A Novel PID Gain Tuning for a System with Target Specifications using Evolutionary Constrained Optimization

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***Abstract*—The PID control is one of the most widely applied control techniques in the control industry. To control the system precisely, PID gain parameters should be tuned finely based on the system model. Since complex or unknown system is hard to model, there are many methods proposed to tune PID gain parameters by checking error of the system without any modeling. These methods usually estimate minimum error of the control system without considering any other system specifications. In this paper, we propose a novel PID gain tuning method with satisfied target specifications. In this paper, two-phase evolutionary programming (TPEP) algorithm is utilized to optimize PID gains of a non-linear and constrained system. In the first-phase of TPEP, general evolutionary programming is applied to a non-linear system. To satisfy constraints, augmented Lagrangian method is utilized in the second-phase. The proposed algorithm optimizes PID gains by establishing inequality constraints from target specifications. Thus, PID parameters are estimated so that the system has minimum error and the target specifications are satisfied at the same time. We demonstrate our proposed algorithm with DC motor simulator by comparing with a conventional gain tuning method.**

Keywords*—PID tuning, PID control, two-phase evolutionary programming, evolutionary constrained optimization.

I. Introduction

The PID (proportional-integral-derivative) controller is one of the most widely used control techniques in automatic system [1]. The most important factor in the PID controller is to decide a proportional, a integral, and a derivative gain parameters. Therefore, the PID gain parameters have to be determined in accordance with a target system of the controller. The gain parameters are generally determined from the system model. If the system, however, is hard to estimate its model, it is difficult to determine the PID gain parameters without the system model. Thus, there are PID tuning methods to estimate suitable PID gain parameters for a complex or unknown system. Ziegler-Nicholas and Cohen-Coon tuning [2] methods are widely used PID tuning methods. Since these methods are heuristic method of tuning a PID controller, it is unsuitable to use in complex and non-linear system. Thus, some intelligent optimization techniques of PID tuning are proposed to complex or unknown system. These techniques estimate best PID gain parameters by trial and error automatically using computational intelligence methods for optimization such as genetic algorithm, evolution programming, particle swarm optimization, fuzzy, etc. [3 - 10] for minimizing of the system error. However, these methods are inappropriate to utilize a specific system which satisfy target specifications such as rising time, maximum overshoot, and settling time. Therefore, we propose a novel PID gain tuning method by considering target specifications based on evolutionary constrained optimization.

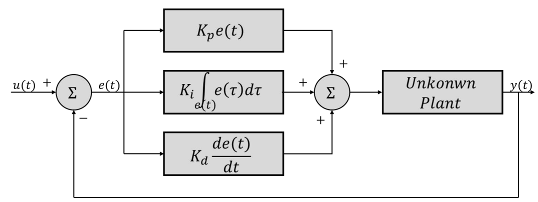

Fig. 1. PID control system.

Two-phase evolutionary programming(TPEP) [11] is a optimization method of evolutionary programming in the cases of inequality and equality constraints. This paper utilizes TPEP algorithm to estimation best PID gain parameters with minimizing error and satisfying target specifications. As the target specifications set inequality constrains of TPEP, the PID gain parameters are tuned as minimizing error and satisfying target specifications at the same time. The target specifications are able to define variously by a user. In this paper, we define target specifications as rising time, maximum overshoot, and settling time in a step response of the system. To demonstrate the proposed algorithm, we compare the results of PID gain parameters with constraints and without constraints.

The rest of this paper is structured as follows. Section II explains the PID control system. In Section III. the proposed gain tuning method is introduced. Section IV shows simulation results of the proposed algorithm to demonstrate. In Section V, we explain conclusion and feature works.

II. PID Control System

To demonstrate the proposed algorithm, a basic closed loop PID controller is designed as Figure 1. $u(t)$ and $y(t)$ in Figure 1 are s input and observation value of the system, respectively. A basic equation of the closed loop PID controller is expressed as follows:

$$MV(t) = K_p e(t) + K_i \int_0^t e(\tau)d\tau + K_d \frac{d}{dt} e(t) \tag{1}$$

where $MV(t)$ is a control value of this system such as manipulated variable or controled variable. K_p, K_i, and K_d are proportional, integral, and derivative gain parameters, respectively. $e(t)$ is a error of current status as a difference between the input value, $u(t)$, and the observation value, $y(t)$. K_p, K_i, and K_d should be tuned parameters of the PID controller.

