

Worksheet 20 Solution

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April 14, 2020

Question 1

a. *Proof.* Let $V = \{1, 2, 3, 4, 5, 6\}$, $E = \{(1, 2), (1, 6), (2, 3), (3, 4), (4, 5), (5, 6)\}$.

We need to prove the graph $G = (V, E)$ is bipartite by proving the following properties:

1. There exists subsets $V_1, V_2 \subset V$ such that $V_1 \neq \emptyset, V_2 \neq \emptyset$, and V_1 and V_2 form a partition of V .
2. Every edge in E has exactly one endpoint in V_1 and one in V_2 .

We will prove the properties in parts.

Part 1 (Proving $V_1 \neq \emptyset, V_2 \neq \emptyset$, and V_1 and V_2 form a partition of V):

Let $V_1 = \{1, 3, 5\}$ and $V_2 = \{2, 4, 6\}$.

We need to prove $V_1 \neq \emptyset, V_2 \neq \emptyset$, and V_1 and V_2 form a partition of V , i.e $V_1 \cup V_2 = V$ and $V_1 \cap V_2 = \emptyset$.

First, we need to show the subsets V_1 and V_2 are non-empty.

The header tells us both subsets V_1 and V_2 have more than 1 elements.

Then, using these facts, we can conclude $V_1 \neq \emptyset$ and $V_2 \neq \emptyset$.

Finally, we need to show $V_1 \cup V_2 = V$ and $V_1 \cap V_2 = \emptyset$.

The header tells us $V_1 = \{1, 3, 5\}$ and $V_2 = \{2, 4, 6\}$.

Then, we can calculate

$$V_1 \cup V_2 = \{1, 2, 3, 4, 5, 6\} = V \quad (1)$$

$$V_1 \cap V_2 = \emptyset \quad (2)$$

Part 2 (Proving every edge in E has exactly one endpoint in V_1 and one in V_2):

Let $V_1 = \{1, 3, 5\}$ and $V_2 = \{2, 4, 6\}$.

We need to show every edge in E has exactly one endpoint in V_1 and one in V_2 .

The header tells us $V_1 = \{1, 3, 5\}$, $V_2 = \{2, 4, 6\}$, and $E = \{(1, 2), (1, 6), (2, 3), (3, 4), (4, 5), (5, 6)\}$.

Using these facts, we can generate the following table.

Edge (1,2)	- 1 is in V_1 - 2 is in V_2	Edge (3,4)	- 3 is in V_1 - 4 is in V_2
Edge (1,6)	- 1 is in V_1 - 6 is in V_2	Edge (4,5)	- 4 is in V_2 - 5 is in V_1
Edge (2,3)	- 2 is in V_2 - 3 is in V_1	Edge (5,6)	- 5 is in V_1 - 6 is in V_2

Then, it follows from observation that every edge in E has one endpoint in V_1 and one in V_2 .

□

Pseudoproof:

Let $V = \{1, 2, 3, 4, 5, 6\}$, $E = \{(1, 2), (1, 6), (2, 3), (3, 4), (4, 5), (5, 6)\}$.

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We will prove the properties in parts.

1. Show there exists subsets $V_1, V_2 \subset V$ such that $V_1 \neq \emptyset, V_2 \neq \emptyset$, and V_1 and V_2 form a partition of V

Let $V_1 = \{1, 3, 5\}$ and $V_2 = \{2, 4, 6\}$.

We need to prove $V_1 \neq \emptyset, V_2 \neq \emptyset$, and V_1 and V_2 form a partition of V , i.e $V_1 \cup V_2 = V \wedge V_1 \cap V_2 = \emptyset$.

1. Show $V_1 \neq \emptyset, V_2 \neq \emptyset$

First, we need to show the subsets V_1 and V_2 are non-empty.

The header tells us both subsets V_1 and V_2 have more than 1 elements.

Then, using these facts, we can conclude $V_1 \neq \emptyset$ and $V_2 \neq \emptyset$.

2. Show $V_1 \cup V_2 = V \wedge V_1 \cap V_2 = \emptyset$

Second, we need to show $V_1 \cup V_2 = V$ and $V_1 \cap V_2 = \emptyset$.

The header tells us $V_1 = \{1, 3, 5\}$ and $V_2 = \{2, 4, 6\}$.

Then, we can calculate

$$V_1 \cup V_2 = \{1, 2, 3, 4, 5, 6\} = V \quad (3)$$

$$V_1 \cap V_2 = \emptyset \quad (4)$$

Part 1:

Let $V_1 = \{1, 3, 5\}$ and $V_2 = \{2, 4, 6\}$.

We need to prove $V_1 \neq \emptyset, V_2 \neq \emptyset$, and V_1 and V_2 form a partition of V , i.e $V_1 \cup V_2 = V \wedge V_1 \cap V_2 = \emptyset$.

First, we need to show the subsets V_1 and V_2 are non-empty.

The header tells us both subsets V_1 and V_2 have more than 1 elements.

Then, using these facts, we can conclude $V_1 \neq \emptyset$ and $V_2 \neq \emptyset$.

Finally, we need to show $V_1 \cup V_2 = V$ and $V_1 \cap V_2 = \emptyset$.

The header tells us $V_1 = \{1, 3, 5\}$ and $V_2 = \{2, 4, 6\}$.

Then, we can calculate

$$V_1 \cup V_2 = \{1, 2, 3, 4, 5, 6\} = V \quad (5)$$

$$V_1 \cap V_2 = \emptyset \quad (6)$$

2. Show every edge in E has exactly one endpoint in V_1 and one in V_2 .

Let $V_1 = \{1, 3, 5\}$ and $V_2 = \{2, 4, 6\}$.

We need to show every edge in E has exactly one endpoint in V_1 and one in V_2 .

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Then, it follows from observation that every edge in E has one endpoint in V_1 and one in V_2 .

Part 2:

Let $V_1 = \{1, 3, 5\}$ and $V_2 = \{2, 4, 6\}$.

We need to show every edge in E has exactly one endpoint in V_1 and one in V_2 .

The header tells us $V_1 = \{1, 3, 5\}$, $V_2 = \{2, 4, 6\}$, and $E = \{(1, 2), (1, 6), (2, 3), (3, 4), (4, 5), (5, 6)\}$.

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Then, it follows from observation that every edge in E has one endpoint in V_1 and one in V_2 .