Midterm 2 Version 1 Solution

April 3, 2020

Question 1

a.

$$
\begin{aligned}
100 \div 2 &= 50, \textbf{ Remainders } 0 \\
50 \div 2 &= 25, \textbf{ Remainders } 0 \\
25 \div 2 &= 12, \textbf{ Remainders } 1 \\
12 \div 2 &= 6, \textbf{ Remainders } 0 \\
6 \div 2 &= 3, \textbf{ Remainders } 0 \\
3 \div 2 &= 1, \textbf{ Remainders } 1 \\
1 \div 2 &= 0, \textbf{ Remainders } 1
\end{aligned}
$$

Then, it follows from above that the binary representation of 100 is $(1100100)_2$.

b. The smallest number that can be expressed by an n-digit balanced ternary representation is

$$\sum_{i=0}^{n-1} d_i \cdot 3^i, \text{ where } d_i \in \{0, 1, 2\} \tag{1}$$

Correct Solution:

The smallest number that can be expressed by an n-digit balanced ternary representation is

$$-\left[\sum_{i=0}^{n-1} 3^i\right] \tag{2}$$

Notes:

- Realized professor is asking for an example of the smallest number.
- Learned a negative number could be expressed in in ternary or binary representation of numbers.

c.

$f(n) \in \Omega(n)$	True	$g(n) \in \Omega(n)$	False	$f(n) \in \mathcal{O}(g(n))$	False
$f(n) \in \Theta(g(n))$	False	$g(n) \in \Theta(\log_3 n)$	True	$f(n) + g(n) \in \Theta(f(n))$	True

Notes:

- $\forall g : \mathbb{N} \to \mathbb{R}^{\geq 0}$, and all numbers $a \in \mathbb{R}^{\geq 0}$, if $g \in \mathcal{O}(f)$, then $f + g \in \mathcal{O}(f)$
- $g \in \Theta(f) : \; g \in \mathcal{O}(f) \wedge g \in \Omega(f)$

  or

  $g \in \Theta(f) : \; \exists c_1, c_2, n_1 \in \mathbb{R}^+, \forall n \in \mathbb{N}, n \geq n_1 \Rightarrow c_1 g(n) \leq f(n) \leq c_2 g(n)$, where $f, g : \; \mathbb{N} \to \mathbb{R}^{\geq 0}$
- $g \in \Omega(f) : \; \exists c, n_o \in \mathbb{R}^+, \; \forall n \in \mathbb{N}, \; n \geq n_0 \Rightarrow g(n) \geq cf(n)$, where $f, g : \mathbb{N} \to \mathbb{R}^{\geq 0}$
- $g \in \mathcal{O}(f) : \; \exists c, n_o \in \mathbb{R}^+, \; \forall n \in \mathbb{N}, \; n \geq n_0 \Rightarrow g(n) \leq cf(n)$, where $f, g : \mathbb{N} \to \mathbb{R}^{\geq 0}$

d.

k	0	1	2
i_k	$3 = 3^1$	$9 = 3^2$	$81 = 3^4$

The value of i_k is

$$3^{2^k} \tag{3}$$

Question 2

Question 3

Question 4