

# CSC343 Worksheet 12 Solution

July 1, 2020

1.
  - Keys
    - {id of molecule}
    - {x position, y position, z position}
  - Functional Dependencies
    - 1. id of molecule  $\rightarrow$  x position, y position, z position, x velocity, y velocity, z velocity
    - 2. x position, y position, z position  $\rightarrow$  id of molecule, x velocity, y velocity, z velocity

## Notes:

- Function Dependencies
  - *Functional Dependency* is a relationship between two attributes typically between the key and other non-key attributes within a table.

### Example:

SIN  $\rightarrow$  Name, Address, Birthdate

### Example 2:

ISBN  $\rightarrow$  Title

- Key of Relations
  - One or more attributes  $\{A_1, A_2, \dots, A_n\}$  is a key for a relation R if
    1. Those attributes functionally determine all other attributes of the relation
    2. No proper subset of  $\{A_1, A_2, \dots, A_n\}$  functionally determines all other attributes of R

### Example:

Given relation

$R = \text{Movies1}(\text{title, year, length, genre, studioName, starName})$

- i. {title, year, starName } form a key for the relation **Movies1**

- ii.  $\{ \text{year}, \text{starName} \}$  is not a key. Same star can be in multiple movies per year
- Superkeys
  - \* Means a set of attributes that contains a key
  - \* Don't need to be minimal

**Example:**

Given relation

 $R = \text{Movies1}(\text{title}, \text{year}, \text{length}, \text{genre}, \text{studioName}, \text{starName})$ 

- $\{ \text{title}, \text{year}, \text{starName} \}$  is a key and superkey
- $\{ \text{title}, \text{year}, \text{starName}, \text{title}, \text{year}, \text{length} \}$  is a superkey

**References:**

- 1) OpenTextBC, Chapter 11 Functional Dependencies, link
2. a)
  1.  $AB \rightarrow C$
  2.  $AB \rightarrow D$
  3.  $C \rightarrow A$
  4.  $C \rightarrow B$
  5.  $D \rightarrow B$
  6.  $D \rightarrow C$
  7.  $C \rightarrow D$
  8.  $D \rightarrow A$

**Second Attempt:**

$\{A, B\}^+ = \{A, B, C, D\}$ , so the following non-trivial FDs follows:  $AB \rightarrow C$  and  $AB \rightarrow D$ .

$\{C\}^+ = \{D, A\}$ , so the following non-trivial FDs follows  $C \rightarrow D$  and  $C \rightarrow A$ .

$\{D\}^+ = \{A\}$ , so the following non-trivial FDs follows:  $D \rightarrow A$ .

**Notes:**

- The Splitting / Combining Rule
  - Combining Rule
    - \*  $A_1, A_2, \dots, A_n \rightarrow B_i$  for  $i = 1, 2, \dots, m$
    - to
    - $A_1, A_2, \dots, A_n \rightarrow B_1, B_2, \dots, B_m$

**Example:**

Given

title year  $\rightarrow$  length  
 title year  $\rightarrow$  genre  
 title year  $\rightarrow$  studioName

it's combined form is

title year  $\rightarrow$  length genre studioName

– Splitting Rule

\*

\*  $A_1, A_2, \dots, A_n \rightarrow B_1, B_2, \dots, B_m$

to

$A_1, A_2, \dots, A_n \rightarrow B_i$  for  $i = 1, 2, \dots, m$

**Example:**

Given

title year  $\rightarrow$  length

It's splitted form is

title  $\rightarrow$  length  
 year  $\rightarrow$  length

• Trivial Functional Dependencies

- A functional dependency  $FD : X \rightarrow Y$  is **trivial** if  $Y$  is a subset of  $X$

**Exmample:**

title year  $\rightarrow$  title

**Example 2:**

title  $\rightarrow$  title

• Non-trivial Functional Dependencies

- is a case where some but not all of the attributes on the R.H.S of an FD are also on L.H.S

**Example:**

title year  $\rightarrow$  title movieLength

- Can be simplified using **trivial-dependency rule**
  - \* The FD  $A_1A_2 \cdots A_n \rightarrow B_1B_2 \cdots B_m$  is equivalent to  $A_1A_2 \cdots A_n \rightarrow C_1C_2 \cdots C_k$

where  $C$ 's are all those  $B$ 's that are not in  $A$ 's.



