

CSC236 Worksheet 7 Solution

Hyungmo Gu

May 9, 2020

Question 1

-

Notes:

- **Divide and Conquer:** Partitions problem into b roughly equal subproblems, solve, and recombine:

$$T(n) = \begin{cases} k & \text{if } n \leq B \\ a_1 T(\lceil n/b \rceil) + a_2 T(\lfloor n/b \rfloor) + f(n) & \text{if } n > B \end{cases} \quad (1)$$

where $b, k > 0$, $a_1, a_2 \geq 0$, and $a = a_1 + a_2 > 0$. $f(n)$ is the cost of splitting and recombining.

Note:

k : non-recursive cost, when $n < b$

b : number of almost-equal parts we divide problem into

a_1 : number of recursive calls to ceiling

a_2 : number of recursive calls to floor

f : cost of splitting and later recombining (should be n^d for master theorem)

- **Divide and Conquer Master Theorem:**

If $f \in \Theta(n^d)$, then

$$T(n) \in \begin{cases} \Theta(n^d) & \text{if } a \leq b^d \\ \Theta(n^d \log_b n) & \text{if } a = b^d \\ \Theta(n^{\log_b a}) & \text{if } a > b^d \end{cases} \quad (2)$$

- The master theorem is for master method.
- The master method provides a cookbook method for solving recurrences of the form

$$T(n) = aT(n/b) + f(n) \quad (3)$$

where $a \geq 1$ and $b > 1$.