

CSC343 Worksheet 12 Solution

July 1, 2020

1.
 - Keys
 - {id of molecule}
 - {x position, y position, z position}
 - Functional Dependencies
 - 1. id of molecule \rightarrow x position, y position, z position, x velocity, y velocity, z velocity
 - 2. x position, y position, z position \rightarrow id of molecule, x velocity, y velocity, z velocity

Notes:

- Function Dependencies
 - *Functional Dependency* is a relationship between two attributes typically between the key and other non-key attributes within a table.

Example:

SIN \rightarrow Name, Address, Birthdate

Example 2:

ISBN \rightarrow Title

- Key of Relations
 - One or more attributes $\{A_1, A_2, \dots, A_n\}$ is a key for a relation R if
 1. Those attributes functionally determine all other attributes of the relation
 2. No proper subset of $\{A_1, A_2, \dots, A_n\}$ functionally determines all other attributes of R

Example:

Given relation

$R = \text{Movies1}(\text{title, year, length, genre, studioName, starName})$

- i. {title, year, starName } form a key for the relation **Movies1**

- ii. $\{ \text{year}, \text{starName} \}$ is not a key. Same star can be in multiple movies per year
- Superkeys
 - * Means a set of attributes that contains a key
 - * Don't need to be minimal

Example:

Given relation

 $R = \text{Movies1}(\text{title}, \text{year}, \text{length}, \text{genre}, \text{studioName}, \text{starName})$

- $\{ \text{title}, \text{year}, \text{starName} \}$ is a key and superkey
- $\{ \text{title}, \text{year}, \text{starName}, \text{title}, \text{year}, \text{length} \}$ is a superkey

References:

- 1) OpenTextBC, Chapter 11 Functional Dependencies, link
2. a)
 1. $AB \rightarrow C$
 2. $AB \rightarrow D$
 3. $C \rightarrow A$
 4. $C \rightarrow B$
 5. $D \rightarrow B$
 6. $D \rightarrow C$
 7. $C \rightarrow D$
 8. $D \rightarrow A$

Second Attempt:

$\{A, B\}^+ = \{A, B, C, D\}$, so the following non-trivial FDs follows: $AB \rightarrow C$ and $AB \rightarrow D$.

$\{C\}^+ = \{D, A\}$, so the following non-trivial FDs follows $C \rightarrow D$ and $C \rightarrow A$.

$\{D\}^+ = \{A\}$, so the following non-trivial FDs follows: $D \rightarrow A$.

Notes:

- The Splitting / Combining Rule
 - Combining Rule
 - * $A_1, A_2, \dots, A_n \rightarrow B_i$ for $i = 1, 2, \dots, m$
to
 $A_1, A_2, \dots, A_n \rightarrow B_1, B_2, \dots, B_m$

Example:

Given

title year \rightarrow length
 title year \rightarrow genre
 title year \rightarrow studioName

it's combined form is

title year \rightarrow length genre studioName

– Splitting Rule

*

* $A_1, A_2, \dots, A_n \rightarrow B_1, B_2, \dots, B_m$

to

$A_1, A_2, \dots, A_n \rightarrow B_i$ for $i = 1, 2, \dots, m$

Example:

Given

title year \rightarrow length

It's splitted form is

title \rightarrow length
 year \rightarrow length

• Trivial Functional Dependencies

- A functional dependency $FD : X \rightarrow Y$ is **trivial** if Y is a subset of X

Exmample:

title year \rightarrow title

Example 2:

title \rightarrow title

• Non-trivial Functional Dependencies

- is a case where some but not all of the attributes on the R.H.S of an FD are also on L.H.S

Example:

title year \rightarrow title movieLength

- Can be simplified using **trivial-dependency rule**
 - * The FD $A_1A_2 \cdots A_n \rightarrow B_1B_2 \cdots B_m$ is equivalent to $A_1A_2 \cdots A_n \rightarrow C_1C_2 \cdots C_k$

where C 's are all those B 's that are not in A 's.



