CSC263 Worksheet 1 Solution

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Question 1

a. *Proof.* Assume the statement $P(115)$ is true. That is, $\sum_{i=0}^{i=115} 2^i = 2^{115+1}$.

We need to prove $\sum_{i=0}^{i=116} 2^i = 2^{116+1}$.

Starting from the left, we can write

$$\sum_{i=0}^{i=116} 2^i = \sum_{i=0}^{i=115} 2^i + 2 \tag{1}$$

Then, using the assumption $\sum_{i=0}^{i=115} 2^i = 2^{115+1}$, we can conclude

$$\begin{aligned}
\sum_{i=0}^{i=116} 2^i &= 2^{115+1} + 2^{116} && (2)\\
&= 2^{116} + 2^{116} && (3)\\
&= 2^{116}(1+1) && (4)\\
&= 2 \cdot 2^{116} && (5)\\
&= 2^{116+1} && (6)
\end{aligned}$$

$\square$

b. *Proof.* No. The statement is not true for every natural natural number.

We will prove this by counter example. That is, $\exists n \in \mathbb{N}, \sum_{i=0}^{i=n} 2^i \neq 2^{n+1}$.

Let $n = 0$.

Then, starting from the left hand side, it follows from the fact $n = 0$ that

$$\begin{align} \sum_{i=0}^{i=n} 2^i &= \sum_{i=0}^{i=0} 2^i \tag{1} \\ &= 0 \tag{2} \end{align}$$

Now, for the right hand side, using the same fact, we can write

$$\begin{align} 2^{0+1} &= 2^1 \tag{3} \\ &= 2 \tag{4} \end{align}$$

$\square$

Question 2

- **<u>Statement:</u>** $\forall n \in \mathbb{N}, \exists d \in \mathbb{Z}, 8^n - 1 = 7d$

Proof. We will prove this statement by induction on n.

<u>Base Case:</u>

Let $n = 0$.

We need to prove $8^n - 1 = 7 \cdot 0$.

Starting from the left hand side, using the fact $n = 0$, we can conclude,

$$\begin{align} 8^0 - 1 &= 1 - 1 \tag{1} \\ &= 0 \tag{2} \\ &= 7 \cdot 0 \tag{3} \end{align}$$

<u>Inductive Step:</u>

Let $n \in \mathbb{N}$. Assume there is an integer d such that $8^n - 1 = 7d$.

We need to prove there is an integer $\tilde{d}$ such that $8^{n+1} - 1 = 7\tilde{d}$.

Let $\tilde{d} = 8^n + d$.

Starting from the left hand side, we can write

$$
\begin{aligned}
8^{n+1} - 1 &= 8^n + 8^n + 8^n + 8^n + 8^n + 8^n + 8^n + 8^n - 1 && (4) \\
&= 8^n + 8^n + 8^n + 8^n + 8^n + 8^n + 8^n + (8^n - 1) && (5)
\end{aligned}
$$

Then, using inductive hypothesis, i.e. $8^n - 1 = 7d$, we can conclude

$$
\begin{aligned}
8^{n+1} - 1 &= 8^n + 8^n + 8^n + 8^n + 8^n + 8^n + 8^n + 7d && (6) \\
&= 7 \cdot 8^n + 7d && (7) \\
&= 7 \cdot (8^n + d) && (8) \\
&= 7 \cdot \tilde{d} && (9)
\end{aligned}
$$

$\square$

Question 3

Question 4