

# Worksheet 5 Solution

March 15, 2020

## Question 1

- $\forall n, p \in \mathbb{N}, \text{Odd}(n) \wedge \text{Odd}(p) \Rightarrow \text{Odd}(n \times p)$

Let  $n, p \in \mathbb{Z}$ , and assume  $n, p$  are odd numbers.

Then,  $\exists k, m \in \mathbb{Z}, n = 2k - 1, p = 2m - 1$  by the definition of odd numbers

Then,

$$n \times p = (2k - 1)(2m - 1) \tag{1}$$

$$= 2k2m - 2k - 2m + 1 \tag{2}$$

$$= (2k2m - 2k - 2m + 2) - 1 \tag{3}$$

$$= 2(2km - k - m + 1) - 1 \tag{4}$$

$$= 2l - 1 \tag{5}$$

where  $l = 2km - k - m + 1$ .

Since  $l \in \mathbb{Z}$ , it follows from the definition of odd number that the product of two odd numbers is odd.

## Question 2

- a.  $\forall m, n \in \mathbb{Z}, \text{Even}(m) \wedge \text{Odd}(n) \Rightarrow m^2 - n^2 = m + n$
- b. The flaw is in the same value  $k$ . This implies that the statement is true only when  $n$  is 1 less than  $m$ . This doesn't mean it's true for all even and odd numbers.

## Question 3

- a.  $\text{Dom}(f, g) : \forall n \in \mathbb{N}, g(n) \leq f(n)$ , where  $f, g : \mathbb{N} \rightarrow \mathbb{R}^{\geq 0}$
- b. Let  $n \in \mathbb{R}^{\geq 0}$ ,  $f(n) = 3n$  and  $g(n) = n$ .

Then,

$$g(n) = n \leq n + n + n \tag{6}$$

$$\leq 3n \tag{7}$$

$$\leq f(n) \tag{8}$$

Then, it follows from the definition that  $f$  dominates  $g$ .

- c. Predicate Logic:  $\exists n \in \mathbb{N}, g(n) > f(n)$ .

Let  $n = 1$ .

Then,

$$g(1) = (1) + 165 = 166 > 1 \tag{9}$$

$$> f(1) \tag{10}$$

Then, it follows from negation of the definition that  $f$  does not dominate  $g$ .

d. Predicate Logic:  $\exists x \in \mathbb{R}^{\geq 0}, \exists n \in \mathbb{N}, g(n) > f(n)$

Let  $x = 1$  and  $n = 1$ .

Then,

$$g(1) = (1) + 1 = 2 > 1 \tag{11}$$

$$> f(1) \tag{12}$$

Then, it follows from negation of the definition that  $f$  does not dominate  $g$ .

## Question 4

- Let  $x \in \mathbb{R}^{\geq 0}$ ,  $\epsilon = x - \lfloor x \rfloor$ , and assume  $x \geq 4$ .

Then,

$$(\lfloor x \rfloor)^2 \geq 4 \tag{13}$$

$$(x + \epsilon)^2 \geq 4 \tag{14}$$

by the fact that  $\epsilon$  can be rewritten as  $\lfloor x \rfloor = x + \epsilon$ .

Then,

$$(x + \epsilon)^2 \geq 4 \tag{15}$$

$$x^2 + 2x\epsilon + \epsilon^2 \geq 4 \tag{16}$$

$$\tag{17}$$