

CSC373 Worksheet 4 Solution

August 4, 2020

1. • Calculating out-degree

Let $G = (V, E)$ be a directed graph. Let $[v_1, \dots, v_n]$ be a list of vertices in graph G .

I need to calculate the outdegree of every vertex using adjacency list.

We know that in addition to counting each v_i in adjacency list where $i = 1, \dots, n$, we are also counting $|Adj[v_i]|$ edges.

Since there are $|V| = n$ many vertices, we can write that the total count is $|V| + \sum_{i=1}^n |Adj[v_i]| = |V| + |E|$, which is $\mathcal{O}(|V| + |E|)$.

• Calculating In-degree

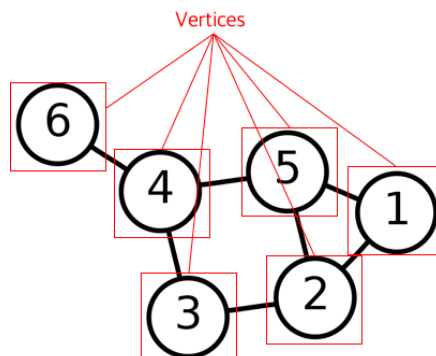
The outdegree of a vertex is indegree of another vertex.

Using this fact, we can conclude the running time of computing indegree of every vertex is $\mathcal{O}(|V| + |E|)$.

Notes:

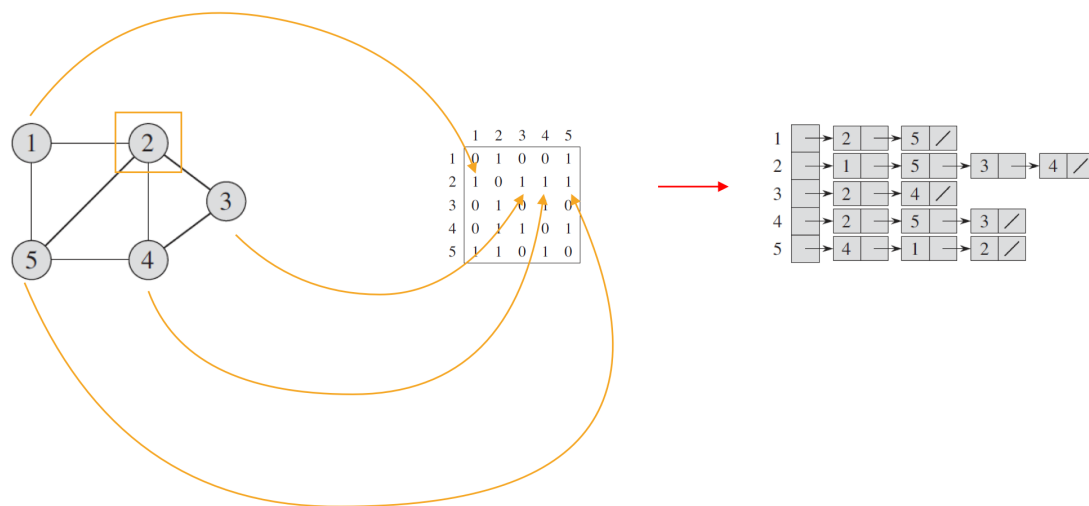
• **Vertex**

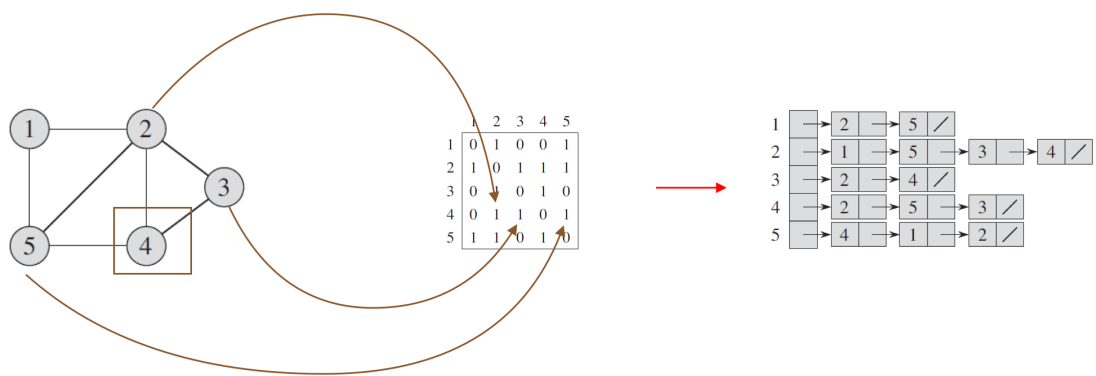
- Is a fundamental unit of which graphs are formed
- Also means node



