CSC373 Worksheet 7 Solution

August 15, 2020

1. **My Work**

   The longest simple cycle problem is the problem of finding a cycle of maximum length in a graph [5].

The decision problem is, given k, to determine whether or not the instance graph has a simple cycle of length at least k. If yes, output 1. Otherwise, output 0.

My Work

The language corresponding to the decision problem is as follows:

$$\begin{aligned}\text{LONGEST-SIMPLE-CYCLE} = \{\langle G, v_0, v_1, ..., v_k, k\rangle :\, & G = (V, E) \text{ is an undirected graph} \\ & k \geq 3 \text{ is an integer,} \\ & v_0, v_1, ..., v_k \in V \text{ are distinct,} \\ & v_0 = v_k, \\ & \text{There should exist a simple cycle in G} \\ & \text{with at least } k \text{ edges}\}\end{aligned}$$

Correct Solution:

The problem LONGEST-SIMPLE-CYCLE is a relation that associates each instacne of a graph with the longest simple cycle in that graph .

The decision problem is, given k, to determine whether or not the instance graph has a simple cycle of length at least k. If yes, output 1. Otherwise, output 0.

The language corresponding to the decision problem is as follows:

$$\text{LONGEST-SIMPLE-CYCLE} = \{\langle G, k\rangle : G = (V, E) \text{ is an undirected graph}$$
$$k \geq 0 \text{ is an integer},$$
$$\text{There should exist a simple cycle in G}$$
$$\text{with at least } k \text{ edges}\}$$

Notes

- **A Cycle in an Undirected Graph**
  - A path $\langle v_0, v_1, ..., v_k$ forms a cycle if $k \geq 3$, and $v_0 = v_k$.
- **Simple Cycle**
  - A cycle is simple if $v_1, v2, ..., v_k$ are distinct
- **Decision Problem**
  - Is the problem with yes/no solution
- **Alphabet**
  - Is a finite set of symbols
  - Is denoted Σ

    Example:

    For decision problem, its alphabet is: $\Sigma = \{0, 1\}$
    - 1 means 'yes'
    - 0 means 'no'
- **Language**
  - Is any set of strings made of symbols from Σ
  - Is denoted L

    Example:

    $L = \{10, 11, 101, 111, 1011, 1101, 10001\}$
  - Is denoted Σ^* for language of all strings over Σ plus empty string ϵ.

    Example:

    $\Sigma^* = \{\epsilon, 0, 1, 00, 01, 11, 000, ...\}$

    Example 2:

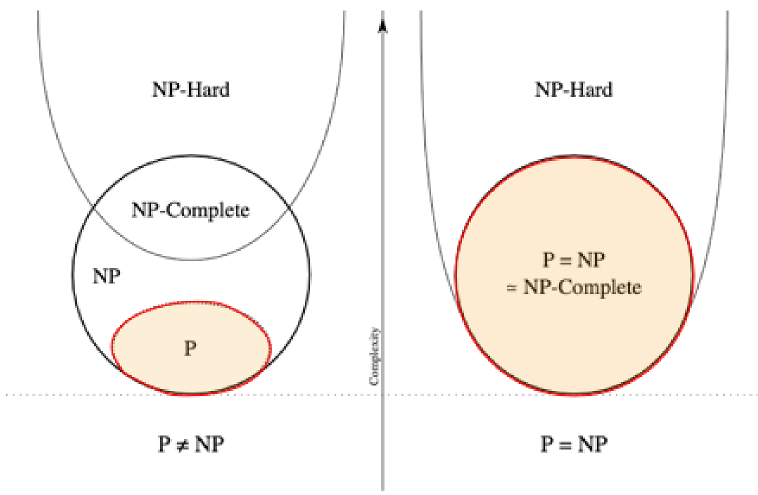
The decision problem PATH has the corresponding language

$$\begin{aligned}\text{PATH} = \{\langle G, U, v, k\rangle : & G = (V, E) \text{ is an undirected graph,}\\ & u, v \in V,\\ & k \geq 0 \text{ is an integer, and}\\ & \text{tere exists a path from } u \text{ to } v \text{ in } G\\ & \text{consisting of at most } k \text{ edges}\}\end{aligned}$$

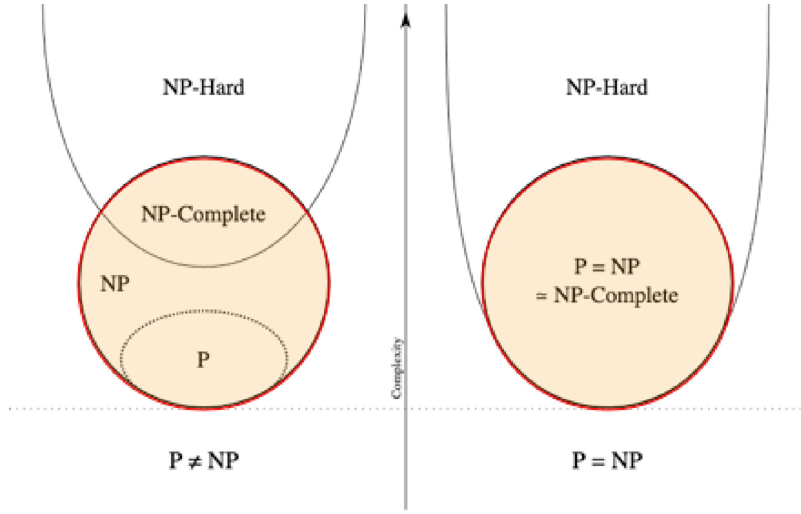
- **P**
  - Is set of problems that can be solved by a deterministic Turing machine in Polynomial time (i.e. $\mathcal{O}(n^k)$) [2].

