

Worksheet 6 Solution

March 16, 2020

Question 1

- a. $P(123) \wedge (\forall n \in \mathbb{N}, P(n) \Rightarrow n \leq 123)$
- b. $isCD(x, y, d): \exists x, y, d \in \mathbb{Z}, d \mid x \wedge d \mid y$
 $isGCD(x, y, d): \exists x, y, d \in \mathbb{Z}, (x = 0 \wedge y = 0 \wedge d = 0) \vee ((x \neq 0 \vee y \neq 0) \wedge isCD(x, y, d) \wedge \forall e \in \mathbb{Z}, e > d \Rightarrow \neg isCD(x, y, e))$
- c. Statement: $\forall x \in \mathbb{Z}^+, IsGCD(x, 0, x)$

For the value x , because we know $x \mid x$, and $\forall n \in \mathbb{Z}^+$ and $\forall d \in \mathbb{Z}, d \mid n \Rightarrow d \leq n$, x is the biggest divisor of x

For the value 0, because we know anything that divides 0 is 0, and $\exists k \in \mathbb{Z}, 0 = k \times 0$, k can be chosen to be x .

Then, it follows from the definition of GCD that the statement $IsGCD(x, 0, x)$ is true.

- d. $\forall a, b \in \mathbb{Z}, (a \neq 0 \vee b \neq 0) \Rightarrow \exists p, q \in \mathbb{Z}, gcd(a, b) = ap + qb \wedge \forall m \in \mathbb{Z}, m < gcd(a, b) \wedge m \neq ap + qb$

Question 2

- a. Let $n \in \mathbb{Z}$. Assume $\exists l \in \mathbb{Z}, n = 2l$.

Then,

$$n^2 - 3n = (2l)^2 = 3(2l) \tag{1}$$

$$= 4l^2 - 6l \tag{2}$$

$$= 2(2l^2 - 3l) \tag{3}$$

Since $2l^2 - 3l \in \mathbb{Z}$, it follows from the definition of even number that $n^2 - 3n$ is even

Question 3