CSC373 Worksheet 7 Solution

August 14, 2020

1. **Rough Works:**

   The longest simple cycle problem is the problem of finding a cycle of maximum length in a graph [5].

   Notes

   - **Decision Problem**
     - Is the problem with yes/no solution

   - **Alphabet**
     - Is a finite set of symbols
     - Is denoted Σ

       Example:

       For decision problem, its alphabet is: $\Sigma = \{0, 1\}$
       * 1 means 'yes'
       * 0 means 'no'

   - **Language**
     - Is any set of strings made of symbols from Σ
     - Is denoted L

       Example:

       $L = \{10, 11, 101, 111, 1011, 1101, 10001\}$

– Is denoted Σ^* for language of all strings over Σ plus empty string ϵ.

Example:

$\Sigma^* = \{\epsilon, 0, 1, 00, 01, 11, 000, ...\}$

Example 2:

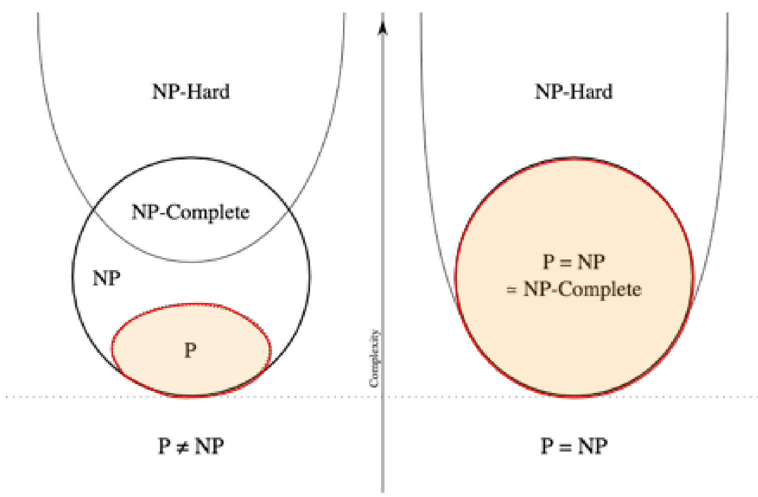
The decision problem PATH has the corresponding language

$$\begin{aligned} \text{PATH} = \{\langle G, U, v, k\rangle : & G = (V, E) \text{ is an undirected graph,} \\ & u, v \in V, \\ & k \geq 0 \text{ is an integer, and} \\ & \text{tere exists a path from } u \text{ to } v \text{ in } G \\ & \text{consisting of at most } k \text{ edges}\} \end{aligned}$$

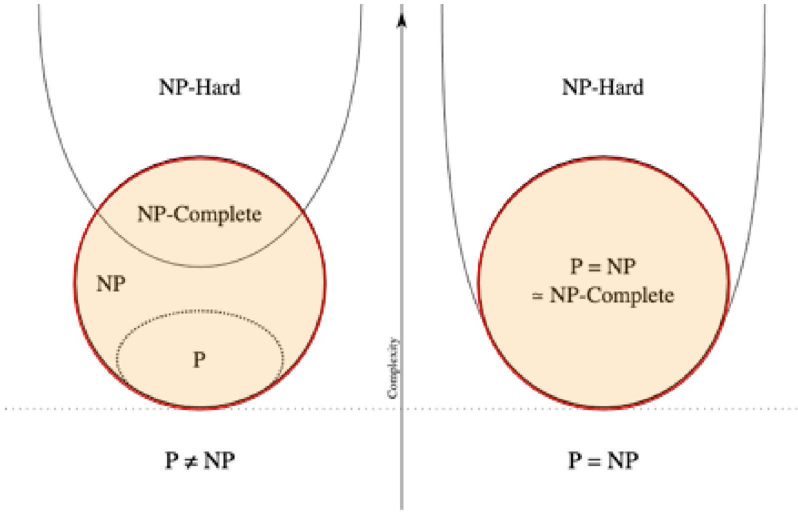
- **P**
  – Is set of problems that can be solved by a deterministic Turing machine in Polynomial time (i.e. $\mathcal{O}(n^k)$) [2].

