

# Worksheet 6 Review

March 24, 2020

## Question 1

- a.  $\forall n \in \mathbb{N}, P(123) \wedge \neg(n > 123 \Rightarrow P(n))$

**Correct Solution:**

$$P(123) \wedge (\forall n \in \mathbb{N}, P(n) \Rightarrow n \leq 123)$$

- b.  $IsCD(x, y) : \exists x, y, d \in \mathbb{Z}, d \mid x \wedge d \mid y$

$$IsGCD(x, y) : \exists x, y, d \in \mathbb{Z}, d \mid x \wedge d \mid y \wedge (\forall n \in \mathbb{N}, n > d \Rightarrow n \nmid x \vee n \nmid y)$$

**Correct Solution:**

$$IsGCD(x, y, d) : \exists x, y, d \in \mathbb{Z}, (x = 0 \wedge y = 0 \Rightarrow d = 0) \wedge (x \neq 0 \vee y \neq 0 \Rightarrow IsCD(x, y, d) \Rightarrow \forall d' \in \mathbb{Z}, IsCD(x, y, d') \Rightarrow d' \leq d)$$

- c. Let  $a = x$ ,  $b = 0$ ,  $d = x$  and  $d' \in \mathbb{Z}$ . Assume  $IsCD(x, y, d')$ .

Because we know  $x \mid x$  and  $x \mid 0$ , we can conclude that  $d$  is a common divisor to  $a$  and  $b$ .

Since  $d' \mid a$  and  $d' \mid b$ , and since  $\forall n \in \mathbb{Z}^+, \forall d \in \mathbb{Z}, d \mid n \Rightarrow d \leq n$ , we can conclude that

$$d' \leq a \tag{1}$$

Then,

$$d' \leq d \tag{2}$$

by the fact that  $d = a$ .

Then it follows from above that the statement  $\forall x \in \mathbb{Z}^+, IsGCD(x, 0, x)$  is true.

d. **Attempt 1:**

$$a, b \in \mathbb{Z}, a \neq 0 \vee b \neq 0 \Rightarrow (\exists d \in \mathbb{Z}, d = GCD(a, b) \Rightarrow \forall d' \in \mathbb{Z}^+, \exists p, q \in \mathbb{Z})$$

**Attempt 2:**

$$a, b \in \mathbb{Z}, \exists d \in \mathbb{Z}, (a \neq 0 \vee b \neq 0) \wedge d = GCD(a, b) \Rightarrow \exists p, q \in \mathbb{Z}, d = ap + bq \wedge d > 0 \wedge (\forall d' \in \mathbb{Z}^+, d' = ap + bq \Rightarrow d' \geq d)$$

## Question 2

a. Let  $n \in \mathbb{Z}$ . Assume that  $\exists k \in \mathbb{Z}, n = 2k$ .

Then,

$$n^2 - 3n = 4n^2 - 6n \tag{1}$$

$$= 2(n^2 - 3n) \tag{2}$$

$$= 2m \tag{3}$$

where  $m = n^2 - 3n \in \mathbb{Z}$ .

Then, by definition of divisibility,  $n^2 - 3n$  is even.

## Question 3