2}{\partial W} + (\hat{y}_2 - y_2) \frac{\partial o_2}{\partial h_2} \frac{\partial h_2}{\partial h_1} \frac{\partial h_1}{\partial W} \\
 &= (\hat{y}_2 - y_2) V \frac{\partial h_2}{\partial W} + (\hat{y}_2 - y_2) V \frac{\partial h_2}{\partial h_1} \frac{\partial h_1}{\partial W} \\
 &= (\hat{y}_2 - y_2) V \frac{\partial h_2}{\partial z_2} \frac{\partial z_2}{\partial W} + (\hat{y}_2 - y_2) V \frac{\partial h_2}{\partial z_2} \frac{\partial z_2}{\partial h_1} \frac{\partial h_1}{\partial z_1} \frac{\partial z_1}{\partial W} \\
 &= (\hat{y}_2 - y_2) V (1 - h_2^2) h_1 + (\hat{y}_2 - y_2) V \frac{\partial h_2}{\partial z_2} \frac{\partial z_2}{\partial h_1} \frac{\partial h_1}{\partial z_1} \frac{\partial z_1}{\partial W} \\
 &= \boxed{(\hat{y}_2 - y_2) V (1 - h_2^2) h_1} + \boxed{(\hat{y}_2 - y_2) V (1 - h_2^2) W \frac{\partial h_1}{\partial z_1} \frac{\partial z_1}{\partial W}}
 \end{aligned}$$

$\begin{pmatrix} 0.068 \\ -0.015 \end{pmatrix}$



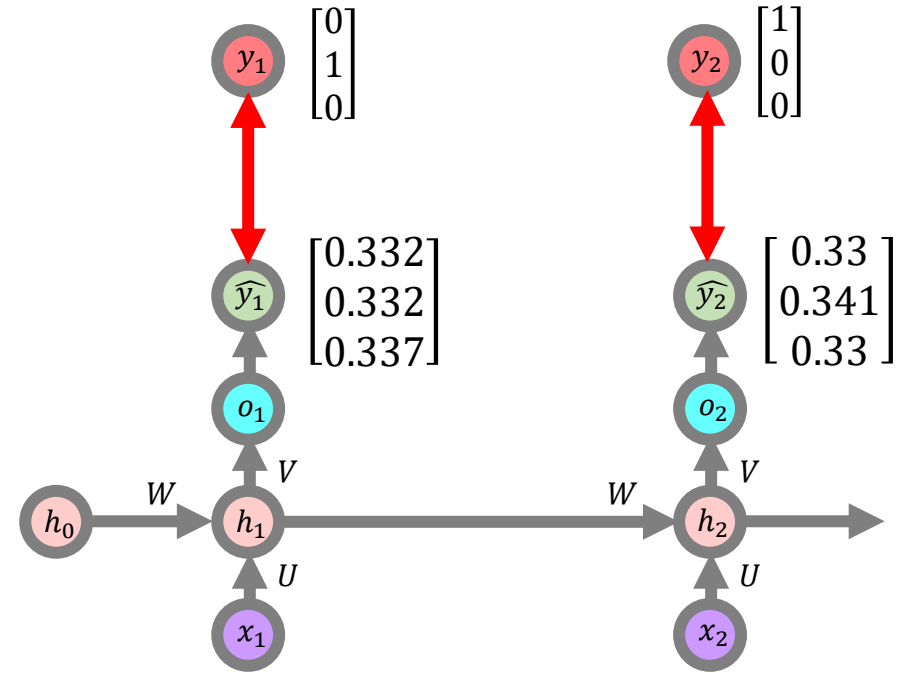
이렇게 값을 넣고

$$\begin{aligned}
 \frac{\partial L_2}{\partial W} &= \frac{\partial L_2}{\partial \hat{y}_2} \frac{\partial \hat{y}_2}{\partial o_2} \frac{\partial o_2}{\partial h_2} \frac{\partial h_2}{\partial W} + \frac{\partial L_2}{\partial \hat{y}_2} \frac{\partial \hat{y}_2}{\partial o_2} \frac{\partial o_2}{\partial h_2} \frac{\partial h_2}{\partial h_1} \frac{\partial h_1}{\partial W} \\
 &= (\hat{y}_2 - y_2) \frac{\partial o_2}{\partial h_2} \frac{\partial h_2}{\partial W} + (\hat{y}_2 - y_2) \frac{\partial o_2}{\partial h_2} \frac{\partial h_2}{\partial h_1} \frac{\partial h_1}{\partial W} \\
 &= (\hat{y}_2 - y_2) V \frac{\partial h_2}{\partial W} + (\hat{y}_2 - y_2) V \frac{\partial h_2}{\partial h_1} \frac{\partial h_1}{\partial W} \\
 &= (\hat{y}_2 - y_2) V \frac{\partial h_2}{\partial z_2} \frac{\partial z_2}{\partial W} + (\hat{y}_2 - y_2) V \frac{\partial h_2}{\partial z_2} \frac{\partial z_2}{\partial h_1} \frac{\partial h_1}{\partial z_1} \frac{\partial z_1}{\partial W} \\
 &= (\hat{y}_2 - y_2) V (1 - h_2^2) h_1 + (\hat{y}_2 - y_2) V \frac{\partial h_2}{\partial z_2} \frac{\partial z_2}{\partial h_1} \frac{\partial h_1}{\partial z_1} \frac{\partial z_1}{\partial W} \\
 &= (\hat{y}_2 - y_2) V (1 - h_2^2) h_1 + (\hat{y}_2 - y_2) V (1 - h_2^2) W \frac{\partial h_1}{\partial z_1} \frac{\partial z_1}{\partial W} \\
 &= \begin{pmatrix} 0.068 \\ -0.015 \end{pmatrix} h_1 + \begin{pmatrix} 0.068 \\ -0.015 \end{pmatrix} W \frac{\partial h_1}{\partial z_1} \frac{\partial z_1}{\partial W}
 \end{aligned}$$



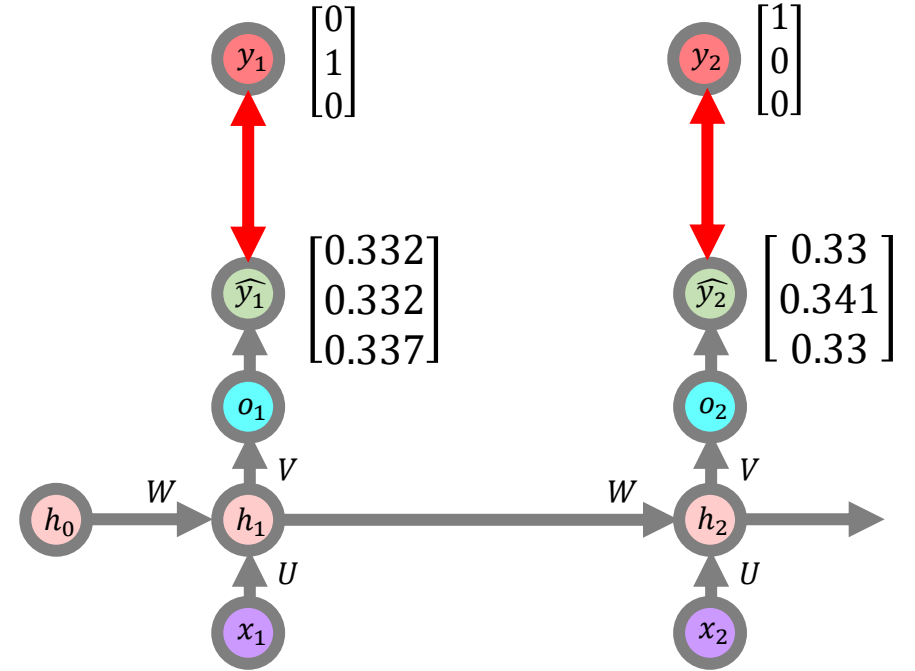
이렇게 값을 넣고

$$\begin{aligned}
 \frac{\partial L_2}{\partial W} &= \frac{\partial L_2}{\partial \hat{y}_2} \frac{\partial \hat{y}_2}{\partial o_2} \frac{\partial o_2}{\partial h_2} \frac{\partial h_2}{\partial W} + \frac{\partial L_2}{\partial \hat{y}_2} \frac{\partial \hat{y}_2}{\partial o_2} \frac{\partial o_2}{\partial h_2} \frac{\partial h_2}{\partial h_1} \frac{\partial h_1}{\partial W} \\
 &= (\hat{y}_2 - y_2) \frac{\partial o_2}{\partial h_2} \frac{\partial h_2}{\partial W} + (\hat{y}_2 - y_2) \frac{\partial o_2}{\partial h_2} \frac{\partial h_2}{\partial h_1} \frac{\partial h_1}{\partial W} \\
 &= (\hat{y}_2 - y_2) V \frac{\partial h_2}{\partial W} + (\hat{y}_2 - y_2) V \frac{\partial h_2}{\partial h_1} \frac{\partial h_1}{\partial W} \\
 &= (\hat{y}_2 - y_2) V \frac{\partial h_2}{\partial z_2} \frac{\partial z_2}{\partial W} + (\hat{y}_2 - y_2) V \frac{\partial h_2}{\partial z_2} \frac{\partial z_2}{\partial h_1} \frac{\partial h_1}{\partial z_1} \frac{\partial z_1}{\partial W} \\
 &= (\hat{y}_2 - y_2) V (1 - h_2^2) h_1 + (\hat{y}_2 - y_2) V \frac{\partial h_2}{\partial z_2} \frac{\partial z_2}{\partial h_1} \frac{\partial h_1}{\partial z_1} \frac{\partial z_1}{\partial W} \\
 &= (\hat{y}_2 - y_2) V (1 - h_2^2) h_1 + (\hat{y}_2 - y_2) V (1 - h_2^2) W \frac{\partial h_1}{\partial z_1} \frac{\partial z_1}{\partial W} \\
 &= \begin{pmatrix} 0.068 \\ -0.015 \end{pmatrix} h_1 + \begin{pmatrix} 0.068 \\ -0.015 \end{pmatrix} W \frac{\partial h_1}{\partial z_1} \frac{\partial z_1}{\partial W} \\
 &= \begin{pmatrix} 0.068 \\ -0.015 \end{pmatrix} (0.094 \quad 0.134) + \begin{pmatrix} -0.011 & 0.13 \\ -0.123 & 0.014 \end{pmatrix}^T \begin{pmatrix} 0.068 \\ -0.015 \end{pmatrix} \frac{\partial h_1}{\partial z_1} \frac{\partial z_1}{\partial W}
 \end{aligned}$$



계산하면 다음과 같이 됩니다

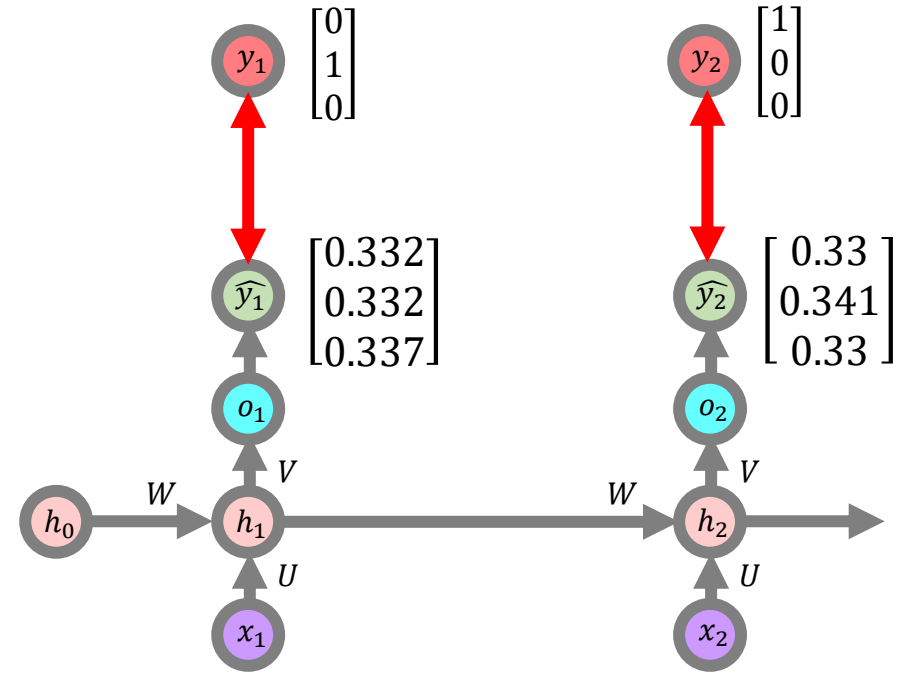
$$\begin{aligned}
 \frac{\partial L_2}{\partial W} &= \frac{\partial L_2}{\partial \hat{y}_2} \frac{\partial \hat{y}_2}{\partial o_2} \frac{\partial o_2}{\partial h_2} \frac{\partial h_2}{\partial W} + \frac{\partial L_2}{\partial \hat{y}_2} \frac{\partial \hat{y}_2}{\partial o_2} \frac{\partial o_2}{\partial h_2} \frac{\partial h_2}{\partial h_1} \frac{\partial h_1}{\partial W} \\
 &= (\hat{y}_2 - y_2) \frac{\partial o_2}{\partial h_2} \frac{\partial h_2}{\partial W} + (\hat{y}_2 - y_2) \frac{\partial o_2}{\partial h_2} \frac{\partial h_2}{\partial h_1} \frac{\partial h_1}{\partial W} \\
 &= (\hat{y}_2 - y_2) V \frac{\partial h_2}{\partial W} + (\hat{y}_2 - y_2) V \frac{\partial h_2}{\partial h_1} \frac{\partial h_1}{\partial W} \\
 &= (\hat{y}_2 - y_2) V \frac{\partial h_2}{\partial z_2} \frac{\partial z_2}{\partial W} + (\hat{y}_2 - y_2) V \frac{\partial h_2}{\partial z_2} \frac{\partial z_2}{\partial h_1} \frac{\partial h_1}{\partial z_1} \frac{\partial z_1}{\partial W} \\
 &= (\hat{y}_2 - y_2) V (1 - h_2^2) h_1 + (\hat{y}_2 - y_2) V \frac{\partial h_2}{\partial z_2} \frac{\partial z_2}{\partial h_1} \frac{\partial h_1}{\partial z_1} \frac{\partial z_1}{\partial W} \\
 &= (\hat{y}_2 - y_2) V (1 - h_2^2) h_1 + (\hat{y}_2 - y_2) V (1 - h_2^2) W \frac{\partial h_1}{\partial z_1} \frac{\partial z_1}{\partial W} \\
 &= \begin{pmatrix} 0.068 \\ -0.015 \end{pmatrix} h_1 + \begin{pmatrix} 0.068 \\ -0.015 \end{pmatrix} W \frac{\partial h_1}{\partial z_1} \frac{\partial z_1}{\partial W} \\
 &= \begin{pmatrix} 0.068 \\ -0.015 \end{pmatrix} (0.094 \quad 0.134) + \begin{pmatrix} -0.011 & 0.13 \\ -0.123 & 0.014 \end{pmatrix}^T \begin{pmatrix} 0.068 \\ -0.015 \end{pmatrix} \frac{\partial h_1}{\partial z_1} \frac{\partial z_1}{\partial W} \\
 &= \begin{pmatrix} 0.006 & 0.009 \\ -0.001 & -0.002 \end{pmatrix} + (0.001 \quad 0.009) \frac{\partial h_1}{\partial z_1} \frac{\partial z_1}{\partial W}
 \end{aligned}$$



그리고 $\frac{\partial h_1}{\partial z_1}$ 은 다시 $1 - h_1^2$ 가 됩니다

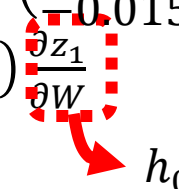
$$\begin{aligned}
 \frac{\partial L_2}{\partial W} &= \frac{\partial L_2}{\partial \hat{y}_2} \frac{\partial \hat{y}_2}{\partial o_2} \frac{\partial o_2}{\partial h_2} \frac{\partial h_2}{\partial W} + \frac{\partial L_2}{\partial \hat{y}_2} \frac{\partial \hat{y}_2}{\partial o_2} \frac{\partial o_2}{\partial h_2} \frac{\partial h_2}{\partial h_1} \frac{\partial h_1}{\partial W} \\
 &= (\hat{y}_2 - y_2) \frac{\partial o_2}{\partial h_2} \frac{\partial h_2}{\partial W} + (\hat{y}_2 - y_2) \frac{\partial o_2}{\partial h_2} \frac{\partial h_2}{\partial h_1} \frac{\partial h_1}{\partial W} \\
 &= (\hat{y}_2 - y_2) V \frac{\partial h_2}{\partial W} + (\hat{y}_2 - y_2) V \frac{\partial h_2}{\partial h_1} \frac{\partial h_1}{\partial W} \\
 &= (\hat{y}_2 - y_2) V \frac{\partial h_2}{\partial z_2} \frac{\partial z_2}{\partial W} + (\hat{y}_2 - y_2) V \frac{\partial h_2}{\partial z_2} \frac{\partial z_2}{\partial h_1} \frac{\partial h_1}{\partial z_1} \frac{\partial z_1}{\partial W} \\
 &= (\hat{y}_2 - y_2) V (1 - h_2^2) h_1 + (\hat{y}_2 - y_2) V \frac{\partial h_2}{\partial z_2} \frac{\partial z_2}{\partial h_1} \frac{\partial h_1}{\partial z_1} \frac{\partial z_1}{\partial W} \\
 &= (\hat{y}_2 - y_2) V (1 - h_2^2) h_1 + (\hat{y}_2 - y_2) V (1 - h_2^2) W \frac{\partial h_1}{\partial z_1} \frac{\partial z_1}{\partial W} \\
 &= \begin{pmatrix} 0.068 \\ -0.015 \end{pmatrix} h_1 + \begin{pmatrix} 0.068 \\ -0.015 \end{pmatrix} W \frac{\partial h_1}{\partial z_1} \frac{\partial z_1}{\partial W} \\
 &= \begin{pmatrix} 0.068 \\ -0.015 \end{pmatrix} \begin{pmatrix} 0.094 & 0.134 \end{pmatrix} + \begin{pmatrix} -0.011 & 0.13 \\ -0.123 & 0.014 \end{pmatrix}^T \begin{pmatrix} 0.068 \\ -0.015 \end{pmatrix} \frac{\partial h_1}{\partial z_1} \frac{\partial z_1}{\partial W} \\
 &= \begin{pmatrix} 0.006 & 0.009 \\ -0.001 & -0.002 \end{pmatrix} + \begin{pmatrix} 0.001 & 0.009 \end{pmatrix} \frac{\partial h_1}{\partial z_1} \frac{\partial z_1}{\partial W}
 \end{aligned}$$

