Worksheet 5 Review 2

April 13, 2020

Question 1

- **Statement:** $\forall m, n \in \mathbb{Z},\ (\exists k_1 \in \mathbb{Z},\ m = 2k_1 + 1) \wedge (\exists k_2 \in \mathbb{Z},\ n = 2k_2 + 1) \Rightarrow (\exists k_3 \in \mathbb{Z},\ mn = 2k_3 + 1)$

  Proof. Let $m, n \in \mathbb{Z}$. Assume there is an integer k_1 such that $m = 2k_1 + 1$. Assume there is an integer k_2 such that $n = 2k_2 + 1$. Let $k_3 = (2k_1k_2) + k_1 + k_2$.

  We need to prove $mn = 2k_3 + 1$.

  The assumption tells us $m = 2k_1 + 1$ and $n = 2k_2 + 1$.

  By using these facts and then multiplying them together, we can conclude

$$\begin{aligned} mn &= (2k_1 + 1)(2k_2 + 1) && (1) \\ &= 4k_1k_2 + 2k_1 + 2k_2 + 1 && (2) \\ &= 2[(2k_1k_2) + k_1 + k_2] + 1 && (3) \\ &= 2k_3 + 1 && (4) \end{aligned}$$

  $\square$

  Notes:

  - Noticed professor pre-calculates the value of k_3 as roughwork before writing proof

Question 2

a. **Predicate Logic:** $\forall m, n \in \mathbb{Z},\ Even(m) \wedge Odd(n) \Rightarrow m^2 - n^2 = m + n$

Predicate Logic Expanded: $\forall m, n \in \mathbb{Z},\ (\exists k_1 \in \mathbb{Z},\ m = 2k_1) \wedge (\exists k_2 \in \mathbb{Z},\ n = 2k_2 + 1) \Rightarrow m^2 - n^2 = m + n$

b. The value of k used for m and n must not be under the same variable.

Question 3

a. $Dom(f, g) :\ \forall n \in \mathbb{N},\ g(n) \leq f(n)$, where $f, g : \mathbb{N} \to \mathbb{R}^{\geq 0}$

Notes:

- **Definition of is Dominated By:** Let $f, g : \mathbb{N} \to \mathbb{R}^{\geq 0}$. We say that g is **is dominated by** f (or f **dominates** g) when for every natural number n, $g(n) \leq f(n)$.

Question 4