CSC236 Worksheet 9 Solution

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Question 1

a. I need to evalulate the reg. expressions for

$$L = \{x \in \Sigma \mid x\text{has even number of 1s or an odd number of 0s}\}$$

I will do so in parts.

Part 1 (Finding reg. expressions for even number of 1s):

In this part, I will find the reg. expressions for even number of 1's.

I will do so by finding patterns in series of small examples.

Starting with $L = \{x \in \Sigma \mid x \text{ has 0 number of 1s}\}$, it's reg. expressions is

$$0^* \tag{1}$$

Now for $L = \{x \in \Sigma \mid x \text{ has 2 number of 1s}\}$, it's reg. expressions is

$$0^*10^*10^* \tag{2}$$

Now for $L = \{x \in \Sigma \mid x \text{ has 4 number of 1s}\}$, it's reg. expressions is

$$0^*10^*10^*10^*10^* \tag{3}$$

From above, I see a pattern that

$$(0^*10^*1)(0^*10^*1)0^* \tag{4}$$

Using the pattern, I can conclude that the regular expression for even number of 1s is

$$(0^*10^*1)^*0^* \tag{5}$$

Part 2 (Finding reg. expressions for odd number of 0s):

In this part, I will find the reg. expressions for odd number of 0's.

I will do so by finding patterns in series of small examples.

Starting with $L = \{x \in \Sigma \mid x \text{ has 1 number of 0s}\}$, it's reg. expressions is

$$1^*01^* \tag{6}$$

Now for $L = \{x \in \Sigma \mid x \text{ has 3 number of 0s}\}$, it's reg. expressions is

$$1^*01^*01^*01^* \tag{7}$$

Now for $L = \{x \in \Sigma \mid x \text{ has 5 number of 0s}\}$, it's reg. expressions is

$$1^*01^*01^*01^*01^*01^* \tag{8}$$

From above, I see a pattern that

$$1^*(01^*)(01^*)(01^*)(01^*)(01^*) \tag{9}$$

Using the pattern, I can conclude that the regular expression for odd number of 0s is

$$1^*(01^*)^* \tag{10}$$

Thus, by combining the two parts with union, we have

$$(0^*10^*1)^*0^* + 1^*(01^*)^* \tag{11}$$

Notes:

- Regular Expression
  - Quick Guide

$$(0+1)((01)^*0) \tag{12}$$

The expression implies that

1. Starts with 0 **or** 1
   - indicated by $(0 + 1)$
2. Are then followed by **one or more repeatitions** of 01
   - indicated by $(01)^*$
3. Ends with 0
   - indicated by the final 0

- Examples
  1. $L = \{w \in \{a, b\}^* \mid w \text{ has an } a\}$

Answer:

$$(a+b)^*a(a+b)^* \tag{13}$$

- Means there is one or more repeatitions of a or b at front
- Means there is a in the middle

- Means there is zero or more repeatitions of a or b at end

2. $L = \{w \in \{a, b\}^* \mid w \text{ has at lest two } a\text{s}\}$

Answer:

$$(a + b)^* a (a + b)^* a (a + b)^* \tag{14}$$

3. $L = \{w \in \{a, b\}^* \mid |w| \geq 2\}$

Answer:

$$(0 + 1)(0 + 1)(0 + 1)^* \tag{15}$$

In this example,

- Two characters are created (indicated by $(0 + 1)(0 + 1)$)
- And more :D!! (indicated by$(0 + 1)^*$)