# CSC373 Worksheet 6 Solution

August 13, 2020

1. 1. Multiply objective function by - 1

   Maximize

   $$-2x_1 - 7x_2 - x_3$$

   Subject to

   $$\begin{aligned} x_1 - x_3 &= 7 \\ 3x_1 + x_2 &\geq 7 \\ x_2 &\geq 0 \\ x_3 &\leq 0 \end{aligned}$$

   2. Replace non-nonnegative constraints $x_1$

   Maximize

   $$-2x_1' + 2x_1'' - 7x_2 - x_3$$

   Subject to

   $$\begin{aligned} x_1' - x_1'' - x_3 &= 7 \\ 3x_1' - 3x_1'' + x_2 &\geq 7 \\ x_1', x_1'', x_2 &\geq 0 \\ x_3 &\leq 0 \end{aligned}$$

   3. Replace non-nonnegative constraints $x_3$

Maximize

$$-2x_1' + 2x_1'' - 7x_2 - x_3' + x_3''$$

Subject to

$$\begin{aligned} x_1' - x_1'' - x_3' + x_3'' &= 7 \\ 3x_1' - 3x_1'' + x_2 &\geq 7 \\ x_1', x_1'', x_2, x_3', x_3'' &\geq 0 \end{aligned}$$

4. Replace equality constraints with $\geq$ and $\leq$

Maximize

$$-2x_1' + 2x_1'' - 7x_2 - x_3' + x_3''$$

Subject to

$$\begin{aligned} x_1' - x_1'' - x_3' + x_3'' &\leq 7 \\ x_1' - x_1'' - x_3' + x_3'' &\geq 7 \\ 3x_1' - 3x_1'' + x_2 &\geq 7 \\ x_1', x_1'', x_2, x_3', x_3'' &\geq 0 \end{aligned}$$

5. Correct greater-than-or-equal-to inequality constraints

Maximize

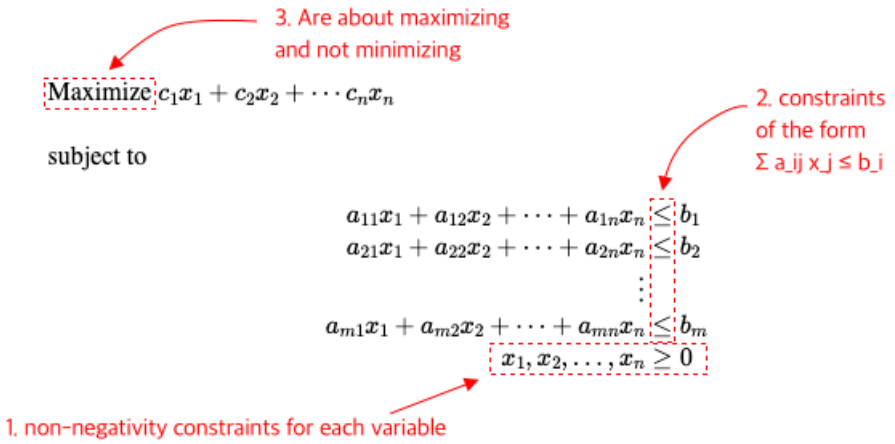
$$-2x_1' + 2x_1'' - 7x_2 - x_3' + x_3''$$

Subject to

$$
\begin{aligned}
x_1' - x_1'' - x_3' + x_3'' &\leq 7 \\
-x_1' + x_1'' + x_3' - x_3'' &\leq -7 \\
-3x_1' + 3x_1'' - x_2 &\leq 7 \\
x_1', x_1'', x_2, x_3', x_3'' &\geq 0
\end{aligned}
$$

