CSC343 Worksheet 12 Solution

July 2, 2020

1. - Keys
     - {id of molecule}
     - {x position, y position, z position}
   - Functional Dependencies
     1. id of molecule $\rightarrow$ x position, y position, z position, x velocity, y velocity, z velocity
     2. x position, y position, z position $\rightarrow$ id of molecule, x velocity, y velocity, z velocity

Notes:

- Function Dependencies
  - *Functional Dependency* is a relationship between two attributes typically between the key and other non-key attributes within a table.

    Example:

    SIN $\rightarrow$ Name, Address, Birthdate

    Example 2:

    ISBN $\rightarrow$ Title
- Key of Relations
  - One or more attributes $\{A_1, A_2, ..., A_n\}$ is a key for a relation R if
    1. Those attributes functionally determine all other attributes of the relation
    2. No proper subset of $\{A_1, A_2, ...A_n\}$ functionally determines all other attributes of R

    Example:

    Given relation

    $R =$ Movies1(title, year, length, genre, studioName, starName)

    i. {title, year, starName } form a key for the relation **Movies1**

ii. { year, starName } is not a key. Same star can be in multiple movies per year

- Superkeys
  * Means a a set of attributes that contains a key
  * Don't need to be minimal

  Example:
  Given relation

  $R =$ Movies1(title, year, length, genre, studioName, starName)
  · { title, year, starName } is a key and superkey
  · { title, year, starName, title, year, length} is a superkey

References:

1) OpenTextBC, Chapter 11 Functional Dependencies, link

2. a) 1. $AB \to C$
   2. $AB \to D$
   3. $C \to A$
   4. $C \to B$
   5. $D \to B$
   6. $D \to C$
   7. $C \to D$
   8. $D \to A$

Second Attempt:
$\{A, B\}^+ = \{A, B, C, D\}$, so the following non-trivial FDs follows: $AB \to C$ and $AB \to D$.

$\{C\}^+ = \{D, A\}$, so the following non-trivial FDs follows $C \to D$ and $C \to A$.

$\{D\}^+ = \{A\}$, so the following non-trivial FDs follows: $D \to A$.

Notes:

- The Splitting / Combining Rule
  - Combining Rule
    * $A_1, A_2, \cdots, A_n \to B_i$ for $i = 1, 2, ..., m$
      to
      $A_1, A_2, \cdots A_n \to B_1, B_2, \cdots B_m$

Example:

Given

title year $\rightarrow$ length
title year $\rightarrow$ genre
title year $\rightarrow$ studioName

it's combined form is

title year $\rightarrow$ length genre studioName

- Splitting Rule
  *
  * $A_1, A_2, \cdots A_n \rightarrow B_1, B_2, \cdots B_m$
    to
    $A_1, A_2, \cdots, A_n \rightarrow B_i$ for $i = 1, 2, ..., m$

  Example:

  Given

  title year $\rightarrow$ length

  It's splitted form is

  title $\rightarrow$ length
  year $\rightarrow$ length

- Trivial Functional Dependencies

  - A functional dependency $FD : X \rightarrow Y$ is **trivial** if Y is a subset of X

    Exmaple:

    title year $\rightarrow$ title

    Example 2:

    title $\rightarrow$ title

- Non-trivial Functional Dependencies
  - is a case where some but not all of the attributes on the R.H.S of an FD are also on L.H.S

    Example:

    title year $\rightarrow$ title movieLength

– Can be simplified using **tirivial-dependency rule**

* The FD $A_1A_2\cdots A_n \to B_1B_2\cdots B_m$ is equivalent to $A_1A_2\cdots A_n \to C_1C_2\cdots C_k$

  where C's are all those B's that are not in A's.

