

Worksheet 6 Review 2

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Question 1

a. $\forall x \in \mathbb{N}, P(123) \wedge P(x) \Rightarrow x \leq 123$

Correct Solution:

$$P(123) \wedge (\forall x \in \mathbb{N}, P(x) \Rightarrow x \leq 123)$$

b. $IsCD(x, y, d) : d \mid x \wedge d \mid y$, where $x, y, d \in \mathbb{Z}$

$$IsGCD(x, y, d) : \forall n \in \mathbb{N}, IsCD(x, y, n) \Rightarrow \exists d \in \mathbb{N}, IsCD(x, y, d) \wedge n \leq d$$

Correct Solution:

$$IsCD(x, y, d) : d \mid x \wedge d \mid y, \text{ where } x, y, d \in \mathbb{Z}$$

$$IsGCD(x, y, d) : (x = 0 \wedge y = 0 \Rightarrow d = 0) \wedge (x \neq 0 \wedge y \neq 0 \Rightarrow IsCD(x, y, d) \wedge (\forall d_1 \in \mathbb{Z}, IsCD(x, y, d_1) \Rightarrow d_1 \leq d)), \text{ where } x, y, d \in \mathbb{Z}$$

Notes:

- Realized the definition of $IsGCD$ extends from previous question
- Noticed professor defines if...else conditions in a predicate logic the following way

$$(\text{case 1} \Rightarrow \text{statement 1}) \wedge (\text{case 2} \Rightarrow \text{statement 2})$$

- Hm... I feel puzzled about \wedge operator used in between cases (i.e. $(x = 0 \wedge y = 0 \Rightarrow d = 0) \wedge (x \neq 0 \wedge y \neq 0 \Rightarrow IsCD(x, y, d) \wedge (\forall d_1 \in \mathbb{Z}, IsCD(x, y, d_1) \Rightarrow d_1 \leq d))$). At glimpse, I felt \vee is more appropriate since if this case is not true, then we want other case should be true.

c. **Statement:** $IsCD(x, 0, x) \wedge (\forall d_1 \in \mathbb{Z}, IsCD(x, 0, d_1) \Rightarrow d_1 \leq x)$

Proof. Let $x \in \mathbb{Z}^+$

We need to prove x is a common divisor to both 0 and x , and we need to prove all common divisors d_1 of 0 and x is less than or equal to x .

First, we need to show there is $k_1 \in \mathbb{Z}$ such that $x = k_1 \cdot x$ and we need to show $k_2 \in \mathbb{Z}$ such that $0 = k_2 \cdot x$.

Let $k_1 = 1$ and $k_2 = 0$.

Then, we can calculate that

$$x = 1 \cdot x = k_1 \cdot x \quad (1)$$

$$0 = 0 \cdot x = k_2 \cdot x \quad (2)$$

Now, we need to show all integers d_1 that is a common divisor to both 0 and x is less than equal to x .

Let $d_1 \in \mathbb{Z}$ and assume $d_1 \mid x$ and $d_1 \mid 0$.

We need to show $d_1 \leq x$.

The hint tells us

$$\forall n \in \mathbb{Z}^+, \forall d \in \mathbb{Z}, d \mid n \Rightarrow d \leq n \quad (3)$$

Because we know from assumption that $d_1 \mid x$, by using the hint, we can conclude

$$d_1 \leq x \quad (4)$$

□

Pseudoproof:

Let $x \in \mathbb{Z}^+$

We need to prove x is a common divisor to both 0 and x , and we need to prove all common divisors d_1 of 0 and x is less than or equal to x .

1. Show $IsCD(x, 0, x)$

We need to show there is $k_1 \in \mathbb{Z}$ such that $x = k_1 \cdot x$ and we need to show $k_2 \in \mathbb{Z}$ such that $0 = k_2 \cdot x$.

Let $k_1 = 1$ and $k_2 = 0$.

- Show $x = k_1 \cdot x$ and $0 = k_2 \cdot 0$

Then, we can calculate that

$$x = 1 \cdot x = k_1 \cdot x \quad (5)$$

$$0 = 0 \cdot x = k_2 \cdot x \quad (6)$$

2. Show $\forall d_1 \in \mathbb{Z}, IsCD(x, 0, d_1) \Rightarrow d_1 \leq x$

Let $d_1 \in \mathbb{Z}$ and assume $d_1 \mid x$ and $d_1 \mid 0$.

We need to show $d_1 \leq x$.

1. Use fact ' $\forall n \in \mathbb{Z}^+, \forall d \in \mathbb{Z}, d \mid n \Rightarrow d \leq n$ ' to show $d_1 \leq x$.

The hint tells us

$$\forall n \in \mathbb{Z}^+, \forall d \in \mathbb{Z}, d \mid n \Rightarrow d \leq n \quad (7)$$

Because we know from assumption that $d_1 \mid x$, by using the hint, we can conclude

$$d_1 \leq x \quad (8)$$

d. $\forall a, b \in \mathbb{Z}, (a \neq 0) \vee (b \neq 0) \Rightarrow \exists p, q \in \mathbb{Z}, pa + qb = gcd(a, b)$

Question 2

Question 3