Figure 3.3: The trivial-dependency rule

- Computing the Closure of Attributes
  - Closure of attribute set  $\{X\}$  is denoted as  $\{X\}^+$ .
  - The closure means a given set of attributes  $A$  satisfying FD, are a sets of all attributes  $B$  such that  $A \rightarrow B$

### Example:

Given attributes  $A, B, C, D, E, F$  and FDs  $AB \rightarrow C$ ,  $BC \rightarrow AD$ ,  $D \rightarrow E$  and  $CF \rightarrow B$ , What is the closure of  $\{A, B\}$  or  $\{A, B\}^+$

1. Start with  $\{A, B\}$ .
2. Split  $BC \rightarrow AD$ 
  - \* We have  $BC \rightarrow A$  and  $BC \rightarrow D$
  - \* Since  $A$  is in  $\{A, B\}$ , this is not included
  - \* Since  $D$  is not in  $\{A, B\}$ , this IS included

So, we have  $\{A, B, D\}$

3. Since  $C$  in  $AB \rightarrow C$  is NOT in  $\{A, B, C, D\}$ ,  $C$  is included and we have  $\{A, B, C, D\}$
4. Since  $A$  in  $BC \rightarrow A$  is in  $\{A, B, C, D\}$ , this is skipped
5. Since  $E$  is not in  $D \rightarrow E$ ,  $E$  is included and we have  $\{A, B, C, D, E\}$  as our solution

- Why the Closure Algorithm Works
- Transitive Rule
  - Definition

If  $A_1A_2 \cdots A_n \rightarrow B_1B_2 \cdots B_m$  and  $B_1B_2 \cdots B_m \rightarrow C_1C_2 \cdots C_k$  hold in relation  $R$ ,  $A_1A_2 \cdots A_n \rightarrow C_1C_2 \cdots C_k$  also holds in  $R$ .

### Example:

Given

title year  $\rightarrow$  studioName  
 studioName  $\rightarrow$  studioAddr

Transitive rule says the above is equal to the following

title year  $\rightarrow$  studioAddr

- Inference Rules
  - Is also called **Armstrong's Axioms**
  - Has 3 axioms
    1. *Reflexivity*
      - \* If  $\{B_1, B_2, \dots, B_n\} \subseteq \{A_1, A_2, \dots, A_n\}$  then  $A_1A_2 \cdots A_n \rightarrow B_1B_2 \cdots B_m$
      - \* also called **trivial FDs**
    2. *Augmentation*
      - \* If  $A_1A_2 \cdots A_n \rightarrow B_1B_2 \cdots B_m$  then  $A_1A_2 \cdots A_nC_1C_2 \cdots C_k \rightarrow B_1B_2 \cdots B_mC_1C_2 \cdots C_k$
      - \*  $C_1C_2 \cdots C_k$  are any set of attributes
    3. *Transitivity*
      - \* If  $A_1A_2 \cdots A_n \rightarrow B_1B_2 \cdots B_m$  and  $B_1B_2 \cdots B_m \rightarrow C_1C_2 \cdots C_k$  then  $A_1A_2 \cdots A_n \rightarrow C_1C_2 \cdots C_k$

b)  $A, B$  is the only key of  $R$ .

