

Midterm 2 Version 2 Solution

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Question 1

a.

$$100 \div 3 = 33, \text{ Remainder } \mathbf{1}$$

$$33 \div 3 = 11, \text{ Remainder } \mathbf{0}$$

$$11 \div 3 = 3, \text{ Remainder } \mathbf{2}$$

$$3 \div 3 = 1, \text{ Remainder } \mathbf{0}$$

$$1 \div 3 = 0, \text{ Remainder } \mathbf{1}$$

It follows from above that the ternary representation of 100 is $(10201)_3$.

Attempt 2:

$$100 + (-1 \cdot 3^4) = 100 - 81 = 19$$

$$19 + (-1 \cdot 3^3) = 19 - 27 = -8$$

$$-8 + (+1 \cdot 3^2) = -8 + 9 = 1$$

$$1 + (0 \cdot 3^1) = 1 + 0 = 1$$

$$1 + (-1 \cdot 3^0) = 1 - 1 = 0$$

So by flipping the signs, and reading from top to bottom, we can conclude the balanced ternary representation of 100 is $(11T101)_{bt}$

Notes:

- Balanced ternary representation expresses a decimal using 1, 0 and -1
- **T** represents negative sign in balanced ternary representation.
- Is my way of calculating balanced ternary representation correct? My approach was ‘which sign should be used given 3^n so the calculation stops at 3^0 ?’

b. The largest number expressible by an n-digit binary representation is

$$\sum_{i=0}^{n-1} 2^i \quad (1)$$

Correct Solution:

$$\sum_{i=0}^{n-1} 2^i = \frac{1 - 2^{n-1+1}}{1 - 2} = 2^n - 1 \quad (1)$$

Notes:

- Noticed professor simplified solution using geometric series
- Geometric series with finite sum

$$\sum_{i=0}^n r^k = \frac{1 - r^{n+1}}{1 - r}, \text{ where } |r| > 1 \quad (2)$$

c.

$f(n) \in \mathcal{O}(n)$	True	$g(n) \in \Omega(n)$	False	$f(n) \in \Omega(g(n))$	True
$f(n) \in \Theta(g(n))$	False	$g(n) \in \Theta(\log_3 n)$	False	$f(n) + g(n) \in \Theta(f(n))$	True

Notes:

- Learned \sqrt{n} rises faster than $\log n$.
- Learned if $g(n) \in \Theta(f(n))$ is true then $f(n) + g(n) \in \Theta(f(n))$ is true.

d.

k	0	1	2	3
$i \cdot i \cdot i$	$2 = 2^{3^0}$	$2^3 = 2^{3^1}$	$2^9 = 2^{3^2}$	$2^{27} = 2^{3^3}$

We can deduce from above that $i_k = 2^{3^k}$

e. $\lceil \log_3(\log_2(n) - 1) \rceil$

Correct Solution:

We want to find the smallest value of k satisfying $2 \cdot i_k \geq n$, and the value is

$$\lceil \log_3(\log_2(n) - 1) \rceil$$

Question 2

Question 3

Question 4