

CSC236 Worksheet 3

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Question 1

Rough Work:

Predicate Logic: $\forall A \subseteq \mathbb{N}, A \neq \emptyset \Rightarrow (\exists a \in A, \forall x \in A, a \leq x)$

Given the statement to prove

$P(x, y, z)$: There are no positive integers x, y, z such that $x^3 + 3y^3 = 9z^3$

I will prove $P(x, y, z)$ using proof by contradiction.

Assume $\exists x, y, z \in \mathbb{N}^+, x^3 + 3y^3 = 9z^3$.

1. Show that the set $X = \{x \mid x \in \mathbb{N}^+, \exists y, z \in \mathbb{N}^+, x^3 + 3y^3 = 9z^3\}$ is not empty.
2. State that there is a minimum element in set satisfying $x^3 + 3y^3 = 9z^3$, using well-ordering principle
3. Find next elements x_1, y_1, z_1 satisfying $x^3 + 3y^3 = 9z^3$, and check to see if x_1, y_1, z_1 are all in \mathbb{N}^+
4. Repeat until finding a value not in \mathbb{N}^+ .

Notes:

- **Proof By Contradiction:** $\neg P \Rightarrow \neg Q \wedge Q$ (Assuming we are proving $P \Rightarrow Q$)
- **Principle of Well-Ordering:** Any nonempty subset A of \mathbb{N} contains a minimum element; i.e. for any $A \subseteq \mathbb{N}$ such that $A \neq \emptyset$, there is some $a \in A$ such that for all $a' \in A, a \leq a'$.
- examples of well-ordered sets

1. $\mathbb{N} \cup \{0\}$
2. $\mathbb{N} \cup \{1, 2\}$
3. $\{n \in \mathbb{N} : n > 5\}$

- examples of non-well-ordered sets

1. \mathbb{R} and the open interval $(0, 2)$
2. \mathbb{Z}

Question 2

Question 3