# Worksheet 11 Solution

March 21, 2020

## Question 1

a. $\forall a, b \in \mathbb{R}^+,\ a \leq b \Rightarrow \exists c, n_0 \in \mathbb{R}^+,\ \forall n \in \mathbb{N},\ n \geq n_0 \Rightarrow n^a \leq cn^b$

b. Let $a, b \in \mathbb{R}^+$, $n \in \mathbb{N}$, $c = 1$, $n_0 = 1$. Assume $a \leq b$, and $n \geq n_0$.

Then,

$$
\begin{aligned}
n^a &\leq [n^a]^k & (1)\\
&\leq n^{ak} & (2)\\
&\leq n^b & (3)
\end{aligned}
$$

by the fact that $k = \frac{b}{a}$, and $k \in \mathbb{R}^+$.

Then,

$$
\begin{aligned}
n^a &\leq n^b & (4)\\
&\leq cn^b & (5)
\end{aligned}
$$

Then, it follows from above that the statement $\forall a, b \in \mathbb{R}^+,\ a \leq b \Rightarrow n^a \in \mathcal{O}(n^b)$ is true.

# Question 2

a. Let $c = \frac{1}{log_b a}$, $n_0 = 1$, $a \in \mathbb{R}^+$, $b \in \mathbb{R}^+$. Assume $a > 1$ and $b > 1$. We want to show that $\log_a n \leq c \log_b n$.

Then,

$$c \log_b n = \frac{1}{\log_b a} log_b n \tag{1}$$

$$= log_a n \tag{2}$$

by change of base rule for logarithms.

Then it follows from the definition of Big-Oh that $\log_a n \in \mathcal{O}(\log_b n)$

# Question 3