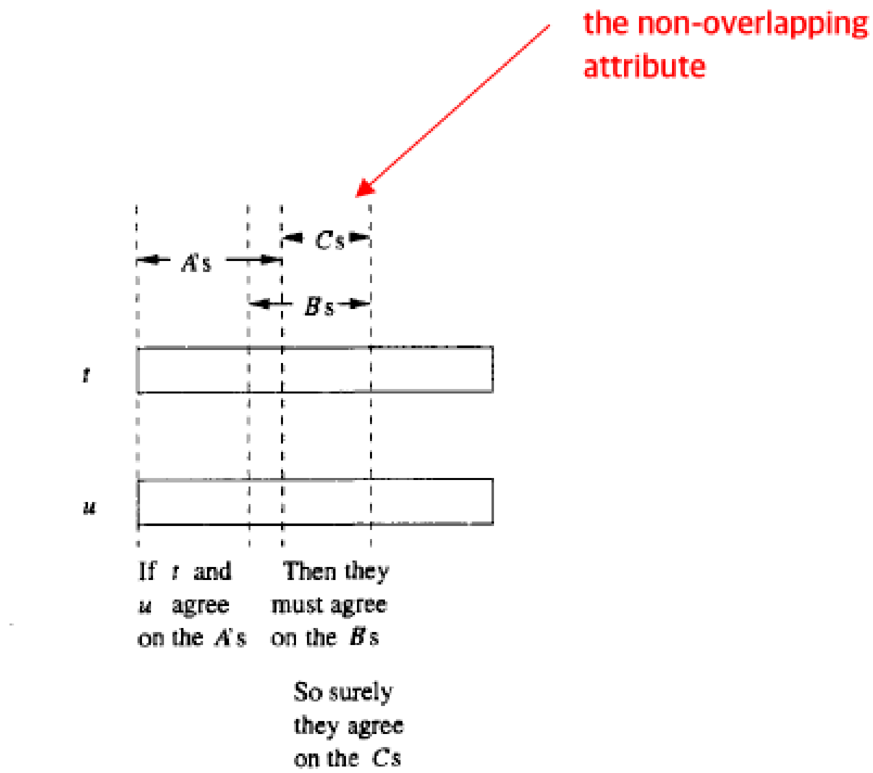


Figure 3.3: The trivial-dependency rule

- Computing the Clousre of Attributes
  – Closure of attribute set $\{X\}$ is denoted as $\{X\}^+$.
  – The closure means a given set of attributes A satisfying FD, are a sets of all attributes B such that $A \to B$

  Example:

  Given attributes A, B, C, D, E, F and FDs $AB \to C$, $BC \to AD$, $D \to E$ and $CF \to B$, What is the closure of $\{A, B\}$ or $\{A, B\}^+$

  1. Start with $\{A, B\}$.
  2. Split $BC \to AD$
     * We have $BC \to A$ and $BCtoD$
     * Since A is in $\{A, B\}$, this is not included
     * Since D is not in $\{A, B\}$, this IS included

     So, we have $\{A, B, D\}$

3. Since C in $AB \rightarrow C$ is NOT in $\{A, B, C, D\}$, C is included and we have $\{A, B, C, D\}$
4. Since A in $BC \rightarrow A$ is in $\{A, B, C, D\}$, this is skipped
5. Since E is not in $D \rightarrow E$, E is included and we have $\{A, B, C, D, E\}$ as our solution

- Why the Closure Algorithm Works
- Transitive Rule
  - Definition

    If $A_1A_2 \cdots A_n \rightarrow B_1B_2 \cdots B_m$ and $B_1B_2 \cdots B_m \rightarrow C_1C_2 \cdots C_k$ hold in relation R, $A_1A_2 \cdots A_n \rightarrow C_1C_2 \cdots C_k$ also holds in R.

    Example:

    Given

    title year $\rightarrow$ studioName
    studioName $\rightarrow$ studioAddr

    Transitive rule says the above is equal to the following

    title year $\rightarrow$ studioAddr
- Inference Rules
  - Is allso called **Armstrong's Axioms**
  - Has 3 axioms
    1. *Reflexivity*
       * If $\{B_1, B_2, ..., B_n\} \subseteq \{A_1, A_2, ..., A_n\}$ then $A_1A_2 \cdots A_n \rightarrow B_1B_2 \cdots B_m$
       * also called **trivial FDs**
    2. *Augmentation*
       * If $A_1A_2 \cdots A_n \rightarrow B_1B_2 \cdots B_m$ then $A_1A_2 \cdots A_nC_1C_2 \cdots C_k \rightarrow B_1B_2 \cdots B_mC_1C_2 \cdots C_k$
       * $C_1C_2 \cdots C_k$ are any set of attributes
    3. *Transitivity*
       * If $A_1A_2 \cdots A_n \rightarrow B_1B_2 \cdots B_m$ and $B_1B_2 \cdots B_m \rightarrow C_1C_2 \cdots C_k$ then $A_1A - 2 \cdots A_n \rightarrow C_1C_2 \cdots C_k$

b) A, B is the only key of R.

Notes:

- Key of Attributes
  - **Definition:** A set of attributes $\{A_1, A_2, \cdots, A_n\}$ is a key for a relation R if
    1. Those attributes functionally determine all other attributes

2. No proper subset of $\{A_1, A_2, ..., A_n\}$ functionally determines all other attributes of R.

c) The superkeys that are not keys are: $\{A, B, C\}$, $\{A, B, D\}$, $\{A, B, C, D\}$

3. i) a) $\{A\}^+ = \{A, B, C, D\}$, so we have $A \to A$, $A \to B$, $A \to C$, $A \to D$

$\{B\}^+ = \{C, D\}$, so we have $B \to C$ and $B \to D$

b) $\{A\}$ is the key of S.

c) The super keys that are not keys are:

$\{A, B\}$, $\{A, C\}$, $\{A, D\}$, $\{A, B, C\}$, $\{A, B, D\}$, $\{A, B, C, D\}$

ii) a) $\{A\}^+ = \{A\}$, so this FD is trivial.

