CSC373 Worksheet 4 Solution

August 4, 2020

1.
   - Calculating out-degree

     Let $G = (V, E)$ be a directed graph. Let $[v_1, ..., v_n]$ be a list of vertices in graph G.

     I need to calculate the outdegree of every vertex using adjacency list.

     We know that in addition to counting each v_i in adjacency list where $i = 1, .., n$, we are also counting $|Adj[v_i]|$ edges.

     Since there are $|V| = n$ many vertices, we can write that the total count is $|V| + \sum_{i=1}^{n} |Adj[v_i]| = |V| + |E|$, which is $\mathcal{O}(|V| + |E|)$.

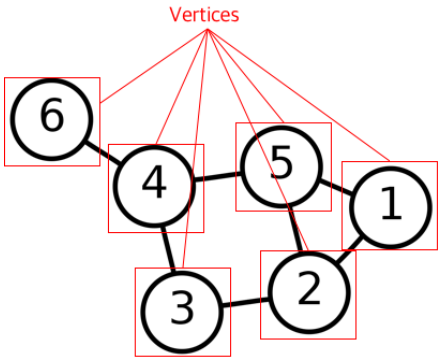
   - Calculating In-degree

     The outdegree of a vertex is indegree of another vertex.

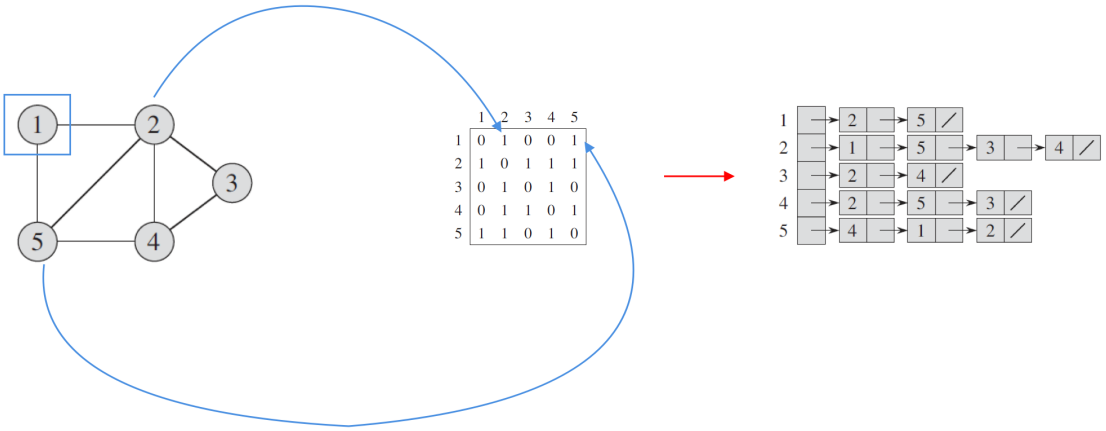
     Using this fact, we can conclude the running time of computing indegree of every vertax is $\mathcal{O}(|V| + |E|)$.

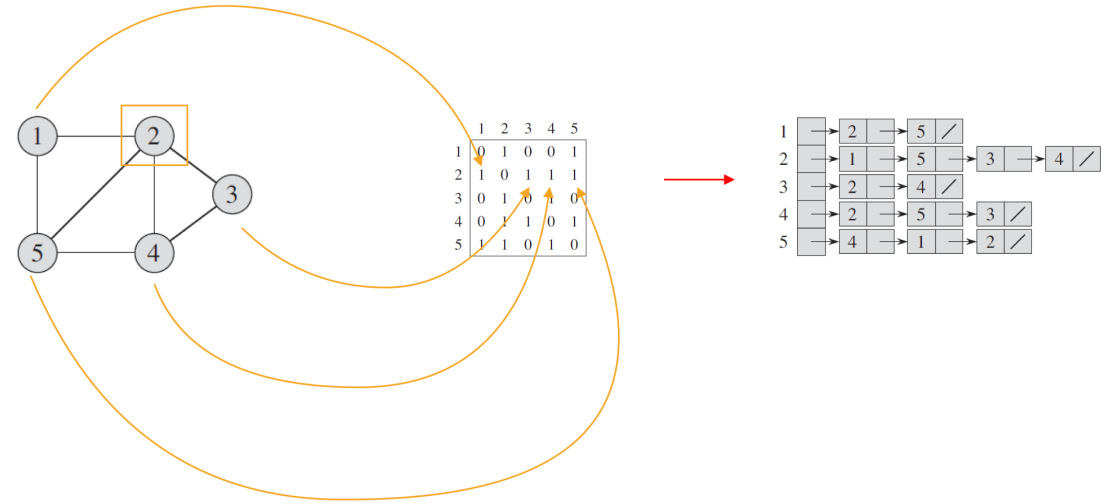
   Notes:

