CSC373 Worksheet 5 Solution

August 8, 2020

1. **Rough Works:**

Assume that a flow network $G = (V, E)$ violates the assumption that the network contains a path $s \rightsquigarrow v \rightsquigarrow t$ for all vertices $v \in V$. Let u be a vertex for which there is no path $s \rightsquigarrow u \rightsquigarrow t$.

I must show such that there exists a maximum flow f in G such that $f(u, v) = f(v, u) = 0$ for all vertices $v \in V$

I will do so in cases

1. **Case 1:** When u only has flow going in
2. **Case 2:** When u only has flow going out
3. **Case 3:** When u only has no flow going in and out

Assume u only has no flow going in and out.

I need to show

Then, we can write

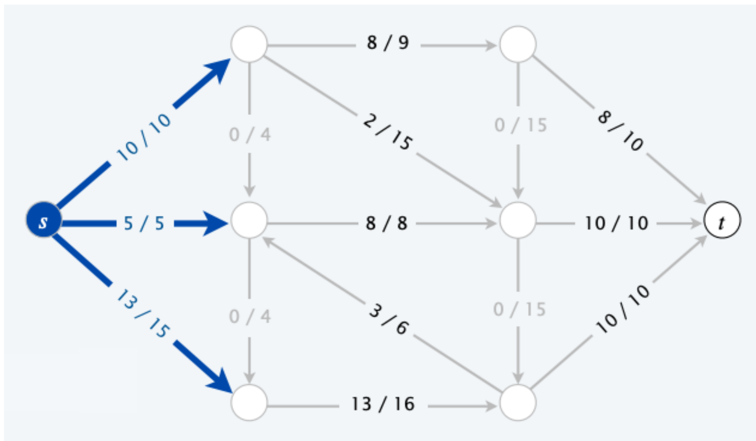
$$\begin{aligned} \sum_{v \in V} f(u, v) &= 0 && \text{[Since } (u, v) \notin E \text{ for all } v \in V\text{]} && (1) \\ &= \sum_{v \in V} f(v, u) && \text{[By flow conservation]} && (2) \end{aligned}$$

Then it follows from the capacity constraint $\forall a, b \in V$, $0 \leq f(a, b)$ that $f(u, v) = f(v, u) = 0$ for all $v \in V$

Notes

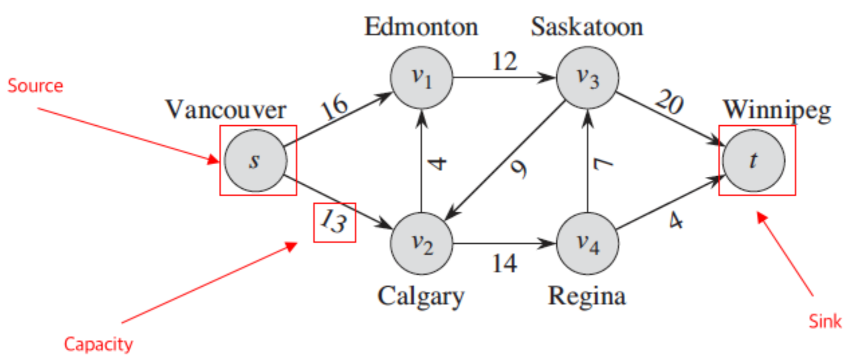
- **Maximum Flow:**
  - Finds a flow of maximum value [1]

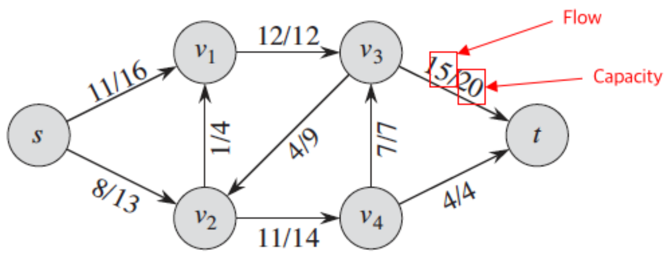
    Example

    Here, the maximum flow is $10 + 5 + 13 = 28$

- **Flow Network:**
  - $G = (V, E)$ is a directed graph in which each edge $(u, v) \in E$ has a nonnegative capacity $c(u, v) \geq 0$.
  - Two vertices must exist: **source** s and **sink** t
  - **path** from source s to vertax v to sink t is represented by $s \rightsquigarrow v \rightsquigarrow t$





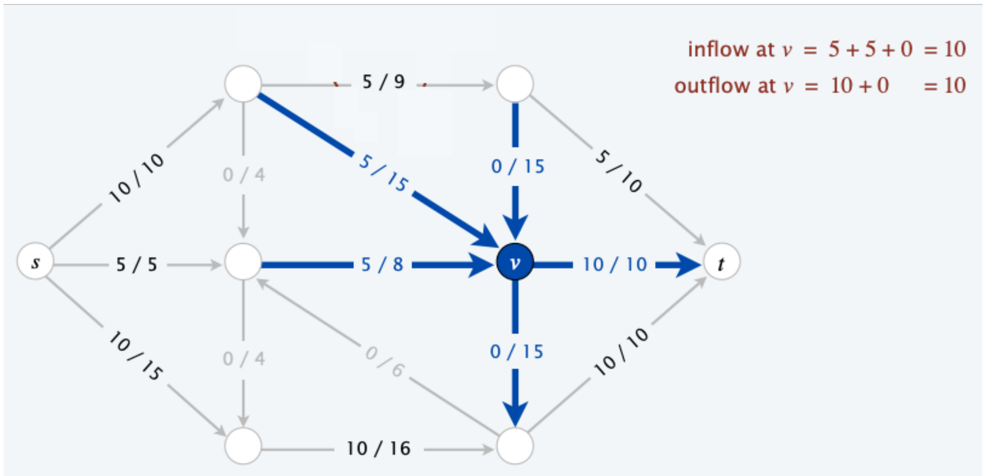
- **Capacity:**
  - Is a non-negative function $f : V \times V \to \mathbb{R}_{\geq 0}$
  - Has **capacity constraint** where for all $u, v \in V$ $0 \leq f(u,v) \leq c(u,v)$
    - Means flow cannot be above capacity constraint
- **Flow:**
  - Is a real valued function $f : V \times V \to \mathbb{R}$ in G
  - Satisfies **capacity constraint** (i.e for all $u, v \in V$, $0 \leq f(u,v) \leq c(u,v)$)
  - Satisfies **flow conservation**

  For all $u \in V - \{s, t\}$, we require

$$\sum_{v \in V} f(v,u) = \sum_{v \in V} f(u,v) \tag{3}$$

Means flow into vertex u is the same as flow going out of vertex u. [1]

Example:



References

1) Princeton University, Network Flow 1, link