CSC373 Worksheet 7 Solution

August 14, 2020

1. **Rough Works:**

   The longest simple cycle problem is the problem of finding a cycle of maximum length in a graph [5].

   Formally, the language corresponding to the decision problem is as follows:

$$
\begin{aligned}
\text{LONGEST-SIMPLE-CYCLE} = \{\langle G, v_0, v_1, ..., v_k, k\rangle :\ & G = (V, E) \text{ is an undirected graph} \\
& k \geq 3 \text{ is an integer,} \\
& v_0, v_1, ..., v_k \in V \text{ are distinct,} \\
& v_0 = v_k, \\
& \text{There should exist a simple cycle in G} \\
& \text{with at least } k \text{ edges}\}
\end{aligned}
$$

   Notes

   - **A Cycle in an Undirected Graph**
     - A path $\langle v_0, v_1, ..., v_k$ forms a cycle if $k \geq 3$, and $v_0 = v_k$.
   - **Simple Cycle**
     - A cycle is simple if $v_1, v2, ..., v_k$ are distinct
   - **Decision Problem**
     - Is the problem with yes/no solution
   - **Alphabet**
     - Is a finite set of symbols

  - Is denoted Σ

    Example:

    For decision problem, its alphabet is: $\Sigma = \{0, 1\}$
    * 1 means 'yes'
    * 0 means 'no'

- **Language**
  - Is any set of strings made of symbols from Σ
  - Is denoted L

    Example:

    $L = \{10, 11, 101, 111, 1011, 1101, 10001\}$

  - Is denoted Σ^* for language of all strings over Σ plus empty string ϵ.

    Example:

    $\Sigma^* = \{\epsilon, 0, 1, 00, 01, 11, 000, ...\}$

    Example 2:

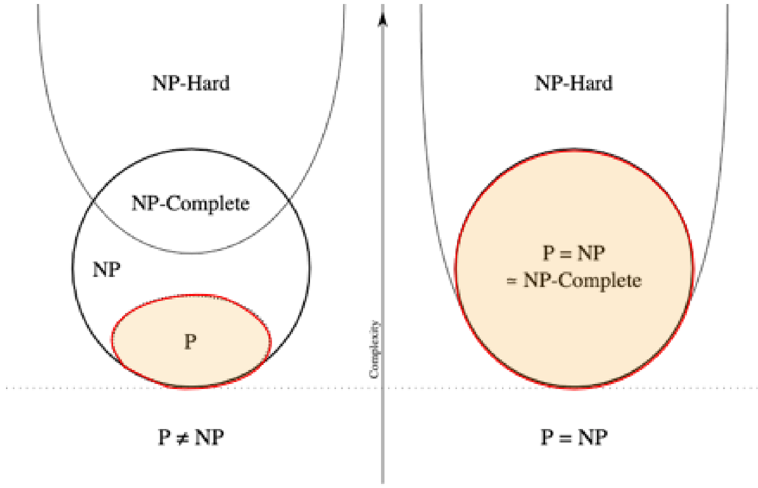
    The decision problem PATH has the corresponding language

$$\begin{aligned}\text{PATH} = \{\langle G, U, v, k\rangle : &G = (V, E) \text{ is an undirected graph,}\\ &u, v \in V,\\ &k \geq 0 \text{ is an integer, and}\\ &\text{tere exists a path from } u \text{ to } v \text{ in } G\\ &\text{consisting of at most } k \text{ edges}\}\end{aligned}$$

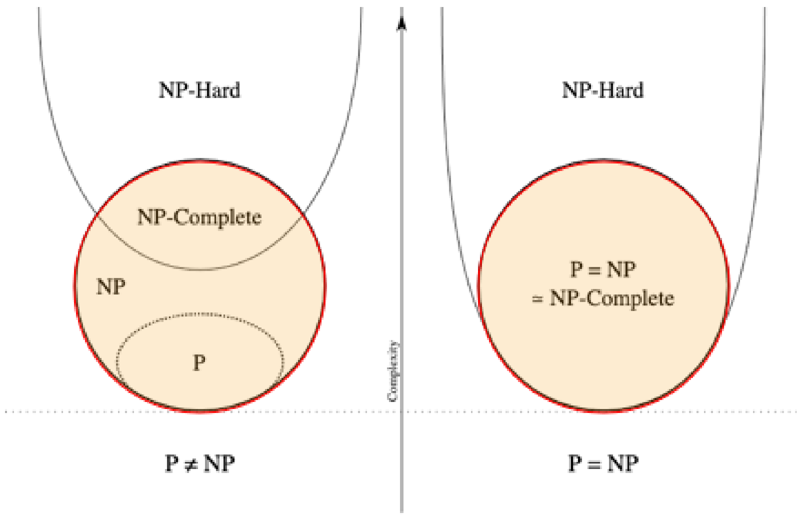
- **P**
  - Is set of problems that can be solved by a deterministic Turing machine in Polynomial time (i.e. $\mathcal{O}(n^k)$) [2].

