Prep 3 quiz

(!) This is a preview of the published version of the quiz

Started: Jan 16 at 9:29am

Quiz Instructions

Readings

Please read the following part of the **Course Notes (https://www.teach.cs.toronto.edu/~csc165h/winter/resources/csc165_notes.pdf)** :

- Chapter 1, pp. 20–31 (this should be review)
- Chapter 2, pp. 33–40 (this introduces the basic idea of proofs, with some simple examples and then a focus on writing a good *proof header*)

General instructions

You can review the general instructions for all prep quizzes at **this page (https://www.teach.cs.toronto.edu/~csc165h/winter/homework/index.html)** . Remember that you can submit multiple times! We have posted a PDF version of the quiz on the course website. You might consider printing this quiz out so that you can work on paper first.

Question 1 **1 pts**

True or False: We will use the following precedence levels, in decreasing order of precedence:

1. $\neg$

2. $\land, \lor$

3. $\Rightarrow, \Leftrightarrow$

4. $\forall, \exists$

- True
- False

Question 2

1 pts

Review the definition of *divisibility* from lecture, and recall that we can define the predicate $d \mid n$ to mean "d divides n". Using this definition, select all of the **True** statements below.

- ☐ $\forall n \in \mathbb{Z}, \exists m \in \mathbb{Z},\ n \mid m$
- ☐ $\forall n \in \mathbb{Z},\ \ -1 \mid n$
- ☐ $\forall n \in \mathbb{Z},\ n \mid 0$
- ☐ $\forall n, m \in \mathbb{Z},\ n \mid m$
- ☐ $\forall n \in \mathbb{Z},\ 0 \mid n$
- ☐ $\exists n \in \mathbb{Z},\ 0 \mid n$
- ☐ $\exists m \in \mathbb{Z}, \forall n \in \mathbb{Z}, n \mid m$

Question 3

1 pts

Review the negation rules on page 25 of the Course Notes. Then, select the correct negation of this statement:

$\forall x \in \mathbb{R},\ \big(\exists y \in \mathbb{R}, P(y) \land Q(x, y)\big) \Rightarrow x > 5$

- ○ $\forall x \in \mathbb{R},\ \big(\exists y \in \mathbb{R},\ P(y) \land Q(x, y)\big) \Rightarrow x \leq 5$
- ○ $\exists x \in \mathbb{R},\ \big(\forall y \in \mathbb{R},\ \neg P(y) \lor \neg Q(x, y)\big) \lor x > 5$
- ○ $\exists x \in \mathbb{R},\ \big(\forall y \in \mathbb{R},\ P(y) \land \neg Q(x, y)\big) \land x \leq 5$
- ○ $\forall x \in \mathbb{R},\ \big(\forall y \in \mathbb{R},\ \neg P(y) \lor \neg Q(x, y)\big) \Rightarrow x \leq 5$
- ○ $\exists x \in \mathbb{R}, \big(\exists y \in \mathbb{R}, P(y) \land Q(x, y)\big) \land x \leq 5$

Question 4

1 pts

True or False: The following propositional formula is a tautology.

$\neg(p \wedge q) \vee q$

- ○ True
- ○ False

Question 5

1 pts

Suppose we want to **prove** the statement $\exists k \in \mathbb{N},\ P(k)$ (assume that we've previously defined a predicate P).

Which of the following sentences could we use to introduce k in our proof header?

- ☐ Let $k = 165$.
- ☐ Let $P(k)$.
- ☐ Let $k = 1$.
- ☐ Let k be a natural number such that $P(k)$.
- ☐ Let $k = -4$.

Question 6

1 pts

Suppose we want to **prove** the statement $\forall x, y \in \mathbb{R},\ P(x, y)$ (assume that we've previously defined a predicate P).

Which of the following statements could we use to introduce x and y in our proof header?

- ☐ Let x be an arbitrary real number, and let $y = x + 1$.

- ☐ Let x and y be arbitrary real numbers such that $P(x, y)$ is true.
- ☐ Let $x, y \in \mathbb{R}$.
- ☐ Let $x = 1$ and $y = 3$.
- ☐ Let x and y be arbitrary real numbers.

Question 7

1 pts

Suppose we have a proof with the following proof header:

> Let x be an arbitrary natural number. Assume that x is greater than 3 and that x is even (i.e., that 2 divides x). We will now prove that $Q(x)$ is true.
>
> *[...proof body omitted...]*

What is the statement being proven?

- ○ $\forall x \in \mathbb{N},\ x > 3 \land 2 \mid x \Rightarrow Q(x)$
- ○ $\forall x \in \mathbb{N}, x > 3 \land 2 \mid x \land Q(x)$
- ○ $Q(x)$
- ○ $\forall x \in \mathbb{N}, Q(x)$
- ○ $\exists x \in \mathbb{N}, x > 3 \land 2 \mid x \land Q(x)$

Question 8

1 pts

Suppose we want to prove the statement $\forall x \in \mathbb{N},\ P(x) \Rightarrow Q(x+1)$.

Select the assumption we should make in our proof header (after we've introduced the variable x).

- ○ Assume that $P(x)$ is true.
- ○ Assume that $Q(x+1)$ is true.

- ○ Assume that for all $x \in \mathbb{N}$, $P(x)$ is true.
- ○ Assume that $P(0)$ is true.
- ○ We should not make any assumptions in our proof header.

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