CSC148 Worksheet 11 Solution

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Question 1

a. Here, the constant time means the running time of accessing and assigning element by index doesn't depend on the length of the list.

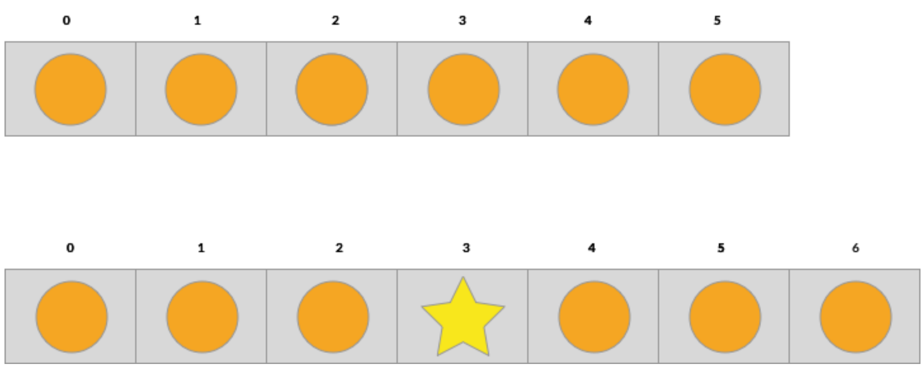
b.

$$n - i$$

many elements need to be shifted to right.

Notes:

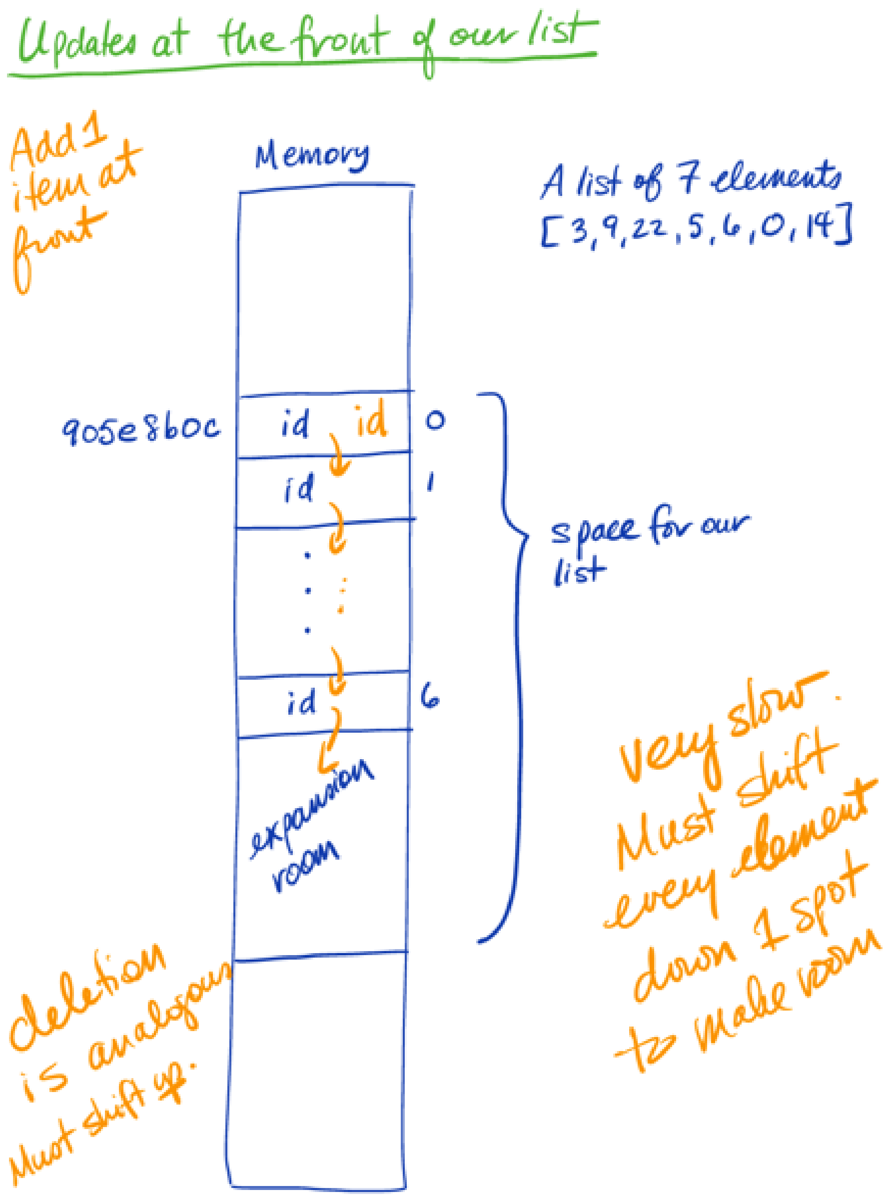
- The following example tells us



  to position an element at index $i = 3$ of the list, $n - i = 6 - 3 = 3$ elements must be moved over.

