

Worksheet 14 Solution

March 25, 2020

Question 1

a. Inner Loop Iterations (upper bound): n

Inner Loop Step Size: 1

Inner Loop Steps Total: n

Outer Loop Iterations (upper bound): n

Outer Loop Step Size: 1

Outer Loop Steps Total: n

Steps Total: $n \cdot n = n^2$

Correct Solution:

Since the inner loop starts at $i + 1$ and ends at $n - 1$, where i represents the variable in outer loop, the inner loop has $(n - 1 - (i + 1) + 1) = n - i - 1$ iterations.

Since each iteration takes 1 step, the total steps taken by inner loop is:

$$(n - i - 1) \cdot 1 = (n - i - 1) \quad (1)$$

Now, we will evaluate total steps taken by outer loop.

Since the outer loop starts at $i = 0$, and ends at $n - 1$, the loop runs at most n iterations.

Since each iteration takes $(n - i - 1)$ steps, the total steps of outer loop is:

$$\sum_{i=0}^{n-1} (n - i - 1) = \sum_{i=0}^{n-1} [(n - 1) - i] \quad (2)$$

$$= \sum_{i=0}^{n-1} (n - 1) - \sum_{i=0}^{n-1} i \quad (3)$$

$$= n(n - 1) - \frac{n(n - 1)}{2} \quad (4)$$

$$= \frac{n^2 - n}{2} \quad (5)$$

Then, it follows from above that the total number of steps of this algorithm is at most $\frac{n^2 - n}{2}$, or $\mathcal{O}(n^2)$.

Question 2