

CSC236 Worksheet 2 Solution

Hyungmo Gu

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Question 1

- Notes:

- Complete Induction

- * **Statement:** $\forall i \in \mathbb{N}, \forall n \in \mathbb{N}, n < i \Rightarrow A(n) \Rightarrow \forall i \in \mathbb{N}, A(i)$

- * **Statement Alt.:** $\left(\forall n \in \mathbb{N}, \left[\bigwedge_{k=0}^{k=n-1} P(k) \right] \Rightarrow P(n) \right) \Rightarrow \forall n \in \mathbb{N}, P(n)$

Simple Example 1:

Statement: $\forall n \in \mathbb{N}, n \geq 0 \Rightarrow 10 \mid (n^5 - n)$

We will prove the statement by strong induction on n .

1. Base Case ($n = 0$)

Let $n = 0$.

We need to prove $10 \mid (n^5 - n)$ is true when $n = 0$. That is, there exists $k \in \mathbb{Z}$ such that $(n^5 - n) = 10k$.

Let $k = 0$.

Starting from the left hand side, using the fact $n = 0$, we can write

$$(n^5 - n) = 0 \tag{1}$$

Then, because we know $10k = 0$, we can conclude

$$(n^5 - n) = 10k \tag{2}$$

2. Base Case ($n = 1$)

3. Inductive Step

Question 2

Question 3