Worksheet 8 Review

March 27, 2020

Question 1

a. $\forall n \in \mathbb{N}, (0 \leq 1) \wedge (n \leq 2^n) \Rightarrow (n+1) \leq 2^{n+1}$

Note:

- **Induction:** $\forall n \in \mathbb{N},\ P(0) \wedge P(n) \Rightarrow P(n+1)$

Question 2

a. We will prove this statement by induction on n.

Base Case:

Let $n = 0$.

Then,

$$0 \leq 2^0 \tag{1}$$
$$0 \leq 1 \tag{2}$$

Since the above inequality is true, the base case holds.

Inductive Case:

Let $n \in \mathbb{N}$. Assume $P(n)$.

Then,

$$\begin{aligned} n &\leq 2^n && (3) \\ n+1 &\leq 2^n + 1 && (4) \\ n+1 &\leq 2^n + 2^n && (5) \\ n+1 &\leq 2^{n+1} && (6) \end{aligned}$$

by the fact $2^k + 2^k = 2^{k+1}$.

Then, it follows from proof by induction that the statement $n \leq 2^n$ is true for all n.

Correct Solution:

We will prove this statement by induction on n.

Base Case:

Let $n = 0$.

Then,

$$\begin{aligned} 0 &\leq 2^0 && (7) \\ 0 &\leq 1 && (8) \end{aligned}$$

Since the above inequality is true, the base case holds.

Inductive Case:

Let $n \in \mathbb{N}$. Assume $P(n)$.

We want to show $n + 1 \leq 2^{n+1}$.

Then,

$$n \leq 2^n \tag{9}$$
$$n + 1 \leq 2^n + 1 \tag{10}$$
$$n + 1 \leq 2^n + 2^n \tag{11}$$
$$n + 1 \leq 2^{n+1} \tag{12}$$

by the fact $2^n + 2^n = 2^{n+1}$.

Then, it follows from proof by induction that the statement $n \leq 2^n$ is true for all n.

Notes:

- professor specifically states what we want to show in inductive case part of the proof. I thought it was obvious, and not necessary.
- When are the times 'we want to show x' in proof can be omitted?

Question 3