UNIVERSITY OF TORONTO

Faculty of Arts and Science

term test #1, Version 2
CSC236F

Date: Friday October 5, 11:10–12:00pm or 12:10–1:00pm
Duration: 50 minutes
Instructor(s): Danny Heap

Examination Aids: pencils, pens, erasers, drinks, snacks

---

first and last names:

utorid:

student number:

Please read the following guidelines carefully!

- **Please write your name, utorid, and student number on the front of this exam.**
- This examination has **3** questions. There are a total of **4 pages, DOUBLE-SIDED**.
- Answer questions clearly and completely.
- You will receive 20% of the marks for any question you leave blank or indicate "I cannot answer this question."

Take a deep breath.
This is your chance to show us
How much you've learned.

We **WANT** to give you the credit

Good luck!

1. **[7 marks]** ($\approx$ 10 minutes)

   Define $f(n)$ by:

$$f(n) = \begin{cases} 1 & \text{if } n = 0 \\ 3 & \text{if } n = 1 \\ 9 & \text{if } n = 2 \\ f(n-1) + 3f(n-2) + 9f(n-3) & \text{if } n > 2 \end{cases}$$

   Define $P(n) : f(n) = 3^n$. Use complete induction to prove $\forall n \in \mathbb{N}, P(n)$. Be sure to introduce names, hypothesis, and all necessary cases. Also be sure to indicate when you use an inductive hypothesis, and why you are justified in using it.

2. **[7 marks]** ($\approx$ 20 minutes) Use contradiction and the Principle of Well Ordering to prove that there are no positive integers x, y, z, w such that $x^4 + 3y^4 + 9z^4 = 27w^4$. Be sure to make it clear when you introduce any assumption(s), where you use the Principle of Well Ordering, and where you think you have derived a contradiction. You may assume, without proof, that $\forall p, k \in \mathbb{N}$, if p is prime and $p \mid k^4$, then $p \mid k$.

3. **[7 marks]** ($\approx$ 20 minutes) Define $\mathcal{T}$ as the smallest set such that:

   (a) $() \in \mathcal{T}$

   (b) If $t_1, t_2 \in \mathcal{T}$, then $(t_1 t_2) \in \mathcal{T}$

   Some examples of elements of $\mathcal{T}$ are (), (()()), and (((()())())). For $t \in \mathcal{T}$, define left(t) as the number of (characters in t. Define:

   $$P(t) : \text{left}(t) \text{ is odd.}$$

   Use structural induction to prove $\forall t \in \mathcal{T}, P(t)$. Be sure to indicate the cases you present, when you introduce names, where you introduce assumptions,and when you have derived a conclusion.