CSC343 Worksheet 12 Solution

July 2, 2020

1.
- Keys
  - {id of molecule}
  - {x position, y position, z position}
- Functional Dependencies
  1. id of molecule → x position, y position, z position, x velocity, y velocity, z velocity
  2. x position, y position, z position → id of molecule, x velocity, y velocity, z velocity

Notes:

- Function Dependencies
  - *Functional Dependency* is a relationship between two attributes typically between the key and other non-key attributes within a table.

    Example:

    SIN → Name, Address, Birthdate

    Example 2:

    ISBN → Title
- Key of Relations
  - One or more attributes $\{A_1, A_2, ..., A_n\}$ is a key for a relation R if
    1. Those attributes functionally determine all other attributes of the relation
    2. No proper subset of $\{A_1, A_2, ...A_n\}$ functionally determines all other attributes of R

    Example:

    Given relation

    $R =$ Movies1(title, year, length, genre, studioName, starName)

    i. {title, year, starName } form a key for the relation **Movies1**

ii. { year, starName } is not a key. Same star can be in multiple movies per year

- Superkeys
  * Means a a set of attributes that contains a key
  * Don't need to be minimal

  Example:
  Given relation

  $R =$ Movies1(title, year, length, genre, studioName, starName)
  · { title, year, starName } is a key and superkey
  · { title, year, starName, title, year, length} is a superkey

References:

1) OpenTextBC, Chapter 11 Functional Dependencies, link

2. a) 1. $AB \to C$
   2. $AB \to D$
   3. $C \to A$
   4. $C \to B$
   5. $D \to B$
   6. $D \to C$
   7. $C \to D$
   8. $D \to A$

Second Attempt:
$\{A, B\}^+ = \{A, B, C, D\}$, so the following non-trivial FDs follows: $AB \to C$ and $AB \to D$.

$\{C\}^+ = \{D, A\}$, so the following non-trivial FDs follows $C \to D$ and $C \to A$.

$\{D\}^+ = \{A\}$, so the following non-trivial FDs follows: $D \to A$.

Notes:

- The Splitting / Combining Rule
  - Combining Rule
    * $A_1, A_2, \cdots, A_n \to B_i$ for $i = 1, 2, ..., m$
      to
      $A_1, A_2, \cdots A_n \to B_1, B_2, \cdots B_m$

Example:

Given

title year → length
title year → genre
title year → studioName

it's combined form is

title year → length genre studioName

- Splitting Rule
  *
  * $A_1, A_2, \cdots A_n \rightarrow B_1, B_2, \cdots B_m$
    to
    $A_1, A_2, \cdots, A_n \rightarrow B_i$ for $i = 1, 2, ..., m$

Example:

Given

title year → length

It's splitted form is

title → length
year → length

- Trivial Functional Dependencies
  - A functional dependency $FD : X \rightarrow Y$ is **trivial** if Y is a subset of X

Exmaple:

title year → title

Example 2:

title → title

- Non-trivial Functional Dependencies
  - is a case where some but not all of the attributes on the R.H.S of an FD are also on L.H.S

Example:

title year → title movieLength

- Can be simplified using **tirivial-dependency rule**
  * The FD $A_1A_2\cdots A_n \rightarrow B_1B_2\cdots B_m$ is equivalent to $A_1A_2\cdots A_n \rightarrow C_1C_2\cdots C_k$

    where C's are all those B's that are not in A's.

the non-overlapping attribute

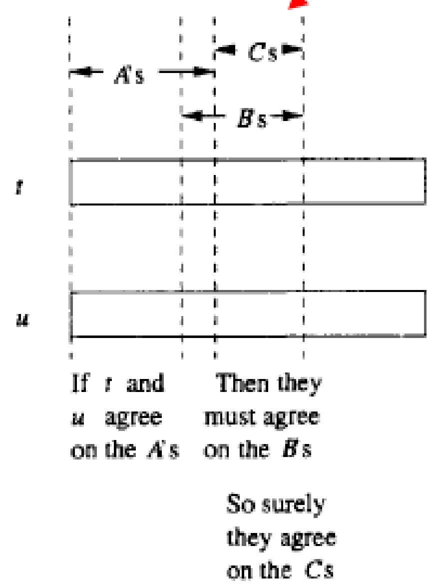


Figure 3.3: The trivial-dependency rule

- Computing the Clousre of Attributes
  - Closure of attribute set $\{X\}$ is denoted as $\{X\}^+$.
  - The closure means a given set of attributes A satisfying FD, are a sets of all attributes B such that $A \rightarrow B$

    Example:

    Given attributes A, B, C, D, E, F and FDs $AB \rightarrow C$, $BC \rightarrow AD$, $D \rightarrow E$ and $CF \rightarrow B$, What is the closure of $\{A, B\}$ or $\{A, B\}^+$

    1. Start with $\{A, B\}$.
    2. Split $BC \rightarrow AD$
       * We have $BC \rightarrow A$ and $BCtoD$
       * Since A is in $\{A, B\}$, this is not included
       * Since D is not in $\{A, B\}$, this IS included