$\{B\}^+ = \{B\}$, so this FD is trivial.

$\{C\}^+ = \{C\}$, so this FD is trivial.

$\{D\}^+ = \{D\}$, so this FD is trivial.

$\{A, B\}^+ = \{A, B, C, D\}$, so we have $AB \to A$, $AB \to B$, $AB \to C$, $AB \to D$

$\{A, C\}^+ = \{A, C\}$, so we have $AC \to A$, $AC \to C$

$\{A, D\}^+ = \{A, D, B\}$, so we have $AD \to A$, $AD \to D$, $AD \to B$

$\{B, C\}^+ = \{B, C, D, A\}$, so we have $BC \to A$, $BC \to B$, $BC \to C$, $BC \to D$

$\{D, C\}^+ = \{D, C, A, B\}$, so we have $DC \to D$, $DC \to C$, $DC \to A$, $DC \to B$

$\{A, B, C\}^+ = \{A, B, C, D\}$, so we have $ABC \to A$, $ABC \to B$, $ABC \to C$, $ABC \to D$

$\{B, C, D\}^+ = \{B, C, D, A\}$, so we have $BCD \to A$, $BCD \to B$, $BCD \to C$, $BCD \to D$

$\{C, D, A\}^+ = \{C, D, A, B\}$, so we have $CDA \to A$, $CDA \to B$, $CDA \to C$, $CDA \to D$

$\{D, A, B\}^+ = \{D, A, B, C\}$, so we have $DAB \to A$, $DAB \to B$, $DAB \to C$, $DAB \to D$

$\{D, A, B\}^+ = \{D, A, B, C\}$, so we have $DAB \to A$, $DAB \to B$, $DAB \to C$, $DAB \to D$

$\{A, B, C, D\}^+ = \{A, B, C, D\}$, so this FD is trivial.

b) $\{A, B\}$, $\{A, C\}$, $\{B, C\}$, $\{D, C\}$ are the keys of T.

c) The super keys that are not keys are:

$\{A, B, C\}$, $\{A, B, D\}$, $\{B, C, D\}$, $\{A, D, C\}$, $\{A, B, D\}$, $\{A, B, C, D\}$

iii) a) $\{A\}^+ = \{A, B, C, D\}$, so we have $A \to C$, $A \to D$

$\{B\}^+ = \{A, B, C, D\}$, so we have $B \to A$, $B \to D$

$\{C\}^+ = \{A, B, C, D\}$, so we have $C \to A$, $C \to B$

$\{D\}^+ = \{A, B, C, D\}$, so we have $D \to B$, $D \to C$

$\{A, B\}^+ = \{A, B, C, D\}$, so we have $AB \to C$, $AB \to D$

$\{B, C\}^+ = \{A, B, C, D\}$, so we have $BC \to A$, $BC \to D$

$\{B, D\}^+ = \{A, B, C, D\}$, so we have $BD \to A$, $BD \to C$

$\{C, D\}^+ = \{A, B, C, D\}$, so we have $CD \to A$, $CD \to B$

$\{C, D\}^+ = \{A, B, C, D\}$, so we have $CD \to A$, $CD \to B$

$\{A, B, C\}^+ = \{A, B, C, D\}$, so we have $ABC \to D$

$\{B, C, D\}^+ = \{A, B, C, D\}$, so we have $BCD \to A$

$\{C, D, A\}^+ = \{A, B, C, D\}$, so we have $CDA \to B$

$\{D, A, B\}^+ = \{A, B, C, D\}$, so we have $DAB \to C$

<u>Correct Solution:</u>

$\{A\}^+ = \{A, B, C, D\}$, so we have $A \to C$, $A \to D$

$\{B\}^+ = \{A, B, C, D\}$, so we have $B \to A$, $B \to D$

$\{C\}^+ = \{A, B, C, D\}$, so we have $C \to A$, $C \to B$

$\{D\}^+ = \{A, B, C, D\}$, so we have $D \to B$, $D \to C$

$\{A, B\}^+ = \{A, B, C, D\}$, so we have $AB \to C$, $AB \to D$

$\{A, C\}^+ = \{A, B, C, D\}$, so we have $AC \to B$, $AC \to D$

$\{A, D\}^+ = \{A, B, C, D\}$, so we have $AD \to B$, $AD \to C$

$\{B, C\}^+ = \{A, B, C, D\}$, so we have $BC \to A$, $BC \to D$

$\{B, D\}^+ = \{A, B, C, D\}$, so we have $BD \to A$, $BD \to C$

$\{C, D\}^+ = \{A, B, C, D\}$, so we have $CD \to A$, $CD \to B$

$\{A, B, C\}^+ = \{A, B, C, D\}$, so we have $ABC \to D$

$\{B, C, D\}^+ = \{A, B, C, D\}$, so we have $BCD \to A$

$\{C, D, A\}^+ = \{A, B, C, D\}$, so we have $CDA \to B$

$\{D, A, B\}^+ = \{A, B, C, D\}$, so we have $DAB \to C$

b) $\{A\}, \{B\}, \{C\}, \{D\}$ are the keys of U.

