Worksheet 14 Solution

March 25, 2020

Question 1

a. **Inner Loop:** n

    Outer Loop: $n - 5$

    Theta Expressions: $\Theta(n^2)$

    Correct Solution:

    Inner Loop: n

    Outer Loop: $n \cdot \left\lceil \dfrac{n}{5} \right\rceil$

    Theta Expressions: $\Theta(n^2)$

b. **Inner Loop:** $\dfrac{n}{3} + (n - 2)$

    Outer Loop: $n - 4$

    Theta Expressions: $\Theta(n^2)$

    Correct Solution:

    Inner Loop: $\lceil \log_3 n \rceil + \left\lceil \dfrac{n}{2} \right\rceil$

    Outer Loop: $max(0, n - 4) \cdot \left[\lceil \log_3 n \rceil + \left\lceil \dfrac{n}{2} \right\rceil \right]$

    Theta Expressions: $\Theta(n^2)$

c. **Inner Loop #2:** $\sum_{i=1}^{n} i = \dfrac{n(n+1)}{2}$

Inner Loop #1: $n \cdot \dfrac{n(n+1)}{2} = \dfrac{n^3 + n^2}{2}$

Outer Loop: $\dfrac{n^3 + n^2}{2} \cdot (n-4) = \dfrac{n^4 - 3n^3 + 4n^2}{2}$

Theta Expressions: $\Theta(n^4)$

Correct Solution:

Inner Loop #2: j

Inner Loop #1: $\sum_{j=1}^{n} j = \dfrac{n(n+1)}{2}$

Outer Loop: $\left\lceil \dfrac{n}{4} \right\rceil \cdot \dfrac{n(n+1)}{2}$

Theta Expressions: $\Theta(n^3)$

d. **Inner Loop:** 2^n

Outer Loop: $\sum_{i=0}^{\frac{n}{2}-1} 2^i = 2^{\frac{n}{2}-1}$

Theta Expressions: $\Theta(2^n)$

Question 2