# Worksheet 14 Review

April 1, 2020

## Question 1

a. Since the inner loop starts at $j = 0$ and finishes at $j = n - 1$ with $j$ increasing by 1 per iteration, we can conclude that the inner loop has

$$\lceil n - 1 - 0 + 1 \rceil = n \tag{1}$$

iterations.

Since the inner loop takes 1 step per iteration, we can conclude that the inner loop has the total cost of

$$n \cdot 1 = n \tag{2}$$

steps.

For the outer loop, because it starts at $i = 0$ and ends at $i = n - 1$ with $i$ increasing by 5 per iteration, we can conclude that the outer loop has

$$\left\lceil \frac{n - 1 - 0 + 1}{5} \right\rceil = \left\lceil \frac{n}{5} \right\rceil \tag{3}$$

iterations.

Since each iteration in the outer loop takes $n$ steps, we can conclude the outer loop has the total cost of

$$n \cdot n = n^2 \tag{4}$$

steps.

Since we are ignoring the cost of the loop variables, the total cost of the algorithm is $n^2 + n$ steps.

Then, because we know the algorithm takes total of $n^2 + n$ steps, we can conclude the algorithm has the runtime of $\Theta(n^2)$.

b. We will determine the exact cost and theta of this algorithm by first calculating the exact cost of inner loop 1

```
    j = 1
    while j < n:
        j = j * 3

```

and then, calculating the exact cost of inner loop 2

```
    k = 0
    while k < n:
        k = k + 2

```

and then, calculating the exact cost of the outer loop using the information from the exact cost of inner loop 1 and inner loop 2

```
    i = 4
    while i < n:
        j = 1
        while j < n:
            j = j * 3
        k = 0
        while k < n:
            k = k + 2
        i = i + 1

```

and then, we will finish off by calculating the theta of the outer loop.

**Part 1 (Calculating the exact cost of loop 1):**

Because we kow $j = j \cdot 3$, we can calculate

$$\begin{aligned} i_1 &= 3 \\ i_2 &= 9 \\ i_3 &= 27 \\ &\vdots \\ i_j &= 3^j \end{aligned}$$

Then, using the fact that loop termination occurs when $i_j \geq n$, we can conclude

$$3^j \geq n \tag{1}$$
$$j \geq \log_3 n \tag{2}$$

Since we are looking for the smallest value of $j$ resulting in loop termination, we can conclude the value of $j$ is $\lceil \log_3 n \rceil$.

Since the inner loop 1 takes constant step per iteration, we can conclude that the loop has exact cost of

$$\lceil \log_3 n \rceil \cdot 1 = \lceil \log_3 n \rceil \tag{3}$$

steps.

**Part 2 (Calculating the exact cost of loop 2):**

Since the loop starts from $k = 0$ and ends at $k = n - 1$, with $k$ increasing by 2 per iteration, we can conclude that the loop has

$$\left\lceil \frac{n - 1 - 0 + 1}{2} \right\rceil = \left\lceil \frac{n}{2} \right\rceil \tag{4}$$

iterations.

Since the loop takes 1 step per iteration, the loop has total cost of

$$\left\lceil \frac{n}{2} \right\rceil \cdot 1 = \left\lceil \frac{n}{2} \right\rceil \tag{5}$$

steps.

**Part 3 (Calculating the exact cost of outer loop):**

Since the loop runs from $i = 4$ to $i = n - 1$ with $i$ increasing by 1 per iteration, we can conclude the loop has

$$\left\lceil \frac{n - 1 - 4 + 1}{1} \right\rceil = n - 4 \tag{6}$$

iterations.

Since each iteration takes $\lceil \log_3 n \rceil + \left\lceil \frac{n}{2} \right\rceil$ steps, we can conclude the outer loop has total of

$$(n - 4) \cdot \left( \lceil \log_3 n \rceil + \left\lceil \frac{n}{2} \right\rceil \right) \tag{7}$$

steps.

**Part 4 (Calculating Theta):**

Because we know the loop in total has exact cost of $(n-4) \cdot \left( \lceil \log_3 n \rceil + \left\lceil \frac{n}{2} \right\rceil \right)$, we can conclude that the algorithm has total runtime of $\Theta(n^2)$.

## Question 2