  Using this fact, we can generalize that to position an element at index i of the list, $n - i$ many elements must be shifted.

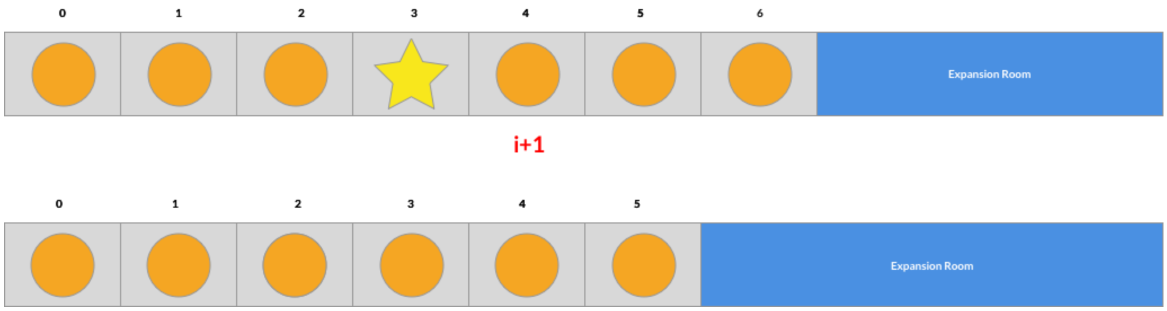
- Learned that when items shifts, it shifts into the expansion room.



c. Because we know the list size stays as is when an element is removed, we can conclude 0 many list elements must be moved.

Correct Solution:

The following example tells us



when an element at index $i = 3$ is removed from the list $n - (i + 1) = 7 - (3 - 1) = 3$ many elements must be moved.

Using this fact, we can generalize that when an element is removed, $n-(i+1) = n-i-1$ many elements must be shifted to left.

d. i. A solution is *LIST.remove(...)*.

The answer to question 1.d tells us when an element is removed, $n - i$ must be shifted to left.

Using this fact, we can write a list of smaller size needs to shift elements less.

Then, it follows from this fact that $n = 100$ works faster than $n = 1,000,000$.

ii. A solution is *LIST.append(...)*

The definition of append tells us that upon call, an element is added to the end of a list, it takes a constant time to add an element as long as the expansion room is not filled.

It follows from this fact that $n = 100$ and $n = 1,000,000$ takes roughly the same amount of time.

e. The definition of *Queue* tells us *Queue* is FIFO. That is, the first element inserted is the first to come out.

Since we know the front of *Queue* is the front of the list, we can conclude *LIST.insert(...)* and *LIST.pop(0)* are used to support *QUEUE.enqueue* and *QUEUE.dequeue*, respectively.

Since we know *LIST.insert(...)* requires shifting of elements by $n - i = n - 0 = n$ and *LIST.pop(0)* requires shifting of $n - i - 1 = n - 1$ many elements, we can conclude both *QUEUE.enqueue* and *QUEUE.dequeue* takes longer time as size increases.

Correct Solution:

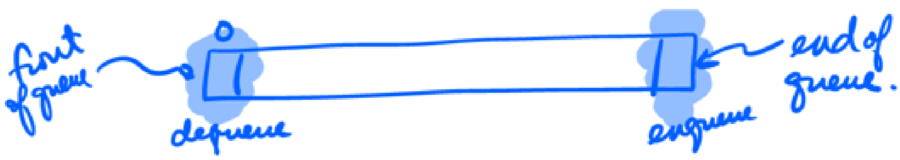
The definition of *Queue* tells us *Queue* is FIFO. That is, the first element inserted is the first to come out.

Since we know the front of *Queue* is the front of the list, we can conclude *LIST.append(...)* and *LIST.pop(0)* are used to support *QUEUE.enqueue* and *QUEUE.dequeue*, respectively.

