# Midterm 2 Version 1 Solution

April 3, 2020

## Question 1

a.

$$\begin{aligned} 100 \div 2 &= 50, \text{ Remainders } \mathbf{0} \\ 50 \div 2 &= 25, \text{ Remainders } \mathbf{0} \\ 25 \div 2 &= 12, \text{ Remainders } \mathbf{1} \\ 12 \div 2 &= 6, \text{ Remainders } \mathbf{0} \\ 6 \div 2 &= 3, \text{ Remainders } \mathbf{0} \\ 3 \div 2 &= 1, \text{ Remainders } \mathbf{1} \\ 1 \div 2 &= 0, \text{ Remainders } \mathbf{1} \end{aligned}$$

Then, it follows from above that the binary representation of 100 is $(1100100)_2$.

b. The smallest number that can be expressed by an n-digit balanced ternary representation is

$$\sum_{i=0}^{n-1} d_i \cdot 3^i, \text{ where } d_i \in \{0, 1, 2\} \tag{1}$$

**Correct Solution:**

The smallest number that can be expressed by an n-digit balanced ternary representation is

$$-\left[\sum_{i=0}^{n-1} 3^i\right] \tag{1}$$

**Notes:**

- Realized professor is asking for an example of the smallest number.
- Ternary representation of a number

$$\sum_{i=0}^{n-1} d_i \cdot 3^i, \text{ where } d_i \in \{0, 1, 2\}$$

- Learned a negative number could be expressed in in ternary or binary representation of numbers.

c.

| | | | | | |
|---|---|---|---|---|---|
| $f(n) \in \Omega(n)$ | True | $g(n) \in \Omega(n)$ | False | $f(n) \in \mathcal{O}(g(n))$ | False |
| $f(n) \in \Theta(g(n))$ | False | $g(n) \in \Theta(\log_3 n)$ | True | $f(n) + g(n) \in \Theta(f(n))$ | True |

**Notes:**

- $\forall g : \mathbb{N} \to \mathbb{R}^{\geq 0}$, and all numbers $a \in \mathbb{R}^{\geq 0}$, if $g \in \mathcal{O}(f)$, then $f + g \in \mathcal{O}(f)$
- $g \in \Theta(f) : \; g \in \mathcal{O}(f) \wedge g \in \Omega(f)$
  or
  $g \in \Theta(f) : \; \exists c_1, c_2, n_1 \in \mathbb{R}^+, \forall n \in \mathbb{N}, n \geq n_1 \Rightarrow c_1 g(n) \leq f(n) \leq c_2 g(n)$, where $f, g : \; \mathbb{N} \to \mathbb{R}^{\geq 0}$
- $g \in \Omega(f) : \; \exists c, n_o \in \mathbb{R}^+, \; \forall n \in \mathbb{N}, \; n \geq n_0 \Rightarrow g(n) \geq cf(n)$, where $f, g : \mathbb{N} \to \mathbb{R}^{\geq 0}$

- $g \in \mathcal{O}(f) : \exists c, n_o \in \mathbb{R}^+, \forall n \in \mathbb{N}, n \geq n_0 \Rightarrow g(n) \leq cf(n)$, where $f, g : \mathbb{N} \rightarrow \mathbb{R}^{\geq 0}$

d.

| k | 0 | 1 | 2 |
|---|---|---|---|
| $i_k$ | $3 = 3^1$ | $9 = 3^2$ | $81 = 3^4$ |

The value of $i_k$ is

$$3^{2^k} \tag{1}$$

**Notes:**

- Realized we are only concerned with the lines **i = i * i** and **i = 3**

e. The number of iterations the function's loop will run is

$$\lceil \log_2 \log_3 n \rceil - 1 \tag{1}$$

**Notes:**

- The loop terminates when $3^{2^{(k+1)}} = i_{k+1} = i_k \cdot i_k \geq n$.
- $\forall x \in \mathbb{Z}, \forall y \in \mathbb{R}, \lfloor x + y \rfloor = x + \lfloor y \rfloor$
- Feel more confident there is no need to add an extra **+1**. Done by playing with examples (i.e is $\lceil \log \log_3(82) \rceil - 1$ true? Would the loop run only once?)

# Question 2

# Question 3

# Question 4