CSC373 Worksheet 4 Solution

August 3, 2020

1. Let $G = (V, E)$ be a directed graph. Let $[v_1, ..., v_n]$ be a list of vertices in graph G.

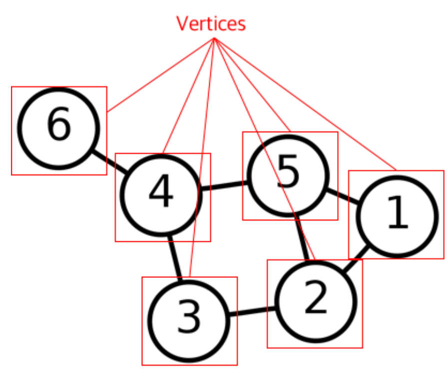
   I need to calculate the outdegree of every vertex using adjacency list.

   We know that in addition to counting v_i where $i = 1, .., n$ in adjacency list, we are counting $|Adj[v_i]|$ edges.

   Since we are traversing $|V| = n$ many vertices, we can write that in addition to there are total of $\sum_{i=1}^{n} |Adj[v_i]| = |E|$ edges.

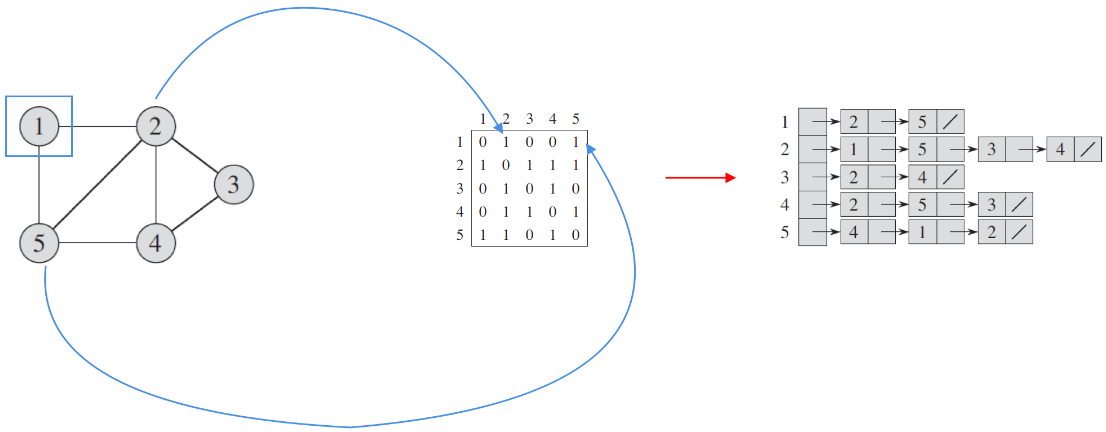
   Notes:

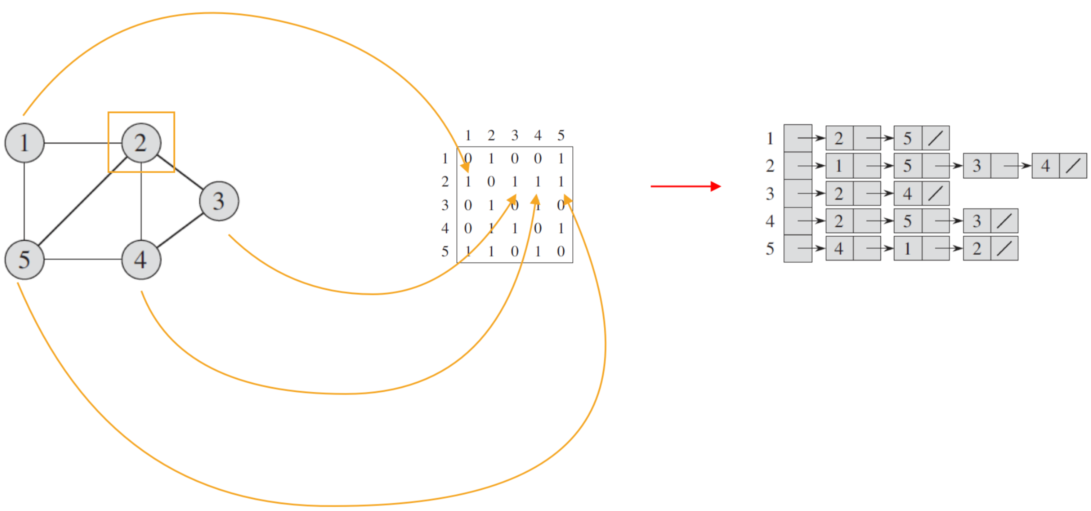
   - **Vertex**
     - Is a fundamental unit of which graphs are formed
     - Also means node

