CSC373 Worksheet 4 Solution

August 4, 2020

1. • Calculating out-degree

   Let $G = (V, E)$ be a directed graph. Let $[v_1, ..., v_n]$ be a list of vertices in graph G.

   I need to calculate the outdegree of every vertex using adjacency list.

   We know that in addition to counting each v_i in adjacency list where $i = 1, .., n$, we are also counting $|Adj[v_i]|$ edges.

   Since there are $|V| = n$ many vertices, we can write that the total count is $|V| + \sum_{i=1}^{n} |Adj[v_i]| = |V| + |E|$, which is $\mathcal{O}(|V| + |E|)$.

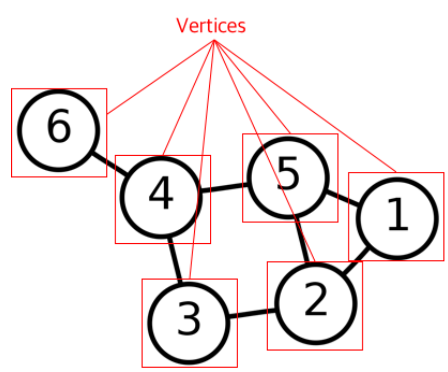
   • Calculating In-degree

   The outdegree of a vertex is indegree of another vertex.

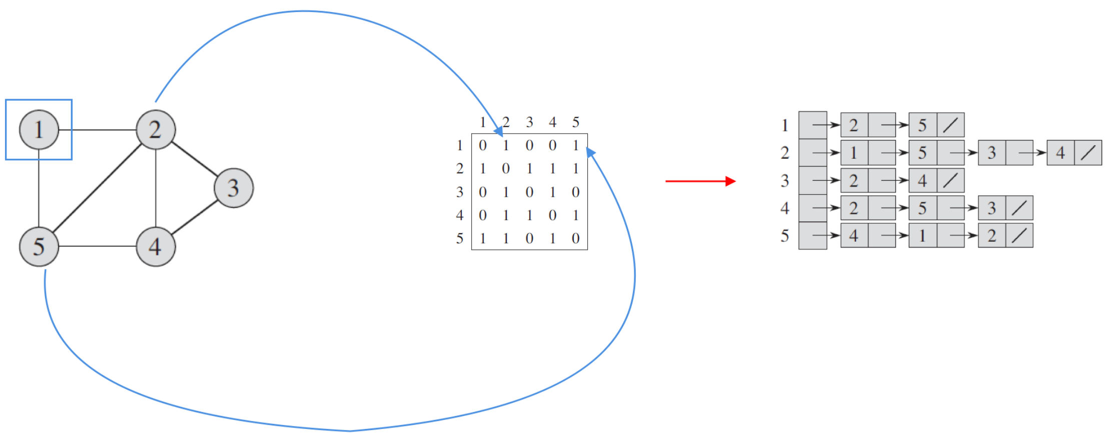
   Using this fact, we can conclude the running time of computing indegree of every vertax is $\mathcal{O}(|V| + |E|)$.

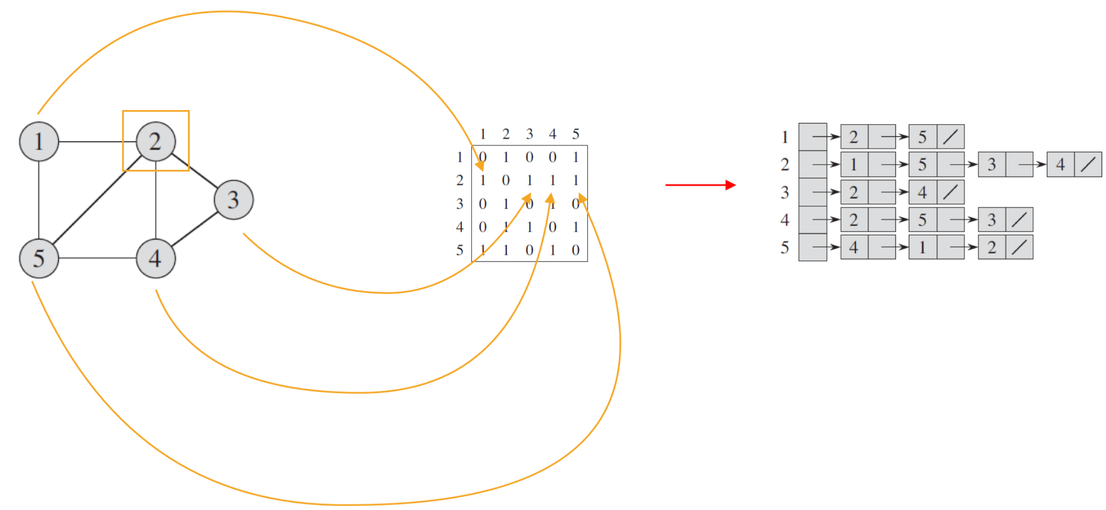
<u>**Notes:**</u>

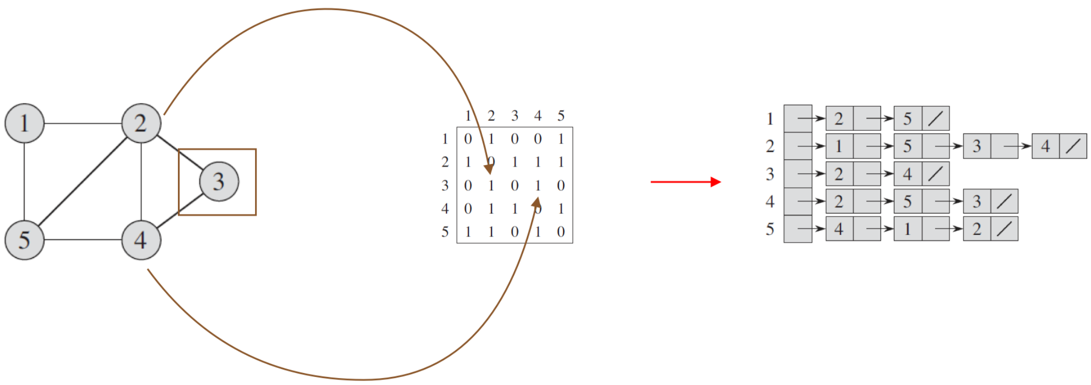
- **Vertex**
  - Is a fundamental unit of which graphs are formed
  - Also means node

