

Worksheet 5 Review

March 22, 2020

Question 1

- **Predicate Logic:** $\forall x, y \in \mathbb{Z}, \text{Odd}(x) \wedge \text{Odd}(y) \Rightarrow \text{Odd}(xy)$

Let $x, y \in \mathbb{Z}$. Assume $\text{Odd}(x)$ and $\text{Odd}(y)$.

Then, $\exists k, m \in \mathbb{Z}$,

$$x = 2k - 1 \tag{1}$$

$$y = 2m - 1 \tag{2}$$

Then,

$$xy = (2k - 1)(2m - 1) \tag{3}$$

$$xy = (4km - 2k - 2m + 2) - 1 \tag{4}$$

$$xy = 2(2km - k - m + 1) - 1 \tag{5}$$

$$xy = 2o - 1 \tag{6}$$

by setting $o = 2km - k - m + 1$.

Since, $o \in \mathbb{Z}$, it follows from the definition of odd that the statement $\forall x, y \in \mathbb{Z}, \text{Odd}(x) \wedge \text{Odd}(y) \Rightarrow \text{Odd}(xy)$ is true.

Question 2

- a. $\forall n, m \in \mathbb{Z}, \text{Even}(n) \wedge \text{Odd}(m) \Rightarrow m^2 - n^2 = m + n$
- b. The flaw is that the value k in $n = 2k$ and $m = 2k + 1$ cannot be the same.

Question 3

- a. $\text{Dom}(f, g) : \forall n \in \mathbb{Z}, g(n) \leq f(n)$, where $f, g : \mathbb{N} \rightarrow \mathbb{R}^{\geq 0}$
- b. Let $f(n) = 3n$, $g(n) = n$, and $n \in \mathbb{N}$.

Then,

$$g(n) = n \leq n + n + n \tag{1}$$

$$\leq 3n \tag{2}$$

$$\leq f(n) \tag{3}$$

Then, it follows from the definition of '**is dominated by**' that g is dominated by f .

Question 4