Midterm 1 Version 1 Review

March 29, 2020

Question 1

a. Because we know

$S_1 = \{aa, bb, cc, aab, aac, aaa, bba, bbb, bbc, cca, ccb, ccc, aaaa, \dots\}$ and S_2 is a set of all strings over U with length 3, we can conclude

$$S_1 \cap S_2 = \{aaa, aab, aac, bba, bbb, bbc, cca, ccb, ccc\}$$

b. See table below

p	q	r	$\neg r$	$p \vee q$	$p \vee q \Rightarrow \neg r$
T	T	T	F	T	F
T	T	F	T	T	T
T	F	T	F	T	F
F	T	T	F	T	F
T	F	F	T	T	T
F	T	F	T	T	T
F	F	T	F	F	T
F	F	F	T	F	T

c. Let $x \in \mathbb{N}$, and $y = $ _____.

We will prove that predicate $P(x, y)$ is true, or predicate $Q(x, y)$ is true.

> **Correct Solution:**
>
> **Let $x = $ _____, and** $y \in \mathbb{N}$**.**
>
> We will prove that **both predicates $P(x, y)$ and $Q(x, y)$ are false.**

Notes:

- How can I proceed a proof when there is $\lor$ on R.H.S of the statement? What's the general structure of proof given this symbol?

Question 2

a. $\exists x \in P, \ Student(x) \land Attends(x)$

b. $\forall x \in P, \ \exists y \in P, Student(y) \land Attends(y) \Rightarrow Loves(x, y)$

> **Correct Solution:**
>
> $\forall x \in P, \ \exists y \in P, Student(y) \land Attends(y) \land Loves(x, y)$

Notes:

- When should $\Rightarrow$ be used, and when should $\land$ be used?

c. $\forall x \in P, \ Student(x) \land Attends(x) \Rightarrow Loves(x, x)$

d. $\forall x_1, x_2 \in P, \ x_1 \neq x_2 \Rightarrow Loves(x_1, x_2) \land Loves(x_2, x_1) \Rightarrow \neg Attends(x_1) \lor \neg Attends(x_2)$

> **Correct Solution:**
>
> $\forall x_1, x_2 \in P, x_1 \neq x_2 \land Loves(x_1, x_2) \land Loves(x_2, x_1) \Rightarrow \neg Attends(x_1) \lor \neg Attends(x_2)$

Question 3

a. $\forall a, b, c \in \mathbb{Z}, \exists l, m, n \in \mathbb{Z}, b = la \land c = mb \Rightarrow c = na$

Question 4