• Adjacency-list Representation

- Associates each vertex in a graph with the collection of its neighbouring vertices or edges
- Is represented by $Adj[v]$
 - * Means all vertices that are neighbour to vertex v
 - * In a directed graph, $Adj[v]$ are all out-degree vertices of vertex v
 - * $|Adj[v]|$ means the total number of outdegree of vertex v







• Directed graph

- Is a graph that is made up of a set of vertices connected by edges, where the edges have a direction associated with them



• Out-degrees

- For a directed graph $G = (V(G), E(G))$ and a vertex $x_1 \in V(G)$, the Out-Degree of x_1 refers to the number of arcs incident from x_1 . That is, the number of arcs directed away from the vertex x_1 .

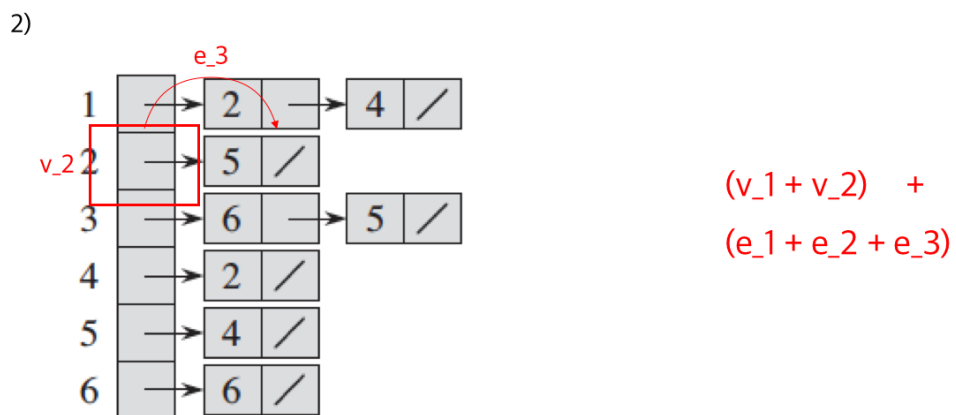


• In-degrees

- For a directed graph $G = (V(G), E(G))$ and a vertex $x_1 \in V(G)$, the In-Degree of x_1 refers to the number of arcs incident to x_1 . That is, the number of arcs directed towards the vertex x_1 .



- Computing the outdegree of every vertex using adjacency list



3)



$$(v_1 + v_2 + v_3) + (e_1 + e_2 + e_3 + e_4 + e_5)$$

3)



$$(v_1 + v_2 + v_3) + (e_1 + e_2 + e_3 + e_4 + e_5)$$

4)



$$(v_1 + v_2 + v_3 + v_4) + (e_1 + e_2 + e_3 + e_4 + e_5)$$

4)



$$(v_1 + v_2 + v_3 + v_4) + (e_1 + e_2 + e_3 + e_4 + e_5)$$

6)



$$(v_1 + v_2 + v_3 + v_4 + v_5 + v_6) + (e_1 + e_2 + e_3 + e_4 + e_5 + e_6 + e_7 + e_8)$$

So it has $\mathcal{O}(V + E)$

- Computing the outdegree of every vertex using adjacency list

The outdegree of a vertex is indegree of another vertex.

Using this fact, we can conclude the running time of computing indegree of every vertex is $\mathcal{O}(V + E)$.

2. • Computing G^T from G in Adjacency List



```

1  COMPUTE-G-TRANSPOSE-ADJ-MATRIX( $Adj, V$ )
2      Let  $Adj'$  be a new adjacency list containing keys  $v_1 \dots v_n$ 
3
4      for  $i = 1$  to  $|V|$ 
5          for every vertex  $w$  in  $Adj[v_i]$ 
6              Insert( $Adj'[w], v_i$ )
7
8      return  $Adj'$ 
9

```

- Computing G^T from G in Adjacency-Matrix




```

1  COMPUTE-G-TRANSPOSE-ADJ-MATRIX(A,V)
2      Let A'[1..|V|, 1..|V|] be a new adjacency matrix
3
4      for i = 1 to |V|
5          for j = 1 to |V|
6              A'[j,i] = A[i,j]
7
8      return A'
9