III. GAIN TUNING METHOD

The purpose of this paper is to estimate K_p, K_i, and K_d of the PID controller to fit the unknown plant by using a optimization method. In general methods of PID gain tuning, there are several solutions of gain tuning in search space. Therefore, this paper estimates a optimal solution by considering target specifications of a system as using the constrained optimization method.

A. Two-phase Evolutionary Programing

Since the PID controller deal with unknown plant as well as satisfies the target specifications, non-linear and constraints condition optimization method is necessary for the PID gain tuning method. Although there are many non-linear and constraints condition optimization method such as active-set algorithm and trust-region method, TPEP algorithm is utilized in this paper because this unknown plant is not able to obtain gradients of a cost function. TPEP consists of two-phase during optimization. The first phase uses the standard EP (evolution programming), while an EP formulation of the augmented Lagrangian method is employed in the second phase. The optimization problem of TPEP is defined by a general constrained optimization problem as follows:

$$\text{minimize} \quad : \quad f(x) \tag{2}$$

$$\text{subject to} \quad : \quad h_i = 0 \qquad (i = 1, ..., n) \tag{3}$$

$$g_j \leq 0 \qquad (j = 1, ..., m) \tag{4}$$

where $f(x)$ denotes a cost function, and $h_i(x)$ and $g_i(x)$ denote equality and inequality constraints, respectively. n and m denote number of equality and inequality constraints, respectively.

Through TPEP algorithm, the cost function, $f(x)$, is minimized while $h_i(x) = 0$ and $g_j(x) \leq 0$ conditions are satisfied. The first phase of TPEP algorithm uses the standard EP. When certain halting conditions are satisfied in the first phase, the elitist EP with deterministic ranking strategy is performed as the second phase for the EP formulation of augmented Lagrangian method. A population of the second phase is initialized from the best solution of the first phase. As elitist EP is employed, the best solution always survives in the subsequent generation through a deterministic ranking strategy.

B. Proposed Method

To find the gain parameters of the PID controller likewise other conventional gain tuning methods, the cost function from equation 2 is designed as minimizing error of the system as follows:

$$f(x) = \sum_{t=t_0}^{t_1} |e(t)| \tag{5}$$

TABLE I. ESSENTIAL PARAMETERS OF TPEP.

Parameter	value
Maximum generation 1	100
Maximum generation 2	200
Parents	250
Parents offspring	500

TABLE II. OPTIMIZATION RESULTS WITHOUT ANY CONSTRAINT.

variable	1	2	3	4	5
K_p	8.24	8.48	7.74	7.86	8.34
K_i	0.15	0.21	0.43	0.39	0.36
K_d	8.19	8.27	9.33	10.35	8.17
C_{rt}	3.02s	3.07s	3.1s	3.19s	3.07s
C_{os}	18%	21%	21%	23%	23%
C_{st}	88s	76s	48s	51s	56s
$f_(x)$	2.37	2.39	2.45	2.44	2.44

where $e(t)$ denotes errors between input and observation values at t status. A range of cost function sets from t_0 to t_1. t_0 usually sets at the time of an incoming signal and t_1 sets after stabilizing of this controller. As finding minimum x of $f(x)$ while $x = (K_p, K_i, K_d)^T$, the optimal gain parameters are estimated through TPEP algorithm.

The main idea of the proposed method satisfies the target specifications using inequality constraints. The reason why inequality constraints are used rather than equality constraints is to find a better solution satisfied with the target specifications. As the target specifications consist of rising time, maximum overshoot, and settling time in this paper, three inequality constraints are designed as follows:

$$\eta_{rt}(C_{rt}(x) - T_{rt}) \leq 0 \tag{6}$$

$$\eta_{os}(C_{os}(x) - T_{os}) \leq 0 \tag{7}$$

$$\eta_{st}(C_{st}(x) - T_{st}) \leq 0 \tag{8}$$

where η denotes a scale factor. C and T denote a current and target value of each specifications. Let rs, os, and st define as rising time, maximum overshoot, and settling time, respectively. C_{rs} means the required time to pass to the 99% goal of rising time. C_{os} means a maximum overshoot percentage and C_{st} means the required time to settle to the 99% goal of settling time. ηs of each inequality constraint means weights of optimization for each constraint.

IV. SIMULATIONS

To perform simulations, a simple DC motor simulator is applied as the unknown plant. The DC motor simulator is utilized the fourth order of a difference equation. A step response is utilized for a input signal to DC motor simulator. Essential parameters of TPEP are initialized as table I. In table I, Maximum generation 1 and 2 denote number of generation in first-phase and second-phase, respectively. Parents and parents offspring denote number of vectors in fir-phase and second-phase, respectively. As setting of the initial vectors, the ranges of K_p, K_i, and K_d are configured as $2 \sim 20$, $0 \sim 5$, and $0 \sim 20$, respectively.