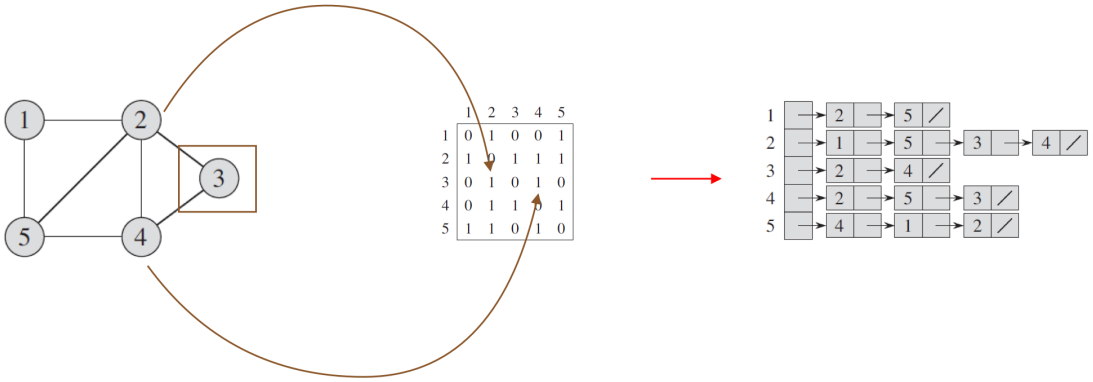
   - **Vertex**
     - Is a fundamental unit of which graphs are formed
     - Also means node