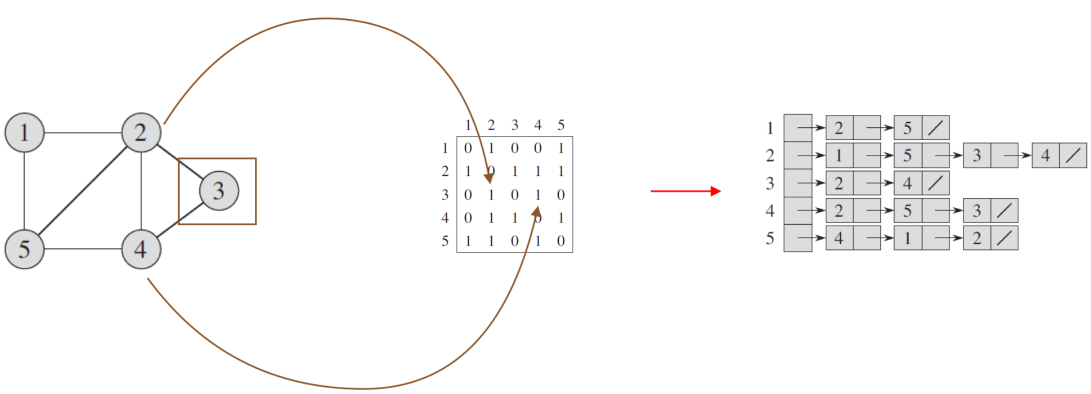
   

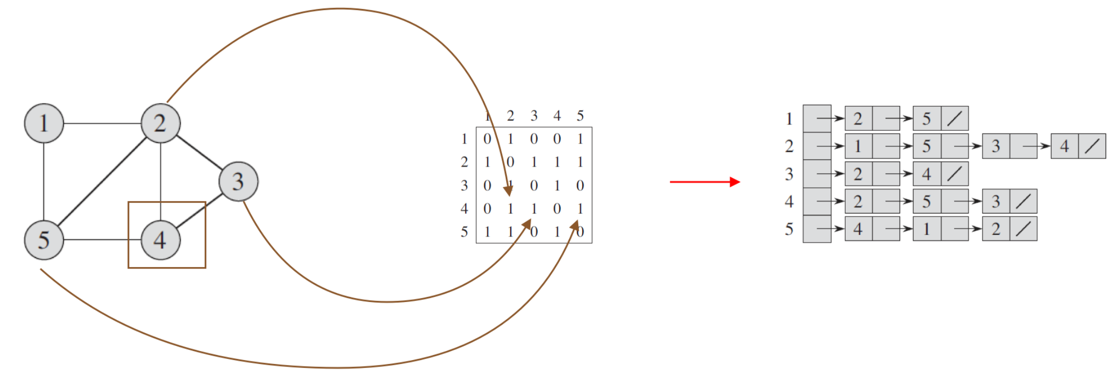
   - **Adjacency-list Representation**
     - Associates each vertax in a graph with the collection of its neighbouring vertices or edges
     - Is represented by $Adj[v]$

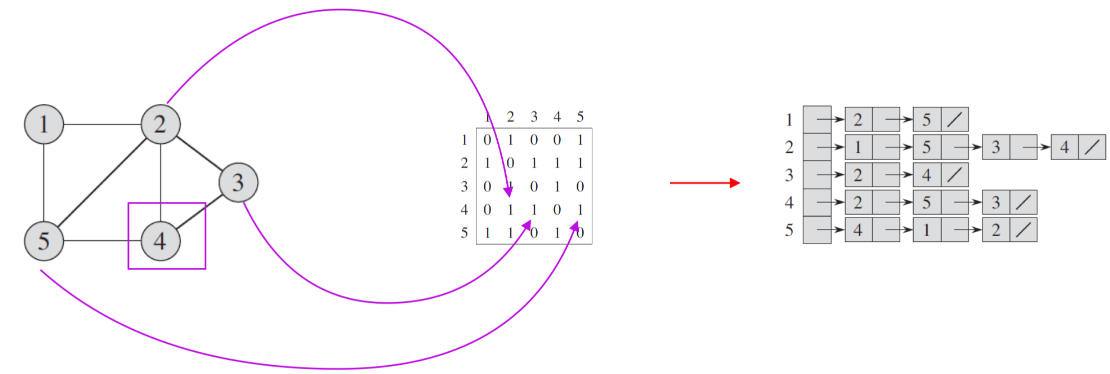
- ∗ Means all vertices that are neighbour to vertex v
- ∗ In a directed graph, $Adj[v]$ are all out-degree vertices of vertax v
- ∗ $|Adj[v]|$ means the total number of outdegree of vertax v

