CSC236 Worksheet 5 Review

Hyungmo Gu

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Question 1

a. *Proof.* Define $P(k) : R(3^k) = k3^k$. Note that when $n = 3^k$, this is equivalent to $R(n) = n \log_3 n$. I will use simple induction to prove $P(k)$.

Base Case ($k = 0$):

Let $k = 0$.

Then,

$$\begin{align} R(3^k) &= 0 && \text{[By def., since } n = 3^0 = 1\text{]} \tag{1}\\ &= 0 \cdot 3^0 \tag{2}\\ &= k \cdot 3^k \tag{3} \end{align}$$

Thus, $P(k)$ is verified in this step.

Inductive Step:

Let $k \in \mathbb{N}$. Assume $P(k)$. That is, $R(3^k) = k \cdot 3^k$. I need to prove $P(k+1)$ follows. That is, $R(3^{k+1}) = (k+1) \cdot 3^{k+1}$

Starting from $R(3^{k+1})$, we have

$$\begin{align} R(3^{k+1}) &= 3^{k+1} + 3R(\lceil 3^{k+1}/3 \rceil) && \text{[By def., since } 0 < k+1, \text{ and } 1 < 3^{k+1}\text{]} \tag{4}\\ &= 3^{k+1} + 3R(\lceil 3^k \rceil) \tag{5}\\ &= 3^{k+1} + 3R(3^k) && \text{[Since } \lceil 3^k \rceil = 3^k\text{]} \tag{6}\\ &= 3^{k+1} + 3(k \cdot 3^k) && \text{[By I.H]} \tag{7}\\ &= 3^{k+1} + (k \cdot 3^{k+1}) \tag{8}\\ &= (k+1) \cdot 3^{k+1} \tag{9} \end{align}$$

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