Worksheet 9 Review

March 28, 2020

Question 1

a. For every set of size 0 has 0 subsets of size 2.

b. Let $n = 0$. Let S be an arbitrary set. Assume S has size 0.

   Since S has size 0, empty subsets are the **only** subsets that can be included in S.

   Then, because we know empty subsets have size 0, we can conclude there are 0 subsets of size 2.

   It follows from above that the base case holds.

   Correct Solution:

   We want to show every set S of size 0 has 0 subsets of size 2.

   Since S has size 0, empty subsets are the **only** subsets that can be included in S.

   Then, because we know an empty subset have size 0, we can conclude there are 0 subsets of size 2.

   Notes:

- Professor specifically mentions **We want to show every set S of size 0 has 0 subsets of size 2**
- Professor doesn't include conclusion at the end of proof
- Under which cases conclusion to a proof are included.

c. Now we will prove inductive step.

Let $k \in \mathbb{N}$. Assume every set of size k has $\frac{k(k-1)}{2}$ subsets of size 2.

We want to show a set of size $k+1$ has $\frac{(k+1)k}{2}$ subsets of size 2.

Part 1: counting subsets of S of size 2 that contain s_{k+1}

It follows from the table below,

k	Sets	Subsets of Size 2 with s_{k+1}
0	$\{0, 1\}$	1
1	$\{0, 1, 2\}$	2
2	$\{0, 1, 2, 3\}$	3
2	$\{0, 1, 2, 3, 4\}$	4

that the number of subsets of size 2 that contain s_{k+1} is $k+1$.

Part 2: counting subsets of S of size 2 that doesn't contain s_{k+1}

Because we know the subset of S that doesn't contain s_{k+1} is a set S of size k, we can conclude using induction hypothesis that there are

$$\frac{k(k+1)}{2} \tag{1}$$

subsets of size 2.

Part 3: Putting the counts together

Then,

$$\frac{k(k+1)}{2} + (k+1) = \frac{(k+1)(k+2)}{2} \tag{2}$$

Then, it follows from proof by induction that the statement '$\forall n \in \mathbb{N}$, every set of size n has $\frac{n(n-1)}{2}$ subsets of size 2' is true for all natural numbers n.

Correct Solution:

Part 1: counting subsets of S of size 2 that contain s_{k+1}

It follows from the table below,

k	Sets	Subsets of Size 2 with s_{k+1}
2	$\{0, 1\}$	1
3	$\{0, 1, 2\}$	2
4	$\{0, 1, 2, 3\}$	3
5	$\{0, 1, 2, 3, 4\}$	4

that the number of subsets of size 2 that contain s_{k+1} is k.

Part 2: counting subsets of S of size 2 that doesn't contain s_{k+1}

Because we know the subset of S that doesn't contain s_{k+1} is a set S of size k, we can conclude using induction hypothesis that there are

$$\frac{k(k-1)}{2} \tag{3}$$

subsets of size 2.

Part 3: Putting the counts together

Then,

$$\frac{k(k-1)}{2} + k = \frac{(k+1)k}{2} \tag{4}$$

Then, it follows from proof by induction that the statement '$\forall k \in \mathbb{N}$, every set of size $k$has $\frac{k(k-1)}{2}$subsets of size 2' is true for all natural numbers k.

Notes:

- I forgot that k represents number of elements in a set.

d. **Statement:** For every $n \in \mathbb{N}$, every finite set S of size n, has

$$\frac{n(n-1)(n-2)}{6} \tag{1}$$

subsets of size 3.

We will prove this statement by using induction on n.

Base Case:

Let $n = 0$.

Then, only the empty subsets can be included in S.

Because an empty subset has size 0, there are 0 subsets of size 3 in S.

Since

$$\frac{0 \cdot (0-1)(0-2)}{6} = 0 \tag{2}$$

the base case holds.

Question 2

Question 3