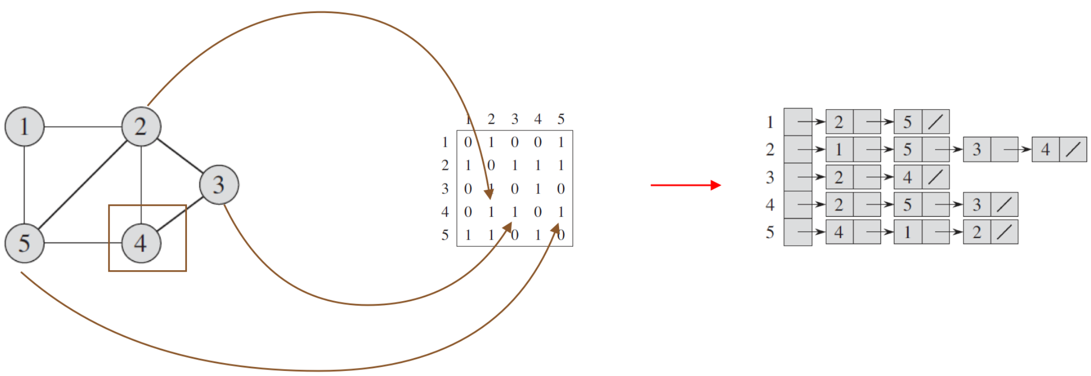


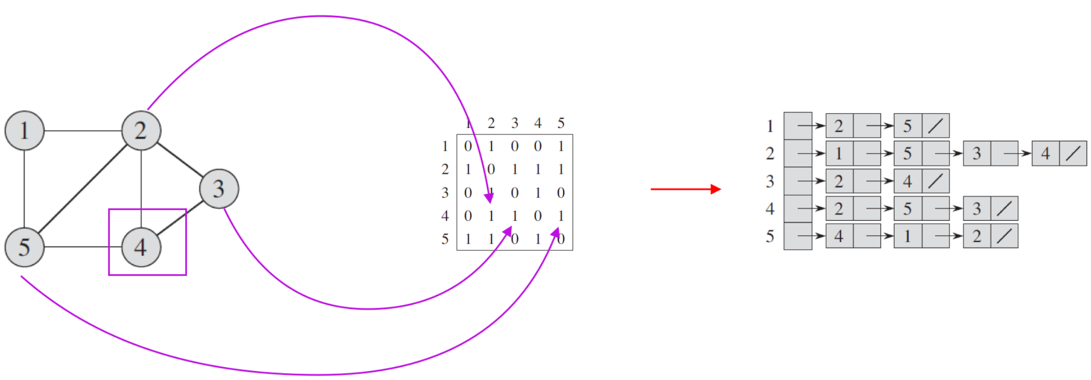
- **Adjacency-list Representation**
  - Associates each vertax in a graph with the collection of its neighbouring vertices or edges
  - Is represented by $Adj[v]$
    - ∗ Means all vertices that are neighbour to vertex v
    - ∗ In a directed graph, $Adj[v]$ are all out-degree vertices of vertax v
    - ∗ $|Adj[v]|$ means the total number of outdegree of vertax v

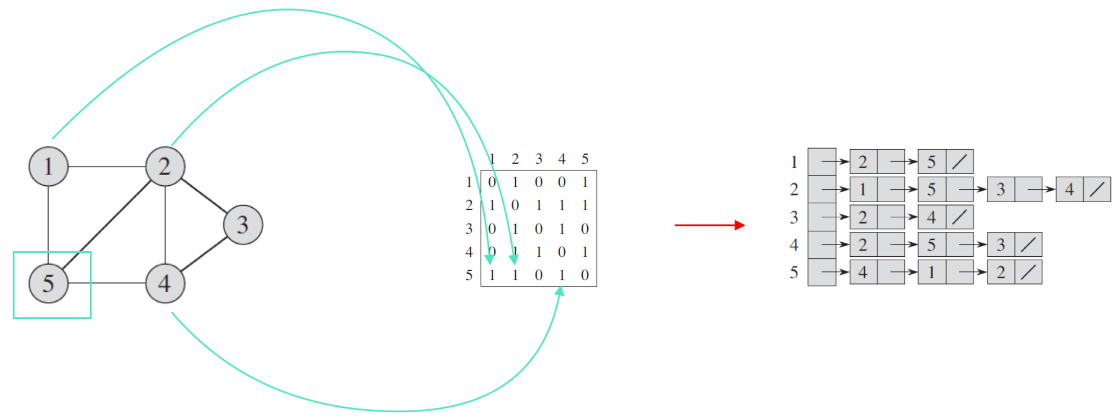




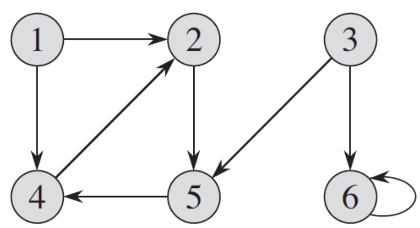

1 2 3 4 5
1 0 1 0 0 1
2 1 0 1 1 1
3 0 1 0 1 0
4 0 1 1 0 1
5 1 1 0 1 0
1 2 5
2 1 5 3 4
3 2 4
4 2 5 3
5 4 1 2


1 2 3 4 5
1 0 1 0 0 1
2 1 0 1 1 1
3 0 1 0 1 0
4 0 1 1 0 1
5 1 1 0 1 0
1 2 5
2 1 5 3 4
3 2 4
4 2 5 3
5 4 1 2

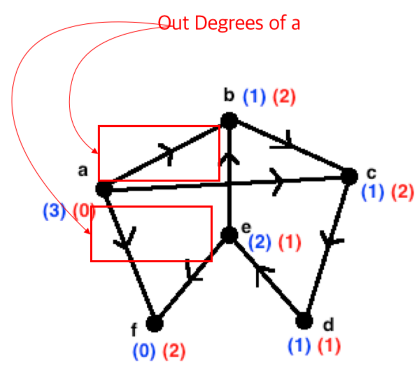

1 2 3 4 5
1 0 1 0 0 1
2 1 0 1 1 1
3 0 1 0 1 0
4 0 1 1 0 1
5 1 1 0 1 0
1 2 5
2 1 5 3 4
3 2 4
4 2 5 3
5 4 1 2



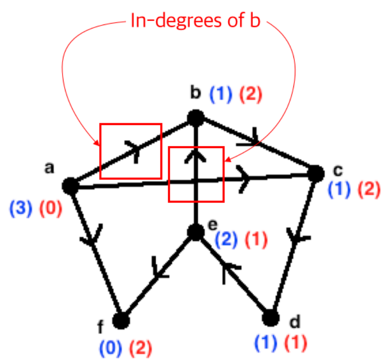
- **Directed graph**
  - Is a graph that is made up of a set of vertices connected by edges, where the edges have a direction associated with them



- **Out-degrees**
  - For a directed graph $G = (V(G), E(G))$ and a vertex $x_1 \in V(G)$, the Out-Degree of x_1 refers to the number of arcs incident from x_1. That is, the number of arcs directed <u>away</u> from the vertex x_1.