Figure 3.3: The trivial-dependency rule

- Computing the Closure of Attributes
 - Closure of attribute set $\{X\}$ is denoted as $\{X\}^+$.
 - The closure means a given set of attributes A satisfying FD, are a sets of all attributes B such that $A \rightarrow B$

Example:

Given attributes A, B, C, D, E, F and FDs $AB \rightarrow C$, $BC \rightarrow AD$, $D \rightarrow E$ and $CF \rightarrow B$, What is the closure of $\{A, B\}$ or $\{A, B\}^+$

1. Start with $\{A, B\}$.
2. Split $BC \rightarrow AD$
 - * We have $BC \rightarrow A$ and $BC \rightarrow D$
 - * Since A is in $\{A, B\}$, this is not included
 - * Since D is not in $\{A, B\}$, this IS included

So, we have $\{A, B, D\}$

3. Since C in $AB \rightarrow C$ is NOT in $\{A, B, C, D\}$, C is included and we have $\{A, B, C, D\}$
4. Since A in $BC \rightarrow A$ is in $\{A, B, C, D\}$, this is skipped
5. Since E is not in $D \rightarrow E$, E is included and we have $\{A, B, C, D, E\}$ as our solution

- Why the Closure Algorithm Works
- Transitive Rule
 - Definition

If $A_1A_2 \cdots A_n \rightarrow B_1B_2 \cdots B_m$ and $B_1B_2 \cdots B_m \rightarrow C_1C_2 \cdots C_k$ hold in relation R , $A_1A_2 \cdots A_n \rightarrow C_1C_2 \cdots C_k$ also holds in R .

Example:

Given

title year \rightarrow studioName
 studioName \rightarrow studioAddr

Transitive rule says the above is equal to the following

title year \rightarrow studioAddr

- Inference Rules
 - Is also called **Armstrong's Axioms**
 - Has 3 axioms
 1. *Reflexivity*
 - * If $\{B_1, B_2, \dots, B_n\} \subseteq \{A_1, A_2, \dots, A_n\}$ then $A_1A_2 \cdots A_n \rightarrow B_1B_2 \cdots B_m$
 - * also called **trivial FDs**
 2. *Augmentation*
 - * If $A_1A_2 \cdots A_n \rightarrow B_1B_2 \cdots B_m$ then $A_1A_2 \cdots A_nC_1C_2 \cdots C_k \rightarrow B_1B_2 \cdots B_mC_1C_2 \cdots C_k$
 - * $C_1C_2 \cdots C_k$ are any set of attributes
 3. *Transitivity*
 - * If $A_1A_2 \cdots A_n \rightarrow B_1B_2 \cdots B_m$ and $B_1B_2 \cdots B_m \rightarrow C_1C_2 \cdots C_k$ then $A_1A_2 \cdots A_n \rightarrow C_1C_2 \cdots C_k$

b) A, B is the only key of R .

Notes:

- Key of Attributes
 - **Definition:** A set of attributes $\{A_1, A_2, \dots, A_n\}$ is a key for a relation R if
 1. Those attributes functionally determine all other attributes

2. No proper subset of $\{A_1, A_2, \dots, A_n\}$ functionally determines all other attributes of R .

c) The superkeys that are not keys are: $\{A, B, C\}$, $\{A, B, D\}$, $\{A, B, C, D\}$

3. i) a) $\{A\}^+ = \{A, B, C, D\}$, so we have $A \rightarrow A$, $A \rightarrow B$, $A \rightarrow C$, $A \rightarrow D$

$\{B\}^+ = \{C, D\}$, so we have $B \rightarrow C$ and $B \rightarrow D$

b) $\{A\}$ is the key of S .

c) The super keys that are not keys are:

$\{A, B\}$, $\{A, C\}$, $\{A, D\}$, $\{A, B, C\}$, $\{A, B, D\}$, $\{A, B, C, D\}$

ii) a) $\{A\}^+ = \{A\}$, so this FD is trivial.

$\{B\}^+ = \{B\}$, so this FD is trivial.

$\{C\}^+ = \{C\}$, so this FD is trivial.

$\{D\}^+ = \{D\}$, so this FD is trivial.

