# CSC373 Worksheet 6 Solution

# August 12, 2020

## 1. 1. Multiply objective function by - 1

Maximize

$$-2x_1 - 7x_2 - x_3$$

Subject to

$$x_1 - x_3 = 7$$
$$3x_1 + x_2 \ge 7$$

$$+x_2 \ge 7$$

$$x_2 \ge 0$$

$$x_3 \le 0$$

2. Replace non-nonnegative constraints  $x_1$ 

Maximize

$$-2x_1' + 2x_1'' - 7x_2 - x_3$$

Subject to

$$x_1' - x_1'' - x_3 = 7$$
$$3x_1' - 3x_1'' + x_2 \ge 7$$

$$x_1', x_1'', x_2 \ge 0$$

$$x_3 \le 0$$

3. Replace non-nonnegative constraints  $x_3$ 

Maximize

$$-2x_1' + 2x_1'' - 7x_2 - x_3' + x_3''$$

Subject to

$$x'_1 - x''_1 - x'_3 + x''_3 = 7$$
$$3x'_1 - 3x''_1 + x_2 \ge 7$$
$$x'_1, x''_1, x_2, x'_3, x''_3 \ge 0$$

4. Replace equality constraints with  $\geq$  and  $\leq$ 

Maximize

$$-2x_1' + 2x_1'' - 7x_2 - x_3' + x_3''$$

Subject to

$$\begin{aligned} x_1' - x_1'' - x_3' + x_3'' &\leq 7 \\ x_1' - x_1'' - x_3' + x_3'' &\geq 7 \\ 3x_1' - 3x_1'' + x_2 &\geq 7 \\ x_1', x_1'', x_2, x_3', x_3'' &\geq 0 \end{aligned}$$

5. Correct greater-than-or-equal-to inequality constraints

Maximize

$$-2x_1' + 2x_1'' - 7x_2 - x_3' + x_3''$$

Subject to

$$x'_1 - x''_1 - x'_3 + x''_3 \le 7$$

$$-x'_1 + x''_1 + x'_3 - x''_3 \le -7$$

$$-3x'_1 + 3x''_1 - x_2 \le 7$$

$$x'_1, x''_1, x_2, x'_3, x''_3 \ge 0$$

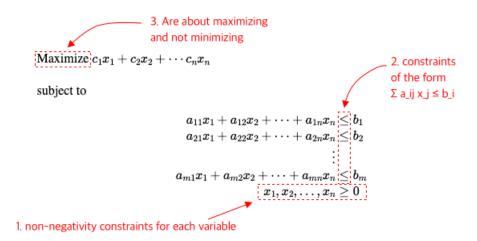
#### Notes:

#### • Linear Programming

- Is a method to achieve the best outcome (such as maximum profit or lowest cost) in a mathematical model whose requirements are represented by linear relationships. [1]
- Is named to make it sound cool for government funding
  - \* Like dynamic programming
- Applications
  - \* Microeconomics (maximize profits, minimize costs)
  - \* Company management

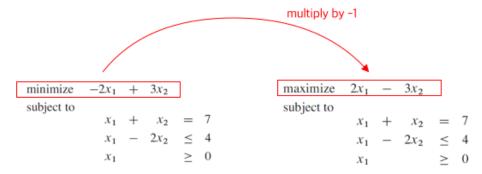
#### • Standard Form

- Is a form of linear programming
- Are about maximizing, not minimizing [2]
- All have a positivity constraint for each variable [2]
- All other constraints are all of the form "linear combination of variables  $\leq$  constant".  $^{[2]}$

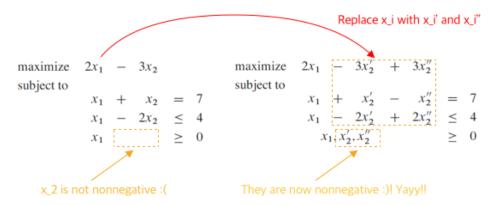


#### • Converting Linear Programming to Standard Form

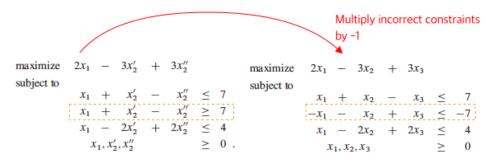
- 1) The objective function might be a minimization rather than a maximization
  - Negate coefficients of the objective function



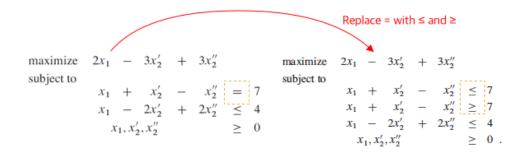
- 2) There might be variables without nonnegativity constraints
  - Replace each non-nonnegative variable  $x_i$  with  $x_i'$  and  $x_i''$
  - Modify linear program



- 3) There might be **equality constraints**, which have an equal sign rather than a less-than-or-equal-to sign
  - Replace equality constraint  $f(x_1, x_2, ..., x_n) = b$  with  $f(x_1, x_2, ..., x_n) \le b$  and  $f(x_1, x_2, ..., x_n) \ge b$



- 4) There might be **inequality constraints**, but instead of having a less-than-or-equal-to-sign
  - Multiply incorrect inequality constraints by -1



### References:

- 1) Wikipedia, Linear Programming, link
- 2) Instituto de Mathematicas, Standard form for Linear Programs, link

### 2. <u>Notes:</u>

#### • Slack Form

- Is a form of linear programming
- Is for efficent solving of liner program using simplex algorithm

#### • Converting Linear Programs into Slack Form

- Start from the standard form of linear programming