CSC373 Worksheet 7 Solution

August 14, 2020

1. **Notes**

   - **Decision Problem**
     - Is the problem if determining ansewr to a class of yes/no questions about some objects of interest

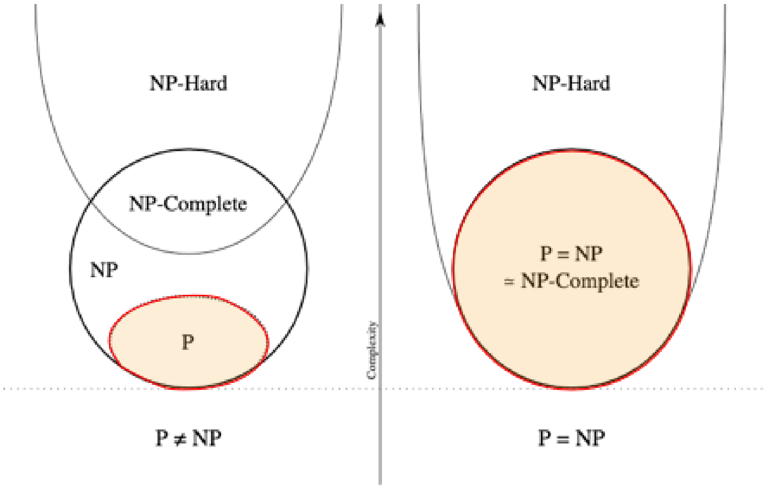
   - **Reductions**
     -

     Example:

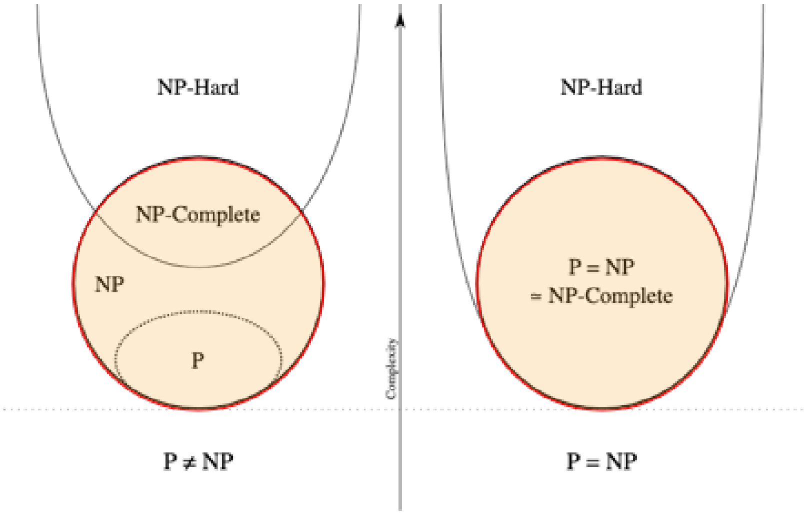
   - **P**
     - Is set of problems that can be solved by a deterministic Turing machine in Polynomial time (i.e. $\mathcal{O}(n^k)$) [2].

       Example:

       1) Shortest path problems
       2) Calculating the greatest common divisor
       3) Finding maximum bipartite matching



- **NP (Non-deterministic Polynominal):**

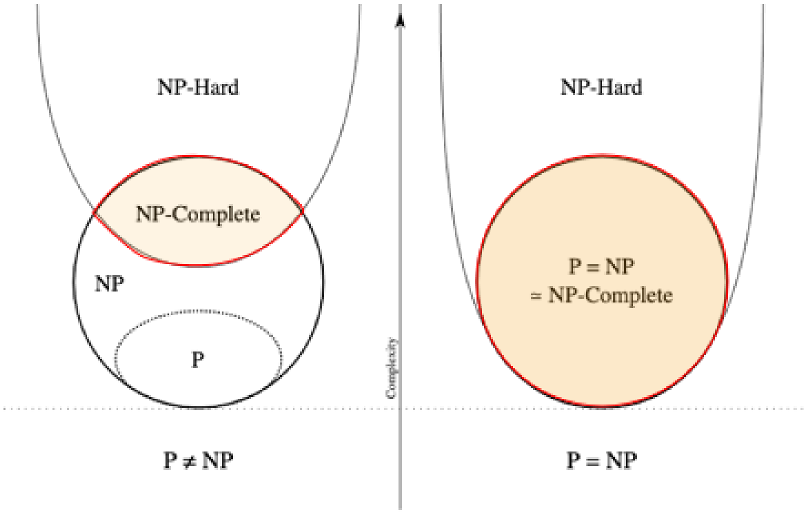


  - Is set of decision problems that can be solved by a Non-deterministic Turing Machine in Polynomial time.[2]
  - Has no particular rule is followed to make a guess [1].
  - Can be solved in polynominal time via a "lucky algorithm", a magical algorithm that always make a right guess [2]
  - $P \subseteq NP$

Examples:

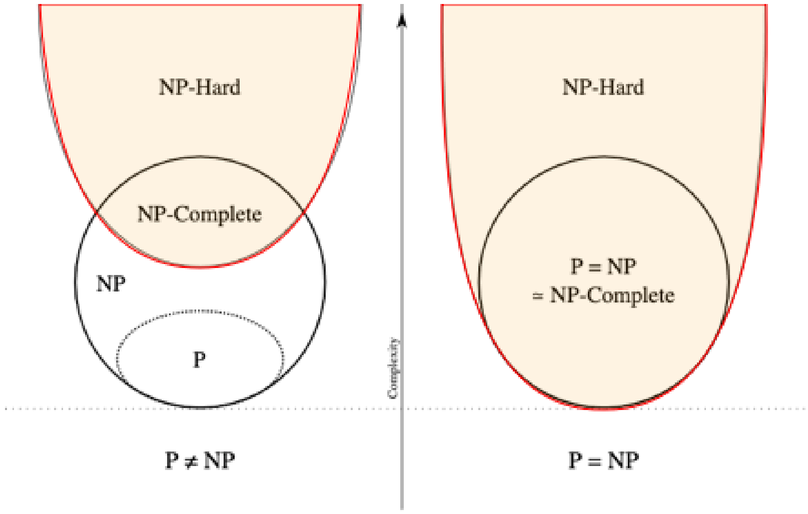
  - Longest-path problems
  - Hamiltonian Cycle
  - Graph coloring

- **NP-Complete Problems:**



- A decision problem A is NP-complete (NPC) if
  1) $A \in NP$ and
  2) Every (other) problems A' in NP is reducible to A
- Has no efficient solution in polynominal number of steps (not yet) [3]
- Is not likely that there is an algorithm to make it efficient [3]

- **NP-Hard:**



- A decision problem A is NP-hard if
  1) $A \in NP$ (Not necessarily) and
  2) Every (other) problems A' in NP is reducible to A
- NP-Hard means "at least as hard as any problems in NP"
- Does not have to be about decision problems

Example:

1) Alan Turing's Halting Problem

References

1) Encyclopedia Britannica, NP-Complete Problem, link
2) Geeks for Geeks, NP-Completeness, link
3) Wikipedia, NP-complete, link
4) UCLA UC-Davis, ECS122A Handout on NP-Completeness, link