```

Correct Solution:

- Computing G^T from G in Adjacency List



```

1  COMPUTE-G-TRANSPOSE-ADJ-MATRIX(Adj,V)
2      Let Adj' be a new adjacency list containing keys
   v1...vn
3
4      for i = 1 to |V|
5          for every vertex w in Adj[vi]
6              Insert(Adj'[w], vi)
7
8      return Adj'
9

```

The running time is $\mathcal{O}(|V| + |E|)$

- Computing G^T from G in Adjacency-Matrix



```

1  COMPUTE-G-TRANSPOSE-ADJ-MATRIX(A,V)
2      Let A'[1..|V|, 1..|V|] be a new adjacency matrix
3
4      for i = 1 to |V|
5          for j = 1 to |V|
6              A'[j,i] = A[i,j]
7
8      return A'
9

```

The running time is $\mathcal{O}(|V|^2)$

```

31 Breadth-First-Search(V, v_i)
32     d = 0
33     for each v_i ∈ V
34         while performing BFS(V, v_i)
35             let w be the current node in BFS
36             if δ(v_i, w) > d
37                 d = δ(v_i, w)
38
39     return d
10

```

Finding Runtime of Algorithm

Since the graph iterates $\sum_{i=1}^n Adj[v_i] = |E|$ times for each $v_i \in V$, the algorithm iterates total of $|V| \cdot |E|$ times, which is $\mathcal{O}(|V||E|)$.

Notes:

- **Breadth First Search**

- Is an algorithm for searching or traversing a graph
- Is one of the simplest algorithm

- **Largest of All Shortest Path Distance**

- Means the shortest distance between two furthest apart nodes



OR



References

1) McGill University, 308-360 Tutorial, link



4.



(e)



(f)



(g)



(g)



(i)



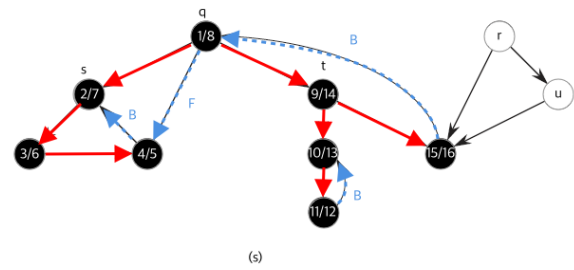
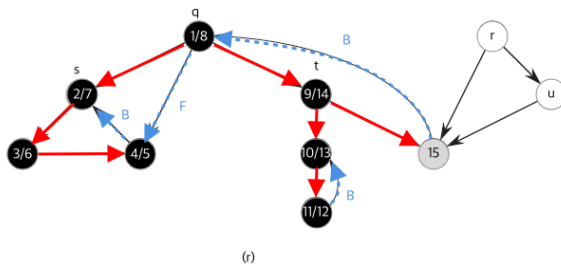
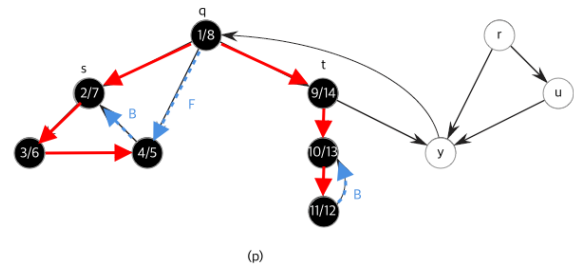
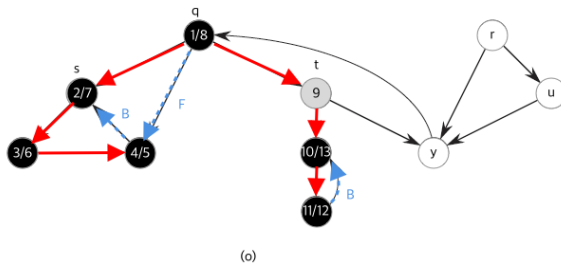
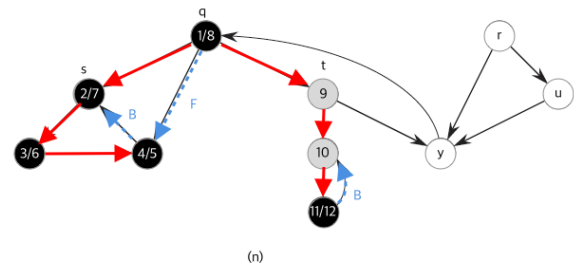
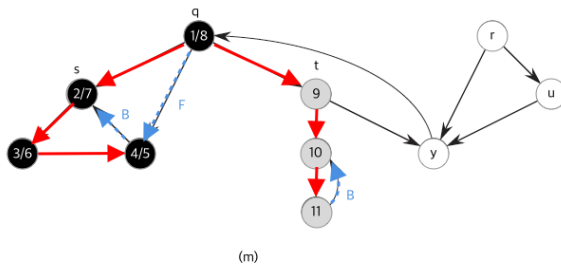
(j)