Since we know *LIST.append(...)* requires the shifting of elements by $n - i = n - n = 0$ and *LIST.pop(0)* requires shifting of $n - i - 1 = n - 1$ many elements, we can conclude *QUEUE.dequeue* takes longer time as size increases.

Notes:

- Learned that the **front of queue** is where *QUEUE.dequeue* occurs.



Question 2

a. **Stack 1:**

$$\begin{aligned}\text{Number of steps for } \textit{s.push(1)} + \text{Number of steps for } \textit{s.pop()} &= 1 + 1 \\ &= 2\end{aligned}$$

Stack 2:

$$\begin{aligned}\text{Number of steps for } \textit{s.push(1)} + \text{Number of steps for } \textit{s.pop()} &= (n+1) + (n+1) \\ &= 2n + 1\end{aligned}$$

b. **Stack 1:**

We need to determine the total number of steps taken in the code using *Stack 1*.

First, we need to find the number of steps taken per iteration.

The code tells us there is only *s.push(i)* in for loop.

Since we know the *push* operation in *Stack 1* takes 1 step, we can conclude 1 step is taken per iteration.

Finally, we need to determine the total number of steps taken.

Because we know the loop performs 5 iterations and each iteration takes 1 step, we can conclude the code takes total of

$$5 \cdot 1 = 5 \tag{1}$$

steps.

<u>Stack 2:</u>

We need to determine the total number of steps taken in the code using *Stack 2*.

First, we need to find the number of steps taken per iteration.

The code tells us there is only *s.push(i)* in for loop.

Since we know the list in s starts empty, and the size of list grows to i at i^{th} iteration, we can conclude the size of list n is i.

Since we know the *push* operation in *Stack 2* takes $n+1$ step, and since we know $n = i$, we can conclude $i+1$ step is taken per iteration.

Finally, we need to determine the total number of steps taken.

Because we know the loop starts at $i = 0$, ends at $i = 4$ with each iteration taking $i+1$ steps, we can conclude the code has total of

$$\begin{aligned}
\sum_{i=0}^{4}(i+1) &= \sum_{i'=1}^{5} i' && (1)\\
&= \frac{5(5+1)}{2} && (2)\\
&= \frac{30}{2} && (3)\\
&= 15 && (4)
\end{aligned}$$

steps.

c. <u>**Stack 1:**</u>

We need to determine the total number of steps taken in the code using *Stack 1*.

First, we need to find the number of steps taken per iteration.

The code tells us there is only *s.push(i)* in for loop.

Since we know the *push* operation in *Stack 1* takes 1 step, we can conclude 1 step is taken per iteration.

Finally, we need to determine the total number of steps taken.

Because we know the loop performs $(k-1)-0+1=k$ iterations and each iteration takes 1 step, we can conclude the code takes total of

$$k \cdot 1 = k \tag{5}$$

steps.

<u>**Stack 2:**</u>

We need to determine the total number of steps taken in the code using *Stack 2*.

First, we need to find the number of steps taken per iteration.

The code tells us there is only *s.push(i)* in for loop.

Since we know the list in s starts empty, and the size of list grows to i at i^{th} iteration, we can conclude the size of list n is i.

Since we know the *push* operation in *Stack 2* takes $n+1$ step, and since we know $n=i$, we can conclude $i+1$ step is taken per iteration.

Finally, we need to determine the total number of steps taken.

Because we know the loop starts at $i=0$, ends at $i=k-1$ with each iteration taking $i+1$ steps, we can conclude the code has total of

$$\begin{aligned} \sum_{i=0}^{k-1}(i+1) &= \sum_{i'=1}^{k} i' && (1) \\ &= \frac{k(k+1)}{2} && (2) \end{aligned}$$

steps.

Correct Solution:

Stack 1:

We need to determine the total number of steps taken in the code using *Stack 1*.

First, we need to find the number of steps taken per iteration.

The code tells us there is only *s.push(i)* in for loop.

Since we know the *push* operation in *Stack 1* takes 1 step, we can conclude 1 step is taken per iteration.

Finally, we need to determine the total number of steps taken.

Because we know the loop performs $(k-1)-0+1=k$ iterations and each iteration takes 1 step, we can conclude the code takes total of

$$k \cdot 1 = k \tag{1}$$

steps.

Stack 2:

We need to determine the total number of steps taken in the code using *Stack 2*.

First, we need to find the number of steps taken per iteration.

The code tells us there is only *s.push(i)* in for loop.