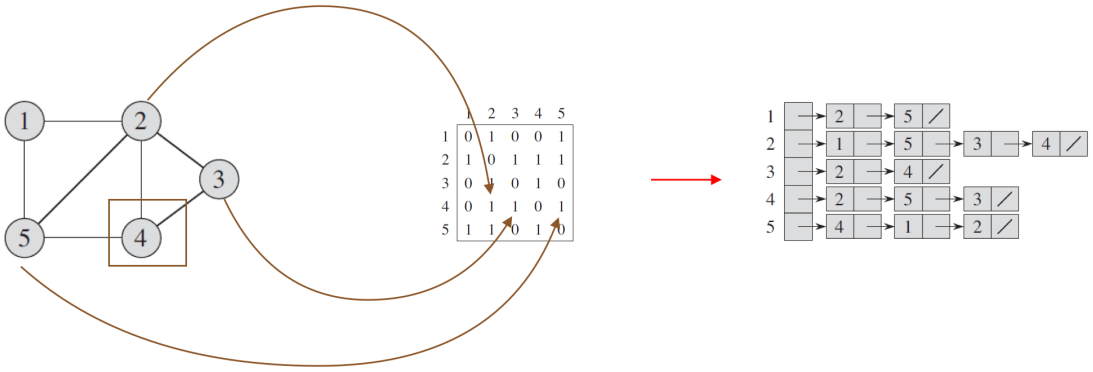


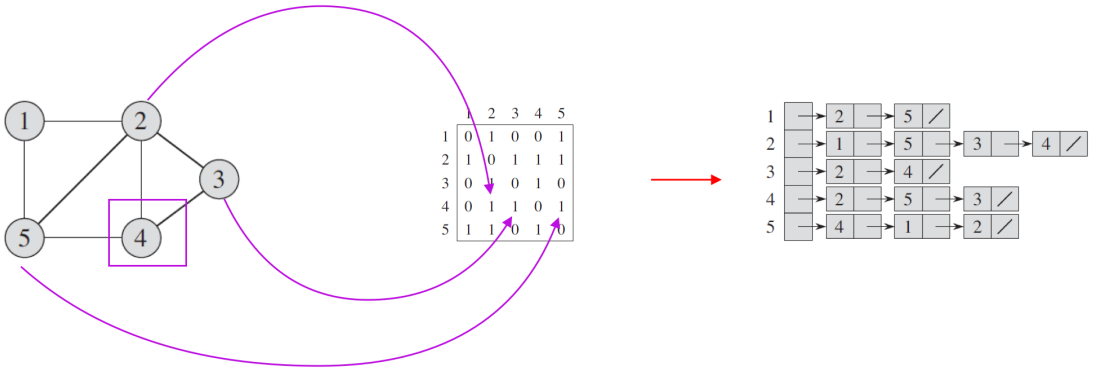
- **Adjacency-list Representation**
  - Associates each vertax in a graph with the collection of its neighbouring vertices or edges
  - Is represented by $Adj[v]$
    - Means all vertices that are neighbour to vertex v
    - In a directed graph, $Adj[v]$ are all out-degree vertices of vertax v
    - $|Adj[v]|$ means the total number of outdegree of vertax v

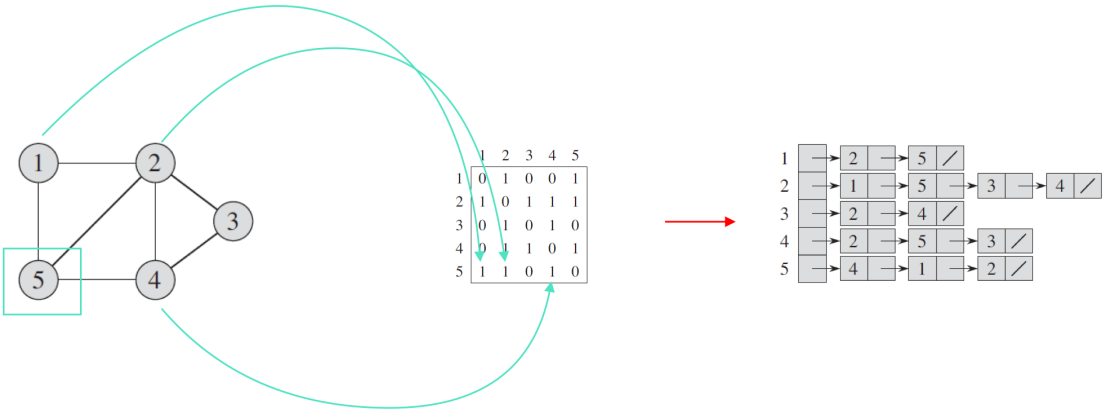




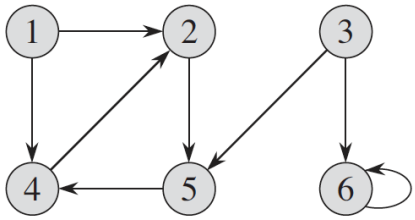

1
2
3
4
5
1 2 3 4 5
1 0 1 0 0 1
2 1 0 1 1 1
3 0 1 0 1 0
4 0 1 1 0 1
5 1 1 0 1 0
1 → 2 → 5 /
2 → 1 → 5 → 3 → 4 /
3 → 2 → 4 /
4 → 2 → 5 → 3 /
5 → 4 → 1 → 2 /


1
2
3
4
5
1 2 3 4 5
1 0 1 0 0 1
2 1 0 1 1 1
3 0 1 0 1 0
4 0 1 1 0 1
5 1 1 0 1 0
1 → 2 → 5 /
2 → 1 → 5 → 3 → 4 /
3 → 2 → 4 /
4 → 2 → 5 → 3 /
5 → 4 → 1 → 2 /