       So, we have $\{A, B, D\}$

3. Since C in $AB \rightarrow C$ is NOT in $\{A, B, C, D\}$, C is included and we have $\{A, B, C, D\}$
4. Since A in $BC \rightarrow A$ is in $\{A, B, C, D\}$, this is skipped
5. Since E is not in $D \rightarrow E$, E is included and we have $\{A, B, C, D, E\}$ as our solution

- Why the Closure Algorithm Works
- Transitive Rule
  - Definition

    If $A_1A_2 \cdots A_n \rightarrow B_1B_2 \cdots B_m$ and $B_1B_2 \cdots B_m \rightarrow C_1C_2 \cdots C_k$ hold in relation R, $A_1A_2 \cdots A_n \rightarrow C_1C_2 \cdots C_k$ also holds in R.

    Example:

    Given

    title year $\rightarrow$ studioName
    studioName $\rightarrow$ studioAddr

    Transitive rule says the above is equal to the following

    title year $\rightarrow$ studioAddr
- Inference Rules
  - Is allso called **Armstrong's Axioms**
  - Has 3 axioms
    1. *Reflexivity*
       * If $\{B_1, B_2, ..., B_n\} \subseteq \{A_1, A_2, ..., A_n\}$ then $A_1A_2 \cdots A_n \rightarrow B_1B_2 \cdots B_m$
       * also called **trivial FDs**
    2. *Augmentation*
       * If $A_1A_2 \cdots A_n \rightarrow B_1B_2 \cdots B_m$ then $A_1A_2 \cdots A_nC_1C_2 \cdots C_k \rightarrow B_1B_2 \cdots B_mC_1C_2 \cdots C_k$
       * $C_1C_2 \cdots C_k$ are any set of attributes
    3. *Transitivity*
       * If $A_1A_2 \cdots A_n \rightarrow B_1B_2 \cdots B_m$ and $B_1B_2 \cdots B_m \rightarrow C_1C_2 \cdots C_k$ then $A_1A - 2 \cdots A_n \rightarrow C_1C_2 \cdots C_k$

b) A, B is the only key of R.

Notes:

- Key of Attributes
  - **Definition:** A set of attributes $\{A_1, A_2, \cdots, A_n\}$ is a key for a relation R if
    1. Those attributes functionally determine all other attributes

2. No proper subset of $\{A_1, A_2, ..., A_n\}$ functionally determines all other attributes of R.

c) The superkeys that are not keys are: $\{A, B, C\}$, $\{A, B, D\}$, $\{A, B, C, D\}$

3. i) a) $\{A\}^+ = \{A, B, C, D\}$, so we have $A \to A$, $A \to B$, $A \to C$, $A \to D$

$\{B\}^+ = \{C, D\}$, so we have $B \to C$ and $B \to D$

b) $\{A\}$ is the key of S.

c) The super keys that are not keys are:

$\{A, B\}$, $\{A, C\}$, $\{A, D\}$, $\{A, B, C\}$, $\{A, B, D\}$, $\{A, B, C, D\}$

ii) a) $\{A\}^+ = \{A\}$, so this FD is trivial.

$\{B\}^+ = \{B\}$, so this FD is trivial.

$\{C\}^+ = \{C\}$, so this FD is trivial.

$\{D\}^+ = \{D\}$, so this FD is trivial.

$\{A, B\}^+ = \{A, B, C, D\}$, so we have $AB \to A$, $AB \to B$, $AB \to C$, $AB \to D$

$\{A, C\}^+ = \{A, C\}$, so we have $AC \to A$, $AC \to C$

$\{A, D\}^+ = \{A, D, B\}$, so we have $AD \to A$, $AD \to D$, $AD \to B$

$\{B, C\}^+ = \{B, C, D, A\}$, so we have $BC \to A$, $BC \to B$, $BC \to C$, $BC \to D$

$\{D, C\}^+ = \{D, C, A, B\}$, so we have $DC \to D$, $DC \to C$, $DC \to A$, $DC \to B$

$\{A, B, C\}^+ = \{A, B, C, D\}$, so we have $ABC \to A$, $ABC \to B$, $ABC \to C$, $ABC \to D$

$\{B, C, D\}^+ = \{B, C, D, A\}$, so we have $BCD \to A$, $BCD \to B$, $BCD \to C$, $BCD \to D$

$\{C, D, A\}^+ = \{C, D, A, B\}$, so we have $CDA \to A$, $CDA \to B$, $CDA \to C$, $CDA \to D$

$\{D, A, B\}^+ = \{D, A, B, C\}$, so we have $DAB \to A$, $DAB \to B$, $DAB \to C$, $DAB \to D$

$\{D, A, B\}^+ = \{D, A, B, C\}$, so we have $DAB \to A$, $DAB \to B$, $DAB \to C$, $DAB \to D$

$\{A, B, C, D\}^+ = \{A, B, C, D\}$, so this FD is trivial.