  Example:

  1) Shortest path problems
  2) Calculating the greatest common divisor
  3) Finding maximum bipartite matching



- **NP (Non-deterministic Polynominal):**

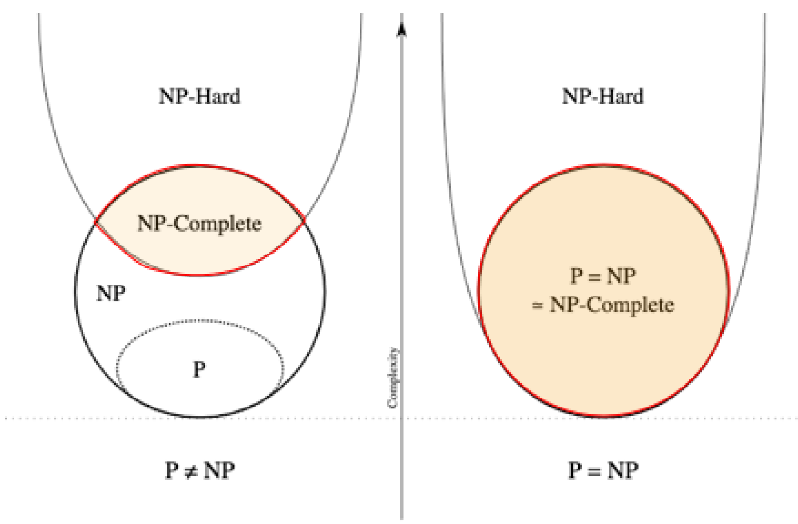


- Is set of decision problems that can be solved by a Non-deterministic Turing Machine in Polynomial time.[2]
- Has no particular rule is followed to make a guess [1].
- Can be solved in polynominal time via a "lucky algorithm", a magical algorithm that always make a right guess [2]
- $P \subseteq NP$

Examples:

- Longest-path problems
- Hamiltonian Cycle
- Graph coloring

- **NP-Complete Problems:**

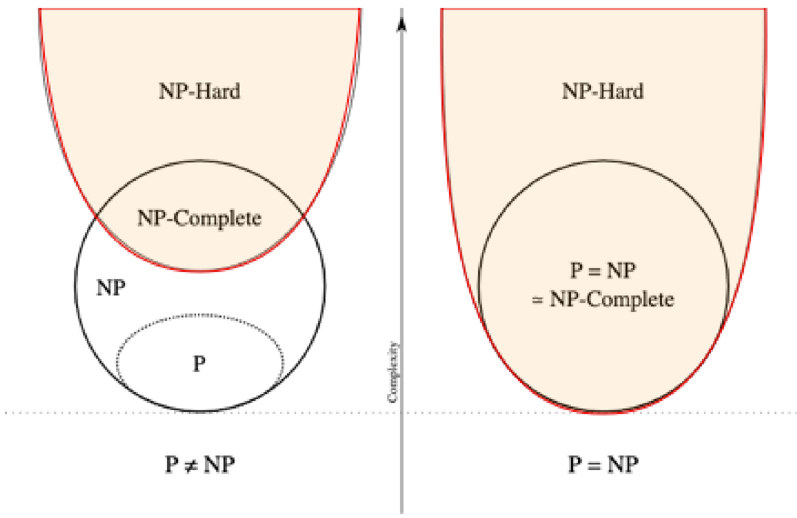


- A decision problem A is NP-complete (NPC) if

1) $A \in NP$ and
2) Every (other) problems A' in NP is reducible to A

- Has no efficient solution in polynominal number of steps (not yet) [3]
- Is not likely that there is an algorithm to make it efficient [3]

- **NP-Hard:**



- A decision problem A is NP-hard if
  1) $A \in NP$ (Not necessarily) and
  2) Every (other) problems A' in NP is reducible to A
- NP-Hard means "at least as hard as any problems in NP"
- Does not have to be about decision problems

Example:

1) Alan Turing's Halting Problem

References

1) Encyclopedia Britannica, NP-Complete Problem, link
2) Geeks for Geeks, NP-Completeness, link
3) Wikipedia, NP-complete, link
4) UCLA UC-Davis, ECS122A Handout on NP-Completeness, link