CSC236 Worksheet 3

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Question 1

Rough Work:

Predicate Logic: $\forall A \subseteq \mathbb{N},\ A \neq \emptyset \Rightarrow (\exists a \in A,\ \forall x \in A,\ a \leq x)$

Given the statement to prove

$$P(x,y,z) : \text{There are no positive integers } x,y,z \text{ such that } x^3 + 3y^3 = 9z^3$$

I will prove $P(x,y,z)$ using proof by contradiction.

Assume $\exists x,y,z \in \mathbb{N}^+,\ x^3 + 3y^3 = 9z^3$.

1. State that the set $X = \{x \mid x \in \mathbb{N}^+,\ \exists y,z \in \mathbb{N}^+,\ x^3 + 3y^3 = 9z^3\}$ is not empty.

   Then, we can write the set $X = \{x \mid x \in \mathbb{N}^+,\ \exists y,z \in \mathbb{N}^+,\ x^3 + 3y^3 = 9z^3\}$ is not empty.

2. State that there are elements x_0, y_0, z_0 satisfying $x^3 + 3y^3 = 9z^3$.
3. Find next elements x_1, y_1, z_1 satisfying $x^3 + 3y^3 = 9z^3$, and check to see if x_1, y_1, z_1 are all in $\mathbb{N}^+$
   - First, show that $x_0 = 3 \cdot x_1$, using $x_0^3 = 9z_0^3 - 3y_0^3$
   - Second, show that $y_0 = 3 \cdot y_1$, using $x_0^3 = 3^3 x_1^3 = 9z_0^3 - 3y_0^3$
   - Third, show that $z_0 = 3 \cdot z_1$, using $x_0^3 = 3^3 x_1^3 = 9z_0^3 - 3y_0^3 = 9z_0^3 - 3^4 y_1^3$
4. Repeat until finding a value not in $\mathbb{N}^+$.

Notes:

- **Proof By Contradiction:** $\neg P \Rightarrow \neg Q \wedge Q$ (Assuming we are proving $P \Rightarrow Q$)
- **Principle of Well-Ordering:** Any nonempty subset A of $\mathbb{N}$ contains a minimum element; i.e. for any $A \subseteq \mathbb{N}$ such that $A \neq \emptyset$, there is some $a \in A$ such that for all $a' \in A$, $a \leq a'$.
- examples of well-ordered sets
  1. $\mathbb{N} \cup \{0\}$
  2. $\mathbb{N} \cup \{1, 2\}$
  3. $\{n \in \mathbb{N} : n > 5\}$
- examples of non-well-ordered sets
  1. $\mathbb{R}$ and the open interval $(0, 2)$
  2. $\mathbb{Z}$

Question 2

Question 3