

CSC373 Worksheet 7 Solution

August 14, 2020

1. My Work

The longest simple cycle problem is the problem of finding a cycle of maximum length in a graph [5].

The decision problem is, given k , to determine whether or not the instance graph has a simple cycle of length at least k . If yes, output 1. Otherwise, output 0.

My Work

The language corresponding to the decision problem is as follows:

LONGEST-SIMPLE-CYCLE = $\{ \langle G, v_0, v_1, \dots, v_k, k \rangle : G = (V, E) \text{ is an undirected graph}$
 $k \geq 3 \text{ is an integer,}$
 $v_0, v_1, \dots, v_k \in V \text{ are distinct,}$
 $v_0 = v_k,$
There should exist a simple cycle in G
with at least k edges }

Correct Solution:

The problem LONGEST-SIMPLE-CYCLE is a relation that associates each instance of a graph with the longest simple cycle in that graph .

The decision problem is, given k , to determine whether or not the instance graph has a simple cycle of length at least k . If yes, output 1. Otherwise, output 0.

The language corresponding to the decision problem is as follows:

LONGEST-SIMPLE-CYCLE = $\{\langle G, k \rangle : G = (V, E) \text{ is an undirected graph}$
 $k \geq 0$ is an integer,
 There should exist a simple cycle in G
 with at least k edges $\}$

Notes

- **A Cycle in an Undirected Graph**

- A path $\langle v_0, v_1, \dots, v_k \rangle$ forms a cycle if $k \geq 3$, and $v_0 = v_k$.

- **Simple Cycle**

- A cycle is simple if v_1, v_2, \dots, v_k are distinct

- **Decision Problem**

- Is the problem with yes/no solution

- **Alphabet**

- Is a finite set of symbols

- Is denoted Σ

Example:

For decision problem, its alphabet is: $\Sigma = \{0, 1\}$

- * 1 means 'yes'

- * 0 means 'no'

- **Language**

- Is any set of strings made of symbols from Σ

- Is denoted L

Example:

$L = \{10, 11, 101, 111, 1011, 1101, 10001\}$

- Is denoted Σ^* for language of all strings over Σ plus empty string ϵ .

Example:

$\Sigma^* = \{\epsilon, 0, 1, 00, 01, 11, 000, \dots\}$

Example 2:

The decision problem PATH has the corresponding language

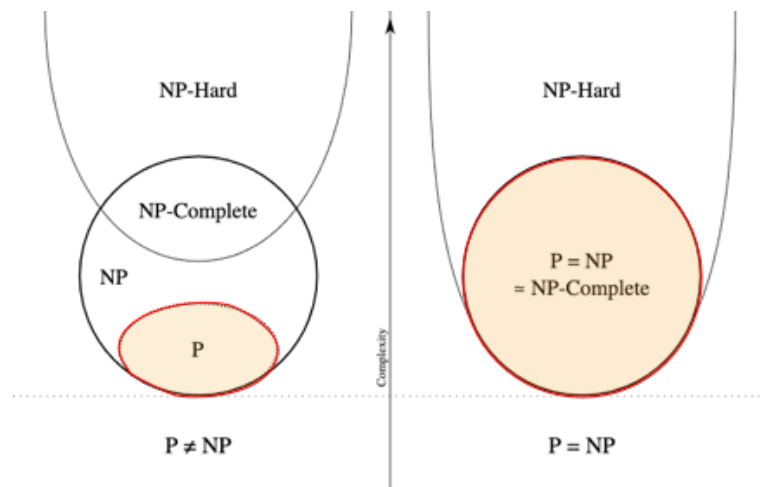
$$\text{PATH} = \{ \langle G, U, v, k \rangle : G = (V, E) \text{ is an undirected graph,} \\ u, v \in V, \\ k \geq 0 \text{ is an integer, and} \\ \text{there exists a path from } u \text{ to } v \text{ in } G \\ \text{consisting of at most } k \text{ edges} \}$$

- **P**

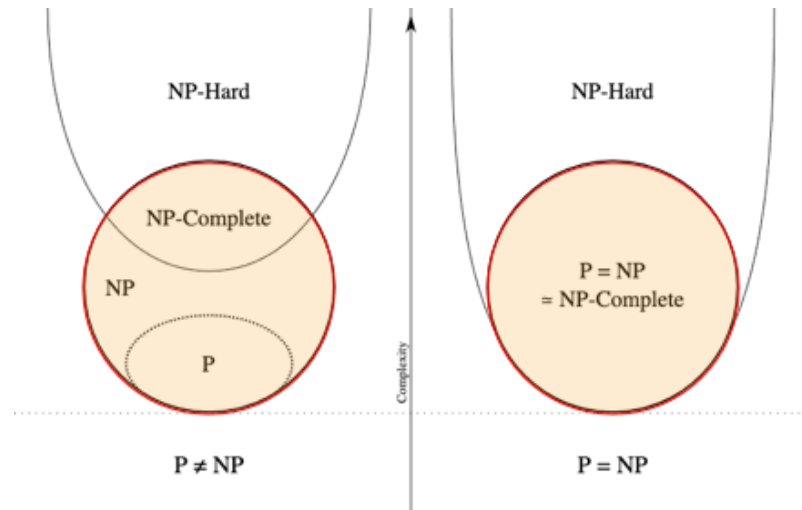
- Is set of problems that can be solved by a deterministic Turing machine in Polynomial time (i.e. $\mathcal{O}(n^k)$) [2].