### Notes:

- Key of Attributes
  - **Definition:** A set of attributes  $\{A_1, A_2, \dots, A_n\}$  is a key for a relation  $R$  if
    1. Those attributes functionally determine all other attributes

2. No proper subset of  $\{A_1, A_2, \dots, A_n\}$  functionally determines all other attributes of  $R$ .

c) The superkeys that are not keys are:  $\{A, B, C\}$ ,  $\{A, B, D\}$ ,  $\{A, B, C, D\}$

3. i) a)  $\{A\}^+ = \{A, B, C, D\}$ , so we have  $A \rightarrow A$ ,  $A \rightarrow B$ ,  $A \rightarrow C$ ,  $A \rightarrow D$

$\{B\}^+ = \{C, D\}$ , so we have  $B \rightarrow C$  and  $B \rightarrow D$

b)  $\{A\}$  is the key of  $S$ .

c) The super keys that are not keys are:

$\{A, B\}$ ,  $\{A, C\}$ ,  $\{A, D\}$ ,  $\{A, B, C\}$ ,  $\{A, B, D\}$ ,  $\{A, B, C, D\}$

ii) a)  $\{A\}^+ = \{A\}$ , so this FD is trivial.

$\{B\}^+ = \{B\}$ , so this FD is trivial.

$\{C\}^+ = \{C\}$ , so this FD is trivial.

$\{D\}^+ = \{D\}$ , so this FD is trivial.

$\{A, B\}^+ = \{A, B, C, D\}$ , so we have  $AB \rightarrow A$ ,  $AB \rightarrow B$ ,  $AB \rightarrow C$ ,  $AB \rightarrow D$

$\{A, C\}^+ = \{A, C\}$ , so we have  $AC \rightarrow A$ ,  $AC \rightarrow C$

$\{A, D\}^+ = \{A, D, B\}$ , so we have  $AD \rightarrow A$ ,  $AD \rightarrow D$ ,  $AD \rightarrow B$

$\{B, C\}^+ = \{B, C, D, A\}$ , so we have  $BC \rightarrow A$ ,  $BC \rightarrow B$ ,  $BC \rightarrow C$ ,  $BC \rightarrow D$

$\{D, C\}^+ = \{D, C, A, B\}$ , so we have  $DC \rightarrow D$ ,  $DC \rightarrow C$ ,  $DC \rightarrow A$ ,  $DC \rightarrow B$

$\{A, B, C\}^+ = \{A, B, C, D\}$ , so we have  $ABC \rightarrow A$ ,  $ABC \rightarrow B$ ,  $ABC \rightarrow C$ ,  $ABC \rightarrow D$

$\{B, C, D\}^+ = \{B, C, D, A\}$ , so we have  $BCD \rightarrow A$ ,  $BCD \rightarrow B$ ,  $BCD \rightarrow C$ ,  $BCD \rightarrow D$

$\{C, D, A\}^+ = \{C, D, A, B\}$ , so we have  $CDA \rightarrow A$ ,  $CDA \rightarrow B$ ,  $CDA \rightarrow C$ ,  $CDA \rightarrow D$

$\{D, A, B\}^+ = \{D, A, B, C\}$ , so we have  $DAB \rightarrow A$ ,  $DAB \rightarrow B$ ,  $DAB \rightarrow C$ ,  $DAB \rightarrow D$

$\{D, A, B\}^+ = \{D, A, B, C\}$ , so we have  $DAB \rightarrow A$ ,  $DAB \rightarrow B$ ,  $DAB \rightarrow C$ ,  $DAB \rightarrow D$

$\{A, B, C, D\}^+ = \{A, B, C, D\}$ , so this FD is trivial.

b)  $\{A, B\}$ ,  $\{A, C\}$ ,  $\{B, C\}$ ,  $\{D, C\}$  are the keys of  $T$ .

c) The super keys that are not keys are:

$\{A, B, C\}$ ,  $\{A, B, D\}$ ,  $\{B, C, D\}$ ,  $\{A, D, C\}$ ,  $\{A, B, D\}$ ,  $\{A, B, C, D\}$

iii) a)  $\{A\}^+ = \{A, B, C, D\}$ , so we have  $A \rightarrow C$ ,  $A \rightarrow D$

