

Worksheet 3 Review 2

Hyungmo Gu

April 11, 2020

Question 1

- a. $Correct(my_prog) \wedge Python(my_prog)$
- b. $\forall x \in P, \neg Correct(x) \Rightarrow Python(x)$

Correct Solution:

$\exists x \in P, \neg Correct(x) \wedge Python(x)$

Notes:

- I feel that ‘ \wedge ’ operator is used instead of ‘ \Rightarrow ’ if ‘is’ is used with an existential quantifier

Example:

An incorrect program is written in Python

- I also feel ‘ \Rightarrow ’ is used when ‘is’ is used with universal quantifier

Example:

Every incorrect program is written in python

- c. $\forall x \in P, Python(x) \Rightarrow \neg Correct(x)$
- d. $\forall x \in P, \neg Correct(x) \Rightarrow Python(x)$
- e. There is a program that is written in *Python* and is *Correct*
- f. All programs are not written in *Python* and is *Correct*
- g. There is a program that is *Correct* and not written in *Python*
- h. All programs that are correct is not written in *Python*, and all programs that are *Correct* is not written in *Python*.

Question 2

- a. Either all programs that are written in *Python* is *Correct*, or all programs that are written in *Python* are not *Correct*
- b. $(\exists x \in P, \text{Python}(x) \wedge \text{Correct}(x)) \Rightarrow (\forall x \in P, \text{Python}(x) \wedge \text{Correct}(x))$
- c. The difference is that in statement 1, each divisibility claims can be validated with different natural numbers where as in statement 2, the two claims must be validated with a single natural number.

The statement 1 is true, where as statement 2 is false (consider counter example of $x = 7$)

Question 3

- a. $\text{Odd}(n) : \forall n \in \mathbb{Z}, \exists \in \mathbb{Z}, n + 1 = 2k$

Correct Solution:

$$\text{Odd}(n) : \exists \in \mathbb{Z}, n + 1 = 2k, \text{ where } n \in \mathbb{Z}$$

Notes:

- Noticed professor defines variable in predicate (i.e. n in $P(n)$) in where (i.e where $n \in \mathbb{Z}$)
- b. $\forall m, n \in \mathbb{Z}, \text{Odd}(m) \wedge \text{Odd}(n) \Rightarrow \text{Odd}(mn)$
- c. $\forall m, n \in \mathbb{Z}, \exists k_1, k_2 \in \mathbb{Z}, (n + 1 = 2k_1) \wedge (m + 1 = 2k_2) \Rightarrow \exists k_3 \in \mathbb{Z}, (mn + 1 = 2k_3)$

Correct Solution:

$$\forall m, n \in \mathbb{Z}, (\exists k_1 \in \mathbb{Z}, n + 1 = 2k_1) \wedge (\exists k_1 \in \mathbb{Z}, m + 1 = 2k_2) \Rightarrow \exists k_3 \in \mathbb{Z}, mn + 1 = 2k_3$$

Notes:

- Noticed professor didn't pull out existential quantifier from parenthesis
- d. $\forall m, n \in \mathbb{Z}, \exists k_1 \in \mathbb{Z}, mn + 1 = 2k_1 \Rightarrow (\exists k_2 \in \mathbb{Z}, m + 1 = 2k_2) \wedge (\exists k_3 \in \mathbb{Z}, n + 1 = 2k_3)$

Question 4

- a. $((a \wedge b) \wedge \neg c) \vee ((\neg a \vee \neg b) \wedge c)$
- b. $\exists x, y \in S, \forall x \in S, \neg P(x, y) \vee \neg Q(x, z)$
- c. $(\exists x \in S, P(x)) \wedge (\forall y \in S, \neg Q(y))$

Question 5