Worksheet 4 Review 2

April 12, 2020

Question 1

a. $\exists n \in \mathbb{N},\ n > 3 \wedge n^2 - 1.5n \geq 5$

b. The variable is existentially quantified

c. Because the variable is existentially quantified, the variable's value should be a *concrete* natural number

d. **Statement:** $\exists n \in \mathbb{N},\ n > 3 \wedge n^2 - 1.5n \geq 5$

Proof. Let $n = 5$.

We will prove $n > 3 \wedge n^2 - 1.5n \geq 5$.

First, we need to prove $n > 3$.

The header tells us $n = 5$.

Using this fact, we can conclude $n > 3$.

Now, we need to show $n^2 - 1.5n \geq 5$.

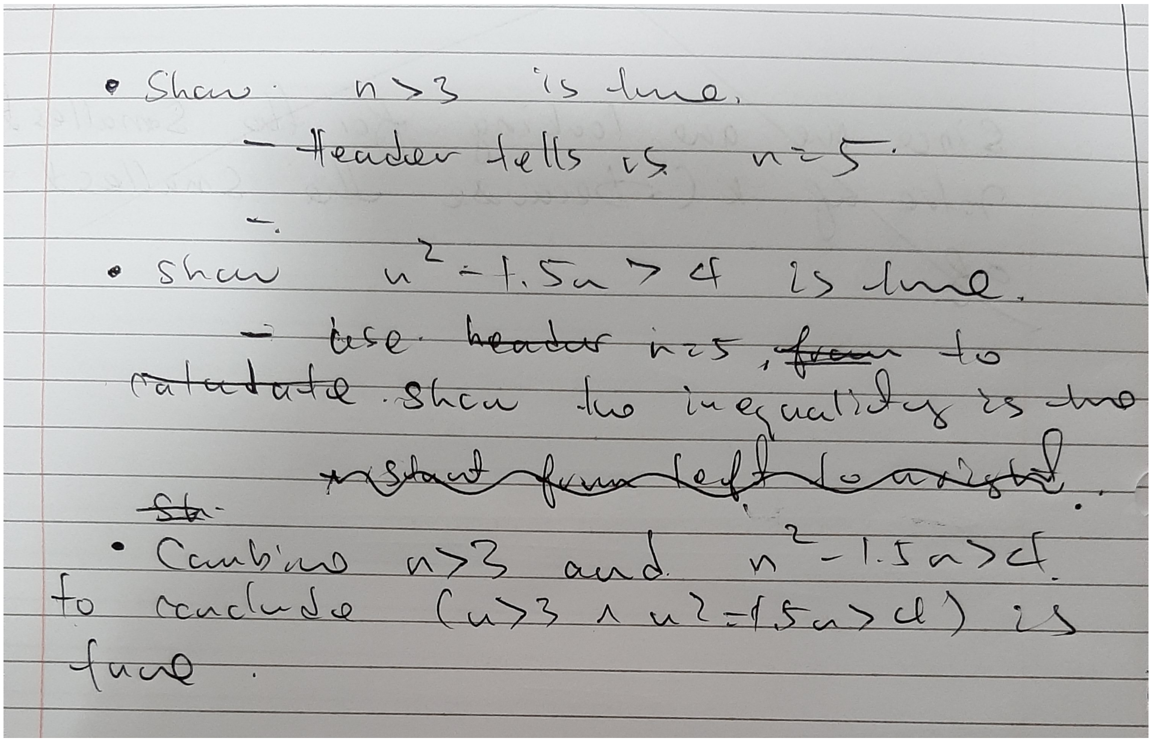
Using the fact $n = 5$, we can calculate

$$\begin{aligned} n^2 - 1.5n &= 25 - 7.5 && (1) \\ &= 17.5 && (2) \\ &\geq 5 && (3) \end{aligned}$$

Finally, since $n > 3$ and $n^2 - 1.5n \geq 5$ are true, we can conclude $n > 3 \wedge n^2 - 1.5n \geq 5$ are true. $\square$

Notes:

- Used the following pseudoproof used for this problem. Proof really feels smoother.



- Show $n>3$ is true.
  - Header tells us $n \geq 5$.
  - .
- Show $n^2 - 1.5n > 4$ is true.
  - Use ~~header~~ $n \geq 5$, ~~from~~ to ~~calculate~~. Show the inequality is true ~~start from left to right~~.

~~Sta~~

- Combine $n>3$ and $n^2 - 1.5n > 4$ to conclude $(n>3 \land n^2 - 1.5n > 4)$ is true.

e. $\forall n \in \mathbb{N},\ n \geq 3 \Rightarrow n^2 - 1.5n > 4$

f. The variable is universally quantified.

g. Because the variable is universally quantified, the variable's value should be an arbitrary natural number.

h. The assumption made is $n > 3$.

This conclusion is made by looking at the L.H.S of the $\Rightarrow$ operator.

i. **Statement:** $\forall n \in \mathbb{N},\ n > 3 \Rightarrow n^2 - 1.5n > 4$

Proof. Let $n \in \mathbb{N}$. Assume $n \geq 3$.

We will prove $n^2 - 1.5n > 4$.

Using the fact $n \geq 3$, we can conclude

$$
\begin{align}
n^2 - 1.5n &\geq (3)^2 - 1.5(3) \tag{1}\\
&= 9 - 4.5 \tag{2}\\
&= 4.5 \tag{3}\\
&> 4 \tag{4}
\end{align}
$$

$\square$

Question 2

a. $\forall n \in \mathbb{N},\ n > 5 \Rightarrow 2 \mid n \lor 3 \mid n$

b. $\exists n \in \mathbb{N},\ n > 5 \land (2 \nmid n) \land (3 \nmid n)$

c. **Statement:** $\exists n \in \mathbb{N},\ n > 5 \land (2 \nmid n) \land (3 \nmid n)$

Proof. Let $n = 7$.

We will prove $n > 5 \land (2 \nmid n) \land (3 \nmid n)$.

First, we will to prove $n > 5$.

The header tells us $n = 7$.

Using this fact, we can conclude $n > 5$.

Now, we will prove $2 \nmid n$.

7 is a prime number, so we know the number can only be divisible by 1 and 7.

Using this fact, we can conclude $2 \nmid 7$.

Now, we will prove $3 \nmid n$.

Since 7 is divisible by 1 and 7 only, we can conclude $3 \nmid 7$.

So, since $n > 5$, $2 \nmid n$ and $3 \nmid n$ are true, we can conclude $n > 5 \land (2 \nmid n) \land (3 \nmid n)$ holds. $\square$

Question 3

a. Let $x \in \mathbb{R}$, and $y = 165$.

Correct Solution:

Let $x \in \mathbb{R}$, and $y = 166 - x$

b. Let $y = 166$ and $x \in \mathbb{N}$.

c. **Negation of Statement:** $\forall y \in \mathbb{R},\ x + y \leq 165$

Let $y \in \mathbb{R}$, and $x = 164 - y$.

We will prove $x + y \leq 165$.

Because we know $x = 164 - y$, we can calculate

$$
\begin{aligned}
x + y &= 164 - y + y && (1)\\
&= 164 && (2)\\
&\leq 165 && (3)
\end{aligned}
$$