$\{B\}^+ = \{A, B, C, D\}$ , so we have  $B \rightarrow A$ ,  $B \rightarrow D$

$\{C\}^+ = \{A, B, C, D\}$ , so we have  $C \rightarrow A$ ,  $C \rightarrow B$

$\{D\}^+ = \{A, B, C, D\}$ , so we have  $D \rightarrow B$ ,  $D \rightarrow C$

$\{A, B\}^+ = \{A, B, C, D\}$ , so we have  $AB \rightarrow C$ ,  $AB \rightarrow D$

$\{B, C\}^+ = \{A, B, C, D\}$ , so we have  $BC \rightarrow A$ ,  $BC \rightarrow D$

$\{B, D\}^+ = \{A, B, C, D\}$ , so we have  $BD \rightarrow A$ ,  $BD \rightarrow C$

$\{C, D\}^+ = \{A, B, C, D\}$ , so we have  $CD \rightarrow A$ ,  $CD \rightarrow B$

$\{C, D\}^+ = \{A, B, C, D\}$ , so we have  $CD \rightarrow A$ ,  $CD \rightarrow B$

$\{A, B, C\}^+ = \{A, B, C, D\}$ , so we have  $ABC \rightarrow D$

$\{B, C, D\}^+ = \{A, B, C, D\}$ , so we have  $BCD \rightarrow A$

$\{C, D, A\}^+ = \{A, B, C, D\}$ , so we have  $CDA \rightarrow B$

$\{D, A, B\}^+ = \{A, B, C, D\}$ , so we have  $DAB \rightarrow C$

**Correct Solution:**

$\{A\}^+ = \{A, B, C, D\}$ , so we have  $A \rightarrow C$ ,  $A \rightarrow D$

$\{B\}^+ = \{A, B, C, D\}$ , so we have  $B \rightarrow A$ ,  $B \rightarrow D$

$\{C\}^+ = \{A, B, C, D\}$ , so we have  $C \rightarrow A$ ,  $C \rightarrow B$

$\{D\}^+ = \{A, B, C, D\}$ , so we have  $D \rightarrow B$ ,  $D \rightarrow C$

$\{A, B\}^+ = \{A, B, C, D\}$ , so we have  $AB \rightarrow C$ ,  $AB \rightarrow D$

$\{A, C\}^+ = \{A, B, C, D\}$ , so we have  $AC \rightarrow B$ ,  $AC \rightarrow D$

$\{A, D\}^+ = \{A, B, C, D\}$ , so we have  $AD \rightarrow B$ ,  $AD \rightarrow C$

$\{B, C\}^+ = \{A, B, C, D\}$ , so we have  $BC \rightarrow A$ ,  $BC \rightarrow D$

$\{B, D\}^+ = \{A, B, C, D\}$ , so we have  $BD \rightarrow A$ ,  $BD \rightarrow C$

$\{C, D\}^+ = \{A, B, C, D\}$ , so we have  $CD \rightarrow A$ ,  $CD \rightarrow B$ 
 $\{A, B, C\}^+ = \{A, B, C, D\}$ , so we have  $ABC \rightarrow D$ 
 $\{B, C, D\}^+ = \{A, B, C, D\}$ , so we have  $BCD \rightarrow A$ 
 $\{C, D, A\}^+ = \{A, B, C, D\}$ , so we have  $CDA \rightarrow B$ 
 $\{D, A, B\}^+ = \{A, B, C, D\}$ , so we have  $DAB \rightarrow C$ 

b)  $\{A\}, \{B\}, \{C\}, \{D\}$  are the keys of  $U$ .

c) The super keys that are not keys are:

$\{A, B\}, \{A, C\}, \{A, D\}, \{B, C\}, \{B, D\}, \{C, D\}, \{A, B, C\}, \{B, C, D\}, \{C, D, A\}, \{D, A, B\}, \{A, B, C, D\}$

4. a) We need to show the closure of attributes  $\{A_1, A_2, \dots, A_n, C\}$  in  $FD$   $A_1, A_2, \dots, A_n, C \rightarrow B$  is  $\{A_1, A_2, \dots, A_n, C, B\}$ , that is  $\{A_1, A_2, \dots, A_n, C\}^+ = \{A_1, A_2, \dots, A_n, C, B\}$

Since we know  $\{A_1, A_2, \dots, A_n\}$  functionally determines  $B$ , we can conclude  $B$  can be added to  $\{A_1, A_2, \dots, A_n, C\}$ .

