Midterm 1 Version 1 Review 2

July 17, 2020

1. a) $aaa, aab, aac, bba, bbb, bbc, cca, ccb, ccc$

   b) **Solution:**

p	q	r	$p \vee q$	$(p \vee q) \Rightarrow \neg r$
T	T	T	T	T
T	T	F	T	F
T	F	T	T	T
F	T	T	T	T
T	F	F	T	F
F	T	F	T	F
F	F	T	F	T
F	F	F	F	T

   c) $\exists x \in \mathbb{N}, \forall y \in \mathbb{N}, \neg P(x,y) \wedge \neg Q(x,y)$

Correct Solution:

Let $x =$ _____. Let $y \in P$.

We need to prove $\neg P(x,y)$ and $\neg Q(x,y)$ are true.

2. a) $\exists x \in P, Student(x) \wedge Attends(x)$

   b) $\forall x \in P, \exists y \in P, Student(y) \wedge Attends(y) \Rightarrow Loves(x,y)$

Correct Solution:

$\forall x \in P, \exists y \in P, Student(y) \wedge Attends(y) \wedge Loves(x,y)$

   c) $\forall x \in P, Student(x) \wedge Attends(x) \Rightarrow Loves(x,x)$

   d) $\forall x, y \in P, x \neq y \Rightarrow Loves(x,y) \Rightarrow Attends(x) \vee Attends(y)$

3. a) $\forall a, b, c \in \mathbb{Z}, \exists k_1, k_2, k_3 \in \mathbb{Z}, a = k_1 b \wedge b = k_2 c \Rightarrow a = k_3 c$

b) *Proof.* Let $a, b, c \in \mathbb{Z}$, and there is some $k1, k2, k3 \in \mathbb{Z}$ such that $a = k_1 b, b = k_2 c$.

I need to prove $a = k_3 c$.

Let $k_3 = k_1 k_2$.

Then, we can conclude

$$\begin{aligned} a &= k_1 b && \text{[By header]} && (1) \\ &= k_1 k_2 c && \text{[By replacing } b \text{ with } k_2 c\text{]} && (2) \\ &= k_3 c && \text{[By } k_3 = k_1 k_2\text{]} && (3) \end{aligned}$$

□

4. *Proof.* Let $x, y \in \mathbb{R}$. Assume $\lfloor x + y \rfloor$.

I need to show $\lfloor x + y \rfloor \geq \lfloor x \rfloor + \lfloor y \rfloor$.

Indeed we have

$$\begin{aligned} \lfloor x + y \rfloor &= \lfloor \lfloor x \rfloor + \epsilon + y \rfloor && \text{[By fact 1]} && (1) \\ &= \lfloor x \rfloor + \lfloor \epsilon + y \rfloor && \text{[By fact 2]} && (2) \\ &\geq \lfloor x \rfloor + \lfloor y \rfloor && \text{[Since } \epsilon \geq 0\text{]} && (3) \end{aligned}$$

□

Correct Solution:

Proof. Let $x, y \in \mathbb{R}$. Assume $\lfloor x + y \rfloor$.

I need to show $\lfloor x + y \rfloor \geq \lfloor x \rfloor + \lfloor y \rfloor$.

Indeed we have

$$\begin{aligned} \lfloor x + y \rfloor &= \lfloor \lfloor x \rfloor + \epsilon + y \rfloor && \text{[By fact 1]} && (1) \\ &= \lfloor x \rfloor + \lfloor \epsilon + y \rfloor && \text{[By fact 2]} && (2) \\ &\geq \lfloor x \rfloor + \lfloor y \rfloor && \text{[Since } \epsilon \geq 0\text{]} && (3) \end{aligned}$$

□