

Worksheet 14 Solution

March 25, 2020

Question 1

a. Inner Loop Iterations (upper bound): n

Inner Loop Step Size: 1

Inner Loop Steps Total: n

Outer Loop Iterations (upper bound): n

Outer Loop Step Size: 1

Outer Loop Steps Total: n

Steps Total: $n \cdot n = n^2$

Correct Solution:

Since the inner loop starts at $i + 1$ and ends at $n - 1$, where i represents the variable in outer loop, the inner loop has $(n - 1 - (i + 1) + 1) = n - i - 1$ iterations.

Since each iteration takes 1 step, the total steps taken by inner loop is:

$$(n - i - 1) \cdot 1 = (n - i - 1) \tag{1}$$

Now, we will evaluate total steps taken by outer loop.

Since the outer loop starts at $i = 0$, and ends at $n - 1$, the loop runs at most n iterations.

Since each iteration takes $(n - i - 1)$ steps, the total steps of outer loop is:

$$\sum_{i=0}^{n-1} (n - i + 1) = \sum_{i=0}^{n-1} [(n - 1) - i] \quad (2)$$

$$= \sum_{i=0}^{n-1} (n - 1) - \sum_{i=0}^{n-1} i \quad (3)$$

$$= n(n - 1) - \frac{n(n - 1)}{2} \quad (4)$$

$$= \frac{n^2 - n}{2} \quad (5)$$

Then, since the last **return** statement takes 1 step, it follows that the total number of steps of this algorithm is at most $\frac{n^2 - n}{2} + 1$, or $\mathcal{O}(n^2)$.

- b. Consider the input family where none of the values in a list are the same (i.e. $[1, 2, 3, 4, 5, 6, 7, 8, 9]$).

Since all values in the input list are not matching, both the inner and the outer loop will run, giving the loops the total number of steps of $\frac{n^2 - n}{2}$.

Since the last **return** statement takes 1 step, the total number of steps of this algorithm is $\frac{n^2 - n}{2} + 1$, or $\Omega(n^2)$.

Correct Solution:

Let $n \in \mathbb{N}$ and $lst = [1, 2, 3, \dots, n - 1, n - 1]$.

Since the inner loop will run without interruptions until the end, the inner loop has

$$n - 1 - (i + 1) + 1 = n - i - 1 \quad (1)$$

iterations.

Then, since the inner loop takes 1 step per iteration, the total steps taken by the inner loop is

$$(n - i - 1) \cdot 1 = (n - i - 1) \quad (2)$$

Since the **if condition** $lst[i] == lst[j]$ and the **return** statement are activated when $i = n - 2$, the outer loop will run until $i = n - 2$, where j is the variable of the inner loop and i is the variable of the outer loop.

Since the outer loop starts at 0 and ends at $n - 2$, it has

$$n - 2 + 1 = n - 1 \quad (3)$$

iterations.

Since each iteration in the outer loop takes $(n - i - 1)$ steps, the outer loop has total cost of

$$\sum_{i=0}^{n-2} (n - i - 1) = \sum_{i=0}^{n-2} (n - 1) + \sum_{i=0}^{n-2} i \quad (4)$$

$$= (n - 1)(n - 1) - \frac{(n - 2)(n - 1)}{2} \quad (5)$$

$$= \frac{(n - 1)n}{2} \quad (6)$$

Since each of the **if condition** and **return** statement has cost of 1, the total cost of algorithm is $\frac{n(n-1)}{2} + 2$, or $\Omega(n^2)$

Question 2