Worksheet 5 Solution

March 15, 2020

Question 1

- $\forall n, p \in \mathbb{N}, Odd(n) \wedge Odd(p) \Rightarrow Odd(n \times p)$

Let $n, p \in \mathbb{Z}$, and assume n, p are odd numbers.

Then, $\exists k, m \in \mathbb{Z}$, $n = 2k - 1$, $p = 2m - 1$ by the definition of odd numbers

Then,

$$
\begin{align}
n \times p &= (2k - 1)(2m - 1) \\
&= 2k2m - 2k - 2m + 1 \\
&= (2k2m - 2k - 2m + 2) - 1 \\
&= 2(2km - k - m + 1) - 1 \\
&= 2l - 1
\end{align}
$$

where $l = 2km - k - m + 1$.

Since $l \in \mathbb{Z}$, it follows from the definition of odd number that the product of two odd numbers is odd.

Question 2

a. $\forall m, n \in \mathbb{Z},\ Even(m) \land Odd(n) \Rightarrow m^2 - n^2 = m + n$

b. The flaw is in the same value k. This implies that the statement is true only when n is 1 less than m. This doesn't mean it's true for all even and odd numbers.

Question 3

a. $Dom(f, g) : \forall n \in \mathbb{N}, g(n) \leq f(n)$, where $f, g : \mathbb{N} \to \mathbb{R}^{\geq 0}$

b. Let $n \in \mathbb{R}^{\geq 0}$, $f(n) = 3n$ and $g(n) = n$.

Then,

$$g(n) = n \leq n + n + n \tag{6}$$
$$\leq 3n \tag{7}$$
$$\leq f(n) \tag{8}$$

It follows from the definition that f dominates g.

c. Predicate Logic: $\exists n \in \mathbb{N},\ g(n) > f(n)$.

Let $n = 1$.

Then,

$$g(1) = (1) + 165 = 166 > 1 \tag{9}$$
$$> f(1) \tag{10}$$

It follows from the negation of definition that f does not dominate g.

Question 4