Midterm 1 Version 2 Solution

March 19, 2020

Question 1

a. Since

$S_1 = \{1, 2, 3, 5, 7, 11, 13, 17, 19, 23, 29\}$, and $S_2 = \{1, 2, 3, 5, 6, 10, 15, 30\}$,

$S_1 \cap S_2 = \{1, 2, 3, 5\}$

b. See the table below

p	q	r	$\neg p$	$\neg p \Leftrightarrow q$	$(\neg p \Leftrightarrow q) \Rightarrow r$
T	T	T	F	F	T
T	T	F	F	F	T
T	F	T	F	T	T
F	T	T	T	T	F
T	F	F	F	T	F
F	F	T	T	F	T
F	F	F	T	F	T

Correct Solution:

p	q	r	$\neg p$	$\neg p \Leftrightarrow q$	$(\neg p \Leftrightarrow q) \Rightarrow r$
T	T	T	F	F	T
T	T	F	F	F	T
T	F	T	F	T	T
*F	*T	*T	*T	*T	*T
T	F	F	F	T	F
*F	*T	*F	*T	*T	*F
F	F	T	T	F	T
F	F	F	T	F	T

$*$ = Incorrect/missing solution

c. Let $x \in \mathbb{N}$. Assume $P(x)$.

We will prove that there is a natural number y such that the predicate $Q(x, y)$ is true.

Correct Solution:

Let $x \in \mathbb{N}$, and $y =$ ________. Assume $P(x)$.

We will prove that the predicate $Q(x, y)$ is true.

Question 2

a. $\forall x \in P,\ Cat(x) \wedge Loves(x, x)$

Correct Solution:

$\forall x \in P,\ Cat(x) \Rightarrow Loves(x, x)$

b. $\forall x \in P,\ \exists y \in P,\ Cat(x) \wedge Cute(y) \wedge Loves(x, y)$

Correct Solution:

$\forall x \in P,\ Cat(x) \wedge Cute(x) \Rightarrow (\forall y \in P, Cat(y) \Rightarrow Cute(y))$

c. $\exists x \in P,\ Cat(x) \wedge Cute(x) \Rightarrow \forall y \in P, Cat(y) \wedge Cute(y)$

d. $\forall p_1, p_2 \in P, p_1 \neq p_2 \wedge Loves(p_1, p_2) \wedge Loves(p_2, p_1) \Rightarrow (Cat(p_1) \wedge \neg Cat(p_2)) \vee (\neg Cat(p_1) \wedge Cat(p_2))$

Question 3

a. $\exists n \in \mathbb{N},\ n > 1 \Rightarrow \forall x \in \mathbb{R},\ \lfloor nx \rfloor = n\lfloor x \rfloor$

Correct Solution:

$\exists n \in \mathbb{N},\ n > 1 \wedge (\forall x \in \mathbb{R}^+,\ \lfloor nx \rfloor = n\lfloor x \rfloor)$

b. **Negation:** $\forall n \in \mathbb{N},\ n > 1 \land (\exists x \in \mathbb{R},\ \lfloor nx \rfloor \neq n\lfloor x \rfloor)$

Let $n = 2,\ x = 0.5$.

Then,

$$\begin{aligned} \lfloor nx \rfloor &= \lfloor 2(0.5) \rfloor && (1) \\ &= 1 && (2) \end{aligned}$$

And,

$$\begin{aligned} n\lfloor x \rfloor &= 2\lfloor 0.5 \rfloor && (3) \\ &= 2(0) && (4) \\ &= 0 && (5) \end{aligned}$$

Since $\lfloor nx \rfloor \neq n\lfloor x \rfloor$,the predicate logic is false.

Correction, First Case ($n \leq 1$):

Assume $n \leq 1$. Then, the first case of the negation is true.

Correction, Second Case ($\exists x \in \mathbb{R}, n\lfloor x \rfloor \neq \lfloor nx \rfloor$):

Let $n \in \mathbb{N},\ x = \frac{1}{n}$. Assume $n > 1$.

Then,

$$\begin{aligned} n\lfloor x \rfloor &= n\lfloor \frac{1}{n} \rfloor && (1) \\ &= 0 && (2) \end{aligned}$$

And,

$$
\begin{aligned}
\lfloor nx \rfloor &= \lfloor \frac{1}{n} \rfloor && (3)\\
&= \lfloor 1 \rfloor && (4)\\
&= 1 && (5)
\end{aligned}
$$

Since $n\lfloor x \rfloor \neq \lfloor nx \rfloor$, the second case of negation is also false.

Then, it follows from the negation that the statement is false.

Question 4

- Let $a, b \in \mathbb{N}$. Assume $b \mid a$ and $b \mid (a + 2)$.

  Then, $\exists k, l \in \mathbb{Z}$,

  $$
  \begin{aligned}
  a &= kb && (1)\\
  (a + 2) &= lb && (2)
  \end{aligned}
  $$

  by the definition of divisibility.

  Then,

  $$2 = (l - k)b \qquad (3)$$

  Since $(l - k) \in \mathbb{Z}$ and $b \in \mathbb{N}$, the only possible combinations that make up 2 are 1 and 2.

  Then it follows from above that $b = 1$ or $b = 2$.