b) $\{A, B\}$, $\{A, C\}$, $\{B, C\}$, $\{D, C\}$ are the keys of T.

c) The super keys that are not keys are:

$\{A, B, C\}$, $\{A, B, D\}$, $\{B, C, D\}$, $\{A, D, C\}$, $\{A, B, D\}$, $\{A, B, C, D\}$

iii) a) $\{A\}^+ = \{A, B, C, D\}$, so we have $A \to C$, $A \to D$

$\{B\}^+ = \{A, B, C, D\}$, so we have $B \to A$, $B \to D$

$\{C\}^+ = \{A, B, C, D\}$, so we have $C \to A$, $C \to B$

$\{D\}^+ = \{A, B, C, D\}$, so we have $D \to B$, $D \to C$

$\{A, B\}^+ = \{A, B, C, D\}$, so we have $AB \to C$, $AB \to D$

$\{B, C\}^+ = \{A, B, C, D\}$, so we have $BC \to A$, $BC \to D$

$\{B, D\}^+ = \{A, B, C, D\}$, so we have $BD \to A$, $BD \to C$

$\{C, D\}^+ = \{A, B, C, D\}$, so we have $CD \to A$, $CD \to B$

$\{C, D\}^+ = \{A, B, C, D\}$, so we have $CD \to A$, $CD \to B$

$\{A, B, C\}^+ = \{A, B, C, D\}$, so we have $ABC \to D$

$\{B, C, D\}^+ = \{A, B, C, D\}$, so we have $BCD \to A$

$\{C, D, A\}^+ = \{A, B, C, D\}$, so we have $CDA \to B$

$\{D, A, B\}^+ = \{A, B, C, D\}$, so we have $DAB \to C$

Correct Solution:

$\{A\}^+ = \{A, B, C, D\}$, so we have $A \to C$, $A \to D$

$\{B\}^+ = \{A, B, C, D\}$, so we have $B \to A$, $B \to D$

$\{C\}^+ = \{A, B, C, D\}$, so we have $C \to A$, $C \to B$

$\{D\}^+ = \{A, B, C, D\}$, so we have $D \to B$, $D \to C$

$\{A, B\}^+ = \{A, B, C, D\}$, so we have $AB \to C$, $AB \to D$

$\{A, C\}^+ = \{A, B, C, D\}$, so we have $AC \to B$, $AC \to D$

$\{A, D\}^+ = \{A, B, C, D\}$, so we have $AD \to B$, $AD \to C$

$\{B, C\}^+ = \{A, B, C, D\}$, so we have $BC \to A$, $BC \to D$

$\{B, D\}^+ = \{A, B, C, D\}$, so we have $BD \to A$, $BD \to C$

$\{C, D\}^+ = \{A, B, C, D\}$, so we have $CD \to A$, $CD \to B$

$\{A, B, C\}^+ = \{A, B, C, D\}$, so we have $ABC \to D$

$\{B, C, D\}^+ = \{A, B, C, D\}$, so we have $BCD \to A$

$\{C, D, A\}^+ = \{A, B, C, D\}$, so we have $CDA \to B$

$\{D, A, B\}^+ = \{A, B, C, D\}$, so we have $DAB \to C$

b) $\{A\}, \{B\}, \{C\}, \{D\}$ are the keys of U.

c) The super keys that are not keys are:

$\{A, B\}$,$\{A, C\}$,$\{A, D\}$,$\{B, C\}$, $\{B, D\}$,$\{C, D\}$, $\{A, B, C\}$, $\{B, C, D\}$, $\{C, D, A\}$, $\{D, A, B\}$. $\{A, B, C, D\}$

4. a) We need to show the closure of attributes $\{A_1, A_2, \cdots, A_n, C\}$ in FD $A_1, A_2, \cdots, A_n, C \to B$ is $\{A_1, A_2, \cdots, A_n, C, B\}$, that is $\{A_1, A_2, \cdots, A_n, C\}^+ = \{A_1, A_2, \cdots, A_n, C, B\}$

Since we know $\{A_1, A_2, \cdots, A_n\}$ functionally determines B, we can conclude B can be added to $\{A_1, A_2, \cdots, A_n, C\}$.

Thus, it follows from above that $\{A_1, A_2, \cdots, A_n, C\}^+ = \{A_1, A_2, \cdots, A_n, C, B\}$.

b) Let $A_1A_2 \cdots A_n \to B$ is FD. That is, $\{A_1A_2 \cdots A_n\}^+ = \{A_1A_2 \cdots A_n, B\}$

We need to show $A_1A_2 \cdots A_nC \to BC$ follows. That is, $\{A_1, A_2, \cdots, A_n, C\}^+ = \{A_1, A_2, \cdots, A_n, C, B\}$

It follows from the combine and split rule that $A_1A_2 \cdots A_nC \to BC$ can be spliited into $A_1A_2 \cdots A_nC \to B$ and $A_1A_2 \cdots, A_nC \to C$.

So, we need to show $A_1A_2 \cdots A_nC \to B$ and $A_1A_2 \cdots, A_nC \to C$ follows from the given.

