Worksheet 3 Review 2

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Question 1

a. $Correct(my\_prog) \wedge Python(my\_prog)$

b. $\forall x \in P,\ \neg Correct(x) \Rightarrow Python(x)$

> **Correct Solution:**
>
> $\exists x \in P,\ \neg Correct(x) \wedge Python(x)$

Notes:

- I feel that '$\wedge$' operator is used instead of '$\Rightarrow$' if 'is' is used with an existential quantifier

> **Example:**
>
> An incorrect program is written in Python

- I also feel '$\Rightarrow$' is used when 'is' is used with universal quantifier

> **Example:**
>
> Every incorrect program is written in python

c. $\forall x \in P, Python(x) \Rightarrow \neg Correct(x)$

d. $\forall x \in P, \neg Correct(x) \Rightarrow Python(x)$

e. There is a program that is written in *Python* and is *Correct*

f. All programs are not written in *Python* and is *Correct*

g. There is a program that is *Correct* and not written in *Python*

h. All programs that are correct is not written in *Python*, and all programs that are *Correct* is not written in *Python*.

Question 2

a. Either all programs that are written in *Python* is *Correct*, or all programs that are written in *Python* are not *Correct*

b. $(\exists x \in P,\ Python(x) \wedge Correct(x)) \Rightarrow (\forall x \in P, Python(x) \wedge Correct(x))$

c. The difference is that in statement 1, each divisibility claims can be validated with different natural numbers where as in statement 2, the two claims must be validated with a single natural number.

   The statement 1 is true, where as statement 2 is false (consider counter example of $x = 7$)

Question 3

a. $Odd(n):\ \forall n \in \mathbb{Z},\ \exists \in \mathbb{Z},\ n + 1 = 2k$

   <u>Correct Solution:</u>

   $Odd(n):\ \exists \in \mathbb{Z},\ n + 1 = 2k,$ where $n \in \mathbb{Z}$

   Notes:

   - Noticed professor defines variable in predicate (i.e. n in $P(n)$) in where (i.e where $n \in \mathbb{Z}$)

b. $\forall m, n \in \mathbb{Z},\ Odd(m) \wedge Odd(n) \Rightarrow Odd(mn)$

c. $\forall m, n \in \mathbb{Z},\ \exists k_1, k_2 \in \mathbb{Z},\ (n + 1 = 2k_1) \wedge (m + 1 = 2k_2) \Rightarrow \exists k_3 \in \mathbb{Z},\ (mn + 1 = 2k_3)$

   <u>Correct Solution:</u>

   $\forall m, n \in \mathbb{Z},\ (\exists k_1 \in \mathbb{Z},\ n+1 = 2k_1) \wedge (\exists k_1 \in \mathbb{Z},\ m+1 = 2k_2) \Rightarrow \exists k_3 \in \mathbb{Z},\ mn+1 = 2k_3$

   Notes:

   - Noticed professor didn't pull out existential quantifier from parenthesis

d. $\forall m, n \in \mathbb{Z},\ \exists k_1 \in \mathbb{Z},\ mn + 1 = 2k_1 \Rightarrow (\exists k_2 \in \mathbb{Z},\ m + 1 = 2k_2) \wedge (\exists k_3 \in \mathbb{Z},\ n + 1 = 2k_3)$

Question 4

a. $((a \wedge b) \wedge \neg c) \vee ((\neg a \vee \neg b) \wedge c)$

b. $\exists x, y \in S,\ \forall x \in S, \neg P(x, y) \vee \neg Q(x, z)$

c. $(\exists x \in S, P(x)) \wedge (\forall y \in S, \neg Q(y))$

Question 5

- A solution that returns the statement as false is:
  - $U:\ \mathbb{N}$
  - $P(x):\ x \neq 1 \wedge x \mid 5$, where $x \in \mathbb{N}$
  - $Q(y):\ y \neq 1 \wedge y \mid 7$, where $y \in \mathbb{N}$