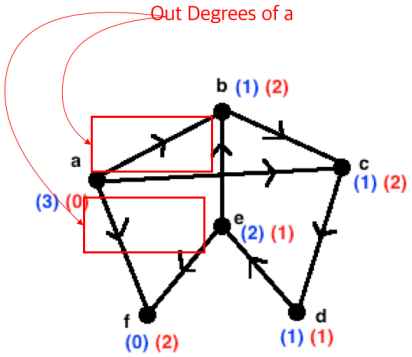

1
2
3
4
5
1 2 3 4 5
1 0 1 0 0 1
2 1 0 1 1 1
3 0 1 0 1 0
4 0 1 1 0 1
5 1 1 0 1 0
1 → 2 → 5 /
2 → 1 → 5 → 3 → 4 /
3 → 2 → 4 /
4 → 2 → 5 → 3 /
5 → 4 → 1 → 2 /



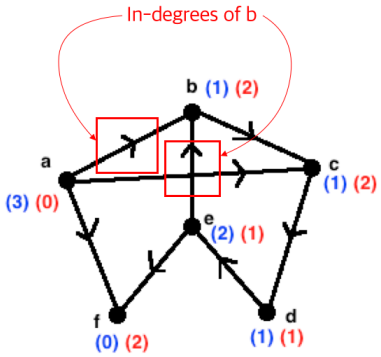
- **Directed graph**
  - Is a graph that is made up of a set of vertices connected by edges, where the edges have a direction associated with them



- **Out-degrees**
  - For a directed graph $G = (V(G), E(G))$ and a vertex $x_1 \in V(G)$, the Out-Degree of x_1 refers to the number of arcs incident from x_1. That is, the number of arcs directed away from the vertex x_1.

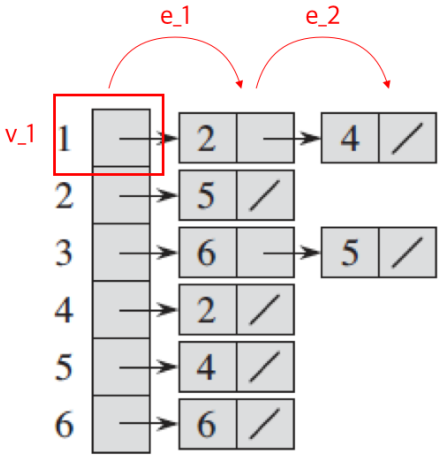


- **In-degrees**
  - For a directed graph $G = (V(G), E(G))$ and a vertex $x_1 \in V(G)$, the In-Degree of x_1 refers to the number of arcs incident to x_1. That is, the number of arcs directed towards the vertex x_1.



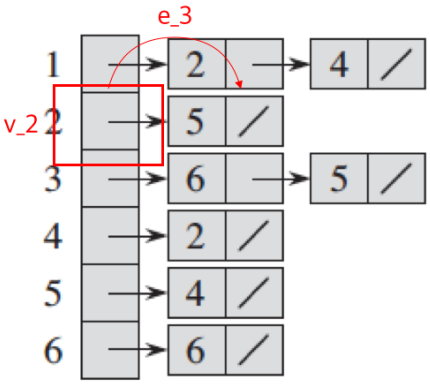
- Computing the outdegree of every vertex using adjacency list

1)



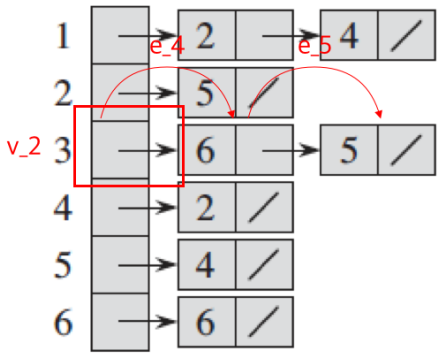
(v_1) + (e_1 + e_2)

