Midterm 1 Version 1 Solution

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Question 1

a. $S_1 = \{aa, bb, cc, aaa, aab, aac, bba, bbb, bbc, cca, ccb, ccc, \dots\}$

Since S_2 is a set of elements with length 3,

$S_1 \cap S_2 = \{aaa, aab, aac, bba, bbb, bbc, cca, ccb, ccc\}$

b. See below

p	q	r	$\neg r$	$(p \vee q)$	$(p \vee q) \Rightarrow \neg r$
T	T	T	F	T	F
T	F	F	T	T	T
F	T	F	T	T	T
F	F	T	F	F	T
T	T	F	T	T	T
T	F	T	F	T	T
F	T	T	F	T	T
F	F	F	T	F	F

c. **Negation:** $\exists x \in \mathbb{N}, \forall y \in \mathbb{N}, \neg P(x, y) \wedge \neg Q(x, y)$.

Let $x =$ ________, and $y \in \mathbb{N}$.

We will prove that predicate P and Q are not true.

Question 2

a. $\exists x \in P,\ Student(x) \wedge Attends(x)$

b. $\forall x \in P,\ \exists y \in P,\ Student(y) \wedge Attends(y) \wedge Loves(x, y)$

c. $\forall x \in P,\ Student(x) \wedge Attends(x) \Rightarrow Loves(x, x)$

d. $\forall x_1, x_2 \in P,\ x_1 \neq x_2 \wedge Loves(x_1, x_2) \Rightarrow (Attends(x_1) \wedge \neg Attends(x_2)) \vee (\neg Attends(x_2) \wedge Attends(x_1))$

Correct Solution:

$\forall x_1, x_2 \in P,\ x_1 \neq x_2 \wedge Loves(x_1, x_2) \wedge Loves(x_1, x_2) \Rightarrow \neg Attends(x_1) \vee \neg Attends(x_2)$

Question 3

a. $\forall a, b, c \in \mathbb{Z},\ \exists k, l \in \mathbb{Z},\ b = ka \wedge c = lb \Rightarrow \exists m \in \mathbb{Z},\ c = ma$

b. Let $a, b, c \in \mathbb{Z}$, and $k = \frac{b}{a}$, $l = \frac{c}{b} \in \mathbb{Z}$. Assume, $b = ka$ and $c = lb$.

Then,

$$\begin{align} c &= lb \tag{1} \\ &= \left(\frac{c}{b}\right) a \tag{2} \\ &= \left(\frac{c}{b}\right)\left(\frac{b}{a}\right) a \tag{3} \\ &= \left[\left(\frac{c}{b}\right)\left(\frac{b}{a}\right)\right] a \tag{4} \end{align}$$

Since $\left(\frac{c}{b}\right), \left(\frac{b}{a}\right) \in \mathbb{Z}$, $\left(\frac{c}{b}\right)\left(\frac{b}{a}\right) \in \mathbb{Z}$.

Then, it follows from the definition of divisibility that a divides c.

Question 4

- Let $x, y \in \mathbb{R}$.

  Then, there exists $\epsilon_1, \epsilon_2 \in \mathbb{R}$, $0 \leq \epsilon_1, \epsilon_2 < 1 \wedge x = \lfloor x \rfloor + \epsilon_1 \wedge y = \lfloor y \rfloor + \epsilon_2$ by fact 1.

  Then,

$$\lfloor x + y \rfloor = \left\lfloor \lfloor x \rfloor + \epsilon_1 = \lfloor y \rfloor + \epsilon_2 \right\rfloor \tag{1}$$
$$= \left\lfloor (\lfloor x \rfloor + \lfloor y \rfloor) + (\epsilon_1 + \epsilon_2) \right\rfloor \tag{2}$$

  Then,

$$\left\lfloor (\lfloor x \rfloor + \lfloor y \rfloor) + (\epsilon_1 + \epsilon_2) \right\rfloor = (\lfloor x \rfloor + \lfloor y \rfloor) + \left\lfloor \epsilon_1 + \epsilon_2 \right\rfloor \tag{3}$$

  by fact 2.

  Then,

$$\lfloor x + y \rfloor \geq \lfloor x \rfloor + \lfloor y \rfloor \tag{4}$$

  Then, it follows from above that the statement $\forall x, y \in \mathbb{R}, \lfloor x + y \rfloor \geq \lfloor x \rfloor + \lfloor y \rfloor$ is true.