

# Midterm 2 Version 2 Solution

Hyungmo Gu

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## Question 1

a.

$$100 \div 3 = 33, \text{ Remainder } \mathbf{1}$$

$$33 \div 3 = 11, \text{ Remainder } \mathbf{0}$$

$$11 \div 3 = 3, \text{ Remainder } \mathbf{2}$$

$$3 \div 3 = 1, \text{ Remainder } \mathbf{0}$$

$$1 \div 3 = 0, \text{ Remainder } \mathbf{1}$$

It follows from above that the ternary representation of 100 is  $(10201)_3$ .

### Attempt 2:

$$100 + (-1 \cdot 3^4) = 100 - 81 = 19$$

$$19 + (-1 \cdot 3^3) = 19 - 27 = -8$$

$$-8 + (+1 \cdot 3^2) = -8 + 9 = 1$$

$$1 + (0 \cdot 3^1) = 1 + 0 = 1$$

$$1 + (-1 \cdot 3^0) = 1 - 1 = 0$$

So by flipping the signs, and reading from top to bottom, we can conclude the balanced ternary representation of 100 is  $(11T101)_{bt}$

### Notes:

- Balanced ternary representation expresses a decimal using 1, 0 and  $-1$
- **T** represents negative sign in balanced ternary representation.
- Is my way of calculating balanced ternary representation correct? My approach was ‘which sign should be used given  $3^n$  so the calculation stops at  $3^0$ ?’

b. The largest number expressible by an n-digit binary representation is

$$\sum_{i=0}^{n-1} 2^i \quad (1)$$

**Correct Solution:**

$$\sum_{i=0}^{n-1} 2^i = \frac{1 - 2^{n-1+1}}{1 - 2} = 2^n - 1 \quad (1)$$

**Notes:**

- Noticed professor simplified solution using geometric series
- Geometric series with finite sum

$$\sum_{i=0}^n r^k = \frac{1 - r^{n+1}}{1 - r}, \text{ where } |r| > 1 \quad (2)$$

c.

$f(n) \in \mathcal{O}(n)$	True	$g(n) \in \Omega(n)$	False	$f(n) \in \Omega(g(n))$	True
$f(n) \in \Theta(g(n))$	False	$g(n) \in \Theta(\log_3 n)$	False	$f(n) + g(n) \in \Theta(f(n))$	True

**Notes:**

- Learned  $\sqrt{n}$  rises faster than  $\log n$ .
- Learned if  $g(n) \in \Theta(f(n))$  is true then  $f(n) + g(n) \in \Theta(f(n))$  is true.

d.

$k$	0	1	2	3
$i \cdot i \cdot i$	$2 = 2^{3^0}$	$2^3 = 2^{3^1}$	$2^9 = 2^{3^2}$	$2^{27} = 2^{3^3}$

We can deduce from above that  $i_k = 2^{3^k}$

e.  $\lceil \log_3(\log_2(n) - 1) \rceil$

**Correct Solution:**

We want to find the smallest value of  $k$  satisfying  $2 \cdot i_k \geq n$ , and the value is

$$\lceil \log_3(\log_2(n) - 1) \rceil$$

## Question 2

- **Statement:**  $\forall n \in \mathbb{N}, n \geq 2 \Rightarrow \prod_{i=1}^n \frac{2^i-1}{2^i} \geq \frac{1}{2n}$

Let  $n \in \mathbb{N}$ . Assume  $n \geq 2$ .

We will prove the statement using induction on  $n$ .

**Base Case ( $n = 2$ ):**

Let  $n = 2$ .

We want to show  $\prod_{i=1}^2 \frac{2^i-1}{2^i} \geq \frac{1}{2 \cdot (2)}$

Starting from  $\prod_{i=1}^2 \frac{2^i-1}{2^i}$ , we can conclude

$$\prod_{i=1}^2 \frac{2^i-1}{2^i} = \left(\frac{1}{2}\right) \cdot \left(\frac{3}{4}\right) = \frac{3}{8} \tag{1}$$

$$\geq \frac{2}{8} \tag{2}$$

$$\geq \frac{1}{4} \tag{3}$$

**Inductive Case:**

## Question 3

## Question 4