CSC373 Worksheet 2 Solution

July 25, 2020

1) $[a\_11 = [12,16)]$

i	1	2	3	4	5	6	7	8	9	10	11
s_i	1	3	0	5	3	5	6	8	8	2	12
f_i	4	5	6	7	9	9	10	11	12	14	16

2) $[a\_11 = [12,16), a\_2 = [8,11)]$

i	1	2	3	4	5	6	7	8	9	10	11
s_i	1	3	0	5	3	5	6	8	8	2	12
f_i	4	5	6	7	9	9	10	11	12	14	16

3) $[a\_11 = [12,16), a\_2 = [8,11), a\_4 = [5,7)]$

i	1	2	3	4	5	6	7	8	9	10	11
s_i	1	3	0	5	3	5	6	8	8	2	12
f_i	4	5	6	7	9	9	10	11	12	14	16

3) $[a\_11 = [12,16), a\_2 = [8,11), a\_4 = [5,7), a\_1 = [1,4)]$

i	1	2	3	4	5	6	7	8	9	10	11
s_i	1	3	0	5	3	5	6	8	8	2	12
f_i	4	5	6	7	9	9	10	11	12	14	16
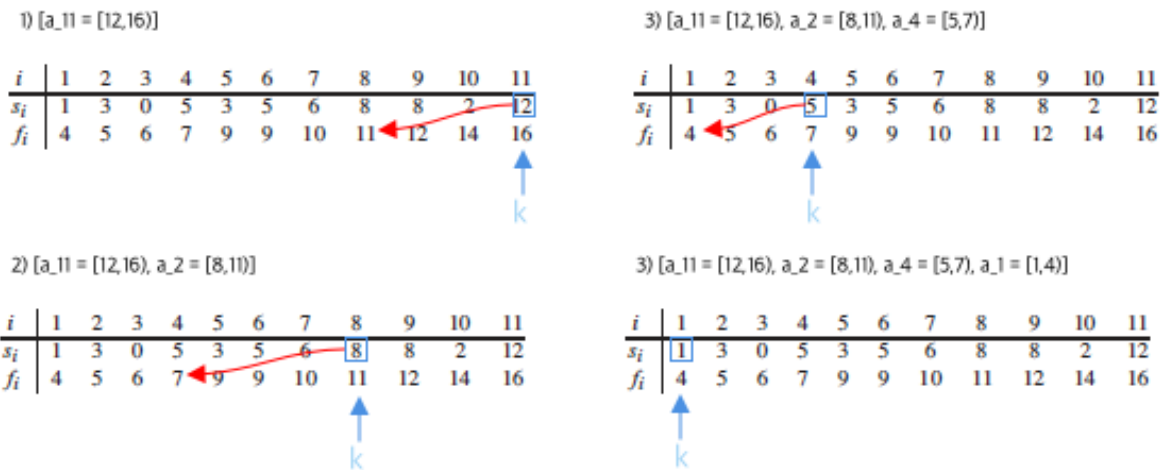
1.

This approach is a greedy algorithm because algorithm

1) Has the greedy choice: selecting the last activity to start that is compatible with all previously selected activites

2) Has the greedy choice that is always part of optimal solution:

Claim:

Consider any nonempty subproblem S_k. Let a_m be an activity in S_k with the last activity to start that is compatible with all previously selected activities. Then a_m is included in some maximum-size subset of mutually compatible activities of S_k

Proof. Let A_k be a maximum-size subset of mutually compatible activities in S_k, and let a_j be the activity in A_k with the last activity to start that is compatible with all previously selected activities.

If $a_j = a_m$, we are done, since we have shown that a_m is the maximum-size subset of mutually compatible activities of S_k.

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If $a_j \neq a_m$, let the set $A'_k = A_k = \{a_j\} \cup \{a_m\}$ be A_k but subtituting a_m for a_j. The activities in A'_k are disjoint, which follow because the activities in A_k are disjoint, a_j is the first activity in A_k to finish, and $s_j \leq s_m$.

Since $|A'_k| = |A_k|$, we conclude that A'_k is a maximum-size subset of mutually compatible activities of S_k, and it includes a_m. $\square$

Notes:

- Greedy Algorithm
  - Always makes the choice that looks best at the moment
    * Locally optimal solution leads to globally optimal solution
- Activity-selection Problem (Greedy algorithm using dynamic programming)
  - Goal: Selecting maximum size set of mutually compatible activities

  ### Example:

i	1	2	3	4	5	6	7	8	9	10	11
s_i	1	3	0	5	3	5	6	8	8	2	12
f_i	4	5	6	7	9	9	10	11	12	14	16

  - Suppose a set exists $S = \{a_1 = [s_1, f_1), a_2 = [s_2, f_2), ..., a_n = [s_n, f_n)\}$
    * a_i represents an ith activity
    * s_i represents starting time
    * f_i represents finishing time
    * $0 \leq s_i < f_i < \infty$
    * $a_1, ..., a_n$ sorted in monotonically increasing order of finish time

      i.e.

      $$f_1 \leq f_2 \leq f_3 \leq ... \leq f_{n-1} \leq f_n$$

    * a_i and a_j are **compatible**, if intervals $[s_i, f_i)$ and $[s_j, f_j)$ don't overlap

      i.e

      $$s_i \geq f_j \text{ and } s_j \geq f_i$$

  - Steps
    1. Think about dynamic programming solution
       * Construct optimal solution using two subproblems

S_{ij}: activities that start after activity a_i finishes and before activity a_j starts

i.e.

