CSC236 Worksheet 2 Review

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Question 3

- *Proof.* For convenience, define $P(n) : f(n) \leq 3^n$. I will use complete induction to prove that $\forall n \in \mathbb{N}$, $P(n)$.

  <u>**Inductive Step:**</u>

  Let $n \in \mathbb{N}$. Assume $H(n) : \bigwedge_{i=0}^{n-1} P(i)$. I will show $P(n)$ follows. That is $f(n) \leq 3^n$.

  <u>**Base Case ($n = 0$):**</u>

  Let $n = 0$.

  Then,

$$\begin{aligned} f(n) &= 1 && \text{[By def.]} && (1)\\ &= 3^0 && && (2)\\ &\leq 3^0 && && (3)\\ &= 3^n && && (4) \end{aligned}$$

  Thus, $P(n)$ follows.

  <u>**Base Case ($n = 1$):**</u>

  Let $n = 1$.

  Then,

$$
\begin{aligned}
f(n) &= 3 && \text{[By def.]} && (5)\\
&= 3^1 && && (6)\\
&\leq 3^1 && && (7)\\
&= 3^n && && (8)
\end{aligned}
$$

Thus, $P(n)$ follows.

<u>Case ($n > 1$):</u>

Let $n \in \mathbb{N} \setminus \{0\}$.

Then, we have

$$
\begin{aligned}
f(n) &= 2(f(n-2) + f(n-1)) + 1 && \text{[By def., since } 1 < n\text{]} && (9)\\
&\leq 2(3^{n-2} + 3^{n-1}) + 1 && \text{[By I.H, since } 1 \leq n-2 < n-1 < n\text{]} && (10)\\
&= 2 \cdot 3^{n-2}(1+3) + 1 && && (11)\\
&= 8 \cdot 3^{n-2} + 1 && && (12)\\
&\leq 8 \cdot 3^{n-2} + 3^{n-2} && \text{[Since } 1 < n \text{ and } 0 \leq 3^{n-2}\text{]} && (13)\\
&= 9 \cdot 3^{n-2} && && (14)\\
&= 3^n && && (15)
\end{aligned}
$$

Thus, $P(n)$ follows. $\square$