

CSC236 Worksheet 5 Solution

Hyungmo Gu

May 7, 2020

Question 1

a. Rough Works:

For convenience, define $H(k) : R(3^k) = 3^k k$. Notice that when $n = 3^k$, $3^k k$ is the same as $n \log_3 n$. I will use simple induction to prove $\forall k \in \mathbb{N}, H(k)$.

1. Base Case ($k = 0$)

Let $k = 0$.

Then,

$$3^k \cdot k = 3^0 \cdot 0 \tag{1}$$

$$= 0 \tag{2}$$

$$= R(n) \quad \text{[By def.]} \tag{3}$$

Thus, $H(0)$ is verified.

2. Inductive Step

Let $k \in \mathbb{N}$. Assume $H(k)$. That is $R(3^k) = 3^k k$.

I will show that $H(k+1)$ follows. That is, $R(3^{k+1}) = (k+1)3^{k+1}$.

The definition of $R(3^{k+1})$ tells us,

$$R(3^{k+1}) = 3^{k+1} + 3R(\lceil 3^{k+1}/3 \rceil) \quad (4)$$

$$= 3^{k+1} + 3R(3^k) \quad (5)$$

Then, using this fact, we can conclude

$$R(3^{k+1}) = 3^{k+1} + 3 \cdot 3^k k \quad [\text{By I.H}] \quad (6)$$

$$= 3^{k+1} + 3^{k+1} k \quad (7)$$

$$= 3^{k+1}(k+1) \quad (8)$$