Midterm 1 Version 1 Review

March 29, 2020

Question 1

a. Because we know

$S_1 = \{aa, bb, cc, aab, aac, aaa, bba, bbb, bbc, cca, ccb, ccc, aaaa, \dots\}$ and S_2 is a set of all strings over U with length 3, we can conclude

$$S_1 \cap S_2 = \{aaa, aab, aac, bba, bbb, bbc, cca, ccb, ccc\}$$

b. See table below

p	q	r	$\neg r$	$p \vee q$	$p \vee q \Rightarrow \neg r$
T	T	T	F	T	F
T	T	F	T	T	T
T	F	T	F	T	F
F	T	T	F	T	F
T	F	F	T	T	T
F	T	F	T	T	T
F	F	T	F	F	T
F	F	F	T	F	T

c. Let $x \in \mathbb{N}$, and $y =$ ________.

We will prove that predicate $P(x, y)$ is true, or predicate $Q(x, y)$ is true.

Correct Solution:

Let $x =$ _______, and $y \in \mathbb{N}$.

We will prove that **both predicates $P(x, y)$ and $Q(x, y)$ are false.**

Notes:

- How can I proceed a proof when there is $\lor$ on R.H.S of the statement? What's the general structure of proof given this symbol?

Question 2

a. $\exists x \in P,\ Student(x) \land Attends(x)$

b. $\forall x \in P,\ \exists y \in P, Student(y) \land Attends(y) \Rightarrow Loves(x, y)$

Correct Solution:

$\forall x \in P,\ \exists y \in P, Student(y) \land Attends(y) \land Loves(x, y)$

Notes:

- When should $\Rightarrow$ be used, and when should $\land$ be used?

c. $\forall x \in P,\ Student(x) \land Attends(x) \Rightarrow Loves(x, x)$

d. $\forall x_1, x_2 \in P,\ x_1 \neq x_2 \Rightarrow Loves(x_1, x_2) \land Loves(x_2, x_1) \Rightarrow \neg Attends(x_1) \lor \neg Attends(x_2)$

Correct Solution:

$\forall x_1, x_2 \in P, x_1 \neq x_2 \land Loves(x_1, x_2) \land Loves(x_2, x_1) \Rightarrow \neg Attends(x_1) \lor \neg Attends(x_2)$

Question 3

a. $\forall a, b, c \in \mathbb{Z}, \exists l, m, n \in \mathbb{Z}, b = la \wedge c = mb \Rightarrow c = na$

b. Let $a, b, c \in \mathbb{Z}$. Assume there is some $l, m, n \in \mathbb{Z}$, $b = la$ and $c = mb$.

We want to show there is some $n \in \mathbb{Z}$, $c = na$.

Because we know $c = mb$ and $b = la$, we can conclude that

$$\begin{aligned} c &= mb && (1) \\ &= (ml)a && (2) \end{aligned}$$

Since $ml \in \mathbb{Z}$, we can choose $n = ml$.

Then,

$$c = na \qquad (3)$$

Question 4

- Let $x, y \in \mathbb{R}$.

  We want to show $\lfloor x + y \rfloor \geq \lfloor x \rfloor + \lfloor y \rfloor$.

  Because we know $x \in \mathbb{R}$, by using fact 1 on x, we can conclude there is $\epsilon \in \mathbb{R}$ and $0 \leq \epsilon < 1$, $x = \lfloor x \rfloor + \epsilon$.

  Then,

$$\lfloor x + y \rfloor \geq \big\lfloor \lfloor x \rfloor + \epsilon + y \big\rfloor \qquad (1)$$

Then, because we know $\lfloor x \rfloor \in \mathbb{Z}$ and $y \in \mathbb{R}$, by using fact 2, we can conclude

$$\lfloor x + y \rfloor \geq \lfloor x \rfloor + \lfloor \epsilon + y \rfloor \tag{2}$$

Then, since $\lfloor \epsilon + y \rfloor \geq \lfloor y \rfloor$,

$$\lfloor x + y \rfloor \geq \lfloor x \rfloor + \lfloor y \rfloor \tag{3}$$

<u>Correct Solution:</u>

Let $x, y \in \mathbb{R}$.

We want to show $\lfloor x + y \rfloor \geq \lfloor x \rfloor + \lfloor y \rfloor$.

Because we know $x \in \mathbb{R}$, by using fact 1 on x, we can conclude there is **some** $\epsilon \in \mathbb{R}$ and $0 \leq \epsilon < 1$, $x = \lfloor x \rfloor + \epsilon$.

Then,

$$\lfloor x + y \rfloor \geq \big\lfloor \lfloor x \rfloor + \epsilon + y \big\rfloor \tag{1}$$

Then, because we know $\lfloor x \rfloor \in \mathbb{Z}$ and $y \in \mathbb{R}$, by using fact 2, we can conclude

$$\lfloor x + y \rfloor \geq \lfloor x \rfloor + \lfloor \epsilon + y \rfloor \tag{2}$$

Then, since $\lfloor \epsilon + y \rfloor \geq \lfloor y \rfloor$,

$$\lfloor x + y \rfloor \geq \lfloor x \rfloor + \lfloor y \rfloor \tag{3}$$