CSC373 Worksheet 6 Solution

August 13, 2020

1. 1. Multiply objective function by - 1

Maximize

$$-2x_1 - 7x_2 - x_3$$

Subject to

$$x_1 - x_3 = 7$$

$$3x_1 + x_2 \ge 7$$

$$x_2 \ge 0$$

$$x_3 \le 0$$

2. Replace non-nonnegative constraints \boldsymbol{x}_1

Maximize

$$-2x_1' + 2x_1'' - 7x_2 - x_3$$

Subject to

$$x_1' - x_1'' - x_3 = 7$$

$$3x_1' - 3x_1'' + x_2 \ge 7$$

$$x_1', x_1'', x_2 \ge 0$$

$$x_3 \le 0$$

3. Replace non-nonnegative constraints x_3

Maximize

$$-2x_1' + 2x_1'' - 7x_2 - x_3' + x_3''$$

Subject to

$$x'_1 - x''_1 - x'_3 + x''_3 = 7$$
$$3x'_1 - 3x''_1 + x_2 \ge 7$$
$$x'_1, x''_1, x_2, x'_3, x''_3 \ge 0$$

4. Replace equality constraints with \geq and \leq

Maximize

$$-2x_1' + 2x_1'' - 7x_2 - x_3' + x_3''$$

Subject to

$$\begin{aligned} x_1' - x_1'' - x_3' + x_3'' &\leq 7 \\ x_1' - x_1'' - x_3' + x_3'' &\geq 7 \\ 3x_1' - 3x_1'' + x_2 &\geq 7 \\ x_1', x_1'', x_2, x_3', x_3'' &\geq 0 \end{aligned}$$

5. Correct greater-than-or-equal-to inequality constraints

Maximize

$$-2x_1' + 2x_1'' - 7x_2 - x_3' + x_3''$$

Subject to

$$x'_1 - x''_1 - x'_3 + x''_3 \le 7$$

$$-x'_1 + x''_1 + x'_3 - x''_3 \le -7$$

$$-3x'_1 + 3x''_1 - x_2 \le 7$$

$$x'_1, x''_1, x_2, x'_3, x''_3 \ge 0$$

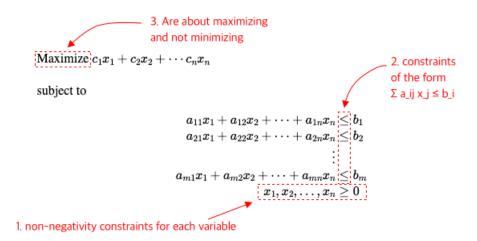
Notes:

• Linear Programming

- Is a method to achieve the best outcome (such as maximum profit or lowest cost) in a mathematical model whose requirements are represented by linear relationships. [1]
- Is named to make it sound cool for government funding
 - * Like dynamic programming
- Applications
 - * Microeconomics (maximize profits, minimize costs)
 - * Company management

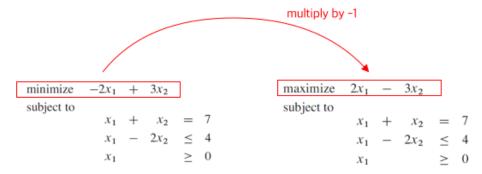
• Standard Form

- Is a form of linear programming
- Are about maximizing, not minimizing [2]
- All have a positivity constraint for each variable [2]
- All other constraints are all of the form "linear combination of variables \leq constant". $^{[2]}$

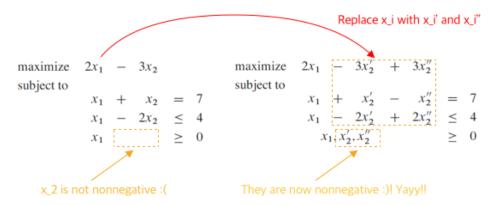


• Converting Linear Programming to Standard Form

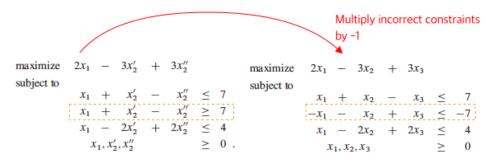
- 1) The objective function might be a minimization rather than a maximization
 - Negate coefficients of the objective function



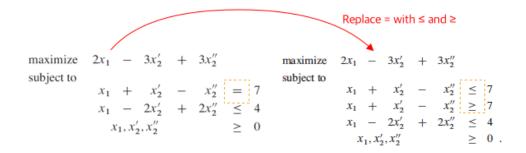
- 2) There might be variables without nonnegativity constraints
 - Replace each non-nonnegative variable x_i with x_i' and x_i''
 - Modify linear program



- 3) There might be **equality constraints**, which have an equal sign rather than a less-than-or-equal-to sign
 - Replace equality constraint $f(x_1, x_2, ..., x_n) = b$ with $f(x_1, x_2, ..., x_n) \le b$ and $f(x_1, x_2, ..., x_n) \ge b$



- 4) There might be **inequality constraints**, but instead of having a less-than-or-equal-to-sign
 - Multiply incorrect inequality constraints by -1



References:

- 1) Wikipedia, Linear Programming, link
- 2) Instituto de Mathematicas, Standard form for Linear Programs, link

2.

$$z = 2x_1 - 6x_3$$

$$x_4 = 7 - x_1 - x_2 + x_3$$

$$x_5 = -8 + 3x_1 - x_2$$

$$x_6 = -x_1 + 2x_2 + 2x_3$$

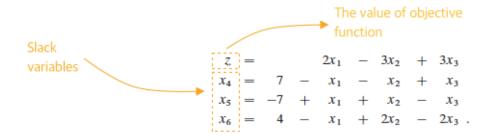
The basic variables are variables on the lhs (i.e B = 4, 5, 6), and the non-basic variables are the variables on the rhs of the expressions (i.e. N = 1, 2, 3).



Notes:

• Slack Form

- Is a form of linear programming
- Is for efficient solving of liner programming problem using simplex algorithm

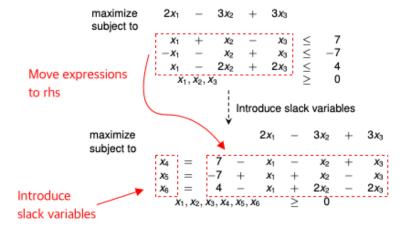


• Converting Linear Programs into Slack Form

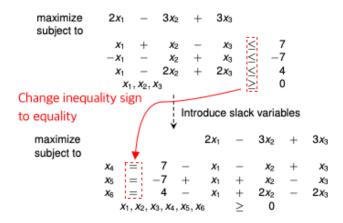
- 1) Start from the standard form of linear programming
- 2) Shift objective functions to right



3) Introduce slack variable x_i to lhs and move expressions $\sum_{j=1}^n a_{ij}x_j$ to rhs



4) Change inequalities in linear programming to equality



5) Use Variable z to denote objective function



6) Omit the nonnegativivty constraints



References:

- 1) Cambridge University, Linear Programming, link
- 3. Multiplying the first expression (under subject to) by 2, and summing the inequality constraints, we have

$$0 \le -6 \tag{1}$$

which is impossible.

Notes

- Infeasible
 - A Linear Program is infeasible if there is no solution that satisfies all of the constraints