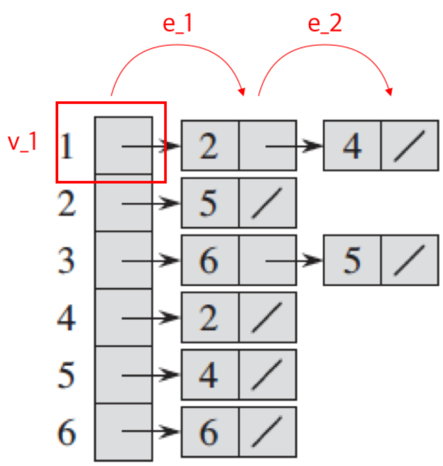


- **In-degrees**
  - For a directed graph $G = (V(G), E(G))$ and a vertex $x_1 \in V(G)$, the In-Degree of x_1 refers to the number of arcs incident to x_1. That is, the number of arcs directed <u>towards</u> the vertex x_1.



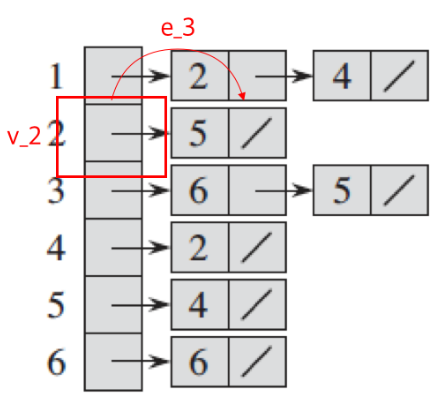
- Computing the outdegree of every vertex using adjacency list

1)



$(v_1) + (e_1 + e_2)$

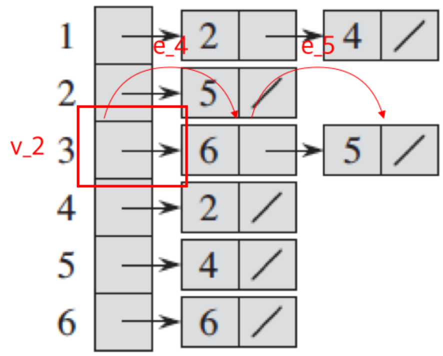
2)



$(v_1 + v_2) +$

$(e_1 + e_2 + e_3)$

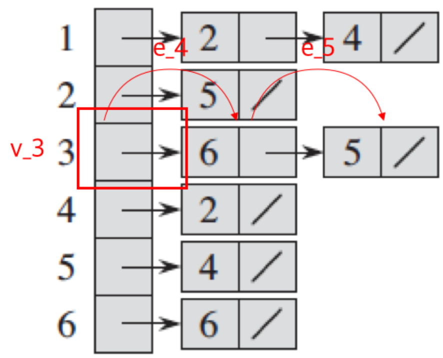
3)



(v_1 + v_2 + v3) +

(e_1 + e_2 + e_3 + e_4 + e_5)

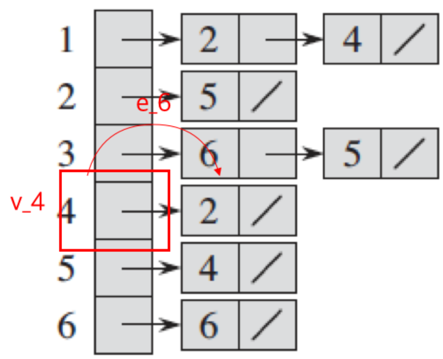
3)



(v_1 + v_2 + v3) +

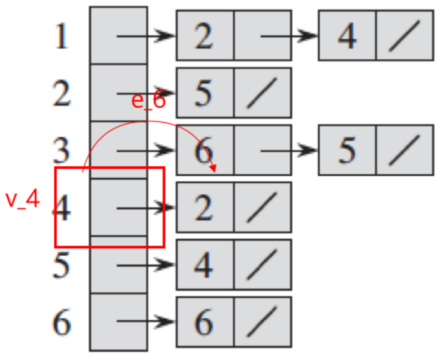
(e_1 + e_2 + e_3 + e_4 + e_5)

4)



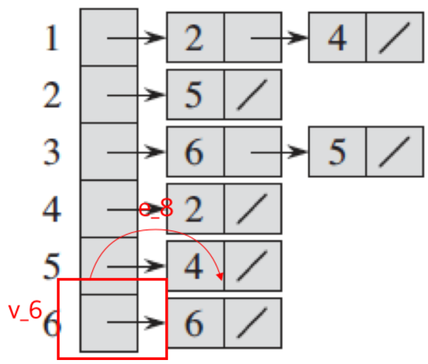
(v_1 + v_2 + v_3 + v_4) +
(e_1 + e_2 + e_3 + e_4 + e_5)

4)



(v_1 + v_2 + v_3 + v_4) +
(e_1 + e_2 + e_3 + e_4 + e_5)

6)



(v_1 + v_2 + v_3 + v_4 + v_5 _ v_6) +
(e_1 + e_2 + e_3 + e_4 + e_5 + e_6 + e_7 + e_8)