Thus, it follows from above that  $\{A_1, A_2, \dots, A_n, C\}^+ = \{A_1, A_2, \dots, A_n, C, B\}$ .

- b) Let  $A_1A_2 \dots A_n \rightarrow B$  is FD. That is,  $\{A_1A_2 \dots A_n\}^+ = \{A_1A_2 \dots A_n, B\}$

We need to show  $A_1A_2 \dots A_nC \rightarrow BC$  follows. That is,  $\{A_1, A_2, \dots, A_n, C\}^+ = \{A_1, A_2, \dots, A_n, C, B\}$

It follows from the combine and split rule that  $A_1A_2 \dots A_nC \rightarrow BC$  can be splitted into  $A_1A_2 \dots A_nC \rightarrow B$  and  $A_1A_2 \dots A_nC \rightarrow C$ .

So, we need to show  $A_1A_2 \dots A_nC \rightarrow B$  and  $A_1A_2 \dots A_nC \rightarrow C$  follows from the given.

We will do so in parts.

### 1. Part 1 (Showing $A_1A_2 \dots A_nC \rightarrow B$ ):

Here, we need to show  $A_1A_2 \dots A_nC \rightarrow B$  follows.

And indeed, this follows from the work of *augmenting left sides*. That is the solution to previous problem.



## 2. Part 2 (Showing $A_1A_2 \cdots A_nC \rightarrow C$ ):

Here, we need to show  $A_1A_2 \cdots A_nC \rightarrow C$  follows.

The definition of trivial FD tells us  $A_1A_2 \cdots A_n \rightarrow B_1B_2 \cdots B_m$  holds when  $\{B_1, B_2, \dots, B_m\} \subseteq \{A_1, A_2, \dots, A_n\}$

Since  $\{C\} \subseteq \{A_1, A_2 \cdots, A_n, C\}$ , we can conclude this FD follows trivially.

- c) Let  $A_1A_2 \cdots A_n \rightarrow B_1B_2 \cdots B_m$  and  $C_1C_2 \cdots C_k \rightarrow D$ , where  $B$  are each among the  $C$ 's.

We need to show  $A_1A_2 \cdots A_nE_1E_2 \cdots E_j \rightarrow D$  follows, where the  $E$ 's are all of those  $C$ 's not found among the  $B$ 's.

The transitive rule tells us if  $A_1A_2 \cdots A_n \rightarrow B_1B_2 \cdots B_m$  and  $B_1B_2 \cdots B_m \rightarrow C_1C_2 \cdots C_k$ , then  $A_1A_2 \cdots A_n \rightarrow C_1C_2 \cdots C_k$  also holds in  $R$ .

Since we know  $A_1A_2 \cdots A_n \rightarrow B_1B_2 \cdots B_m$  and  $C_1C_2 \cdots C_k \rightarrow D$  where  $B$ 's are each among the  $C$ 's, we can conclude from the transitive rule that  $A_1A_2 \cdots A_n \rightarrow D$ .

Then using **augmenting left sides** to all  $C$ 's not found among the  $B$ 's on  $A_1A_2 \cdots A_n \rightarrow D$ , we can conclude  $A_1A_2 \cdots A_nE_1E_2 \cdots E_j \rightarrow D$  follows.

- d) Assume  $FD$ 's  $A_1A_2 \cdots A_n \rightarrow B_1B_2 \cdots B_m$  and  $C_1C_2 \cdots C_k \rightarrow D_1D_2 \cdots D_j$  holds.

We need to show  $FD$   $A_1A_2 \cdots A_nC_1C_2 \cdots C_k \rightarrow B_1B_2 \cdots B_mD_1D_2 \cdots D_k$  follows.

