

# Worksheet 14 Review

April 1, 2020

## Question 1

- a. Since the inner loop starts at  $j = 0$  and finishes at  $j = n - 1$  with  $j$  increasing by 1 per iteration, we can conclude that the inner loop has

$$\lceil n - 1 - 0 + 1 \rceil = n \quad (1)$$

iterations.

Since the inner loop takes 1 step per iteration, we can conclude that the inner loop has the total cost of

$$n \cdot 1 = n \quad (2)$$

steps.

For the outer loop, because it starts at  $i = 0$  and ends at  $i = n - 1$  with  $i$  increasing by 5 per iteration, we can conclude that the outer loop has

$$\left\lceil \frac{n - 1 - 0 + 1}{5} \right\rceil = \left\lceil \frac{n}{5} \right\rceil \quad (3)$$

iterations.

Since each iteration in the outer loop takes  $n$  steps, we can conclude the outer loop has the total cost of

$$n \cdot n = n^2 \quad (4)$$

steps.

Since we are ignoring the cost of the loop variables, the total cost of the algorithm is  $n^2 + n$  steps.

Then, because we know the algorithm takes total of  $n^2 + n$  steps, we can conclude the algorithm has the runtime of  $\Theta(n^2)$ .

- b. We will determine the exact cost and theta of this algorithm by first calculating the exact cost of inner loop 1

```
1  j = 1
2  while j < n:
3      j = j * 3
4
```

and then, calculating the exact cost of inner loop 2

```
1  k = 0
2  while k < n:
3      k = k + 2
4
```

and then, calculating the exact cost of the outer loop using the information from the exact cost of inner loop 1 and inner loop 2

```
1  i = 4
2  while i < n:
3      j = 1
4      while j < n:
5          j = j * 3
6      k = 0
7      while k < n:
8          k = k + 2
9      i = i + 1
10
```

and then, we will finish off by calculating the theta of the outer loop.

**Part 1 (Calculating the exact cost of loop 1):**

Because we know  $i = i \cdot 3$ , we can calculate

$$\begin{aligned} i_1 &= 3 \\ i_2 &= 9 \\ i_3 &= 27 \\ &\vdots \\ i_j &= 3^j \end{aligned}$$

Then, using the fact that loop termination occurs when  $i_j \geq n$ , we can conclude

$$3^j \geq n \tag{1}$$

$$j \geq \log_3 n \tag{2}$$

Since we are looking for the smallest value of  $j$  resulting in loop termination, we can conclude the value of  $j$  is  $\lceil \log_3 n \rceil$ .

Since the inner loop 1 takes constant step per iteration, we can conclude that the loop has exact cost of

$$\lceil \log_3 n \rceil \cdot 1 = \lceil \log_3 n \rceil \tag{3}$$

steps.

**Part 2 (Calculating the exact cost of loop 2):**

Since the loop starts from  $k = 0$  and ends at  $k = n - 1$ , with  $k$  increasing by 2 per iteration, we can conclude that the loop has

$$\left\lceil \frac{n - 1 - 0 + 1}{2} \right\rceil = \left\lceil \frac{n}{2} \right\rceil \quad (4)$$

iterations.

Since the loop takes 1 step per iteration, the loop has total cost of

$$\left\lceil \frac{n}{2} \right\rceil \cdot 1 = \left\lceil \frac{n}{2} \right\rceil \quad (5)$$

steps.

**Part 3 (Calculating the exact cost of outer loop):**

**Part 4 (Calculating Theta):**

## Question 2