2)



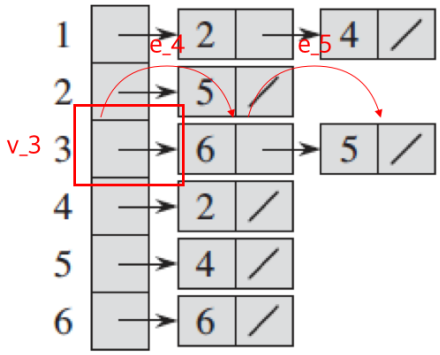
(v_1 + v_2) +
(e_1 + e_2 + e_3)

3)



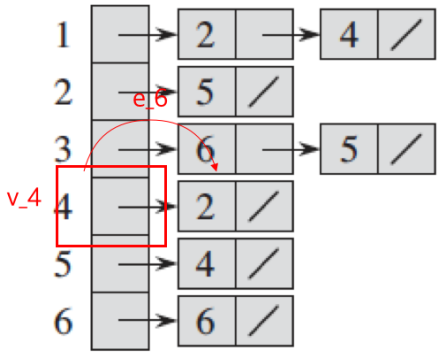
(v_1 + v_2 + v3) +
(e_1 + e_2 + e_3 + e_4 + e_5)

3)



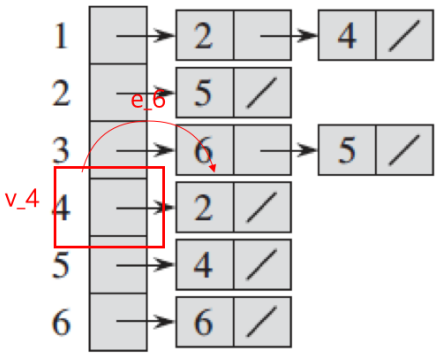
(v_1 + v_2 + v3) +
(e_1 + e_2 + e_3 + e_4 + e_5)

4)



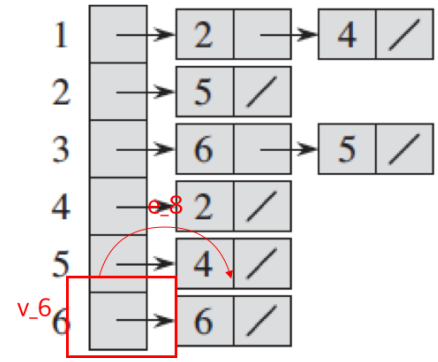
(v_1 + v_2 + v_3 + v_4) +
(e_1 + e_2 + e_3 + e_4 + e_5)

4)



(v_1 + v_2 + v_3 + v_4) +
(e_1 + e_2 + e_3 + e_4 + e_5)

6)



(v_1 + v_2 + v_3 + v_4 + v_5 _ v_6) +
(e_1 + e_2 + e_3 + e_4 + e_5 + e_6 + e_7 + e_8)

