CSC373 Worksheet 6

August 12, 2020

1. **CLRS 29.1-4:** Convert the following linear program into standard form:

   Minimize

$$2x_1 + 7x_2 + x_3$$

   Subject to

$$\begin{aligned} x_1 - x_3 &= 7 \\ 3x_1 + x_2 &\geq 7 \\ x_2 &\geq 0 \\ x_3 &\leq 0 \end{aligned}$$

2. **CLRS 29.1-5:** Convert the following linear program into slack form:

   Maximize

$$2x_1 - 6x_3$$

   Subject to

$$\begin{aligned} x_1 + x_2 - x_3 &\leq 7 \\ 3x_1 - x_2 &\geq 7 \\ -x_1 + 2x_2 + 2x_3 &\geq 0 \\ x_1, x_2, x_3 &\geq 0 \end{aligned}$$

3. **CLRS 29.1-6:** Show the following linear program is infeasible:

   Maximize

   $$3x_1 - 2x_2$$

   Subject to

   $$\begin{aligned} x_1 + x_2 &\leq 2 \\ -2x_1 - 2x_2 &\leq -10 \\ x_1, x_2 &\geq 0 \end{aligned}$$

4. **CLRS 29.1-7:** Show that the following linear program is unbounded:

   Maximize

   $$x_1 - x_2$$

   Subject to

   $$\begin{aligned} -2x_1 + x_2 &\leq -1 \\ -x_1 - 2x_2 &\leq -2 \\ x_1, x_2 &\geq 0 \end{aligned}$$

5. **CLRS 29.1-8:** Suppose that we have a general linear program with n variables and m constraints, and suppose that we convert it into standard form. Give an upper bound on the number of variables and constraints in the resulting linear program.

6. **CLRS 29.1-9:** Give an example of a linear program for which the feasible region is not bounded, but the optimal objective value is finite.

7. **CLRS 29.2-3:** In the single-source shortest-paths problem, we want to find the shortest-path weights from a source vertex s to all vertices $v \in V$. Given a graph G, write a linear program for which the solution has the property that d_v is the shortest-path weight from s to v for each vertex $v \in V$.

8. **CLRS 29.2-7:** In the **minimum-cost multicommodity-folow problem**, we are given directed graph $G = (V, E)$ in which each edge $(u, v) \in E$ has a nonnegative capacity $c(u, v) \geq 0$ and a cost $a(u, v)$. As in the multicommodity-flow problem, we are given k different commodities, $K_1, K_2, ..., K_k$, where we specify commodify i by the triple $K_i = (s_i, t_i, d_i)$. We define the flow f_i for commodity i and the aggregate flow f_{uv} in which the aggregate flow on each ege (u, v) is no more than the capacity of edge (u, v). The cost of a flow is $\sum_{u,v \in V} a(u, v) f_{uv}$, and the goal is to find the feasible flow of minimum cost. Express this problem as a linear program.