

# Worksheet 16 Solution

March 27, 2020

## Question 1

- a. **Part 1.a - Finding minimum possible change for a loop in a single iteration**

The minimum possible change in a loop occurs when  $i$  increments by 1.

- Part 1.b - Finding maximum possible change for a loop in a single iteration**

The maximum possible change in a loop occurs when  $i$  increments by 6.

- Part 2.a - Determine formula for an exact lower bound on the value**

Since the loop starts at  $i = 0$  and ends at  $n - 1$ , the loop has

$$n - 1 + 1 = n \tag{1}$$

iterations.

Since the smallest step increases by 1 per iteration, the total cost of the loop at minimum possible change is

$$(n) \cdot 1 = n \tag{2}$$

steps.

**Part 2.a - Determine formula for an exact upper bound on the value**

Since the loop starts at  $i = 0$  and ends at  $n - 1$ , the loop has

$$n - 1 + 1 = n \tag{3}$$

iterations.

**Part 2.b - Determine formula for an exact lower bound on the value**

Since the largest step increases by 6 per iteration, the total cost of the loop at minimum possible change is

$$\left\lceil \frac{n}{6} \right\rceil \tag{4}$$

steps.

**Part 3.a - Determine formula for an exact upper bound on the value** Is it  $n$ ?

**Part 3.a - Determine formula for an exact upper bound on the value** Is it  $\left\lceil \frac{n}{6} \right\rceil$ ?

**Part 4 - Determine Big Oh and Big Omega**

The big Oh bound of running time is  $\mathcal{O}(n)$ , and the big theta of running time is  $\Omega(n)$ .

Since  $n$  in  $\mathcal{O}(n)$  and  $\Omega(n)$  are the same,  $\Theta(n)$  is also true.

**Correct Solution:**

**Part 1.a - Finding minimum possible change for a loop in a single iteration**

The minimum possible change in a loop occurs when  $i$  increments by 1.

**Part 1.b - Finding maximum possible change for a loop in a**

### single iteration

The maximum possible change in a loop occurs when  $i$  increments by 6.

#### Part 2.a - Determine formula for an exact upper bound on the value

The upper bound of loop termination is when  $k \geq n$

#### Part 2.b - Determine formula for an exact lower bound on the value

The lower bound of loop termination is when  $6k \leq n$

#### Part 3.a - Use the formula to determine the exact number of loops that will occur for upper bound

Since the loop starts from 0 and ends at  $n - 1$ , the loop has total of

$$n - 1 - 0 + 1 = n \quad (5)$$

iterations.

Since 1 step is taken for each iteration, the upper bound total cost of loop iteration is

$$n \cdot 1 = n \quad (6)$$

Since the statement on line 2 has cost of 1, the upper bound total cost of the algorithm is  $n + 1$ , or  $\mathcal{O}(n)$ .

#### Part 3.b - Use the formula to determine the exact number of loops that will occur for lower bound

Since the loop starts from 0 and ends at  $n - 1$ , the loop has total of

$$n - 1 - 0 + 1 = n \quad (7)$$

iterations.

Since 6 steps are taken for each iteration, the lower bound total cost of loop iteration is

$$\left\lceil \frac{n}{6} \right\rceil \quad (8)$$

Since the statement on line 2 has cost of 1, the lower bound total cost of the algorithm is  $\left\lceil \frac{n}{6} \right\rceil + 1$ , or  $\Omega(n)$

#### **Part 4 - Determine Big Oh and Big Omega**

The big Oh bound of running time is  $\mathcal{O}(n)$ , and the big theta of running time is  $\Omega(n)$ .

Since  $n$  in  $\mathcal{O}(n)$  and  $\Omega(n)$  are the same,  $\Theta(n)$  is also true.

#### **b. Part 1.a - Finding minimum possible change for a loop in a single iteration**

The minimum possible change for a loop in a single iteration is when  $i$  increases by a factor of 2

#### **Part 1.b - Finding maximum possible change for a loop in a single iteration**

The maximum possible change for a loop in a single iteration is when  $i$  increases by a factor of 3

#### **Part 2.a - Determine formula for an exact upper bound of the loop variable after k iterations**

The exact upper bound of the loop variable after k iteration is  $2^k \geq n$

**Part 2.b - Determine formula for an exact lower bound of the loop variable after k iterations**

The exact lower bound of the loop variable after k iteration is  $3^k \geq n$

**Part 3.a - Use the formula to determine the exact number of loops that will occur for upper bound**

The upper bound of loop iteration is  $\lceil \log n \rceil$ , or  $\mathcal{O}(\log n)$

**Part 3.b - Use the formula to determine the exact number of loops that will occur for lower bound**

The lower bound of loop iteration is  $\lceil \log_3 n \rceil$ , or  $\Omega(\log n)$

**Part 4 - Determine Big Oh and Big Omega**

For the upper bound, we have  $\mathcal{O}(\log n)$ .

For the lower bound, we have  $\Omega(\log n)$

Since Big Oh and Big Omega have the same value,  $\Theta(\log n)$  is also true.

## Question 2

## Question 3