Worksheet 20 Solution

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Question 1

a. **Pseudoproof:**

Let $V = \{1, 2, 3, 4, 5, 6\}$, $E = \{(1, 2), (1, 6), (2, 3), (3, 4), (4, 5), (5, 6)\})$.

We need to prove the graph $G = (V, E)$ is bipartite by proving the following properties:

1. There exists subsets $V_1, V_2 \subset V$ such that $V_1 \neq \emptyset, V_2 \neq \emptyset$, and V_1 and V_2 form a partition of V.
2. Every edge in E has exactly one endpoint in V_1 and one in V_2.

We will prove the properties in parts.

1. Show there exists subsets $V_1, V_2 \subset V$ such that $V_1 \neq \emptyset, V_2 \neq \emptyset$, and V_1 and V_2 form a partition of V

   Let $V_1 = \{1, 3, 5\}$ and $V_2 = \{2, 4, 6\}$.

   We need to prove $V_1 \neq \emptyset, V_2 \neq \emptyset$, and V_1 and V_2 form a partition of V, i.e $V_1 \cup V_2 = V \wedge V_1 \cap V_2 = \emptyset$.

   1. Show $V_1 \neq \emptyset, V_2 \neq \emptyset$
      First, we need to show the subsets V_1 and V_2 are non-empty.

      The header tells us both subsets V_1 and V_2 have more than 1 elements.

      Then, using these facts, we can conclude $V_1 \neq \emptyset$ and $V_2 \neq \emptyset$.

2. Show $V_1 \cup V_2 = V \land V_1 \cap V_2 = \emptyset$

Second, we need to show $V_1 \cup V_2 = V$ and $V_1 \cap V_2 = \emptyset$.

The header tells us $V_1 = \{1, 3, 5\}$ and $V_2 = \{2, 4, 6\}$.

Then, we can calculate

$$V_1 \cup V_2 = \{1, 2, 3, 4, 5, 6\} = V \tag{1}$$
$$V_1 \cap V_2 = \emptyset \tag{2}$$

<u>Part 1:</u>

Let $V_1 = \{1, 3, 5\}$ and $V_2 = \{2, 4, 6\}$.

We need to prove $V_1 \neq \emptyset, V_2 \neq \emptyset$, and V_1 and V_2 form a partition of V, i.e $V_1 \cup V_2 = V \land V_1 \cap V_2 = \emptyset$.

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Then, using these facts, we can conclude $V_1 \neq \emptyset$ and $V_2 \neq \emptyset$.

Finally, we need to show $V_1 \cup V_2 = V$ and $V_1 \cap V_2 = \emptyset$.

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Then, we can calculate

$$V_1 \cup V_2 = \{1, 2, 3, 4, 5, 6\} = V \tag{3}$$
$$V_1 \cap V_2 = \emptyset \tag{4}$$

2. Show every edge in E has exactly one endpoint in V_1 and one in V_2.