

# Worksheet 4 Review 2

April 12, 2020

## Question 1

- a.  $\exists n \in \mathbb{N}, n > 3 \wedge n^2 - 1.5n \geq 5$
- b. The variable is existentially quantified
- c. Because the variable is existentially quantified, the variable's value should be a *concrete* natural number
- d. **Statement:**  $\exists n \in \mathbb{N}, n > 3 \wedge n^2 - 1.5n \geq 5$

*Proof.* Let  $n = 5$ .

We will prove  $n > 3 \wedge n^2 - 1.5n \geq 5$ .

First, we need to prove  $n > 3$ .

The header tells us  $n = 5$ .

Using this fact, we can conclude  $n > 3$ .

Now, we need to show  $n^2 - 1.5n \geq 5$ .

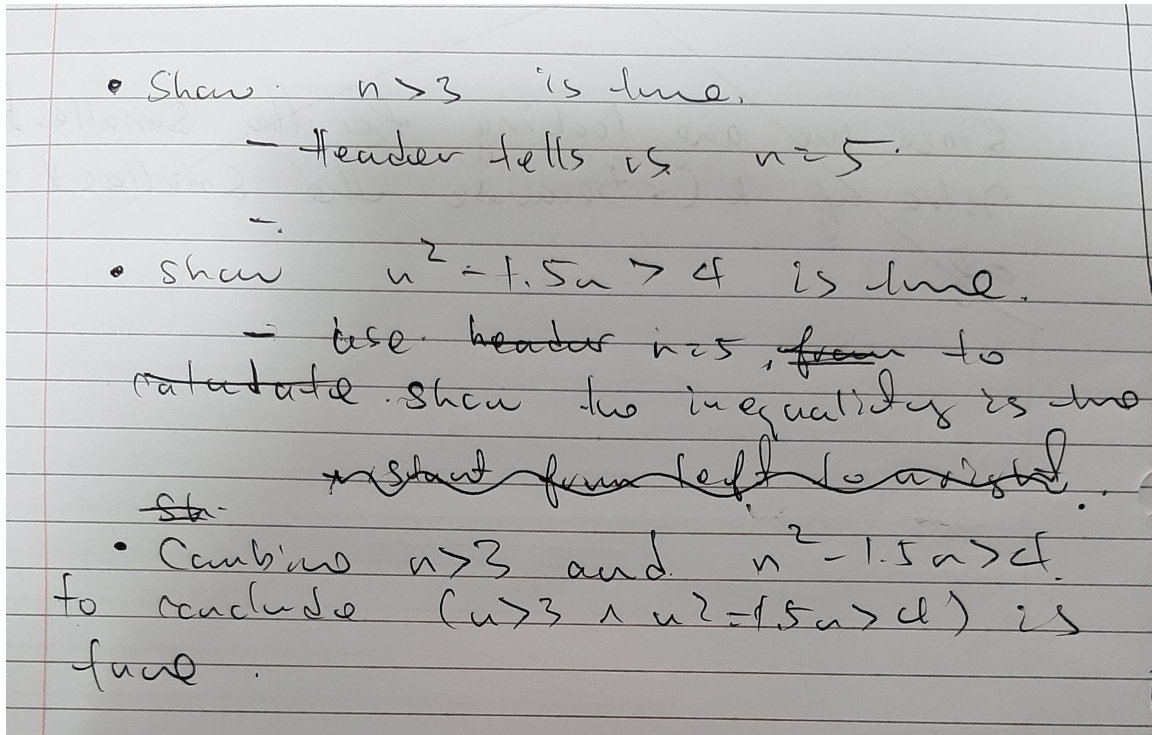
Using the fact  $n = 5$ , we can calculate

$$\begin{aligned} n^2 - 1.5n &= 25 - 7.5 & (1) \\ &= 17.5 & (2) \\ &\geq 5 & (3) \end{aligned}$$

Finally, since  $n > 3$  and  $n^2 - 1.5n \geq 5$  are true, we can conclude  $n > 3 \wedge n^2 - 1.5n \geq 5$  are true.  $\square$

### Notes:

- Used the following pseudoproof used for this problem. Proof really feels smoother.



e.  $\forall n \in \mathbb{N}, n \geq 3 \Rightarrow n^2 - 1.5n > 4$

f. The variable is universally quantified.

g. Because the variable is universally quantified, the variable's value should be an arbitrary natural number.

h. The assumption made is  $n > 3$ .

This conclusion is made by looking at the L.H.S of the  $\Rightarrow$  operator.

i. **Statement:**  $\forall n \in \mathbb{N}, n > 3 \Rightarrow n^2 - 1.5n > 4$

*Proof.* Let  $n \in \mathbb{N}$ . Assume  $n \geq 3$ .

We will prove  $n^2 - 1.5n > 4$ .

Using the fact  $n \geq 3$ , we can conclude

$$n^2 - 1.5n \geq (3)^2 - 1.5(3) \quad (1)$$

$$= 9 - 4.5 \quad (2)$$

$$= 4.5 \quad (3)$$

$$> 4 \quad (4)$$

□

## Question 2

a.  $\forall n \in \mathbb{N}, n > 5 \Rightarrow 2 \mid n \vee 3 \mid n$

b.  $\exists n \in \mathbb{N}, n > 5 \wedge (2 \nmid n) \wedge (3 \nmid n)$

c. **Statement:**  $\exists n \in \mathbb{N}, n > 5 \wedge (2 \nmid n) \wedge (3 \nmid n)$

*Proof.* Let  $n = 7$ .

We will prove  $n > 5 \wedge (2 \nmid n) \wedge (3 \nmid n)$ .

First, we will to prove  $n > 5$ .

The header tells us  $n = 7$ .

Using this fact, we can conclude  $n > 5$ .

Now, we will prove  $2 \nmid n$ .

7 is a prime number, so we know the number can only be divisible by 1 and 7.

Using this fact, we can conclude  $2 \nmid 7$ .

Now, we will prove  $3 \nmid n$ .

Since 7 is divisible by 1 and 7 only, we can conclude  $3 \nmid 7$ .

So, since  $n > 5$ ,  $2 \nmid n$  and  $3 \nmid n$  are true, we can conclude  $n > 5 \wedge (2 \nmid n) \wedge (3 \nmid n)$  holds. □

## Question 3

a. Let  $x \in \mathbb{R}$ , and  $y = 165$ .