Worksheet 6 Review

March 24, 2020

Question 1

a. $\forall n \in \mathbb{N},\ P(123) \wedge \neg(n > 123 \Rightarrow P(n))$

Correct Solution:

$P(123) \wedge (\forall n \in \mathbb{N}, P(n) \Rightarrow n \leq 123)$

b. $IsCD(x, y) :\ \exists x, y, d \in \mathbb{Z},\ d \mid x \wedge d \mid y$

$IsGCD(x, y) :\ \exists x, y, d \in \mathbb{Z}, d \mid x \wedge d \mid y \wedge (\forall n \in \mathbb{N}, n > d \Rightarrow n \nmid x \vee n \nmid y)$

Correct Solution:

$IsGCD(x, y, d) : \exists x, y, d \in \mathbb{Z}, (x = 0 \wedge y = 0 \Rightarrow d = 0) \wedge (x \neq 0 \vee y \neq 0 \Rightarrow IsCD(x, y, d) \Rightarrow \forall d' \in \mathbb{Z}, IsCD(x, y, d') \Rightarrow d' \leq d)$

c. Let $a = x$, $b = 0$, $d = x$ and $d' \in \mathbb{Z}$. Assume $IsCD(x, y, d')$.

Because we know $x \mid x$ and $x \mid 0$, we can conclude that d is a common divisor to a and b.

Since $d' \mid a$ and $d' \mid b$, and since $\forall n \in \mathbb{Z}^+, \forall d \in \mathbb{Z},\ d \mid n \Rightarrow d \leq n$, we can conclude that

$$d' \leq a \tag{1}$$

Then,

$$d' \leq d \tag{2}$$

by the fact that $d = a$.

Then it follows from above that the statement $\forall x \in \mathbb{Z}^+,\ IsGCD(x, 0, x)$ is true.

d. **Attempt 1:**

$a, b \in \mathbb{Z},\ a \neq 0 \lor b \neq 0 \Rightarrow (\exists d \in \mathbb{Z},\ d = GCD(a, b) \Rightarrow \forall d' \in \mathbb{Z}^+, \exists p, q \in \mathbb{Z})$

Attempt 2:

$a, b \in \mathbb{Z},\ \exists d \in \mathbb{Z},\ (a \neq 0 \lor b \neq 0) \land d = GCD(a, b) \Rightarrow \exists p, q \in \mathbb{Z},\ d = ap + bq \land d > 0 \land (\forall d' \in \mathbb{Z}^+, d' = ap + bq \Rightarrow d' \geq d))$

Question 2

Question 3