

Day 1 - The goat is more likely to be behind the other door

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Learning Objective 1

Apply basic rules of probability. Determine when random variables are independent or dependent. Interpret distribution plots.

0.1 Conditional Probability

The problem of false positives

No diagnostic test is perfect in the real world. For example, in a flow cytometry test, we may have issues with unspecific binding and specific cross-reactivity with off targets resulting in false positives, as well as lack of reactivity with the stated target antigen resulting in false negatives.

Mathematically, these can be described using **sensitivity** and **specificity**.

- Sensitivity is the probability of a positive test result, conditioned on the individual truly being positive.
- Specificity is the probability of a negative test result, conditioned on the individual truly being negative.

Suppose we have a test designed to identify the presence of a pathogen in patient blood samples. The sensitivity of the test is 80% and the specificity is 99%.

1. Suppose that 10% of the population is infected with the pathogen.
 - (a) What is the probability that a randomly selected patient will be tested positive?
 - (b) If a randomly selected patient is tested and the test result is **positive**, what is the probability that the patient truly has the pathogen?
 - (c) If a randomly selected patient is tested and the test result is **negative**, what is the probability that the patient, in fact, has the pathogen?
2. Suppose now that 0.1% of the population is infected with the pathogen. Does the probability above change? If so, how?

Monty Hall puzzle

The Monty Hall problem is a (in)famous probability puzzle, loosely based on the American television game show *Let's Make a Deal* and named after its original host, *Monty Hall*.

Suppose you're on a game show, and you're given the choice of three doors:

- Behind one door is a car; behind the others, goats.
- The player pick a door. Then, the host Monty Hall, who knows what's behind the doors, will open one of the doors that has not been chosen, thus revealing a goat (he obviously won't open the car door).
- He then says to you, "Do you want to switch to the other door?"

1. What is the probability of winning the car if you stick with original choice? Does it change from the fact that Monty opens one of the doors?

2. What is the probability of winning the car if you switch?

The Two Children puzzle

Let's say you meet four parents - Mrs. Taylor, Mrs. Jones, Mr. Robinson, and Mr. Smith - and they all have exactly two children. Apart from following pieces of information, you are not told anything about them.

- Mrs. Taylor : Has at least one boy.
- Mrs. Jones : The elder one is a boy.
- Mr. Robinson : Has at least one boy born on Monday.
- Mr. Smith : One of them is a boy, Oscar, and he was born on Tuesday.

For the simplicity, following assumptions are made:

- Each child is either a boy or a girl.
 - Each child has the equal chance of being a boy or a girl.
 - The sex of each child is independent of the sex of the other.
1. What is the probability that both children are boys in each of the family?

Homework 1: Simulate Monty Hall puzzle and answer questions in `day01_activity01_montyhall.ipynb`.

Homework 2: Following the instructions in `day01_activity02_twoChildren.ipynb`, complete a code for Two-Children puzzle simulation and answer questions