

Worksheet 7 Solution

March 16, 2020

Question 1

a. **Case 1** ($n \geq 1$):

No more proof required. This is exactly what we want to show.

Case 2 ($\exists d \in \mathbb{N}, d \mid n \wedge d \neq 1 \wedge d \neq n$):

Let $a = d$ and $b = k$.

Because we know $\forall n \in \mathbb{Z}^+, \text{ and } l \in \mathbb{Z}, l \mid n \Rightarrow l \leq n, a \leq n$.

Then $n \mid a$ is true only when $a = n$ and $b = 1$, by the fact that any lower value of a results in non-integer value.

Then it follows from the assumption $a \neq 1 \wedge a \neq n$ that $n \nmid a$.

The same logic holds for $n \nmid b$.

Lastly, since $n = ab$, and $\forall x \in \mathbb{Z}, x \mid x, n \mid ab$.

Question 2

a. Let $n, m \in \mathbb{N}$. Assume $\text{Prime}(n)$, and $n \nmid m$.

Then,

$$\gcd(n, m) = 1 \tag{1}$$

by fact 2 (i.e. $\forall n, p \in \mathbb{Z}, \text{Prime}(p) \wedge p \nmid n \Rightarrow \gcd(p, n) = 1$).

Then $\exists r, s \in \mathbb{Z}$,

$$1 = \gcd(n, m) = rn + sm \tag{2}$$

by fact 6 (i.e. $\forall n, m \in \mathbb{N}, \exists r, s \in \mathbb{Z}, rn + sm = \gcd(n, m)$).

Then, it follows from above that the statement $\forall n, m \in \mathbb{N}, \text{Prime}(n) \wedge n \nmid m \Rightarrow (\exists r, s \in \mathbb{Z}, rn + sm = 1)$ is true.

b. Let $n, m \in \mathbb{N}$. Assume $\text{Prime}(n)$ and $(\exists r, s \in \mathbb{Z}, rn + sm = 1)$.

Then,

$$\gcd(n, m) = 1 \tag{3}$$

by fact 6 (i.e. $\forall n, m \in \mathbb{N}, \exists r, s \in \mathbb{Z}, rn + sm = \gcd(n, m)$).

Then, 1 is the maximum number that divides both n and m , by the definition of GCD.

It follows from the above that $n \mid m$ only when $n = 1$.

Since n is prime and $n > 1$, the above is not possible, and $n \nmid m$.

Question 3

a. **Fact 1:**

Let $x \in \mathbb{Z}$.

Then,

$$x = x \tag{4}$$

$$x = (1)x \tag{5}$$

Then, it follows from the definition of divisibility that x divides x .