Worksheet 10 Review

March 28, 2020

Question 1

a.

$$
\begin{align}
(165)_8 &= 5 \cdot 8^0 + 6 \cdot 8^1 + 1 \cdot 8^2 \tag{1} \\
&= 5 + 48 + 64 \tag{2} \\
&= 53 + 64 \tag{3} \\
&= 117 \tag{4}
\end{align}
$$

b.

$$
\begin{align}
(B4)_{16} &= 4 \cdot 16^0 + 11 \cdot 16^1 \tag{1} \\
&= 4 + (11 \cdot 16) \tag{2} \\
&= 4 + 176 \tag{3} \\
&= 180 \tag{4}
\end{align}
$$

Question 2

a.

$$
\begin{aligned}
357 \div 2 &= 178\text{,remainder } \mathbf{1}, \\
178 \div 2 &= 89\text{,remainder } \mathbf{0}, \\
89 \div 2 &= 44\text{,remainder } \mathbf{1}, \\
44 \div 2 &= 22\text{,remainder } \mathbf{0}, \\
22 \div 2 &= 11\text{,remainder } \mathbf{0}, \\
11 \div 2 &= 5\text{,remainder } \mathbf{1}, \\
5 \div 2 &= 2\text{,remainder } \mathbf{1}, \\
2 \div 2 &= 1\text{,remainder } \mathbf{0}, \\
1 \div 2 &= 0\text{,remainder } \mathbf{1}
\end{aligned}
$$

Combining it together, the binary representation of 357 is $(101100101)_2$

Notes:

- **Converting decimal to binary**

$$
\begin{aligned}
29 \div 2 &= 14 + \text{remainder } \mathbf{1}, && (1) \\
14 \div 2 &= 7 + \text{remainder } \mathbf{0}, && (2) \\
7 \div 2 &= 3 + \text{remainder } \mathbf{1}, && (3) \\
3 \div 2 &= 1 + \text{remainder } \mathbf{1}, && (4) \\
1 \div 2 &= 0 + \text{remainder } \mathbf{1} && (5)
\end{aligned}
$$

  Binaries read *bottom to top*

b.

$$
\begin{aligned}
1 \cdot 2^0 + 0 \cdot 2^1 + 1 \cdot 2^2 &= \frac{1+0+4}{8^0} = 5 \\
0 \cdot 2^3 + 0 \cdot 2^4 + 1 \cdot 2^5 &= \frac{0+0+32}{8^1} = 4 \\
1 \cdot 2^6 + 0 \cdot 2^7 + 1 \cdot 2^8 &= \frac{64+0+256}{8^2} = 5
\end{aligned}
$$

Combining it together, the octal representation of $(101100101)_2$ is $(545)_8$.

c.

$$\begin{aligned} 357 \div 16 &= 22\text{,remainder } \mathbf{5}, \\ 22 \div 16 &= 1\text{,remainder } \mathbf{5}, \\ 1 \div 16 &= 0\text{,remainder } \mathbf{1} \end{aligned}$$

Combining it together, the hexadecimal representation of 357 is $(155)_{16}$.

<u>Correct Solution:</u>

$$\begin{aligned} 357 \div 16 &= 22\text{,remainder } \mathbf{5}, \\ 22 \div 16 &= 1\text{,remainder } \mathbf{6}, \\ 1 \div 16 &= 0\text{,remainder } \mathbf{1} \end{aligned}$$

Combining it together, the hexadecimal representation of 357 is $(165)_{16}$.

Question 3

a.

$$0.375 \times 2 = 0.75 + \mathbf{0} \quad (1)$$
$$0.75 \times 2 = 0.5 + \mathbf{1} \quad (2)$$
$$0.5 \times 2 = 0 + \mathbf{1} \quad (3)$$

Combining the above, the binary representation of 0.375 is $(0.011)_2$.

Notes:

- **Converting fractional decimal to binary**

$$0.8125 \times 2 = 0.625 + \mathbf{1} \quad (4)$$
$$0.625 \times 2 = 0.25 + \mathbf{1} \quad (5)$$
$$0.25 \times 2 = 0.5 + \mathbf{0} \quad (6)$$
$$0.5 \times 2 = 0 + \mathbf{1} \quad (7)$$

Binaries read *top to bottom*

b.

$$0.1 \times 2 = 0.2 + \mathbf{0} \tag{1}$$
$$0.2 \times 2 = 0.4 + \mathbf{0} \tag{2}$$
$$0.4 \times 2 = 0.8 + \mathbf{0} \tag{3}$$
$$0.8 \times 2 = 0.6 + \mathbf{1} \tag{4}$$
$$0.6 \times 2 = 0.2 + \mathbf{1} \tag{5}$$
$$0.2 \times 2 = 0.4 + \mathbf{0} \tag{6}$$
$$0.4 \times 2 = 0.8 + \mathbf{0} \tag{7}$$
$$0.8 \times 2 = 0.6 + \mathbf{1} \tag{8}$$

Combining the above, the binary representation of 0.1 is $(0.0\overline{0011})_2$.

Question 4

a.

$$\sum_{i=0}^{\infty} \left(\frac{1}{2}\right) = \frac{\frac{1}{2}}{1 - \frac{1}{2}} \tag{1}$$
$$= \frac{\frac{1}{2}}{\frac{1}{2}} \tag{2}$$
$$= 1 \tag{3}$$

b. Since 1^{st} 1 repeats every 4 decimal places, and 2^{nd} 1 repeats every 5 decimal places, we have

$$
\begin{aligned}
(0.0\overline{0011})_2 &= \sum_{i=1}^{\infty}\left(\frac{1}{2}\right)^{4i} + \sum_{i=1}^{\infty}\left(\frac{1}{2}\right)^{4i+1} && (1)\\
&= \sum_{i=1}^{\infty}\frac{1}{16} + \frac{1}{2}\cdot\sum_{i=1}^{\infty}\frac{1}{16} && (2)\\
&= \frac{1}{15} + \frac{1}{30} && (3)\\
&= \frac{3}{30} && (4)\\
&= \frac{1}{10} && (5)
\end{aligned}
$$