Midterm 2 Version 3 Solution

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Question 1

a.

$$\begin{aligned}
165 \div 2 &= 82, \text{ remainders } \mathbf{1} \\
82 \div 2 &= 41, \text{ remainders } \mathbf{0} \\
41 \div 2 &= 20, \text{ remainders } \mathbf{1} \\
20 \div 2 &= 10, \text{ remainders } \mathbf{0} \\
10 \div 2 &= 5, \text{ remainders } \mathbf{0} \\
5 \div 2 &= 2, \text{ remainders } \mathbf{1} \\
2 \div 2 &= 1, \text{ remainders } \mathbf{0} \\
1 \div 2 &= 0, \text{ remainders } \mathbf{1}
\end{aligned}$$

From the above, we can conclude the binary representation of the decimal number 165 is $(10100101)_2$

b. The largest number that can be expressed by an n-digit balanced ternary representation is

$$\sum_{i=0}^{n-1} 3^i = \frac{1}{2} \cdot (3^n - 1) \tag{1}$$

Notes:

- Geometric Series

$$\sum_{i=0}^{n} r^i = \frac{1 - r^{n+1}}{1 - r}, \text{ where } |r| > 1$$

c.

$f(n) \in \mathcal{O}(n)$	True	$g(n) \in \Omega(n)$	True	$f(n) \in \Omega(g(n))$	True
$f(n) \in \Theta(g(n))$	False	$g(n) \in \Theta(n)$	False	$f(n) + g(n) \in \Theta(g(n))$	False

Question 2

Question 3

Question 4