Worksheet 4 Solution

March 13, 2020

Question 1

a. $\exists n \in \mathbb{N}, (n > 3) \wedge (n^2 - 1.5n \geq 5)$

b. The variable is existentially quantified

c. Concrete natural number

d. Let n = 5.

Then,

$$(5)^2 - 1.5(5) \tag{1}$$

Then,

$$25 - 7.5 \tag{2}$$

Then,

$$17.5 \tag{3}$$

which is greater than 5. So, the statement is True

e. $\forall n \in \mathbb{N}, n > 3 \Rightarrow n^2 - 1.5n > 4$

Here $\Rightarrow$ should be used because $n > 3$ is a given, and we are using it to show that the statement $n^2 - 1.5n > 4$ is True

f. The variable is universally quantified

g. In this proof the variable must be **arbitrary** natural number

h. The assumption made is that the any natural number greater than 3 satisfies the statement $n^2 - 1.5n > 4$.

   This assumption is made since the predicate logic is the proof of an implication

i. Let $n \in \mathbb{N}$ be an arbitrary number of $\mathbb{N}$, and assume $n > 3$. Then,

$$n^2 > 3n \tag{1}$$

$$n^2 - 1.5n > 3n - 1.5n \tag{2}$$

$$n^2 - 1.5n > 1.5n \tag{3}$$

   Because we know that $n > 3$, we can conclude

$$n^2 - 1.5n > 1.5(3) \tag{4}$$

$$n^2 - 1.5n > 4.5 \tag{5}$$

   It follows that the statement $\forall n \in \mathbb{N}, n > 3 \Rightarrow n^2 - 1.5n > 4$ is true.

Question 2

a. $\forall n \in \mathbb{N}, n > 5 \Rightarrow 2 \mid n \vee 3 \mid n$

b. $\exists n \in \mathbb{N}, (n > 5) \wedge (2 \nmid n \vee 3 \nmid n)$

c. Let $n = 7$.

   Since 7 is a prime number, 7 is not divisible by both 2 and 3.

   It follows from the above that the original statement $\forall n \in \mathbb{N}, n > 5 \Rightarrow 2 \mid n \vee 3 \mid n$ is False

Question 3

a. Let x be an arbitrary number of $\mathbb{R}$. Let $y = 165 - x + 1$

b. Let $y = 166$. Let x be an arbitrary number of $\mathbb{N}$