Since we know the list in s starts empty, and the size of list grows to i at i^{th} iteration, we can conclude the size of list n is i.

Since we know the *push* operation in *Stack 2* takes $n+1$ step, and since we know $n=i$, we can conclude $i+1$ step is taken per iteration.

Finally, we need to determine the total number of steps taken.

Because we know the loop starts at $i=0$, ends at $i=k-1$ with each iteration taking $i+1$ steps, we can conclude the code has total of

$$\sum_{i=0}^{k-1}(i+1) = \sum_{i'=1}^{k} i' \tag{1}$$
$$= \frac{k(k+1)}{2} \tag{2}$$

steps.

d. **Stack 1:**

We need to determine the total number of steps taken using *Stack 1*.

First, we need to determine the number of steps taken by *s2.push(s1.pop())*.

The problem tells us both *s2.push(...)* and *s1.pop()* takes 1 step.

Using this fact, we can conclude *s2.push(s1.pop())* takes 1 step.

Second, we need to determine the total number of steps taken by loop 1.

The code tells us that for loop 1, an element popped from *s1* is inserted to *s2*, and this is repeated until no elements are left in *s1*.

Since we know *s1* starts with n many elements, and its size decreases by 1 until its length is 0, we can write that with k representing k^{th} iteration in loop 1, the terminating condition is reached when

$$n - k \leq 0 \tag{1}$$
$$n \leq k \tag{2}$$

Since we are looking for the smallest value of k (because it represents the number of iterations), we can conclude loop 1 has

$$\lceil n \rceil = n \tag{3}$$

many iterations.

Since we know each iteration in loop 1 takes 1 step, we can conclude loop 1 takes total of

$$n \cdot 1 = n \tag{4}$$

steps.

Third, we need to show that the second loop takes total of 0 steps.

The code tells us *s2* starts a stack of size 0.

Since we know loop 2 doesn't run when the size of stack in *s2* is 0, we can conclude loop 2 has 0 iterations.

Using this fact, we can conclude loop 2 has total of 0 steps.

Finally, we need to determine the total number of steps taken in this code.

Since we know loop 1 takes total of n steps and loop 2 takes total of 0 step, we can conclude the total number of steps taken by the code is

$$n + 0 = n \tag{5}$$

steps.

<u>Stack 2:</u>

We need to determine the total number of steps taken using *Stack 2*.

First, we need to determine the number of steps taken by *s2.push(s1.pop())* in loop 1.

The code tells us that *s1.pop()* operation takes $n_1 + 1$ steps and *s2.push(...)* takes $n_2 + 1$ steps, where n_1 and n_2 be the size of stack for *s1* and *s2*, respectively.

Since we know $s2$ starts as an empty stack, and $s1$ starts as a stack with the size of n, and since we know *s2.push()* and *s1.pop(...)* causes the stack size of *s2* and *s1* to increase and decrease by 1 per iteration, respectively, we can conclude that at k^{th} iteration, *s2.push(s1.pop())* takes total of

$$(n - k + 1) + (k + 1) = n + 2 \tag{6}$$

steps.

Second, we need to determine the total number of steps taken by loop 1.

The code tells us that for loop 1, an element popped from *s1* is inserted to *s2*, and this is repeated until no elements are left in *s1*.

Since we know *s1* starts with n many elements, and its size decreases by 1 until its length is 0, we can write that with k representing k^{th} iteration in loop 1, the terminating condition is reached when

$$n - k \leq 0 \tag{7}$$
$$n \leq k \tag{8}$$

Since we are looking for the smallest value of k (because it represents the number of iterations), we can conclude loop 1 has

$$\lceil n \rceil = n \tag{9}$$

many iterations.

Since we know each iteration in loop 1 takes $n+2$ step, we can conclude loop 1 takes total of

$$n \cdot (n+2) = n(n+2) \tag{10}$$

steps.

Third, we need to show that the second loop takes total of 0 steps.

The code tells us *s2* starts a stack of size 0.

Since we know loop 2 doesn't run when the size of stack in *s2* is 0, we can conclude loop 2 has 0 iterations.

Using this fact, we can conclude loop 2 has total of 0 steps.

Finally, we need to determine the total number of steps taken in this code.

Since we know loop 1 takes total of n steps and loop 2 takes total of 0 step, we can conclude the total number of steps taken by the code is

$$n(n+2) + 0 = n(n+2) \tag{11}$$

steps.