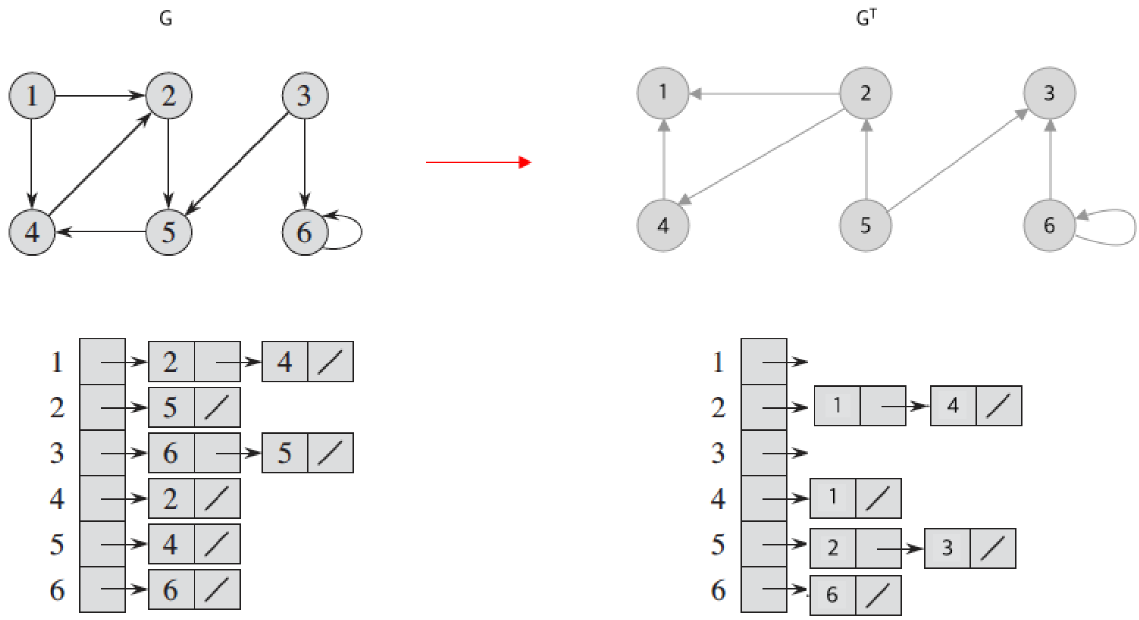
So it has $\mathcal{O}(V + E)$

- Computing the outdegree of every vertex using adjacency list

  The outdegree of a vertex is indegree of another vertex.

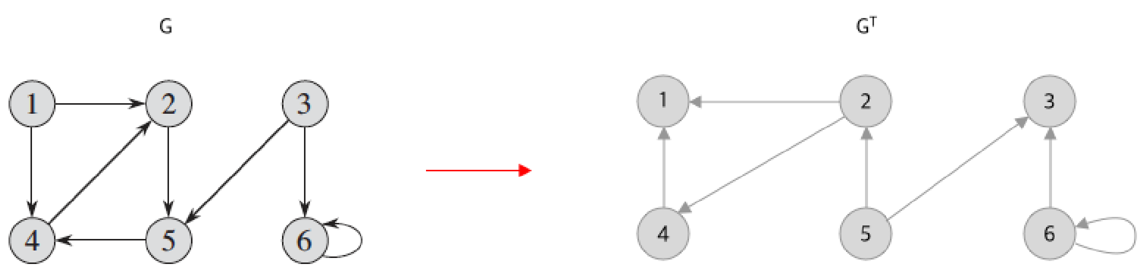
  Using this fact, we can conclude the running time of computing indegree of every vertax is $\mathcal{O}(V + E)$.

2. - Computing G^T from G in Adjacency List




```
COMPUTE-G-TRANSPOSE-ADJ-MATRIX(Adj,V)
    Let Adj' be a new adjacency list containing keys v_i...v_n

    for i = 1 to |V|
        for every vertex w in Adj[v_i]
            Insert(Adj'[w], v_i)

    return Adj'
```

- Computing G^T from G in Adjacency-Matrix



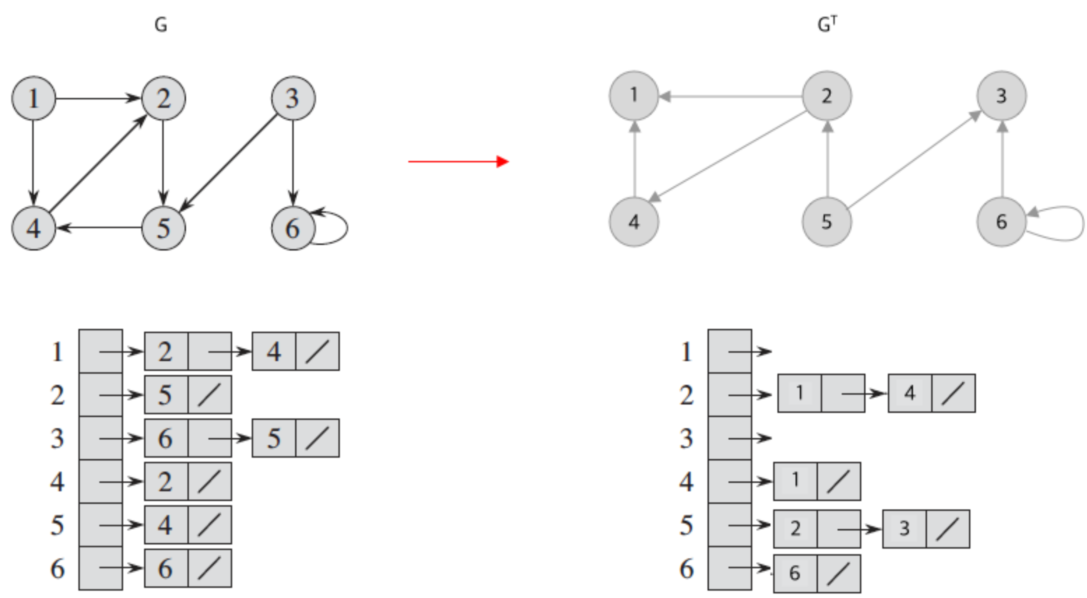
	1	2	3	4	5	6
1	0	1	0	1	0	0
2	0	0	0	0	1	0
3	0	0	0	0	1	1
4	0	1	0	0	0	0
5	0	0	0	1	0	0
6	0	0	0	0	0	1

	1	2	3	4	5	6
1	0	0	0	0	0	0
2	1	0	0	1	0	0
3	0	0	0	0	0	0
4	1	0	0	0	1	0
5	0	1	1	0	0	0
6	0	0	1	0	0	1

```
COMPUTE-G-TRANSPOSE-ADJ-MATRIX(A,V)
    Let A'[1..|V|, 1..|V|] be a new adjacency matrix

    for i = 1 to |V|
        for j = 1 to |V|
            A'[j,i] = A[i,j]

    return A'
```

Correct Solution:

