

# CSC236 Worksheet 9 Solution

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May 15, 2020

## Question 1

a. Part 1 (Finding reg. expressions for even number of 1s):

First, I will find the reg. expressions for even number of 1's.

I will do so by finding patterns in series of small examples.

Starting with  $L = \{x \in \Sigma \mid x \text{ has 0 number of 1s}\}$ , it's reg. expressions is

$$0^* \tag{1}$$

Now for  $L = \{x \in \Sigma \mid x \text{ has 2 number of 1s}\}$ , it's reg. expressions is

$$0^*10^*10^* \tag{2}$$

Now for  $L = \{x \in \Sigma \mid x \text{ has 4 number of 1s}\}$ , it's reg. expressions is

$$0^*10^*10^*10^*10^* \tag{3}$$

From above, I see a pattern that

$$(0^*10^*1)(0^*10^*1)0^* \tag{4}$$

Using the pattern, I can conclude that the regular expression for odd number of 0s is

$$1^*(01^*)^* \quad (5)$$

## Part 2 (Finding reg. expressions for odd number of 0s):

Second, I will find the reg. expressions for odd number of 0's.

### Rough Works:

1. Find regular expression for even number of 1's

First, I will find the reg. expressions for even number of 1's.

I will do so by finding patterns in series of small examples.

- Find reg. expressions for  $L = \{x \in \Sigma \mid x \text{ has 0 number of 1s}\}$

Starting with  $L = \{x \in \Sigma \mid x \text{ has 0 number of 1s}\}$ , it's reg. expressions is

$$0^* \quad (6)$$

- Find reg. expressions for  $L = \{x \in \Sigma \mid x \text{ has 2 number of 1s}\}$

Now for  $L = \{x \in \Sigma \mid x \text{ has 2 number of 1s}\}$ , it's reg. expressions is

$$0^*10^*10^* \quad (7)$$

- Find reg. expressions for  $L = \{x \in \Sigma \mid x \text{ has 4 number of 1s}\}$

Now for  $L = \{x \in \Sigma \mid x \text{ has 4 number of 1s}\}$ , it's reg. expressions is

$$0^*10^*10^*10^*10^* \quad (8)$$

- Hey I see a pattern!!

From above, I see a pattern that

$$(0^*10^*1)(0^*10^*1)0^* \quad (9)$$

- Conclude :)

Using the pattern, I can conclude that the regular expression for even number of 1s is

$$(0^*10^*1)^*0^* \quad (10)$$

## 2. Find regular expression for odd number of 0's

- Find reg. expressions for  $L = \{x \in \Sigma \mid x \text{ has 1 number of 0s}\}$

Starting with  $L = \{x \in \Sigma \mid x \text{ has 1 number of 0s}\}$ , it's reg. expressions is

$$1^*01^* \quad (11)$$

- Find reg. expressions for  $L = \{x \in \Sigma \mid x \text{ has 3 number of 0s}\}$

Now for  $L = \{x \in \Sigma \mid x \text{ has 3 number of 0s}\}$ , it's reg. expressions is

$$1^*01^*01^*01^* \quad (12)$$

- Find reg. expressions for  $L = \{x \in \Sigma \mid x \text{ has 5 number of 0s}\}$

Now for  $L = \{x \in \Sigma \mid x \text{ has 5 number of 0s}\}$ , it's reg. expressions is

$$1^*01^*01^*01^*01^* \quad (13)$$

- Hey I see a pattern!!

From above, I see a pattern that

$$1^*(01^*)^* \quad (14)$$

- Conclude :)

Using the pattern, I can conclude that the regular expression for odd number of 0s is

$$1^*(01^*)^* \quad (15)$$

3. Combine 1 and 2 using +

So, by combining the two with union, we have

$$(0^*10^*1)^*0^* + 1^*(01^*)^* \quad (16)$$

Notes:

- Regular Expression
  - Quick Guide

$$(0 + 1)((01)^*0) \quad (17)$$

The expression implies that

1. Starts with 0 **or** 1  
\* indicated by  $(0 + 1)$
  2. Are then followed by **one or more repetitions** of 01  
\* indicated by  $(01)^*$
  3. Ends with 0  
\* indicated by the final 0
- Examples
    1.  $L = \{w \in \{a, b\}^* \mid w \text{ has an } a\}$

Answer:

$$(a + b)^*a(a + b)^* \quad (18)$$

- Means there is one or more repetitions of  $a$  or  $b$  at front
- Means there is  $a$  in the middle
- Means there is zero or more repetitions of  $a$  or  $b$  at end

2.  $L = \{w \in \{a, b\}^* \mid w \text{ has at least two } as\}$

**Answer:**

$$(a + b)^* a (a + b)^* a (a + b)^* \quad (19)$$

3.  $L = \{w \in \{a, b\}^* \mid |w| \geq 2\}$

**Answer:**

$$(0 + 1)(0 + 1)(0 + 1)^* \quad (20)$$

In this example,

- Two characters are created (indicated by  $(0 + 1)(0 + 1)$ )
- And more :D!! (indicated by  $(0 + 1)^*$ )