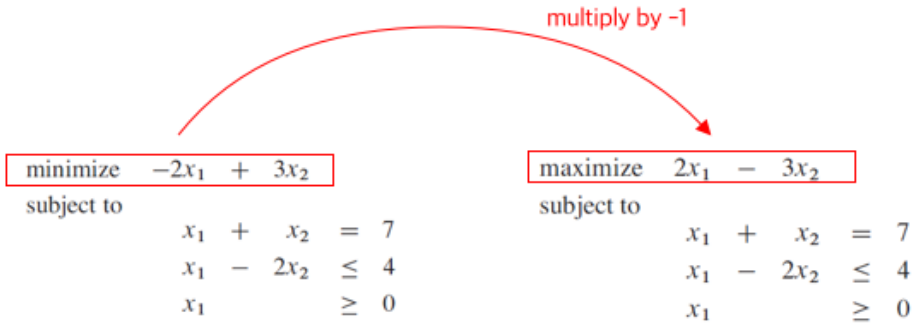
**Notes:**

- **Linear Programming**
  - Is a method to achieve the best outcome (such as maximum profit or lowest cost) in a mathematical model whose requirements are represented by linear relationships. [1]
  - Is named to make it sound cool for government funding
    - Like dynamic programming
  - Applications
    - Microeconomics (maximize profits, minimize costs)
    - Company management
- **Standard Form**
  - Is a form of linear programming
  - Are about maximizing, not minimizing [2]
  - All have a positivity constraint for each variable [2]
  - All other constraints are all of the form "linear combination of variables $\leq$ constant". [2]

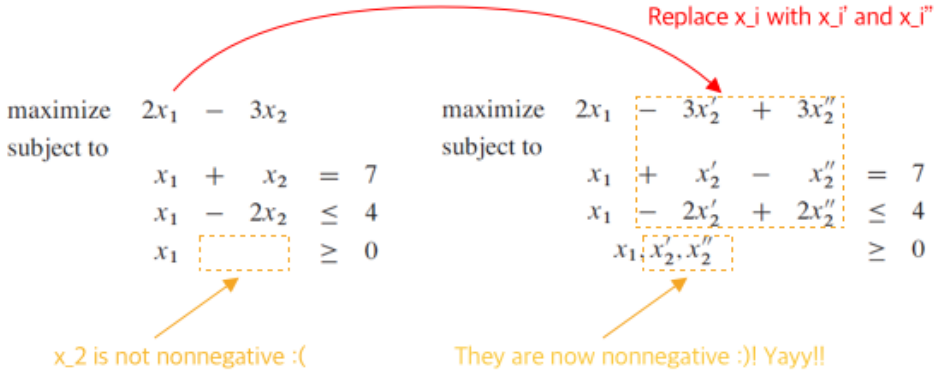



- **Converting Linear Programming to Standard Form**

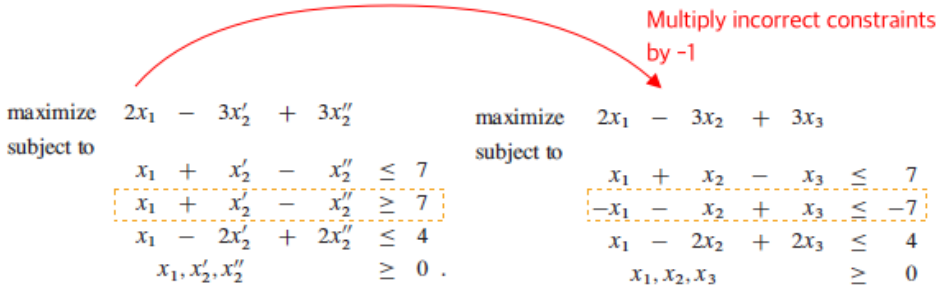
1) The objective function might be a minimization rather than a maximization
   - Negate coefficients of the objective function



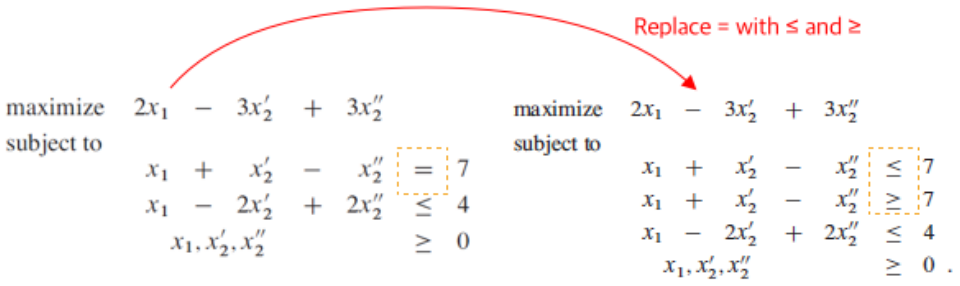

2) There might be variables without nonnegativity constraints
   - Replace each non-nonnegative variable $x_i$ with $x_i'$ and $x_i''$
   - Modify linear program




