# CSC236 Midterm 2 Version 1 Solution

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## Question 1

- Let $n, q \in \mathbb{N}$. Let $r \in \{0, 1\}$

  Assume $n > 2$, and $n = 2q + r$.

  I need to find a closed form for $T(2q + r)$, using repeated subtitution.

  Starting from $T(n)$, we have

$$
\begin{aligned}
T(n) &= n + T(n-2) && \text{[By def. since } n > 2\text{]} && (1)\\
T(2q+r) &= 2q + r + T(2q+r-2) && \text{[By replacing } n \text{ for } 2q+r\text{]} && (2)\\
&= 2q + r + T(2(q-1)+r) && && (3)\\
&\vdots && && (4)\\
&= \sum_{i=0}^{i=q-1} (2(q-i)+r) + T(r) && \text{[After } q-1 \text{ repeatitions]} && (5)\\
&= 2\sum_{i=0}^{i=q-1} (q-i) + \sum_{i=0}^{i=q-1} r + T(r) && && (6)\\
&= 2\sum_{i=0}^{i=q-1} (q-i) + \sum_{i=0}^{i=q-1} r && \text{[Since } T(r) = 0\text{]} && (7)\\
&= 2\sum_{i'=1}^{i=q} i' + \sum_{i=0}^{i=q-1} r && && (8)\\
&= 2\sum_{i'=1}^{i'=q} i' + \sum_{i=0}^{i=q-1} r && && (9)\\
&= 2\sum_{i'=1}^{i'=q} i' + \sum_{i=0}^{i=q-1} r && && (10)\\
&= 2(q(q+1))/2 + \sum_{i=0}^{i=q-1} r && \text{[By using } \sum_{i=1}^{i=n} i = (n(n+1))/2\text{]} && (11)\\
&= q(q+1) + rq && && (12)\\
&= q(q+1+r) && && (13)
\end{aligned}
$$

- **Rough Works:**

For convenience, define $H(q) : q(q+r+1) = T(2q+r)$.

I will use simple induction to prove that $\forall q \in \mathbb{N}$, $H(q)$.

1. Base Case ($q = 0$)

**Base Case ($q = 0$):**

Let $q = 0$.

Then,

$$\begin{aligned} q(q + r + 1) &= 0 && (14) \\ &= T(2 \cdot 0 + r) && [\textbf{By def.}] \quad (15) \\ &= T(2q + r) && (16) \end{aligned}$$

Thus, $T(2q + r)$ verifies in this step.

2. Inductive Step

**Base Case ($q = 0$):**

Let $q \in \mathbb{N}$. Assume $H(q)$.

I need to show $H(q + 1)$ follows. That is, $(q + 1)\Big[(q + 1) + r + 1\Big] = T(2(q + 1) + r)$.

Starting with $(q + 1)\Big[(q + 1) + r + 1\Big]$, we have