We will do so in parts.

1. **Part 1 (Showing $A_1A_2 \cdots A_nC \to B$):**

Here, we need to show $A_1A_2 \cdots A_nC \to B$ follows.

And indeed, this follows from the work of *augmenting left sides*. That is the solution to previous problem.

2. **Part 2 (Showing $A_1A_2 \cdots A_nC \to C$):**

   Here, we need to show $A_1A_2 \cdots A_nC \to C$ follows.

   The definition of trivial FD tells us $A_1A_2 \cdots A_n \to B_1B_2 \cdots B_m$ holds when $\{B_1, B_2, \cdots, B_m\} \subseteq \{A_1, A_2, \cdots, A_n\}$

   Since $\{C\} \subseteq \{A_1, A_2 \cdots, A_n, C\}$, we can conclude this FD follows trivially.

c) Let $A_1A_2 \cdots A_n \to B_1B_2 \cdots B_m$ and $C_1C_2 \cdots C_k \to D$, where B are each among the C's.

We neeed to show $A_1A_2 \cdots A_nE_1E_2 \cdots E_j \to D$ follows, where the E's are all of those C's not found among the B's.

The transitive rule tells us if $A_1A_2 \cdots A_n \to B_1B_2 \cdots B_m$ and $B_1B_2 \cdots B_m \to C_1C_2 \cdots C_k$, then $A_1A_2 \cdots A_n \to C_1C_2 \cdots C_k$ also holds in R.

Since we know $A_1A_2 \cdots A_n \to B_1B_2 \cdots B_m$ and $C_1C_2 \cdots C_k \to D$ where B's are each among the C's, we can conclude from the transitive rule that $A_1A_2 \cdots A_n \to D$.

Then using **augmenting left sides** to all C's not found among the B's on $A_1A_2 \cdots A_n \to D$, we can conclude $A_1A_2 \cdots A_nE_1E_2 \cdots E_j \to D$ follows.

d) Assume FD's $A_1A_2 \cdots A_n \to B_1B_2 \cdots B_m$ and $C_1C_2 \ldots C_k \to D_1D_2 \cdots D_j$ holds.

We need to show FD $A_1A_2 \cdots A_nC_1C_2 \cdots C_k \to B_1B_2 \cdots B_mD_1D_2 \cdots D_k$ follows.

Using the split / combine rule, we can conclude showing $A_1A_2 \cdots A_nC_1C_2 \cdots C_k \to B_1B_2 \cdots B_mD_1D_2 \cdots D_k$ is the same as showing $A_1A_2 \cdots A_nC_1C_2 \cdots C_k \to B_1B_2 \cdots B_m$ and $A_1A_2 \cdots A_nC_1C_2 \cdots C_k \to D_1D_2 \cdots D_k$

So, we will prove the two, in parts

1. **Part 1 (Showing $A_1A_2 \cdots A_nC_1C_2 \cdots C_k \to B_1B_2 \cdots B_m$)**

   Here, we need to show $A_1A_2 \cdots A_nC_1C_2 \cdots C_k \to B_1B_2 \cdots B_m$.

   The header of problem tells us $A_1A_2 \cdots A_n \to B_1B_2 \cdots B_m$ holds.

   Then by using **Augmenting Left Sides** rule to all Cs not found among the As, $A_1A_2 \cdots A_nC_1C_2 \cdots C_k \to B_1B_2 \cdots B_m$ follows.

2. **Part 2 (Showing $A_1A_2 \cdots A_nC_1C_2 \cdots C_k \to D_1D_2 \cdots D_k$ follows)**

   Here, we need to show $A_1A_2 \cdots A_nC_1C_2 \cdots C_k \to D_1D_2 \cdots D_k$.

   The header of problem tells us $C_1C_2 \cdots C_k \to D_1D_2 \cdots D_k$ holds.

   Then by using **Augmenting Left Sides** rule to all As not found among the Cs, $A_1A_2 \cdots A_nC_1C_2 \cdots C_k \to D_1D_2 \cdots D_k$ follows.

5. a) An example is

   A being movieID and
   B being movie length.

   b) An example is

   A being movieID
   B being movieTitle
   C being movieLength

   c) An example is

   A being movieTitle
   B being year
   C being length

6. Assume a relation has no attribute that is functionallty determined by all the other attributes.

   And, assume for the sake of contradiction that there is a relation with non-trivial FD $X \to Y$.

   Then, it follows from the definition of non-trivial functional dependency that $Y \neq\subseteq X$.

   Then, we can conclude the attributes in Y is functionally determined by other attributes in X.

   But this contradicts the assumption that no attribute is functionally determined by all other attributes.

7. Let X and Y be sets of attributes. Assume $X \subseteq Y$.

   I need to show $X^+ \subseteq Y^+$.

   I will do so in cases

   1. **Case 1 ($X = Y$):**

      Assume $X = Y$.

      I need to show $X^+ \subseteq Y^+$ follows.

      The header tells us $X = Y$.

