Midterm 2 Version 3 Solution

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Question 1

a.

$$\begin{aligned}
165 \div 2 &= 82, \text{remainders } \mathbf{1} \\
82 \div 2 &= 41, \text{remainders } \mathbf{0} \\
41 \div 2 &= 20, \text{remainders } \mathbf{1} \\
20 \div 2 &= 10, \text{remainders } \mathbf{0} \\
10 \div 2 &= 5, \text{remainders } \mathbf{0} \\
5 \div 2 &= 2, \text{remainders } \mathbf{1} \\
2 \div 2 &= 1, \text{remainders } \mathbf{0} \\
1 \div 2 &= 0, \text{remainders } \mathbf{1}
\end{aligned}$$

From the above, we can conclude the binary representation of the decimal number 165 is $(10100101)_2$

b. The largest number that can be expressed by an n-digit balanced ternary representation is

$$\sum_{i=0}^{n-1} 3^i = \frac{1}{2} \cdot (3^n - 1) \tag{1}$$

Notes:

- Geometric Series

$$\sum_{i=0}^{n} r^i = \frac{1 - r^{n+1}}{1 - r}, \text{ where } |r| > 1$$

c.

$f(n) \in \mathcal{O}(n)$	True	$g(n) \in \Omega(n)$	True	$f(n) \in \Omega(g(n))$	True
$f(n) \in \Theta(g(n))$	False	$g(n) \in \Theta(n)$	False	$f(n) + g(n) \in \Theta(g(n))$	False

Correct Solution:

$f(n) \in \mathcal{O}(n)$	True	$g(n) \in \Omega(n)$	True	$f(n) \in \Omega(g(n))$	True
$f(n) \in \Theta(g(n))$	False	$g(n) \in \Theta(n)$	False	$f(n) + g(n) \in \Theta(g(n))$	True

Notes:

- Note that for $f(n) + g(n) \in \Theta(g(n))$, large values of n causes $g(n) = n^{\log_2 n}$ to dominate $f(n) = \frac{3n}{\log_2 n+8}$. This causes the inequality to be simplified to

$$c_1 \cdot n^{\log_2 n} \leq n^{\log_2 n} \leq c_2 \cdot n^{\log_2 n} \tag{1}$$

It follows from above the answer is True.

d.

k	0	1	2
$i * i$	$3 = 3^{2^0}$	$9 = 3^{2^1}$	$81 = 3^{2^4}$

From the rough work, we can deduce the value of i after k iterations is

$$3^{2^k} \tag{1}$$

e. Loop termination occurs when $i_k \geq n^3$.

We need to find the smallest value of k, and the value is

$$\lceil \log_2 3 \log_3 n \rceil \tag{1}$$

Question 2

Question 3

Question 4