CSC373 Worksheet 6 Solution

August 12, 2020

1. 1. Multiply objective function by - 1

Maximize

$$-2x_1 - 7x_2 - x_3$$

Subject to

$$x_1 - x_3 = 7 \\ 3x_1 + x_2 \ge 7$$

$$+x_2 \ge 7$$

$$x_2 \ge 0$$

$$x_3 \le 0$$

2. Replace non-nonnegative constraints x_1

Maximize

$$-2x_1' + 2x_1'' - 7x_2 - x_3$$

Subject to

$$x_1' - x_1'' - x_3 = 7$$
$$3x_1' - 3x_1'' + x_2 \ge 7$$

$$x_1', x_1'', x_2 \ge 0$$

$$x_3 \le 0$$

3. Replace non-nonnegative constraints x_3

Maximize

$$-2x_1' + 2x_1'' - 7x_2 - x_3' + x_3''$$

Subject to

$$x'_1 - x''_1 - x'_3 + x''_3 = 7$$
$$3x'_1 - 3x''_1 + x_2 \ge 7$$
$$x'_1, x''_1, x_2, x'_3, x''_3 \ge 0$$

4. Replace equality constraints with \geq and \leq

Maximize

$$-2x_1' + 2x_1'' - 7x_2 - x_3' + x_3''$$

Subject to

$$\begin{aligned} x_1' - x_1'' - x_3' + x_3'' &\leq 7 \\ x_1' - x_1'' - x_3' + x_3'' &\geq 7 \\ 3x_1' - 3x_1'' + x_2 &\geq 7 \\ x_1', x_1'', x_2, x_3', x_3'' &\geq 0 \end{aligned}$$

5. Correct greater-than-or-equal-to inequality constraints

Maximize

$$-2x_1' + 2x_1'' - 7x_2 - x_3' + x_3''$$

Subject to

$$x'_1 - x''_1 - x'_3 + x''_3 \le 7$$

$$-x'_1 + x''_1 + x'_3 - x''_3 \le -7$$

$$-3x'_1 + 3x''_1 - x_2 \le 7$$

$$x'_1, x''_1, x_2, x'_3, x''_3 \ge 0$$

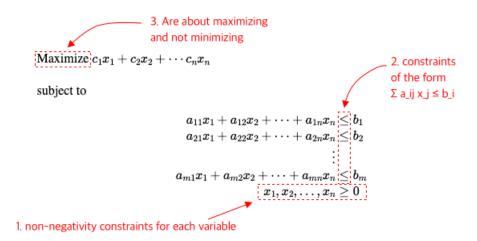
Notes:

• Linear Programming

- Is a method to achieve the best outcome (such as maximum profit or lowest cost) in a mathematical model whose requirements are represented by linear relationships. [1]
- Is named to make it sound cool for government funding
 - * Like dynamic programming
- Applications
 - * Microeconomics (maximize profits, minimize costs)
 - * Company management

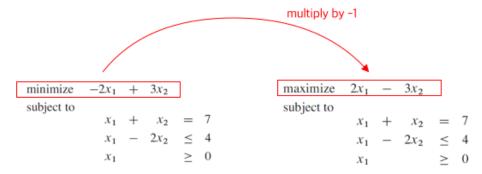
• Standard Form

- Is a form of linear programming
- Are about maximizing, not minimizing [2]
- All have a positivity constraint for each variable [2]
- All other constraints are all of the form "linear combination of variables \leq constant". $^{[2]}$

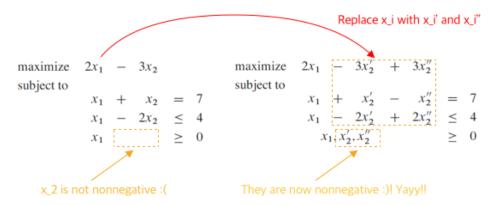


• Converting Linear Programming to Standard Form

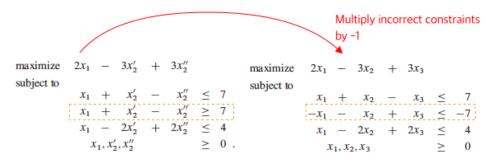
- 1) The objective function might be a minimization rather than a maximization
 - Negate coefficients of the objective function



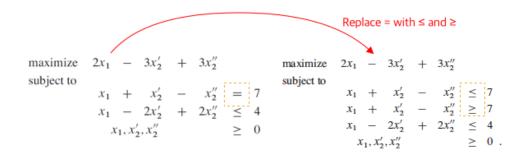
- 2) There might be variables without nonnegativity constraints
 - Replace each non-nonnegative variable x_i with x_i' and x_i''
 - Modify linear program



- 3) There might be **equality constraints**, which have an equal sign rather than a less-than-or-equal-to sign
 - Replace equality constraint $f(x_1, x_2, ..., x_n) = b$ with $f(x_1, x_2, ..., x_n) \le b$ and $f(x_1, x_2, ..., x_n) \ge b$



- 4) There might be **inequality constraints**, but instead of having a less-than-or-equal-to-sign
 - Multiply incorrect inequality constraints by -1



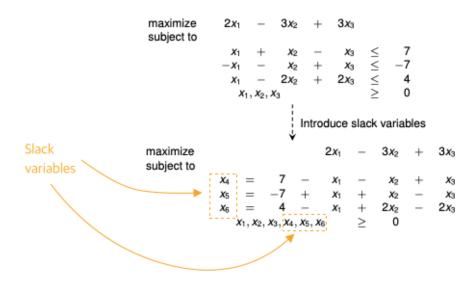
References:

- 1) Wikipedia, Linear Programming, link
- 2) Instituto de Mathematicas, Standard form for Linear Programs, link

2. Notes:

• Slack Form

- Is a form of linear programming
- Is for efficent solving of liner program using simplex algorithm



• Converting Linear Programs into Slack Form

- Start from the standard form of linear programming
- Shift objective functions to right
- Move constraint b_i to lhs, and replace it with slack variable x_i
- Change inequalities in linear programming to equality