CSC373 Worksheet 6 Solution

August 13, 2020

1. 1. Multiply objective function by - 1

Maximize

$$-2x_1 - 7x_2 - x_3$$

Subject to

$$\begin{aligned} x_1 - x_3 &= 7 \\ 3x_1 + x_2 &\geq 7 \\ x_2 &\geq 0 \\ x_3 &\leq 0 \end{aligned}$$

2. Replace non-nonnegative constraints x_1

Maximize

$$-2x_1' + 2x_1'' - 7x_2 - x_3$$

Subject to

$$\begin{aligned} x_1' - x_1'' - x_3 &= 7 \\ 3x_1' - 3x_1'' + x_2 &\geq 7 \\ x_1', x_1'', x_2 &\geq 0 \\ x_3 &\leq 0 \end{aligned}$$

3. Replace non-nonnegative constraints x_3

Maximize

$$-2x_1' + 2x_1'' - 7x_2 - x_3' + x_3''$$

Subject to

$$\begin{aligned} x_1' - x_1'' - x_3' + x_3'' &= 7 \\ 3x_1' - 3x_1'' + x_2 &\geq 7 \\ x_1', x_1'', x_2, x_3', x_3'' &\geq 0 \end{aligned}$$

4. Replace equality constraints with $\geq$ and $\leq$

Maximize

$$-2x_1' + 2x_1'' - 7x_2 - x_3' + x_3''$$

Subject to

$$\begin{aligned} x_1' - x_1'' - x_3' + x_3'' &\leq 7 \\ x_1' - x_1'' - x_3' + x_3'' &\geq 7 \\ 3x_1' - 3x_1'' + x_2 &\geq 7 \\ x_1', x_1'', x_2, x_3', x_3'' &\geq 0 \end{aligned}$$

5. Correct greater-than-or-equal-to inequality constraints

Maximize

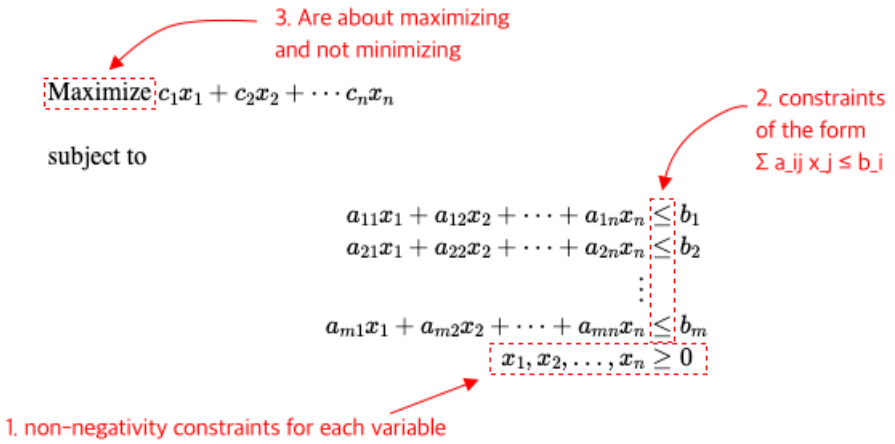
$$-2x_1' + 2x_1'' - 7x_2 - x_3' + x_3''$$

Subject to

$$
\begin{aligned}
x_1' - x_1'' - x_3' + x_3'' &\leq 7 \\
-x_1' + x_1'' + x_3' - x_3'' &\leq -7 \\
-3x_1' + 3x_1'' - x_2 &\leq 7 \\
x_1', x_1'', x_2, x_3', x_3'' &\geq 0
\end{aligned}
$$

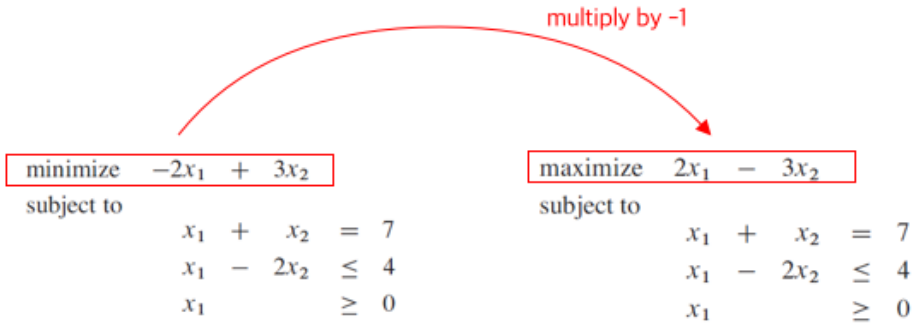
Notes:

