CSC236 Worksheet 9 Solution

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Question 1

a. I need to evalulate the reg. expressions for

$$L = \{x \in \Sigma \mid x \text{has even number of 1s or an odd number of 0s}\}$$

I will do so in parts.

Part 1 (Finding reg. expressions for even number of 1s):

In this part, I will find the reg. expressions for even number of 1's.

I will do so by finding patterns in series of small examples.

Starting with $L = \{x \in \Sigma \mid x \text{ has 0 number of 1s}\}$, it's reg. expressions is

$$0^* \tag{1}$$

Now for $L = \{x \in \Sigma \mid x \text{ has 2 number of 1s}\}$, it's reg. expressions is

$$0^*10^*10^* \tag{2}$$

Now for $L = \{x \in \Sigma \mid x \text{ has 4 number of 1s}\}$, it's reg. expressions is

$$0^*10^*10^*10^*10^* \tag{3}$$

From above, I see a pattern that

$$(0^*10^*1)(0^*10^*1)0^* \tag{4}$$

Using the pattern, I can conclude that the regular expression for even number of 1s is

$$(0^*10^*1)^*0^* \tag{5}$$

Part 2 (Finding reg. expressions for odd number of 0s):

In this part, I will find the reg. expressions for odd number of 0's.

I will do so by finding patterns in series of small examples.

Starting with $L = \{x \in \Sigma \mid x \text{ has 1 number of 0s}\}$, it's reg. expressions is

$$1^*01^* \tag{6}$$

Now for $L = \{x \in \Sigma \mid x \text{ has 3 number of 0s}\}$, it's reg. expressions is

$$1^*01^*01^*01^* \tag{7}$$

Now for $L = \{x \in \Sigma \mid x \text{ has 5 number of 0s}\}$, it's reg. expressions is

$$1^*01^*01^*01^*01^*01^* \tag{8}$$

From above, I see a pattern that

$$1^*(01^*)(01^*)(01^*)(01^*)(01^*) \tag{9}$$

Using the pattern, I can conclude that the regular expression for odd number of 0s is

$$1^*(01^*)^* \tag{10}$$

Thus, by combining the two parts with union, we have

$$(0^*10^*1)^*0^* + 1^*(01^*)^* \tag{11}$$

<u>Notes:</u>

- Regular Expression
  - Quick Guide

$$(0+1)((01)^*0) \tag{12}$$

    The expression implies that

    1. Starts with 0 **or** 1
       * indicated by (0 + 1)
    2. Are then followed by **one or more repeatitions** of 01
       * indicated by $(01)^*$
    3. Ends with 0
       * indicated by the final 0

- Examples
  1. $L = \{w \in \{a, b\}^* \mid w \text{ has an } a\}$

<u>Answer:</u>

$$(a+b)^*a(a+b)^* \tag{13}$$

- Means there is one or more repeatitions of a or b at front
- Means there is a in the middle

- Means there is zero or more repeatitions of a or b at end

2. $L = \{w \in \{a,b\}^* \mid w \text{ has at lest two } a\text{s}\}$

Answer:

$$(a+b)^*a(a+b)^*a(a+b)^* \tag{14}$$

3. $L = \{w \in \{a,b\}^* \mid |w| \geq 2\}$

Answer:

$$(0+1)(0+1)(0+1)^* \tag{15}$$

In this example,

- Two characters are created (indicated by $(0+1)(0+1)$)
- And more :D!! (indicated by$(0+1)^*$)

b. I need to find the reg. expressions for $L = \{x \in \Sigma \mid x \text{ has at least one 1 and at least one 0}\}$. That is, regex expressions for $\{x \in \Sigma \mid x \text{ has at least one 1 followed by at least one 0}\}$ plus $\{x \in \Sigma \mid x \text{ has at least one 0 followed by at least one 1}\}$.

First, I need to find reg. expressions for

$$\{x \in \Sigma \mid x \text{ has at least one 1 followed by at least one 0}\} \tag{1}$$

Since the reg expressions for x with at least one 1 is $(0+1)^*1(0+1)^*$ and the reg expressions for x with at least one 0 is $(0+1)^*0(0+1)^*$, we have

$$(0+1)^*1(0+1)^*0(0+1)^* \tag{2}$$

Second, I need to find reg. expressions for

$$\{x \in \Sigma \mid x \text{ has at least one 0 followed by at least one 1}\} \tag{3}$$

