CSC373 Worksheet 5 Solution

August 8, 2020

1. **Rough Works:**

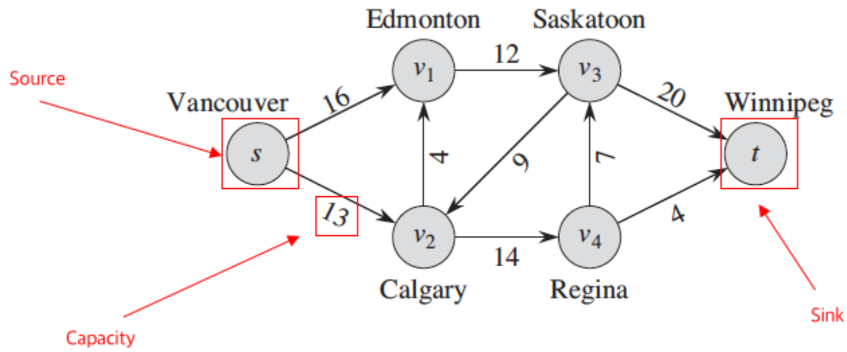
   Assume that a flow network $G = (V, E)$ violates the assumption that the network contains a path $s \rightsquigarrow v \rightsquigarrow t$ for all vertices $v \in V$. Let u be a vertex for which there is no path $s \rightsquigarrow u \rightsquigarrow t$.

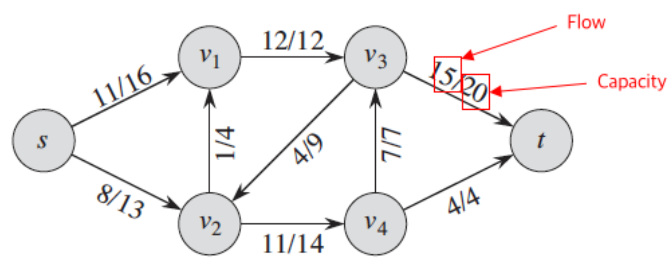
   I must show such that there exists a maximum flow f in G such that $f(u, v) = f(v, u) = 0$ for all vertices $v \in V$

   1.

   Notes

   - **Maximum Flow Problem:**
     - Is about computing the greatest rate at which we can ship material from the source to the sink without violating any capacity constraints
   - **Flow Network:**
     - $G = (V, E)$ is a directed graph in which each edge $(u, v) \in E$ has a nonnegative capacity $c(u, v) \geq 0$.
     - Two vertices must exist: **source** s and **sink** t
     - **path** from source s to vertax v to sink t is represented by $s \rightsquigarrow v \rightsquigarrow t$





- **Capacity:**
  - Is a non-negative function $f : V \times V \to \mathbb{R}_{\geq 0}$
  - Has **capacity constraint** where for all $u, v \in V$ $0 \leq f(u, v) \leq c(u, v)$
    - Means flow cannot be above capacity constraint
- **Flow:**
  - Is a real valued function $f : V \times V \to \mathbb{R}$ in G
  - Satisfies **capacity constraint** (i.e for all $u, v \in V$, $0 \leq f(u, v) \leq c(u, v)$)
  - Satisfies **flow conservation**

    For all $u \in V - \{s, t\}$, we require

$$\sum_{v \in V} f(v, u) = \sum_{v \in V} f(u, v) \tag{1}$$

Means total flow forward is the same as total flow backward