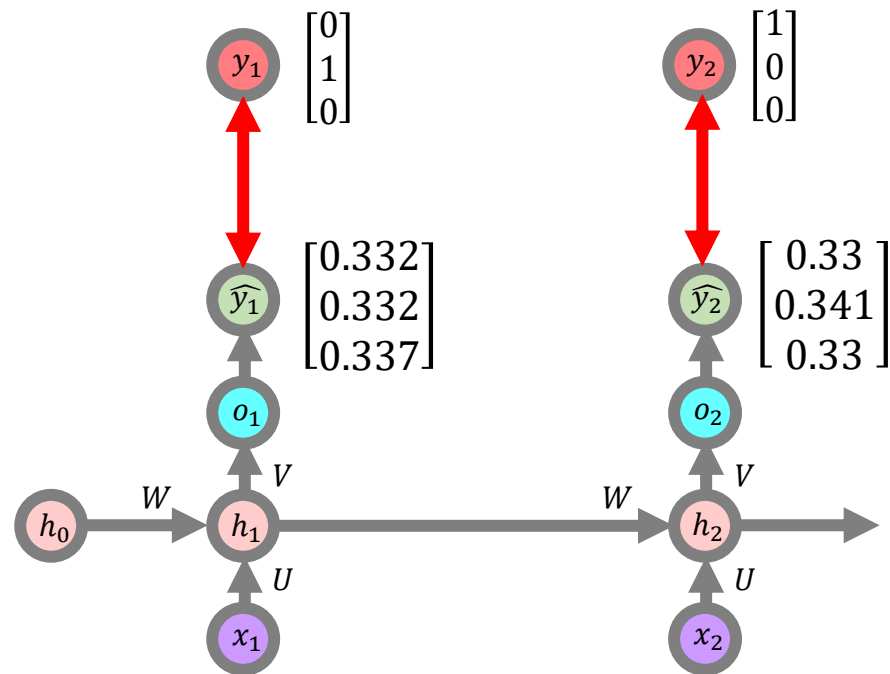
$\frac{\partial h_1}{\partial z_1} = 1 - h_1^2$



그리고 $\frac{\partial z_1}{\partial W}$ 은 공식에 의해서 h_0 가 됩니다

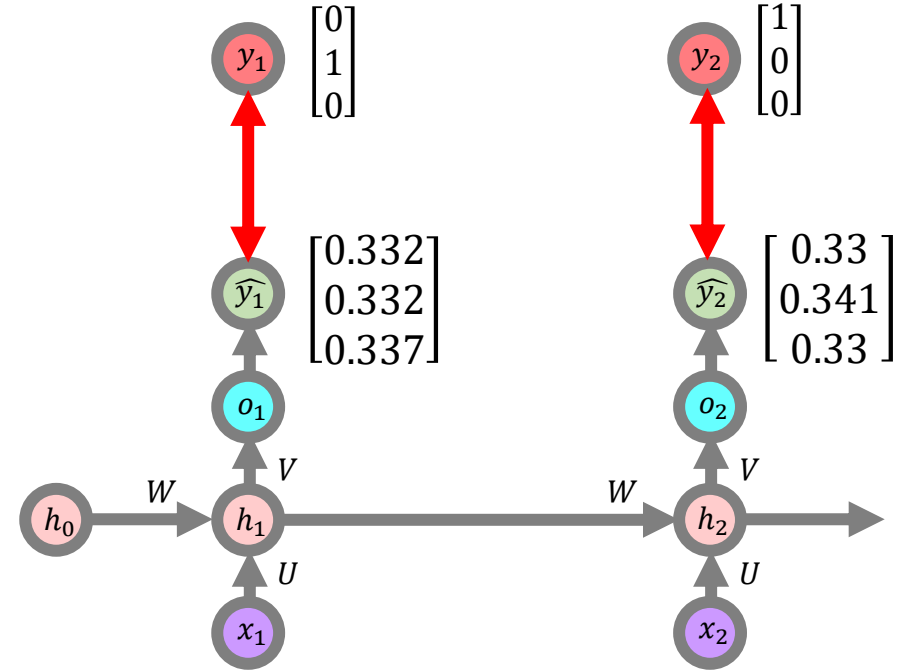
$$\begin{aligned}
 \frac{\partial L_2}{\partial W} &= \frac{\partial L_2}{\partial \hat{y}_2} \frac{\partial \hat{y}_2}{\partial o_2} \frac{\partial o_2}{\partial h_2} \frac{\partial h_2}{\partial W} + \frac{\partial L_2}{\partial \hat{y}_2} \frac{\partial \hat{y}_2}{\partial o_2} \frac{\partial o_2}{\partial h_2} \frac{\partial h_2}{\partial h_1} \frac{\partial h_1}{\partial W} \\
 &= (\hat{y}_2 - y_2) \frac{\partial o_2}{\partial h_2} \frac{\partial h_2}{\partial W} + (\hat{y}_2 - y_2) \frac{\partial o_2}{\partial h_2} \frac{\partial h_2}{\partial h_1} \frac{\partial h_1}{\partial W} \\
 &= (\hat{y}_2 - y_2) V \frac{\partial h_2}{\partial W} + (\hat{y}_2 - y_2) V \frac{\partial h_2}{\partial h_1} \frac{\partial h_1}{\partial W} \\
 &= (\hat{y}_2 - y_2) V \frac{\partial h_2}{\partial z_2} \frac{\partial z_2}{\partial W} + (\hat{y}_2 - y_2) V \frac{\partial h_2}{\partial z_2} \frac{\partial z_2}{\partial h_1} \frac{\partial h_1}{\partial z_1} \frac{\partial z_1}{\partial W} \\
 &= (\hat{y}_2 - y_2) V (1 - h_2^2) h_1 + (\hat{y}_2 - y_2) V \frac{\partial h_2}{\partial z_2} \frac{\partial z_2}{\partial h_1} \frac{\partial h_1}{\partial z_1} \frac{\partial z_1}{\partial W} \\
 &= (\hat{y}_2 - y_2) V (1 - h_2^2) h_1 + (\hat{y}_2 - y_2) V (1 - h_2^2) W \frac{\partial h_1}{\partial z_1} \frac{\partial z_1}{\partial W} \\
 &= \begin{pmatrix} 0.068 \\ -0.015 \end{pmatrix} h_1 + \begin{pmatrix} 0.068 \\ -0.015 \end{pmatrix} W \frac{\partial h_1}{\partial z_1} \frac{\partial z_1}{\partial W} \\
 &= \begin{pmatrix} 0.068 \\ -0.015 \end{pmatrix} (0.094 \quad 0.134) + \begin{pmatrix} -0.011 & 0.13 \\ -0.123 & 0.014 \end{pmatrix}^T \begin{pmatrix} 0.068 \\ -0.015 \end{pmatrix} \frac{\partial h_1}{\partial z_1} \frac{\partial z_1}{\partial W} \\
 &= \begin{pmatrix} 0.006 & 0.009 \\ -0.001 & -0.002 \end{pmatrix} + (0.001 \quad 0.009) (1 - h_1^2) \frac{\partial z_1}{\partial W}
 \end{aligned}$$





그런데 h_0 은 우리가 $[0 \ 0]$ 로 했었기 때문에..

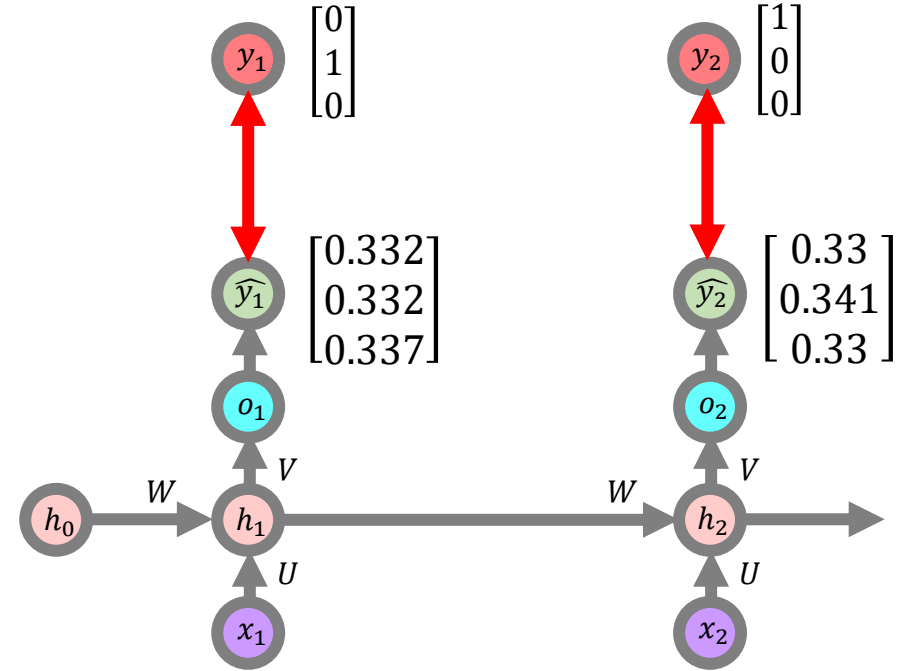
$$\begin{aligned}
 \frac{\partial L_2}{\partial W} &= \frac{\partial L_2}{\partial \hat{y}_2} \frac{\partial \hat{y}_2}{\partial o_2} \frac{\partial o_2}{\partial h_2} \frac{\partial h_2}{\partial W} + \frac{\partial L_2}{\partial \hat{y}_2} \frac{\partial \hat{y}_2}{\partial o_2} \frac{\partial o_2}{\partial h_2} \frac{\partial h_2}{\partial h_1} \frac{\partial h_1}{\partial W} \\
 &= (\hat{y}_2 - y_2) \frac{\partial o_2}{\partial h_2} \frac{\partial h_2}{\partial W} + (\hat{y}_2 - y_2) \frac{\partial o_2}{\partial h_2} \frac{\partial h_2}{\partial h_1} \frac{\partial h_1}{\partial W} \\
 &= (\hat{y}_2 - y_2) V \frac{\partial h_2}{\partial W} + (\hat{y}_2 - y_2) V \frac{\partial h_2}{\partial h_1} \frac{\partial h_1}{\partial W} \\
 &= (\hat{y}_2 - y_2) V \frac{\partial h_2}{\partial z_2} \frac{\partial z_2}{\partial W} + (\hat{y}_2 - y_2) V \frac{\partial h_2}{\partial z_2} \frac{\partial z_2}{\partial h_1} \frac{\partial h_1}{\partial z_1} \frac{\partial z_1}{\partial W} \\
 &= (\hat{y}_2 - y_2) V (1 - h_2^2) h_1 + (\hat{y}_2 - y_2) V \frac{\partial h_2}{\partial z_2} \frac{\partial z_2}{\partial h_1} \frac{\partial h_1}{\partial z_1} \frac{\partial z_1}{\partial W} \\
 &= (\hat{y}_2 - y_2) V (1 - h_2^2) h_1 + (\hat{y}_2 - y_2) V (1 - h_2^2) W \frac{\partial h_1}{\partial z_1} \frac{\partial z_1}{\partial W} \\
 &= \begin{pmatrix} 0.068 \\ -0.015 \end{pmatrix} h_1 + \begin{pmatrix} 0.068 \\ -0.015 \end{pmatrix} W \frac{\partial h_1}{\partial z_1} \frac{\partial z_1}{\partial W} \\
 &= \begin{pmatrix} 0.068 \\ -0.015 \end{pmatrix} (0.094 \quad 0.134) + \begin{pmatrix} -0.011 & 0.13 \\ -0.123 & 0.014 \end{pmatrix}^T \begin{pmatrix} 0.068 \\ -0.015 \end{pmatrix} \frac{\partial h_1}{\partial z_1} \frac{\partial z_1}{\partial W} \\
 &= \begin{pmatrix} 0.006 & 0.009 \\ -0.001 & -0.002 \end{pmatrix} + (0.001 \quad 0.009) (1 - h_1^2) h_0
 \end{aligned}$$



뒤에 부분은 0로 처리할 수 있습니다

$$\begin{aligned}
 \frac{\partial L_2}{\partial W} &= \frac{\partial L_2}{\partial \hat{y}_2} \frac{\partial \hat{y}_2}{\partial o_2} \frac{\partial o_2}{\partial h_2} \frac{\partial h_2}{\partial W} + \frac{\partial L_2}{\partial \hat{y}_2} \frac{\partial \hat{y}_2}{\partial o_2} \frac{\partial o_2}{\partial h_2} \frac{\partial h_2}{\partial h_1} \frac{\partial h_1}{\partial W} \\
 &= (\hat{y}_2 - y_2) \frac{\partial o_2}{\partial h_2} \frac{\partial h_2}{\partial W} + (\hat{y}_2 - y_2) \frac{\partial o_2}{\partial h_2} \frac{\partial h_2}{\partial h_1} \frac{\partial h_1}{\partial W} \\
 &= (\hat{y}_2 - y_2) V \frac{\partial h_2}{\partial W} + (\hat{y}_2 - y_2) V \frac{\partial h_2}{\partial h_1} \frac{\partial h_1}{\partial W} \\
 &= (\hat{y}_2 - y_2) V \frac{\partial h_2}{\partial z_2} \frac{\partial z_2}{\partial W} + (\hat{y}_2 - y_2) V \frac{\partial h_2}{\partial z_2} \frac{\partial z_2}{\partial h_1} \frac{\partial h_1}{\partial z_1} \frac{\partial z_1}{\partial W} \\
 &= (\hat{y}_2 - y_2) V (1 - h_2^2) h_1 + (\hat{y}_2 - y_2) V \frac{\partial h_2}{\partial z_2} \frac{\partial z_2}{\partial h_1} \frac{\partial h_1}{\partial z_1} \frac{\partial z_1}{\partial W} \\
 &= (\hat{y}_2 - y_2) V (1 - h_2^2) h_1 + (\hat{y}_2 - y_2) V (1 - h_2^2) W \frac{\partial h_1}{\partial z_1} \frac{\partial z_1}{\partial W} \\
 &= \begin{pmatrix} 0.068 \\ -0.015 \end{pmatrix} h_1 + \begin{pmatrix} 0.068 \\ -0.015 \end{pmatrix} W \frac{\partial h_1}{\partial z_1} \frac{\partial z_1}{\partial W} \\
 &= \begin{pmatrix} 0.068 \\ -0.015 \end{pmatrix} (0.094 \quad 0.134) + \begin{pmatrix} -0.011 & 0.13 \\ -0.123 & 0.014 \end{pmatrix}^T \begin{pmatrix} 0.068 \\ -0.015 \end{pmatrix} \frac{\partial h_1}{\partial z_1} \frac{\partial z_1}{\partial W} \\
 &= \begin{pmatrix} 0.006 & 0.009 \\ -0.001 & -0.002 \end{pmatrix} + (0.001 \quad 0.009) (1 - h_1^2) h_0
 \end{aligned}$$