c) The super keys that are not keys are:

$\{A, B\}$,$\{A, C\}$,$\{A, D\}$,$\{B, C\}$, $\{B, D\}$,$\{C, D\}$, $\{A, B, C\}$, $\{B, C, D\}$, $\{C, D, A\}$, $\{D, A, B\}$. $\{A, B, C, D\}$

4. a) We need to show the closure of attributes $\{A_1, A_2, \cdots, A_n, C\}$ in FD $A_1, A_2, \cdots, A_n, C \to B$ is $\{A_1, A_2, \cdots, A_n, C, B\}$, that is $\{A_1, A_2, \cdots, A_n, C\}^+ = \{A_1, A_2, \cdots, A_n, C, B\}$

Since we know $\{A_1, A_2, \cdots, A_n\}$ functionally determines B, we can conclude B can be added to $\{A_1, A_2, \cdots, A_n, C\}$.

Thus, it follows from above that $\{A_1, A_2, \cdots, A_n, C\}^+ = \{A_1, A_2, \cdots, A_n, C, B\}$.

b) Let $A_1A_2 \cdots A_n \to B$ is FD. That is, $\{A_1A_2 \cdots A_n\}^+ = \{A_1A_2 \cdots A_n, B\}$

We need to show $A_1A_2 \cdots A_nC \to BC$ follows. That is, $\{A_1, A_2, \cdots, A_n, C\}^+ = \{A_1, A_2, \cdots, A_n, C, B\}$

It follows from the combine and split rule that $A_1A_2 \cdots A_nC \to BC$ can be spliited into $A_1A_2 \cdots A_nC \to B$ and $A_1A_2 \cdots, A_nC \to C$.

So, we need to show $A_1A_2 \cdots A_nC \to B$ and $A_1A_2 \cdots, A_nC \to C$ follows from the given.

We will do so in parts.

1. **Part 1 (Showing $A_1A_2 \cdots A_nC \to B$):**

   Here, we need to show $A_1A_2 \cdots A_nC \to B$ follows.

   And indeed, this follows from the work of *augmenting left sides*. That is the solution to previous problem.

2. **Part 2 (Showing $A_1A_2\cdots A_nC \to C$):**

   Here, we need to show $A_1A_2\cdots A_nC \to C$ follows.

   The definition of trivial FD tells us $A_1A_2\cdots A_n \to B_1B_2\cdots B_m$ holds when $\{B_1, B_2, \cdots, B_m\} \subseteq \{A_1, A_2, \cdots, A_n\}$

   Since $\{C\} \subseteq \{A_1, A_2\cdots, A_n, C\}$, we can conclude this FD follows trivially.

c) Let $A_1A_2\cdots A_n \to B_1B_2\cdots B_m$ and $C_1C_2\cdots C_k \to D$, where B are each among the C's.

We neeed to show $A_1A_2\cdots A_nE_1E_2\cdots E_j \to D$ follows, where the E's are all of those C's not found among the B's.

The transitive rule tells us if $A_1A_2\cdots A_n \to B_1B_2\cdots B_m$ and $B_1B_2\cdots B_m \to C_1C_2\cdots C_k$, then $A_1A_2\cdots A_n \to C_1C_2\cdots C_k$ also holds in R.

Since we know $A_1A_2\cdots A_n \to B_1B_2\cdots B_m$ and $C_1C_2\cdots C_k \to D$ where B's are each among the C's, we can conclude from the transitive rule that $A_1A_2\cdots A_n \to D$.

Then using **augmenting left sides** to all C's not found among the B's on $A_1A_2\cdots A_n \to D$, we can conclude $A_1A_2\cdots A_nE_1E_2\cdots E_j \to D$ follows.

d) Assume FD's $A_1A_2\cdots A_n \to B_1B_2\cdots B_m$ and $C_1C_2\ldots C_k \to D_1D_2\cdots D_j$ holds.

We need to show FD $A_1A_2\cdots A_nC_1C_2\cdots C_k \to B_1B_2\cdots B_mD_1D_2\cdots D_k$ follows.

Using the split / combine rule, we can conclude showing $A_1A_2\cdots A_nC_1C_2\cdots C_k \to B_1B_2\cdots B_mD_1D_2\cdots D_k$ is the same as showing $A_1A_2\cdots A_nC_1C_2\cdots C_k \to B_1B_2\cdots B_m$ and $A_1A_2\cdots A_nC_1C_2\cdots C_k \to D_1D_2\cdots D_k$

So, we will prove the two, in parts

1. **Part 1 (Showing $A_1A_2\cdots A_nC_1C_2\cdots C_k \to B_1B_2\cdots B_m$)**

   Here, we need to show $A_1A_2\cdots A_nC_1C_2\cdots C_k \to B_1B_2\cdots B_m$.

