CSC236 Term Test 1 Version 2 Solution

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Question 1

- *Proof.* Define $P(n) : f(n) = 3^n$.

  I will use complete induction to prove that $\forall n \in \mathbb{N}, n > 2 \Rightarrow\ P(n)$.

  Base Case ($n = 0$):

  Let $n = 0$.

  Then, the definition of $f(n)$ tells us $f(n) = 1$.

  Then, we have

  $$\begin{align} f(n) &= 3^0 \tag{1} \\ &= 3^n \tag{2} \end{align}$$

  Thus, $P(n)$ follows.

  Base Case ($n = 1$):

  Let $n = 1$.

  Then, the definition of $f(n)$ tells us $f(n) = 3$.

  Then, we have

  $$\begin{align} f(n) &= 3^1 \tag{3} \\ &= 3^n \tag{4} \end{align}$$

Thus, $P(n)$ follows.

Base Case ($n = 2$):

Let $n = 2$.

Then, the definition of $f(n)$ tells us $f(n) = 9$.

Then, we have

$$\begin{aligned} f(n) &= 3^2 && (5)\\ &= 3^n && (6) \end{aligned}$$

Thus, $P(n)$ follows.

Case ($n > 2$):

Assume $n > 2$.

Then, since $0 \leq n-1 < n$, $0 \leq n-2 < n$, and $0 \leq n-3 < n$, the complete induction tells us $P(n-1)$, $P(n-2)$, and $P(n-3)$, i.e. $f(n-1) = 3^{n-1}$, $f(n-2) = 3^{n-2}$, and $f(n-3) = 3^{n-3}$, respectively.

Then, using these facts, we can write

$$\begin{aligned} f(n) &= f(n-1) + 3f(n-2) + 9f(n-3) && (7)\\ &= 3^{n-1} + 3 \cdot 3^{n-2} + 3^2 \cdot 3^{n-3} && (8)\\ &= 3^{n-1} + 3^{n-1} + 3^{n-1} && (9)\\ &= 3^{n-1}(1+1+1) && (10)\\ &= 3^{n-1}3 && (11)\\ &= 3^n && (12) \end{aligned}$$

Thus, $P(n)$ follows. $\square$

Question 2

Question 3