

CSC373 Worksheet 6 Solution

August 12, 2020

1. Notes:

• Linear Programming

- Is a method to achieve the best outcome (such as maximum profit or lowest cost) in a mathematical model whose requirements are represented by linear relationships. ^[1]
- Is named to make it sound cool for government funding
 - * Like dynamic programming
- Applications
 - * Microeconomics (maximize profits, minimize costs)
 - * Company management

• Standard Form

- Is a form of linear programming
- Are about maximizing, not minimizing ^[2]
- All have a positivity constraint for each variable ^[2]
- All other constraints are all of the form “linear combination of variables \leq constant”. ^[2]

The diagram shows a linear programming problem in standard form with three annotations:

- 3. Are about maximizing and not minimizing**: Points to the word "Maximize" in the objective function.
- 2. constraints of the form $\sum a_{ij} x_j \leq b_i$** : Points to the constraints $a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n \leq b_1$, $a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n \leq b_2$, and $a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mn}x_n \leq b_m$.
- 1. non-negativity constraints for each variable**: Points to the constraint $x_1, x_2, \dots, x_n \geq 0$.

The objective function is $\text{Maximize } c_1x_1 + c_2x_2 + \dots + c_nx_n$.

subject to

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n &\leq b_1 \\ a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n &\leq b_2 \\ &\vdots \\ a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mn}x_n &\leq b_m \\ x_1, x_2, \dots, x_n &\geq 0 \end{aligned}$$

• **Converting Linear Programming to Standard Form**

- 1) The objective function might be a minimization rather than a maximization
 - Negate coefficients of the objective function

multiply by -1

minimize $-2x_1 + 3x_2$ subject to $x_1 + x_2 = 7$ $x_1 - 2x_2 \leq 4$ $x_1 \geq 0$	maximize $2x_1 - 3x_2$ subject to $x_1 + x_2 = 7$ $x_1 - 2x_2 \leq 4$ $x_1 \geq 0$
---	--

- 2) There might be variables without nonnegativity constraints
 - Replace each non-nonnegative variable x_i with x'_i and x''_i
 - Modify linear program

Replace x_i with x'_i and x''_i

maximize $2x_1 - 3x_2$ subject to $x_1 + x_2 = 7$ $x_1 - 2x_2 \leq 4$ $x_1 \geq 0$	maximize $2x_1 - 3x'_2 + 3x''_2$ subject to $x_1 + x'_2 - x''_2 = 7$ $x_1 - 2x'_2 + 2x''_2 \leq 4$ $x_1, x'_2, x''_2 \geq 0$
--	--

x_2 is not nonnegative :(They are now nonnegative :)! Yayy!!

- 3) There might be **equality constraints**, which have an equal sign rather than a less-than-or-equal-to sign
 - Replace equality constraint $f(x_1, x_2, \dots, x_n) = b$ with $f(x_1, x_2, \dots, x_n) \leq b$ and $f(x_1, x_2, \dots, x_n) \geq b$

Replace $=$ with \leq and \geq

maximize $2x_1 - 3x'_2 + 3x''_2$ subject to $x_1 + x'_2 - x''_2 = 7$ $x_1 - 2x'_2 + 2x''_2 \leq 4$ $x_1, x'_2, x''_2 \geq 0$	maximize $2x_1 - 3x'_2 + 3x''_2$ subject to $x_1 + x'_2 - x''_2 \leq 7$ $x_1 + x'_2 - x''_2 \geq 7$ $x_1 - 2x'_2 + 2x''_2 \leq 4$ $x_1, x'_2, x''_2 \geq 0$
--	--

- 4) There might be **inequality constraints**, but instead of having a less-than-or-equal-to-sign

Example:**References:**

- 1) Wikipedia, Linear Programming, [link](#)
- 2) Instituto de Matematicas, Standard form for Linear Programs, [link](#)