(k)



(l)





Notes:

- **Depth First Search**

- Searches deeper in the graph whenever possible

- **Forward Edge**

- Is an edge (u, v) such that v is descendant but not part of the DFS tree. Edge $1 \rightarrow 8$ is a forward edge



- **Back Edge**

- It is an edge (u, v) such that v is ancestor of edge u but not part of DFS tree. Edge from $6 \rightarrow 2$ is a back edge.
- Indicates a cycle in a graph



- **Cross Edge**

- It is a edge which connects two node such that they do not have any ancestor and a descendant relationship between them. Edge from node $5 \rightarrow 4$ is cross edge.



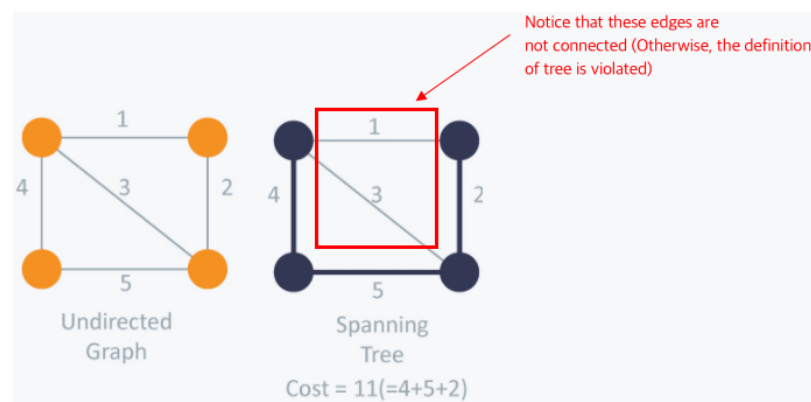
References

- 1) Geeks For Geeks, Tree, Back, Edge and Cross Edges in DFS of Graph, link

5. Notes:

• Spanning Tree

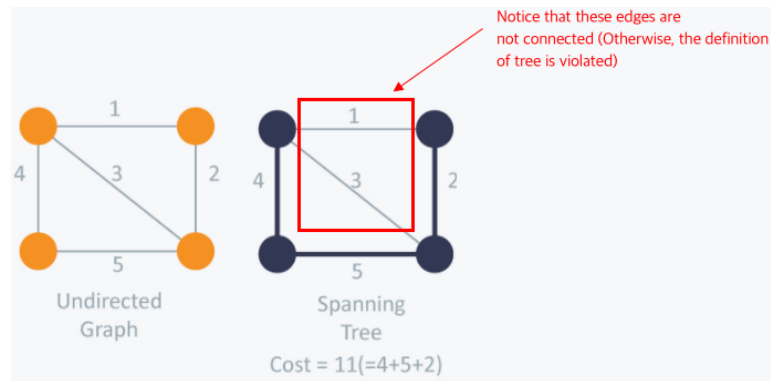
Given an undirected and connected graph $G = (V, E)$, a spanning tree of the graph G is a tree that spans G (that is, it includes every vertex of G) and is a subgraph of G (every edge in the tree belongs to G)



• Minimum Spanning Tree

- Is the spanning tree where the cost is minimum among all the spanning trees.
 - * The cost of the spanning tree is the sum of the weights of all the edges in the tree.

- There can be many minimum spanning trees.



- Is used in
 1. Network design (Telephone, electrical, hydraulic, TV cable, computer, road)
 2. Approximation algorithm for NP-hard problems
 3. Learning salient features for real-time face verification
 4. Reducing data storage in sequencing amino acids in a protein

- **Cut**

- A cut of an undirected graph $G = (V, E)$ is denoted $(S, V - S)$
- Is a partition of V