3) There might be **equality constraints**, which have an equal sign rather than a less-than-or-equal-to sign
   - Replace equality constraint $f(x_1, x_2, ..., x_n) = b$ with $f(x_1, x_2, ..., x_n) \leq b$ and $f(x_1, x_2, ..., x_n) \geq b$




4) There might be **inequality constraints**, but instead of having a less-than-or-equal-to-sign
   - Multiply incorrect inequality constraints by -1




**References:**

1) Wikipedia, Linear Programming, link
2) Instituto de Mathematicas, Standard form for Linear Programs, link

## 2. Rough Works:

1) Start from the standard form of linear programming

Maximize

$$2x_1 - 6x_3$$

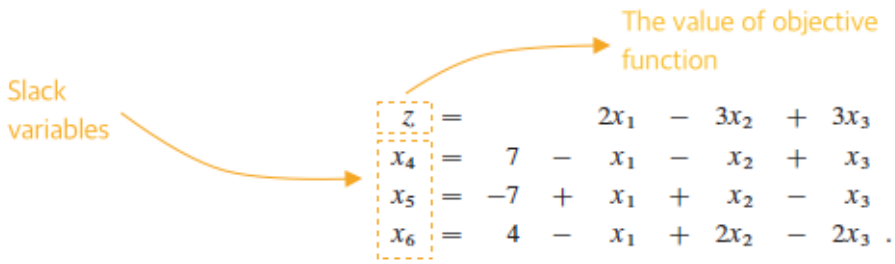
Subject to

$$\begin{aligned} x_1 + x_2 - x_3 &\leq 7 \\ -3x_1 + x_2 &\leq -7 \\ x_1 - 2x_2 - 2x_3 &\leq 0 \\ x_1, x_2, x_3 &\geq 0 \end{aligned}$$

2) Shift objective functions to right
3) Introduce slack variable $x_i$ to lhs and move expressions $\sum_{j=1}^{n} a_{ij}x_j$ to rhs
4) Change inequalities in linear programming to equality
5) Use Variable $z$ to denote objective function
6) Omit the nonnegativivty constraints

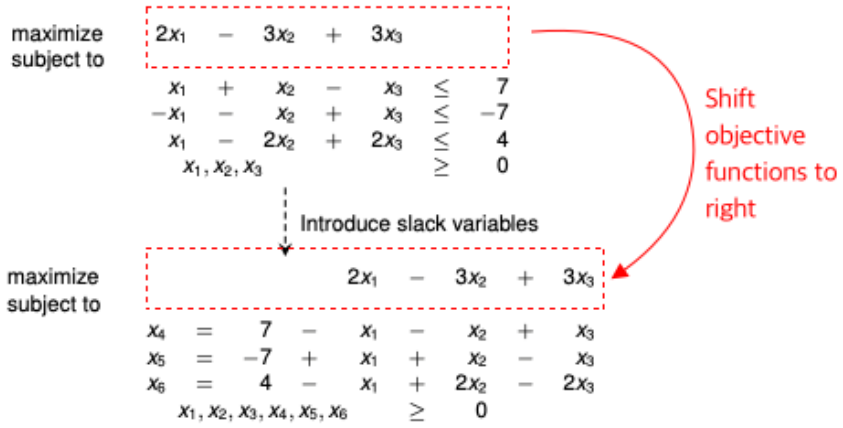
**Notes:**

- **Slack Form**
  - Is a form of linear programming
  - Is for efficient solving of liner programming problem using simplex algorithm

