CSC373 Worksheet 0

July 18, 2020

1. **CLRS 4.3-1:** Show that the solution of $T(n) = T(n-1) + n$ is $\mathcal{O}(n^2)$.
2. **CLRS 4.3-2:** Show that the solution of $T(n) = T(\lceil n/2 \rceil) + 1$ is $\mathcal{O}(\lg n)$.
3. **CLRS 4.3-3:** We saw that the solution of $T(n) = 2T(\lfloor n/2 \rfloor) + n$ is $O(n \lg n)$. Show that the solution of this recurrence is also $\Omega(n \lg n)$. Conclude that the solution is $\Theta(n \lg n)$.
4. **CLRS 4.3-5:** Show that $\Theta(n \lg n)$ is the solution to the "exact" recurrence (4.3) for merge sort.
5. **CLRS 4.3-6:** Show that the solution to $T(n) = 2T(\lfloor n/2 \rfloor + 17) + n$ is $O(n \lg n)$.
6. **CLRS 4.3-7:** Using the master method in Section 4.5, you can show that the solution to the recurrence $T(n) = 4T(n/3) + n$ is $T(n) = \Theta(n^{\log_3 4})$. Show that a substitution proof with the assumption $T(n) \leq cn^{\log_3 4}$ fails. Then show how to subtract off a lower-order term to make a substitution proof work.
7. **CLRS 4.3-8:** Using the master method in Section 4.5, you can show that the solution to the recurrence $T(n) = 4T(n/2) + n$ is $T(n) = \Theta(n^2)$. Show that a substitution proof with the assumption $T(n) \leq cn^2$ fails. Then show how to subtract off a lower-order term to make a substitution proof work.
8. **CLRS 4.4-1:** Use a recursion tree to determine a good asymptotic upper bound on the recurrrence $T(n) = 3T(\lfloor n/2 \rfloor) + n$. Use the subtitution method to verify your answer.
9. **CLRS 4.4-2:** Use a recursion tree to determine a good asymptotic upper bound on the recurrrence $T(n) = 3T(n/2) + n^2$. Use the subtitution method to verify your answer.
10. **CLRS 4.4-3:** Use a recursion tree to determine a good asymptotic upper bound on the recurrence $T(n) = 4T(n/2 + 2) + n$. Use the subtitution method to verify your answer.
11. **CLRS 4.4-4:** Use a recursion tree to determine a good asymptotic upper bound on the recurrence $T(n) = 2T(n-1) + 1$. Use the subtitution method to verify your answer.
12. **CLRS 4.4-5:** Use a recursion tree to determine a good asymptotic upper bound on the recurrence $T(n) = 2T(n-1) + T(n/2) + n$. Use the subtitution method to verify your answer.

13. **CLRS 4.4-6:** Argue that the solution to the recurrent $T(n) = 4T(\lfloor n/2 \rfloor) + cn$ where c is a constant, and provide a tight asymptotic bound on its solution. Verify your bound on the subtitution method.

14. **CLRS 4.4-7:** Draw the recursion tree for $T(n) = 4T(n/2) + cn$, where c is a constant, and provide a tight asymptotic bound on its solution. Verify your bound by the substitution method.

15. **CLRS 4.5-1:** Use the master method to give tight asymptotic bounds for the following recurrences

    a) $T(n) = 2T(n/4) + 1$
    b) $T(n) = 2T(n/4) + \sqrt{n}$
    c) $T(n) = 2T(n/4) + n$
    d) $T(n) = 2T(n/4) + n^2$

16. **CLRS 4.5-3:** Use the master method to show that the solution to the binary-search recurrence $T(n) = T(n/2) + \Theta(1)$ is $T(n) = \Theta(\lg n)$. (See Exercise 2.3-5 for a desscription of binary search)

17. **CLRS 4.5-4:** Can the master method be applied to the recurrence $T(n) = 4T(n/2) + n^2 \lg n$? Why or why not? Give an asymptotic upper bound for this recurrence.