

# Midterm 1 Version 1 Review 2

July 17, 2020

1. a)  $aaa, aab, aac, bba, bbb, bbc, cca, ccb, ccc$

b) **Solution:**

$p$	$q$	$r$	$p \vee q$	$(p \vee q) \Rightarrow \neg r$
T	T	T	T	T
T	T	F	T	F
T	F	T	T	T
F	T	T	T	T
T	F	F	T	F
F	T	F	T	F
F	F	T	F	T
F	F	F	F	T

- c)  $\exists x \in \mathbb{N}, \forall y \in \mathbb{N}, \neg P(x, y) \wedge \neg Q(x, y)$

**Correct Solution:**

Let  $x = \dots$ . Let  $y \in P$ .

We need to prove  $\neg P(x, y)$  and  $\neg Q(x, y)$  are true.

2. a)  $\exists x \in P, Student(x) \wedge Attends(x)$

- b)  $\forall x \in P, \exists y \in P, Student(y) \wedge Attends(y) \Rightarrow Loves(x, y)$

**Correct Solution:**

$\forall x \in P, \exists y \in P, Student(y) \wedge Attends(y) \wedge Loves(x, y)$

- c)  $\forall x \in P, Student(x) \wedge Attends(x) \Rightarrow Loves(x, x)$

- d)  $\forall x, y \in P, x \neq y \Rightarrow Loves(x, y) \Rightarrow Attends(x) \vee Attends(y)$

3. a)  $\forall a, b, c \in \mathbb{Z}, \exists k_1, k_2, k_3 \in \mathbb{Z}, a = k_1 b \wedge b = k_2 c \Rightarrow a = k_3 c$

b) *Proof.* Let  $a, b, c \in \mathbb{Z}$ , and there is some  $k_1, k_2, k_3 \in \mathbb{Z}$  such that  $a = k_1 b, b = k_2 c$ .

I need to prove  $a = k_3 c$ .

Let  $k_3 = k_1 k_2$ .

Then, we can conclude

$$a = k_1 b \quad \text{[By header]} \quad (1)$$

$$= k_1 k_2 c \quad \text{[By replacing } b \text{ with } k_2 c] \quad (2)$$

$$= k_3 c \quad \text{[By } k_3 = k_1 k_2] \quad (3)$$

□