Worksheet 11 Solution

March 21, 2020

Question 1

a. $\forall a, b \in \mathbb{R}^+,\ a \leq b \Rightarrow \exists c, n_0 \in \mathbb{R}^+,\ \forall n \in \mathbb{N},\ n \geq n_0 \Rightarrow n^a \leq cn^b$

b. Let $a, b \in \mathbb{R}^+$, $n \in \mathbb{N}$, $c = 1$, $n_0 = 1$. Assume $a \leq b$, and $n \geq n_0$.

Then,

$$\begin{aligned} n^a &\leq [n^a]^k && (1)\\ &\leq n^{ak} && (2)\\ &\leq n^b && (3) \end{aligned}$$

by the fact that $k = \frac{b}{a}$, and $k \in \mathbb{R}^+$.

Then,

$$\begin{aligned} n^a &\leq n^b && (4)\\ &\leq cn^b && (5) \end{aligned}$$

Then, it follows from above that the statement $\forall a, b \in \mathbb{R}^+,\ a \leq b \Rightarrow n^a \in \mathcal{O}(n^b)$ is true.

Question 2

Question 3