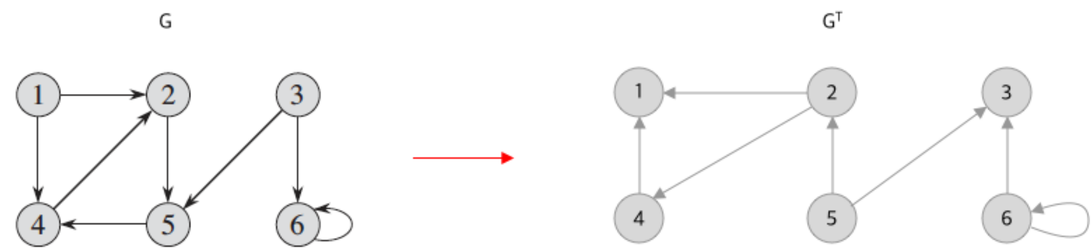
- Computing G^T from G in Adjacency List




```
COMPUTE-G-TRANSPOSE-ADJ-MATRIX(Adj,V)
    Let Adj' be a new adjacency list containing keys
v_i...v_n

    for i = 1 to |V|
        for every vertax w in Adj[v_i]
            Insert(Adj'[w], v_i)

    return Adj'
```

The running time is $\mathcal{O}(|V| + |E|)$

- Computing G^T from G in Adjacency-Matrix



	1	2	3	4	5	6
1	0	1	0	1	0	0
2	0	0	0	0	1	0
3	0	0	0	0	1	1
4	0	1	0	0	0	0
5	0	0	0	1	0	0
6	0	0	0	0	0	1

	1	2	3	4	5	6
1	0	0	0	0	0	0
2	1	0	0	1	0	0
3	0	0	0	0	0	0
4	1	0	0	0	1	0
5	0	1	1	0	0	0
6	0	0	1	0	0	1

```
COMPUTE-G-TRANSPOSE-ADJ-MATRIX(A,V)
    Let A'[1..|V|, 1..|V|] be a new adjacency matrix

    for i = 1 to |V|
        for j = 1 to |V|
            A'[j,i] = A[i,j]

    return A'
```

The running time is $\mathcal{O}(|V|^2)$

3.

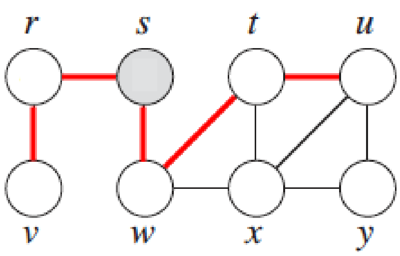
```
Breadth-First-Search(V,v_i)
    d = 0
    for each v_i ∈ V
        while performing BFS(V, v_i)
            let w be the current node in BFS
                if δ(v_i,w) > d
                    d = δ(v_i,w)

    return d
```

Finding Runtime of Algorithm

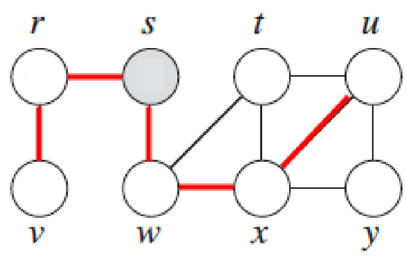
Since the graph iterates $\sum_{i=1}^{n} Adj[v_i] = |E|$ times for each $v_i \in V$, the algorithm iterates total of $|V| \cdot |E|$ times, which is $\mathcal{O}(|V||E|)$.

Notes:

- **Breadth First Search**
  - Is an algorithm for searching or traversing a graph
  - Is one of the simplest algorithm
- **Largest of All Shortest Path Distance**
  - Means the shortest distance between two furthest apart nodes

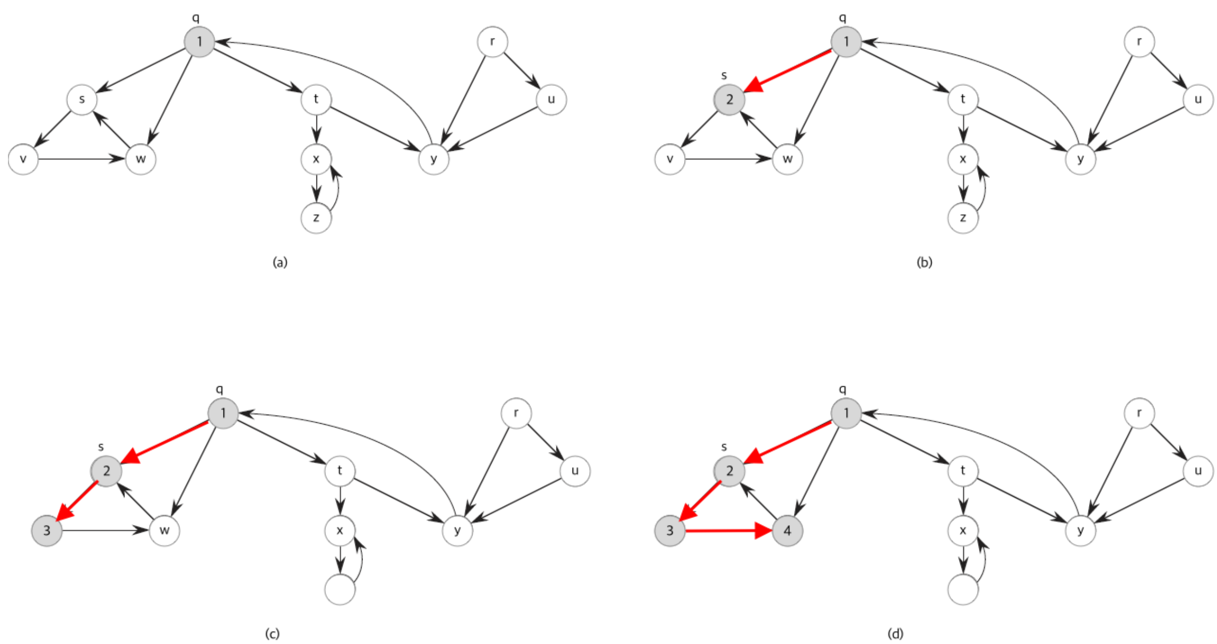


OR

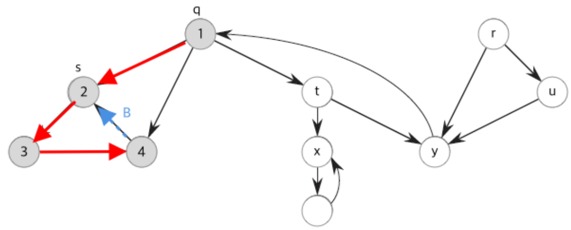


References

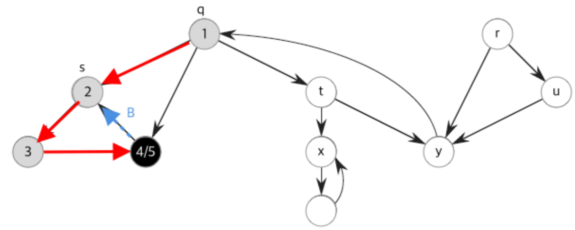
1) McGill University, 308-360 Tutorial, link