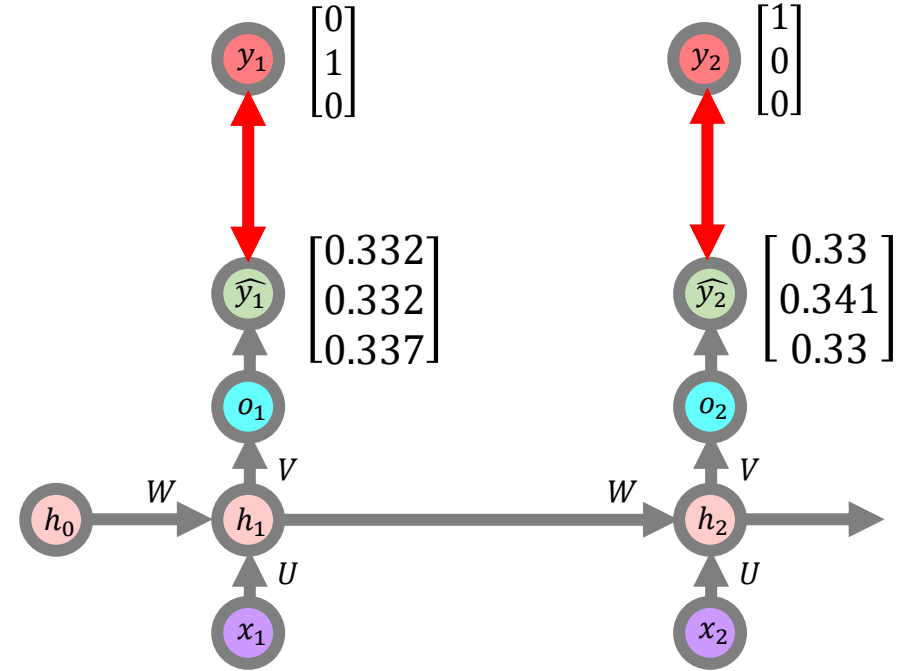
0



그러나 대부분의 경우 h_0 가 이전 입력 시퀀스의 마지막 hidden state가 될 수도 있습니다

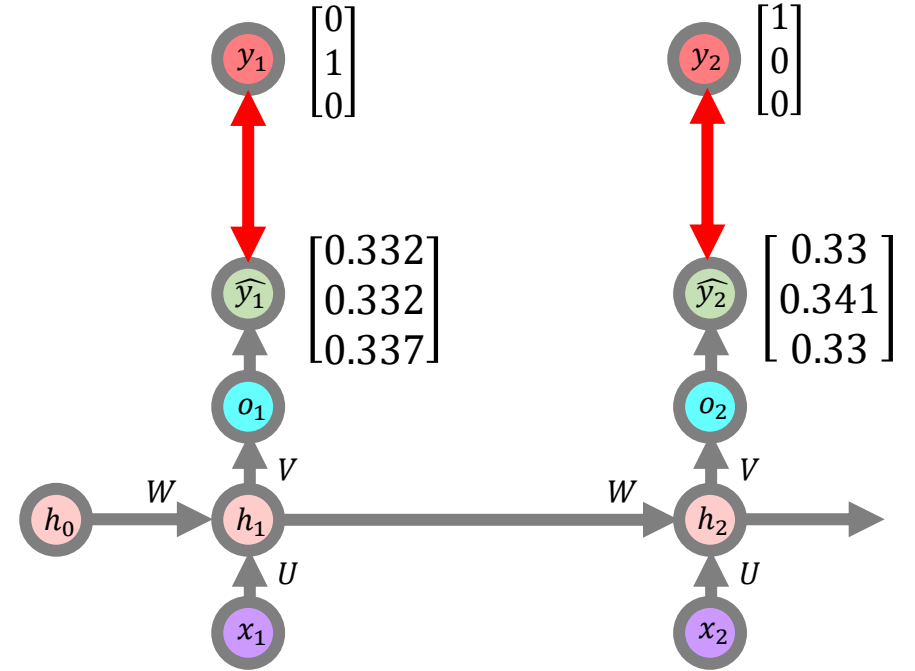
$$\begin{aligned}
 \frac{\partial L_2}{\partial W} &= \frac{\partial L_2}{\partial \hat{y}_2} \frac{\partial \hat{y}_2}{\partial o_2} \frac{\partial o_2}{\partial h_2} \frac{\partial h_2}{\partial W} + \frac{\partial L_2}{\partial \hat{y}_2} \frac{\partial \hat{y}_2}{\partial o_2} \frac{\partial o_2}{\partial h_2} \frac{\partial h_2}{\partial h_1} \frac{\partial h_1}{\partial W} \\
 &= (\hat{y}_2 - y_2) \frac{\partial o_2}{\partial h_2} \frac{\partial h_2}{\partial W} + (\hat{y}_2 - y_2) \frac{\partial o_2}{\partial h_2} \frac{\partial h_2}{\partial h_1} \frac{\partial h_1}{\partial W} \\
 &= (\hat{y}_2 - y_2) V \frac{\partial h_2}{\partial W} + (\hat{y}_2 - y_2) V \frac{\partial h_2}{\partial h_1} \frac{\partial h_1}{\partial W} \\
 &= (\hat{y}_2 - y_2) V \frac{\partial h_2}{\partial z_2} \frac{\partial z_2}{\partial W} + (\hat{y}_2 - y_2) V \frac{\partial h_2}{\partial z_2} \frac{\partial z_2}{\partial h_1} \frac{\partial h_1}{\partial z_1} \frac{\partial z_1}{\partial W} \\
 &= (\hat{y}_2 - y_2) V (1 - h_2^2) h_1 + (\hat{y}_2 - y_2) V \frac{\partial h_2}{\partial z_2} \frac{\partial z_2}{\partial h_1} \frac{\partial h_1}{\partial z_1} \frac{\partial z_1}{\partial W} \\
 &= (\hat{y}_2 - y_2) V (1 - h_2^2) h_1 + (\hat{y}_2 - y_2) V (1 - h_2^2) W \frac{\partial h_1}{\partial z_1} \frac{\partial z_1}{\partial W} \\
 &= \begin{pmatrix} 0.068 \\ -0.015 \end{pmatrix} h_1 + \begin{pmatrix} 0.068 \\ -0.015 \end{pmatrix} W \frac{\partial h_1}{\partial z_1} \frac{\partial z_1}{\partial W} \\
 &= \begin{pmatrix} 0.068 \\ -0.015 \end{pmatrix} \begin{pmatrix} 0.094 & 0.134 \end{pmatrix} + \begin{pmatrix} -0.011 & 0.13 \\ -0.123 & 0.014 \end{pmatrix}^T \begin{pmatrix} 0.068 \\ -0.015 \end{pmatrix} \frac{\partial h_1}{\partial z_1} \frac{\partial z_1}{\partial W} \\
 &= \begin{pmatrix} 0.006 & 0.009 \\ -0.001 & -0.002 \end{pmatrix} + (0.001 \quad 0.009) (1 - h_1^2) h_0
 \end{aligned}$$

0



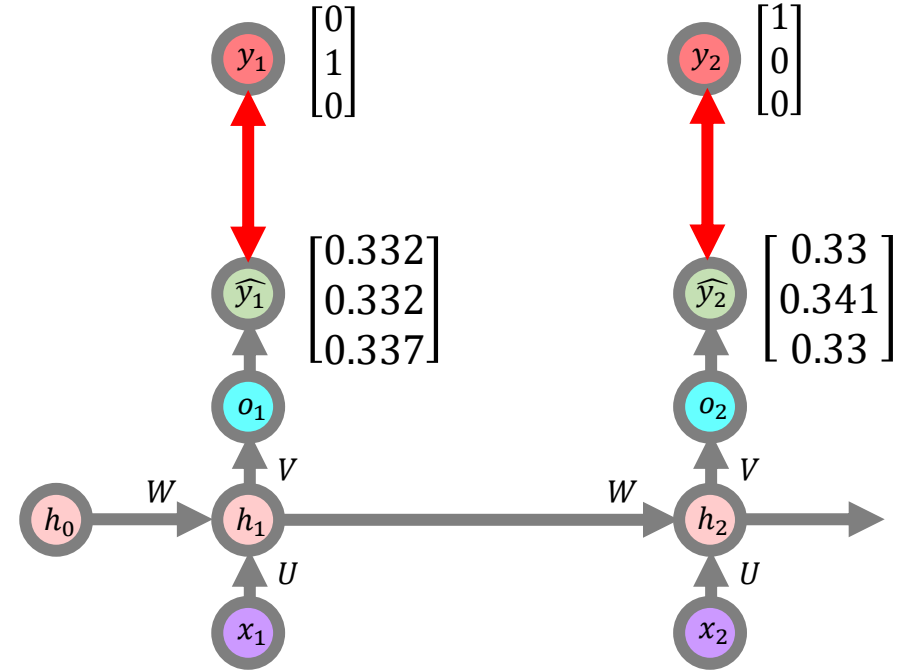
그럴 경우, 뒤의 부분도 계산을 해주어야 합니다

$$\begin{aligned}
 \frac{\partial L_2}{\partial W} &= \frac{\partial L_2}{\partial \hat{y}_2} \frac{\partial \hat{y}_2}{\partial o_2} \frac{\partial o_2}{\partial h_2} \frac{\partial h_2}{\partial W} + \frac{\partial L_2}{\partial \hat{y}_2} \frac{\partial \hat{y}_2}{\partial o_2} \frac{\partial o_2}{\partial h_2} \frac{\partial h_2}{\partial h_1} \frac{\partial h_1}{\partial W} \\
 &= (\hat{y}_2 - y_2) \frac{\partial o_2}{\partial h_2} \frac{\partial h_2}{\partial W} + (\hat{y}_2 - y_2) \frac{\partial o_2}{\partial h_2} \frac{\partial h_2}{\partial h_1} \frac{\partial h_1}{\partial W} \\
 &= (\hat{y}_2 - y_2) V \frac{\partial h_2}{\partial W} + (\hat{y}_2 - y_2) V \frac{\partial h_2}{\partial h_1} \frac{\partial h_1}{\partial W} \\
 &= (\hat{y}_2 - y_2) V \frac{\partial h_2}{\partial z_2} \frac{\partial z_2}{\partial W} + (\hat{y}_2 - y_2) V \frac{\partial h_2}{\partial z_2} \frac{\partial z_2}{\partial h_1} \frac{\partial h_1}{\partial z_1} \frac{\partial z_1}{\partial W} \\
 &= (\hat{y}_2 - y_2) V (1 - h_2^2) h_1 + (\hat{y}_2 - y_2) V \frac{\partial h_2}{\partial z_2} \frac{\partial z_2}{\partial h_1} \frac{\partial h_1}{\partial z_1} \frac{\partial z_1}{\partial W} \\
 &= (\hat{y}_2 - y_2) V (1 - h_2^2) h_1 + (\hat{y}_2 - y_2) V (1 - h_2^2) W \frac{\partial h_1}{\partial z_1} \frac{\partial z_1}{\partial W} \\
 &= \begin{pmatrix} 0.068 \\ -0.015 \end{pmatrix} h_1 + \begin{pmatrix} 0.068 \\ -0.015 \end{pmatrix} W \frac{\partial h_1}{\partial z_1} \frac{\partial z_1}{\partial W} \\
 &= \begin{pmatrix} 0.068 \\ -0.015 \end{pmatrix} (0.094 \quad 0.134) + \begin{pmatrix} -0.011 & 0.13 \\ -0.123 & 0.014 \end{pmatrix}^T \begin{pmatrix} 0.068 \\ -0.015 \end{pmatrix} \frac{\partial h_1}{\partial z_1} \frac{\partial z_1}{\partial W} \\
 &= \begin{pmatrix} 0.006 & 0.009 \\ -0.001 & -0.002 \end{pmatrix} + \boxed{\begin{pmatrix} 0.001 & 0.009 \end{pmatrix} (1 - h_1^2) h_0}
 \end{aligned}$$



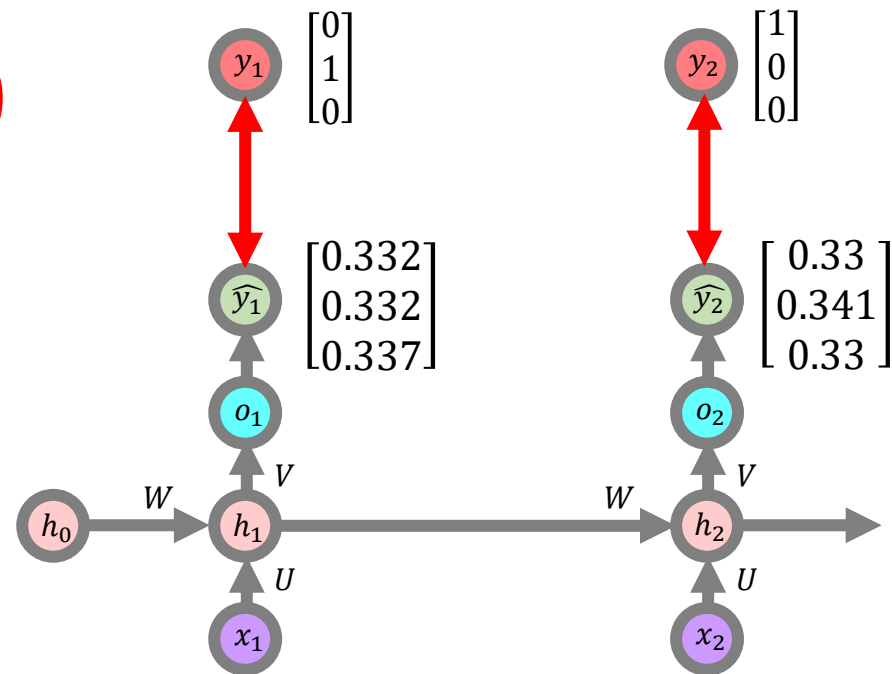
여기서는 h_0 가 $[0 \ 0]$ 이기 때문에 간단하게 0이 됩니다

$$\begin{aligned}
 \frac{\partial L_2}{\partial W} &= \frac{\partial L_2}{\partial \hat{y}_2} \frac{\partial \hat{y}_2}{\partial o_2} \frac{\partial o_2}{\partial h_2} \frac{\partial h_2}{\partial W} + \frac{\partial L_2}{\partial \hat{y}_2} \frac{\partial \hat{y}_2}{\partial o_2} \frac{\partial o_2}{\partial h_2} \frac{\partial h_2}{\partial h_1} \frac{\partial h_1}{\partial W} \\
 &= (\hat{y}_2 - y_2) \frac{\partial o_2}{\partial h_2} \frac{\partial h_2}{\partial W} + (\hat{y}_2 - y_2) \frac{\partial o_2}{\partial h_2} \frac{\partial h_2}{\partial h_1} \frac{\partial h_1}{\partial W} \\
 &= (\hat{y}_2 - y_2) V \frac{\partial h_2}{\partial W} + (\hat{y}_2 - y_2) V \frac{\partial h_2}{\partial h_1} \frac{\partial h_1}{\partial W} \\
 &= (\hat{y}_2 - y_2) V \frac{\partial h_2}{\partial z_2} \frac{\partial z_2}{\partial W} + (\hat{y}_2 - y_2) V \frac{\partial h_2}{\partial z_2} \frac{\partial z_2}{\partial h_1} \frac{\partial h_1}{\partial z_1} \frac{\partial z_1}{\partial W} \\
 &= (\hat{y}_2 - y_2) V (1 - h_2^2) h_1 + (\hat{y}_2 - y_2) V \frac{\partial h_2}{\partial z_2} \frac{\partial z_2}{\partial h_1} \frac{\partial h_1}{\partial z_1} \frac{\partial z_1}{\partial W} \\
 &= (\hat{y}_2 - y_2) V (1 - h_2^2) h_1 + (\hat{y}_2 - y_2) V (1 - h_2^2) W \frac{\partial h_1}{\partial z_1} \frac{\partial z_1}{\partial W} \\
 &= \begin{pmatrix} 0.068 \\ -0.015 \end{pmatrix} h_1 + \begin{pmatrix} 0.068 \\ -0.015 \end{pmatrix} W \frac{\partial h_1}{\partial z_1} \frac{\partial z_1}{\partial W} \\
 &= \begin{pmatrix} 0.068 \\ -0.015 \end{pmatrix} (0.094 \quad 0.134) + \begin{pmatrix} -0.011 & 0.13 \\ -0.123 & 0.014 \end{pmatrix}^T \begin{pmatrix} 0.068 \\ -0.015 \end{pmatrix} \frac{\partial h_1}{\partial z_1} \frac{\partial z_1}{\partial W} \\
 &= \begin{pmatrix} 0.006 & 0.009 \\ -0.001 & -0.002 \end{pmatrix} + (0.001 \quad 0.009) (1 - h_1^2) h_0
 \end{aligned}$$



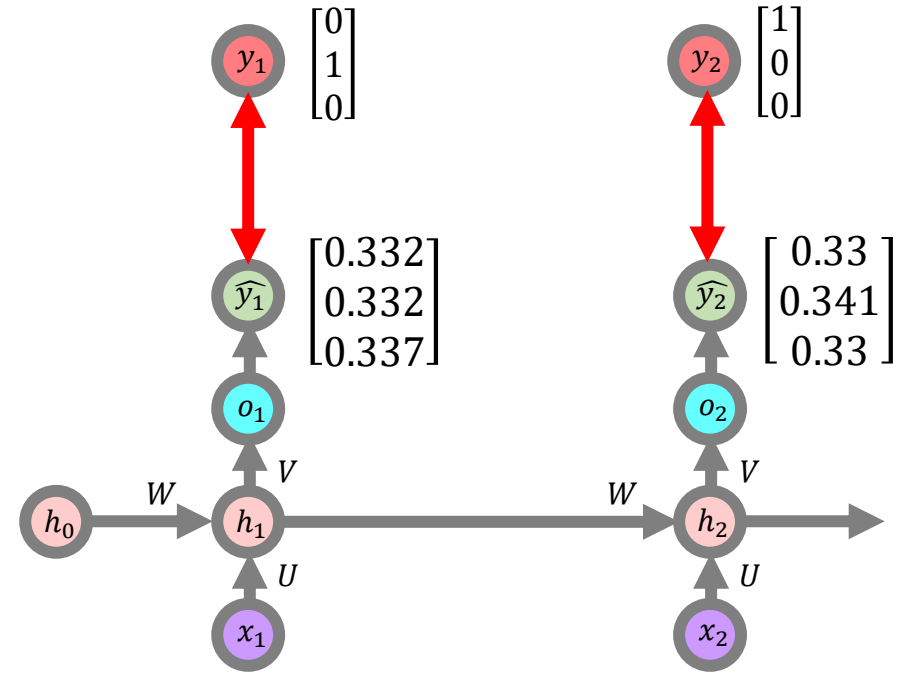
그리하여 우리는 $\frac{\partial L_2}{\partial W}$ 값을 얻게 되었습니다

$$\frac{\partial L_2}{\partial W} = \frac{\partial L_2}{\partial \hat{y}_2} \frac{\partial \hat{y}_2}{\partial o_2} \frac{\partial o_2}{\partial h_2} \frac{\partial h_2}{\partial W} + \frac{\partial L_2}{\partial \hat{y}_2} \frac{\partial \hat{y}_2}{\partial o_2} \frac{\partial o_2}{\partial h_2} \frac{\partial h_2}{\partial h_1} \frac{\partial h_1}{\partial W} = \begin{pmatrix} 0.006 & 0.009 \\ -0.001 & -0.002 \end{pmatrix}$$



