CSC343 Worksheet 12 Solution

July 1, 2020

1.
- Keys
  - {id of molecule}
  - {x position, y position, z position}
- Functional Dependencies
  1. id of molecule → x position, y position, z position, x velocity, y velocity, z velocity
  2. x position, y position, z position → id of molecule, x velocity, y velocity, z velocity

Notes:

- Function Dependencies
  - *Functional Dependency* is a relationship between two attributes typically between the key and other non-key attributes within a table.

    Example:

    SIN → Name, Address, Birthdate

    Example 2:

    ISBN → Title
- Key of Relations
  - One or more attributes $\{A_1, A_2, ..., A_n\}$ is a key for a relation R if
    1. Those attributes functionally determine all other attributes of the relation
    2. No proper subset of $\{A_1, A_2, ...A_n\}$ functionally determines all other attributes of R

    Example:

    Given relation

    $R = $ Movies1(title, year, length, genre, studioName, starName)
    i. {title, year, starName } form a key for the relation **Movies1**

    ii. { year, starName } is not a key. Same star can be in multiple movies per year

  – Superkeys

     * Means a a set of attributes that contains a key

     * Don't need to be minimal

     Example:

     Given relation

     $R = $ Movies1(title, year, length, genre, studioName, starName)

      · { title, year, starName } is a key and superkey

      · { title, year, starName, title, year, length} is a superkey

References:

1) OpenTextBC, Chapter 11 Functional Dependencies, link

2. a) 1. $AB \rightarrow C$

    2. $AB \rightarrow D$

    3. $C \rightarrow A$

    4. $C \rightarrow B$

    5. $D \rightarrow B$

    6. $D \rightarrow C$

    7. $C \rightarrow D$

    8. $D \rightarrow A$

Second Attempt:

$\{A, B\}^+ = \{A, B, C, D\}$, so the following non-trivial FDs follows: $AB \rightarrow C$ and $AB \rightarrow D$.

$\{C\}^+ = \{D, A\}$, so the following non-trivial FDs follows $C \rightarrow D$ and $C \rightarrow A$.

$\{D\}^+ = \{A\}$, so the following non-trivial FDs follows: $D \rightarrow A$.

Notes:

- The Splitting / Combining Rule
  - Combining Rule
    * $A_1, A_2, \cdots, A_n \rightarrow B_i$ for $i = 1, 2, ..., m$
      to
      $A_1, A_2, \cdots A_n \rightarrow B_1, B_2, \cdots B_m$

Example:

Given

title year $\rightarrow$ length
title year $\rightarrow$ genre
title year $\rightarrow$ studioName

it's combined form is

title year $\rightarrow$ length genre studioName

- Splitting Rule
  *
  * $A_1, A_2, \cdots A_n \rightarrow B_1, B_2, \cdots B_m$
    to
    $A_1, A_2, \cdots, A_n \rightarrow B_i$ for $i = 1, 2, ..., m$

Example:

Given

title year $\rightarrow$ length

It's splitted form is

title $\rightarrow$ length
year $\rightarrow$ length

- Trivial Functional Dependencies

  - A functional dependency $FD : X \rightarrow Y$ is **trivial** if Y is a subset of X

  #### Exmaple:

  title year $\rightarrow$ title

  #### Example 2:

  title $\rightarrow$ title

- Non-trivial Functional Dependencies
  - is a case where some but not all of the attributes on the R.H.S of an FD are also on L.H.S

  #### Example:

  title year $\rightarrow$ title movieLength

– Can be simplified using **tirivial-dependency rule**
  * The FD $A_1 A_2 \cdots A_n \to B_1 B_2 \cdots B_m$ is equivalent to
    $A_1 A_2 \cdots A_n \to C_1 C_2 \cdots C_k$

    where C's are all those B's that are not in A's.

the non-overlapping attribute

Figure 3.3: The trivial-dependency rule

- Computing the Clousre of Attributes
  – Closure of attribute set $\{X\}$ is denoted as $\{X\}^+$.
  – The closure means a given set of attributes A satisfying FD, are a sets of all attributes B such that $A \to B$

  Example:

  Given attributes A, B, C, D, E, F and FDs $AB \to C$, $BC \to AD$, $D \to E$ and $CF \to B$, What is the closure of $\{A, B\}$ or $\{A, B\}^+$

  1. Start with $\{A, B\}$.
  2. Split $BC \to AD$
     * We have $BC \to A$ and $BCtoD$
     * Since A is in $\{A, B\}$, this is not included
     * Since D is not in $\{A, B\}$, this IS included

     So, we have $\{A, B, D\}$

4

3. Since C in $AB \rightarrow C$ is NOT in $\{A, B, C, D\}$, C is included and we have $\{A, B, C, D\}$

4. Since A in $BC \rightarrow A$ is in $\{A, B, C, D\}$, this is skipped

5. Since E is not in $D \rightarrow E$, E is included and we have $\{A, B, C, D, E\}$ as our solution

- Why the Closure Algorithm Works
- Transitive Rule
  - Definition

    If $A_1 A_2 \cdots A_n \rightarrow B_1 B_2 \cdots B_m$ and $B_1 B_2 \cdots B_m \rightarrow C_1 C_2 \cdots C_k$ hold in relation R, $A_1 A_2 \cdots A_n \rightarrow C_1 C_2 \cdots C_k$ also holds in R.

