Worksheet 4 Review

March 22, 2020

Question 1

a. $\exists n \in \mathbb{N},\ n > 3 \wedge n^2 - 1.5n \geq 5$

b. The variable is existentially quantified

c. When introduced, the variable's value should be a **concrete natural number**.

d. Let $n = 5$.

Then $n > 3$, and

$$\begin{aligned} n^2 - 1.5n &= 25 - 7.5 &(1)\\ &= 17.5 \geq 5 &(2) \end{aligned}$$

Then, it follows from above that the statement $\exists n \in \mathbb{N},\ n > 3 \wedge n^2 - 1.5n \geq 5$ is true.

e. $\forall n \in \mathbb{N}, n > 3 \Rightarrow n^2 - 1.5n > 4$

$\Rightarrow$ should be used, because it allows the scoping of the set $\mathbb{N}$.

f. Universally Quantified

g. The variable's value should be an arbitrary natural number.

h. The assumption made is $n > 3$. It is determined by seeing the lhs of $\Rightarrow$.

i. Let $n \in \mathbb{N}$. Assume $n > 3$.

Then,

$$
\begin{align}
n &> 3 \tag{1}\\
(n - 0.75)^2 &> (3 - 0.75)^2 \tag{2}\\
n^2 - 1.5n + 0.5625 &> 5.0625 \tag{3}\\
n^2 - 1.5n &> 4.5 \tag{4}\\
n^2 - 1.5n &> 4 \tag{5}
\end{align}
$$

Then, it follows from above that the statement $\forall n \in \mathbb{N},\ n > 3 \Rightarrow n^2 - 1.5n > 4$.

Question 2

a. $\forall n \in \mathbb{N},\ n > 5 \Rightarrow 2 \mid n \land 3 \mid n$

b. $\exists n \in \mathbb{N}, (n > 5) \land (2 \nmid n \lor 3 \nmid n)$

c. Let $n = 7$.

Then, $2 \nmid n \lor 3 \nmid n$.

Then, it follows from the negation that the statement $\forall n \in \mathbb{N},\ n > 5 \Rightarrow 2 \mid n \land 3 \mid n$ is false.

Question 3

a. Let $x \in \mathbb{R}$, and $y = -x + 165$.

Then, the statement $\forall x \in \mathbb{R},\ \exists y \in \mathbb{R},\ x + y < 165$ is true.

b. Let $y = 166$, and $x \in \mathbb{N}$.

Then the statement $\exists y \in \mathbb{N},\ \forall x \in \mathbb{N},\ x + y > 165$ is true.

c. **Negation:** $\exists y \in \mathbb{R}, \forall x \in \mathbb{R}, x + y \leq 165$

Let $y \in \mathbb{R}$, and let $x = -y + 164$.

Then,

$$\begin{align} x + y &= y - y + 164 \tag{1} \\ &= 164 \tag{2} \\ &\leq 165 \tag{3} \end{align}$$

Then, by negation, the statement $\exists y \in \mathbb{N}, \forall x \in \mathbb{N}, x + y < 165$ is true.