Worksheet 7 Solution

March 26, 2020

Question 1

1. Assume that $n \leq 1$.

   Then, it follows from the assumption that the statement holds for the case $n \leq 1$.

   Correct Solution:

   Assume that $n \leq 1$.

   Then, the assumption satisfies the first part of the OR we want to prove.

   Notes:

   - the professor specifically states the assumption satisfies the first part of the OR we want to prove.

2. Assume $\exists k, d \in \mathbb{N}$, $n = kd \wedge d \neq 1 \wedge d \neq n$.

   Let $a = d$ and $b = k$.

   We will divide proof into parts and combine them together.

   Part 1 ($n \nmid a$):

Since $\frac{1}{k} \cdot n = d$, k must be 1 for n to divide d.

Then, because we know $d \neq n$, we can conclude that $n \nmid a$.

Part 2 $(n \nmid b)$:

Since $\frac{1}{d} \cdot n = k$, d must be 1 for n to divide k.

Then, because we know $d \neq 1$, we can conclude $n \nmid b$.

Part 3 $(n \mid ab)$:

Since $ab = n$ and $\forall n \in \mathbb{N}$, $n \mid n$, we can conclude that $n \mid ab$.

Then, it follows from the result of part 1, part 2 and part 3 that the second part of the OR is true.

Correct Solution:

Assume $\exists d \in \mathbb{N}$, $k \in \mathbb{Z}$, $n = dk \wedge d \neq 1 \wedge d \neq n$, and n ¿ 1.

Let $a = d$ and $b = k$.

We will prove this statement by dividing into cases and combining them together.

Case 1 $(n \mid ab)$:

Because we know $n = ab$ and $n \mid n$ by fact 1 , we can conclude $n \mid ab$.

Case 2 $(n \nmid a)$:

Because we know $d \geq 1$ from $d \in \mathbb{N}$ and $n > 1$ in assumption, we can conclude $k \geq 1$.

Then,

$$n = dk \tag{1}$$
$$n > d \tag{2}$$

where $'>'$ sign is due to the assumption $d \neq n$.

Then,

$$d < 1 \vee n \nmid d \tag{3}$$

by contrapositive of fact 2.

Since the first part of OR is not true, we can conclude $n \nmid a$.

Case 3 $(n \nmid b)$:

Because we know $n = dk$, $d \geq 1$ from $d \in \mathbb{N}$ and $n > 1$ in assumption, we can conclude $k \geq 1$.

Then because we know $d \neq n \wedge d \neq 1$ and $n = dk$, we can conclude $k \neq n \wedge k \neq 1$.

Then,

$$n = dk \tag{4}$$
$$n > k \tag{5}$$

where $'>'$ sign is due to the fact $k \neq n \wedge k \neq 1$.

Then,

$$b < 1 \vee n \nmid y \tag{6}$$

by contrapositive of fact 2.

Since the first part of OR is not true, and we can conclude $n \nmid b$.

Notes:

- **Definition of Divisibility:** Let $a, d \in \mathbb{Z}$. There exists $k \in \mathbb{Z}$, $n = dk$
- **Contrapositive of Fact 2:** $\forall x, y \in \mathbb{N}, 1 > x \vee x > y \Rightarrow y < 1 \vee x \nmid y$

Question 2

Question 3