- **Linear Programming**
  - Is a method to achieve the best outcome (such as maximum profit or lowest cost) in a mathematical model whose requirements are represented by linear relationships. [1]
  - Is named to make it sound cool for government funding
    - Like dynamic programming
  - Applications
    - Microeconomics (maximize profits, minimize costs)
    - Company management
- **Standard Form**
  - Is a form of linear programming
  - Are about maximizing, not minimizing [2]
  - All have a positivity constraint for each variable [2]
  - All other constraints are all of the form "linear combination of variables $\leq$ constant". [2]

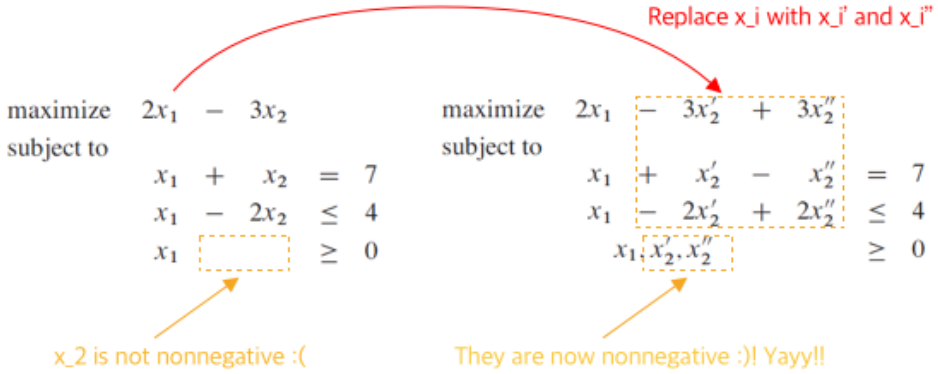


- **Converting Linear Programming to Standard Form**

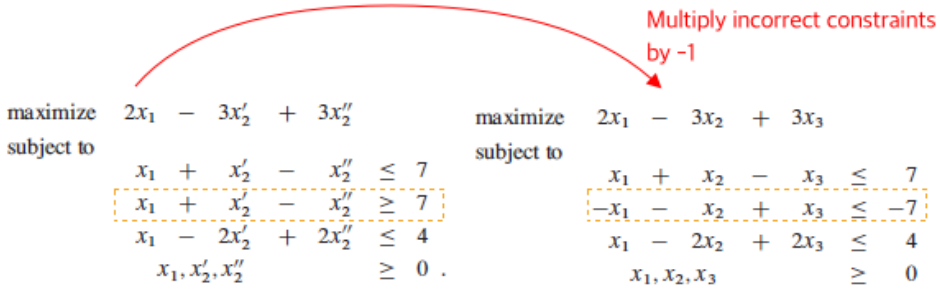
1) The objective function might be a minimization rather than a maximization
   - Negate coefficients of the objective function



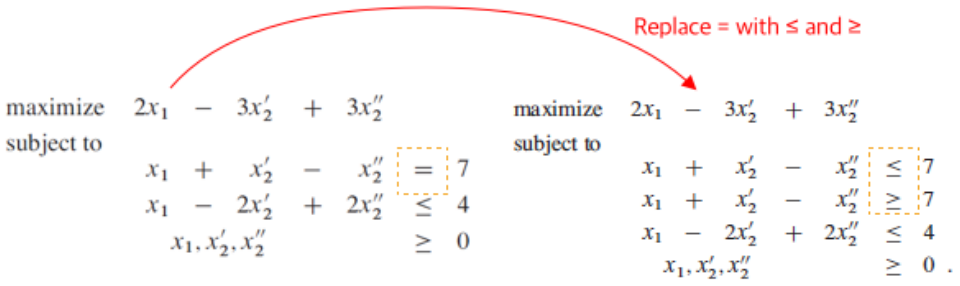
2) There might be variables without nonnegativity constraints
   - Replace each non-nonnegative variable x_i with x_i' and x_i''
   - Modify linear program



3) There might be **equality constraints**, which have an equal sign rather than a less-than-or-equal-to sign
   - Replace equality constraint $f(x_1, x_2, ..., x_n) = b$ with $f(x_1, x_2, ..., x_n) \leq b$ and $f(x_1, x_2, ..., x_n) \geq b$



4) There might be **inequality constraints**, but instead of having a less-than-or-equal-to-sign
   - Multiply incorrect inequality constraints by -1



