Worksheet 14 Solution

March 25, 2020

Question 1

a. **Inner Loop:** n

Outer Loop: $n - 5$

Theta Expressions: $\Theta(n^2)$

Correct Solution:

Inner Loop: n

Outer Loop: $n \cdot \left\lceil \frac{n}{5} \right\rceil$

Theta Expressions: $\Theta(n^2)$

b. **Inner Loop:** $\frac{n}{3} + (n - 2)$

Outer Loop: $n - 4$

Theta Expressions: $\Theta(n^2)$

Correct Solution:

Inner Loop: $\lceil \log_3 n \rceil + \left\lceil \frac{n}{2} \right\rceil$

Outer Loop: $max(0, n - 4) \cdot \left[\lceil \log_3 n \rceil + \left\lceil \frac{n}{2} \right\rceil \right]$

Theta Expressions: $\Theta(n^2)$

c. **Inner Loop #2:** $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$

Inner Loop #1: $n \cdot \frac{n(n+1)}{2} = \frac{n^3+n^2}{2}$

Outer Loop: $\frac{n^3+n^2}{2} \cdot (n-4) = \frac{n^4-3n^3+4n^2}{2}$

Theta Expressions: $\Theta(n^4)$

Correct Solution:

Inner Loop #2: j

Inner Loop #1: $\sum_{j=1}^{n} j = \frac{n(n+1)}{2}$

Outer Loop: $\left\lceil \frac{n}{4} \right\rceil \cdot \frac{n(n+1)}{2}$

Theta Expressions: $\Theta(n^3)$

d. **Inner Loop:** 2^n

Outer Loop: $\sum_{i=0}^{\frac{n}{2}-1} 2^i = 2^{\frac{n}{2}-1}$

Theta Expressions: $\Theta(2^n)$

Correct Solution:

Inner Loop: i

Outer Loop: $\sum_{i=0}^{\log n-1} 2^i = \frac{1-2^{\log n-1+1}}{1-2} = 2^{\log n} - 1 = n-1$

Theta Expressions: $\Theta(n)$

Question 2