

CSC236 Worksheet 1 Review

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Question 4

- *Proof.* Base Case ($n = 7$):

Let $n = 7$.

Then,

$$4^n = 16384 \tag{1}$$

$$\geq 12011 \tag{2}$$

$$= 5(7)^4 + 6 \tag{3}$$

$$= 5n^4 + 6 \tag{4}$$

So, $H(n)$ is verified.

Inductive Step:

Let $n \in \mathbb{N}$. Assume $H(n)$.

I need to show $H(n+1)$ follows. That is $4^{n+1} \geq (5n^4 + 6)$.

Starting from 4^{n+1} , we have

$$4^{n+1} = 4^n + 4^n + 4^n + 4^n \tag{5}$$

$$\geq (5n^4 + 6) + (5n^4 + 6) + (5n^4 + 6) + (5n^4 + 6) \tag{6}$$

[By I.H]

$$= 5(n^4 + n^4 + n^4 + n^4) + 24 \tag{7}$$

$$= 5(n^4 + n \cdot n^3 + n^2 \cdot n^2 + n^3 \cdot n) + 24 \tag{8}$$

$$> 5(n^4 + 7 \cdot n^3 + 7^2 \cdot n^2 + 7^3 \cdot n) + 24 \tag{9}$$

[Since $n > 7$]

$$> 5(n^4 + 4 \cdot n^3 + 6 \cdot n^2 + 4 \cdot n) + 6 \tag{10}$$

$$> 5(n+1)^4 + 6 \tag{11}$$

□

Correct Solution:

Base Case ($n = 7$):

Let $n = 7$.

Then,

$$4^n = 16384 \quad (12)$$

$$\geq 12011 \quad (13)$$

$$= 5(7)^4 + 6 \quad (14)$$

$$= 5n^4 + 6 \quad (15)$$

So, $H(n)$ is verified.

Inductive Step:

Let $n \in \mathbb{N} \setminus \{0, 1, 2, 3, 4, 5, 6\}$. Assume $H(n)$.

I need to show $H(n+1)$ follows. That is $4^{n+1} \geq (5n^4 + 6)$.

Starting from 4^{n+1} , we have

$$4^{n+1} = 4^n + 4^n + 4^n + 4^n \quad (16)$$

$$\geq (5n^4 + 6) + (5n^4 + 6) + (5n^4 + 6) + (5n^4 + 6) \quad [\text{By I.H}] \quad (17)$$

$$= 5(n^4 + n^4 + n^4 + n^4) + 24 \quad (18)$$

$$= 5(n^4 + n \cdot n^3 + n^2 \cdot n^2 + n^3 \cdot n) + 24 \quad (19)$$

$$> 5(n^4 + 7 \cdot n^3 + 7^2 \cdot n^2 + 7^3 \cdot n) + 24 \quad [\text{Since } n \geq 7] \quad (20)$$

$$> 5(n^4 + 4 \cdot n^3 + 6 \cdot n^2 + 4 \cdot n) + 6 \quad (21)$$

$$> 5(n+1)^4 + 6 \quad (22)$$