CSC373 Worksheet 0 Solution

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1. **Notes:**

- Substitution method
  - Solves recurrences
    - Recurrence characters the running time of divide-and-conquer algorithm
  - How it works:
    1. Make a guess for the solution
    2. Use mathematical induction to prove the guess is correct or incorrect.

    Example:

    Recurrence: $T(n) = 2T(\lfloor n/2 \rfloor) + n$

    Guess: $T(n) = \mathcal{O}(n \log n)$,

    We need to show $T(n) \leq cn \lg n$.

    1. Assume the bound holds for all positive $m < n$, in particular $m = \lfloor n/2 \rfloor$
    2. Find the upper bound of $T(m)$

       $T(\lfloor n/2 \rfloor) \leq c\lfloor n/2 \rfloor \lg(\lfloor n/2 \rfloor)$

    3. Show $T(n) = 2T(\lfloor n/2 \rfloor) + n$ leads to $T(n) \leq cn \lg n$

$$
\begin{aligned}
T(n) &\leq 2(c\lfloor n/2 \rfloor \lg(\lfloor n/2 \rfloor)) + n && (1)\\
&\leq cn \lg(n/2) + n && (2)\\
&= cn \lg(n) - cn \lg 2 + n && (3)\\
&= cn \lg(n) - cn + n && (4)\\
&\leq cn \lg(n) - cn + cn && (5)\\
&\leq cn \lg(n) && (6)
\end{aligned}
$$

4. Show that the boundary holds using mathematical induction

   Doesn't have information in detail. Skipping this for now.

- Making good guess
  - Three suggestions
    1. Using recursion tree
    2. Through practice
    3. prove loose upper and lower bounds on the recurrence and then reduce the range of uncertainty