

# Problem Set 4 Solution

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## Question 1

- a. **Statement:**  $\forall f, g : \mathbb{N} \rightarrow \mathbb{R}^+, b \in \mathbb{R}^+, (g(n) \in \Theta(f(n))) \wedge (n_0 \in \mathbb{N}, n \geq n_0 \Rightarrow f(n) \geq b \wedge g(n) \geq b) \wedge (b > 1) \Rightarrow \log_b(g(n)) \in \Theta(\log_b(f(n)))$

**Statement Expanded:**  $\forall f, g : \mathbb{N} \rightarrow \mathbb{R}^+, b \in \mathbb{R}^+, \left( \exists c_1, c_2, n_0 \in \mathbb{R}^+, \forall n \in \mathbb{N}, n \geq n_0 \Rightarrow c_1 g(n) \leq f(n) \leq c_2 g(n) \right) \wedge \left( \exists n_1 \in \mathbb{N}, n \geq n_1 \Rightarrow f(n) \geq b \wedge g(n) \geq b \right) \wedge (b > 1) \Rightarrow \left( \exists d_1, d_2, n_2 \in \mathbb{R}^+, \forall n \in \mathbb{N}, n \geq n_2 \Rightarrow d_1 \log_b(g(n)) \leq \log_b(f(n)) \leq d_2 \log_b(g(n)) \right)$

**Notes:**

- $\forall x, y \in \mathbb{R}^+, x \geq y \Leftrightarrow \log x \geq \log y$
- $\exists c_1, c_2, n_0 \in \mathbb{R}^+, \forall n \in \mathbb{N}, n \geq n_0 \Rightarrow c_1 \cdot g(n) \leq f(n) \leq c_2 \cdot g(n)$
- **Definition of Eventually:**  $\exists n_0 \in \mathbb{N}, n \geq n_0 \Rightarrow P$ , where  $P : \mathbb{N} \rightarrow \{\text{True}, \text{False}\}$

## Question 2

## Question 3

## Question 4