Worksheet 16 Solution

March 27, 2020

Question 1

a. Since the inner loop runs from $i + 1$ and ends at $n - 1$, the loop has at most

$$((n-1) - (i+1) + 1) = (n - i - 1) \tag{1}$$

iterations.

Since the inner loop takes 1 step per iteration, the inner loop has total cost of at most

$$(n - i - 1) \cdot 1 = (n - i - 1) \tag{2}$$

steps.

Since the outer loop takes $(n - i - 1)$ steps per iteration, the total cost of the outer loop is at most

$$\sum_{i=0}^{n-1}(n-i-1) = \sum_{i=0}^{n-1}[(n-1)-i] \tag{3}$$

$$= \sum_{i=0}^{n-1}(n-1) - \sum_{i=0}^{n-1} i \tag{4}$$

$$= n \cdot (n-1) - \frac{n(n-1)}{2} \tag{5}$$

$$= \frac{2n(n-1)}{2} - \frac{n(n-1)}{2} \tag{6}$$

$$= \frac{n(n-1)}{2} \tag{7}$$

steps.

Then, since the return statement on line 7 and len statement on line 2 each has cost of 1 step, the total cost of algorithm is at most

$$\frac{n(n-1)}{2} + 2 \tag{8}$$

steps, or $\mathcal{O}(n^2)$

Question 2

Question 3