

CSC373 Worksheet 6 Solution

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1. Notes:

• Linear Programming

- Is a method to achieve the best outcome (such as maximum profit or lowest cost) in a mathematical model whose requirements are represented by linear relationships. ^[1]
- Is named to make it sound cool for government funding
 - * Like dynamic programming
- Applications
 - * Microeconomics (maximize profits, minimize costs)
 - * Company management

• Standard Form

- Is a form of linear programming
- Are about maximizing, not minimizing ^[2]
- All have a positivity constraint for each variable ^[2]
- All other constraints are all of the form “linear combination of variables \leq constant”. ^[2]

3. Are about maximizing and not minimizing

Maximize $c_1x_1 + c_2x_2 + \cdots c_nx_n$

subject to

2. constraints of the form $\sum a_{ij}x_j \leq b_i$

$$\begin{array}{rcl} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n & \leq & b_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n & \leq & b_2 \\ \vdots & & \vdots \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n & \leq & b_m \end{array}$$

1. non-negativity constraints for each variable

$$x_1, x_2, \dots, x_n \geq 0$$

- **Converting Linear Programming to Standard Form**

- 1) The objective function might be a minimization rather than a maximization
 - Negate coefficients of the objective function

The diagram illustrates the conversion of a minimization problem to a maximization problem. A red curved arrow labeled "multiply by -1" points from the objective function of the minimization problem to the objective function of the maximization problem.

<div style="border: 1px solid red; padding: 5px; display: inline-block; margin-bottom: 5px;"> minimize $-2x_1 + 3x_2$ </div> <p>subject to</p> $\begin{array}{rcl} x_1 + x_2 & = & 7 \\ x_1 - 2x_2 & \leq & 4 \\ x_1 & \geq & 0 \end{array}$	<div style="border: 1px solid red; padding: 5px; display: inline-block; margin-bottom: 5px;"> maximize $2x_1 - 3x_2$ </div> <p>subject to</p> $\begin{array}{rcl} x_1 + x_2 & = & 7 \\ x_1 - 2x_2 & \leq & 4 \\ x_1 & \geq & 0 \end{array}$
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- 2) There might be variables without nonnegativity constraints
- 3) There might be **equality constraints**, which have an equal sign rather than a less-than-or-equal-to sign
- 4) There might be **inequality constraints**, but instead of having a less-than-or-equal-to sign

Example:

References:

- 1) Wikipedia, Linear Programming, [link](#)
- 2) Instituto de Matematicas, Standard form for Linear Programs, [link](#)