    Example:

    1) Shortest path problems
    2) Calculating the greatest common divisor
    3) Finding maximum bipartite matching



- **NP (Non-deterministic Polynominal):**

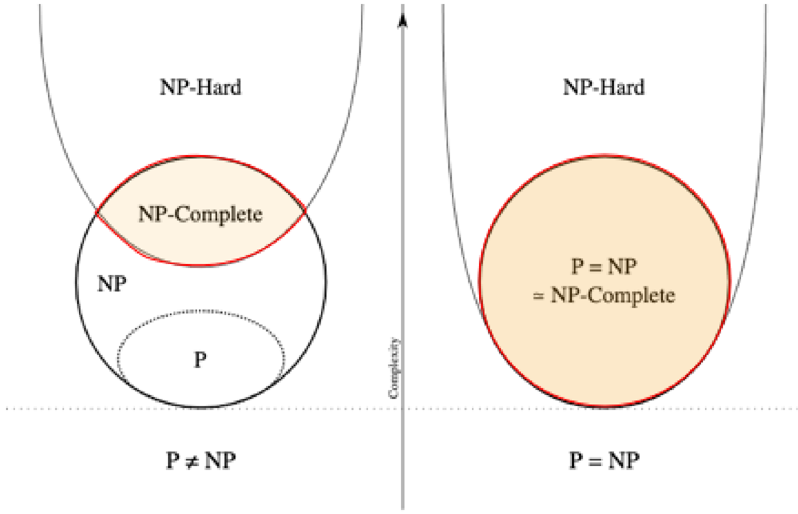


  - Is set of decision problems that can be solved by a Non-deterministic Turing Machine in Polynomial time.[2]
  - Has no particular rule is followed to make a guess [1].
  - Can be solved in polynominal time via a "lucky algorithm", a magical algorithm that always make a right guess [2]
  - $P \subseteq NP$

  <u>**Examples:**</u>

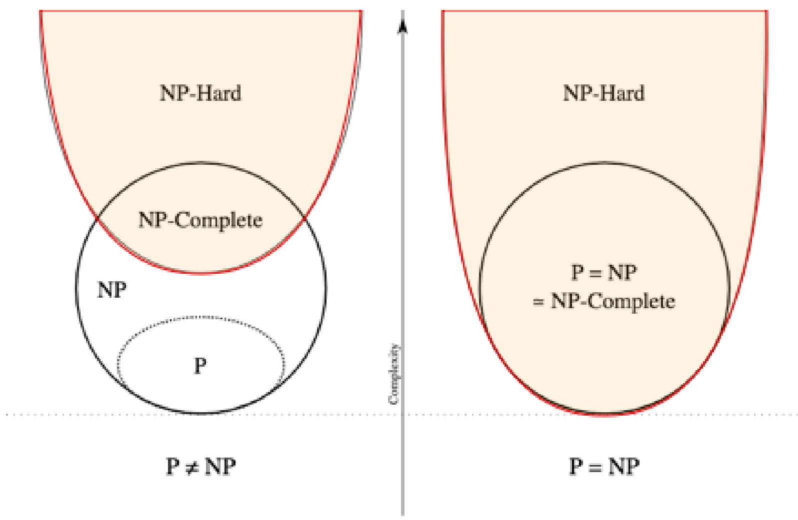
  - Longest-path problems
  - Hamiltonian Cycle
  - Graph coloring

- **NP-Complete Problems:**



- A decision problem A is NP-complete (NPC) if
  1) $A \in NP$ and
  2) Every (other) problems A' in NP is reducible to A
- Has no efficient solution in polynominal number of steps (not yet) [3]
- Is not likely that there is an algorithm to make it efficient [3]

- **NP-Hard:**



- A decision problem A is NP-hard if
  1) $A \in NP$ (Not necessarily) and
  2) Every (other) problems A' in NP is reducible to A
- NP-Hard means "at least as hard as any problems in NP"
- Does not have to be about decision problems

Example:

1) Alan Turing's Halting Problem

References

1) Encyclopedia Britannica, NP-Complete Problem, link
2) Geeks for Geeks, NP-Completeness, link
3) Wikipedia, NP-complete, link
4) UCLA UC-Davis, ECS122A Handout on NP-Completeness, link