$\{A, B\}^+ = \{A, B, C, D\}$, so we have $AB \rightarrow A$, $AB \rightarrow B$, $AB \rightarrow C$, $AB \rightarrow D$

$\{A, C\}^+ = \{A, C\}$, so we have $AC \rightarrow A$, $AC \rightarrow C$

$\{A, D\}^+ = \{A, D, B\}$, so we have $AD \rightarrow A$, $AD \rightarrow D$, $AD \rightarrow B$

$\{B, C\}^+ = \{B, C, D, A\}$, so we have $BC \rightarrow A$, $BC \rightarrow B$, $BC \rightarrow C$, $BC \rightarrow D$

$\{D, C\}^+ = \{D, C, A, B\}$, so we have $DC \rightarrow D$, $DC \rightarrow C$, $DC \rightarrow A$, $DC \rightarrow B$

$\{A, B, C\}^+ = \{A, B, C, D\}$, so we have $ABC \rightarrow A$, $ABC \rightarrow B$, $ABC \rightarrow C$, $ABC \rightarrow D$

$\{B, C, D\}^+ = \{B, C, D, A\}$, so we have $BCD \rightarrow A$, $BCD \rightarrow B$, $BCD \rightarrow C$, $BCD \rightarrow D$

$\{C, D, A\}^+ = \{C, D, A, B\}$, so we have $CDA \rightarrow A$, $CDA \rightarrow B$, $CDA \rightarrow C$, $CDA \rightarrow D$

$\{D, A, B\}^+ = \{D, A, B, C\}$, so we have $DAB \rightarrow A$, $DAB \rightarrow B$, $DAB \rightarrow C$, $DAB \rightarrow D$

$\{D, A, B\}^+ = \{D, A, B, C\}$, so we have $DAB \rightarrow A$, $DAB \rightarrow B$, $DAB \rightarrow C$, $DAB \rightarrow D$

$\{A, B, C, D\}^+ = \{A, B, C, D\}$, so this FD is trivial.

b) $\{A, B\}, \{A, C\}, \{B, C\}, \{D, C\}$ are the keys of T .

c) The super keys that are not keys are:

$\{A, B, C\}, \{A, B, D\}, \{B, C, D\}, \{A, D, C\}, \{A, B, D\}, \{A, B, C, D\}$

iii) a) $\{A\}^+ = \{A, B, C, D\}$, so we have $A \rightarrow C, A \rightarrow D$

$\{B\}^+ = \{A, B, C, D\}$, so we have $B \rightarrow A, B \rightarrow D$

$\{C\}^+ = \{A, B, C, D\}$, so we have $C \rightarrow A, C \rightarrow B$

$\{D\}^+ = \{A, B, C, D\}$, so we have $D \rightarrow B, D \rightarrow C$

$\{A, B\}^+ = \{A, B, C, D\}$, so we have $AB \rightarrow C, AB \rightarrow D$

$\{B, C\}^+ = \{A, B, C, D\}$, so we have $BC \rightarrow A, BC \rightarrow D$

$\{B, D\}^+ = \{A, B, C, D\}$, so we have $BD \rightarrow A, BD \rightarrow C$

$\{C, D\}^+ = \{A, B, C, D\}$, so we have $CD \rightarrow A, CD \rightarrow B$

$\{C, D\}^+ = \{A, B, C, D\}$, so we have $CD \rightarrow A, CD \rightarrow B$

$\{A, B, C\}^+ = \{A, B, C, D\}$, so we have $ABC \rightarrow D$

$\{B, C, D\}^+ = \{A, B, C, D\}$, so we have $BCD \rightarrow A$

$\{C, D, A\}^+ = \{A, B, C, D\}$, so we have $CDA \rightarrow B$

$\{D, A, B\}^+ = \{A, B, C, D\}$, so we have $DAB \rightarrow C$

Correct Solution:

$\{A\}^+ = \{A, B, C, D\}$, so we have $A \rightarrow C, A \rightarrow D$

$\{B\}^+ = \{A, B, C, D\}$, so we have $B \rightarrow A, B \rightarrow D$

$\{C\}^+ = \{A, B, C, D\}$, so we have $C \rightarrow A, C \rightarrow B$

$\{D\}^+ = \{A, B, C, D\}$, so we have $D \rightarrow B, D \rightarrow C$

$\{A, B\}^+ = \{A, B, C, D\}$, so we have $AB \rightarrow C, AB \rightarrow D$

$\{A, C\}^+ = \{A, B, C, D\}$, so we have $AC \rightarrow B, AC \rightarrow D$

$\{A, D\}^+ = \{A, B, C, D\}$, so we have $AD \rightarrow B, AD \rightarrow C$

$\{B, C\}^+ = \{A, B, C, D\}$, so we have $BC \rightarrow A, BC \rightarrow D$

$\{B, D\}^+ = \{A, B, C, D\}$, so we have $BD \rightarrow A, BD \rightarrow C$

$$\{C, D\}^+ = \{A, B, C, D\}, \text{ so we have } CD \rightarrow A, CD \rightarrow B$$

$$\{A, B, C\}^+ = \{A, B, C, D\}, \text{ so we have } ABC \rightarrow D$$

$$\{B, C, D\}^+ = \{A, B, C, D\}, \text{ so we have } BCD \rightarrow A$$

$$\{C, D, A\}^+ = \{A, B, C, D\}, \text{ so we have } CDA \rightarrow B$$

$$\{D, A, B\}^+ = \{A, B, C, D\}, \text{ so we have } DAB \rightarrow C$$

b) $\{A\}, \{B\}, \{C\}, \{D\}$ are the keys of U .