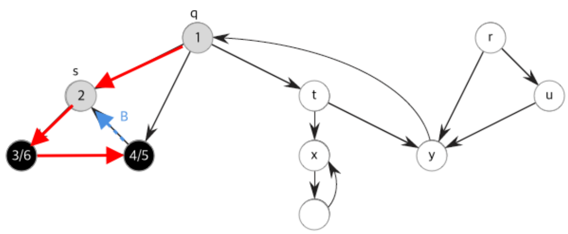
4.


q
1
s
2
3
4
B
t
x
y
r
u

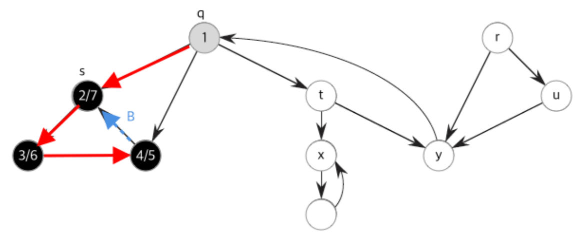
(e)


q
1
s
2
3
4/5
B
t
x
y
r
u

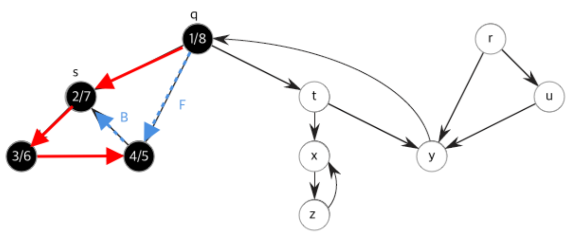
(f)


q
1
s
2
3/6
4/5
B
t
x
y
r
u

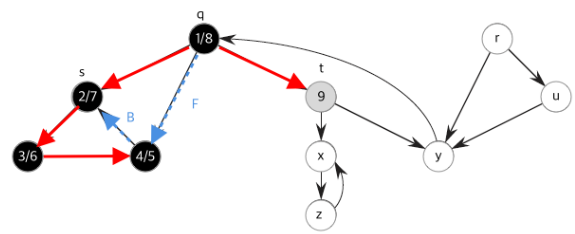
(g)


q
1
s
2/7
3/6
4/5
B
t
x
y
r
u

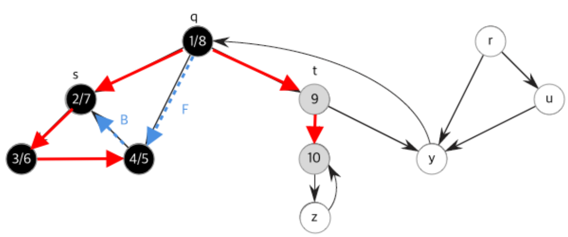
(g)


q
1/8
s
2/7
3/6
4/5
B
F
t
x
z
y
r
u

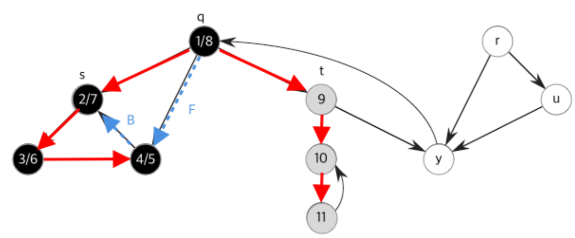
(i)


q
1/8
s
2/7
3/6
4/5
B
F
t
9
x
z
y
r
u

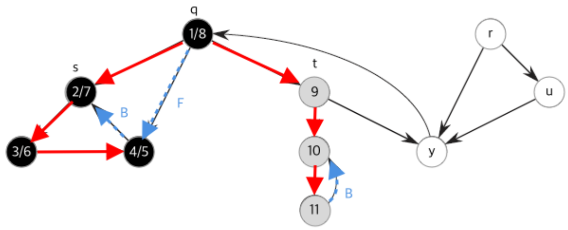
(j)


q
1/8
s
2/7
3/6
4/5
B
F
t
9
10
z
y
r
u

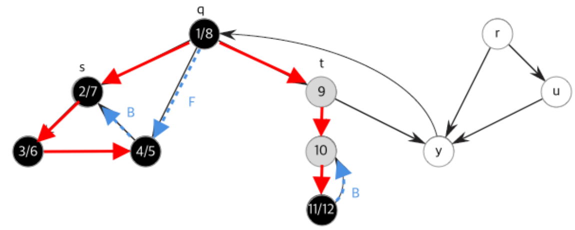
(k)


q
1/8
s
2/7
3/6
4/5
B
F
t
9
10
11
y
r
u

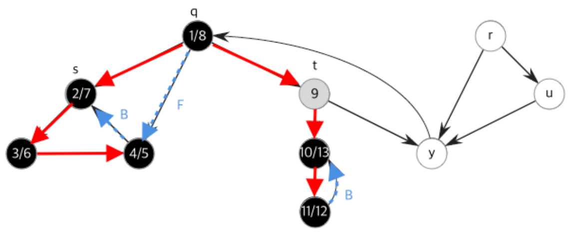
(l)


q
1/8
s
2/7
3/6
4/5
B
F
t
9
10
11
B
y
r
u

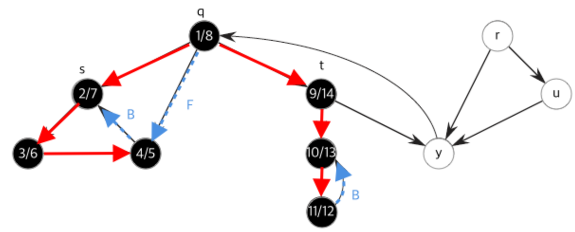
(m)


q
1/8
s
2/7
3/6
4/5
B
F
t
9
10
11/12
B
y
r
u

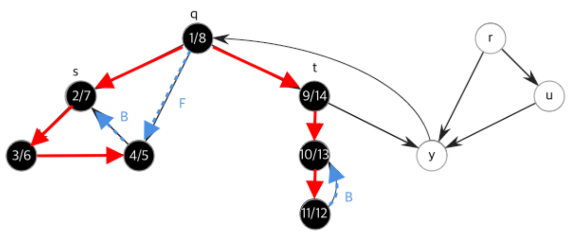
(n)


