CSC236 Assignment 1

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Question 1

a. Yes. We can prove $P(235)$ follows from $P(234)$.

Proof. Let b be the bipartite graph with 235 vertices where 117 vertices are in one partition and 118 vertices in the other partition (Note this is the configuration where maximum number of edges form).

The bipartite graph with 117 vertices on both sides of partition has $\frac{234^2}{4}$ edges, and the assumption tells us this is the maximum number of edges the bipartite graph could form.

Since we know b has 117 more edges than the bipartite graph with 117 vertices on both sides, using these facts, we can conclude the upper bound number of edges for the bipartite graph with 235 vertices is

$$\begin{aligned}
\frac{234^2}{4} + 117 &= \frac{234^2}{4} + \frac{4 \cdot 117}{4} && (1)\\
&= \frac{234^2 + 2 \cdot 234}{4} && (2)\\
&\leq \frac{234^2 + 2 \cdot 234 + 1}{4} && (3)\\
&= \frac{(234+1)^2}{4} && (4)\\
&= \frac{(235)^2}{4} && (5)
\end{aligned}$$

□

Question 2

Question 3

Question 4