

# CSC373 Worksheet 0

July 18, 2020

1. **CLRS 4.3-1:** Show that the solution of  $T(n) = T(n-1) + n$  is  $\mathcal{O}(n^2)$ .
2. **CLRS 4.3-2:** Show that the solution of  $T(n) = T(\lceil n/2 \rceil) + 1$  is  $\mathcal{O}(\lg n)$ .
3. **CLRS 4.3-3:** We saw that the solution of  $T(n) = 2T(\lfloor n/2 \rfloor) + n$  is  $\mathcal{O}(n \lg n)$ . Show that the solution of this recurrence is also  $\Omega(n \lg n)$ . Conclude that the solution is  $\Theta(n \lg n)$ .
4. **CLRS 4.3-5:** Show that  $\Theta(n \lg n)$  is the solution to the “exact” recurrence (4.3) for merge sort.
5. **CLRS 4.3-6:** Show that the solution to  $T(n) = 2T(\lfloor n/2 \rfloor + 17) + n$  is  $\mathcal{O}(n \lg n)$ .
6. **CLRS 4.3-7:** Using the master method in Section 4.5, you can show that the solution to the recurrence  $T(n) = 4T(n/3) + n$  is  $T(n) = \Theta(n^{\log_3 4})$ . Show that a substitution proof with the assumption  $T(n) \leq cn^{\log_3 4}$  fails. Then show how to subtract off a lower-order term to make a substitution proof work.
7. **CLRS 4.3-8:** Using the master method in Section 4.5, you can show that the solution to the recurrence  $T(n) = 4T(n/2) + n$  is  $T(n) = \Theta(n^2)$ . Show that a substitution proof with the assumption  $T(n) \leq cn^2$  fails. Then show how to subtract off a lower-order term to make a substitution proof work.
8. **CLRS 4.4-1:** Use a recursion tree to determine a good asymptotic upper bound on the recurrence  $T(n) = 3T(\lfloor n/2 \rfloor) + n$ . Use the substitution method to verify your answer.
9. **CLRS 4.4-2:** Use a recursion tree to determine a good asymptotic upper bound on the recurrence  $T(n) = 3T(n/2) + n^2$ . Use the substitution method to verify your answer.
10. **CLRS 4.4-3:** Use a recursion tree to determine a good asymptotic upper bound on the recurrence  $T(n) = 4T(n/2 + 2) + n$ . Use the substitution method to verify your answer.
11. **CLRS 4.4-4:** Use a recursion tree to determine a good asymptotic upper bound on the recurrence  $T(n) = 2T(n-1) + 1$ . Use the substitution method to verify your answer.
12. **CLRS 4.4-5:** Use a recursion tree to determine a good asymptotic upper bound on the recurrence  $T(n) = 2T(n-1) + T(n/2) + n$ . Use the substitution method to verify your answer.

13. **CLRS 4.4-6:** Argue that the solution to the recurrent  $T(n) = 4T(\lfloor n/2 \rfloor) + cn$  where  $c$  is a constant, and provide a tight asymptotic bound on its solution. Verify your bound on the substitution method.
14. **CLRS 4.4-7:** Draw the recursion tree for  $T(n) = 4T(n/2) + cn$ , where  $c$  is a constant, and provide a tight asymptotic bound on its solution. Verify your bound by the substitution method.