   The header of problem tells us $A_1A_2\cdots A_n \to B_1B_2\cdots B_m$ holds.

   Then by using **Augmenting Left Sides** rule to all Cs not found among the As, $A_1A_2\cdots A_nC_1C_2\cdots C_k \to B_1B_2\cdots B_m$ follows.

2. **Part 2 (Showing $A_1A_2\cdots A_nC_1C_2\cdots C_k \to D_1D_2\cdots D_k$ follows)**

   Here, we need to show $A_1A_2\cdots A_nC_1C_2\cdots C_k \to D_1D_2\cdots D_k$.

   The header of problem tells us $C_1C_2\cdots C_k \to D_1D_2\cdots D_k$ holds.

   Then by using **Augmenting Left Sides** rule to all As not found among the Cs, $A_1A_2\cdots A_nC_1C_2\cdots C_k \to D_1D_2\cdots D_k$ follows.

5. a) An example is

   A being movieID and
   B being movie length.

   b) An example is

   A being movieID
   B being movieTitle
   C being movieLength

   c) An example is

   A being movieTitle
   B being year
   C being length

6. Assume a relation has no attribute that is functionallty determined by all the other attributes.

   And, assume for the sake of contradiction that there is a relation with non-trivial FD $X \to Y$.

   Then, it follows from the definition of non-trivial functional dependency that $Y \neq\subseteq X$.

   Then, we can conclude the attributes in Y is functionally determined by other attributes in X.

   But this contradicts the assumption that no attribute is functionally determined by all other attributes.

7. Let X and Y be sets of attributes. Assume $X \subseteq Y$.

   I need to show $X^+ \subseteq Y^+$.

   I will do so in cases

   1. **Case 1 ($X = Y$):**

      Assume $X = Y$.

      I need to show $X^+ \subseteq Y^+$ follows.

      The header tells us $X = Y$.

      Using this fact, $X^+ = Y^+$ is true.

      Then it follows from above that $X^+ \subseteq Y^+$ is also true.

2. **Case 2** ($X \subset Y$)

   Assume $X \subset Y$.

   I need to show $X^+ \subseteq Y^+$ follows.

   Since the attributes in X is in Y, we can conclude the attributes in X^+ is also in Y^+.

   And, since Y has attributes not in X, we can conclude Y^+ may contain attributes not in X^+.

   Thus, we can conclude $X^+ \subseteq Y^+$.

8. 1. Only one solution will be included for now :)
   The following

   1. $A \to C$
   2. $B \to A$
   3. $B \to C$
   4. $C \to A$
   5. $C \to B$
   6. $AB \to C$
   7. $AC \to B$
   8. $BC \to A$
   9. $A \to BC$
   10. $A \to A$

   can be simplified to

   1. $A \to C$
   2. $B \to A$
   3. $B \to C$
   4. $C \to A$
   5. $C \to B$
   6. $A \to C$ B removed from here!!
   7. $AC \to B$
   8. $BC \to A$
   9. $A \to BC$
   10. $A \to A$

   since **augmenting left sides** rule tells us $AB \to C$ can be attained by adding B to L.H.S of $A \to C$.

   Then, the following

1. $A \to C$
2. $B \to A$
3. $B \to C$
4. $C \to A$
5. $C \to B$
6. $A \to C$
7. $AC \to B$
8. $BC \to A$
9. $A \to BC$
10. $A \to A$

can be simplified to

1. $A \to C$
2. $B \to A$
3. $B \to C$
4. $C \to A$
5. $C \to B$
6. $AC \to B$
7. $BC \to A$
8. $A \to BC$
9. $A \to A$

by removing redundant $A \to C$.

Then, the following

1. $A \to C$
2. $B \to A$
3. $B \to C$
4. $C \to A$
5. $C \to B$
6. $AC \to B$
7. $BC \to A$
8. $A \to BC$
9. $A \to A$

can be simplified to

1. $A \to C$
2. $B \to A$
3. $B \to C$
4. $C \to A$
5. $C \to B$

6. $AC \to B$
7. $BC \to A$
8. $A \to B$ Splitted from $A \to BC$
9. $A \to C$ Splitted from $A \to BC$
10. $A \to A$

by using **splitting rule** on $A \to BC$.

Then, the following

1. $A \to C$
2. $B \to A$
3. $B \to C$
4. $C \to A$
5. $C \to B$
6. $AC \to B$
7. $BC \to A$
8. $A \to B$
9. $A \to C$
10. $A \to A$

can be simplified to

1. $A \to C$
2. $B \to A$
3. $B \to C$
4. $C \to A$
5. $C \to B$
6. $AC \to B$
7. $BC \to A$
8. $A \to B$
9. $A \to A$

by removing redundant $A \to C$.

