# CSC236 Worksheet 1 Review

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## Question 4

- *Proof.* **Base Case ($n = 7$):**

  Let $n = 7$.

  Then,

$$
\begin{aligned}
4^n &= 16384 && (1)\\
&\geq 12011 && (2)\\
&= 5(7)^4 + 6 && (3)\\
&= 5n^4 + 6 && (4)
\end{aligned}
$$

  So, $H(n)$ is verified.

  **Inductive Step:**

  Let $n \in \mathbb{N}$. Assume $H(n)$.

  I need to show $H(n+1)$ follows. That is $4^{n+1} \geq (5n^4 + 6)$.

  Starting from $4^{n+1}$, we have

$$
\begin{aligned}
4^{n+1} &= 4^n + 4^n + 4^n + 4^n && && (5)\\
&\geq (5n^4 + 6) + (5n^4 + 6) + (5n^4 + 6) + (5n^4 + 6 && \text{[By I.H]} && (6)\\
&= 5(n^4 + n^4 + n^4 + n^4) + 24 && && (7)\\
&= 5(n^4 + n \cdot n^3 + n^2 \cdot n^2 + n^3 \cdot n) + 24 && && (8)\\
&> 5(n^4 + 7 \cdot n^3 + 7^2 \cdot n^2 + 7^3 \cdot n) + 24 && \text{[Since } n > 7\text{]} && (9)\\
&> 5(n^4 + 4 \cdot n^3 + 6 \cdot n^2 + 4 \cdot n) + 6 && && (10)\\
&> 5(n+1)^4 + 6 && && (11)
\end{aligned}
$$

$\square$