CSC373 Worksheet 0 Solution

July 18, 2020

1. <u>Recurrence:</u> $T(n) = T(n-1) + n$

   <u>Guess:</u> $T(n) = \mathcal{O}(n^2)$.

   I need to show $T(n) \leq c \cdot n^2$.

$$
\begin{align}
T(n) &\leq c(n-1)^2 + n \tag{1}\\
&= c(n^2 - 2n + 1) + n \tag{2}\\
&= cn^2 - c2n + c + n \tag{3}\\
&\leq cn^2 - c2n + cn + n \tag{4}\\
&= cn^2 - cn + n \tag{5}\\
&\leq cn^2 - cn + cn \tag{6}\\
&= cn^2 \tag{7}
\end{align}
$$

   Notes:

   - Substitution method
     - Solves recurrences
       * Recurrence characters the running time of divide-and-conquer algorithm
     - How it works:
       1. Make a guess for the solution
       2. Use mathematical induction to prove the guess is correct or incorrect.

     Example:

     <u>Recurrence:</u> $T(n) = 2T(\lfloor n/2 \rfloor) + n$

     <u>Guess:</u> $T(n) = \mathcal{O}(n \log n)$,

     We need to show $T(n) \leq cn \lg n$.

1. Assume the bound holds for all positive $m < n$, in particular $m = \lfloor n/2 \rfloor$
2. Find the upper bound of $T(m)$

   $$T(\lfloor n/2 \rfloor) \leq c\lfloor n/2 \rfloor \lg(\lfloor n/2 \rfloor)$$

3. Show $T(n) = 2T(\lfloor n/2 \rfloor) + n$ leads to $T(n) \leq cn \lg n$

$$
\begin{align}
T(n) &\leq 2(c\lfloor n/2 \rfloor \lg(\lfloor n/2 \rfloor)) + n \tag{8} \\
&\leq cn \lg(n/2) + n \tag{9} \\
&= cn \lg(n) - cn \lg 2 + n \tag{10} \\
&= cn \lg(n) - cn + n \tag{11} \\
&\leq cn \lg(n) - cn + cn \tag{12} \\
&\leq cn \lg(n) \tag{13}
\end{align}
$$

4. Show that the boundary holds using mathematical induction

   Doesn't have information in detail. Skipping this for now.

– Making good guess
   * Three suggestions
   1. Using recursion tree
   2. Through practice
   3. prove loose upper and lower bounds on the recurrence and then reduce the range of uncertainty