Worksheet 12 Review

March 30, 2020

Question 1

a. $g \in \mathcal{O}(1) : \exists c, n_0 \in \mathbb{R}^+, \forall n \in \mathbb{N}, n \geq n_0 \Rightarrow g(n) \leq c$, where $g : \mathbb{N} \to \mathbb{R}^{\geq 0}$

Notes:

- $g \in \mathcal{O}(f) : \exists c, n_o \in \mathbb{R}^+, \forall n \in \mathbb{N}, n \geq n_0 \Rightarrow g(n) \leq cf(n)$, where $f, g : \mathbb{N} \to \mathbb{R}^{\geq 0}$

b. **Predicate Logic** $\exists c, n_0 \in \mathbb{R}^+, \forall n \in \mathbb{N}, n \geq n_0 \Rightarrow g(n) \leq c$, where $g : \mathbb{N} \to \mathbb{R}^{\geq 0}$

Proof. Let $n_0 = 1$, $c = 200$ and $g(n) = 100 + \frac{77}{n+1}$. Assume $n \geq n_0$.

We will prove the statement by showing

$$100 + \frac{77}{n+1} \leq c \tag{1}$$

It follows from the fact $n_0 \geq 1$ that we can write

$$
\begin{aligned}
100 + \frac{77}{n+1} &\leq 100 + \frac{77}{1+1} && (2)\\
&\leq 100 + \frac{77}{2} && (3)\\
&\leq 100 + 77 && (4)\\
&\leq 100 + 100 && (5)\\
&\leq 200 && (6)
\end{aligned}
$$

Then,

$$
100 + \frac{77}{n+1} \leq c \tag{7}
$$

by the fact that $c = 200$. □

Question 2

Question 3

Question 4