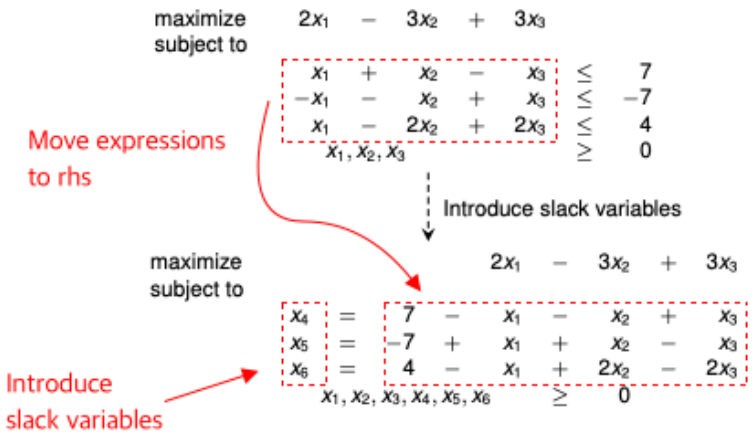



- **Converting Linear Programs into Slack Form**

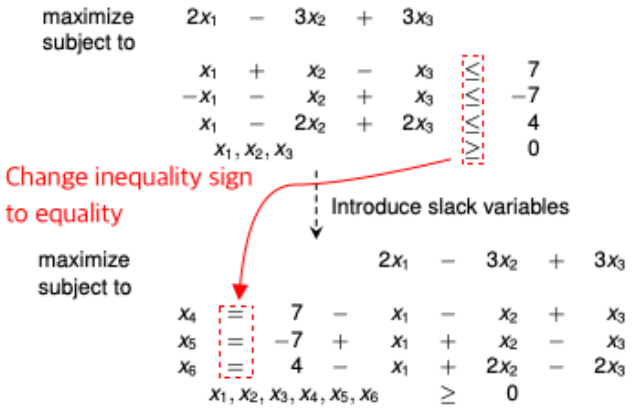
1) Start from the standard form of linear programming
2) Shift objective functions to right



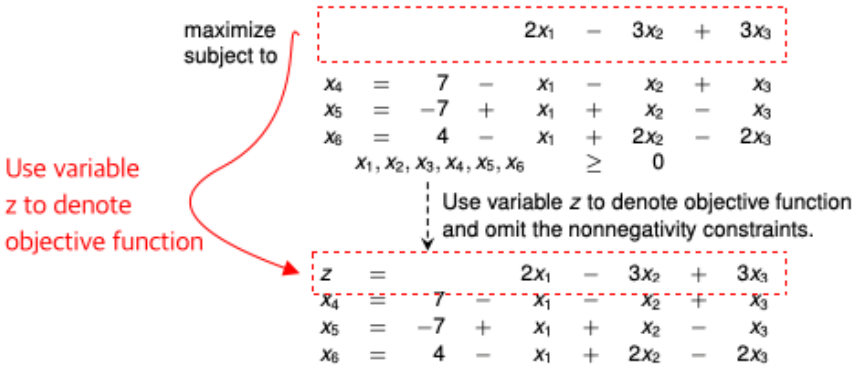

3) Introduce slack variable $x_i$ to lhs and move expressions $\sum_{j=1}^{n} a_{ij}x_j$ to rhs



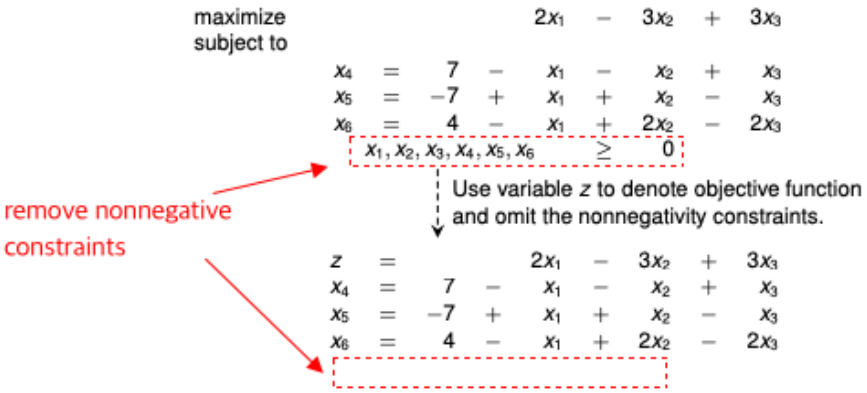

4) Change inequalities in linear programming to equality




5) Use Variable $z$ to denote objective function




6) Omit the nonnegativivty constraints




**References:**

1) Cambridge University, Linear Programming, link