Worksheet 3 Review

March 20, 2020

Question 1

a. $Correct(my\_prog) \wedge Python(my\_prog)$

b. $\exists x \in P,\ \neg Correct(x) \Rightarrow Python(x)$

c. $\forall x \in P,\ Python(x) \Rightarrow \neg Correct(x)$

d. $\forall x \in P,\ \neg Correct(x) \Rightarrow Python(x)$

e. There is a program that is written in Python and is correct.

f. Every program is not written in Python and is correct.

g. There is a program that is correct but is not written in Python.

h. All running programs not written in Python are correct, and all correctly running programs are not written in Python.

Question 2

a. All programs written in Python is correct, or all program written in python is not correct.

b. $(\exists x \in P, Python(x) \Rightarrow Correct(x)) \Rightarrow (\forall y \in P, Python(y) \Rightarrow Correct(y))$

Correct Solution:

$(\exists x \in P,\ Python(x) \wedge Correct(x)) \Rightarrow (\forall y \in P,\ Python(y) \Rightarrow Correct(y))$

c. x_1 and x_2 in first statement could have different values. Where as the x in second statement has the same value.

The first statement is true, but the second is false. Assume x is divisible by 7. Then, 165 must be divisible by 7, but this is false.

Question 3

a. $\forall n \in \mathbb{N},\ Odd(n) \Rightarrow \exists k \in \mathbb{Z}, n = 2k + 1$

Correct Solution:

$Odd(n) : \exists k \in \mathbb{Z}, n = 2k + 1$, where $n \in \mathbb{Z}$

b. $\forall n, m \in \mathbb{N},\ Odd(n) \wedge Odd(m) \Rightarrow Odd(nm)$

c. $\forall n, m \in \mathbb{N},\ \exists k, l \in \mathbb{Z},\ n = 2k - 1 \wedge m = 2l - 1 \Rightarrow \exists o \in \mathbb{Z},\ nm = 2o - 1$

d. $\forall n, m \in \mathbb{N},\ \exists o \in \mathbb{Z},\ nm = 2o - 1 \Rightarrow \exists k, l \in \mathbb{Z},\ n = 2k - 1 \wedge m = 2k - 1$

Question 4

a. $((\neg a \vee \neg b) \wedge c) \vee ((a \wedge b) \wedge \neg c)$

b. $\exists x, y \in S,\ \forall z \in S,\ \neg P(x, y) \vee \neg Q(x, z)$

c. $(\exists x \in S, P(x)) \wedge (\forall y \in S, \neg Q(y))$

Question 5

- Define

  $P(x)$: x is an odd number, where $x \in \mathbb{Z}$

  $Q(x) : x$ is an even number, where $x \in \mathbb{Z}$

  $U \in \mathbb{R}^{\geq 1}$

  Then the statement $[(\exists x \in U,\ P(x)) \wedge (\exists y \in U,\ Q(y))] \Rightarrow [\exists z \in U,\ P(z) \wedge Q(z)]$ is false.