    Example:

    1) Shortest path problems
    2) Calculating the greatest common divisor
    3) Finding maximum bipartite matching



- **NP (Non-deterministic Polynominal):**

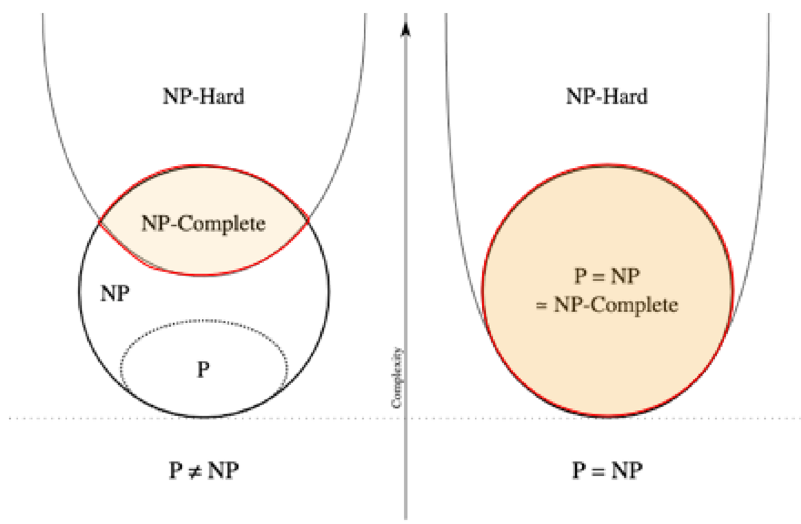


- Is set of decision problems that can be solved by a Non-deterministic Turing Machine in Polynomial time.[2]
- Has no particular rule is followed to make a guess [1].
- Can be solved in polynominal time via a “lucky algorithm”, a magical algorithm that always make a right guess [2]
- $P \subseteq NP$

Examples:

- Longest-path problems
- Hamiltonian Cycle
- Graph coloring

- **NP-Complete Problems:**

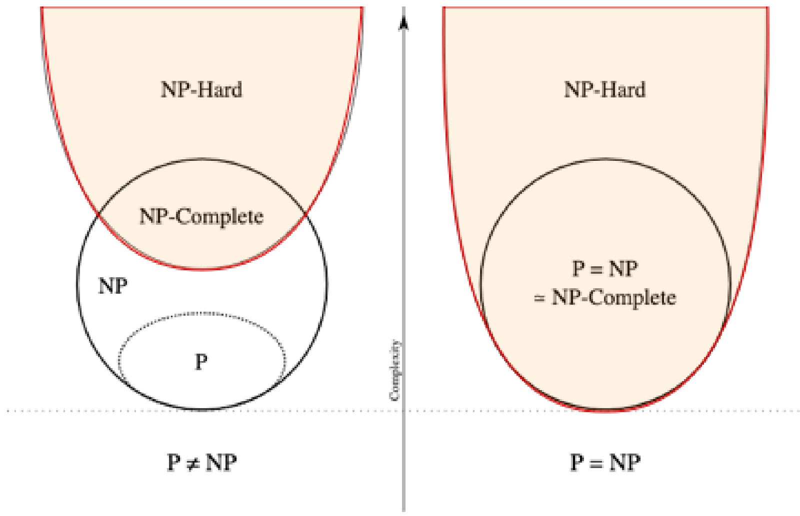


- A decision problem A is NP-complete (NPC) if

1) $A \in NP$ and
2) Every (other) problems A' in NP is reducible to A

- Has no efficient solution in polynominal number of steps (not yet) [3]
- Is not likely that there is an algorithm to make it efficient [3]

- **NP-Hard:**



- A decision problem A is NP-hard if
  1) $A \in NP$ (Not necessarily) and
  2) Every (other) problems A' in NP is reducible to A
- NP-Hard means "at least as hard as any problems in NP"
- Does not have to be about decision problems

Example:

1) Alan Turing's Halting Problem

References

1) Encyclopedia Britannica, NP-Complete Problem, link
2) Geeks for Geeks, NP-Completeness, link
3) Wikipedia, NP-complete, link
4) UCLA UC-Davis, ECS122A Handout on NP-Completeness, link

2. **Rough Works**

Notes

- **Encoding**
  - Represents problem instances in a way that the program understands
  - Encoding of a set S is a mapping e from S to the set of binary strings.

  Example

  Given natural numbers $\mathbb{N} = \{0, 1, 2, 3, 4\}$,

  it's encoding is $\{0, 1, 10, 11, 100, ...\}$.

  Using this encoding, $e(17) = 10001$.

References

1)