      Using this fact, $X^+ = Y^+$ is true.

      Then it follows from above that $X^+ \subseteq Y^+$ is also true.

2. **Case 2** ($X \subset Y$)

   Assume $X \subset Y$.

   I need to show $X^+ \subseteq Y^+$ follows.

   Since the attributes in X is in Y, we can conclude the attributes in X^+ is also in Y^+.

   And, since Y has attributes not in X, we can conclude Y^+ may contain attributes not in X^+.

   Thus, we can conclude $X^+ \subseteq Y^+$.

8. 1. Only one solution will be included for now :)
   The following

   1. $A \to C$
   2. $B \to A$
   3. $B \to C$
   4. $C \to A$
   5. $C \to B$
   6. $AB \to C$
   7. $AC \to B$
   8. $BC \to A$
   9. $A \to BC$
   10. $A \to A$

   can be simplified to

   1. $A \to C$
   2. $B \to A$
   3. $B \to C$
   4. $C \to A$
   5. $C \to B$
   6. $A \to C$ B removed from here!!
   7. $AC \to B$
   8. $BC \to A$
   9. $A \to BC$
   10. $A \to A$

   since **augmenting left sides** rule tells us $AB \to C$ can be attained by adding B to L.H.S of $A \to C$.

   Then, the following

1. $A \rightarrow C$
2. $B \rightarrow A$
3. $B \rightarrow C$
4. $C \rightarrow A$
5. $C \rightarrow B$
6. $A \rightarrow C$
7. $AC \rightarrow B$
8. $BC \rightarrow A$
9. $A \rightarrow BC$
10. $A \rightarrow A$

can be simplified to

1. $A \rightarrow C$
2. $B \rightarrow A$
3. $B \rightarrow C$
4. $C \rightarrow A$
5. $C \rightarrow B$
6. $AC \rightarrow B$
7. $BC \rightarrow A$
8. $A \rightarrow BC$
9. $A \rightarrow A$

by removing redundant $A \rightarrow C$.

Then, the following

1. $A \rightarrow C$
2. $B \rightarrow A$
3. $B \rightarrow C$
4. $C \rightarrow A$
5. $C \rightarrow B$
6. $AC \rightarrow B$
7. $BC \rightarrow A$
8. $A \rightarrow BC$
9. $A \rightarrow A$

can be simplified to

1. $A \rightarrow C$
2. $B \rightarrow A$
3. $B \rightarrow C$
4. $C \rightarrow A$
5. $C \rightarrow B$

6. $AC \to B$
7. $BC \to A$
8. $A \to B$ Splitted from $A \to BC$
9. $A \to C$ Splitted from $A \to BC$
10. $A \to A$

by using **splitting rule** on $A \to BC$.

Then, the following

1. $A \to C$
2. $B \to A$
3. $B \to C$
4. $C \to A$
5. $C \to B$
6. $AC \to B$
7. $BC \to A$
8. $A \to B$
9. $A \to C$
10. $A \to A$

can be simplified to

1. $A \to C$
2. $B \to A$
3. $B \to C$
4. $C \to A$
5. $C \to B$
6. $AC \to B$
7. $BC \to A$
8. $A \to B$
9. $A \to A$

by removing redundant $A \to C$.

Then, the following

1. $A \to C$
2. $B \to A$
3. $B \to C$
4. $C \to A$
5. $C \to B$
6. $AC \to B$
7. $BC \to A$

8. $A \to B$
9. $A \to A$

can be simplified to

1. $A \to C$
2. $B \to A$
3. $B \to C$
4. $C \to A$
5. $C \to B$
6. $A \to B$ C removed here!!
7. $BC \to A$
8. $A \to B$
9. $A \to A$

since **augmenting left sides** tells us $AC \to B$ can be attained by adding C to $A \to B$.

Then, the following

1. $A \to C$
2. $B \to A$
3. $B \to C$
4. $C \to A$
5. $C \to B$
6. $A \to B$
7. $BC \to A$
8. $A \to B$
9. $A \to A$

can be simplified to

1. $A \to C$
2. $B \to A$
3. $B \to C$
4. $C \to A$
5. $C \to B$
6. $A \to B$
7. $BC \to A$
8. $A \to A$

by removing redundant $A \to B$.

Then, the following

1. $A \to C$

2. $B \to A$
3. $B \to C$
4. $C \to A$
5. $C \to B$
6. $A \to B$
7. $BC \to A$
8. $A \to A$

can be simplified to

1. $A \to C$
2. $B \to A$
3. $B \to C$
4. $C \to A$
5. $C \to B$
6. $A \to B$
7. $BC \to A$

since $A \to A$ can be attained by using **transitivity** rule on $A \to C$ and $C \to A$.

Then, the following

1. $A \to C$
2. $B \to A$
3. $B \to C$
4. $C \to A$
5. $C \to B$
6. $A \to B$
7. $BC \to A$

can be simplified to

1. $A \to C$
2. $B \to A$
3. $B \to C$
4. $C \to A$
5. $C \to B$
6. $A \to B$
7. $B \to A$ C removed here!!

since **augmenting let sides** rule tells us $BC \to A$ can be attained by adding C to L.H.S of $B \to A$.

