Worksheet 19 Solution

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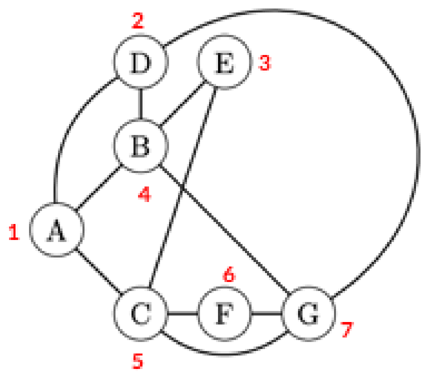
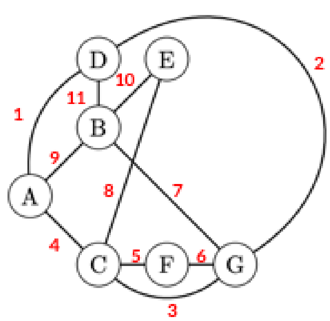
Question 1

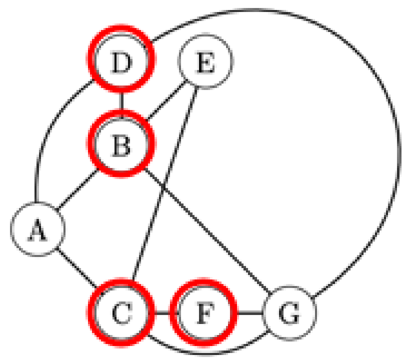
a. By the figure below, we can conclude there are 7 vertices.



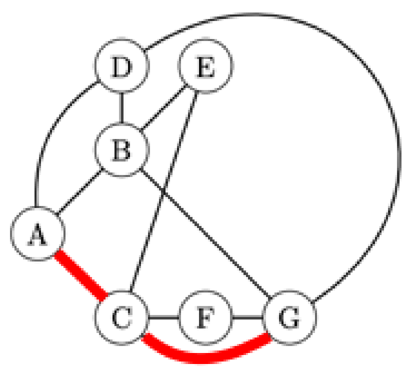
b. By the figure below, we can conclude there are 11 edges.



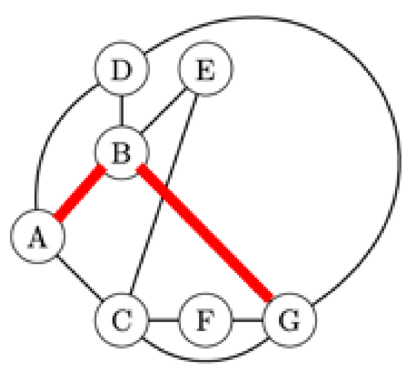
c. By the figure below, we can conclude there are 4 vertices adjacent to G.



d. By the figure below, we can conclude the distance between A ang G is 2.

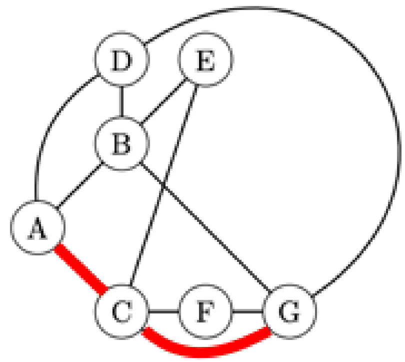


There are 2 shortest paths between A and G. One is the path from A to C to G as shown above, and the other is the path from A to B to G

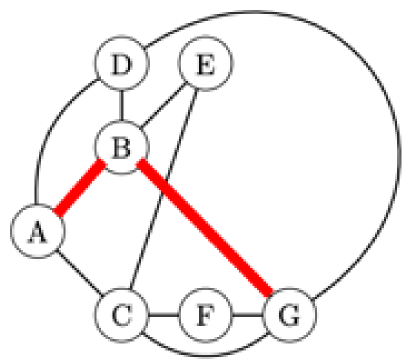


Correct Solution:

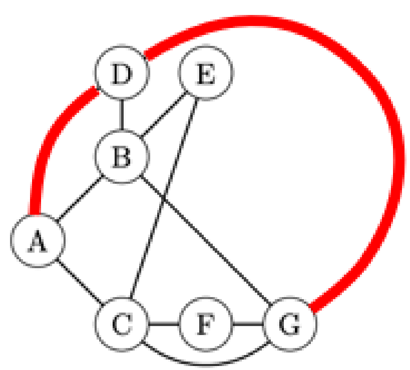
By the figure below, we can conclude the distance between A ang G is 2.