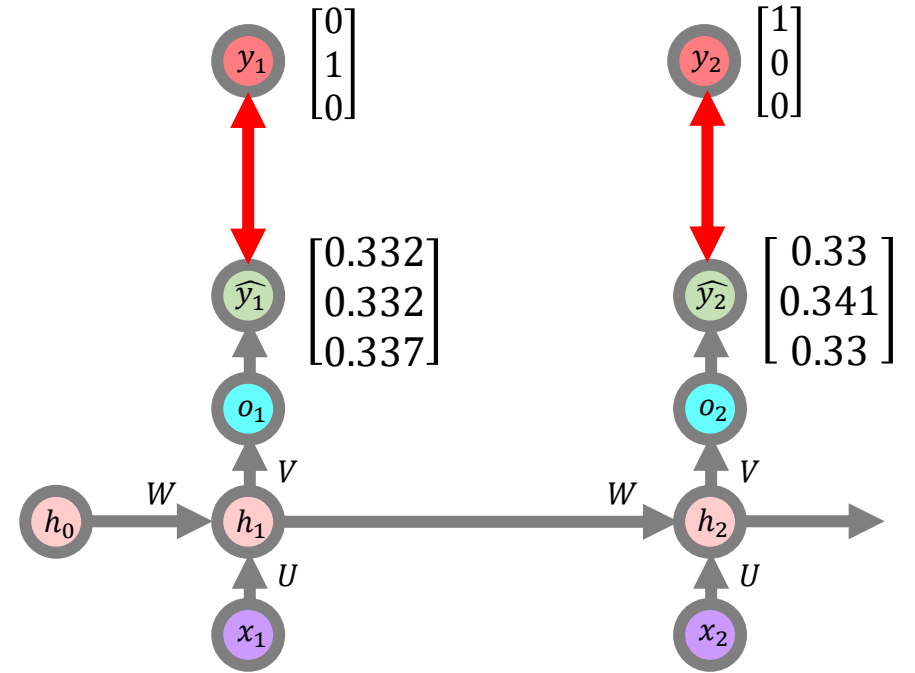
계속하여 $\frac{\partial L_1}{\partial W}$ 도 마찬가지로 방법을 사용하여 구할 수 있습니다

$$\begin{aligned}\frac{\partial L_1}{\partial W} &= \frac{\partial L_1}{\partial \hat{y}_1} \frac{\partial \hat{y}_1}{\partial o_1} \frac{\partial o_1}{\partial h_1} \frac{\partial h_1}{\partial W} \\ &= \frac{\partial L_1}{\partial \hat{y}_1} \frac{\partial \hat{y}_1}{\partial o_1} \frac{\partial o_1}{\partial h_1} \frac{\partial h_1}{\partial z_1} \frac{\partial z_1}{\partial W} \\ &= (\hat{y}_1 - y_1) V (1 - h_1^2) h_0\end{aligned}$$



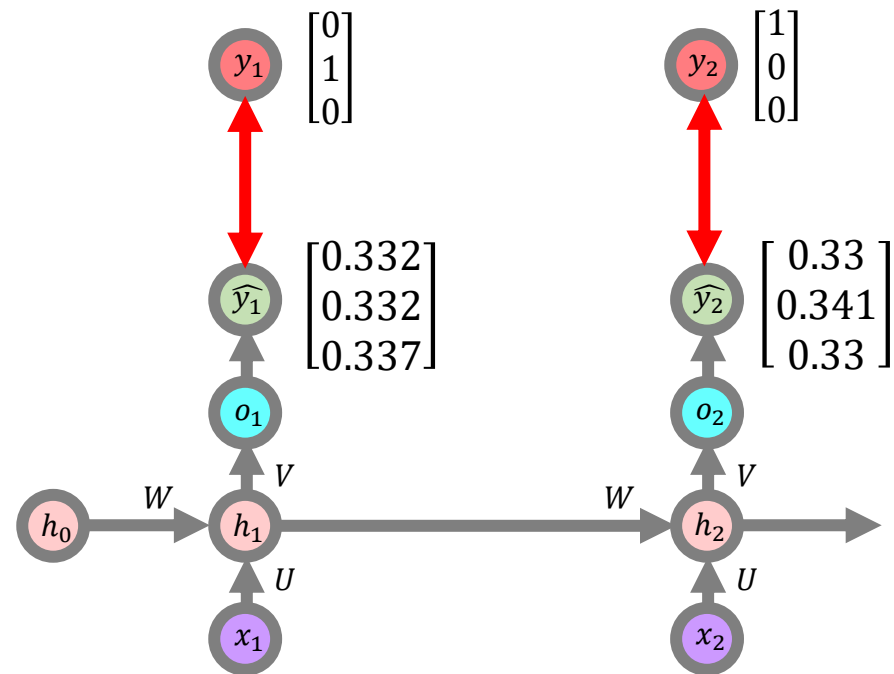
여기서도 h_0 의 값을 $[0 \ 0]$ 으로 하였기 때문에

$$\begin{aligned}\frac{\partial L_1}{\partial W} &= \frac{\partial L_1}{\partial \hat{y}_1} \frac{\partial \hat{y}_1}{\partial o_1} \frac{\partial o_1}{\partial h_1} \frac{\partial h_1}{\partial W} \\ &= \frac{\partial L_1}{\partial \hat{y}_1} \frac{\partial \hat{y}_1}{\partial o_1} \frac{\partial o_1}{\partial h_1} \frac{\partial h_1}{\partial z_1} \frac{\partial z_1}{\partial W} \\ &= (\hat{y}_1 - y_1) V (1 - h_1^2) h_0\end{aligned}$$



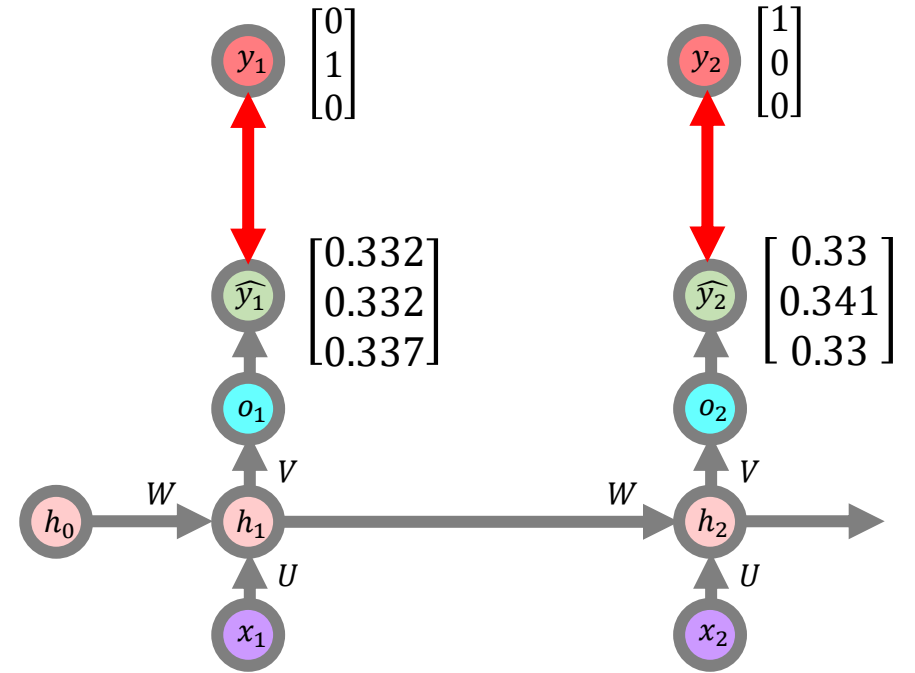
$\frac{\partial L_1}{\partial W}$ 의 값은 0이 되지만,

$$\begin{aligned}\frac{\partial L_1}{\partial W} &= \frac{\partial L_1}{\partial \hat{y}_1} \frac{\partial \hat{y}_1}{\partial o_1} \frac{\partial o_1}{\partial h_1} \frac{\partial h_1}{\partial W} \\ &= \frac{\partial L_1}{\partial \hat{y}_1} \frac{\partial \hat{y}_1}{\partial o_1} \frac{\partial o_1}{\partial h_1} \frac{\partial h_1}{\partial z_1} \frac{\partial z_1}{\partial W} \\ &= (\hat{y}_1 - y_1) V (1 - h_1^2) h_0 = 0\end{aligned}$$



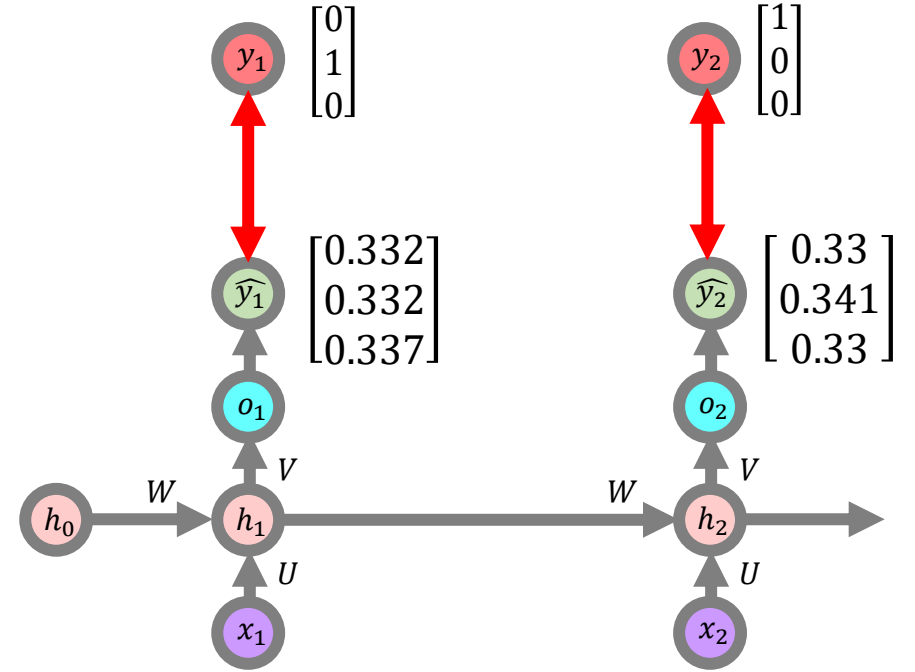
이것은 어디까지나 계산상의 편의를 위해 제가 h_0 의 값을 그렇게 설정했기 때문입니다

$$\begin{aligned}\frac{\partial L_1}{\partial W} &= \frac{\partial L_1}{\partial \hat{y}_1} \frac{\partial \hat{y}_1}{\partial o_1} \frac{\partial o_1}{\partial h_1} \frac{\partial h_1}{\partial W} \\ &= \frac{\partial L_1}{\partial \hat{y}_1} \frac{\partial \hat{y}_1}{\partial o_1} \frac{\partial o_1}{\partial h_1} \frac{\partial h_1}{\partial z_1} \frac{\partial z_1}{\partial W} \\ &= (\hat{y}_1 - y_1) V (1 - h_1^2) h_0 = 0\end{aligned}$$



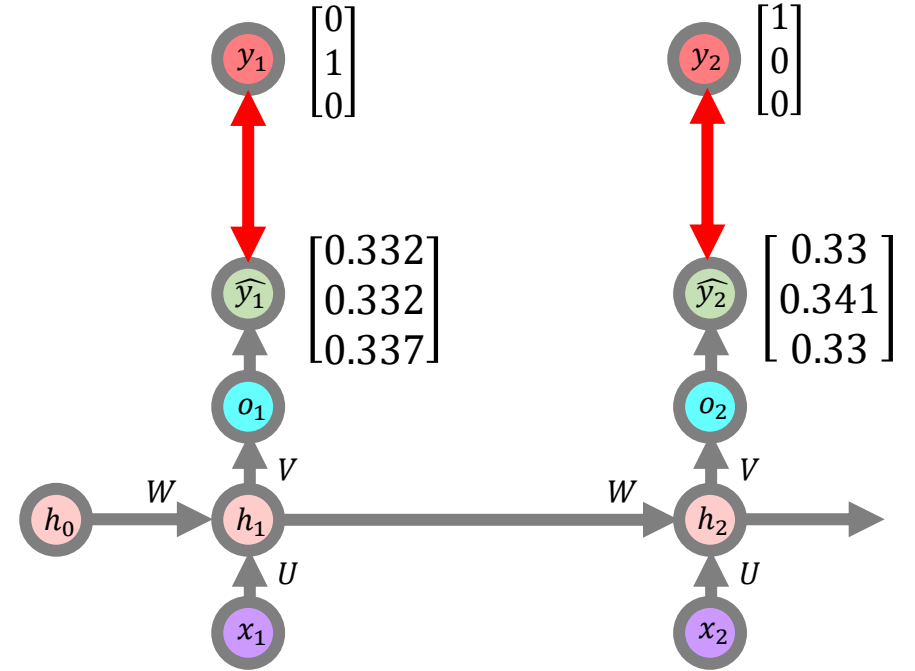
앞서 말씀 드린바 대로, 실제 RNN에서 구현할 때는 h_0 의 값은 앞선 입력 시퀀스에 의해 계산된 마지막 h_{t-1} 의 값이 들어갈 때가 많습니다

$$\begin{aligned}\frac{\partial L_1}{\partial W} &= \frac{\partial L_1}{\partial \hat{y}_1} \frac{\partial \hat{y}_1}{\partial o_1} \frac{\partial o_1}{\partial h_1} \frac{\partial h_1}{\partial W} \\ &= \frac{\partial L_1}{\partial \hat{y}_1} \frac{\partial \hat{y}_1}{\partial o_1} \frac{\partial o_1}{\partial h_1} \frac{\partial h_1}{\partial z_1} \frac{\partial z_1}{\partial W} \\ &= (\hat{y}_1 - y_1) V (1 - h_1^2) h_0 = 0\end{aligned}$$

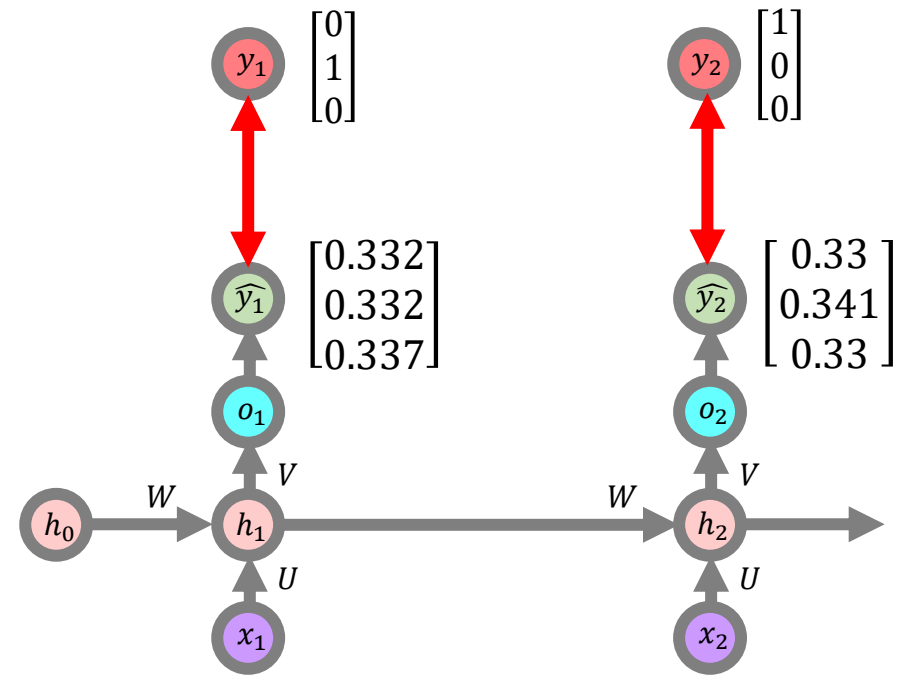


자 이런 계산 과정을 통해 우리는 $\frac{\partial L}{\partial W}$ 까지 구해보았습니다

$$\begin{aligned}\frac{\partial L}{\partial W} &= \frac{\partial L_1}{\partial W} + \frac{\partial L_2}{\partial W} \\ &= 0 + \begin{pmatrix} 0.006 & 0.009 \\ -0.001 & -0.002 \end{pmatrix} \\ &= \begin{pmatrix} 0.006 & 0.009 \\ -0.001 & -0.002 \end{pmatrix}\end{aligned}$$