Then, the following

1. $A \to C$

2. $B \to A$
3. $B \to C$
4. $C \to A$
5. $C \to B$
6. $A \to B$
7. $B \to A$

can be simplified to

1. $A \to C$
2. $B \to A$
3. $B \to C$
4. $C \to A$
5. $C \to B$
6. $A \to B$

by removing redundant $B \to A$.

Then, the following

1. $A \to C$
2. $B \to A$
3. $B \to C$
4. $C \to A$
5. $C \to B$
6. $A \to B$

can be simplified to

1. $A \to C$
2. $B \to A$
3. $C \to A$
4. $C \to B$
5. $A \to B$

since **transitivity** rule tells us $B \to C$ can be attained by using $B \to A$ and $A \to C$.

Then, the following

1. $A \to C$
2. $B \to A$
3. $C \to A$
4. $C \to B$
5. $A \to B$

can be simplified to

1. $A \to C$
2. $B \to A$
3. $C \to A$
4. $C \to B$

since **transitivity** rule tells us $A \to B$ can be attained by using $A \to C$ and $C \to B$.

Rough Works:

1. Add attributes from A^+ to L.H.S of $A_1A_2 \cdots A_n \to A^+$.
2. Show that the R.H.S is still A^+.

Notes:

- Closure (Definition)
  - Suppose $A = \{A_1, A_2, ..., A_n\}$ is a set of attributes of R and S is a set of FD'.

    The closure of A under the set S, denoted by A^+, is the set of attributes B such that any relation that satisfies all the FD's in S is also satisfies $A_1A_2 \cdots A_n \to A^+$.
  - In other words $A_1 \cdots A_n \to A^+$ follows from the FD's of S.
- I wish the definition is a little more clear :(

9. **Notes:**

- Basis
  - Is the set of FD's that represent the full set of FD's of a relation
- Finding minal bases for FD's
  - A minimal basis for a relation satisfies three conditions
    1. All the FD's in B have singleton right sides.
    2. If any FD is removed from B, the result is no longer a basis
    3. If for any FD in B we remove on or more attributes from the left side of F, the result is no longer a basis
  - Steps
    1. Get rid of redundant attributes

       *
    2. Get rid of redundant dependencies
- Example

  The following

1. $A \to B$
2. $ABCD \to E$
3. $EF \to G$
4. $EF \to H$
5. $ACDF \to E$
6. $ACDF \to G$

can be simplified to

1. $A \to B$
2. $ACD \to E$ B removed here!!
3. $EF \to G$
4. $EF \to H$
5. $ACDF \to E$
6. $ACDF \to G$

since by **augmentation rule**, $A \to B$ can be re-written as $ACD \to BCD$. And by **trivial rule**, $ACD \to BCD$ can be re-written as $ACB \to ABCD$, which then can be used to get E from $ABCD \to E$.

Second, the following

1. $A \to B$
2. $ACD \to E$
3. $EF \to G$
4. $EF \to H$
5. $ACDF \to E$
6. $ACDF \to G$

can be simplified to

1. $A \to B$
2. $ACD \to E$
3. $EF \to G$
4. $EF \to H$
5. $ACD \to E$ F Removed here!!
6. $ACDF \to G$

since **augmenting left side** rule tells us $ACDF \to E$ can be attained by adding F to ACD in $ACD \to E$.

Then, the following

1. $A \to B$
2. $ACD \to E$
3. $EF \to G$

4. $EF \rightarrow H$
5. $ACD \rightarrow E$
6. $ACDF \rightarrow G$

can be simplified to

1. $A \rightarrow B$
2. $ACD \rightarrow E$
3. $EF \rightarrow G$
4. $EF \rightarrow H$
5. $ACDF \rightarrow G$

by removing redundant $ACD \rightarrow E$.

Then, the following

1. $A \rightarrow B$
2. $ACD \rightarrow E$
3. $EF \rightarrow G$
4. $EF \rightarrow H$
5. $ACD \rightarrow E$
6. $ACDF \rightarrow G$

can be simplified to

1. $A \rightarrow B$
2. $ACD \rightarrow E$
3. $EF \rightarrow G$
4. $EF \rightarrow H$

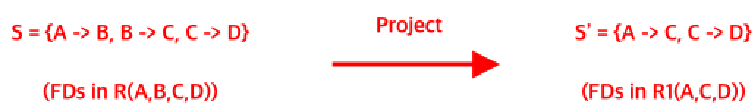
since **augmentation** rule tells us $ACDF \rightarrow G$ can be re-written to get $ACDF \rightarrow EF$ and then use **transtivity rule** on $EF \rightarrow G$ to get $ACDF \rightarrow G$.

