Midterm 2 Version 1 Review

July 17, 2020

1. a) 1100100

   b) $-\sum_{i=0}^{n-1} 3^i$

   Notes:

   - Balanced Ternary
     - is a way of representing numbers
     - balanced ternary is in base 3, and has values 1,0 or -1

$$\sum_{i=0}^{n-1} d_i \cdot 3^i \text{ where } d_i \in \{0, 1, -1\} \tag{1}$$

   c) i. $\underline{f(n) \in \Omega(n)}$

      True (since $n^2 + 10n + 2 \geq cn$)

      ii. $\underline{g(n) \in \Omega(n)}$

      False (Let $c = 100, n_0 = 100$. Then $100 \log_2 n < 100n$)

      iii. $\underline{f(n) \in \mathcal{O}(g(n))}$

      False ($f(n) = n^2 + 10n + 2$ grows faster than $g(n) = 100 \log_2 n$)

      iv. $\underline{f(n) \in \Theta(g(n))}$

      True (Set $c_1 = -1, c_2 = 1, n_1 = 100$. Then $c_1 f(n) \leq g(n) \leq c_2 f(n)$)

      v. $\underline{g(n) \in \Theta(\log_3 n)}$

      True (set $c_1 = -1, c_2 = 1, n_1 = 2$. Then $c_1 g(n) \leq \log_3 n \leq c_2 g(n)$)

vi. $\underline{g(n) \in \Theta(\log_3 n)}$

False (set $c_1 = -1, c_2 = 1, n_1 = 2$. Then $c_1 g(n) \leq \log_3 n \leq c_2 g(n)$)

vii. $\underline{f(n) + g(n) \in \Theta(f(n))}$

True (set $c_1 = -2, c_2 = 2, n_1 = 1$. Then $c_1(f(n) + g(n)) \leq f(n) \leq c_2(f(n) + g(n))$)

Notes:

- $g \in \Omega(f):\ \exists c, n_o \in \mathbb{R}^+,\ \forall n \in \mathbb{N},\ n \geq n_0 \Rightarrow g(n) \geq cf(n)$, where $f, g : \mathbb{N} \to \mathbb{R}^{\geq 0}$
- $g \in \mathcal{O}(f):\ \exists c, n_o \in \mathbb{R}^+,\ \forall n \in \mathbb{N},\ n \geq n_0 \Rightarrow g(n) \leq cf(n)$, where $f, g : \mathbb{N} \to \mathbb{R}^{\geq 0}$
- $g \in \Theta(f):\ g \in \mathcal{O}(f) \wedge g \in \Omega(f)$
  or
  $g \in \Theta(f):\ \exists c_1, c_2, n_1 \in \mathbb{R}^+, \forall n \in \mathbb{N}, n \geq n_1 \Rightarrow c_1 g(n) \leq f(n) \leq c_2 g(n)$, where $f, g:\ \mathbb{N} \to \mathbb{R}^{\geq 0}$

d) $i = 3^{2^k}$

Since

k	0	1	2
i	3	9	81
	3^1	3^2	3^4

e) $k = \lceil \log_3(\log_2 n) - 1 \rceil$

Since

$$
\begin{aligned}
i^2 &\geq n && (1)\\
3^{2^k} &\geq n^{1/2} && (2)\\
2^k &\geq \log_3(n^{1/2}) && (3)\\
2^k &\geq (1/2)\log_3(n) && (4)\\
k &\geq \log_2((1/2)\log_3(n)) && (5)\\
&\geq \log_2(\log_3(n)) - 1 && (6)
\end{aligned}
$$

which gives $k = \lceil \log_2(\log_3(n)) - 1 \rceil$