

Worksheet 15 Review

April 1, 2020

Question 1

a. First, we will evaluate the cost of the inner most loop.

Because the loop runs from $j = i + 1$ to $j = n - 1$, with each iteration costing 1 step, we can conclude that the inner most loop has cost of at most

$$\lceil (n - 1) - (i + 1) + 1 \rceil = n - i - 1 \quad (1)$$

steps.

Next, we will evaluate the cost of the outer most loop.

Because the loop runs from $i = 0$ to $i = n - 1$ with each iteration costing $(n - i - 1)$ steps, we can conclude the outer most loop has cost of at most

$$\sum_{i=0}^{n-1} (n - i - 1) = \left[\sum_{i=0}^{n-1} (n - 1) - \sum_{i=0}^{n-1} i \right] \quad (2)$$

$$= \left[\frac{2n(n - 1)}{2} - \frac{(n - 1)n}{2} \right] \quad (3)$$

$$= \frac{n(n - 1)}{2} \quad (4)$$

steps.

Next, we will bring everything together.

Since the lines **n = len(lst)** and **return False** have cost of 1 step each, the total cost of the algorithm is

$$\frac{n(n-1)}{2} + 2 \tag{5}$$

steps.

Then, it follows from above that the algorithm has runtime of $\Theta(n^2)$.

Question 2