The target specifications about rising time, maximum overshoot, and settling time in Equations 6 to 8 set as follows:

$$T_{rt} = 3s, T_{os} = 10\%, T_{st} = 15s. \tag{9}$$

A simulations are performed as repeating five time with four conditions, which are no constraints, rising time constraints,

TABLE III. OPTIMIZATION RESULTS WITH THE CONSTRAINT OF RISING TIME.

variable	1	2	3	4	5
K_p	7.65	5.25	8.16	6.81	7.7
K_i	0.56	0.42	0.77	1.15	1.31
K_d	7.83	12.04	6.57	7.8	6.14
C_{rt}	**2.98s**	**2.98s**	**2.98s**	**2.98s**	**2.98s**
C_{os}	20%	16%	28%	27%	40%
C_{st}	39s	40s	31s	32s	25s
$f(x)$	2.54	2.65	2.67	2.88	3.04

TABLE IV. OPTIMIZATION RESULTS WITH THE CONSTRAINT OF MAXIMUM OVERSHOOT.

variable	1	2	3	4	5
K_p	6.73	6.97	7.06	6.5	7.45
K_i	0.28	0.01	0.1	0.16	0.15
K_d	9.81	10.55	10.25	11.41	7.33
C_{rt}	4.88s	3.01s	3.02s	3.05s	4.51s
C_{os}	**9%**	**7%**	**9%**	**7%**	**9%**
C_{st}	61s	14s	107s	98s	89s
$f(x)$	2.44	2.34	2.36	2.41	2.41

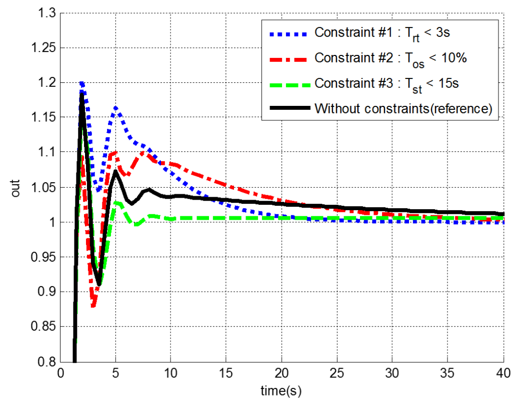

Fig. 2. PID control results of each constraints with reference.

maximum overshoot constraints, and settling time constraints. The representative results of each conditions are shown in Figure 3. Although each plot has different shape, each one well satisfies each condition. Tables II to V show the results of gain parameters and each condition for all data. All data are satisfy target specifications as inequality constraints. All data

TABLE V. OPTIMIZATION RESULTS WITH THE CONSTRAINT OF SETTLING TIME.

variable	1	2	3	4	5
K_p	8.58	9.32	7.47	9.46	10.38
K_i	0.01	1.66	0.02	1.66	0.01
K_d	7.05	8.16	7.13	7.95	4.27
C_{rt}	4.85s	3.36s	4.36s	3.36s	4.92s
C_{os}	16%	57%	6%	58%	29%
C_{st}	**12s**	**15s**	**12s**	**15s**	**14s**
$f(x)$	2.34	3.04	2.36	3.06	2.53

TABLE VI. OPTIMIZATION RESULTS WITH THE MIXED CONSTRAINTS.

variable	1	2	3	4	5
K_p	8.14	7.97	7.97	7.69	8.16
K_i	0.65	0.73	0.87	0.8	0.74
K_d	8.26	8.06	7.87	8.26	7.78
C_{rt}	3.1s	3.08s	3.09s	3.07s	3.08s
C_{os}	27%	26%	29%	26%	28%
C_{st}	36s	32s	27s	28s	32s
$f(x)$	2.64	2.58	2.65	2.62	2.6

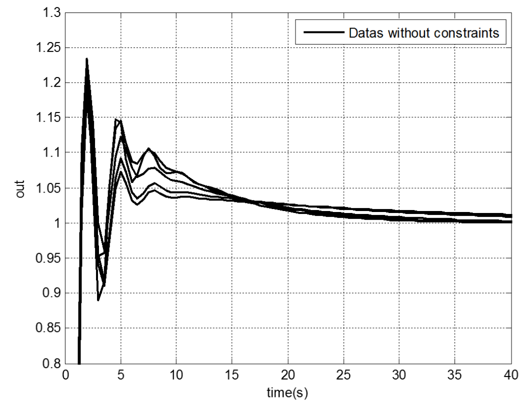

Fig. 3. PID control results without any constraints.