There are 3 shortest paths between A and G. One is the path from A to C to G as shown above, and the other is the path from A to B to G



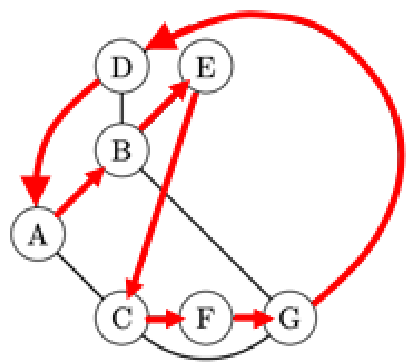
and the last one is from A to D to G



Notes:

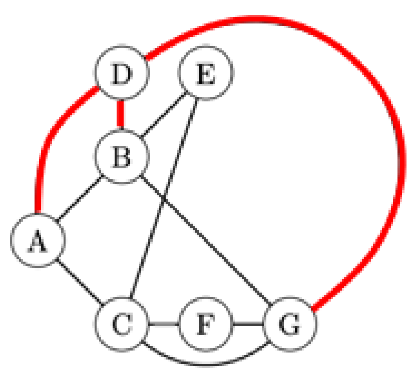
- **Distance** is the number of edges in a shortest path.

e. Path [C,F,G,D,A,B,E] is one example that goes through all vertices of the graph.



Question 2

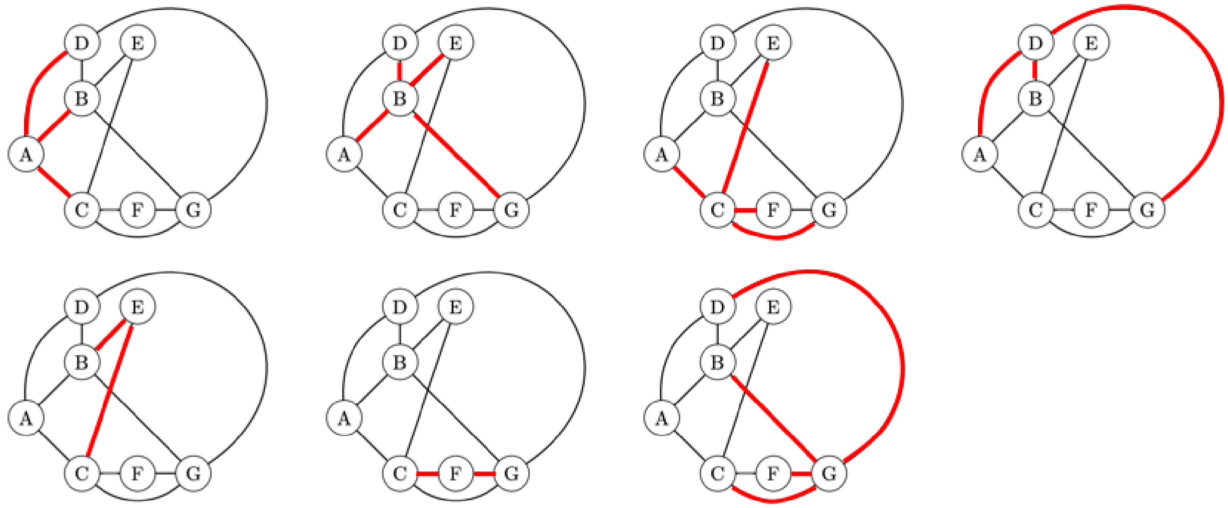
a. By the figure below, we can conclude the degree of vertex D is 3.



b. By the figure below, we can see

- vertex A has degree of 3
- vertex B has degree of 4
- vertex C has degree of 4
- vertex D has degree of 3
- vertex E has degree of 2
- vertex F has degree of 2
- vertex G has degree of 4

Using this fact, we can conclude the vertices with the largest degree are B,C and G.



c. **Statement:** $\forall G = (V, E),\ (\forall v \in V,\ d(v) \leq 5) \Rightarrow |E| \leq \frac{5}{2}|V|$

Proof. Let $G = (V, E)$ be a graph. Assume $v \in V$ and $d(v) \leq 5$.

We need to show $|E| \leq \frac{5}{2}|V|$.

Because we know the number of edges is half of the summation of all degrees of v in V, we can write

$$|E| = \frac{1}{2} \cdot \sum_{v \in V} d(v) \tag{1}$$

Then, using the assumption $d(v) \leq 5$, we can conclude

$$|E| \leq \frac{1}{2} \cdot \sum_{v \in V} 5 \tag{2}$$
$$= \frac{5}{2}|V| \tag{3}$$

□

Notes:

- I should work on improving this proof in future iterations. I feel the beginning has been jumped too quick.

Question 3

a. The adjacency matrix of this graph is

$$\begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \end{bmatrix} \tag{1}$$

b.

```python
    def degree(A,i):
        """ Returns the degree of vertex i from adjacency matrix A

        @type A: list
        @type B: int
        @rtype: int
        >>> A = [[0,1,0,1,0],
            [1,0,1,0,1],
            [0,1,0,0,1],
            [1,0,0,0,1],
            [0,1,1,1,0]]
        >>> i = 0
        >>> degree(A,i)
        2
        """

        return sum(A[i])

```