

CSC373 Worksheet 6 Solution

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1. Notes:

• Linear Programming

- Is a method to achieve the best outcome (such as maximum profit or lowest cost) in a mathematical model whose requirements are represented by linear relationships. ^[1]
- Is named to make it sound cool for government funding
 - * Like dynamic programming
- Applications
 - * Microeconomics (maximize profits, minimize costs)
 - * Company management

• Standard Form

- Is a form of linear programming
- Are about maximizing, not minimizing ^[2]
- All have a positivity constraint for each variable ^[2]
- All other constraints are all of the form “linear combination of variables \leq constant”. ^[2]

The diagram illustrates the standard form of a linear programming problem. It consists of the following parts:

- Objective Function:** $\text{Maximize } c_1x_1 + c_2x_2 + \dots + c_nx_n$. The word "Maximize" is enclosed in a red dashed box. A red arrow points from the annotation "3. Are about maximizing and not minimizing" to this box.
- Subject to:** The text "subject to" is written below the objective function.
- Constraints:** A system of linear inequalities:
$$\begin{aligned}a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n &\leq b_1 \\a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n &\leq b_2 \\&\vdots \\a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mn}x_n &\leq b_m\end{aligned}$$
The right-hand side of these inequalities (b_1, b_2, \dots, b_m) is enclosed in a red dashed box. A red arrow points from the annotation "2. constraints of the form $\sum a_{ij}x_j \leq b_i$ " to this box.
- Non-negativity Constraints:** $x_1, x_2, \dots, x_n \geq 0$. This entire line is enclosed in a red dashed box. A red arrow points from the annotation "1. non-negativity constraints for each variable" to this box.

• **Converting Linear Programming to Standard Form**

- 1) The objective function might be a minimization rather than a maximization
 - Negate coefficients of the objective function
- 2) There might be variables without nonnegativity constraints
- 3) There might be **equality constraints**, which have an equal sign rather than a less-than-or-equal-to sign
- 4) There might be **inequality constraints**, but instead of having a less-than-or-equal-to sign

Example:

Minimize

$$80x + 60y$$

subject to

$$x + y \geq 1$$

$$-.05x + .07y \leq 0$$

$$x, y \geq 0.$$

Introduce two new variables $s_1, s_2 \geq 0$.

Convert inequalities to equalities

Minimize

$$80x + 60y$$

subject to

$$x + y - s_1 = 1$$

$$-.05x + .07y + s_2 = 0$$

$$x, y, s_1, s_2 \geq 0.$$

References:

- 1) Wikipedia, Linear Programming, [link](#)
- 2) Instituto de Matematicas, Standard form for Linear Programs, [link](#)