CSC236 Worksheet 4 Review

Hyungmo Gu

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Question 1

- Let $n \in \mathbb{N}$. Assume that $\exists k \in \mathbb{N},\ n = 3^k$, so $k = \log_3 n$.

  Then,

$$
\begin{aligned}
T(n) &= 2n + T(\lceil n/3 \rceil) && \text{[By def.]} && (1)\\
&= 2n + T(n/3) && \text{[Since } 3 \mid n\text{, and } \lceil n/3 \rceil = n/3\text{]} && (2)\\
&= 2n + (2(n/3) + T(n/3^2)) && \text{[By subtituting } n/3 \text{ for } n \text{ in def.]} && (3)\\
&\ \ \vdots && && (4)\\
&= 2n \sum_{i=0}^{k-1} \frac{1}{3^i} + T(n/3^k) && \text{[After } k \text{ steps]} && (5)\\
&= 2n\Big(\frac{1 - (1/3^k)}{1 - (1/3)}\Big) + T(n/3^k) && \text{[By geometric series]} && (6)\\
&= 2 \cdot 3^k\Big(\frac{1 - (1/3^k)}{1 - (1/3)}\Big) + T(3^k/3^k) && \text{[By replacing } 3^k \text{ for } n\text{]} && (7)\\
&= 3(3^k - 1) + 2 && && (8)\\
&= 3^{k+1} - 1 && && (9)\\
&= 3^{\log_3 n + 1} - 1 && \text{[By replacing } \log_3 n \text{ for } k\text{]} && (10)\\
&= 3^{\log_3 n + \log_3 3} - 1 && && (11)\\
&= 3^{\log_3 3n} - 1 && && (12)\\
&= 3n - 1 && && (13)
\end{aligned}
$$