

Midterm 1 Version 3 Solution

March 20, 2020

Question 1

- a. Since $S_1 = \{ab, ba, aab, bba, baa, \dots\}$ and $S_2 = \{aaa, aab, aba, baa, abb, bab, bba\}$,

$$S_2 \setminus S_1 = \{aaa, aab, aba, bab\}$$

Correct Solution:

Since $S_1 = \{ab, ba, aab, abb, bba, baa, \dots\}$ and $S_2 = \{aaa, aab, aba, baa, abb, bab, bba, bbb\}$,
 $S_2 \setminus S_1 = \{aaa, aba, bab, bbb\}$

- b. See table below

p	q	r	$\neg r$	$p \Rightarrow q$	$(p \Rightarrow q) \Leftrightarrow \neg r$
T	T	T	F	T	F
T	T	F	T	T	T
T	F	T	F	F	T
F	T	T	F	T	F
T	F	F	T	F	F
F	T	F	T	T	T
F	F	T	F	T	F
F	F	F	T	T	T

- c. Let $x = \underline{\hspace{2cm}}$, and $y \in \mathbb{N}$.

We will prove that $P(x)$ is true and $Q(x, y)$ or $Q(x, y + 1)$ is false.

Correct Solution:

Negation: $\exists x \in \mathbb{N}, \forall y \in \mathbb{N}, P(x) \wedge (\neg Q(x, y) \wedge \neg Q(x, y + 1))$

Let $x = \underline{\hspace{2cm}}$ and $y \in \mathbb{N}$.

We will prove that $P(x)$ is true, and both $Q(x, y)$ and $Q(x, y + 1)$ are false.

Question 2

a. $\forall x \in T, \text{Canadian}(x) \wedge \text{Star}(x)$

Correct Solution:

$\forall x \in T, \text{Canadian}(x) \Rightarrow \text{Star}(x)$

b. $\forall x \in T, \text{Canadian}(x) \Rightarrow \forall y \in T, \neg \text{Canadian}(y) \wedge \text{Defeated}(x, y)$

Correct Solution:

$\forall x \in T, \text{Canadian}(x) \Rightarrow (\forall y \in T, \neg \text{Canadian}(y) \Rightarrow \text{Defeated}(x, y))$

c. $\exists x \in T, \text{Canadian}(x) \wedge \text{Star}(x) \Rightarrow \forall y \in T, \exists z \in T, y \neq z \wedge \text{Canadian}(y) \wedge \text{Defeated}(y, z)$

Correct Solution:

$\exists x \in T, \text{Canadian}(x) \wedge \text{Star}(x) \Rightarrow (\forall y \in T, \text{Canadian}(y) \Rightarrow \exists z \in T, y \neq z \wedge \text{Defeated}(y, z))$

d. $\exists x \in T, \text{Canadian}(x) \wedge \text{Star}(x) \wedge (\forall y \in T, x \neq y \wedge \text{Canadian}(y) \wedge \neg \text{Star}(y))$

Correct Solution:

$\exists x \in T, \text{Canadian}(x) \wedge \text{Star}(x) \wedge (\forall y \in T, x \neq y \wedge \text{Canadian}(y) \Rightarrow \neg \text{Star}(y))$

Question 3

a. $\forall n \in \mathbb{N}, \exists k \in \mathbb{N}, n > 1 \wedge n = 2k + 1 \Rightarrow \exists p, q \in \mathbb{Z}^+, n = p^2 - q^2$

Question 4