

CSC373 Worksheet 0 Solution

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1. Notes:

- Substitution method
 - Solves recurrences
 - * Recurrence characterizes the running time of divide-and-conquer algorithm
 - How it works:
 1. Make a guess for the solution
 2. Use mathematical induction to prove the guess is correct or incorrect.

Example:

Recurrence: $T(n) = 2T(\lfloor n/2 \rfloor) + n$

Guess: $T(n) = \mathcal{O}(n \lg n)$,

We need to show $T(n) \leq cn \lg n$.

1. Assume the bound holds for all positive $m < n$, in particular $m = \lfloor n/2 \rfloor$
2. Find the upper bound of $T(m)$

$$T(\lfloor n/2 \rfloor) \leq c \lfloor n/2 \rfloor \lg(\lfloor n/2 \rfloor)$$

3. Show $T(n) = 2T(\lfloor n/2 \rfloor) + n$ leads to $T(n) \leq cn \lg n$

$$T(n) \leq 2(c \lfloor n/2 \rfloor \lg(\lfloor n/2 \rfloor)) + n \tag{1}$$

$$\leq cn \lg(n/2) + n \tag{2}$$

$$= cn \lg(n) - cn \lg 2 + n \tag{3}$$

$$= cn \lg(n) - cn + n \tag{4}$$

$$\leq cn \lg(n) - cn + cn \tag{5}$$

$$\leq cn \lg(n) \tag{6}$$

4. Show that the boundary holds using mathematical induction