

Worksheet 8 Review

March 27, 2020

Question 1

a. $\forall n \in \mathbb{N}, (0 \leq 1) \wedge (n \leq 2^n) \Rightarrow (n+1) \leq 2^{n+1}$

Note:

• **Induction:** $\forall n \in \mathbb{N}, P(0) \wedge P(n) \Rightarrow P(n+1)$

Question 2

a. We will prove this statement by induction on n .

Base Case:

Let $n = 0$.

Then,

$$0 \leq 2^0 \tag{1}$$

$$0 \leq 1 \tag{2}$$

Since the above inequality is true, the base case holds.

Inductive Case:

Let $n \in \mathbb{N}$. Assume $P(n)$.

Then,

$$n \leq 2^n \tag{3}$$

$$n + 1 \leq 2^n + 1 \tag{4}$$

$$n + 1 \leq 2^n + 2^n \tag{5}$$

$$n + 1 \leq 2^{n+1} \tag{6}$$

by the fact $2^k + 2^k = 2^{k+1}$.

Then, it follows from proof by induction that the statement $n \leq 2^n$ is true for all n .

Question 3