Worksheet 1 Solution

March 9, 2020

Question 1

a) A $= \{\, 2,\ 5 \,\}$
$A^c = \{\, 1,\ 3,\ 4,\ 6 \,\}$

b) $A^c = U \setminus A$

c) $A^c \cap B^c = \{\, x \mid x \in U,\ x \leq 0 \text{ and } x \geq 4 \,\}$
$A^c \cap B^c = \{\, x \mid x \in U,\ x < 1 \text{ and } x > 2 \,\}$
$(A \cap B)^c = \{\, x \mid x \in U,\ x < 1 \text{ and } x > 2 \,\}$
$(A \cup B)^c = \{\, x \mid x \in U,\ x \leq 0 \text{ and } x \geq 4 \,\}$

Question 2

a) $T_0 \to 0, 3, 6$
$T_1 \to 1, 4, 7$
$T_2 \to 2, 5, 8$
$T_3 \to 12, 18, 24$

b) $\mathbb{Z}^+ = \{\, T_0, T_1, T_2 \,\}$

T_3 not included. A partition of a set must not have any common elements.

Question 3

a) 000, 110,
001, 010,
011, 100,
101, 111

b) $S_1 = \{\, aa, bb, cc, ab, ca, ba, ac, bc, cb \,\}$

$S_2 = \{\, a, b, c, aa, bb, cc, ab, ca, aaa, aba, aca, bab, bbb, bcb, cac, cbc, ccc \ldots \}$

$S_1 \cap S_2 = \{\, aa, bb, cc \,\}$

$S_1 \setminus S_2 = \{\, ab, ca, ba, ac, bc, cb \,\}$

c) $S_1 = (S_1 \cap S_2) \cup (S_1 \setminus S_2)$

Question 4

a)

x	$\lfloor x \rfloor$	$\lceil x \rceil$
$\frac{25}{4}$	6	7
0.999	0	1
-2.01	-3	-2

b) Domain: $\mathbb{R}$

Codomain: $\mathbb{Z}$

c) False. Consider the following example.

$1 = \lfloor 0.75 + 0.25 \rfloor$

$0 = \lfloor 0.75 \rfloor + \lfloor 0.25 \rfloor$

Question 5

a) $\sum_{k=1}^{3}(k+1) = (1+1)+(2+1)+(3+1)$

$\sum_{m=0}^{1}\frac{1}{2^m} = \frac{1}{2^0}+\frac{1}{2^1}$

$\sum_{k=0}^{2}(k^2+3) = (1^2+3)+(2^2+3)$

$\sum_{j=0}^{4}\frac{(-1)^j\cdot j}{j+1} = \frac{(-1)^0\cdot 0}{0+1}+\frac{(-1)^1\cdot 1}{1+1}+\frac{(-1)^2\cdot 2}{2+1}+\frac{(-1)^3\cdot 3}{3+1}+\frac{(-1)^4\cdot 4}{4+1}$

$\sum_{k=1}^{5}(2k) = (2\cdot 1)+(2\cdot 2)+(2\cdot 3)+(2\cdot 4)+(2\cdot 5)$

$\prod_{i=2}^{4}\frac{i\cdot(i+2)}{(i-1)\cdot(i+1)} = \frac{1\cdot(1+2)}{(1-1)\cdot(1+1)}\times\frac{2\cdot(2+2)}{(2-1)\times(2+1)}\times\frac{3\cdot(3+2)}{(3-1)\times(3+1)}\times\frac{4\cdot(4+2)}{(4-1)\cdot(4+1)}$

b) $3+6+12+24+48+96 = \sum_{i=0}^{6}(3\cdot 2^k)$

$0+1-2+3-4+5 = \sum_{i=0}^{6}(-1)^(i+1)$

$\frac{(1\cdot 2)}{(3\cdot 4)}+\frac{(2\cdot 3)}{(4\cdot 5)}+\frac{(3\cdot 4)}{(5\cdot 6)} = \prod_{i=1}^{3}\frac{i\cdot(i+1)}{(i+2)\cdot(i+3)}$

$\frac{1}{3}+\frac{4}{9}+\frac{9}{27}+\frac{16}{81}+\frac{25}{243}+\frac{36}{729} = \sum_{i=1}^{6}\frac{i^2}{3^i}$

$\frac{1}{(1+1)}\times\frac{2}{(2+1)}\times\frac{3}{(3+1)}\dots\frac{k}{(k+1)} = \prod_{i=1}^{k}\frac{k}{k+1}$

Question 6

a) $3\cdot\sum_{i=1}^{n}(2i+3)+\sum_{i=1}^{n} = (4-5i) = \sum_{i=1}^{n}[2\cdot(2i+3)+(4-5i)]$

$(\prod_{i=1}^{n}\frac{i}{i+1})\cdot(\prod_{i=1}^{n}\frac{i+1}{i+2}) = \prod_{i=1}^{n}(\frac{i}{i+1})\cdot(\frac{i+1}{i+2})$

$$\sum_{i=10}^{15} 2^i + \sum_{i=101}^{106} (i-1) = \sum_{i=0}^{5} [2^{i+10} + (i+100)]$$