Using the facts provided above, we have

$$(0+1)^*0(0+1)^*1(0+1)^* \tag{4}$$

Thus, by combining the two, we can conclude

$$(0+1)^*1(0+1)^*0(0+1)^* + (0+1)^*0(0+1)^*1(0+1)^* \tag{5}$$

c. I need to find reg. expressions for

$$\{x \in \Sigma \mid \text{every 1 in } x \text{ is immediately preceded and followed by a 0}\}$$

An example expresion of the above is

$$0^*0100^*0100^*0100^* \tag{1}$$

From above, we can see the following pattern

$$(0^*010)(0^*010)(0^*010)0^* \tag{2}$$

Thus, we have

$$(0^*010)^*0^* \tag{3}$$

Question 2

a. **Negation:** $\exists r_1, r_2, r_3 \in \mathcal{R}\varepsilon$, $(r_1 r_2 \equiv r_2 r_1) \wedge (r_1 \not\equiv \varepsilon \not\equiv r_2) \wedge (r_1 \not\equiv \emptyset \not\equiv r_2) \wedge (r_1 \not\equiv r_2)$

Let $r_1 = 1$ and $r_2 = 1^*$.

Then, $r_1 r_2 \equiv r_2 r_1$ and $r_1 \not\equiv \varepsilon \not\equiv r_2$ and $r_1 \not\equiv \emptyset \not\equiv r_2$, but $r_1 \not\equiv r_2$.

So, by counter-example, the statement is false.

Notes:

- Equivalence ($\equiv$) in Regular Expressions
  - Two regular expressions $\mathcal{R}$ and $\mathcal{S}$ are equivalent, written $R \equiv S$, if they denote the same language, i.e. $\mathcal{L}(\mathcal{R}) \equiv \mathcal{L}(\mathcal{S})$
    - Example, $(0^*1^*) \equiv (0+1)^*$
  - For all regular expressions $\mathcal{R}$, $\mathcal{S}$, $\mathcal{T}$, the following euivalences hold.
    - **Commutativity of union:** $(\mathcal{R} + \mathcal{S}) \equiv (\mathcal{S} + \mathcal{R})$
    - **Associativity of union:** $((\mathcal{R} + \mathcal{S}) + \mathcal{T}) \equiv (\mathcal{R}) + (\mathcal{S} + \mathcal{T})$
    - **Associativity of concatenation:** $((\mathcal{R}\mathcal{S})\mathcal{T}) \equiv (\mathcal{R}(\mathcal{S}\mathcal{T}))$
    - **Left distributivity:** $(\mathcal{R}(\mathcal{S} + \mathcal{T})) \equiv ((\mathcal{R}\mathcal{S})(\mathcal{R}\mathcal{T}))$
    - **Right distributivity:** $((\mathcal{S} + \mathcal{T})\mathcal{R}) \equiv ((\mathcal{S}\mathcal{R})(\mathcal{T}\mathcal{R}))$
    - **Identity for union:** $(\mathcal{R} + \emptyset) \equiv \mathcal{R}$
    - **Identity for concatenation:** $(\mathcal{R}\epsilon) \equiv R$ and $(\epsilon\mathcal{R}) \equiv \mathcal{R}$
    - **Annihilator for concatenation:** $(\emptyset\mathcal{R}) \equiv \emptyset$ and $(\mathcal{R}\emptyset) \equiv \emptyset$
    - **Idempotence of Kleene star:** $R^{**} \equiv R^*$

b. **Negation:** $(r_1 r_2 \equiv r_1 r_3) \wedge (r_1 \not\equiv \emptyset) \wedge (r_2 \not\equiv r_3)$

Let $r_1 = 1^*$, $r_2 = \varepsilon$, $r_3 = 1^*$.

Then, $r_1 r_2 \equiv r_1 r_3$ and $r_1 \not\equiv \emptyset$, but $r_2 \not\equiv r_3$.

So, by counter-example, the statement is false.

Notes:

- Learned I forgot to consider ε when exploring counter-examples :(

$r_1 = 0$

	0	0^*	1	1^*	$(1+\varepsilon)$	$(1+\varepsilon)^*$	$(0+\varepsilon)$	$(0+\varepsilon)^*$	ε
0	x	x	x	x	x	x			
0^*	x	x	x	x	x	x			
1	x	x	x	x	x	x			
1^*	x	x	x	x	x	x			
$(1+\varepsilon)$	x	x	x	x	x	x			
$(1+\varepsilon)^*$	x	x	x	x	x	x			
$(0+\varepsilon)$							x		
$(0+\varepsilon)^*$								x	
ε									x