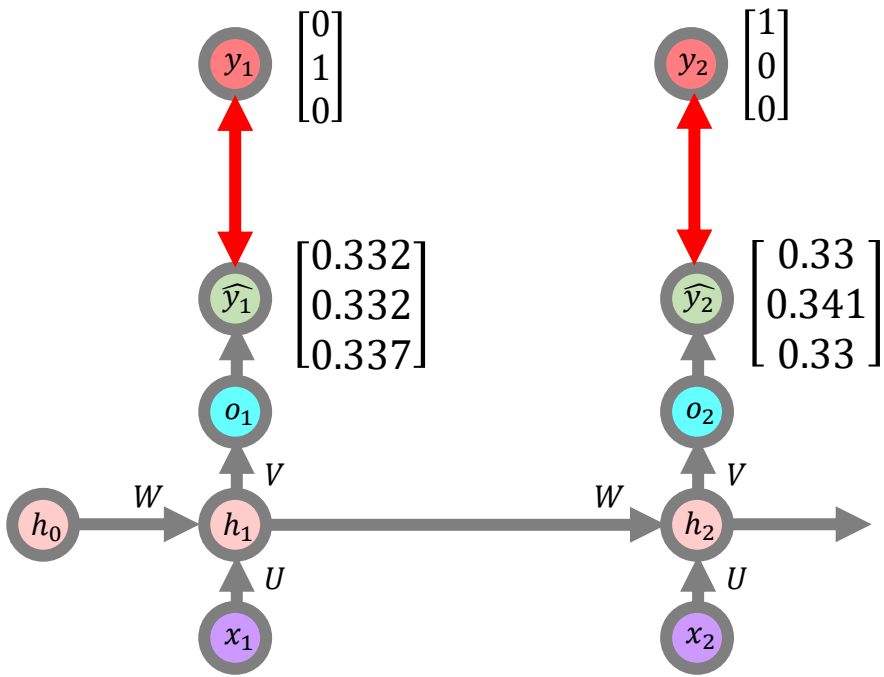


이제 남은 것은 $\frac{\partial L}{\partial U}$ 입니다



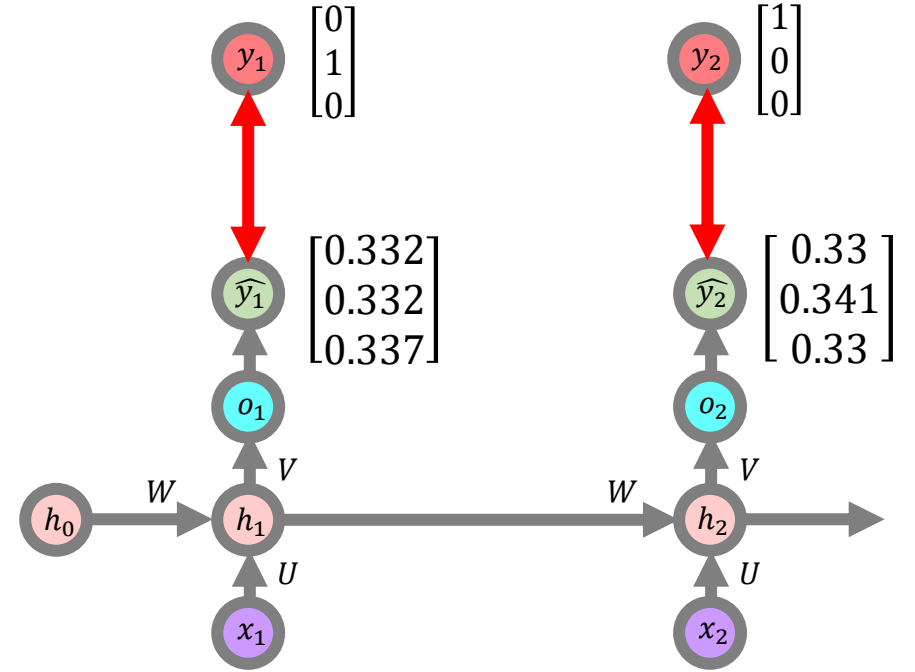
이제 남은 것은 $\frac{\partial L}{\partial U}$ 도 $\frac{\partial L_2}{\partial U}$ 부터 구해보겠습니다

$$\frac{\partial L_2}{\partial U} =$$



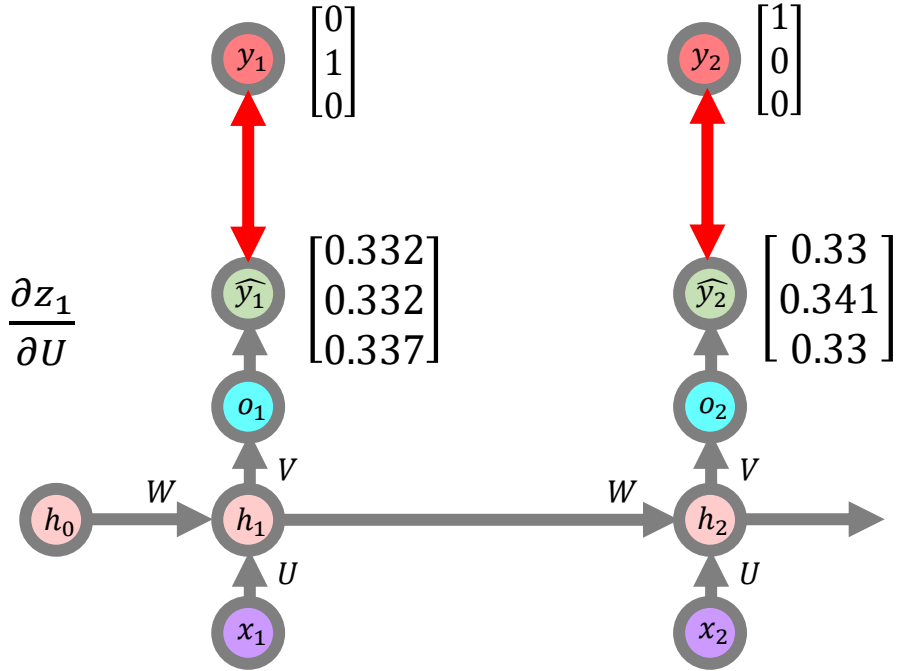
$\frac{\partial L_2}{\partial U}$ 는 $\frac{\partial L}{\partial W}$ 를 계산하는 방식과 크게 다르지 않습니다

$$\frac{\partial L_2}{\partial U} = \frac{\partial L_2}{\partial \hat{y}_2} \frac{\partial \hat{y}_2}{\partial o_2} \frac{\partial o_2}{\partial h_2} \frac{\partial h_2}{\partial U} + \frac{\partial L_2}{\partial \hat{y}_2} \frac{\partial \hat{y}_2}{\partial o_2} \frac{\partial o_2}{\partial h_2} \frac{\partial h_2}{\partial h_1} \frac{\partial h_1}{\partial U}$$



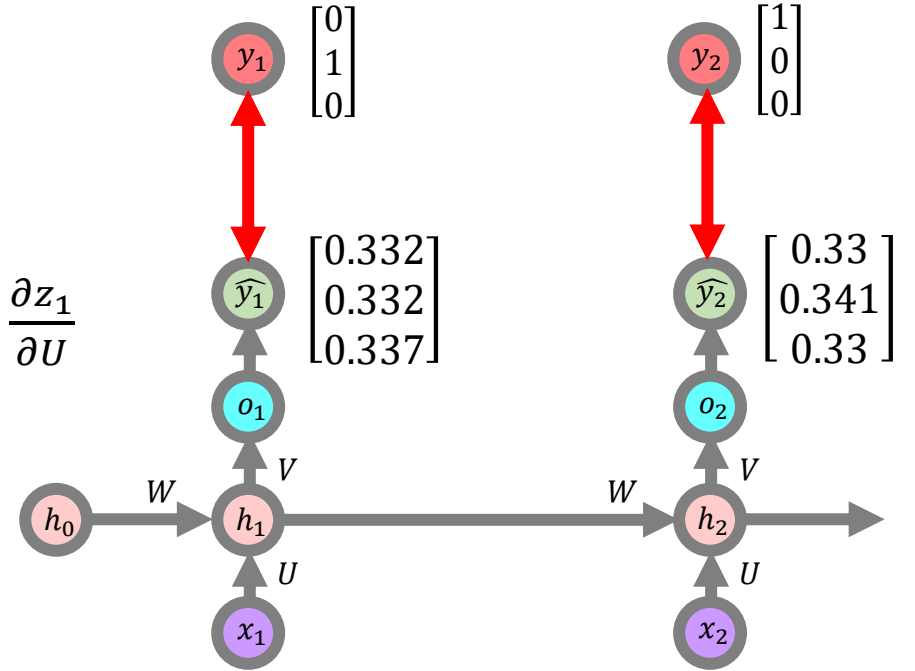
여전히 체인룰을 사용합니다

$$\begin{aligned}
 \frac{\partial L_2}{\partial U} &= \frac{\partial L_2}{\partial \hat{y}_2} \frac{\partial \hat{y}_2}{\partial o_2} \frac{\partial o_2}{\partial h_2} \frac{\partial h_2}{\partial U} + \frac{\partial L_2}{\partial \hat{y}_2} \frac{\partial \hat{y}_2}{\partial o_2} \frac{\partial o_2}{\partial h_2} \frac{\partial h_2}{\partial h_1} \frac{\partial h_1}{\partial U} \\
 &= \frac{\partial L_2}{\partial \hat{y}_2} \frac{\partial \hat{y}_2}{\partial o_2} \frac{\partial o_2}{\partial h_2} \frac{\partial h_2}{\partial z_2} \frac{\partial z_2}{\partial U} + \frac{\partial L_2}{\partial \hat{y}_2} \frac{\partial \hat{y}_2}{\partial o_2} \frac{\partial o_2}{\partial h_2} \frac{\partial h_2}{\partial z_2} \frac{\partial h_2}{\partial h_1} \frac{\partial h_1}{\partial z_1} \frac{\partial z_1}{\partial U} \\
 &= (\hat{y}_2 - y_2) V (1 - h_2^2) \frac{\partial z_2}{\partial U} + (\hat{y}_2 - y_2) V (1 - h_2^2) \frac{\partial z_2}{\partial h_1} \frac{\partial h_1}{\partial z_1} \frac{\partial z_1}{\partial U}
 \end{aligned}$$



다음과 같은 공식에 의해서,

$$\begin{aligned}\frac{\partial L_2}{\partial U} &= \frac{\partial L_2}{\partial \hat{y}_2} \frac{\partial \hat{y}_2}{\partial o_2} \frac{\partial o_2}{\partial h_2} \frac{\partial h_2}{\partial U} + \frac{\partial L_2}{\partial \hat{y}_2} \frac{\partial \hat{y}_2}{\partial o_2} \frac{\partial o_2}{\partial h_2} \frac{\partial h_2}{\partial h_1} \frac{\partial h_1}{\partial U} \\ &= \frac{\partial L_2}{\partial \hat{y}_2} \frac{\partial \hat{y}_2}{\partial o_2} \frac{\partial o_2}{\partial h_2} \frac{\partial h_2}{\partial z_2} \frac{\partial z_2}{\partial U} + \frac{\partial L_2}{\partial \hat{y}_2} \frac{\partial \hat{y}_2}{\partial o_2} \frac{\partial o_2}{\partial h_2} \frac{\partial h_2}{\partial z_2} \frac{\partial h_2}{\partial h_1} \frac{\partial z_2}{\partial h_1} \frac{\partial h_1}{\partial z_1} \frac{\partial z_1}{\partial U} \\ &= (\hat{y}_2 - y_2) V (1 - h_2^2) \frac{\partial z_2}{\partial U} + (\hat{y}_2 - y_2) V (1 - h_2^2) \frac{\partial z_2}{\partial h_1} \frac{\partial h_1}{\partial z_1} \frac{\partial z_1}{\partial U}\end{aligned}$$

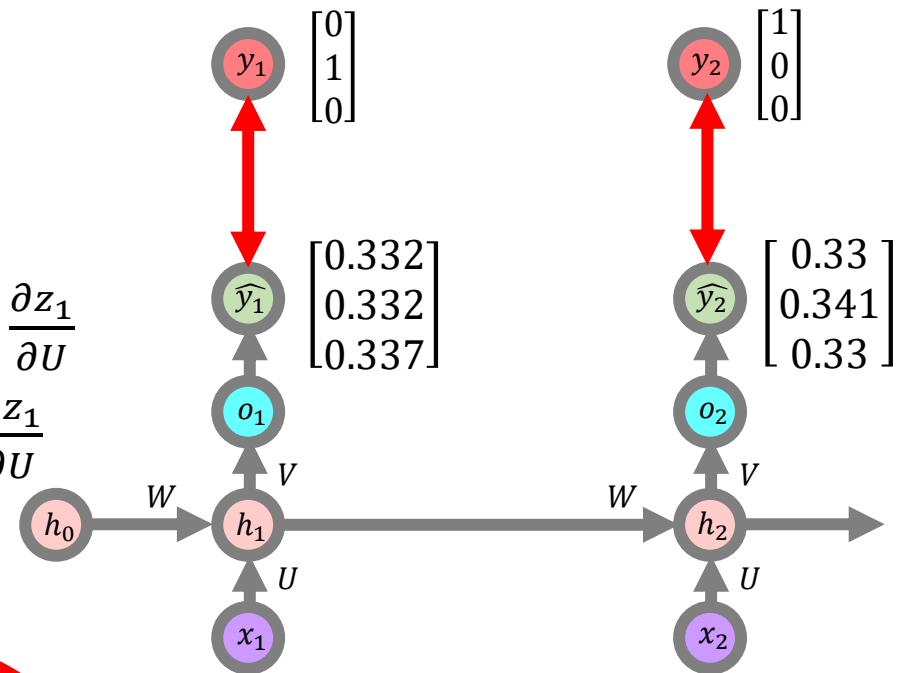


$$h_2 = \tanh(z_2) \quad \rightarrow \quad \frac{\partial h_2}{\partial z_2} = 1 - \tanh^2(z_2) = 1 - h_2^2$$

$$z_2 = W h_1 + U x_2 \quad \rightarrow \quad \frac{\partial z_2}{\partial U} = x_2 \quad \rightarrow \quad \frac{\partial z_2}{\partial h_1} = W$$

$\frac{\partial L_2}{\partial U}$ 에 다음과 같이 대입할 수 있고

$$\begin{aligned}\frac{\partial L_2}{\partial U} &= \frac{\partial L_2}{\partial \hat{y}_2} \frac{\partial \hat{y}_2}{\partial o_2} \frac{\partial o_2}{\partial h_2} \frac{\partial h_2}{\partial U} + \frac{\partial L_2}{\partial \hat{y}_2} \frac{\partial \hat{y}_2}{\partial o_2} \frac{\partial o_2}{\partial h_2} \frac{\partial h_2}{\partial h_1} \frac{\partial h_1}{\partial U} \\ &= \frac{\partial L_2}{\partial \hat{y}_2} \frac{\partial \hat{y}_2}{\partial o_2} \frac{\partial o_2}{\partial h_2} \frac{\partial h_2}{\partial z_2} \frac{\partial z_2}{\partial U} + \frac{\partial L_2}{\partial \hat{y}_2} \frac{\partial \hat{y}_2}{\partial o_2} \frac{\partial o_2}{\partial h_2} \frac{\partial h_2}{\partial z_2} \frac{\partial h_1}{\partial z_1} \frac{\partial z_1}{\partial U} \\ &= (\hat{y}_2 - y_2) V (1 - h_2^2) \frac{\partial z_2}{\partial U} + (\hat{y}_2 - y_2) V (1 - h_2^2) \frac{\partial z_2}{\partial h_1} \frac{\partial h_1}{\partial z_1} \frac{\partial z_1}{\partial U} \\ &= (\hat{y}_2 - y_2) V (1 - h_2^2) x_2 + (\hat{y}_2 - y_2) V (1 - h_2^2) W^T \frac{\partial h_1}{\partial z_1} \frac{\partial z_1}{\partial U}\end{aligned}$$

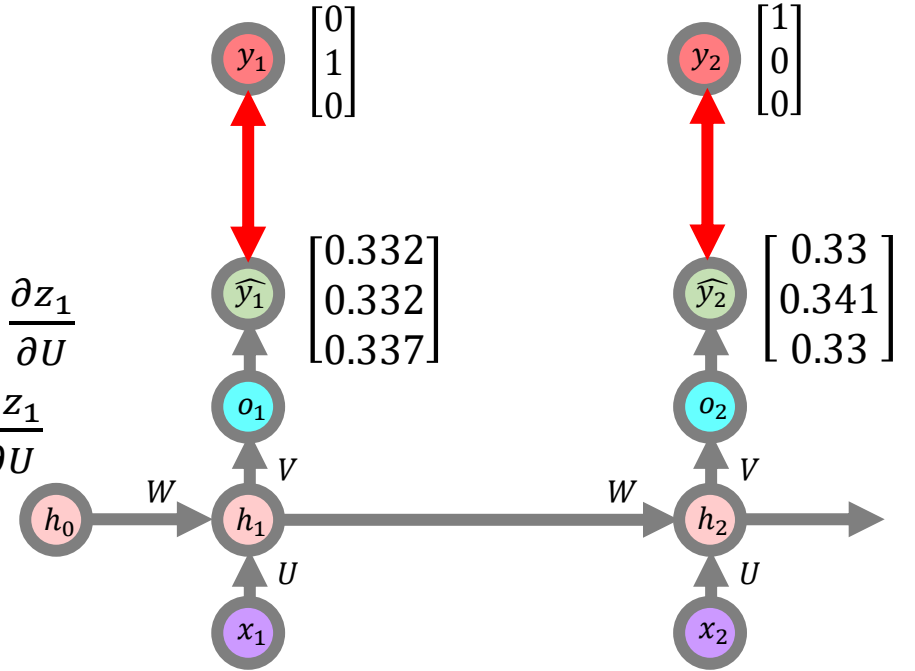


$$h_2 = \tanh(z_2) \quad \rightarrow \quad \frac{\partial h_2}{\partial z_2} = 1 - \tanh^2(z_2) = 1 - h_2^2$$

$$z_2 = W h_1 + U x_2 \quad \rightarrow \quad \frac{\partial z_2}{\partial U} = x_2 \quad \frac{\partial z_2}{\partial h_1} = W$$

다음과 같은 공식에 의해서,

$$\begin{aligned}\frac{\partial L_2}{\partial U} &= \frac{\partial L_2}{\partial \hat{y}_2} \frac{\partial \hat{y}_2}{\partial o_2} \frac{\partial o_2}{\partial h_2} \frac{\partial h_2}{\partial U} + \frac{\partial L_2}{\partial \hat{y}_2} \frac{\partial \hat{y}_2}{\partial o_2} \frac{\partial o_2}{\partial h_2} \frac{\partial h_2}{\partial h_1} \frac{\partial h_1}{\partial U} \\ &= \frac{\partial L_2}{\partial \hat{y}_2} \frac{\partial \hat{y}_2}{\partial o_2} \frac{\partial o_2}{\partial h_2} \frac{\partial h_2}{\partial z_2} \frac{\partial z_2}{\partial U} + \frac{\partial L_2}{\partial \hat{y}_2} \frac{\partial \hat{y}_2}{\partial o_2} \frac{\partial o_2}{\partial h_2} \frac{\partial h_2}{\partial z_2} \frac{\partial h_1}{\partial z_1} \frac{\partial z_1}{\partial U} \\ &= (\hat{y}_2 - y_2) V (1 - h_2^2) \frac{\partial z_2}{\partial U} + (\hat{y}_2 - y_2) V (1 - h_2^2) \frac{\partial z_2}{\partial h_1} \frac{\partial h_1}{\partial z_1} \frac{\partial z_1}{\partial U} \\ &= (\hat{y}_2 - y_2) V (1 - h_2^2) x_2 + (\hat{y}_2 - y_2) V (1 - h_2^2) W^T \frac{\partial h_1}{\partial z_1} \frac{\partial z_1}{\partial U}\end{aligned}$$