10. a)
- Finding subsets
  $X_1 = \{A\}$, $X_2 = \{B\}$, $X_3 = \{C\}$, $X_4 = \{A, B\}$, $X_5 = \{A, C\}$, $X_6 = \{B, C\}$, $X_7 = \{A, B, C\}$, $X_8 = \{\}$
- Finding X_i^+
  1. $X_1^+ = \{A\}$
  2. $X_2^+ = \{B\}$
  3. $X_3^+ = \{C, E, A\}$
  4. $X_4^+ = \{A, B, C, D, E\}$
  5. $X_5^+ = \{A, C, E\}$
  6. $X_6^+ = \{A, B, C, D, E\}$
  7. $X_7^+ = \{A, B, C, D, E\}$
  8. $X_8^+ = \{\}$

- Putting all nontirival FD's in T
  $T = \{C \to E, C \to A, AB \to C, AB \to D, AB \to E, AC \to E, BC \to A, BC \to D, BC \to E, ABC \to D, ABC \to E\}$
- Finding minimal basis for the FD of S
  $T_{\text{minimal}} = \{C \to E, C \to A, B \to C, B \to D, B \to E, B \to A\}$

Notes:

- Projecting Functional Dependency
  - Remember that π is equivalent to SQL's SELECT of columns
  - Answers the question to "given a relation R and a set of FD's S, what FD's hold if we project R by $R_1 = \Pi_L(R)$?
  - The new set S'
    1. Follows from S
    2. Involves only attributes of R_1



- Algorithm for Projecting a set of Functional Dependencies
  - Inputs and Outputs
    * Input
      · **R:** The original relation
      · **R1:** The projection of R
      · **S:** The set of FD's that hold in R
    * Output
      · **T:** The set of FD's that hold in R_1
  - Steps
    1. Initialize $T = \{\}$.
    2. Construct a set of all subsets of attributes of R_1 called X
    3. Compute X_i^+ for all members of X under S.
       * X_i^+ may consist of attributes that are not in $R1$
    4. Add to T all nontirival FD's $X \to A$ such that A is both in X_i^+ and an attributes of R_1
    5. Now, T is a basis for the FD's that hold in $R1$ but may not be a minimal basis. Modify T as follows.
       a) If there is an FD in F in T that follows from the other FD's in T, remove F
       b) Let $Y \to B$ be an FD in T, with at least two attributes in Y. Remove one attribute from Y and call it Z. If $Z \to B$ follows from the FD's in T, then replace $Z \to B$ with $Y \to B$.

– Example

Consider $R(A, B, C, D)$ has FD's$A \rightarrow B$, $B \rightarrow C$, and $C \rightarrow D$.
$R_1(A, C, D)$ is a projection of R. Find FD's for R_1

1. Initialize $T = \{\}$.
   * $T = \{\}$
2. Construct a set of all subsets of attributes of R_1 called X
   * There are 8 subsets

     $X_1 = \{A\}$, $X_2 = \{C\}$, $X_3 = \{D\}$, $X_4 = \{A, C\}$, $X_5 = \{A, D\}$, $X_6 = \{C, D\}$, $X_7 = \{A, C, D\}$, $X_8 = \{\}$
3. Compute X_i^+ for all members of X under S.
   * $X_1 = \{A\}$

     $X_1^+ = \{A, B, C, D\}$
   * $X_2 = \{C\}$

     $X_2^+ = \{C, D\}$
   * $X_3 = \{D\}$

     $X_3^+ = \{D\}$
   * $X_4 = \{A, C\}$

     $X_4^+ = \{A, B, C, D\}$
   * $X_5 = \{A, D\}$

     $X_5^+ = \{A, B, C, D\}$
   * $X_6 = \{C, D\}$

     $X_6^+ = \{C, D\}$
   * $X_7 = \{A, C, D\}$

     $X_7^+ = \{A, B, C, D\}$
   * $X_8 = \{\}$

     $X_8^+ = \{\}$
4. Add to T all nontirival FD's $X \rightarrow A$ such that A is both in X_i^+ and an attributes of R_1
   * $T = \{A \rightarrow C, A \rightarrow D, C \rightarrow D, AC \rightarrow D, AD \rightarrow C\}$
5. Now, T is a basis for the FD's that hold in $R1$ but may not be a minimal basis. Modify T as follows.
   * $T = \{C \rightarrow D, A \rightarrow D, A \rightarrow C\}$

b)
- Finding subsets
  $X_1 = \{A\}$, $X_2 = \{B\}$, $X_3 = \{C\}$, $X_4 = \{A, B\}$, $X_5 = \{A, C\}$, $X_6 = \{B, C\}$, $X_7 = \{A, B, C\}$, $X_8 = \{\}$
- Finding X_i^+
  1. $X_1^+ = \{A, D\}$
  2. $X_2^+ = \{B\}$
  3. $X_3^+ = \{C\}$
  4. $X_4^+ = \{A, B, D, E\}$
  5. $X_5^+ = \{A, B, C, D, E\}$
  6. $X_6^+ = \{B, C\}$
  7. $X_7^+ = \{A, B, C, D, E\}$
  8. $X_8^+ = \{\}$
- Putting all nontirival FD's in T
  $T = \{A \to D, AB \to D, AB \to E, AC \to B, AC \to D, AC \to E, ABC \to D, ABC \to E\}$
- Finding minimal basis for the FD of S
  $T_{\text{minimal}} = \{A \to D, AB \to E, AC \to B, AC \to E\}$

