

Worksheet 3 Solution

March 12, 2020

Question 1

Part 1

- a) $Correct(my_prog) \wedge Python(my_prog)$
- b) $\exists x \in P, \neg Correct(x) \Rightarrow Python(x)$
- c) $\forall x \in P, \neg Python(x) \Rightarrow Correct(x)$
- d) $\forall x \in P, \neg Correct(x) \Rightarrow Correct(x)$
- e) A program is written in Python and is correct
- f) All programs are not written in Python and is correct
- g) It is not true that all programs written in python is correct
- h) All programs that are not written in python is correct, and all correctly running programs are not written in python

Question 2

- a) All program written in Python is correct, or all program written in Python is not correct
- b) $(\exists x \in P, Python(x) \Rightarrow Correct(x)) \Rightarrow (\forall y \in P, Python(x) \Rightarrow Correct(x))$

- c) The first statement considers two different natural numbers, where as the second uses the same number

The first statement is True (with $x_1 = 5$ and $x_2 = 35$), but the second statement is False (165 cannot be in multiples of 7)

Question 3

- a) $Odd(x) : \exists n \in \mathbb{Z}, 2 \mid (n + 1)$
- b) $(\forall m \in \mathbb{Z}, Odd(m)) \wedge (\forall n \in \mathbb{Z}, Odd(n)) \Rightarrow Odd(mn)$
- c) $\forall m, n \in \mathbb{Z}, \exists k, l \in \mathbb{Z}, ((m+1) = 2k) \wedge ((n+1) = 2l) \Rightarrow \exists o \in \mathbb{Z}, (mn+1) = 2o$
- d) $\forall m, n \in \mathbb{Z}, \exists k \in \mathbb{Z}, ((mn+1) = 2k) \Rightarrow \exists l, o \in \mathbb{Z}, ((m+1) = 2l) \wedge ((n+1) = 2o)$

Question 4

a)

$$\neg((a \wedge b) \Leftrightarrow c) \quad (1)$$

Expanding definition of iff:

$$\neg(((a \wedge b) \wedge c) \vee (\neg(a \wedge b) \wedge \neg c)) \quad (2)$$

Using $\neg(p \vee q) \Rightarrow \neg p \wedge \neg q$:

$$\neg((a \wedge b) \wedge c) \wedge \neg(\neg(a \wedge b) \wedge \neg c) \quad (3)$$

Using $\neg(p \wedge q) \Rightarrow \neg p \vee \neg q$:

$$(\neg(a \wedge b) \vee \neg c) \wedge ((a \wedge b) \vee c) \quad (4)$$

- b) Using rule #6 and #7:

$$\exists x, y \in S, \forall z \in S, \neg(P(x, y) \wedge Q(x, z)) \quad (1)$$

Using rule #2:

$$\exists x, y \in S, \forall z \in S, \neg P(x, y) \vee \neg Q(x, z) \quad (2)$$

c) Using rule #4:

$$((\exists x \in S, P(x)) \wedge \neg(\exists y \in S, Q(y))) \quad (1)$$

Using rule #6:

$$((\exists x \in S, P(x)) \wedge (\forall y \in S, \neg Q(y))) \quad (2)$$

Question 5