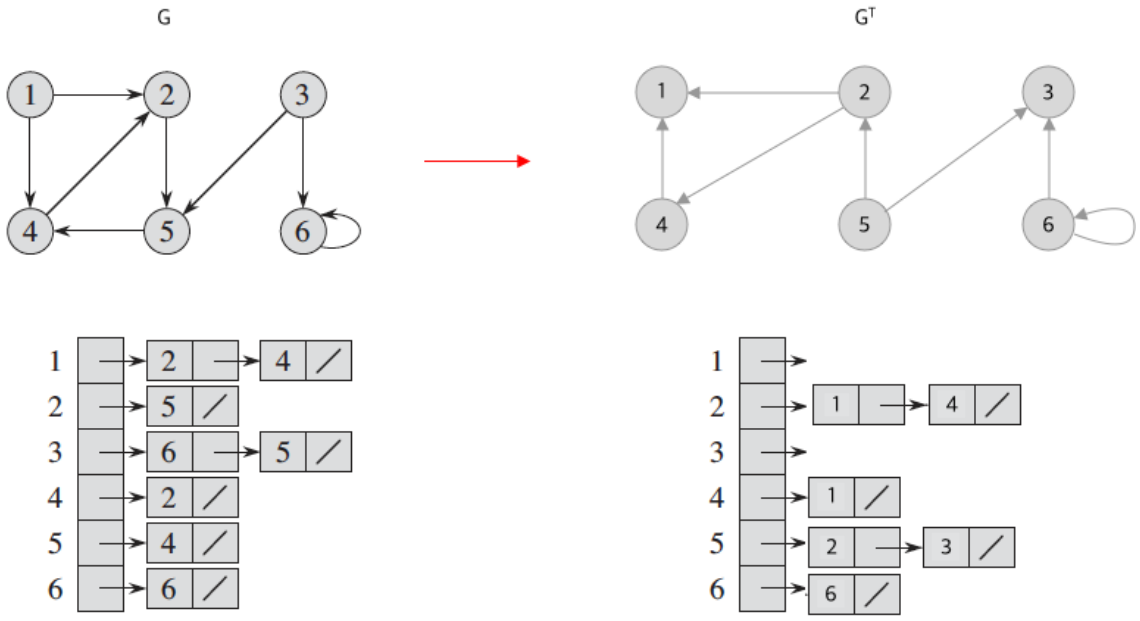
So it has $\mathcal{O}(V + E)$

- Computing the outdegree of every vertex using adjacency list

  The outdegree of a vertex is indegree of another vertex.

  Using this fact, we can conclude the running time of computing indegree of every vertax is $\mathcal{O}(V + E)$.

2. - Computing G^T from G in Adjacency List

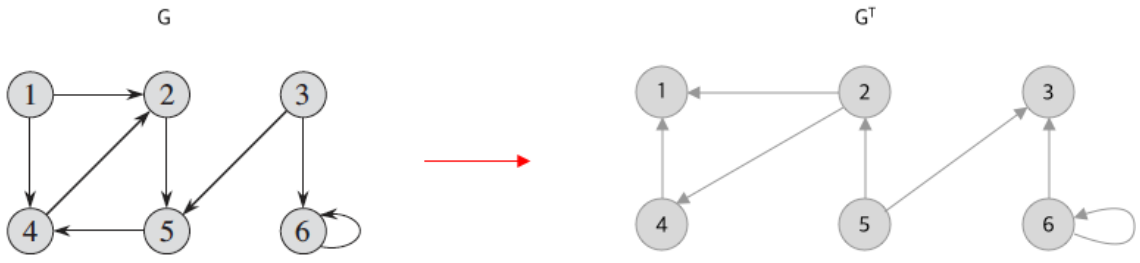



```
COMPUTE-G-TRANSPOSE-ADJ-MATRIX(Adj,V)
    Let Adj' be a new adjacency list containing keys v_i...v_n

    for i = 1 to |V|
        for every vertax w in Adj[v_i]
            Insert(Adj'[w], v_i)

    return Adj'
```

- Computing G^T from G in Adjacency-Matrix



	1	2	3	4	5	6
1	0	1	0	1	0	0
2	0	0	0	0	1	0
3	0	0	0	0	1	1
4	0	1	0	0	0	0
5	0	0	0	1	0	0
6	0	0	0	0	0	1

	1	2	3	4	5	6
1	0	0	0	0	0	0
2	1	0	0	1	0	0
3	0	0	0	0	0	0
4	1	0	0	0	1	0
5	0	1	1	0	0	0
6	0	0	1	0	0	1

