

# Worksheet 6 Review 2

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## Question 1

a.  $\forall x \in \mathbb{N}, P(123) \wedge P(x) \Rightarrow x \leq 123$

**Correct Solution:**

$$P(123) \wedge (\forall x \in \mathbb{N}, P(x) \Rightarrow x \leq 123)$$

b.  $IsCD(x, y, d) : d \mid x \wedge d \mid y$ , where  $x, y, d \in \mathbb{Z}$

$$IsGCD(x, y, d) : \forall n \in \mathbb{N}, IsCD(x, y, n) \Rightarrow \exists d \in \mathbb{N}, IsCD(x, y, d) \wedge n \leq d$$

**Correct Solution:**

$$IsCD(x, y, d) : d \mid x \wedge d \mid y, \text{ where } x, y, d \in \mathbb{Z}$$

$$IsGCD(x, y, d) : (x = 0 \wedge y = 0 \Rightarrow d = 0) \wedge (x \neq 0 \wedge y \neq 0 \Rightarrow IsCD(x, y, d) \wedge (\forall d_1 \in \mathbb{Z}, IsCD(x, y, d_1) \Rightarrow d_1 \leq d)), \text{ where } x, y, d \in \mathbb{Z}$$

**Notes:**

- Realized the definition of  $IsGCD$  extends from previous question
- Noticed professor defines conditions in a predicate logic the following way

$$(\text{case 1} \Rightarrow \text{statement 1}) \wedge (\text{case 2} \Rightarrow \text{statement 2})$$

- Hm... I feel puzzled about  $\wedge$  operator used in between cases ( i.e.  $(x = 0 \wedge y = 0 \Rightarrow d = 0) \wedge (x \neq 0 \wedge y \neq 0 \Rightarrow IsCD(x, y, d) \wedge (\forall d_1 \in \mathbb{Z}, IsCD(x, y, d_1) \Rightarrow d_1 \leq d))$ ). At glimpse, I felt  $\vee$  is more appropriate since if this case is not true, then we want other case should be true.

**Question 2**

**Question 3**