Worksheet 15 Review

April 1, 2020

Question 1

a. First, we will evaluate the cost of he inner most loop.

Because the loop runs from $j = i = 1$ to $j = n - 1$, with each iteration costing 1 step, we can conclude that the inner most loop has cost of at most

$$\lceil (n-1) - (i+1) + 1 \rceil = n - i - 1 \tag{1}$$

steps.

Next, we will evaluate the cost of the outer most loop.

Because the loop runs from $i = 0$ to $i = n - 1$ with each iteration costing $(n - i - 1)$ steps, we can conclude the outer most loop has cost of at most

$$\begin{aligned}
\sum_{i=0}^{n-1}(n-i-1) &= \left[\sum_{i=0}^{n-1}(n-1) - \sum_{i=0}^{n-1} i\right] &(2)\\
&= \left[\frac{2n(n-1)}{2} - \frac{(}{n}(n-1))2\right] &(3)\\
&= \frac{n(n-1)}{2} &(4)
\end{aligned}$$

steps.

Next, we will bring everything together.

Since the lines **n = len(lst)** and **return False** have cost of 1 step each, the total cost of the algorithm is

$$\frac{n(n-1)}{2} + 2 \tag{5}$$

steps.

Then, it follows from above that the algorithm has runtime of $\Theta(n^2)$.

Correct Solution:

First, we will evaluate the cost of he inner most loop.

Because the loop runs from $j = i = 1$ to $j = n - 1$, with each iteration costing 1 step, we can conclude that the inner most loop has cost of at most

$$\lceil (n-1) - (i+1) + 1 \rceil = n - i - 1 \tag{6}$$

steps.

Next, we will evaluate the cost of the outer most loop.

Because the loop runs from $i = 0$ to $i = n - 1$ with each iteration costing $(n - i - 1)$ steps, we can conclude the outer most loop has cost of at most

$$\begin{aligned}
\sum_{i=0}^{n-1}(n-i-1) &= \left[\sum_{i=0}^{n-1}(n-1) - \sum_{i=0}^{n-1} i\right] && (7)\\
&= \left[\frac{2n(n-1)}{2} - \frac{(}{n}(n-1))2\right] && (8)\\
&= \frac{n(n-1)}{2} && (9)
\end{aligned}$$

steps.

Next, we will bring everything together.

Since the lines **n = len(lst)** and **return False** have cost of 1 step each, the total cost of the algorithm is at most

$$\frac{n(n-1)}{2} + 2 \tag{10}$$

steps.

Then, it follows from above that the algorithm has runtime of $\mathcal{O}(n^2)$.

Question 2