

Worksheet 14 Solution

March 25, 2020

Question 1

a. **Inner Loop:** n

Outer Loop: $n - 5$

Theta Expressions: $\Theta(n^2)$

Correct Solution:

Inner Loop: n

Outer Loop: $n \cdot \left\lceil \frac{n}{5} \right\rceil$

Theta Expressions: $\Theta(n^2)$

b. **Inner Loop:** $\frac{n}{3} + (n - 2)$

Outer Loop: $n - 4$

Theta Expressions: $\Theta(n^2)$

Correct Solution:

Inner Loop: $\lceil \log_3 n \rceil + \left\lceil \frac{n}{2} \right\rceil$

Outer Loop: $\max(0, n - 4) \cdot \left[\lceil \log_3 n \rceil + \left\lceil \frac{n}{2} \right\rceil \right]$

Theta Expressions: $\Theta(n^2)$

c. **Inner Loop #2:** $\sum_{i=1}^n i = \frac{n(n+1)}{2}$

Inner Loop #1: $n \cdot \frac{n(n+1)}{2} = \frac{n^3 + n^2}{2}$

Outer Loop: $\frac{n^3 + n^2}{2} \cdot (n-4) = \frac{n^4 - 3n^3 + 4n^2}{2}$

Theta Expressions: $\Theta(n^4)$

Correct Solution:

Inner Loop #2: j

Inner Loop #1: $\sum_{j=1}^n j = \frac{n(n+1)}{2}$

Outer Loop: $\left\lceil \frac{n}{4} \right\rceil \cdot \frac{n(n+1)}{2}$

Theta Expressions: $\Theta(n^3)$

d. **Inner Loop:** 2^n

Outer Loop: $\sum_{i=0}^{\frac{n}{2}-1} 2^i = 2^{\frac{n}{2}-1}$

Theta Expressions: $\Theta(2^n)$

Correct Solution:

Inner Loop: i

Outer Loop: $\sum_{i=0}^{\log n - 1} 2^i = \frac{1 - 2^{\log n - 1 + 1}}{1 - 2} = 2^{\log n} - 1 = n - 1$

Theta Expressions: $\Theta(n)$

Question 2

- Inner Loop #2: $j - i$

Inner Loop #1: $\sum_{j=i}^{n-1} (j - i)$

Outer Loop: $\sum_{i=0}^{n-1} \sum_{j=i}^{n-1} (j - i)$

Calculation:

$$\sum_{i=0}^{n-1} \sum_{j=i}^{n-1} (j - i) = \sum_{i=0}^{n-1} \sum_{j'=0}^{n-1-i} (j' + i - i) \quad (1)$$

$$= \sum_{i=0}^{n-1} \sum_{j'=0}^{n-1-i} j' \quad (2)$$

$$= \sum_{i=0}^{n-1} \frac{(n-1-i)(n-i)}{2} \quad (3)$$

$$= \sum_{i=0}^{n-1} \frac{(n-1-i)(n-i)}{2} \quad (4)$$

$$= \frac{1}{2} \left[\sum_{i=0}^{n-1} n^2 - \sum_{i=0}^{n-1} n - 2n \sum_{i=0}^{n-1} i + \sum_{i=0}^{n-1} i + \sum_{i=0}^{n-1} i^2 \right] \quad (5)$$

$$= \frac{1}{2} \left[\sum_{i=0}^{n-1} n^2 - \sum_{i=0}^{n-1} n - 2n \sum_{i=0}^{n-1} i + \sum_{i=0}^{n-1} i + \sum_{i=0}^{n-1} i^2 \right] \quad (6)$$

$$= \frac{1}{2} \left[\frac{(n^2 - n)}{2} + \frac{2n^3 - n^2 - 2n^2 + n}{6} \right] \quad (7)$$

$$= \frac{1}{2} \left[\frac{3n^2 - 3n}{6} + \frac{2n^3 - 3n^2 + n}{6} \right] \quad (8)$$

$$= \frac{1}{2} \left[\frac{2n^3 - 2n}{6} \right] \quad (9)$$

$$= \frac{n^3 - n}{6} \quad (10)$$

Theta Expressions: $\Theta(n^3)$