q
1/8
s
2/7
3/6
4/5
B
F
t
9
10/13
11/12
B
y
r
u

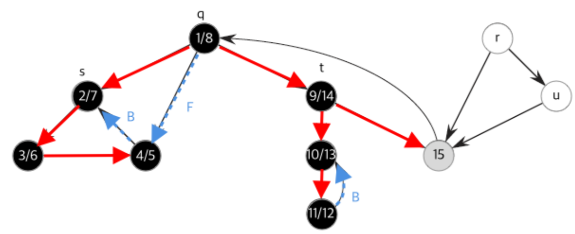
(o)


q
1/8
s
2/7
3/6
4/5
B
F
t
9/14
10/13
11/12
B
y
r
u

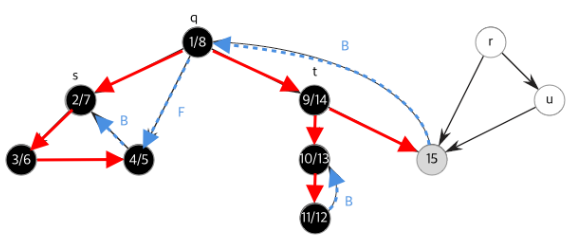
(p)


q
1/8
s
2/7
3/6
4/5
B
F
t
9/14
10/13
11/12
B
y
r
u

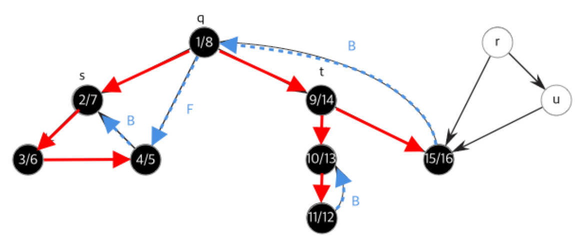
(p)


q
1/8
s
2/7
3/6
4/5
B
F
t
9/14
10/13
11/12
B
15
r
u

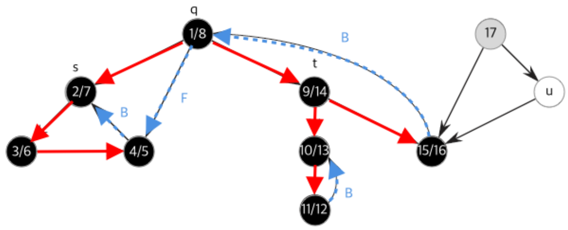
(q)


q
1/8
B
s
2/7
3/6
4/5
B
F
t
9/14
10/13
11/12
B
15
r
u

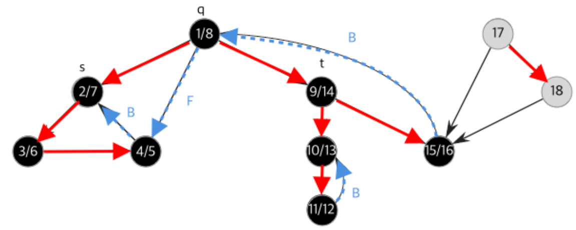
(r)


q
1/8
B
s
2/7
3/6
4/5
B
F
t
9/14
10/13
11/12
B
15/16
r
u

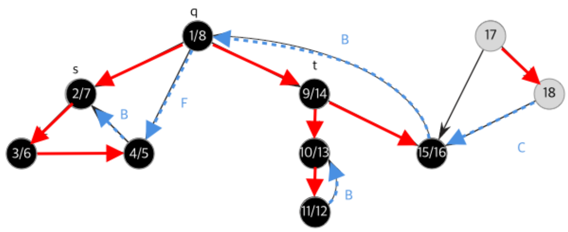
(s)



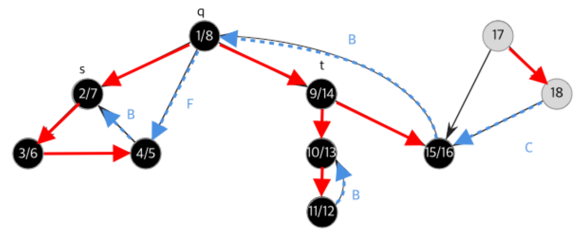
(t)



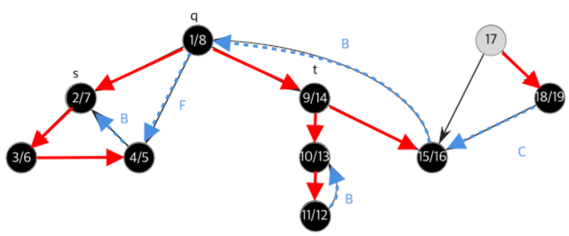
(u)



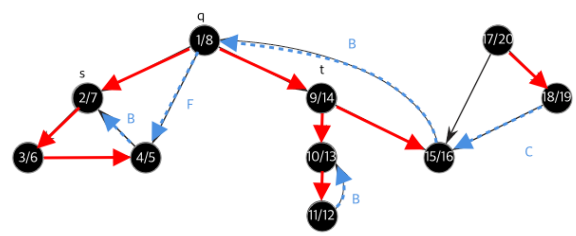
(v)



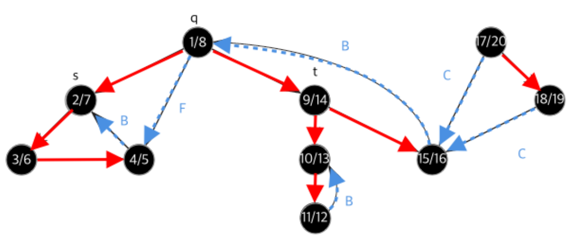
(w)



(x)



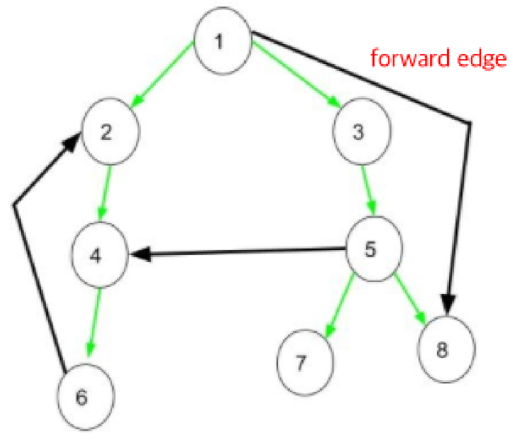
(y)