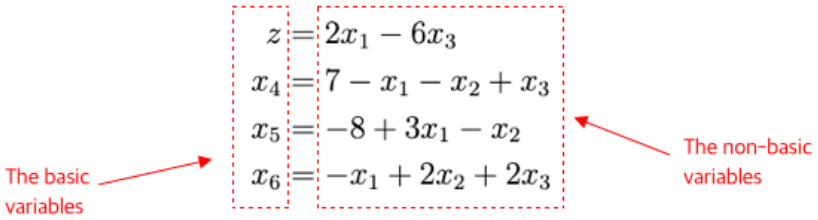
References:

1) Wikipedia, Linear Programming, link
2) Instituto de Mathematicas, Standard form for Linear Programs, link

2.

$$z = 2x_1 - 6x_3$$
$$x_4 = 7 - x_1 - x_2 + x_3$$
$$x_5 = -8 + 3x_1 - x_2$$
$$x_6 = -x_1 + 2x_2 + 2x_3$$

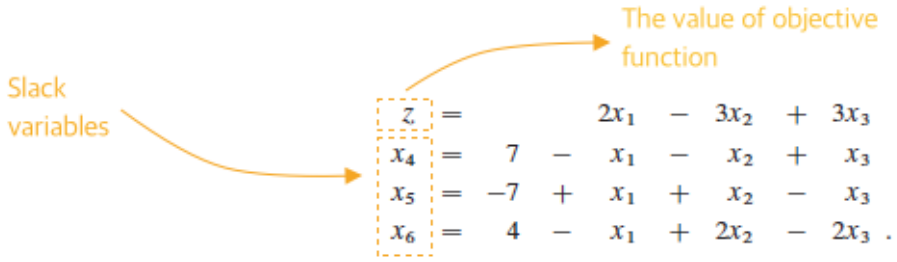
The basic variables are variables on the lhs (i.e $B = 4, 5, 6$), and the non-basic variables are the variables on the rhs of the expressions (i.e. $N = 1, 2, 3$).



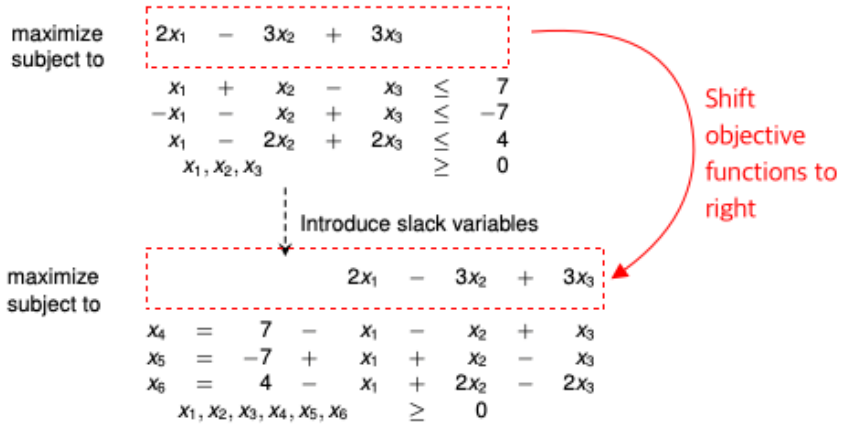
Notes:

- **Slack Form**

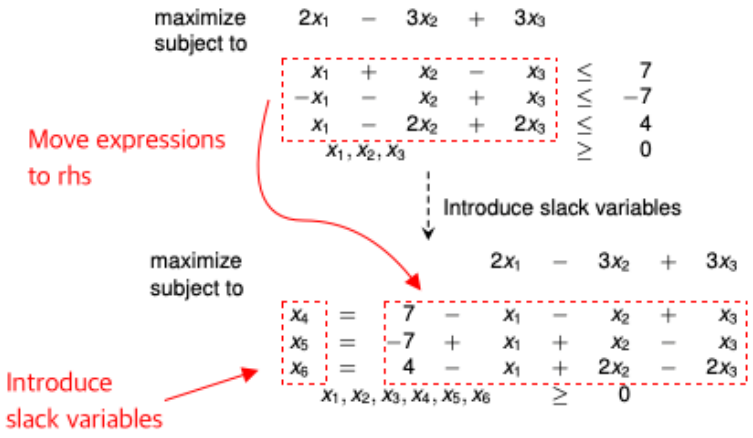
– Is a form of linear programming
– Is for efficient solving of liner programming problem using simplex algorithm