Then, the following

1. $A \to C$
2. $B \to A$
3. $B \to C$
4. $C \to A$
5. $C \to B$
6. $AC \to B$
7. $BC \to A$

8. $A \to B$
9. $A \to A$

can be simplified to

1. $A \to C$
2. $B \to A$
3. $B \to C$
4. $C \to A$
5. $C \to B$
6. $A \to B$ C removed here!!
7. $BC \to A$
8. $A \to B$
9. $A \to A$

since **augmenting left sides** tells us $AC \to B$ can be attained by adding C to $A \to B$.

Then, the following

1. $A \to C$
2. $B \to A$
3. $B \to C$
4. $C \to A$
5. $C \to B$
6. $A \to B$
7. $BC \to A$
8. $A \to B$
9. $A \to A$

can be simplified to

1. $A \to C$
2. $B \to A$
3. $B \to C$
4. $C \to A$
5. $C \to B$
6. $A \to B$
7. $BC \to A$
8. $A \to A$

by removing redundant $A \to B$.

Then, the following

1. $A \to C$

2. $B \to A$
3. $B \to C$
4. $C \to A$
5. $C \to B$
6. $A \to B$
7. $BC \to A$
8. $A \to A$

can be simplified to

1. $A \to C$
2. $B \to A$
3. $B \to C$
4. $C \to A$
5. $C \to B$
6. $A \to B$
7. $BC \to A$

since $A \to A$ can be attained by using **transitivity** rule on $A \to C$ and $C \to A$.

Then, the following

1. $A \to C$
2. $B \to A$
3. $B \to C$
4. $C \to A$
5. $C \to B$
6. $A \to B$
7. $BC \to A$

can be simplified to

1. $A \to C$
2. $B \to A$
3. $B \to C$
4. $C \to A$
5. $C \to B$
6. $A \to B$
7. $B \to A$ C removed here!!

since **augmenting let sides** rule tells us $BC \to A$ can be attained by adding C to L.H.S of $B \to A$.

Then, the following

1. $A \to C$

2. $B \to A$
3. $B \to C$
4. $C \to A$
5. $C \to B$
6. $A \to B$
7. $B \to A$

can be simplified to

1. $A \to C$
2. $B \to A$
3. $B \to C$
4. $C \to A$
5. $C \to B$
6. $A \to B$

by removing redundant $B \to A$.

Then, the following

1. $A \to C$
2. $B \to A$
3. $B \to C$
4. $C \to A$
5. $C \to B$
6. $A \to B$

can be simplified to

1. $A \to C$
2. $B \to A$
3. $C \to A$
4. $C \to B$
5. $A \to B$

since **transitivity** rule tells us $B \to C$ can be attained by using $B \to A$ and $A \to C$.

Then, the following

1. $A \to C$
2. $B \to A$
3. $C \to A$
4. $C \to B$
5. $A \to B$

can be simplified to

1. $A \rightarrow C$
2. $B \rightarrow A$
3. $C \rightarrow A$
4. $C \rightarrow B$

since **transitivity** rule tells us $A \rightarrow B$ can be attained by using $A \rightarrow C$ and $C \rightarrow B$.

Rough Works:

1. Add attributes from A^+ to L.H.S of $A_1A_2 \cdots A_n \rightarrow A^+$.
2. Show that the R.H.S is still A^+.

Notes:

- Closure (Definition)
  - Suppose $A = \{A_1, A_2, ..., A_n\}$ is a set of attributes of R and S is a set of FD'.

    The closure of A under the set S, denoted by A^+, is the set of attributes B such that any relation that satisfies all the FD's in S is also satisfies $A_1A_2 \cdots A_n \rightarrow A^+$.
  - In other words $A_1 \cdots A_n \rightarrow A^+$ follows from the FD's of S.
- I wish the definition is a little more clear :(

9. **Notes:**

- Basis
  - Is the set of FD's that represent the full set of FD's of a relation
- Finding minal bases for FD's
  - A minimal basis for a relation satisfies three conditions
    1. All the FD's in B have singleton right sides.
    2. If any FD is removed from B, the result is no longer a basis
    3. If for any FD in B we remove on or more attributes from the left side of F, the result is no longer a basis
  - Steps
    1. Get rid of redundant attributes
       *
    2. Get rid of redundant dependencies
- Example

  The following

1. $A \to B$
2. $ABCD \to E$
3. $EF \to G$
4. $EF \to H$
5. $ACDF \to E$
6. $ACDF \to G$

can be simplified to

1. $A \to B$
2. $ACD \to E$ B removed here!!
3. $EF \to G$
4. $EF \to H$
5. $ACDF \to E$
6. $ACDF \to G$

since by **augmentation rule**, $A \to B$ can be re-written as $ACD \to BCD$. And by **trivial rule**, $ACD \to BCD$ can be re-written as $ACB \to ABCD$, which then can be used to get E from $ABCD \to E$.