Correct Solution:

Stack 1:

We need to determine the total number of steps taken using *Stack 1*.

First, we need to determine the number of steps taken by *s2.push(s1.pop())* in loop 1.

The problem tells us both *s2.push(...)* and *s1.pop()* takes 1 step.

Using this fact, we can conclude *s2.push(s1.pop())* takes 1 step.

Second, we need to determine the total number of steps taken by loop 1.

The code tells us that for loop 1, an element popped from *s1* is inserted to *s2*, and this is repeated until no elements are left in *s1*.

Since we know *s1* starts with n many elements, and its size decreases by 1 until its length is 0, we can write that with k representing k^{th} iteration in loop 1, the terminating condition is reached when

$$n - k \leq 0 \tag{1}$$
$$n \leq k \tag{2}$$

Since we are looking for the smallest value of k (because it represents the number of iterations), we can conclude loop 1 has

$$\lceil n \rceil = n \tag{3}$$

many iterations.

Since we know each iteration in loop 1 takes 1 step, we can conclude loop 1 takes total of

$$n \cdot 1 = n \tag{4}$$

steps.

Third, we need to show that the second loop also takes total of n steps.

The code tells us that for loop 2, it functions the same as loop 1, and we know from the header that *s1.push()* and *s2.pop()* in loop 2 takes the number of steps as *s2.push()* and *s1.pop()* in loop 1.

Since we know *s2* starts at stack size of n and *s1* starts at size of 0 just like how loop 1 had stack size of n for *s1* and 0 for *s2*, we can conclude loop 2 is loop 1 but working in reverse.

Since we know loop 1 takes total of n steps, we can conclude loop 2 also takes n steps.

Finally, we need to determine the total number of steps taken in this code.

Since we know loop 1 and loop 2 both take total of n steps, we can conclude the total number of steps taken by the code is

$$n + n = 2n \tag{5}$$

steps.

<u>Stack 2:</u>

We need to determine the total number of steps taken using *Stack 2*.

First, we need to determine the number of steps taken by *s2.push(s1.pop())* in loop 1.

The code tells us that *s1.pop()* operation takes $n_1 + 1$ steps and *s2.push(...)* takes $n_2 + 1$ steps, where n_1 and n_2 be the size of stack for *s1* and *s2*, respectively.

Since we know $s2$ starts as an empty stack, and $s1$ starts as a stack with the size of n, and since we know *s2.push()* and *s1.pop(...)* causes the stack size of *s2* and *s1* to increase and decrease by 1 per iteration, respectively, we can conclude that at k^{th} iteration, *s2.push(s1.pop())* takes total of

$$(n - k + 1) + (k + 1) = n + 2 \tag{1}$$

steps.

Second, we need to determine the total number of steps taken by loop 1.

The code tells us that for loop 1, an element popped from *s1* is inserted to *s2*, and this is repeated until no elements are left in *s1*.

Since we know *s1* starts with n many elements, and its size decreases by 1 until its length is 0, we can write that with k representing k^{th} iteration in loop 1, the terminating condition is reached when

$$\begin{aligned} n - k &\leq 0 \qquad (2) \\ n &\leq k \qquad (3) \end{aligned}$$

Since we are looking for the smallest value of k (because it represents the number of iterations), we can conclude loop 1 has

$$\lceil n \rceil = n \qquad (4)$$

many iterations.

Since we know each iteration in loop 1 takes $n + 2$ step, we can conclude loop 1 takes total of

$$n \cdot (n + 2) = n(n + 2) \qquad (5)$$

steps.

Third, we need to show that the second loop also takes total of $n(n + 2)$ steps.

The code tells us that for loop 2, it functions the same as loop 1, and we know from the header that *s1.push()* and *s2.pop()* in loop 2 takes the number of steps as *s2.push()* and *s1.pop()* in loop 1.

Since we know *s2* starts at stack size of n and *s1* starts at size of 0 just like how loop 1 had stack size of n for *s1* and 0 for *s2*, we can conclude loop 2 is loop 1 but working in reverse.

Since we know loop 1 takes total of $n(n + 2)$ steps, we can conclude loop 2 also takes $n(n + 2)$ steps.

Finally, we need to determine the total number of steps taken in this code.

Since we know both loop 1 and loop 2 take total of $n(n+2)$ steps, we can conclude the total number of steps taken by the code is

$$n(n+2) + n(n+2) = 2n(n+2) \tag{6}$$

steps.