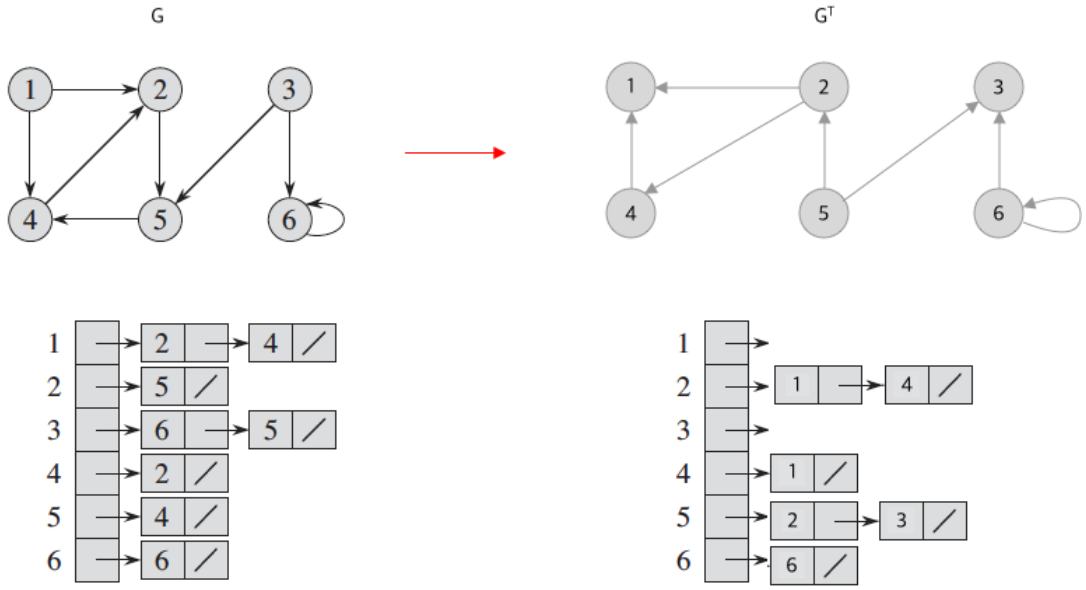
```
COMPUTE-G-TRANSPOSE-ADJ-MATRIX(A,V)
    Let A'[1..|V|, 1..|V|] be a new adjacency matrix

    for i = 1 to |V|
        for j = 1 to |V|
            A'[j,i] = A[i,j]

    return A'
```

Correct Solution:

- Computing G^T from G in Adjacency List

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v_i...v_n

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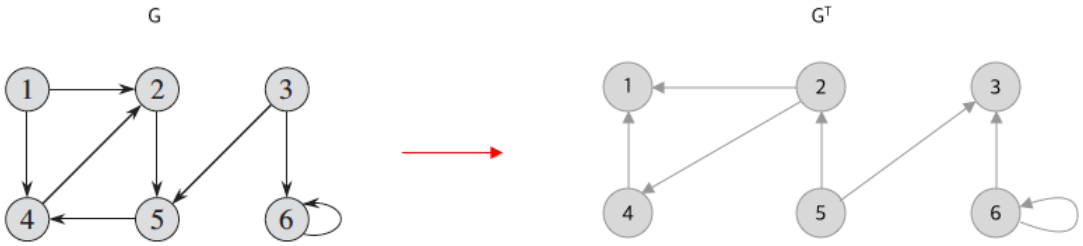
        for every vertax w in Adj[v_i]
            Insert(Adj'[w], v_i)

    return Adj'
```

The running time is $\mathcal{O}(|V| + |E|)$

- Computing G^T from G in Adjacency-Matrix



	1	2	3	4	5	6
1	0	1	0	1	0	0
2	0	0	0	0	1	0
3	0	0	0	0	1	1
4	0	1	0	0	0	0
5	0	0	0	1	0	0
6	0	0	0	0	0	1

	1	2	3	4	5	6
1	0	0	0	0	0	0
2	1	0	0	1	0	0
3	0	0	0	0	0	0
4	1	0	0	0	1	0
5	0	1	1	0	0	0
6	0	0	1	0	0	1

```
1  COMPUTE-G-TRANSPOSE-ADJ-MATRIX(A,V)
2      Let A'[1..|V|, 1..|V|] be a new adjacency matrix
3
4      for i = 1 to |V|
5          for j = 1 to |V|
6              A'[j,i] = A[i,j]
7
8      return A'
9
```

The running time is $\mathcal{O}(|V|^2)$