

CSC236 Worksheet 9 Solution

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Question 1

a. I need to evaluate the reg. expressions for

$$L = \{x \in \Sigma \mid x \text{ has even number of 1s or an odd number of 0s}\}$$

I will do so in parts.

Part 1 (Finding reg. expressions for even number of 1s):

In this part, I will find the reg. expressions for even number of 1's.

I will do so by finding patterns in series of small examples.

Starting with $L = \{x \in \Sigma \mid x \text{ has 0 number of 1s}\}$, it's reg. expressions is

$$0^* \tag{1}$$

Now for $L = \{x \in \Sigma \mid x \text{ has 2 number of 1s}\}$, it's reg. expressions is

$$0^*10^*10^* \tag{2}$$

Now for $L = \{x \in \Sigma \mid x \text{ has 4 number of 1s}\}$, it's reg. expressions is

$$0^*10^*10^*10^*10^* \quad (3)$$

From above, I see a pattern that

$$(0^*10^*1)(0^*10^*1)0^* \quad (4)$$

Using the pattern, I can conclude that the regular expression for even number of 1s is

$$(0^*10^*1)^*0^* \quad (5)$$

Part 2 (Finding reg. expressions for odd number of 0s):

In this part, I will find the reg. expressions for odd number of 0's.

I will do so by finding patterns in series of small examples.

Starting with $L = \{x \in \Sigma \mid x \text{ has 1 number of 0s}\}$, it's reg. expressions is

$$1^*01^* \quad (6)$$

Now for $L = \{x \in \Sigma \mid x \text{ has 3 number of 0s}\}$, it's reg. expressions is

$$1^*01^*01^*01^* \quad (7)$$

Now for $L = \{x \in \Sigma \mid x \text{ has 5 number of 0s}\}$, it's reg. expressions is

$$1^*01^*01^*01^*01^*01^* \quad (8)$$

From above, I see a pattern that

$$1^*(01^*)(01^*)(01^*)(01^*)(01^*) \quad (9)$$

Using the pattern, I can conclude that the regular expression for odd number of 0s is

$$1^*(01^*)^* \quad (10)$$

Thus, by combining the two parts with union, we have

$$(0^*10^*1)^*0^* + 1^*(01^*)^* \quad (11)$$

Notes:

- Regular Expression
 - Quick Guide

$$(0 + 1)((01)^*0) \quad (12)$$

The expression implies that

1. Starts with 0 **or** 1
 - * indicated by $(0 + 1)$
 2. Are then followed by **one or more repetitions** of 01
 - * indicated by $(01)^*$
 3. Ends with 0
 - * indicated by the final 0
- Examples
 1. $L = \{w \in \{a, b\}^* \mid w \text{ has an } a\}$

Answer:

$$(a + b)^*a(a + b)^* \quad (13)$$

- Means there is one or more repetitions of a or b at front
- Means there is a in the middle

– Means there is zero or more repetitions of a or b at end

2. $L = \{w \in \{a, b\}^* \mid w \text{ has at least two } as\}$

Answer:

$$(a + b)^*a(a + b)^*a(a + b)^* \quad (14)$$

3. $L = \{w \in \{a, b\}^* \mid |w| \geq 2\}$

Answer:

$$(0 + 1)(0 + 1)(0 + 1)^* \quad (15)$$

In this example,

- Two characters are created (indicated by $(0 + 1)(0 + 1)$)
- And more :D!! (indicated by $(0 + 1)^*$)

b. I need to find the reg. expressions for $L = \{x \in \Sigma \mid x \text{ has at least one } 1 \text{ and at least one } 0\}$. That is, regex expressions for $\{x \in \Sigma \mid x \text{ has at least one } 1 \text{ followed by at least one } 0\}$ plus $\{x \in \Sigma \mid x \text{ has at least one } 0 \text{ followed by at least one } 1\}$.

