# CSC236 Worksheet 4 Review

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May 8, 2020

## Question 1

- Let $n \in \mathbb{N}$. Assume that $\exists k \in \mathbb{N},\ n = 3^k$, so $k = \log_3 n$.

  Then,

$$
\begin{align}
T(n) &= 2n + T(\lceil n/3 \rceil) && \text{[By def.]} \tag{1}\\
&= 2n + T(n/3) && \text{[Since } 3 \mid n \text{, and } \lceil n/3 \rceil = n/3\text{]} \tag{2}\\
&= 2n + (2(n/3) + T(n/3^2)) && \text{[By subtituting } n/3 \text{ for } n \text{ in def.]} \tag{3}\\
&\vdots \tag{4}\\
&= 2n \sum_{i=0}^{k-1} \frac{1}{3^i} + T(n/3^k) && \text{[After } k \text{ steps]} \tag{5}\\
&= 2n \Big( \frac{1-(1/3^k)}{1-(1/3)} \Big) + T(n/3^k) && \text{[By geometric series]} \tag{6}\\
&= 2 \cdot 3^k \Big( \frac{1-(1/3^k)}{1-(1/3)} \Big) + T(3^k/3^k) && \text{[By replacing } 3^k \text{ for } n\text{]} \tag{7}\\
&= 3(3^k - 1) + 2 \tag{8}\\
&= 3^{k+1} - 1 \tag{9}\\
&= 3^{\log_3 n + 1} - 1 && \text{[By replacing } \log_3 n \text{ for } k\text{]} \tag{10}\\
&= 3^{\log_3 n + \log_3 3} - 1 \tag{11}\\
&= 3^{\log_3 3n} - 1 \tag{12}\\
&= 3n - 1 \tag{13}
\end{align}
$$

**Attempt 2:**

Let $n \in \mathbb{N}$. Assume that $\exists k \in \mathbb{N},\ n = 3^k$, so $k = \log_3 n$.

Then,

$$
\begin{aligned}
T(n) &= 2n + T(\lceil n/3 \rceil) && \text{[By def.]} \quad (14)\\
&= 2n + T(n/3) && \text{[Since } 3 \mid n\text{, and } \lceil n/3 \rceil = n/3\text{]} \quad (15)\\
&= 2n + (2(n/3) + T(n/3^2)) && \text{[By replacing } n/3 \text{ for } n \text{ in def.]} \quad (16)\\
&\vdots && (17)\\
&= 2n \sum_{i=0}^{k-1} \frac{1}{3^i} + T(n/3^k) && \text{[After } k \text{ steps]} \quad (18)\\
&= 2n\Big(\frac{1-(1/3^k)}{1-(1/3)}\Big) + T(n/3^k) && \text{[By geometric series]} \quad (19)\\
&= 2 \cdot 3^k \Big(\frac{1-(1/3^k)}{1-(1/3)}\Big) + T(3^k/3^k) && \text{[By replacing } 3^k \text{ for } n\text{]} \quad (20)\\
&= 3(3^k - 1) + 2 && (21)\\
&= 3^{k+1} - 1 && (22)\\
&= 3^{\log_3 n + 1} - 1 && \text{[By replacing } \log_3 n \text{ for } k\text{]} \quad (23)\\
&= 3^{\log_3 n + \log_3 3} - 1 && (24)\\
&= 3^{\log_3 3n} - 1 && (25)\\
&= 3n - 1 && (26)
\end{aligned}
$$