Worksheet 7 Review 2

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Question 1

a. In this case assume that $n \leq 1$.

We want to show $n \leq 1$.

Since the assumption tells us $n \leq 1$, we can conclude this is true.

b. **Pseudoproof:**

Let $a = d$ and $b = k$. Assume there exists $d \in \mathbb{N}$ where $(\exists k \in \mathbb{Z}, n = dk) \wedge d \neq 1 \wedge d \neq n$. Assume $n > 1$

We need to prove that $n \nmid a$, $n \nmid b$ and $n \mid ab$.

1. Show $n \nmid a$.

   First, we need to show $n \nmid a$.

   1. Show $n \geq d$.

      The fact 2 tells us

      $$\forall x, y \in \mathbb{N}, y \geq 1 \wedge x \mid y \Rightarrow 1 \leq x \leq y \tag{1}$$

      and we know from headers that $d \mid n$, $n > 1$, and $n, d \in \mathbb{N}$.

      Then, by using these facts, we can write

      $$1 \leq d \leq n \tag{2}$$

2. Show that for n to divide d, $n = d$.

   Now, the definition of divisibility tells us for n to divide d, there must be some $k_1 \in \mathbb{Z}$ such that d is equal to $k_1 \cdot n$.

   Then, since we know $n \geq d$, by using these facts, we can conclude the definition of divisibility is satisfied when $k_1 = 1$, or when $n = d$.

3. Conclude $n \nmid a$.

   Then, since we know from header that $n \neq d$, we can conclude $n \nmid d$.

First, we need to show $n \nmid a$.

The fact 2 tells us

$$\forall x, y \in \mathbb{N}, y \geq 1 \land x \mid y \Rightarrow 1 \leq x \leq y \tag{3}$$

and we know from headers that $d \mid n$, $n > 1$, and $n, d \in \mathbb{N}$.

Then, by using these facts, we can write

$$1 \leq d \leq n \tag{4}$$

Now, the definition of divisibility tells us for n to divide d, there must be some $k_1 \in \mathbb{Z}$ such that d is equal to $k_1 \cdot n$.

Then, since we know $n \geq d$, by using these facts, we can conclude the definition of divisibility is satisfied only when $k_1 = 1$, or when $n = d$.

Then, since we know from the header that $n \neq d$, we can conclude $n \nmid d$.

Then, since we know $d = a$ from the header, we can conclude $n \nmid a$.

2. Show $n \nmid b$
   - Show $k \mid n$
     - State $n = kd$.

The assumption tells us $n = kd$.

- Show $k \mid n$ by using the definition of divisibility

Then, it follows from the definition of divisibility that $k \mid d$.

The assumption tells us $n = kd$.

Then, it follows from the definition of divisibility that $k \mid d$.

- Show $k \geq 1$.
- Show $n \geq k$ using the fact $k \mid n$ and $k \geq 1$.
- Show that for n to divide k, $n = k$.

3. Show $n \mid ab$

Question 2

Question 3