Problem Set 2 Solution

March 17, 2020

Question 1

a. **Fact 1:**

Let $a, b \in \mathbb{N}$. Assume $2 \nmid a$ and $2 \nmid b$.

Then, $\exists k, l \in \mathbb{Z}$.

$$a = 2k - 1 \tag{1}$$
$$b = 2l - 1 \tag{2}$$

Then,

$$\begin{aligned} a - b &= (2k-1)(2l-1) && (3)\\ &= 2(k-1) - 2 && (4)\\ &= 2(k-l-1) && (5) \end{aligned}$$

Then, it follows from the definition of divisibility that $2 \mid a - b$.

Fact 2:

Let $a, b, c \in \mathbb{Z}$. Assume $a \mid b$ and $b \mid c$.

Then $\exists k, l \in \mathbb{Z}$,

$$b = ka \tag{1}$$
$$c = lb \tag{2}$$

Then,

$$c = l(ka) \tag{3}$$
$$c = (lk)a \tag{4}$$

Then, it follows from the definition of divisibility that $a \mid c$.

Question 2

Question 3