    Example:

    Given

    title year $\rightarrow$ studioName
    studioName $\rightarrow$ studioAddr

    Transitive rule says the above is equal to the following

    title year $\rightarrow$ studioAddr

- Inference Rules
  - Is allso called **Armstrong's Axioms**
  - Has 3 axioms
    1. *Reflexivity*
       * If $\{B_1, B_2, ..., B_n\} \subseteq \{A_1, A_2, ..., A_n\}$ then $A_1 A_2 \cdots A_n \rightarrow B_1 B_2 \cdots B_m$
       * also called **trivial FDs**
    2. *Augmentation*
       * If $A_1 A_2 \cdots A_n \rightarrow B_1 B_2 \cdots B_m$
         then $A_1 A_2 \cdots A_n C_1 C_2 \cdots C_k \rightarrow B_1 B_2 \cdots B_m C_1 C_2 \cdots C_k$
       * $C_1 C_2 \cdots C_k$ are any set of attributes
    3. *Transitivity*
       * If $A_1 A_2 \cdots A_n \rightarrow B_1 B_2 \cdots B_m$ and $B_1 B_2 \cdots B_m \rightarrow C_1 C_2 \cdots C_k$
         then $A_1 A - 2 \cdots A_n \rightarrow C_1 C_2 \cdots C_k$

b) A, B is the only key of R.

Notes:

- Key of Attributes
  - **Definition:** A set of attributes $\{A_1, A_2, \cdots, A_n\}$ is a key for a relation R if
    1. Those attributes functionally determine all other attributes

        2. No proper subset of $\{A_1, A_2, ..., A_n\}$ functionally determines all other attributes of R.

c) The superkeys that are not keys are: $\{A, B, C\}$, $\{A, B, D\}$, $\{A, B, C, D\}$

3.   i) a) $\{A\}^+ = \{A, B, C, D\}$, so we have $A \to A$, $A \to B$, $A \to C$, $A \to D$

        $\{B\}^+ = \{C, D\}$, so we have $B \to C$ and $B \to D$

    b) $\{A\}$ is the key of S.

    c) The super keys that are not keys are:

      $\{A, B\}$, $\{A, C\}$, $\{A, D\}$, $\{A, B, C\}$, $\{A, B, D\}$, $\{A, B, C, D\}$

  ii) a) $\{A\}^+ = \{A\}$, so this FD is trivial.

      $\{B\}^+ = \{B\}$, so this FD is trivial.

      $\{C\}^+ = \{C\}$, so this FD is trivial.

      $\{D\}^+ = \{D\}$, so this FD is trivial.

      $\{A, B\}^+ = \{A, B, C, D\}$, so we have $AB \to A$, $AB \to B$, $AB \to C$, $AB \to D$

      $\{A, C\}^+ = \{A, C\}$, so we have $AC \to A$, $AC \to C$

      $\{A, D\}^+ = \{A, D, B\}$, so we have $AD \to A$, $AD \to D$, $AD \to B$

      $\{B, C\}^+ = \{B, C, D, A\}$, so we have $BC \to A$, $BC \to B$, $BC \to C$, $BC \to D$

      $\{D, C\}^+ = \{D, C, A, B\}$, so we have $DC \to D$, $DC \to C$, $DC \to A$, $DC \to B$

      $\{A, B, C\}^+ = \{A, B, C, D\}$, so we have $ABC \to A$, $ABC \to B$, $ABC \to C$, $ABC \to D$

      $\{B, C, D\}^+ = \{B, C, D, A\}$, so we have $BCD \to A$, $BCD \to B$, $BCD \to C$, $BCD \to D$

      $\{C, D, A\}^+ = \{C, D, A, B\}$, so we have $CDA \to A$, $CDA \to B$, $CDA \to C$, $CDA \to D$

      $\{D, A, B\}^+ = \{D, A, B, C\}$, so we have $DAB \to A$, $DAB \to B$, $DAB \to C$, $DAB \to D$

      $\{D, A, B\}^+ = \{D, A, B, C\}$, so we have $DAB \to A$, $DAB \to B$, $DAB \to C$, $DAB \to D$

      $\{A, B, C, D\}^+ = \{A, B, C, D\}$, so this FD is trivial.

b) $\{A, B\}$, $\{A, C\}$, $\{B, C\}$, $\{D, C\}$ are the keys of T.

c) The super keys that are not keys are:

$\{A, B, C\}$, $\{A, B, D\}$, $\{B, C, D\}$, $\{A, D, C\}$, $\{A, B, D\}$, $\{A, B, C, D\}$

iii) a) $\{A\}^+ = \{A, B, C, D\}$, so we have $A \rightarrow C$, $A \rightarrow D$

$\{B\}^+ = \{A, B, C, D\}$, so we have $B \rightarrow A$, $B \rightarrow D$

$\{C\}^+ = \{A, B, C, D\}$, so we have $C \rightarrow A$, $C \rightarrow B$

$\{D\}^+ = \{A, B, C, D\}$, so we have $D \rightarrow B$, $D \rightarrow C$

$\{A, B\}^+ = \{A, B, C, D\}$, so we have $AB \rightarrow C$, $AB \rightarrow D$

$\{B, C\}^+ = \{A, B, C, D\}$, so we have $BC \rightarrow A$, $BC \rightarrow D$

$\{B, D\}^+ = \{A, B, C, D\}$, so we have $BD \rightarrow A$, $BD \rightarrow C$

$\{C, D\}^+ = \{A, B, C, D\}$, so we have $CD \rightarrow A$, $CD \rightarrow B$

$\{C, D\}^+ = \{A, B, C, D\}$, so we have $CD \rightarrow A$, $CD \rightarrow B$

$\{A, B, C\}^+ = \{A, B, C, D\}$, so we have $ABC \rightarrow D$

$\{B, C, D\}^+ = \{A, B, C, D\}$, so we have $BCD \rightarrow A$

$\{C, D, A\}^+ = \{A, B, C, D\}$, so we have $CDA \rightarrow B$

$\{D, A, B\}^+ = \{A, B, C, D\}$, so we have $DAB \rightarrow C$

iv)