CSC263 Worksheet 1 Solution

Hyungmo Gu

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Question 1

a. *Proof.* Assume the statement $P(115)$ is true. That is, $\sum_{i=0}^{i=115} 2^i = 2^{115+1}$.

We need to prove $\sum_{i=0}^{i=116} 2^i = 2^{116+1}$.

Starting from the left, we can write

$$\sum_{i=0}^{i=116} 2^i = \sum_{i=0}^{i=115} 2^i + 2 \tag{1}$$

Then, using the assumption $\sum_{i=0}^{i=115} 2^i = 2^{115+1}$, we can conclude

$$\begin{aligned}
\sum_{i=0}^{i=116} 2^i &= 2^{115+1} + 2^{116} && (2)\\
&= 2^{116} + 2^{116} && (3)\\
&= 2^{116}(1+1) && (4)\\
&= 2 \cdot 2^{116} && (5)\\
&= 2^{116+1} && (6)
\end{aligned}$$

□

b. *Proof.* No. The statement is not true for every natural natural number.

We will prove this by counter example. That is, $\exists n \in \mathbb{N}, \sum_{i=0}^{i=n} 2^i \neq 2^{n+1}$.

Let $n = 0$.

Then, starting from the left hand side, it follows from the fact $n = 0$ that

$$\sum_{i=0}^{i=n} 2^i = \sum_{i=0}^{i=0} 2^i \tag{1}$$

$$= 0 \tag{2}$$

Now, for the right hand side, using the same fact, we can write

$$2^{0+1} = 2^1 \tag{3}$$

$$= 2 \tag{4}$$

$\square$

Question 2

Question 3

Question 4