Using the split / combine rule, we can conclude showing  $A_1A_2 \cdots A_nC_1C_2 \cdots C_k \rightarrow B_1B_2 \cdots B_mD_1D_2 \cdots D_k$  is the same as showing  $A_1A_2 \cdots A_nC_1C_2 \cdots C_k \rightarrow B_1B_2 \cdots B_m$  and  $A_1A_2 \cdots A_nC_1C_2 \cdots C_k \rightarrow D_1D_2 \cdots D_k$

So, we will prove the two, in parts

### 1. Part 1 (Showing $A_1A_2 \cdots A_nC_1C_2 \cdots C_k \rightarrow B_1B_2 \cdots B_m$ )

Here, we need to show  $A_1A_2 \cdots A_nC_1C_2 \cdots C_k \rightarrow B_1B_2 \cdots B_m$ .

The header of problem tells us  $A_1A_2 \cdots A_n \rightarrow B_1B_2 \cdots B_m$  holds.

Then by using **Augmenting Left Sides** rule to all  $C$ s not found among the  $A$ s,  $A_1A_2 \cdots A_nC_1C_2 \cdots C_k \rightarrow B_1B_2 \cdots B_m$  follows.

### 2. Part 2 (Showing $A_1A_2 \cdots A_nC_1C_2 \cdots C_k \rightarrow D_1D_2 \cdots D_k$ follows)

Here, we need to show  $A_1A_2 \cdots A_nC_1C_2 \cdots C_k \rightarrow D_1D_2 \cdots D_k$ .

The header of problem tells us  $C_1C_2 \cdots C_k \rightarrow D_1D_2 \cdots D_k$  holds.

Then by using **Augmenting Left Sides** rule to all  $A$ s not found among the  $C$ s,  $A_1A_2 \cdots A_nC_1C_2 \cdots C_k \rightarrow D_1D_2 \cdots D_k$  follows.

5. a) An example is

$A$  being movieID and  
 $B$  being movie length.

b) An example is

$A$  being movieID  
 $B$  being movieTitle  
 $C$  being movieLength

c) An example is

$A$  being movieTitle  
 $B$  being year  
 $C$  being length

6. Assume a relation has no attribute that is functionally determined by all the other attributes.

And, assume for the sake of contradiction that there is a relation with non-trivial FD  $X \rightarrow Y$ .

Then, it follows from the definition of non-trivial functional dependency that  $Y \not\subseteq X$ .

Then, we can conclude the attributes in  $Y$  is functionally determined by other attributes in  $X$ .

But this contradicts the assumption that no attribute is functionally determined by all other attributes.

7. Let  $X$  and  $Y$  be sets of attributes. Assume  $X \subseteq Y$ .

I need to show  $X^+ \subseteq Y^+$ .

I will do so in cases

1. **Case 1** ( $X = Y$ ):

Assume  $X = Y$ .

I need to show  $X^+ \subseteq Y^+$  follows.

The header tells us  $X = Y$ .

Using this fact,  $X^+ = Y^+$  is true.

Then it follows from above that  $X^+ \subseteq Y^+$  is also true.

**2. Case 2 ( $X \subset Y$ )**

Assume  $X \subset Y$ .

I need to show  $X^+ \subseteq Y^+$  follows.

Since the attributes in  $X$  is in  $Y$ , we can conclude the attributes in  $X^+$  is also in  $Y^+$ .

And, since  $Y$  has attributes not in  $X$ , we can conclude  $Y^+$  may contain attributes not in  $X^+$ .

Thus, we can conclude  $X^+ \subseteq Y^+$ .

**8. Notes:**

- Closure (Definition)

- Suppose  $A = \{A_1, A_2, \dots, A_n\}$  is a set of attributes of  $R$  and  $S$  is a set of FD'.

The closure of  $A$  under the set  $S$ , denoted by  $A^+$ , is the set of attributes  $B$  such that any relation that satisfies all the FD's in  $S$  is also satisfies  $A_1 A_2 \cdots A_n \rightarrow B$ .

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