Worksheet 10 Solution

March 19, 2020

Question 1

a.

$$\begin{aligned}(165)_8 &= 5 \times 8^0 + 6 \times 8^1 + 1 \times 8^2 && (1)\\ &= 5 + 48 + 64 && (2)\\ &= 117 && (3)\end{aligned}$$

b. **Reference Table**

Number	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F	G
Value	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16

$$\begin{aligned}(B4)_{16} &= 4 \times 16^0 + 11 \times 16^1 && (1)\\ &= 4 + 176 && (2)\\ &= 180 && (3)\end{aligned}$$

Question 2

a.

$$\begin{aligned}
357 \div 2 &= 178, \text{remainder } \mathbf{1} \\
178 \div 2 &= 89, \text{remainder } \mathbf{0} \\
89 \div 2 &= 44, \text{remainder } \mathbf{1} \\
44 \div 2 &= 22, \text{remainder } \mathbf{0} \\
22 \div 2 &= 11, \text{remainder } \mathbf{0} \\
11 \div 2 &= 5, \text{remainder } \mathbf{1} \\
5 \div 2 &= 2, \text{remainder } \mathbf{1} \\
2 \div 2 &= 1, \text{remainder } \mathbf{0} \\
1 \div 2 &= 0, \text{remainder } \mathbf{1}
\end{aligned}$$

Hence, the binary representation of 357 is (101100101).

b.

$$\begin{aligned}
357 \div 8 &= 44, \text{remainder } \mathbf{5} \\
44 \div 8 &= 5, \text{remainder } \mathbf{4} \\
5 \div 8 &= 0, \text{remainder } \mathbf{5}
\end{aligned}$$

Hence, the octal representation of 357 is $(545)_8$

c.

$$\begin{aligned}
357 \div 16 &= 22, \text{remainder } \mathbf{5} \\
22 \div 16 &= 1, \text{remainder } \mathbf{6} \\
1 \div 16 &= 0, \text{remainder } \mathbf{1}
\end{aligned}$$

Hence, the hexadecimal representation of 357 is $(165)_{16}$

Question 3

a.

$$\begin{aligned}
0.375 \times 2 &= 0.750, +\mathbf{0} \\
0.750 \times 2 &= 0.5, +\mathbf{1} \\
0.5 \times 2 &= 0, +\mathbf{1}
\end{aligned}$$

Hence, the binary representation of 0.375 is $(0.011)_2$.

b.

$$\begin{aligned}
\frac{1}{10} \times 2 &= \frac{2}{10} + \mathbf{0} \\
\frac{2}{10} \times 2 &= \frac{4}{10} + \mathbf{0} \\
\frac{4}{10} \times 2 &= \frac{8}{10} + \mathbf{0} \\
\frac{8}{10} \times 2 &= \frac{6}{10} + \mathbf{1} \\
\frac{6}{10} \times 2 &= \frac{2}{10} + \mathbf{1} \\
\frac{2}{10} \times 2 &= \frac{4}{10} + \mathbf{0} \\
\frac{4}{10} \times 2 &= \frac{8}{10} + \mathbf{0} \\
\frac{8}{10} \times 2 &= \frac{6}{10} + \mathbf{1}
\end{aligned}$$

Hence, the binary representation of $\frac{1}{10}$ is $(0.0\overline{0011})_2$.

c. Let $a = 1$, and $r = \frac{1}{2}$.

$$\sum_{i=1}^{\infty} \frac{1}{2}^i = \frac{1(\frac{1}{2})}{1 - \frac{1}{2}} \tag{1}$$

$$= 1 \tag{2}$$

d. The first 1 in $(0.0\overline{0011})_2$ repeats every 4^{th} position, and the second 1 repeats every $4^{th} + 1$ position.

So,

$$(0.0\overline{0011})_2 = \sum_{i=1}^{\infty}(\frac{1}{2})^{4i} + \sum_{i=1}^{\infty}(\frac{1}{2})^{4i+1} \tag{1}$$

$$= \sum_{i=1}^{\infty}(\frac{1}{16})^{i} + \sum_{i=1}^{\infty}\frac{1}{2}(\frac{1}{16})^{i} \tag{2}$$

$$= \frac{\frac{1}{16}}{1-\frac{1}{16}} + \frac{1}{2}(\frac{\frac{1}{16}}{1-\frac{1}{16}}) \tag{3}$$

$$= \frac{1}{15} + \frac{1}{30} \tag{4}$$

$$= \frac{3}{30} \tag{5}$$

$$= \frac{1}{10} \tag{6}$$