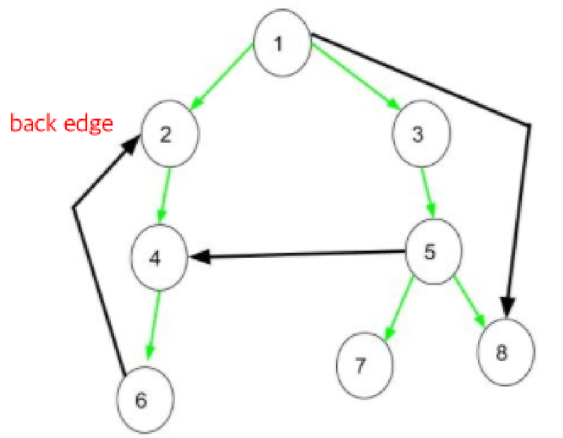
(z)

<u>Notes:</u>

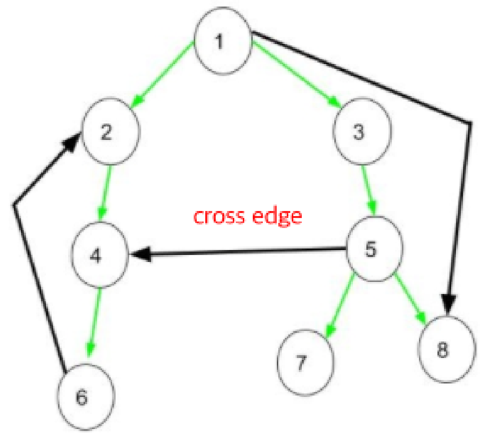
- **Depth First Search**
  - Searches deeper in the graph whenever possible
- **Forward Edge**
  - Is an edge (u, v) such that v is descendant but not part of the DFS tree. Edge $1 \rightarrow 8$ is a foward edge



- **Back Edge**
  - It is an edge (u, v) such that v is ancestor of edge u but not part of DFS tree. Edge from $6 \to 2$ is a back edge.
  - Indicates a cycle in a graph



- **Cross Edge**
  - It is a edge which connects two node such that they do not have any ancestor and a descendant relationship between them. Edge from node $5 \to 4$ is cross edge.



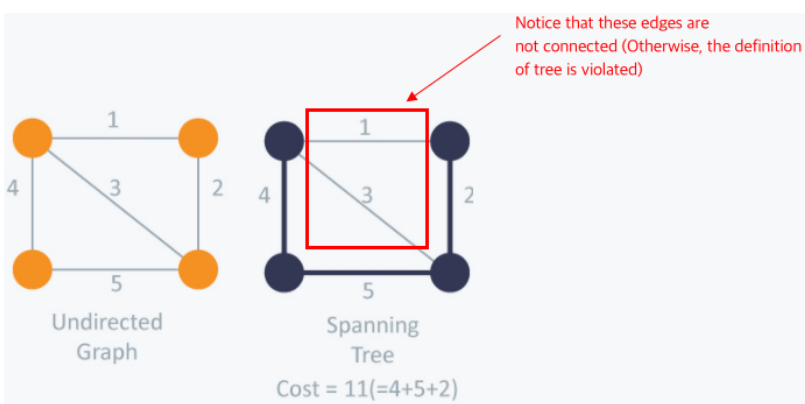
References

1) Geeks For Geeks, Tree, Back, Edge and Cross Edges in DFS of Graph, link

5. **Notes:**

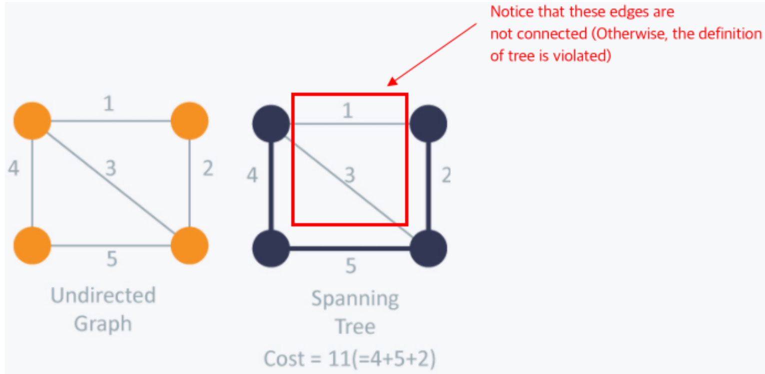
- **Spanning Tree**

  Given an undirected and connected graph $G = (V, E)$, a spanning tree of the graph G is a tree that spans G (that is, it includes every vertex of G) and is a subgraph of G (every edge in the tree belongs to G)



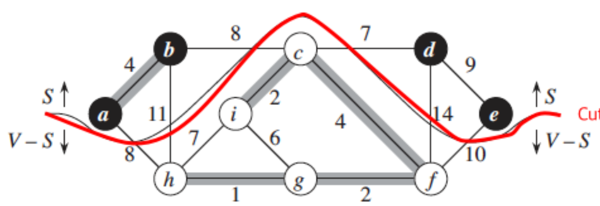
- **Minimum Spanning Tree**
  - Is the spanning tree where the cost is minimum among all the spanning trees.
    * The cost of the spanning tree is the sum of the weights of all the edges in the tree.

- There can be many minimum spanning trees.



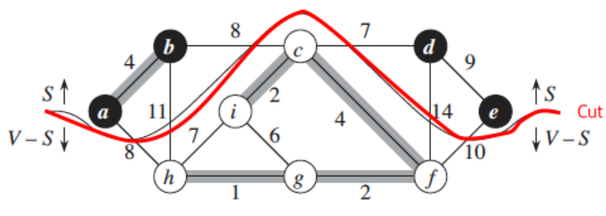
- Is used in
  1. Network design (Telephone, electrical, hydraulic, TV cable, computer, road)
  2. Approximation algorithm for NP-hard problems
  3. Learning sailent features for real-time face verification
  4. Reducing data storage in sequencing amino acids in a protein

- **Cut**
  - A cut of an undirected graph $G = (V, E)$ is denoted $(S, V - S)$
  - Is a partition of V



(a)

- **Light Edge Crossing**
  - An edge is a light edge crossing if its weight is the minimum of any edge crossing the cut



(a)

References:

-