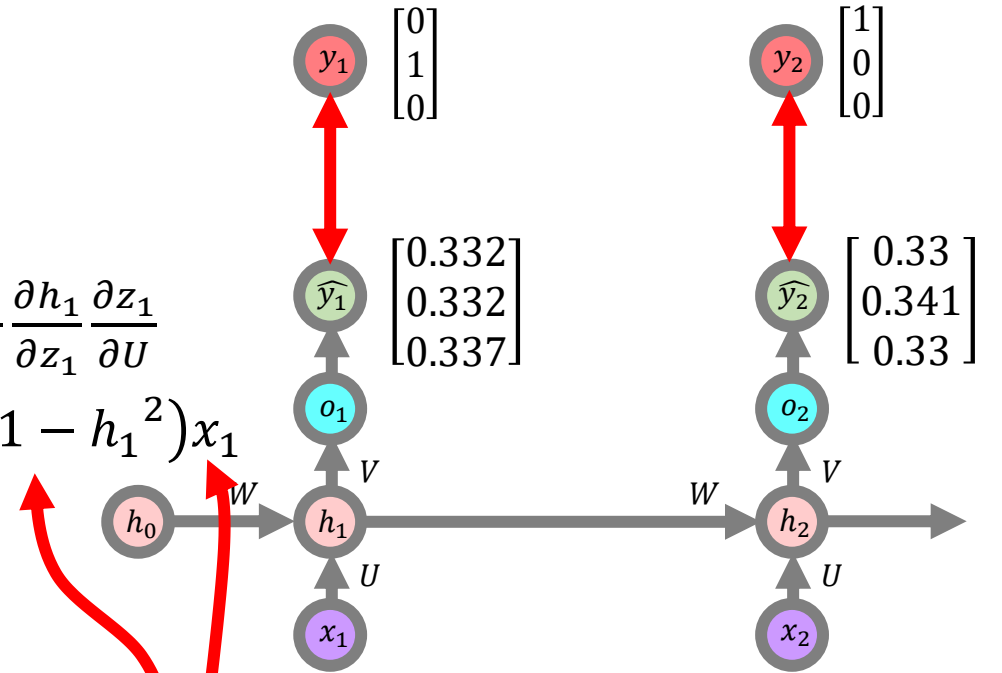


$$h_1 = \tanh(z_1) \quad \rightarrow \quad \frac{\partial h_1}{\partial z_1} = 1 - \tanh^2(z_1) = 1 - h_1^2$$

$$z_1 = W h_1 + U x_2 \quad \rightarrow \quad \frac{\partial z_1}{\partial U} = x_1 \quad \rightarrow \quad \frac{\partial z_1}{\partial h_1} = x_1$$

이렇게 대입할 수가 있습니다

$$\begin{aligned}
 \frac{\partial L_2}{\partial U} &= \frac{\partial L_2}{\partial \hat{y}_2} \frac{\partial \hat{y}_2}{\partial o_2} \frac{\partial o_2}{\partial h_2} \frac{\partial h_2}{\partial U} + \frac{\partial L_2}{\partial \hat{y}_2} \frac{\partial \hat{y}_2}{\partial o_2} \frac{\partial o_2}{\partial h_2} \frac{\partial h_2}{\partial h_1} \frac{\partial h_1}{\partial U} \\
 &= \frac{\partial L_2}{\partial \hat{y}_2} \frac{\partial \hat{y}_2}{\partial o_2} \frac{\partial o_2}{\partial h_2} \frac{\partial h_2}{\partial z_2} \frac{\partial z_2}{\partial U} + \frac{\partial L_2}{\partial \hat{y}_2} \frac{\partial \hat{y}_2}{\partial o_2} \frac{\partial o_2}{\partial h_2} \frac{\partial h_2}{\partial z_2} \frac{\partial h_1}{\partial z_1} \frac{\partial z_1}{\partial U} \\
 &= (\hat{y}_2 - y_2) V (1 - h_2^2) \frac{\partial z_2}{\partial U} + (\hat{y}_2 - y_2) V (1 - h_2^2) \frac{\partial z_2}{\partial h_1} \frac{\partial h_1}{\partial z_1} \frac{\partial z_1}{\partial U} \\
 &= (\hat{y}_2 - y_2) V (1 - h_2^2) x_2 + (\hat{y}_2 - y_2) V (1 - h_2^2) W^T (1 - h_1^2) x_1
 \end{aligned}$$



$$h_1 = \tanh(z_1) \quad \rightarrow \quad \frac{\partial h_1}{\partial z_1} = 1 - \tanh^2(z_1) = 1 - h_1^2$$

$$z_1 = W h_1 + U x_2 \quad \rightarrow \quad \frac{\partial z_1}{\partial U} = x_1 \quad \rightarrow \quad \frac{\partial z_1}{\partial h_1} = x_1$$

이 부분은 앞서 $\frac{\partial L}{\partial W}$ 계산할 때 도출한 값을 재 사용할 수 있습니다

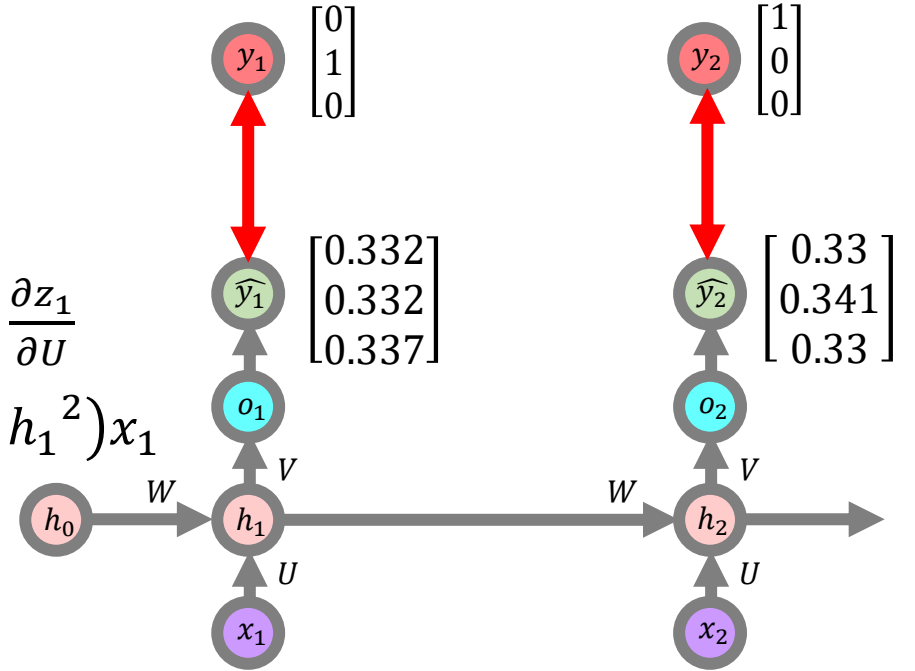
$$\frac{\partial L_2}{\partial U} = \frac{\partial L_2}{\partial \hat{y}_2} \frac{\partial \hat{y}_2}{\partial o_2} \frac{\partial o_2}{\partial h_2} \frac{\partial h_2}{\partial U} + \frac{\partial L_2}{\partial \hat{y}_2} \frac{\partial \hat{y}_2}{\partial o_2} \frac{\partial o_2}{\partial h_2} \frac{\partial h_2}{\partial h_1} \frac{\partial h_1}{\partial U}$$

$$= \frac{\partial L_2}{\partial \hat{y}_2} \frac{\partial \hat{y}_2}{\partial o_2} \frac{\partial o_2}{\partial h_2} \frac{\partial h_2}{\partial z_2} \frac{\partial z_2}{\partial U} + \frac{\partial L_2}{\partial \hat{y}_2} \frac{\partial \hat{y}_2}{\partial o_2} \frac{\partial o_2}{\partial h_2} \frac{\partial h_2}{\partial z_2} \frac{\partial h_1}{\partial z_1} \frac{\partial z_1}{\partial U}$$

$$= (\hat{y}_2 - y_2) V (1 - h_2^2) \frac{\partial z_2}{\partial U} + (\hat{y}_2 - y_2) V (1 - h_2^2) \frac{\partial z_2}{\partial h_1} \frac{\partial h_1}{\partial z_1} \frac{\partial z_1}{\partial U}$$

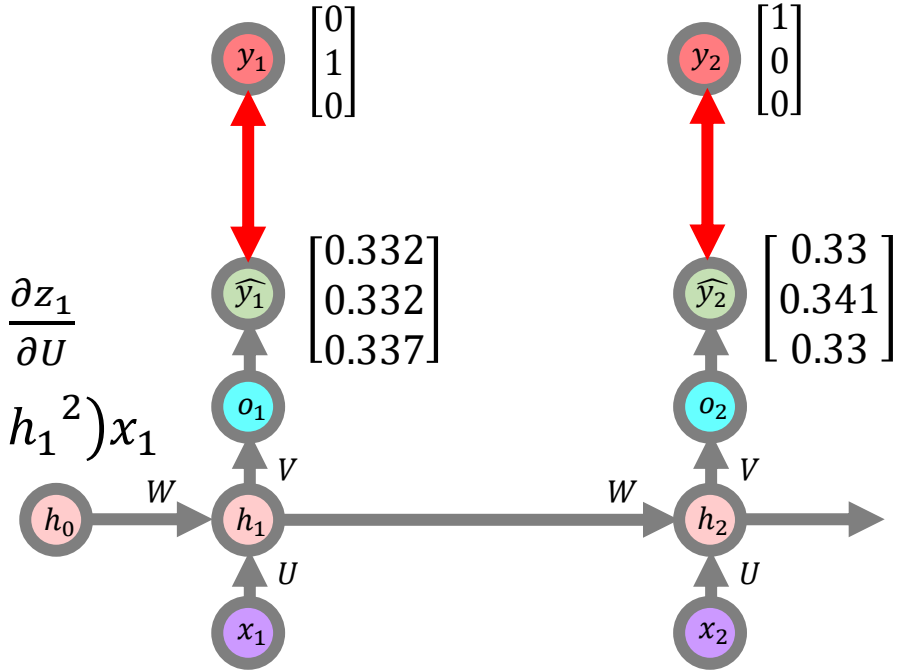
$$= (\hat{y}_2 - y_2) V (1 - h_2^2) x_2 + (\hat{y}_2 - y_2) V (1 - h_2^2) W^T (1 - h_1^2) x_1$$

$$\begin{pmatrix} -0.141 & 0.038 \\ 0.056 & -0.105 \\ -0.132 & 0.14 \end{pmatrix}^T \left(\begin{bmatrix} 0.33 \\ 0.341 \\ 0.33 \end{bmatrix} - \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \right) * (0.977 \quad 1)$$



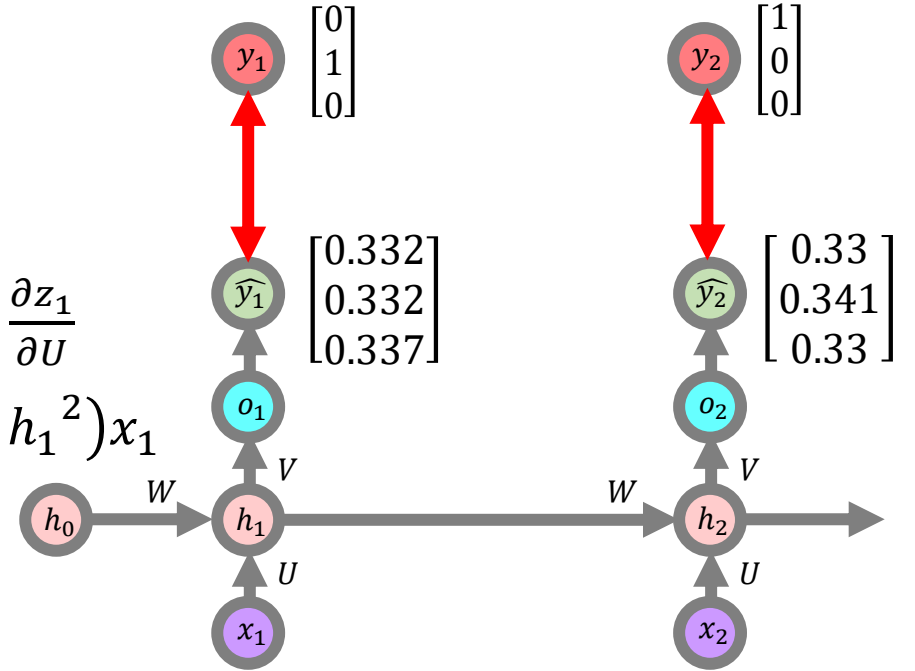
이렇게 계산이 되고

$$\begin{aligned}
 \frac{\partial L_2}{\partial U} &= \frac{\partial L_2}{\partial \hat{y}_2} \frac{\partial \hat{y}_2}{\partial o_2} \frac{\partial o_2}{\partial h_2} \frac{\partial h_2}{\partial U} + \frac{\partial L_2}{\partial \hat{y}_2} \frac{\partial \hat{y}_2}{\partial o_2} \frac{\partial o_2}{\partial h_2} \frac{\partial h_2}{\partial h_1} \frac{\partial h_1}{\partial U} \\
 &= \frac{\partial L_2}{\partial \hat{y}_2} \frac{\partial \hat{y}_2}{\partial o_2} \frac{\partial o_2}{\partial h_2} \frac{\partial h_2}{\partial z_2} \frac{\partial z_2}{\partial U} + \frac{\partial L_2}{\partial \hat{y}_2} \frac{\partial \hat{y}_2}{\partial o_2} \frac{\partial o_2}{\partial h_2} \frac{\partial h_2}{\partial z_2} \frac{\partial h_1}{\partial z_1} \frac{\partial z_1}{\partial U} \\
 &= (\hat{y}_2 - y_2) V (1 - h_2^2) \frac{\partial z_2}{\partial U} + (\hat{y}_2 - y_2) V (1 - h_2^2) \frac{\partial z_2}{\partial h_1} \frac{\partial h_1}{\partial z_1} \frac{\partial z_1}{\partial U} \\
 &= (\hat{y}_2 - y_2) V (1 - h_2^2) x_2 + (\hat{y}_2 - y_2) V (1 - h_2^2) W^T (1 - h_1^2) x_1 \\
 &\quad \begin{pmatrix} 0.068 \\ -0.015 \end{pmatrix}
 \end{aligned}$$



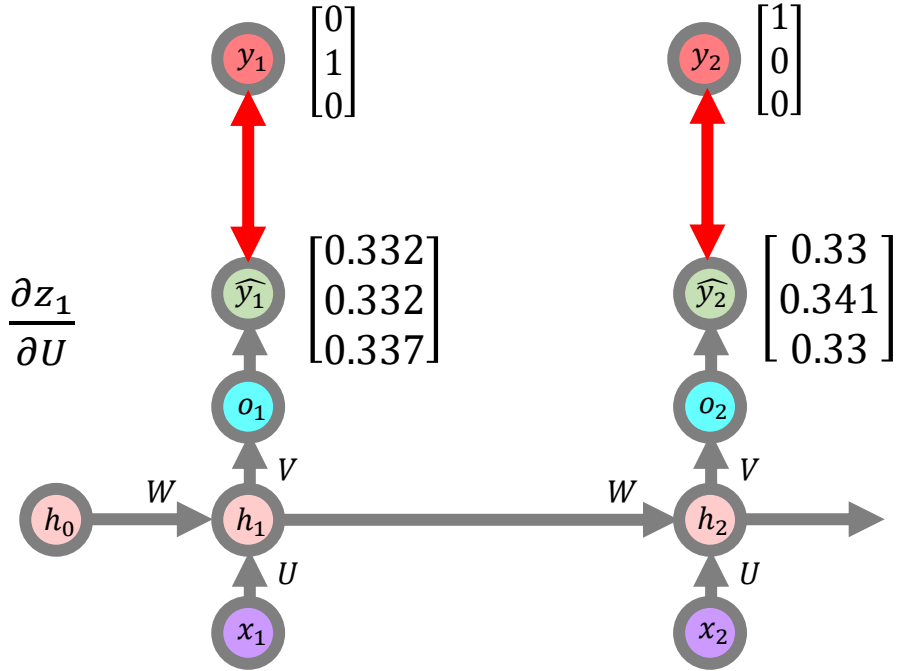
이 두부분에 공통적으로 적용할 수 있습니다

$$\begin{aligned}
 \frac{\partial L_2}{\partial U} &= \frac{\partial L_2}{\partial \hat{y}_2} \frac{\partial \hat{y}_2}{\partial o_2} \frac{\partial o_2}{\partial h_2} \frac{\partial h_2}{\partial U} + \frac{\partial L_2}{\partial \hat{y}_2} \frac{\partial \hat{y}_2}{\partial o_2} \frac{\partial o_2}{\partial h_2} \frac{\partial h_2}{\partial h_1} \frac{\partial h_1}{\partial U} \\
 &= \frac{\partial L_2}{\partial \hat{y}_2} \frac{\partial \hat{y}_2}{\partial o_2} \frac{\partial o_2}{\partial h_2} \frac{\partial h_2}{\partial z_2} \frac{\partial z_2}{\partial U} + \frac{\partial L_2}{\partial \hat{y}_2} \frac{\partial \hat{y}_2}{\partial o_2} \frac{\partial o_2}{\partial h_2} \frac{\partial h_2}{\partial z_2} \frac{\partial h_1}{\partial z_1} \frac{\partial z_1}{\partial U} \\
 &= (\hat{y}_2 - y_2) V (1 - h_2^2) \frac{\partial z_2}{\partial U} + (\hat{y}_2 - y_2) V (1 - h_2^2) \frac{\partial z_2}{\partial h_1} \frac{\partial h_1}{\partial z_1} \frac{\partial z_1}{\partial U} \\
 &= \boxed{(\hat{y}_2 - y_2) V (1 - h_2^2) x_2} + \boxed{(\hat{y}_2 - y_2) V (1 - h_2^2) W^T (1 - h_1^2) x_1} \\
 &\quad \begin{pmatrix} 0.068 \\ -0.015 \end{pmatrix}
 \end{aligned}$$



