

# Worksheet 6 Solution

March 16, 2020

## Question 1

- a.  $P(123) \wedge (\forall n \in \mathbb{N}, P(n) \Rightarrow n \leq 123)$
- b.  $isCD(x, y, d): \exists x, y, d \in \mathbb{Z}, d \mid x \wedge d \mid y$   
 $isGCD(x, y, d): \exists x, y, d \in \mathbb{Z}, (x = 0 \wedge y = 0 \wedge d = 0) \vee ((x \neq 0 \vee y \neq 0) \wedge isCD(x, y, d) \wedge \forall e \in \mathbb{Z}, e > d \Rightarrow \neg isCD(x, y, e))$
- c. Statement:  $\forall x \in \mathbb{Z}^+, IsGCD(x, 0, x)$

For the value  $x$ , because we know  $x \mid x$ , and  $\forall n \in \mathbb{Z}^+$  and  $\forall d \in \mathbb{Z}, d \mid n \Rightarrow d \leq n$ ,  $x$  is the biggest divisor of  $x$

For the value 0, because we know anything that divides 0 is 0, and  $\exists k \in \mathbb{Z}, 0 = k \times 0$ ,  $k$  can be chosen to be  $x$ .

Then, it follows from the definition of GCD that the statement  $IsGCD(x, 0, x)$  is true.

- d.  $\forall a, b \in \mathbb{Z}, (a \neq 0 \vee b \neq 0) \Rightarrow \exists p, q \in \mathbb{Z}, gcd(a, b) = ap + qb \wedge \forall m \in \mathbb{Z}, m < gcd(a, b) \wedge m \neq ap + qb$

## Question 2

- a. Let  $n \in \mathbb{Z}$ . Assume  $\exists l \in \mathbb{Z}, n = 2l$ .

Then,

$$n^2 - 3n = (2l)^2 = 3(2l) \quad (1)$$

$$= 4l^2 - 6l \quad (2)$$

$$= 2(2l^2 - 3l) \quad (3)$$

Since  $2l^2 - 3l \in \mathbb{Z}$ , it follows from the definition of even number that  $n^2 - 3n$  is even

b. Let  $n \in \mathbb{Z}$ . Assume  $\exists l \in \mathbb{Z}, n = 2l - 1$ .

Then,

$$n^2 - 3n = (2l - 1)^2 = 3(2l - 1) \quad (1)$$

$$= 4l^2 - 4l + 1 - 6l + 3 \quad (2)$$

$$= 4l^2 - 10l + 4 \quad (3)$$

$$= 2(2l^2 - 5l + 2) \quad (4)$$

Since  $2l^2 - 5l + 2 \in \mathbb{Z}$ , it follows from the definition of even number that  $n^2 - 3n$  is even

### Question 3