# CSC236 Assignment 1

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## Question 1

a. Yes. We can prove $P(235)$ follows from $P(234)$.

*Proof.* Let $b$ be the bipartite graph with 235 vertices where 117 vertices are in one partition and 118 vertices in the other partition (Note this is the configuration where maximum number of edges form).

The bipartite graph with 117 vertices on both sides of partition has $\frac{234^2}{4}$ edges, and the assumption tells us this is the maximum number of edges the bipartite graph could form.

Since we know $b$ has 117 more edges than the bipartite graph with 117 vertices on both sides, using these facts, we can conclude the upper bound number of edges for the bipartite graph with 235 vertices is

$$\begin{aligned}
\frac{234^2}{4} + 117 &= \frac{234^2}{4} + \frac{4 \cdot 117}{4} && (1)\\
&= \frac{234^2 + 2 \cdot 234}{4} && (2)\\
&\leq \frac{234^2 + 2 \cdot 234 + 1}{4} && (3)\\
&= \frac{(234+1)^2}{4} && (4)\\
&= \frac{(235)^2}{4} && (5)
\end{aligned}$$

□

**Attempt #2:**

Assume $P(234)$. That is, every bipartite graph on 234 vertices has no more than $\frac{234^2}{4}$ edges.

We need to prove $P(235)$ follows. That is, every bipartite graph on 235 vertices has no more than $\frac{235^2}{4}$ edges.

Let $b$ be the bipartite graph with 235 vertices. Let $b'$ be the bipartite graph with a vertex removed from the larger of two partitions in $b$ along with its edges.

Since we know the maximum number of edges occur in $b'$ when there are 117 vertices in both sides of the partitions, and since we know the edges of removed vertex forms edges with partition with bigger number of vertices, we can conclude the removed vertex has at most 117 edges.

Since the assumption tells us $b'$ has at most $\frac{234^2}{4}$ edges, we can conclude the upper bound number of edges for the bipartite graph with 235 vertices is

$$\begin{aligned}
\frac{234^2}{4} + 117 &= \frac{234^2}{4} + \frac{4 \cdot 117}{4} && (6)\\
&= \frac{234^2 + 2 \cdot 234}{4} && (7)\\
&\leq \frac{234^2 + 2 \cdot 234 + 1}{4} && (8)\\
&= \frac{(234+1)^2}{4} && (9)\\
&= \frac{(235)^2}{4} && (10)
\end{aligned}$$

So $P(235)$ follows.

# Question 2

# Question 3

# Question 4