Second, the following

1. $A \to B$
2. $ACD \to E$
3. $EF \to G$
4. $EF \to H$
5. $ACDF \to E$
6. $ACDF \to G$

can be simplified to

1. $A \to B$
2. $ACD \to E$
3. $EF \to G$
4. $EF \to H$
5. $ACD \to E$ F Removed here!!
6. $ACDF \to G$

since **augmenting left side** rule tells us $ACDF \to E$ can be attained by adding F to ACD in $ACD \to E$.

Then, the following

1. $A \to B$
2. $ACD \to E$
3. $EF \to G$

4. $EF \to H$
5. $ACD \to E$
6. $ACDF \to G$

can be simplified to

1. $A \to B$
2. $ACD \to E$
3. $EF \to G$
4. $EF \to H$
5. $ACDF \to G$

by removing redundant $ACD \to E$.

Then, the following

1. $A \to B$
2. $ACD \to E$
3. $EF \to G$
4. $EF \to H$
5. $ACD \to E$
6. $ACDF \to G$

can be simplified to

1. $A \to B$
2. $ACD \to E$
3. $EF \to G$
4. $EF \to H$

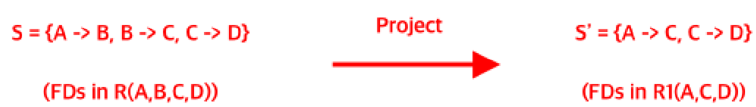
since **augmentation** rule tells us $ACDF \to G$ can be re-written to get $ACDF \to EF$ and then use **transtivity rule** on $EF \to G$ to get $ACDF \to G$.

10. a)
- Finding subsets
  $X_1 = \{A\}$, $X_2 = \{B\}$, $X_3 = \{C\}$, $X_4 = \{A, B\}$, $X_5 = \{A, C\}$, $X_6 = \{B, C\}$, $X_7 = \{A, B, C\}$, $X_8 = \{\}$
- Finding X_i^+
  1. $X_1^+ = \{A\}$
  2. $X_2^+ = \{B\}$
  3. $X_3^+ = \{C, E, A\}$
  4. $X_4^+ = \{A, B, C, D, E\}$
  5. $X_5^+ = \{A, C, E\}$
  6. $X_6^+ = \{A, B, C, D, E\}$
  7. $X_7^+ = \{A, B, C, D, E\}$
  8. $X_8^+ = \{\}$

- Putting all nontirival FD's in T
  $T = \{C \to E, C \to A, AB \to C, AB \to D, AB \to E, AC \to E, BC \to A, BC \to D, BC \to E, ABC \to D, ABC \to E\}$
- Finding minimal basis for the FD of S
  $T_{\text{minimal}} = \{C \to E, C \to A, B \to C, B \to D, B \to E, B \to A\}$

Notes:

- Projecting Functional Dependency
  - Remember that π is equivalent to SQL's SELECT of columns
  - Answers the question to "given a relation R and a set of FD's S, what FD's hold if we project R by $R_1 = \Pi_L(R)$?
  - The new set S'
    1. Follows from S
    2. Involves only attributes of R_1



- Algorithm for Projecting a set of Functional Dependencies
  - Inputs and Outputs
    * Input
      - **R:** The original relation
      - **R1:** The projection of R
      - **S:** The set of FD's that hold in R
    * Output
      - **T:** The set of FD's that hold in R_1
  - Steps
    1. Initialize $T = \{\}$.
    2. Construct a set of all subsets of attributes of R_1 called X
    3. Compute X_i^+ for all members of X under S.
       * X_i^+ may consist of attributes that are not in $R1$
    4. Add to T all nontirival FD's $X \to A$ such that A is both in X_i^+ and an attributes of R_1
    5. Now, T is a basis for the FD's that hold in $R1$ but may not be a minimal basis. Modify T as follows.
       a) If there is an FD in F in T that follows from the other FD's in T, remove F
       b) Let $Y \to B$ be an FD in T, with at least two attributes in Y. Remove one attribute from Y and call it Z. If $Z \to B$ follows from the FD's in T, then replace $Z \to B$ with $Y \to B$.

– Example

Consider $R(A, B, C, D)$ has FD's$A \to B$, $B \to C$, and $C \to D$.
$R_1(A, C, D)$ is a projection of R. Find FD's for R_1

1. Initialize $T = \{\}$.
   * $T = \{\}$
2. Construct a set of all subsets of attributes of R_1 called X
   * There are 8 subsets

     $X_1 = \{A\}$, $X_2 = \{C\}$, $X_3 = \{D\}$, $X_4 = \{A, C\}$, $X_5 = \{A, D\}$, $X_6 = \{C, D\}$, $X_7 = \{A, C, D\}$, $X_8 = \{\}$
3. Compute X_i^+ for all members of X under S.
   * $X_1 = \{A\}$

