Worksheet 1 Review

Hyungmo Gu

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Question 1

a. $A^c = \{1, 3, 4, 6\}$

b. $A = U \setminus A$

c. $A^c \cap B^c = \{x \mid x \in U,\ x \leq 0 \text{ and } x \geq 4\}$
$A^c \cup B^c = \{x \mid x \in U,\ x < 1 \text{ and } x > 2\}$
$(A \cap B)^c = \{x \mid x \in U,\ x < 1 \text{ and } x > 2\}$
$(A \cup B)^c = \{x \mid x \in U,\ x \leq 0 \text{ and } x \geq 4\}$

Correct Solution:

$A^c \cap B^c = \{x \mid x \in U,\ x \leq 0 \text{ or } x \geq 4\}$
$A^c \cup B^c = \{x \mid x \in U,\ x < 1 \text{ or } x > 2\}$
$(A \cap B)^c = \{x \mid x \in U,\ x < 1 \text{ or } x > 2\}$
$(A \cup B)^c = \{x \mid x \in U,\ x \leq 0 \text{ or } x \geq 4\}$
It follows from above that $A^c \cap B^c = (A \cup B)^c$ and $A^c \cup B^c = (A \cap B)^c$

Question 2

a. $T_0 = \{3, 6, 9, \dots\}$
$T_1 = \{1, 4, 7, \dots\}$
$T_2 = \{2, 5, 8, \dots\}$
$T_3 = \{6, 12, 18, \dots\}$

b. A partition of $\mathbb{Z}$ is $\{T_0, T_1, T2\}$.

All four sets can't be used because elements in T_3 overlaps with T_0. A partition cannot have any elements in common.

Notes:

- **Definition of Partition:** Let A be a set. A (finite or infinite) collection of nonempty sets $\{A_1, A_2, A_3\}$ is called a **partition** of A when (1) A is the union of all of the A_i, and (2) the sets $A_1, A_2, A_3, \dots$ do not have any element in common.

Question 3

a. All strings over the alphabet $\{0, 1\}$ of length three are

$$000, 100, 010, 001, 110, 101, 011, 111$$

b. $S_1 = \{aa, ab, ac, ba, bb, bc, ca, cb, cc\}$

$S_2 = \{a, b, c, aa, bb, cc, \dots\}$

$S_1 \cap S_2 = \{aa, bb, cc\}$

$S_1 \setminus S_2 = \{ab, ac, ba, bc, ca, cb\}$

c. $S_1 = (S_1 \cap S_2) \cup (S_1 \setminus S_2)$

d.

	$\lfloor x \rfloor$	$\lceil x \rceil$
$\frac{25}{4}$	6	7
0.99	0	1
-2.01	-3.0	-2.0

Notes:

- floor of a negative number: ceiling but with negative sign
- ceiling of a negative number: floor but with negative sign

e. **Domain of the floor & ceiling function:** $\mathbb{R}$

Codomain of the floor & ceiling function: $\mathbb{N}$

f. The statement is false. Consider example $x = -0.5$ and $y = 0.5$.

Then, $\lfloor x + y \rfloor = 0$ and $\lfloor x \rfloor + \lfloor y \rfloor = -1 + 0 = -1$.

Question 4