c)
- Finding subsets
  $X_1 = \{A\}$, $X_2 = \{B\}$, $X_3 = \{C\}$, $X_4 = \{A, B\}$, $X_5 = \{A, C\}$, $X_6 = \{B, C\}$, $X_7 = \{A, B, C\}$, $X_8 = \{\}$
- Finding X_i^+
  1. $X_1^+ = \{A\}$
  2. $X_2^+ = \{B\}$
  3. $X_3^+ = \{C\}$
  4. $X_4^+ = \{A, B, D\}$
  5. $X_5^+ = \{A, B, C, D, E\}$
  6. $X_6^+ = \{A, B, C, D, E\}$
  7. $X_7^+ = \{A, B, C, D, E\}$
  8. $X_8^+ = \{\}$
- Putting all nontirival FD's in T
  $T = \{AB \to D, AC \to B, AC \to D, AC \to E, BC \to A, BC \to D, BC \to E, ABC \to D, ABC \to E\}$
- Finding minimal basis for the FD of S
  $T_{\text{minimal}} = \{A \to D, AC \to B, C \to A, C \to B, C \to E\}$

d)
- Finding subsets
  $X_1 = \{A\}$, $X_2 = \{B\}$, $X_3 = \{C\}$, $X_4 = \{A, B\}$, $X_5 = \{A, C\}$, $X_6 = \{B, C\}$, $X_7 = \{A, B, C\}$, $X_8 = \{\}$
- Finding X_i^+
  1. $X_1^+ = \{A, B, C, D, E\}$
  2. $X_2^+ = \{A, B, C, D, E\}$

3. $X_3^+ = \{A, B, C, D, E\}$
4. $X_4^+ = \{A, B, C, D, E\}$
5. $X_5^+ = \{A, B, C, D, E\}$
6. $X_6^+ = \{A, B, C, D, E\}$
7. $X_7^+ = \{A, B, C, D, E\}$
8. $X_8^+ = \{\}$

- Putting all nontirival FD's in T
  $T = \{A \to B, A \to C, A \to D, A \to E, B \to A, B \to C, B \to D, B \to E, C \to A, C \to B, C \to D, C \to E, AB \to C, AB \to D, AB \to E, AC \to B, AC \to D, AC \to E, BC \to A, BC \to D, BC \to E, ABC \to D, ABC \to E\}$
- Finding minimal basis for the FD of S
  $T_{\text{minimal}} = \{A \to B, A \to C, A \to D, A \to E, B \to A, B \to C, B \to D, B \to E, C \to A, C \to B, C \to D, C \to E\}$

e) a)

Notes:

- Anomalies
  - means "Something that you don't expect" in layman's terms
  - Three main types of anomalies exist
    * *Redundancy* - Information may be prepeated unnecessarily in several tuples

Redundancy Anomaly

title	*year*	*length*	*genre*	*studioName*	*starName*
Star Wars	1977	124	SciFi	Fox	Carrie Fisher
Star Wars	1977	124	SciFi	Fox	Mark Hamill
Star Wars	1977	124	SciFi	Fox	Harrison Ford
Gone With the Wind	1939	231	drama	MGM	Vivien Leigh
Wayne's World	1992	95	comedy	Paramount	Dana Carvey
Wayne's World	1992	95	comedy	Paramount	Mike Meyers

    * *Update Anomalies* - information changed in one tuple, but the same information is not changed in the other
    * *Deletion Anomalies* - Deletion of one tuple causing undesired deletion of other information

      e.g. deleting movie *Gone with the wind* resulting in loss of information about studio *Fox*
- Decomposing Relations
  - *Decompose Relations*
    * is an accepted way to eliminate anomalies
    * involves splitting the attributes of R to make the schemas of two new relations.
  - The how of decomposing anomalies

      * Given a relation $R(A_1, A_2, ..., A_n)$ we may decompose R into two relations $S(B_1, B_2, \cdots, B_m)$ and $T(C_1, C_2, \cdots, C_k)$

        1. $\{A_1, A_2, \cdots, A_n\} = \{B_1, B_2, \cdots, B_m\} \cup \{C_1, C_2, \cdots, C_k\}$
        2. $S = \pi_{B_1, B_2, \cdots, B_m}(R)$
        3. $T = \pi_{C_1, C_2, \cdots, C_k}(R)$

- Boyce-Codd Normal Form
  - is a simple condition under which the anormalies can be guarenteed NOT to exist

      * A relation R is in BCNF if and only if: whenever there is a non trivial FD $A_1 A_2 A_n \rightarrow B_1 B_2 \cdots B_m$ for R, it is the case that $\{A_1, A_2, \cdots A_n\}$ is a superkey for R.