Worksheet 6 Review 2

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Question 1

a. $\forall x \in \mathbb{N},\ P(123) \wedge P(x) \Rightarrow x \leq 123$

> **Correct Solution:**
>
> $P(123) \wedge (\forall x \in \mathbb{N}, P(x) \Rightarrow x \leq 123)$

b. $IsCD(x, y, d) :\ d \mid x \wedge d \mid y$, where $x, y, d \in \mathbb{Z}$

$IsGCD(x, y, d) :\ \forall n \in \mathbb{N},\ IsCD(x, y, n) \Rightarrow \exists d \in \mathbb{N},\ IsCD(x, y, d) \wedge n \leq d$

> **Correct Solution:**
>
> $IsCD(x, y, d) :\ d \mid x \wedge d \mid y$, where $x, y, d \in \mathbb{Z}$
>
> $IsGCD(x, y, d) : (x = 0 \wedge y = 0 \Rightarrow d = 0) \wedge (x \neq 0 \wedge y \neq 0 \Rightarrow IsCD(x, y, d) \wedge (\forall d_1 \in \mathbb{Z},\ IsCD(x, y, d_1) \Rightarrow d_1 \leq d))$, where $x, y, d \in \mathbb{Z}$

Notes:

- Realized the definition of $IsGCD$ extends from previous question
- Noticed professor defines if...else conditions in a predicate logic the following way

  > (case 1 $\Rightarrow$ statement 1) $\wedge$ (case 2 $\Rightarrow$ statement 2)

- Hm... I feel puzzled about $\wedge$ operator used in between cases (i.e. $(x = 0 \wedge y = 0 \Rightarrow d = 0) \wedge (x \neq 0...)$). At glimpse, I felt $\vee$ is more appropriate since if this case is not true, then we want other case should be true.

c. **Statement:** $IsCD(x, 0, x) \wedge (\forall d_1 \in \mathbb{Z}, IsCD(x, 0, d_1) \Rightarrow d_1 \leq x)$

Proof. Let $x \in \mathbb{Z}^+$

We need to prove x is a common divisor to both 0 and x, and we need to prove all common divisors d_1 of 0 and x is less than or equal to x.

First, we need to show there is $k_1 \in \mathbb{Z}$ such that $x = k_1 \cdot x$ and we need to show $k_2 \in \mathbb{Z}$ such that $0 = k_2 \cdot x$.

Let $k_1 = 1$ and $k_2 = 0$.

Then, we can calculate that

$$x = 1 \cdot x = k_1 \cdot x \tag{1}$$
$$0 = 0 \cdot x = k_2 \cdot x \tag{2}$$

Now, we need to show all integers d_1 that is a common divisor to both 0 and x is less than equal to x.

Let $d_1 \in \mathbb{Z}$ and assume $d_1 \mid x$ and $d_1 \mid 0$.

We need to show $d_1 \leq x$.

The hint tells us

$$\forall n \in \mathbb{Z}^+,\ \forall d \in \mathbb{Z},\ d \mid n \Rightarrow d \leq n \tag{3}$$

Because we know from assumption that $d_1 \mid x$, by using the hint, we can conclude

$$d_1 \leq x \tag{4}$$

□

Pseudoproof:

Let $x \in \mathbb{Z}^+$

We need to prove x is a common divisor to both 0 and x, and we need to prove all common divisors d_1 of 0 and x is less than or equal to x.

1. Show $IsCD(x, 0, x)$

   We need to show there is $k_1 \in \mathbb{Z}$ such that $x = k_1 \cdot x$ and we need to show $k_2 \in \mathbb{Z}$ such that $0 = k_2 \cdot x$.

   Let $k_1 = 1$ and $k_2 = 0$.

   - Show $x = k_1 \cdot x$ and $0 = k_2 \cdot 0$

   Then, we can calculate that

   $$x = 1 \cdot x = k_1 \cdot x \tag{5}$$
   $$0 = 0 \cdot x = k_2 \cdot x \tag{6}$$

2. Show $\forall d_1 \in \mathbb{Z}, IsCD(x, 0, d_1) \Rightarrow d_1 \leq x$

   Let $d_1 \in \mathbb{Z}$ and assume $d_1 \mid x$ and $d_1 \mid 0$.

   We need to show $d_1 \leq x$.

   1. Use fact '$\forall n \in \mathbb{Z}^+,\ \forall d \in \mathbb{Z},\ d \mid n \Rightarrow d \leq n$' to show $d_1 \leq x$.

   The hint tells us

   $$\forall n \in \mathbb{Z}^+,\ \forall d \in \mathbb{Z},\ d \mid n \Rightarrow d \leq n \tag{7}$$

   Because we know from assumption that $d_1 \mid x$, by using the hint, we can conclude

   $$d_1 \leq x \tag{8}$$

Question 2

Question 3