Worksheet 10 Review

March 28, 2020

Question 1

a.

$$
\begin{aligned}
(165)_8 &= 5 \cdot 8^0 + 6 \cdot 8^1 + 1 \cdot 8^2 && (1)\\
&= 5 + 48 + 64 && (2)\\
&= 53 + 64 && (3)\\
&= 117 && (4)
\end{aligned}
$$

b.

$$
\begin{aligned}
(B4)_1 6 &= 4 \cdot 16^0 + 11 \cdot 16^1 && (1)\\
&= 4 + (11 \cdot 16) && (2)\\
&= 4 + 176 && (3)\\
&= 180 && (4)
\end{aligned}
$$

Question 2

a.

$$
\begin{aligned}
357 \div 2 &= 178, \text{remainder } \mathbf{1},\\
178 \div 2 &= 89, \text{remainder } \mathbf{0},\\
89 \div 2 &= 44, \text{remainder } \mathbf{1},\\
44 \div 2 &= 22, \text{remainder } \mathbf{0},\\
22 \div 2 &= 11, \text{remainder } \mathbf{0},\\
11 \div 2 &= 5, \text{remainder } \mathbf{1},\\
5 \div 2 &= 2, \text{remainder } \mathbf{1},\\
2 \div 2 &= 1, \text{remainder } \mathbf{0},\\
1 \div 2 &= 0, \text{remainder } \mathbf{1}
\end{aligned}
$$

Combining it together, the binary representation of 357 is $(101100101)_2$

b.

$$
\begin{aligned}
1 \cdot 2^0 + 0 \cdot 2^1 + 1 \cdot 2^2 &= \frac{1+0+4}{8^0} = 5\\
0 \cdot 2^3 + 0 \cdot 2^4 + 1 \cdot 2^5 &= \frac{0+0+32}{8^1} = 4\\
1 \cdot 2^6 + 0 \cdot 2^7 + 1 \cdot 2^8 &= \frac{64+0+256}{8^2} = 5
\end{aligned}
$$

Combining it together, the octal representation of $(101100101)_2$ is $(545)_8$.

c.

$$
\begin{aligned}
357 \div 16 &= 22, \text{remainder } \mathbf{5},\\
22 \div 16 &= 1, \text{remainder } \mathbf{5},\\
1 \div 16 &= 0, \text{remainder } \mathbf{1},
\end{aligned}
$$

Combining it together, the hexadecimal representation of 357 is $(155)_{16}$.

Correct Solution:

$$
\begin{aligned}
357 \div 16 &= 22, \text{remainder } \mathbf{5},\\
22 \div 16 &= 1, \text{remainder } \mathbf{6},\\
1 \div 16 &= 0, \text{remainder } \mathbf{1},
\end{aligned}
$$

Combining it together, the hexadecimal representation of 357 is $(165)_{16}$.

Question 3

a.

$$
\begin{aligned}
0.375 \times 2 &= 0.75 + \mathbf{0} && (1)\\
0.75 \times 2 &= 0.5 + \mathbf{1} && (2)\\
0.5 \times 2 &= 0 + \mathbf{1} && (3)
\end{aligned}
$$

Combining the above, the binary representation of 0.375 is $(0.011)_2$.

Notes:

- **Converting decimal to binary**

$$
\begin{aligned}
0.8125 \times 2 &= 0.625 + \mathbf{1} && (4)\\
0.625 \times 2 &= 0.25 + \mathbf{1} && (5)\\
0.25 \times 2 &= 0.5 + \mathbf{0} && (6)\\
0.5 \times 2 &= 0 + \mathbf{1} && (7)
\end{aligned}
$$

Binaries read *top to bottom*

b.

$$
\begin{align}
0.1 \times 2 &= 0.2 + \mathbf{0} \tag{1}\\
0.2 \times 2 &= 0.4 + \mathbf{0} \tag{2}\\
0.4 \times 2 &= 0.8 + \mathbf{0} \tag{3}\\
0.8 \times 2 &= 0.6 + \mathbf{1} \tag{4}\\
0.6 \times 2 &= 0.2 + \mathbf{1} \tag{5}\\
0.2 \times 2 &= 0.4 + \mathbf{0} \tag{6}\\
0.4 \times 2 &= 0.8 + \mathbf{0} \tag{7}\\
0.8 \times 2 &= 0.6 + \mathbf{1} \tag{8}
\end{align}
$$

Combining the above, the binary representation of 0.1 is $(0.0\overline{0011})_2$.

Question 4

a.

$$
\begin{align}
\sum_{i=0}^{\infty} \left(\frac{1}{2}\right) &= \frac{\frac{1}{2}}{1 - \frac{1}{2}} \tag{1}\\
&= \frac{\frac{1}{2}}{\frac{1}{2}} \tag{2}\\
&= 1 \tag{3}
\end{align}
$$

b. Since 1^{st} 1 repeats every 4 decimal places, and 2^{nd} 1 repeats every 5 decimal places, we have

$$
\begin{align}
(0.0\overline{0011})_2 &= \sum_{i=1}^{\infty} \left(\frac{1}{2}\right)^{4i} + \sum_{i=1}^{\infty} \left(\frac{1}{2}\right)^{4i+1} \tag{1}\\
&= \sum_{i=1}^{\infty} \frac{1}{16} + \frac{1}{2} \cdot \sum_{i=1}^{\infty} \frac{1}{16} \tag{2}\\
&= \frac{1}{15} + \frac{1}{30} \tag{3}\\
&= \frac{3}{30} \tag{4}\\
&= \frac{1}{10} \tag{5}
\end{align}
$$