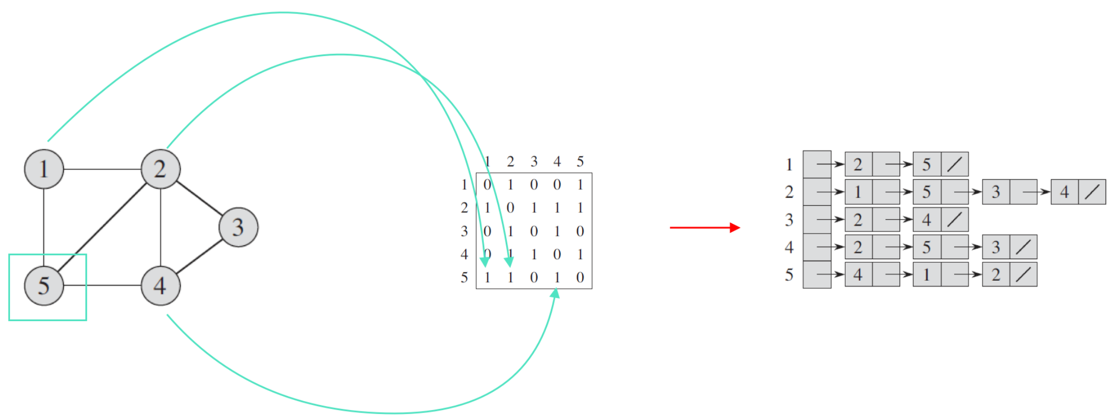




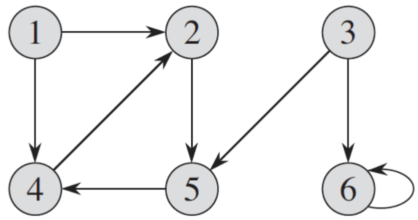




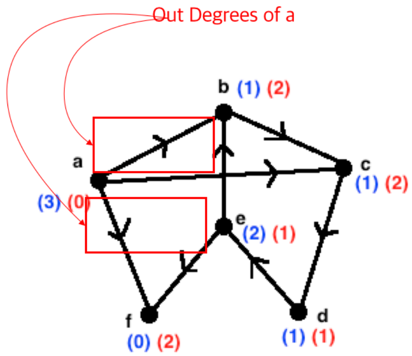




- **Directed graph**
  - Is a graph that is made up of a set of vertices connected by edges, where the edges have a direction associated with them



- **Out-degrees**
  - For a directed graph $G = (V(G), E(G))$ and a vertex $x_1 \in V(G)$, the Out-Degree of x_1 refers to the number of arcs incident from x_1. That is, the number of arcs directed <u>away</u> from the vertex x_1.



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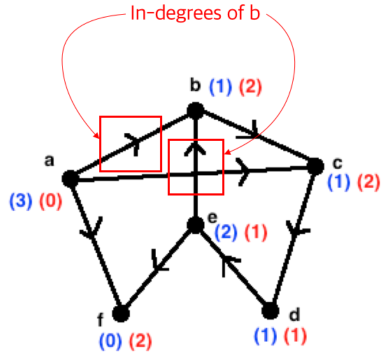
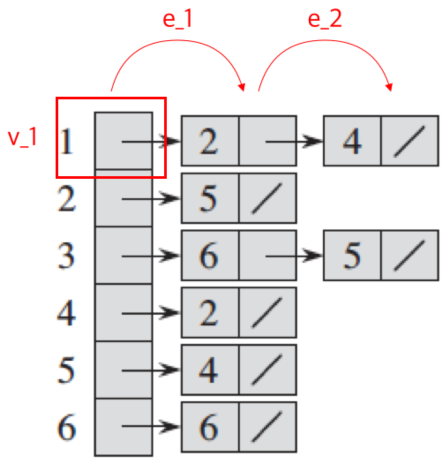


