Worksheet 16 Review

April 2, 2020

Question 1

a. Let $k \in \mathbb{N}$.

Here, the minimum possible change occurs for the loop variable in a single iteration when $i = i + 1$.

The maximum possible change occurs for the loop variable in a single iteration when $i = i + 6$.

The exact upper bound of the variable after k iteration is

$$i_k \leq 6k \tag{1}$$

The exact lower bound of the variable after k iteration is

$$k \leq i_k \tag{2}$$

Using the fact that the termination occurs when $i_k = n$, we can calculate that for the upper bound, the loop terminates when

$$6k \geq n \tag{3}$$

$$k \geq \frac{n}{6} \tag{4}$$

Because we know $\frac{n}{6}$ may be a decimal, we can conclude the closest value at which the loop terminates is when

$$k = \left\lceil \frac{n}{6} \right\rceil \tag{5}$$

Using the same fact, we can calculate that for the lower bound, the loop terminates when

$$k \geq n \tag{6}$$

It follows from above that for the lower bound, the smallest value of k at which the loop termination occurs is when

$$k = n \tag{7}$$

Then, we can conclude the function has asymptotic lower bound of $\Omega(n)$, and asymptotic upper bound of $\mathcal{O}(n)$.

Then, since both Ω and $\mathcal{O}$ have the same value, $\Theta(n)$ is also true.

Correct Solution:

Here, the minimum possible change occurs for the loop variable in a single iteration when $i = i + 1$.

The maximum possible change occurs for the loop variable in a single iteration when $i = i + 6$.

The exact upper bound of the variable after k iteration is

$$i_k \leq 6k \tag{8}$$

The exact lower bound of the variable after k iteration is

$$k \leq i_k \tag{9}$$

Using the fact that the termination occurs when $i_k = n$, we can calculate that for the upper bound, the loop terminates when

$$6k \geq n \tag{10}$$

$$k \geq \frac{n}{6} \tag{11}$$

Because we know $\frac{n}{6}$ may be a decimal, we can conclude the closest value at which the loop terminates is when

$$k = \left\lceil \frac{n}{6} \right\rceil + 1 \tag{12}$$

Using the same fact, we can calculate that for the lower bound, the loop terminates when

$$k \geq n \tag{13}$$

It follows from above that for the lower bound, the smallest value of k at which the loop termination occurs is when

$$k = n + 1 \quad (14)$$

Then, we can conclude the function has asymptotic lower bound of $\Omega(n)$, and asymptotic upper bound of $\mathcal{O}(n)$.

Since both Ω and $\mathcal{O}$ have the same value, $\Theta(n)$ is also true.

Question 2

Question 3