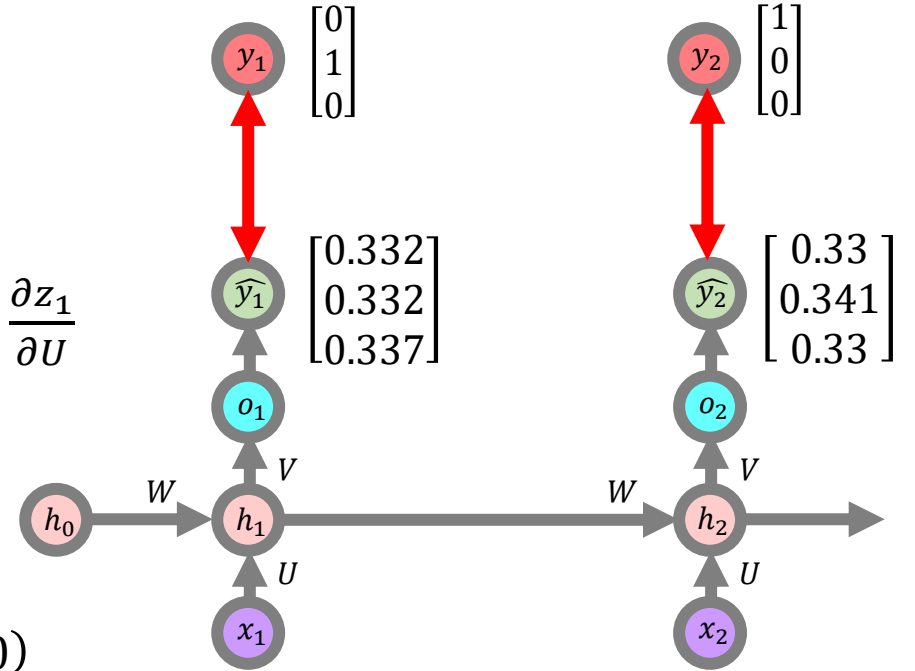
그리고 나머지 값들도 대입해 보도록 하겠습니다

$$\begin{aligned}
 \frac{\partial L_2}{\partial U} &= \frac{\partial L_2}{\partial \hat{y}_2} \frac{\partial \hat{y}_2}{\partial o_2} \frac{\partial o_2}{\partial h_2} \frac{\partial h_2}{\partial U} + \frac{\partial L_2}{\partial \hat{y}_2} \frac{\partial \hat{y}_2}{\partial o_2} \frac{\partial o_2}{\partial h_2} \frac{\partial h_2}{\partial h_1} \frac{\partial h_1}{\partial U} \\
 &= \frac{\partial L_2}{\partial \hat{y}_2} \frac{\partial \hat{y}_2}{\partial o_2} \frac{\partial o_2}{\partial h_2} \frac{\partial h_2}{\partial z_2} \frac{\partial z_2}{\partial U} + \frac{\partial L_2}{\partial \hat{y}_2} \frac{\partial \hat{y}_2}{\partial o_2} \frac{\partial o_2}{\partial h_2} \frac{\partial h_2}{\partial z_2} \frac{\partial h_1}{\partial z_1} \frac{\partial z_1}{\partial U} \\
 &= (\hat{y}_2 - y_2) V (1 - h_2^2) \frac{\partial z_2}{\partial U} + (\hat{y}_2 - y_2) V (1 - h_2^2) \frac{\partial z_2}{\partial h_1} \frac{\partial h_1}{\partial z_1} \frac{\partial z_1}{\partial U} \\
 &= \begin{pmatrix} 0.068 \\ -0.015 \end{pmatrix} x_2 + \begin{pmatrix} 0.068 \\ -0.015 \end{pmatrix} W^T (1 - h_1^2) x_1
 \end{aligned}$$



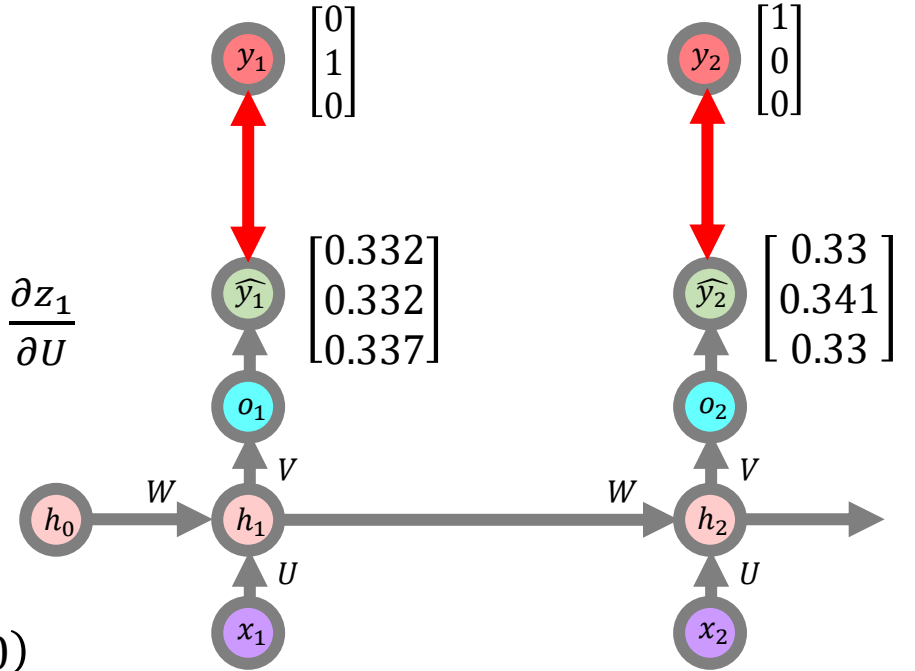
그리고 나머지 값들도 대입해 보도록 하겠습니다

$$\begin{aligned}
 \frac{\partial L_2}{\partial U} &= \frac{\partial L_2}{\partial \hat{y}_2} \frac{\partial \hat{y}_2}{\partial o_2} \frac{\partial o_2}{\partial h_2} \frac{\partial h_2}{\partial U} + \frac{\partial L_2}{\partial \hat{y}_2} \frac{\partial \hat{y}_2}{\partial o_2} \frac{\partial o_2}{\partial h_2} \frac{\partial h_2}{\partial h_1} \frac{\partial h_1}{\partial U} \\
 &= \frac{\partial L_2}{\partial \hat{y}_2} \frac{\partial \hat{y}_2}{\partial o_2} \frac{\partial o_2}{\partial h_2} \frac{\partial h_2}{\partial z_2} \frac{\partial z_2}{\partial U} + \frac{\partial L_2}{\partial \hat{y}_2} \frac{\partial \hat{y}_2}{\partial o_2} \frac{\partial o_2}{\partial h_2} \frac{\partial h_2}{\partial z_2} \frac{\partial h_1}{\partial z_1} \frac{\partial z_1}{\partial U} \\
 &= (\hat{y}_2 - y_2) V (1 - h_2^2) \frac{\partial z_2}{\partial U} + (\hat{y}_2 - y_2) V (1 - h_2^2) \frac{\partial z_2}{\partial h_1} \frac{\partial h_1}{\partial z_1} \frac{\partial z_1}{\partial U} \\
 &= \begin{pmatrix} 0.068 \\ -0.015 \end{pmatrix} x_2 + \begin{pmatrix} 0.068 \\ -0.015 \end{pmatrix} W^T (1 - h_1^2) x_1 \\
 &= \begin{pmatrix} 0.068 \\ -0.015 \end{pmatrix} \begin{pmatrix} 0 & 0 & 1 \end{pmatrix} + \\
 &\quad \begin{pmatrix} -0.011 & 0.13 \\ -0.123 & 0.014 \end{pmatrix}^T \begin{pmatrix} 0.068 \\ -0.015 \end{pmatrix} (1 - (0.094 \quad 0.134)^2) \begin{pmatrix} 1 & 0 & 0 \end{pmatrix}
 \end{aligned}$$



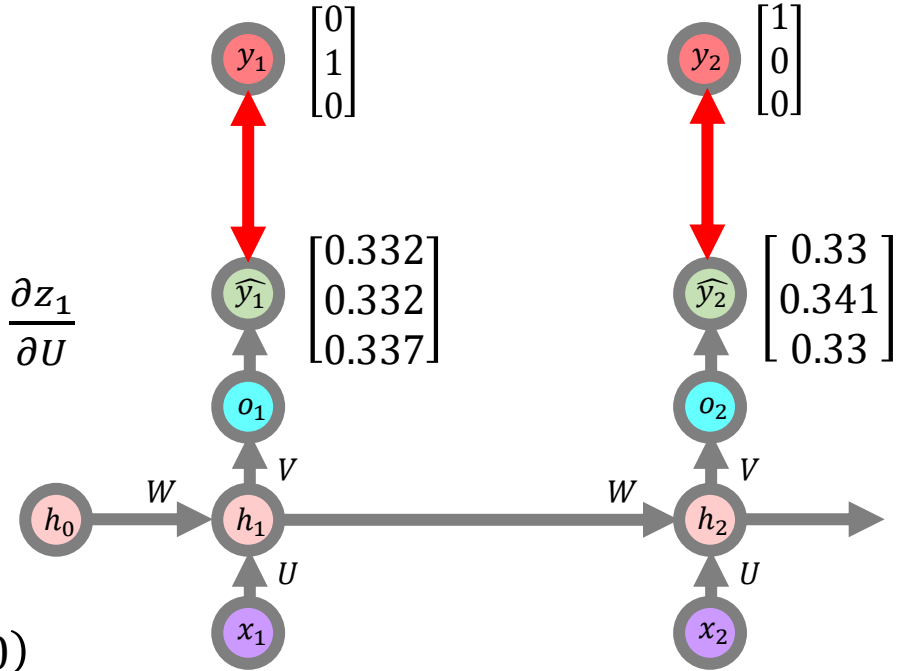
그리고 나머지 값들도 대입해 보도록 하겠습니다

$$\begin{aligned}
 \frac{\partial L_2}{\partial U} &= \frac{\partial L_2}{\partial \hat{y}_2} \frac{\partial \hat{y}_2}{\partial o_2} \frac{\partial o_2}{\partial h_2} \frac{\partial h_2}{\partial U} + \frac{\partial L_2}{\partial \hat{y}_2} \frac{\partial \hat{y}_2}{\partial o_2} \frac{\partial o_2}{\partial h_2} \frac{\partial h_2}{\partial h_1} \frac{\partial h_1}{\partial U} \\
 &= \frac{\partial L_2}{\partial \hat{y}_2} \frac{\partial \hat{y}_2}{\partial o_2} \frac{\partial o_2}{\partial h_2} \frac{\partial h_2}{\partial z_2} \frac{\partial z_2}{\partial U} + \frac{\partial L_2}{\partial \hat{y}_2} \frac{\partial \hat{y}_2}{\partial o_2} \frac{\partial o_2}{\partial h_2} \frac{\partial h_2}{\partial z_2} \frac{\partial h_1}{\partial z_1} \frac{\partial z_1}{\partial U} \\
 &= (\hat{y}_2 - y_2) V (1 - h_2^2) \frac{\partial z_2}{\partial U} + (\hat{y}_2 - y_2) V (1 - h_2^2) \frac{\partial z_2}{\partial h_1} \frac{\partial h_1}{\partial z_1} \frac{\partial z_1}{\partial U} \\
 &= \begin{pmatrix} 0.068 \\ -0.015 \end{pmatrix} x_2 + \begin{pmatrix} 0.068 \\ -0.015 \end{pmatrix} W^T (1 - h_1^2) x_1 \\
 &= \begin{pmatrix} 0.068 \\ -0.015 \end{pmatrix} \begin{pmatrix} 0 & 0 & 1 \end{pmatrix} + \\
 &\quad \begin{pmatrix} -0.011 & 0.13 \\ -0.123 & 0.014 \end{pmatrix}^T \begin{pmatrix} 0.068 \\ -0.015 \end{pmatrix} (1 - (0.094 \quad 0.134)^2) \begin{pmatrix} 1 & 0 & 0 \end{pmatrix} \\
 &= \begin{pmatrix} 0 & 0 & 0.068 \\ 0 & 0 & -0.014 \end{pmatrix} + \begin{pmatrix} 0.001 & 0 & 0 \\ 0.009 & 0 & 0 \end{pmatrix}
 \end{aligned}$$



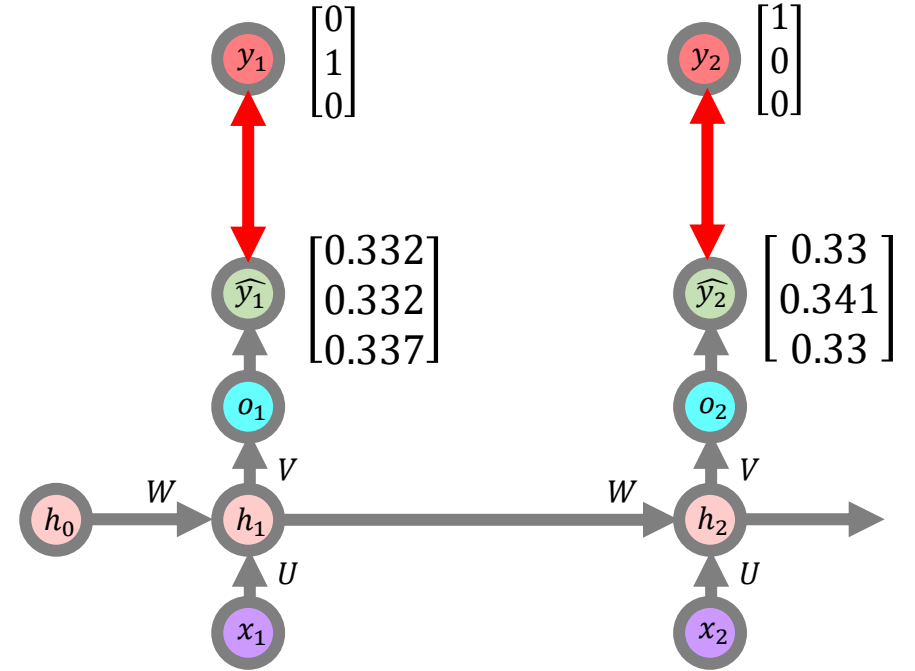
이렇게 값을 구할 수 있습니다

$$\begin{aligned}
 \frac{\partial L_2}{\partial U} &= \frac{\partial L_2}{\partial \hat{y}_2} \frac{\partial \hat{y}_2}{\partial o_2} \frac{\partial o_2}{\partial h_2} \frac{\partial h_2}{\partial U} + \frac{\partial L_2}{\partial \hat{y}_2} \frac{\partial \hat{y}_2}{\partial o_2} \frac{\partial o_2}{\partial h_2} \frac{\partial h_2}{\partial h_1} \frac{\partial h_1}{\partial U} \\
 &= \frac{\partial L_2}{\partial \hat{y}_2} \frac{\partial \hat{y}_2}{\partial o_2} \frac{\partial o_2}{\partial h_2} \frac{\partial h_2}{\partial z_2} \frac{\partial z_2}{\partial U} + \frac{\partial L_2}{\partial \hat{y}_2} \frac{\partial \hat{y}_2}{\partial o_2} \frac{\partial o_2}{\partial h_2} \frac{\partial h_2}{\partial z_2} \frac{\partial h_1}{\partial z_1} \frac{\partial z_1}{\partial U} \\
 &= (\hat{y}_2 - y_2) V (1 - h_2^2) \frac{\partial z_2}{\partial U} + (\hat{y}_2 - y_2) V (1 - h_2^2) \frac{\partial z_2}{\partial h_1} \frac{\partial h_1}{\partial z_1} \frac{\partial z_1}{\partial U} \\
 &= \begin{pmatrix} 0.068 \\ -0.015 \end{pmatrix} x_2 + \begin{pmatrix} 0.068 \\ -0.015 \end{pmatrix} W^T (1 - h_1^2) x_1 \\
 &= \begin{pmatrix} 0.068 \\ -0.015 \end{pmatrix} \begin{pmatrix} 0 & 0 & 1 \end{pmatrix} + \\
 &\quad \begin{pmatrix} -0.011 & 0.13 \\ -0.123 & 0.014 \end{pmatrix}^T \begin{pmatrix} 0.068 \\ -0.015 \end{pmatrix} (1 - (0.094 \quad 0.134)^2) \begin{pmatrix} 1 & 0 & 0 \end{pmatrix} \\
 &= \begin{pmatrix} 0 & 0 & 0.068 \\ 0 & 0 & -0.015 \end{pmatrix} + \begin{pmatrix} 0.001 & 0 & 0 \\ 0.009 & 0 & 0 \end{pmatrix} \\
 &= \begin{pmatrix} 0.001 & 0 & 0.068 \\ 0.009 & 0 & -0.015 \end{pmatrix}
 \end{aligned}$$