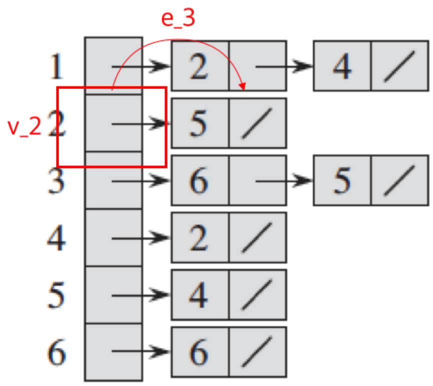
- Computing the outdegree of every vertex using adjacency list

1)



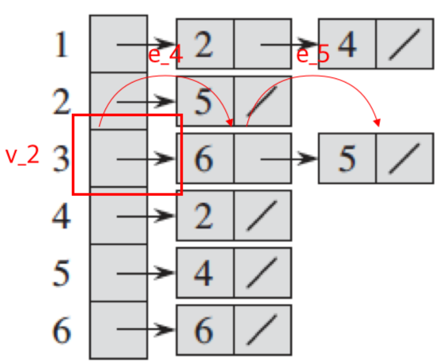
(v_1) + (e_1 + e_2)

2)



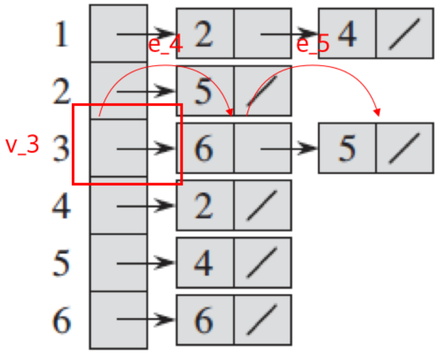
(v_1 + v_2) +
(e_1 + e_2 + e_3)

3)



(v_1 + v_2 + v3) +
(e_1 + e_2 + e_3 + e_4 + e_5)

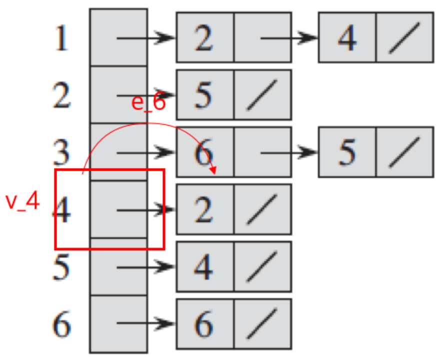
3)



(v_1 + v_2 + v3) +

(e_1 + e_2 + e_3 + e_4 + e_5)

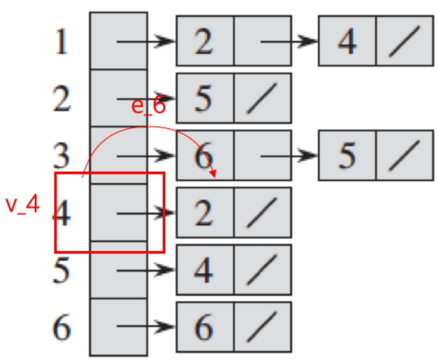
4)



(v_1 + v_2 + v_3 + v_4) +

(e_1 + e_2 + e_3 + e_4 + e_5)

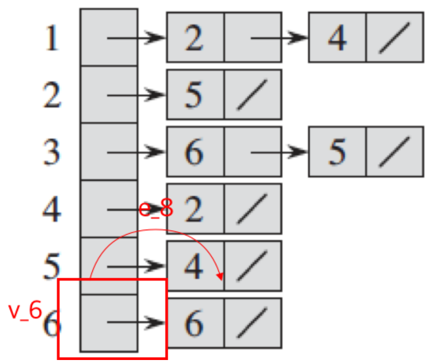
4)



(v_1 + v_2 + v_3 + v_4) +

(e_1 + e_2 + e_3 + e_4 + e_5)

6)



(v_1 + v_2 + v_3 + v_4 + v_5 _ v_6) +
(e_1 + e_2 + e_3 + e_4 + e_5 + e_6 + e_7 + e_8)

So it has $\mathcal{O}(V + E)$

- Computing the outdegree of every vertex using adjacency list

  The outdegree of a vertex is indegree of another vertex.

  Using this fact, we can conclude the running time of computing indegree of every vertax is $\mathcal{O}(V + E)$.