CSC373 Worksheet 2 Solution

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1. **Notes:**

   - Greedy Algorithm
     - Always makes the choice that looks best at the moment
       * Locally optimal solution leads to globally optimal solution
   - Activity-selection Problem (Greedy algorithm using dynamic programming)
     - Goal: Selecting maximum size set of mutually compatible activities

     Example:

i	1	2	3	4	5	6	7	8	9	10	11
s_i	1	3	0	5	3	5	6	8	8	2	12
f_i	4	5	6	7	9	9	10	11	12	14	16

     - Suppose a set exists $S = \{a_1 = [s_1, f_1), a_2 = [s_2, f_2), ..., a_n = [s_n, f_n)\}$
       * a_i represents an ith activity
       * s_i represents starting time
       * f_i represents finishing time
       * $0 \leq s_i < f_i < \infty$
       * $a_1, ..., a_n$ sorted in monotonically increasing order of finish time

         i.e.

         $$f_1 \leq f_2 \leq f_3 \leq ... \leq f_{n-1} \leq f_n$$

       * a_i and a_j are **compatible**, if intervals $[s_i, f_i)$ and $[s_j, f_j)$ don't overlap

         i.e

         $$s_i \geq f_j \text{ and } s_j \geq f_i$$

– Steps

1. Think about dynamic programming solution
   * S_{ij}: activities that start after activity a_i finishes and before activity a_j starts

     i.e.

     $$S_{19} = \{a_4 = [5,7), a_6 = [5,9), a_7 = [6,10)\}$$

   * A_{ij}: maximum set of mutually compatible activities in S_{ij} (including a_k)
     · $A_{ik} = A_{ij} \cap S_{ik}$
     · $A_{kj} = A_{ij} \cap S_{kj}$
     · $A_{ij} = A_{ik} \cup \{a_k\} \cup Akj$
     · So, $|A_{ij}| = |A_{ik}| + |A_{kj}| + 1$
   * The optimal solution A_{ij} must include optimal solution to the two subproblems for S_{ik} and S_{kj}

2. Observe that only one choice - greedy choice, and that when we make the greedy choice, only one subproblem remains

3. Develop recursive greedy solution

4. Convert the recursive algorithm into iterative one