

# Worksheet 5 Solution

March 15, 2020

## Question 1

- $\forall n, p \in \mathbb{N}, \text{Odd}(n) \wedge \text{Odd}(p) \Rightarrow \text{Odd}(n \times p)$

Let  $n, p \in \mathbb{Z}$ , and assume  $n, p$  are odd numbers.

Then,  $\exists k, m \in \mathbb{Z}, n = 2k - 1, p = 2m - 1$  by the definition of odd numbers

Then,

$$n \times p = (2k - 1)(2m - 1) \tag{1}$$

$$= 2k2m - 2k - 2m + 1 \tag{2}$$

$$= (2k2m - 2k - 2m + 2) - 1 \tag{3}$$

$$= 2(2km - k - m + 1) - 1 \tag{4}$$

$$= 2l - 1 \tag{5}$$

where  $l = 2km - k - m + 1$ .

Since  $l \in \mathbb{Z}$ , it follows from the definition of odd number that the product of two odd numbers is odd.

## Question 2

- a.  $\forall m, n \in \mathbb{Z}, \text{Even}(m) \wedge \text{Odd}(n) \Rightarrow m^2 - n^2 = m + n$

**Question 3**

**Question 4**