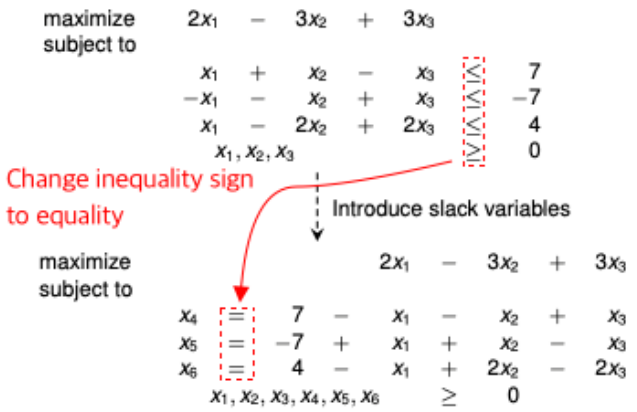
- **Converting Linear Programs into Slack Form**
  1) Start from the standard form of linear programming
  2) Shift objective functions to right



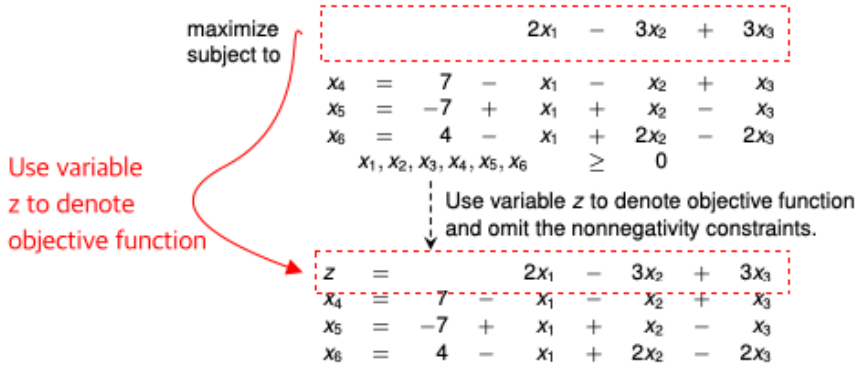
3) Introduce slack variable x_i to lhs and move expressions $\sum_{j=1}^{n} a_{ij}x_j$ to rhs



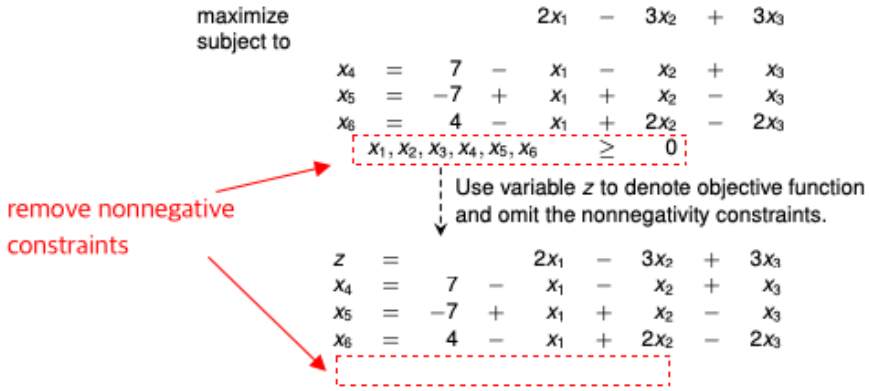
4) Change inequalities in linear programming to equality



5) Use Variable z to denote objective function



6) Omit the nonnegativivty constraints



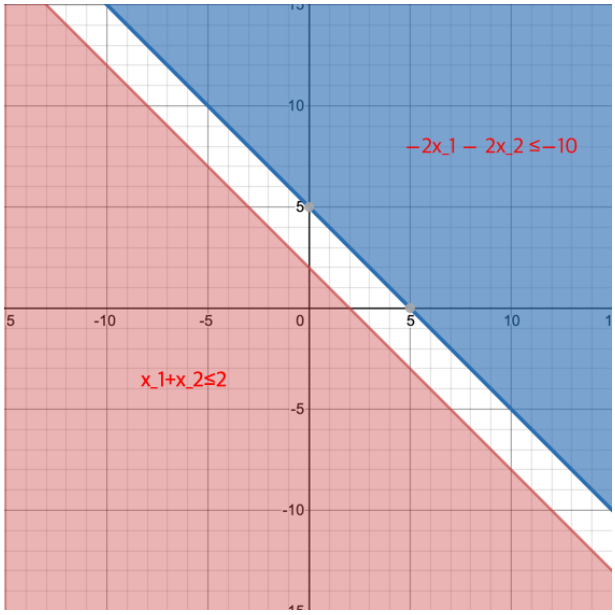
References:

1) Cambridge University, Linear Programming, link

3. Multiplying the first expression (under subject to) by 2, and summing the inequality constraints, we have

$$0 \leq -6 \tag{1}$$

which is impossible.



Notes

- **Infeasible**
  - I noticed that infeasible solution has non-overlapping region
  - A Linear Program is infeasible if there is no solution that satisfies all of the constraints