$S_{19} = \{a_4 = [5,7), a_6 = [5,9), a_7 = [6,10)\}$

A_{ij}: maximum set of mutually compatible activities in S_{ij} (including a_k)

· $A_{ik} = A_{ij} \cap S_{ik}$
· $A_{kj} = A_{ij} \cap S_{kj}$
· $A_{ij} = A_{ik} \cup \{a_k\} \cup Akj$
· So, $|A_{ij}| = |A_{ik}| + |A_{kj}| + 1$

∗ Verify that optimal solution A_{ij} must include optimal solution to the two subproblems for S_{kj}

Let A'_{kj} be another mutually compatible activities in S_{kj} where $|A'_{kj}| > |A_{kj}|$.

Then we could use A'_{kj} in a solution to subproblem of S_{ij}

Then we have $|A_{ik}| + |A'_{kj}| + 1 > |A_{jk}| + |A_{kj}| + 1 = |A_{ij}|$ mutually compatible activites

This contradicts assumption that A_{ij} is an optimal solution

∗ Verify that optimal solution A_{ij} must include optimal solution to the two subproblems for S_{ik}

The same applies for activities in S_{ik}

2. Observe that only one choice - greedy choice, and that when we make the greedy choice, only one subproblem remains
   ∗ Steps
     1. Make a greedy choice
        · Choose an activity that makes the most resource possible (intuition)
        · Choose an acitivty that finishes the earliest (intuition)
     2. Solve a subproblem: Find activities that start after a_1 finishes
     3. Verify that making greedy choices always arrive at optimal solution

Theorem 16.1 (Page 418):

Consider any non-empty subproble S_k, and let a_m be an activity in S_k with the earliest finish time. Then a_m is included in some maximum-size

subset of mutually compatible activities of S_k

3. Develop recursive greedy solution
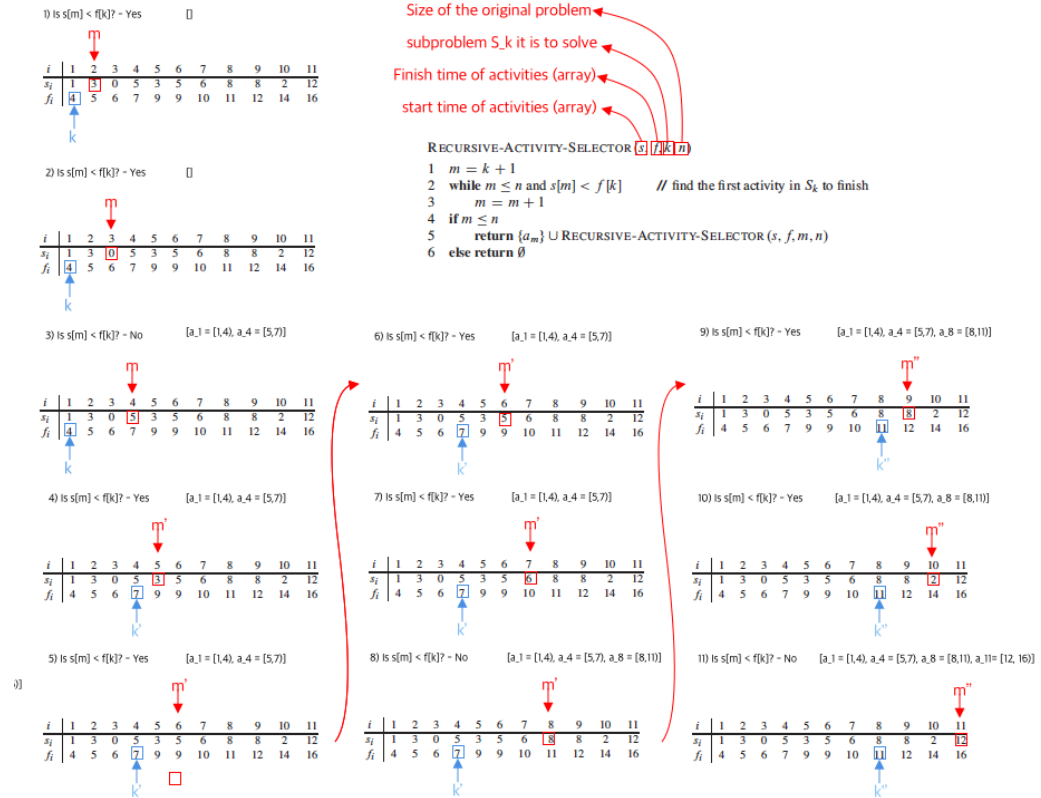
4. Convert the recursive algorithm into iterative one