

# Worksheet 17 Solution

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## Question 1

a. We need to determine  $|\mathcal{I}_n|$ .

The problem tells that the values in inputs are either 1 or 0, and we know  $\mathcal{I}_n$  represents all possible inputs of size  $n$  containing binary values.

After watching lecture videos, and reading notes, I do not yet understand the details of how to evaluate the  $\mathcal{I}_n$ , but from the pattern below

$[0], [1], [1, 0], [0, 1], [1, 1], [0, 0], [0, 0, 0], [0, 0, 1], [0, 1, 0], [1, 0, 0], [1, 1, 0], [1, 0, 1], [0, 1, 1], [1, 1, 1]$

we can see the inputs of size 1 have 2 different inputs, the inputs of size 2 have 4 different inputs, and the inputs of size 3 have 8 different inputs.

Using this pattern, I can make an educated guess that  $|\mathcal{I}_n| = 2^n$ .

### Notes:

- The idea of average-case analysis is that some data structures and algorithms have poor worst-case performance but perform well in vast majority of others.
- Average-case analysis looks at running time on sets of inputs
- Average case:  $AVG_{func}(n) = avg\{\text{runtime of func}(x) \mid x \in \mathcal{I}_n\}$
- Worst case:  $WC_{func}(n) = max\{\text{runtime of func}(x) \mid x \in \mathcal{I}_n\}$

	$n$	$i$	Sets	$ S_{n,i} $
	2	0	$\{[0]\}$	1
	2	0	$\{[0, 1], [0, 0]\}$	2
b.	2	1	$\{[1, 0]\}$	1
	3	0	$\{[0, 1, 1], [0, 0, 1], [0, 0, 0]\}$	3
	3	1	$\{[1, 0, 1], [1, 0, 0]\}$	2
	3	2	$\{[1, 1, 0]\}$	1

By the pattern outlined above, we can deduce that  $|S_{n,i}| = n - i$ .

**Correct Solution:**

$n$	$i$	Sets	$ S_{n,i} $
1	0	$\{[0]\}$	1
2	0	$\{[0, 1], [0, 0]\}$	2
2	1	$\{[1, 0]\}$	1
3	0	$\{[0, 1, 1], [0, 0, 1], [0, 1, 0], [0, 0, 0]\}$	4
3	1	$\{[1, 0, 1], [1, 0, 0]\}$	2
3	2	$\{[1, 1, 0]\}$	1

By the pattern outlined above, we can deduce that  $|S_{n,i}| = 2^{n-i-1}$ .

- c. We will prove the statement informally using proof by cases.

**Case 1 (when list doesn't have 0s):**

The definition of  $S_{n,i}$  tells us  $0 \leq i \leq n$ ,  $S_{n,i}$  contains all lists with 0 starting at  $i$ th position.

Using the fact, we can conclude  $S_{n,n}$  is a set of lists containing 0 at  $n^{th}$  position.

Since  $i$  in a list starts at  $i = 0$  and ends at  $i = n - 1$ , there are no 0 in the list of a set  $S_{n,n}$ .

Then, we can conclude  $S_{n,n}$  is one and the only that contains a list of only 1s.

**Case 2 (when list has one or more 0s):**

Because we know this list has 0 starting at  $i^{th}$  position, we can conclude this list exists in the set  $S_{n,i}$ .