Worksheet 14 Solution

March 25, 2020

Question 1

a. **Inner Loop Iterations (upper bound):** n
   Inner Loop Step Size: 1
   Inner Loop Steps Total: n

   Outer Loop Iterations (upper bound): n
   Outer Loop Step Size: 1
   Outer Loop Steps Total: n

   Steps Total: $n \cdot n = n^2$

Correct Solution:

Since the inner loop starts at $i + 1$ and ends at $n - 1$, where i represents the variable in outer loop, the inner loop has $(n - 1 - (i + 1) + 1) = n - i - 1$ iterations.

Since each iteration takes 1 step, the total steps taken by inner loop is:

$$(n - i - 1) \cdot 1 = (n - i - 1) \tag{1}$$

Now, we will evaluate total steps taken by outer loop.

Since the outer loop starts at $i = 0$, and ends at $n - 1$, the loop runs at most n iterations.

Since each iteration takes $(n - i - 1)$ steps, the total steps of outer loop is:

$$\begin{align}
\sum_{i=0}^{n-1}(n - i + 1) &= \sum_{i=0}^{n-1}\left[(n - 1) - i\right] \tag{2}\\
&= \sum_{i=0}^{n-1}(n - 1) - \sum_{i=0}^{n-1} i \tag{3}\\
&= n(n - 1) - \frac{n(n - 1)}{2} \tag{4}\\
&= \frac{n^2 - n}{2} \tag{5}
\end{align}$$

Then, since the last **return** statement takes 1 step, it follows from that the total number of steps of this algorithm is at most $\frac{n^2-n}{2} + 1$, or $\mathcal{O}(n^2)$.

b. Consider the input family where none of the values in a list are the same (i.e. $[1, 2, 3, 4, 5, 6, 7, 8, 9]$).

Since all values in the input list are not matching, both the inner and the outer loop will run, giving the loops the total number of steps of $\frac{n^2-n}{2}$.

Since the last **return** statement takes 1 step, the total number of steps of this algorithm is $\frac{n^2-n}{2} + 1$, or $\Omega(n^2)$.

Question 2