

Midterm 2 Version 1 Review

July 17, 2020

1. a) 1100100

b) $-\sum_{i=0}^{n-1} 3^i$

Notes:

- Balanced Ternary
 - is a way of representing numbers
 - balanced ternary is in base 3, and has values 1,0 or -1

$$\sum_{i=0}^{n-1} d_i \cdot 3^i \text{ where } d_i \in \{0, 1, -1\} \quad (1)$$

c) i. $f(n) \in \Omega(n)$

True (since $n^2 + 10n + 2 \geq cn$)

ii. $g(n) \in \Omega(n)$

False (Let $c = 100, n_0 = 100$. Then $100 \log_2 n < 100n$)

iii. $f(n) \in \mathcal{O}(g(n))$

False ($f(n) = n^2 + 10n + 2$ grows faster than $g(n) = 100 \log_2 n$)

iv. $f(n) \in \Theta(g(n))$

True (Set $c_1 = -1, c_2 = 1, n_1 = 100$. Then $c_1 f(n) \leq g(n) \leq c_2 f(n)$)

v. $g(n) \in \Theta(\log_3 n)$

True (set $c_1 = -1, c_2 = 1, n_1 = 2$. Then $c_1 g(n) \leq \log_3 n \leq c_2 g(n)$)

vi. $g(n) \in \Theta(\log_3 n)$

False (set $c_1 = -1, c_2 = 1, n_1 = 2$. Then $c_1 g(n) \leq \log_3 n \leq c_2 g(n)$)

vii. $f(n) + g(n) \in \Theta(f(n))$

True (set $c_1 = -2, c_2 = 2, n_1 = 1$. Then $c_1(f(n) + g(n)) \leq f(n) \leq c_2(f(n) + g(n))$)

Notes:

- $g \in \Omega(f) : \exists c, n_o \in \mathbb{R}^+, \forall n \in \mathbb{N}, n \geq n_o \Rightarrow g(n) \geq cf(n)$, where $f, g : \mathbb{N} \rightarrow \mathbb{R}^{\geq 0}$
 - $g \in \mathcal{O}(f) : \exists c, n_o \in \mathbb{R}^+, \forall n \in \mathbb{N}, n \geq n_o \Rightarrow g(n) \leq cf(n)$, where $f, g : \mathbb{N} \rightarrow \mathbb{R}^{\geq 0}$
 - $g \in \Theta(f) : g \in \mathcal{O}(f) \wedge g \in \Omega(f)$
- or
- $g \in \Theta(f) : \exists c_1, c_2, n_1 \in \mathbb{R}^+, \forall n \in \mathbb{N}, n \geq n_1 \Rightarrow c_1 g(n) \leq f(n) \leq c_2 g(n)$, where $f, g : \mathbb{N} \rightarrow \mathbb{R}^{\geq 0}$