Worksheet 5 Review

March 23, 2020

Question 1

- **Predicate Logic:** $\forall x, y \in \mathbb{Z}, Odd(x) \wedge Odd(y) \Rightarrow Odd(xy)$

  Let $x, y \in \mathbb{Z}$. Assume $Odd(x)$ and $Odd(y)$.

  Then, $\exists k, m \in \mathbb{Z}$,

$$x = 2k - 1 \tag{1}$$
$$y = 2m - 1 \tag{2}$$

  Then,

$$xy = (2k-1)(2m-1) \tag{3}$$
$$xy = (4km - 2k - 2m + 2) - 1 \tag{4}$$
$$xy = 2(2km - k - m + 1) - 1 \tag{5}$$
$$xy = 2o - 1 \tag{6}$$

  by setting $o = 2km - k - m + 1$.

  Since, $o \in \mathbb{Z}$, it follows from the definition of odd that the statement $\forall x, y \in \mathbb{Z}, Odd(x) \wedge Odd(y) \Rightarrow Odd(xy)$ is true.

Question 2

a. $\forall n, m \in \mathbb{Z},\ Even(n) \wedge Odd(m) \Rightarrow m^2 - n^2 = m + n$

b. The flaw is that the value k in $n = 2k$ and $m = 2k + 1$ cannot be the same.

Question 3

a. $Dom(f, g) : \forall n \in \mathbb{Z},\ g(n) \leq f(n)$, where $f, g : \mathbb{N} \to \mathbb{R}^{\geq 0}$

b. Let $f(n) = 3n$, $g(n) = n$, and $n \in \mathbb{N}$.

Then,

$$\begin{aligned} g(n) = n &\leq n + n + n && (1)\\ &\leq 3n && (2)\\ &\leq f(n) && (3) \end{aligned}$$

Then, it follows from the definition of '**is dominated by**' that g is dominated by f.

c. **Negation:** $\neg Dom(f, g) : \exists n \in \mathbb{Z},\ g(n) > f(n)$, where $f, g : \mathbb{N} \to \mathbb{R}^{\geq 0}$

Let $n = 1$, $f(n) = 3n$, and $g(n) = n$.

Then,

$$\begin{aligned} n + 165 &= (1) + 165 && (1)\\ &= 166 && (2)\\ &> 1 && (3)\\ &> (1)^2 && (4)\\ &> n^2 && (5) \end{aligned}$$

Then it follows from the negation of $Dom(f, g)$ that g is not dominated by f.

d. **Negation:** $\forall x \in \mathbb{R}, \forall n \in \mathbb{N}, g(n) = n+1 \wedge f(n) = n^2 \Rightarrow (\exists m \in \mathbb{N}, g(m) > f(m))$

Question 4