c) The super keys that are not keys are:

$$\{A, B\}, \{A, C\}, \{A, D\}, \{B, C\}, \{B, D\}, \{C, D\}, \{A, B, C\}, \{B, C, D\}, \{C, D, A\}, \{D, A, B\}, \{A, B, C, D\}$$

4. a) We need to show the closure of attributes $\{A_1, A_2, \dots, A_n, C\}$ in FD $A_1, A_2, \dots, A_n, C \rightarrow B$ is $\{A_1, A_2, \dots, A_n, C, B\}$, that is $\{A_1, A_2, \dots, A_n, C\}^+ = \{A_1, A_2, \dots, A_n, C, B\}$

Since we know $\{A_1, A_2, \dots, A_n\}$ functionally determines B , we can conclude B can be added to $\{A_1, A_2, \dots, A_n, C\}$.

Thus, it follows from above that $\{A_1, A_2, \dots, A_n, C\}^+ = \{A_1, A_2, \dots, A_n, C, B\}$.

- b) Let $A_1A_2 \dots A_n \rightarrow B$ is FD. That is, $\{A_1A_2 \dots A_n\}^+ = \{A_1A_2 \dots A_n, B\}$

We need to show $A_1A_2 \dots A_nC \rightarrow BC$ follows. That is, $\{A_1, A_2, \dots, A_n, C\}^+ = \{A_1, A_2, \dots, A_n, C, B\}$

It follows from the combine and split rule that $A_1A_2 \dots A_nC \rightarrow BC$ can be splitted into $A_1A_2 \dots A_nC \rightarrow B$ and $A_1A_2 \dots A_nC \rightarrow C$.

So, we need to show $A_1A_2 \dots A_nC \rightarrow B$ and $A_1A_2 \dots A_nC \rightarrow C$ follows from the given.

We will do so in parts.

1. Part 1 (Showing $A_1A_2 \dots A_nC \rightarrow B$):

Here, we need to show $A_1A_2 \dots A_nC \rightarrow B$ follows.

And indeed, this follows from the work of *augmenting left sides*. That is the solution to previous problem.

2. Part 2 (Showing $A_1A_2 \cdots A_nC \rightarrow C$):

Here, we need to show $A_1A_2 \cdots A_nC \rightarrow C$ follows.

The definition of trivial FD tells us $A_1A_2 \cdots A_n \rightarrow B_1B_2 \cdots B_m$ holds when $\{B_1, B_2, \dots, B_m\} \subseteq \{A_1, A_2, \dots, A_n\}$

Since $\{C\} \subseteq \{A_1, A_2 \cdots, A_n, C\}$, we can conclude this FD follows trivially.

- c) Let $A_1A_2 \cdots A_n \rightarrow B_1B_2 \cdots B_m$ and $C_1C_2 \cdots C_k \rightarrow D$, where B are each among the C 's.

We need to show $A_1A_2 \cdots A_nE_1E_2 \cdots E_j \rightarrow D$ follows, where the E 's are all of those C 's not found among the B 's.

The transitive rule tells us if $A_1A_2 \cdots A_n \rightarrow B_1B_2 \cdots B_m$ and $B_1B_2 \cdots B_m \rightarrow C_1C_2 \cdots C_k$, then $A_1A_2 \cdots A_n \rightarrow C_1C_2 \cdots C_k$ also holds in R .

Since we know $A_1A_2 \cdots A_n \rightarrow B_1B_2 \cdots B_m$ and $C_1C_2 \cdots C_k \rightarrow D$ where B 's are each among the C 's, we can conclude from the transitive rule that $A_1A_2 \cdots A_n \rightarrow D$.

Then using **augmenting left sides** to all C 's not found among the B 's on $A_1A_2 \cdots A_n \rightarrow D$, we can conclude $A_1A_2 \cdots A_nE_1E_2 \cdots E_j \rightarrow D$ follows.

