Worksheet 13 Solution

March 22, 2020

Question 1

a. **Exact number of iterations:** n

Simplest theta expression: $f_1 \in \Theta(n)$

<u>**Correct Solution:**</u>

Exact number of iterations: $\frac{n}{5}$

Simplest theta expression: $f_1 \in \Theta(n)$

b. **Exact number of iterations:** $n - 4$

Simplest theta expression: $f_2 \in \Theta(n)$

c. **Exact number of iterations:** n

Simplest theta expression: $f_3 \in \Theta(n)$

<u>**Correct Solution:**</u>

Exact number of iterations: $\frac{n}{\frac{n}{10}} = 10$

Simplest theta expression: $f_3 \in \Theta(1)$

d. **Exact number of iterations:** $n^2 - 20$

Simplest theta expression: $f_4 \in \Theta(n^2)$

<u>**Correct Solution:**</u>

Exact number of iterations: $\frac{n^2-20}{3}$

Simplest theta expression: $f_4 \in \Theta(n^2)$

e. **Exact number of iterations:** $n^2 - 20 + n$

Simplest theta expression: $f_5 \in \Theta(n^2)$

<u>**Correct Solution:**</u>

Exact number of iterations: $\frac{n^2-20}{3} + 100n$

Simplest theta expression: $f_5 \in \Theta(n^2)$

Question 2

a. $i_3 = 8$

$i_4 = 16$

$i_k = 2^k$

b. **Exact number of iterations:** $\lceil \sqrt{n} \rceil$

<u>**Correct Solution:**</u>

The goal is to find the smallest k where the condition returns false.

So,

$$
\begin{align}
i_k &\geq n \\
2^k &\geq n \\
k &\geq log(n)
\end{align}
$$

Hence, the exact number of iteration that occurs if $\lceil log(n) \rceil$.

c. $f \in \Theta(n^{\frac{1}{2}})$

<u>**Correct Solution:**</u>

$f \in \Theta(log(n))$

d. With $i = 0$, the i in loop will be forever 0. This will result in the while loop running indefinitely.

Question 3

- The value of $i_k = k^2$ by the pattern ruled in table below.

i_k	i_0	i_1	i_2	i_3	i_4
Value	2	4	9	16	25

Then,

$$k^2 \geq n \tag{1}$$
$$k \geq \sqrt{n} \tag{2}$$

Then, it follows from above that the smallest value of i_k where $i_k < n$ returns false is $\sqrt{n}$.

Correct Solution:

The value of $i_k = 2^{2^k}$ by the pattern ruled in table below.

i_k	i_0	i_1	i_2	i_3	i_4
Value	2	4	16	256	65536

Then,

$$2^{2^k} \geq n \tag{1}$$
$$2^k \geq \log n \tag{2}$$
$$k \geq \log \log n \tag{3}$$
$$\tag{4}$$

Then, it follows from above that the smallest value of i_k where $i_k < n$ returns false is $\log \log n$.