First, I need to find reg. expressions for

$$\{x \in \Sigma \mid x \text{ has at least one } 1 \text{ followed by at least one } 0\} \quad (1)$$

Since the reg expressions for x with at least one 1 is $(0+1)^*1(0+1)^*$ and the reg expressions for x with at least one 0 is $(0 + 1)^*0(0 + 1)^*$, we have

$$(0 + 1)^*1(0 + 1)^*0(0 + 1)^* \quad (2)$$

Second, I need to find reg. expressions for

$$\{x \in \Sigma \mid x \text{ has at least one } 0 \text{ followed by at least one } 1\} \quad (3)$$

Using the facts provided above, we have

$$(0 + 1)^*0(0 + 1)^*1(0 + 1)^* \quad (4)$$

Thus, by combining the two, we can conclude

$$(0 + 1)^*1(0 + 1)^*0(0 + 1)^* + (0 + 1)^*0(0 + 1)^*1(0 + 1)^* \quad (5)$$

c. I need to find reg. expressions for

$$\{x \in \Sigma \mid \text{every } 1 \text{ in } x \text{ is immediately preceded and followed by a } 0\}$$

An example expresion of the above is

$$0^*0100^*0100^*0100^* \quad (1)$$

From above, we can see the following pattern

$$(0^*010)(0^*010)(0^*010)0^* \quad (2)$$

Thus, we have

$$(0^*010)^*0^* \quad (3)$$

Question 2

a. **Negation:** $\exists r_1, r_2, r_3 \in \mathcal{RE}, (r_1 r_2 \equiv r_2 r_1) \wedge (r_1 \neq \varepsilon \neq r_2) \wedge (r_1 \neq \emptyset \neq r_2) \wedge (r_1 \neq r_2)$

Let $r_1 = 1$ and $r_2 = 1^*$.

Then, $r_1 r_2 \equiv r_2 r_1$ and $r_1 \neq \varepsilon \neq r_2$ and $r_1 \neq \emptyset \neq r_2$, but $r_1 \neq r_2$.

So, by counter-example, the statement is false.

Notes:

- Equivalence (\equiv) in Regular Expressions
 - Two regular expressions \mathcal{R} and \mathcal{S} are equivalent, written $R \equiv S$, if they denote the same language, i.e. $\mathcal{L}(\mathcal{R}) \equiv \mathcal{L}(\mathcal{S})$
 - * Example, $(0^*1^*) \equiv (0 + 1)^*$
 - For all regular expressions $\mathcal{R}, \mathcal{S}, \mathcal{T}$, the following euivalences hold.
 - * **Commutativity of union:** $(\mathcal{R} + \mathcal{S}) \equiv (\mathcal{S} + \mathcal{R})$
 - * **Associativity of union:** $((\mathcal{R} + \mathcal{S}) + \mathcal{T}) \equiv (\mathcal{R}) + (\mathcal{S} + \mathcal{T})$
 - * **Associativity of concatenation:** $((\mathcal{R}\mathcal{S})\mathcal{T}) \equiv (\mathcal{R}(\mathcal{S}\mathcal{T}))$
 - * **Left distributivity:** $(\mathcal{R}(\mathcal{S} + \mathcal{T})) \equiv ((\mathcal{R}\mathcal{S})(\mathcal{R}\mathcal{T}))$
 - * **Right distributivity:** $((\mathcal{S} + \mathcal{T})\mathcal{R}) \equiv ((\mathcal{S}\mathcal{R})(\mathcal{T}\mathcal{R}))$
 - * **Identity for union:** $(\mathcal{R} + \emptyset) \equiv \mathcal{R}$
 - * **Identity for concatenation:** $(\mathcal{R}\epsilon) \equiv \mathcal{R}$ and $(\epsilon\mathcal{R}) \equiv \mathcal{R}$
 - * **Annihilator for concatenation:** $(\emptyset\mathcal{R}) \equiv \emptyset$ and $(\mathcal{R}\emptyset) \equiv \emptyset$
 - * **Idempotence of Kleene star:** $R^{**} \equiv R^*$

b. **Negation:** $(r_1 r_2 \equiv r_1 r_3) \wedge (r_1 \neq \emptyset) \wedge (r_2 \neq r_3)$

Let $r_1 = 1^*$, $r_2 = \varepsilon$, $r_3 = 1^*$.

Then, $r_1 r_2 \equiv r_1 r_3$ and $r_1 \neq \emptyset$, but $r_2 \neq r_3$.

So, by counter-example, the statement is false.