

Worksheet 2 Review

April 10, 2020

Question 1

- a. One example is $x = \text{Aizah}$ and $y = \text{Aizah}$.

There are more than one possible answer. The following examples also show truthiness of the statement.

- $x = \text{Carlos}$ and $y = \text{Carlos}$
- $x = \text{Ellen}$ and $y = \text{Ellen}$

- b. One example is $x = \text{Betty}$ and $y = \text{Aizah}$.

There are more than one possible answer. The following examples also show truthiness of the statement.

Part 1 ($\neg \text{Rich}(x)$ - **True**, $\neg \text{SameDept}(x, y)$ - **False**):

- $x = \text{Betty}$, $y = \text{Betty}$
- $x = \text{Betty}$, $y = \text{Doug}$
- $x = \text{Doug}$, $y = \text{Aizah}$
- $x = \text{Doug}$, $y = \text{Betty}$
- $x = \text{Doug}$, $y = \text{Doug}$
- $x = \text{Flo}$, $y = \text{Ellen}$
- $x = \text{Flo}$, $y = \text{Flo}$

Part 2 ($\neg \text{Rich}(x)$ - **False**, $\neg \text{SameDept}(x, y)$ - **True**):

- $x = \text{Aizah}$, $y = \text{Carlos}$
- $x = \text{Aizah}$, $y = \text{Ellen}$
- $x = \text{Aizah}$, $y = \text{Flo}$

- $x = \text{Carlos}, y = \text{Aizah}$
- $x = \text{Carlos}, y = \text{Betty}$
- $x = \text{Carlos}, y = \text{Doug}$
- $x = \text{Carlos}, y = \text{Ellen}$
- $x = \text{Carlos}, y = \text{Flo}$
- $x = \text{Ellen}, y = \text{Aizah}$
- $x = \text{Ellen}, y = \text{Betty}$
- $x = \text{Ellen}, y = \text{Carlos}$
- $x = \text{Ellen}, y = \text{Doug}$

Part 3 ($\neg Rich(x)$ - True, $\neg SameDept(x, y)$ - True):

- $x = \text{Betty}, y = \text{Carlos}$
- $x = \text{Betty}, y = \text{Ellen}$
- $x = \text{Betty}, y = \text{Flo}$
- $x = \text{Doug}, y = \text{Carlos}$
- $x = \text{Doug}, y = \text{Ellen}$
- $x = \text{Doug}, y = \text{Flo}$
- $x = \text{Flo}, y = \text{Aizh}$
- $x = \text{Flo}, y = \text{Betty}$
- $x = \text{Flo}, y = \text{Carlos}$

- c. This statement is true. This is because in each department there is an individual who is rich. For example, in sales, there is Aizah. In HR, there is Carlos. In design, there is Ellen. So, for every employee y , we can choose person who is rich in the same department.
- d. Consider an example where y is not in the same department as x , say $x = \text{Aizah}$ and $y = \text{Carlos}$. This sets the statement $Rich(x) \wedge SameDept(x, y)$ to false.

Notes:

- **Negation of Statement:** $\forall x, \exists y \in E, \neg Rich(x) \vee \neg SameDept(x, y)$
- In above negation, only y needs to be chosen.

Question 2

- a. $f(x) = 10$, where $f : \mathbb{R} \rightarrow \mathbb{R}$

Correct Solution:

$$\exists x \in \mathbb{R}, f(x) = 10, \text{ where } f : \mathbb{R} \rightarrow \mathbb{R}$$

- b. $\forall y \in \text{codomain}(\mathbb{R}), \exists x \in \text{domain}(\mathbb{R}), f(x) = y$, where $f : \mathbb{R} \rightarrow \mathbb{R}$

Correct Solution:

$$\forall y \in \mathbb{R}, \exists x \in \mathbb{R}, f(x) = y, \text{ where } f : \mathbb{R} \rightarrow \mathbb{R}$$

Notes:

- Noticed professor doesn't label sets using *codomain* or *domain*.

- c. **Negation of Onto:** $\exists y \in \mathbb{R}, \forall x \in \mathbb{R}, f(x) \neq y$

A counter example of f not being onto is $y = -1$.

Question 3

- a. $\{n \mid n \in S, n > 1\}$

Correct Solution:

$$S = \{n \mid n \in \mathbb{N} \text{ and } n > 3\}$$

- b. $P(n) : n > 3$

Correct Solution:

$$P(n) : n > 3, n \in \mathbb{N}$$

- c. **Part 1** (Every integer that is great than 10 or less than -40 is not equal to 0):

$$\forall x \in \mathbb{Z}, x > 10 \vee x < -40 \Rightarrow x \neq 0$$

$$\forall y \in \{x \in \mathbb{Z}, x > 10 \vee x < -40\}, y \neq 0$$

Part 2 (Every employee who is in the same department as Doug is rich):

$$\forall x \in E, \text{SameDept}(x, \text{Doug}) \Rightarrow \text{Rich}(x)$$

$$\forall y \in \{x \mid x \in E \text{ and } \text{SameDept}(x, \text{Doug})\}, \text{Rich}(y)$$

Correct Solution:

Part 1 (Every integer that is great than 10 or less than -40 is not equal to 0):

$$\forall x \in \mathbb{Z}, x > 10 \vee x < -40 \Rightarrow x \neq 0$$

$$\forall y \in \{x \in x \in \mathbb{Z} \text{ and } x > 10 \text{ or } x < -40\}, y \neq 0$$

Part 2 (Every employee who is in the same department as Doug is rich):

$$\forall x \in E, \text{SameDept}(x, \text{Doug}) \Rightarrow \text{Rich}(x)$$

$$\forall y \in \{x \mid x \in E \text{ and } \text{SameDept}(x, \text{Doug})\}, \text{Rich}(y)$$

Notes:

- When is , used instead of *and* in set $\{x \in x \in \mathbb{Z} \text{ and } x > 10 \text{ or } x < -40\}$?