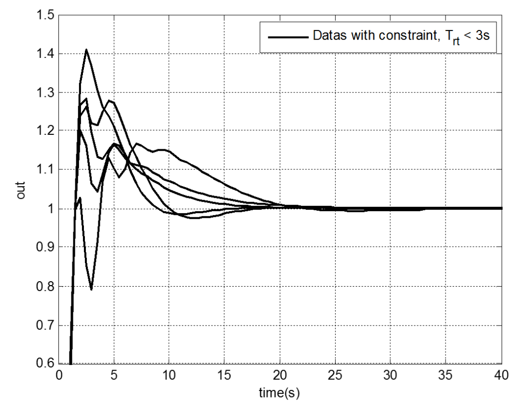

Fig. 4. PID control results with the constraint of rising time.

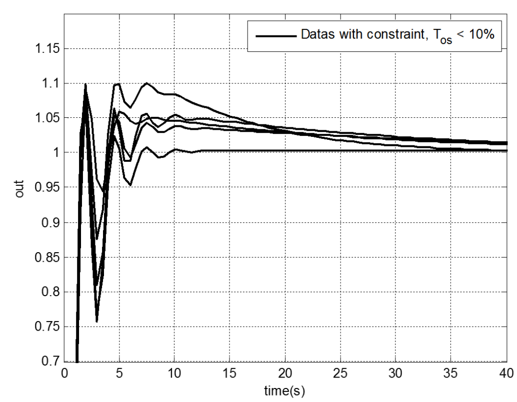

Fig. 5. PID control results with the constraint of maximum overshoot.

results show in Figures 3 to 6.

Simulation with the mixed constraints is also performed to verify the proposed algorithm. The mixed constraints are defined as follows:

$$T_{rt} = 3.1s, T_{os} = 30\%, T_{st} = 40s. \qquad (10)$$

If the constraints are tighter than the target system, the optimization fails. Therefore, the mixed constraints should be well defined. The results of the mixed constraints are shown

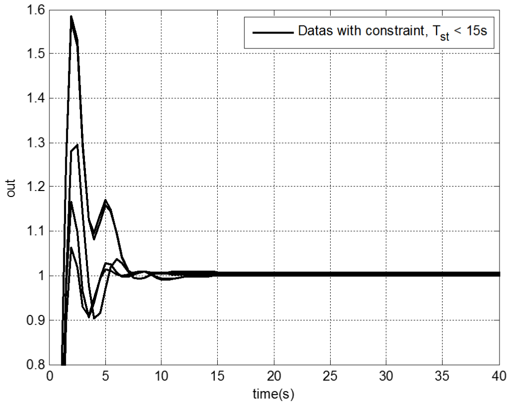

Fig. 6. PID control results with the constraint of settling time.

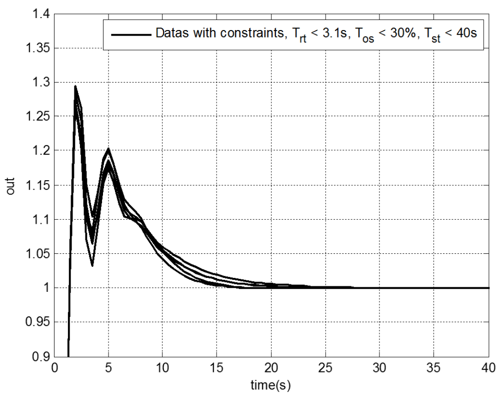

Fig. 7. PID control results with the mixed constraints.

in Figure 7. Tables VI shows the results of gain parameters and each condition for all data of the mixed constraints.

V. CONCLUSION AND FUTURE WORKS

This paper proposes a novel PID gain tuning method using constraints condition to fit the target specifications of a system. evolutionary constrained optimization as TPEP algorithm is applied for non-linear and constraint system. The proposed algorithm finds not only minimum error of the PID controller but also satisfies the target specifications using inequality constrained optimization. As a reference result, the gain parameters are estimated using optimization algorithm without any constraint. To verify the proposed algorithm, the gain parameters are optimized with inequality constraints about rising time, maximum overshoot, and settling time by comparing with the reference result. Finally, ideal PID gain parameters are estimated while conditions of constraints are satisfied.

For the future works, we will verify the proposed algorithm in the real environments using various constraints. Moreover, we will apply to specific systems as multiple and complex constraints.

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