Example:

- 1) Shortest path problems
- 2) Calculating the greatest common divisor
- 3) Finding maximum bipartite matching



- **NP (Non-deterministic Polynomial):**

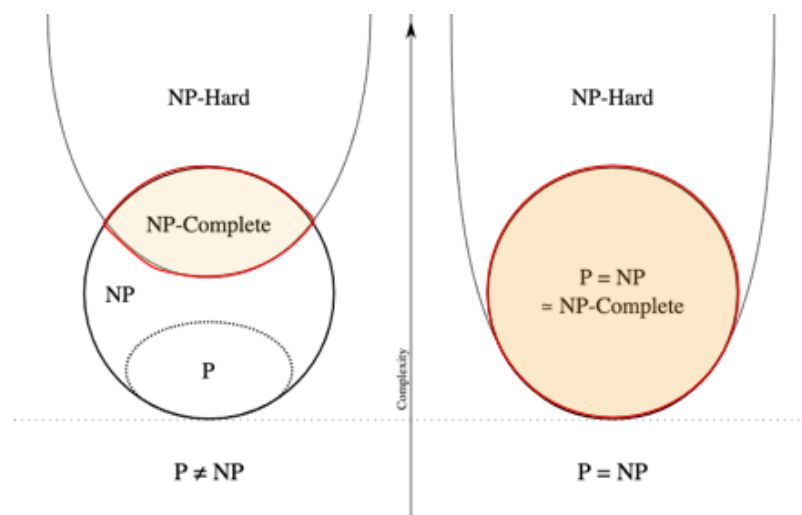


- Is set of decision problems that can be solved by a Non-deterministic Turing Machine in Polynomial time.^[2]
- Has no particular rule is followed to make a guess ^[1].
- Can be solved in polynomial time via a “lucky algorithm”, a magical algorithm that always make a right guess ^[2]
- $P \subseteq NP$

Examples:

- Longest-path problems
- Hamiltonian Cycle
- Graph coloring

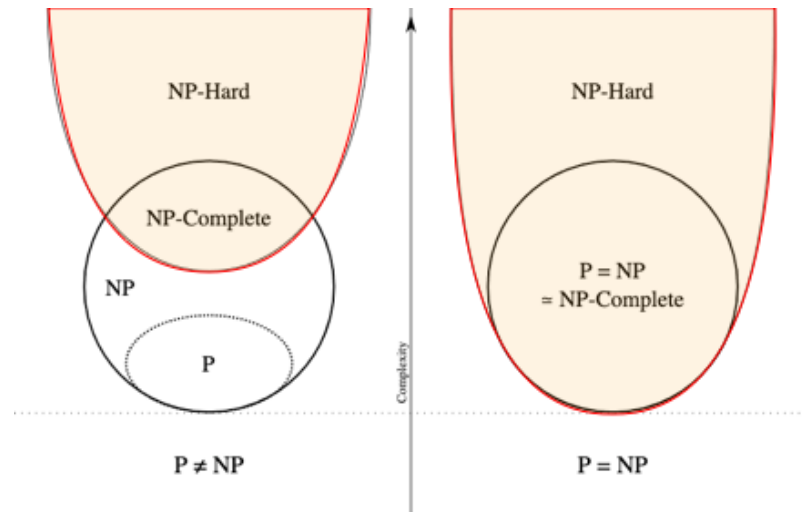
• NP-Complete Problems:



- A decision problem A is NP-complete (NPC) if

- 1) $A \in NP$ and
- 2) Every (other) problems A' in NP is reducible to A
- Has no efficient solution in polynomial number of steps (not yet) ^[3]
- Is not likely that there is an algorithm to make it efficient ^[3]

• **NP-Hard:**



- A decision problem A is NP-hard if
 - 1) $A \in NP$ (Not necessarily) and
 - 2) Every (other) problems A' in NP is reducible to A
- NP-Hard means “at least as hard as any problems in NP”
- Does not have to be about decision problems

Example:

- 1) Alan Turing’s Halting Problem

References

- 1) Encyclopedia Britannica, NP-Complete Problem, [link](#)
- 2) Geeks for Geeks, NP-Completeness, [link](#)
- 3) Wikipedia, NP-complete, [link](#)
- 4) UCLA UC-Davis, ECS122A Handout on NP-Completeness, [link](#)

2. **References**