

Worksheet 1 Solution

March 9, 2020

Question 1

a) $A = \{2, 5\}$

$$A^c = \{1, 3, 4, 6\}$$

b) $A^c = U \setminus A$

c) $A^c \cap B^c = \{x \mid x \in U, x \leq 0 \text{ and } x \geq 4\}$

$$A^c \cap B^c = \{x \mid x \in U, x < 1 \text{ and } x > 2\}$$

$$(A \cap B)^c = \{x \mid x \in U, x < 1 \text{ and } x > 2\}$$

$$(A \cup B)^c = \{x \mid x \in U, x \leq 0 \text{ and } x \geq 4\}$$

Question 2

a) $T_0 \rightarrow 0, 3, 6$

$$T_1 \rightarrow 1, 4, 7$$

$$T_2 \rightarrow 2, 5, 8$$

$$T_3 \rightarrow 12, 18, 24$$

b) $\mathbb{Z}^+ = \{T_0, T_1, T_2\}$

T_3 not included. A partition of a set must not have any common elements.

Question 3

a) 000, 110,
001, 010,
011, 100,
101, 111

b) $S_1 = \{aa, bb, cc, ab, ca, ba, ac, bc, cb\}$
 $S_2 = \{a, b, c, aa, bb, cc, ab, ca, aaa, aba, aca, bab, bbb, bcb, cac, cbc, ccc \dots\}$

$$S_1 \cap S_2 = \{aa, bb, cc\}$$

$$S_1 \setminus S_2 = \{ab, ca, ba, ac, bc, cb\}$$

c) $S_1 = (S_1 \cap S_2) \cup (S_1 \setminus S_2)$

Question 4

a)

x	$\lfloor x \rfloor$	$\lceil x \rceil$
$\frac{25}{4}$	6	7
0.999	0	1
-2.01	-3	-2

b) Domain: \mathbb{R}
Codomain: \mathbb{Z}

c) False. Consider the following example.

$$1 = \lfloor 0.75 + 0.25 \rfloor$$

$$0 = \lfloor 0.75 \rfloor + \lfloor 0.25 \rfloor$$

Question 5

$$\text{a) } \sum_{k=1}^3 (k+1) = (1+1) + (2+1) + (3+1)$$

$$\sum_{m=0}^1 \frac{1}{2^m} = \frac{1}{2^0} + \frac{1}{2^1}$$

$$\sum_{k=0}^2 (k^2 + 3) = (1^2 + 3) + (2^2 + 3)$$

$$\sum_{j=0}^4 \frac{(-1)^j \cdot j}{j+1} = \frac{(-1)^0 \cdot 0}{0+1} + \frac{(-1)^1 \cdot 1}{1+1} + \frac{(-1)^2 \cdot 2}{2+1} + \frac{(-1)^3 \cdot 3}{3+1} + \frac{(-1)^4 \cdot 4}{4+1}$$

$$\sum_{k=1}^5 (2k) = (2 \cdot 1) + (2 \cdot 2) + (2 \cdot 3) + (2 \cdot 4) + (2 \cdot 5)$$

$$\prod_{i=2}^4 \frac{i \cdot (i+2)}{(i-1) \cdot (i+1)} = \frac{1 \cdot (1+2)}{(1-1) \cdot (1+1)} \times \frac{2 \cdot (2+2)}{(2-1) \times (2+1)} \times \frac{3 \cdot (3+2)}{(3-1) \times (3+1)} \times \frac{4 \cdot (4+2)}{(4-1) \cdot (4+1)}$$

$$\text{b) } 3 + 6 + 12 + 24 + 48 + 96 = \sum_{i=0}^6 (3 \cdot 2^i)$$

$$0 + 1 - 2 + 3 - 4 + 5 = \sum_{i=0}^6 (-1)^i (i+1)$$

$$\frac{(1 \cdot 2)}{(3 \cdot 4)} + \frac{(2 \cdot 3)}{(4 \cdot 5)} + \frac{(3 \cdot 4)}{(5 \cdot 6)} = \prod_{i=1}^3 \frac{i \cdot (i+1)}{(i+2) \cdot (i+3)}$$

$$\frac{1}{3} + \frac{4}{9} + \frac{9}{27} + \frac{16}{81} + \frac{25}{243} + \frac{36}{729} = \sum_{i=1}^6 \frac{i^2}{3^i}$$

$$\frac{1}{(1+1)} \times \frac{2}{(2+1)} \times \frac{3}{(3+1)} \cdots \frac{k}{(k+1)} = \prod_{i=1}^k \frac{i}{i+1}$$