     $X_1^+ = \{A, B, C, D\}$
   * $X_2 = \{C\}$

     $X_2^+ = \{C, D\}$
   * $X_3 = \{D\}$

     $X_3^+ = \{D\}$
   * $X_4 = \{A, C\}$

     $X_4^+ = \{A, B, C, D\}$
   * $X_5 = \{A, D\}$

     $X_5^+ = \{A, B, C, D\}$
   * $X_6 = \{C, D\}$

     $X_6^+ = \{C, D\}$
   * $X_7 = \{A, C, D\}$

     $X_7^+ = \{A, B, C, D\}$
   * $X_8 = \{\}$

     $X_8^+ = \{\}$
4. Add to T all nontirival FD's $X \to A$ such that A is both in X_i^+ and an attributes of R_1
   * $T = \{A \to C, A \to D, C \to D, AC \to D, AD \to C\}$
5. Now, T is a basis for the FD's that hold in $R1$ but may not be a minimal basis. Modify T as follows.
   * $T = \{C \to D, A \to D, A \to C\}$

b)
- Finding subsets
  $X_1 = \{A\}$, $X_2 = \{B\}$, $X_3 = \{C\}$, $X_4 = \{A, B\}$, $X_5 = \{A, C\}$, $X_6 = \{B, C\}$, $X_7 = \{A, B, C\}$, $X_8 = \{\}$
- Finding X_i^+
  1. $X_1^+ = \{A, D\}$
  2. $X_2^+ = \{B\}$
  3. $X_3^+ = \{C\}$
  4. $X_4^+ = \{A, B, D, E\}$
  5. $X_5^+ = \{A, B, C, D, E\}$
  6. $X_6^+ = \{B, C\}$
  7. $X_7^+ = \{A, B, C, D, E\}$
  8. $X_8^+ = \{\}$
- Putting all nontirival FD's in T
  $T = \{A \rightarrow D, AB \rightarrow D, AB \rightarrow E, AC \rightarrow B, AC \rightarrow D, AC \rightarrow E, ABC \rightarrow D, ABC \rightarrow E\}$
- Finding minimal basis for the FD of S
  $T_{\text{minimal}} = \{A \rightarrow D, AB \rightarrow E, AC \rightarrow B, AC \rightarrow E\}$

c)
- Finding subsets
  $X_1 = \{A\}$, $X_2 = \{B\}$, $X_3 = \{C\}$, $X_4 = \{A, B\}$, $X_5 = \{A, C\}$, $X_6 = \{B, C\}$, $X_7 = \{A, B, C\}$, $X_8 = \{\}$
- Finding X_i^+
  1. $X_1^+ = \{A\}$
  2. $X_2^+ = \{B\}$
  3. $X_3^+ = \{C\}$
  4. $X_4^+ = \{A, B, D\}$
  5. $X_5^+ = \{A, B, C, D, E\}$
  6. $X_6^+ = \{A, B, C, D, E\}$
  7. $X_7^+ = \{A, B, C, D, E\}$
  8. $X_8^+ = \{\}$
- Putting all nontirival FD's in T
  $T = \{AB \rightarrow D, AC \rightarrow B, AC \rightarrow D, AC \rightarrow E, BC \rightarrow A, BC \rightarrow D, BC \rightarrow E, ABC \rightarrow D, ABC \rightarrow E\}$
- Finding minimal basis for the FD of S
  $T_{\text{minimal}} = \{A \rightarrow D, AC \rightarrow B, C \rightarrow A, C \rightarrow B, C \rightarrow E\}$

d)
- Finding subsets
  $X_1 = \{A\}$, $X_2 = \{B\}$, $X_3 = \{C\}$, $X_4 = \{A, B\}$, $X_5 = \{A, C\}$, $X_6 = \{B, C\}$, $X_7 = \{A, B, C\}$, $X_8 = \{\}$
- Finding X_i^+
  1. $X_1^+ = \{A, B, C, D, E\}$
  2. $X_2^+ = \{A, B, C, D, E\}$

3. $X_3^+ = \{A, B, C, D, E\}$
4. $X_4^+ = \{A, B, C, D, E\}$
5. $X_5^+ = \{A, B, C, D, E\}$
6. $X_6^+ = \{A, B, C, D, E\}$
7. $X_7^+ = \{A, B, C, D, E\}$
8. $X_8^+ = \{\}$

- Putting all nontirival FD's in T
  $T = \{A \to B, A \to C, A \to D, A \to E, B \to A, B \to C, B \to D, B \to E, C \to A, C \to B, C \to D, C \to E, AB \to C, AB \to D, AB \to E, AC \to B, AC \to D, AC \to E, BC \to A, BC \to D, BC \to E, ABC \to D, ABC \to E\}$
- Finding minimal basis for the FD of S
  $T_{\text{minimal}} = \{A \to B, A \to C, A \to D, A \to E, B \to A, B \to C, B \to D, B \to E, C \to A, C \to B, C \to D, C \to E\}$