CSC236 Worksheet 3 Review

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Question 2

- *Proof.* Define $P(e) : S_1(e) = 3(s_2(e) - 1)$

  I will use structural induction to prove $\forall e \in \varepsilon,\ P(e)$.

  Basis:

  Let $\{x, y, z\} \in \varepsilon$.

  In this step, there are following cases to consider: $e = x$, $e = y$, and $e = z$.

  In each of the cases, we have $s_1(e) = 0$ and $s_2(e) = 1$.

  Thus,

$$\begin{aligned} s_1(e) = 0 &= 3(0) && (1)\\ &= 3(1-1) && (2)\\ &= 3(s_2(e) - 1) && (3) \end{aligned}$$

  So, , $P(e)$ holds.

  Inductive Step:

  Let $e_1, e_2 \in \varepsilon$. Assume $H(e) : P(e_1)$ and $P(e_2)$. That is, $s_1(e_1) = 3(s_2(e_1) - 1)$ and $s_2(e_2) = 3(s_2(e_2) - 1)$.

  I need to prove all possible combinations of e_1 and e_2 satisfy the statement. That is $P((e_1 + e_2))$ and $P((e_1 - e_2))$.

In each of the combination, the total number of variables of e is the sum of the number of variables in e_1 and e_2, and the total number of parenthesis and operators in e is the sum of operators and parenthesis in e_1 and e_2 plus 3.

Then, using these facts, we have

$$s_2(e) = s_2(e_1) + s_2(e_2) \tag{4}$$
$$s_1(e) = s_1(e_1) + s_1(e_2) + 3 \tag{5}$$

Thus, we can calculate

$$
\begin{aligned}
s_1(e) &= s_1(e_1) + s_1(e_2) + 3 && \text{[By 5]} && (6)\\
&= 3(s_2(e_1) - 1) + 3(s_2(e_2) - 1) + 3 && \text{[By I.H]} && (7)\\
&= 3(s_2(e_1) + s_2(e_2)) - 6 + 3 && \text{[By I.H]} && (8)\\
&= 3s_2(e) - 3 && \text{[By 4]} && (9)\\
&= 3(s_2(e) - 1) && && (10)
\end{aligned}
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□