- d) Assume FD 's $A_1A_2 \cdots A_n \rightarrow B_1B_2 \cdots B_m$ and $C_1C_2 \cdots C_k \rightarrow D_1D_2 \cdots D_j$ holds.

We need to show FD $A_1A_2 \cdots A_nC_1C_2 \cdots C_k \rightarrow B_1B_2 \cdots B_mD_1D_2 \cdots D_k$ follows.

Using the split / combine rule, we can conclude showing $A_1A_2 \cdots A_nC_1C_2 \cdots C_k \rightarrow B_1B_2 \cdots B_mD_1D_2 \cdots D_k$ is the same as showing $A_1A_2 \cdots A_nC_1C_2 \cdots C_k \rightarrow B_1B_2 \cdots B_m$ and $A_1A_2 \cdots A_nC_1C_2 \cdots C_k \rightarrow D_1D_2 \cdots D_k$

So, we will prove the two, in parts

1. Part 1 (Showing $A_1A_2 \cdots A_nC_1C_2 \cdots C_k \rightarrow B_1B_2 \cdots B_m$)

Here, we need to show $A_1A_2 \cdots A_nC_1C_2 \cdots C_k \rightarrow B_1B_2 \cdots B_m$.

The header of problem tells us $A_1A_2 \cdots A_n \rightarrow B_1B_2 \cdots B_m$ holds.

Then by using **Augmenting Left Sides** rule to all C s not found among the A s, $A_1A_2 \cdots A_nC_1C_2 \cdots C_k \rightarrow B_1B_2 \cdots B_m$ follows.

2. Part 2 (Showing $A_1A_2 \cdots A_nC_1C_2 \cdots C_k \rightarrow D_1D_2 \cdots D_k$ follows)

Here, we need to show $A_1A_2 \cdots A_nC_1C_2 \cdots C_k \rightarrow D_1D_2 \cdots D_k$.

The header of problem tells us $C_1C_2 \cdots C_k \rightarrow D_1D_2 \cdots D_k$ holds.

Then by using **Augmenting Left Sides** rule to all A s not found among the C s, $A_1A_2 \cdots A_nC_1C_2 \cdots C_k \rightarrow D_1D_2 \cdots D_k$ follows.

5. a) An example is

A being movieID and
 B being movie length.

b) An example is

A being movieID
 B being movieTitle
 C being movieLength

c) An example is

A being movieTitle
 B being year
 C being length

6. Assume a relation has no attribute that is functionally determined by all the other attributes.

And, assume for the sake of contradiction that there is a relation with non-trivial FD $X \rightarrow Y$.

Then, it follows from the definition of non-trivial functional dependency that $Y \not\subseteq X$.

Then, we can conclude the attributes in Y is functionally determined by other attributes in X .

But this contradicts the assumption that no attribute is functionally determined by all other attributes.

7. Let X and Y be sets of attributes. Assume $X \subseteq Y$.

I need to show $X^+ \subseteq Y^+$.

I will do so in cases

1. **Case 1** ($X = Y$):

Assume $X = Y$.

I need to show $X^+ \subseteq Y^+$ follows.

The header tells us $X = Y$.

Using this fact, $X^+ = Y^+$ is true.

Then it follows from above that $X^+ \subseteq Y^+$ is also true.

2. Case 2 ($X \subset Y$)

Assume $X \subset Y$.

I need to show $X^+ \subseteq Y^+$ follows.

Since the attributes in X is in Y , we can conclude the attributes in X^+ is also in Y^+ .

And, since Y has attributes not in X , we can conclude Y^+ may contain attributes not in X^+ .

Thus, we can conclude $X^+ \subseteq Y^+$.

8. Rough Works:

1. Add attributes from A^+ to L.H.S of $A_1A_2 \cdots A_n \rightarrow A^+$.
2. Show that the R.H.S is still A^+ .

Notes:

- Closure (Definition)

- Suppose $A = \{A_1, A_2, \dots, A_n\}$ is a set of attributes of R and S is a set of FD'.

The closure of A under the set S , denoted by A^+ , is the set of attributes B such that any relation that satisfies all the FD's in S is also satisfies $A_1A_2 \cdots A_n \rightarrow A^+$.

- In other words $A_1 \cdots A_n \rightarrow A^+$ follows from the FD's of S .