

CSC373 Worksheet 6 Solution

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1. Notes:

• Linear Programming

- Is a method to achieve the best outcome (such as maximum profit or lowest cost) in a mathematical model whose requirements are represented by linear relationships. ^[1]
- Is named to make it sound cool for government funding
 - * Like dynamic programming
- Applications
 - * Microeconomics (maximize profits, minimize costs)
 - * Company management

• Standard Form

- Is a form of linear programming
- Are about maximizing, not minimizing ^[2]
- All have a positivity constraint for each variable ^[2]
- All other constraints are all of the form “linear combination of variables \leq constant”. ^[2]

3. Are about maximizing and not minimizing

Maximize $c_1x_1 + c_2x_2 + \cdots c_nx_n$

subject to

2. constraints of the form $\sum a_{ij} x_j \leq b_i$

$$\begin{array}{rcl} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n & \leq & b_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n & \leq & b_2 \\ \vdots & & \vdots \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n & \leq & b_m \end{array}$$

1. non-negativity constraints for each variable

$$x_1, x_2, \dots, x_n \geq 0$$

• **Converting Linear Programming to Standard Form**

- 1) The objective function might be a minimization rather than a maximization
 - Negate coefficients of the objective function

multiply by -1

minimize $-2x_1 + 3x_2$ subject to $x_1 + x_2 = 7$ $x_1 - 2x_2 \leq 4$ $x_1 \geq 0$	maximize $2x_1 - 3x_2$ subject to $x_1 + x_2 = 7$ $x_1 - 2x_2 \leq 4$ $x_1 \geq 0$
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- 2) There might be variables without nonnegativity constraints
 - Replace each non-nonnegative variable x_i with x'_i and x''_i
 - Modify linear program

Replace x_i with x'_i and x''_i

maximize $2x_1 - 3x_2$ subject to $x_1 + x_2 = 7$ $x_1 - 2x_2 \leq 4$ $x_1 \geq 0$	maximize $2x_1 - 3x'_2 + 3x''_2$ subject to $x_1 + x'_2 - x''_2 = 7$ $x_1 - 2x'_2 + 2x''_2 \leq 4$ $x_1, x'_2, x''_2 \geq 0$
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x_2 is not nonnegative :(They are now nonnegative :)! Yayy!!

- 3) There might be **equality constraints**, which have an equal sign rather than a less-than-or-equal-to sign
- 4) There might be **inequality constraints**, but instead of having a less-than-or-equal-to sign

Example:

References:

- 1) Wikipedia, Linear Programming, [link](#)
- 2) Instituto de Matematicas, Standard form for Linear Programs, [link](#)