CSC236 Worksheet 1 Review

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Question 4

- **Rough Works:**

For convenience, define $H(n) : 4^{\geq} 5n^4 + 6$.

I will prove that $\forall n \in \mathbb{N}, n \geq 7 \Rightarrow 4^n \geq 5n^4 + 6$.

1. Base Case ($n = 7$)

   Let $n = 7$.

   Then,

$$\begin{aligned} 4^n &= 16384 && (1)\\ &\geq 12011 && (2)\\ &= 5(7)^4 + 6 && (3)\\ &= 5n^4 + 6 && (4) \end{aligned}$$

   So, $H(n)$ is verified.

2. Inductive Step

Let $n \in \mathbb{N}$. Assume $H(n)$.

I need to show $H(n+1)$ follows. That is $4^{n+1} \geq (5n^4 + 6)$.

Starting from 4^{n+1}, we have

$$
\begin{aligned}
4^{n+1} &= 4^n + 4^n + 4^n + 4^n && (5)\\
&\geq (5n^4 + 6) + (5n^4 + 6) + (5n^4 + 6) + (5n^4 + 6 && \text{[By I.H]} \quad (6)\\
&= 5(n^4 + n^4 + n^4 + n^4) + 24 && (7)\\
&= 5(n^4 + n \cdot n^3 + n^2 \cdot n^2 + n^3 \cdot n) + 24 && (8)\\
&> 5(n^4 + 7 \cdot n^3 + 7^2 \cdot n^2 + 7^3 \cdot n) + 24 && \text{[Since } n > 7\text{]} \quad (9)\\
&> 5(n^4 + 4 \cdot n^3 + 6 \cdot n^2 + 4 \cdot n) + 6 && (10)\\
&> 5(n+1)^4 + 6 && (11)
\end{aligned}
$$