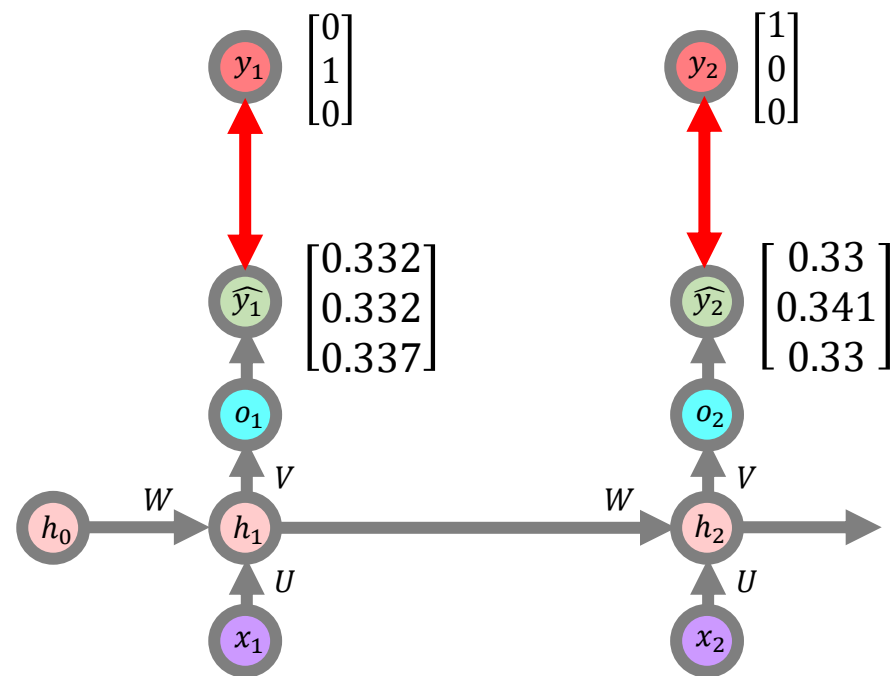
마찬가지 방법으로, $\frac{\partial L_1}{\partial U}$ 도 계산할 수 있습니다

$$\begin{aligned}
 \frac{\partial L_1}{\partial U} &= \frac{\partial L_1}{\partial \hat{y}_1} \frac{\partial \hat{y}_1}{\partial o_1} \frac{\partial o_1}{\partial h_1} \frac{\partial h_1}{\partial U} \\
 &= \frac{\partial L_1}{\partial \hat{y}_1} \frac{\partial \hat{y}_1}{\partial o_1} \frac{\partial o_1}{\partial h_1} \frac{\partial h_1}{\partial z_1} \frac{\partial z_1}{\partial U} \\
 &= (\hat{y}_1 - y_1) V (1 - h_1^2) \frac{\partial z_1}{\partial U} \\
 &= \begin{pmatrix} -0.126 \\ 0.127 \end{pmatrix} x_1 \\
 &= \begin{pmatrix} -0.126 \\ 0.127 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \end{pmatrix} \\
 &= \begin{pmatrix} -0.126 & 0 & 0 \\ 0.127 & 0 & 0 \end{pmatrix}
 \end{aligned}$$



최종 $\frac{\partial L}{\partial U}$ 값은 다음과 같습니다

$$\begin{aligned}\frac{\partial L}{\partial U} &= \frac{\partial L_1}{\partial U} + \frac{\partial L_2}{\partial U} \\ &= \begin{pmatrix} -0.126 & 0 & 0 \\ 0.127 & 0 & 0 \end{pmatrix} + \begin{pmatrix} 0.001 & 0 & 0.068 \\ 0.009 & 0 & -0.015 \end{pmatrix} \\ &= \begin{pmatrix} -0.125 & 0 & 0.068 \\ 0.136 & 0 & -0.015 \end{pmatrix}\end{aligned}$$

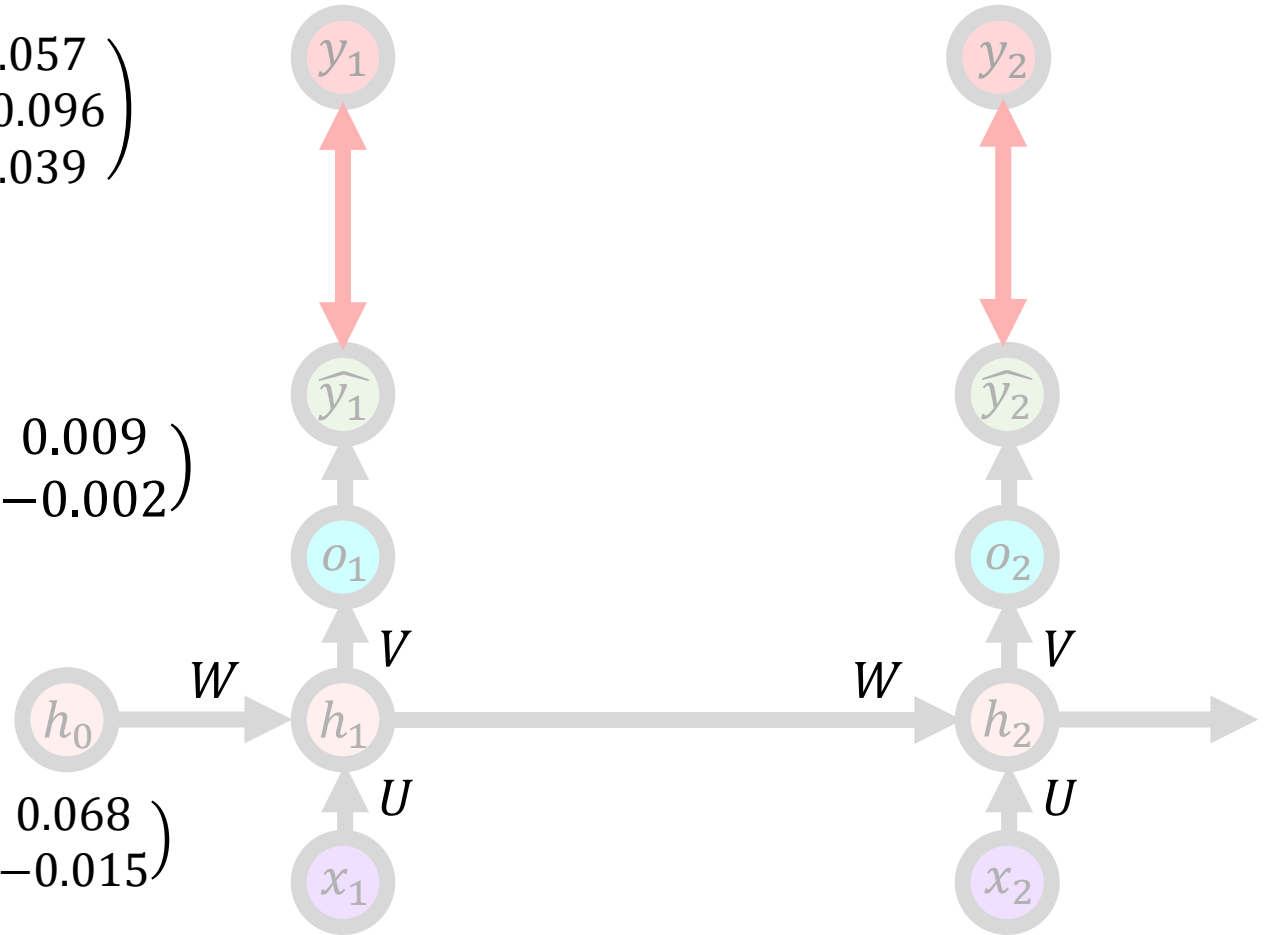


이렇게 $\frac{\partial L}{\partial V}$, $\frac{\partial L}{\partial W}$, $\frac{\partial L}{\partial U}$ 값을 구해보았습니다

$$V^* = V - \alpha \frac{\partial L}{\partial V} = \begin{pmatrix} -0.07 & 0.057 \\ -0.011 & -0.096 \\ 0.081 & 0.039 \end{pmatrix}$$

$$W^* = W - \alpha \frac{\partial L}{\partial W} = \begin{pmatrix} 0.006 & 0.009 \\ -0.001 & -0.002 \end{pmatrix}$$

$$U^* = U - \alpha \frac{\partial L}{\partial U} = \begin{pmatrix} -0.125 & 0 & 0.068 \\ 0.136 & 0 & -0.015 \end{pmatrix}$$

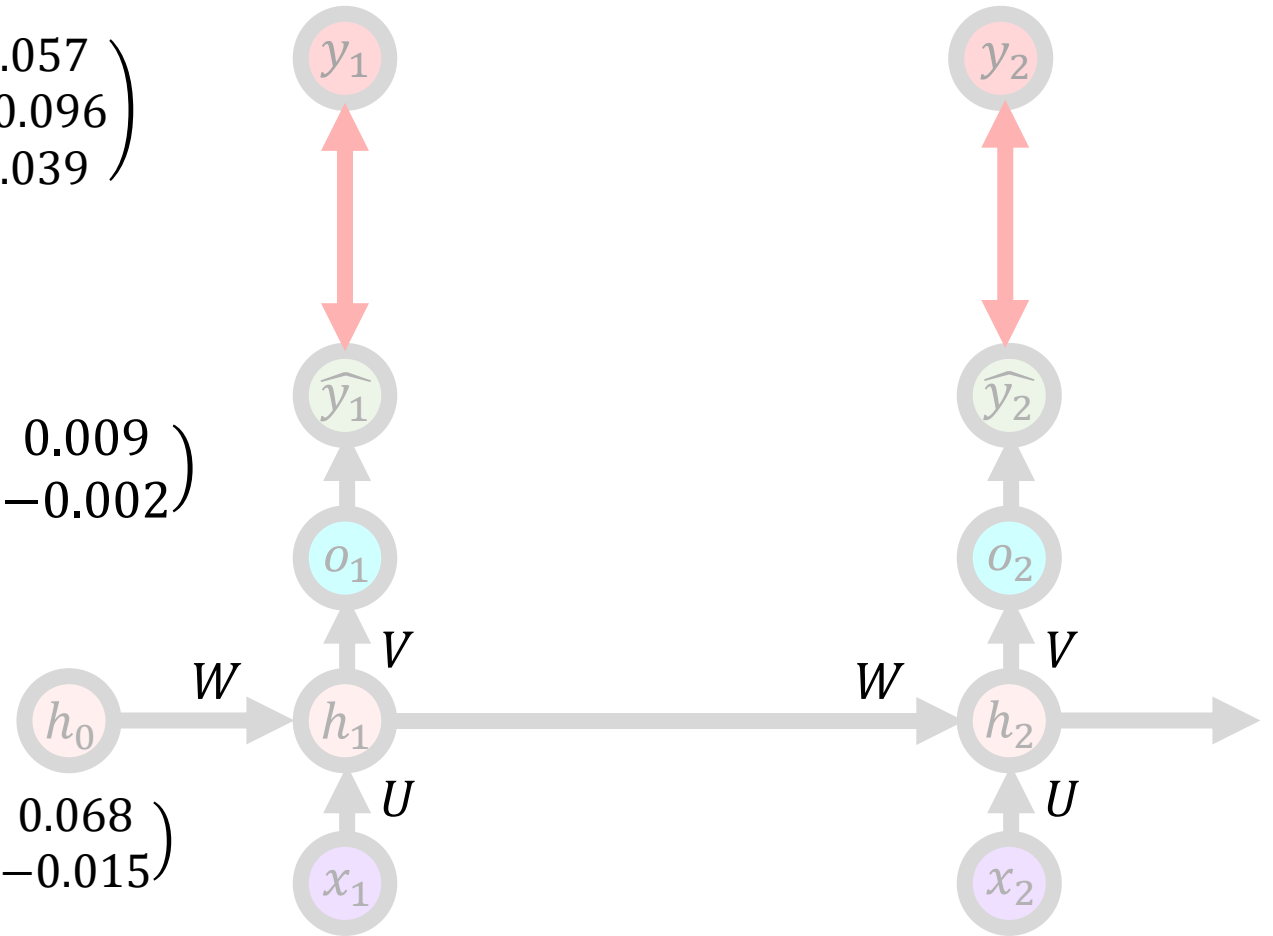


이러한 기울기들을 경사하강법을 사용하여 점진적으로 U , W , V 가중치들을 업데이트 하면 학습이 이루어집니다

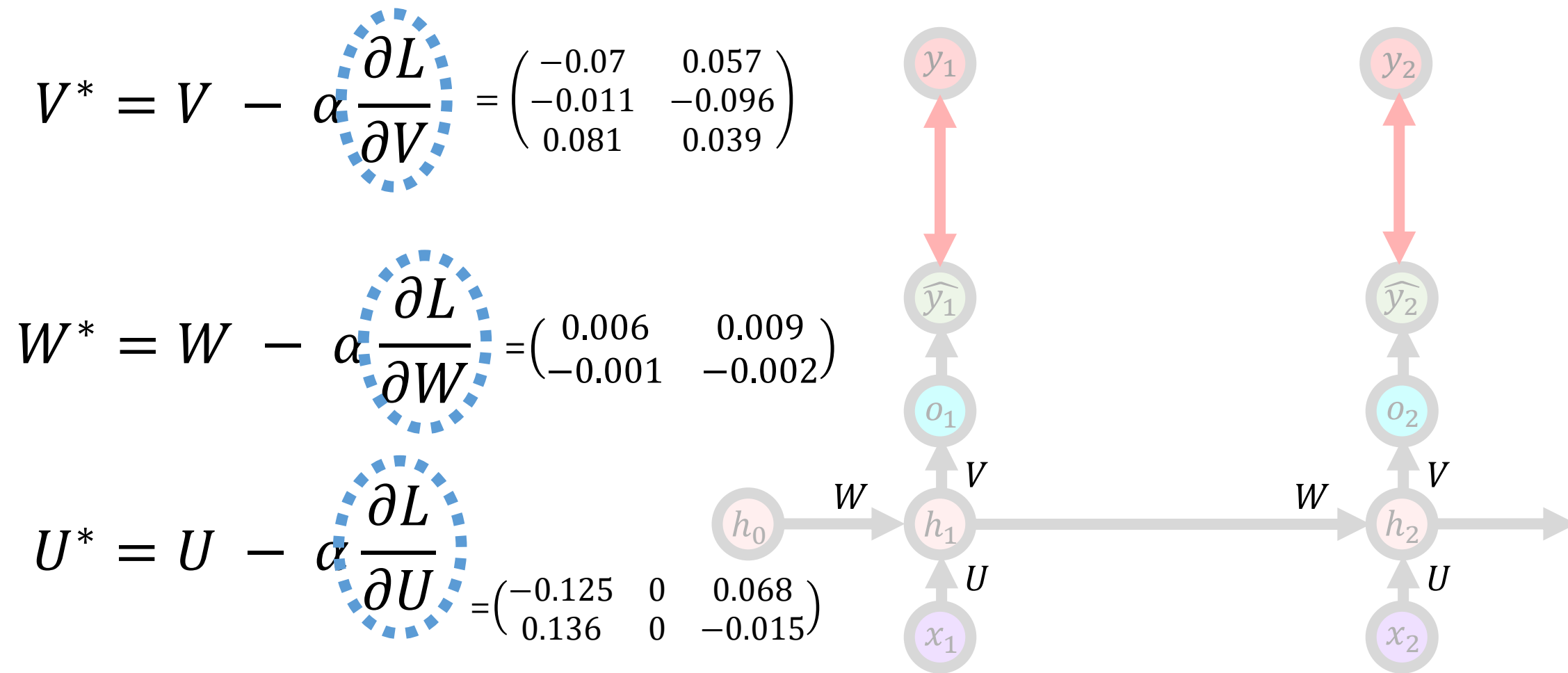
$$V^* = V - \alpha \frac{\partial L}{\partial V} = \begin{pmatrix} -0.07 & 0.057 \\ -0.011 & -0.096 \\ 0.081 & 0.039 \end{pmatrix}$$

$$W^* = W - \alpha \frac{\partial L}{\partial W} = \begin{pmatrix} 0.006 & 0.009 \\ -0.001 & -0.002 \end{pmatrix}$$

$$U^* = U - \alpha \frac{\partial L}{\partial U} = \begin{pmatrix} -0.125 & 0 & 0.068 \\ 0.136 & 0 & -0.015 \end{pmatrix}$$



이것이 RNN의 시간을 통한 역전파 Backpropagation through time의 대략적인 과정입니다



여기까지가 RNN 소개 및 순전파 역전파 과정
에 관한 조금은 깊이가 있는 설명이었습니다

RNN을 그저 피상적으로 이해하지 않고
보다 좀 더 깊은 이해를 돕기 위하여

세세한 공식과 증명과정까지, 다소 긴 영상을
준비하였음을 양해 부탁드립니다

이런 내부적인 연산들을 이해함으로써,
제 영상을 시청해주시는 분들에게

딥러닝을 연구하고 공부하시는데 조금이나마
도움이 되기를 바라는 마음으로
준비하였습니다

다음 영상에서는 오늘 배운 것을 바탕으로 RNN
을 구현해보는 시간을 갖도록 하겠습니다

이 채널은 여러분의 관심과 사랑이 필요합니다

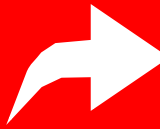
좋아요



댓글



공유



구독



‘좋아요’와 ‘구독’버튼은 강의 준비에 큰 힘이 됩니다!

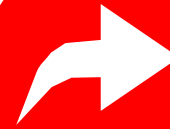
좋아요



댓글



공유



구독



그리고 영상 자료를 사용하실때는
출처 '신박AI'를 밝혀주세요



오늘 긴 시간 시청해 주셔서..

감사합니다!



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