# CSC236 Term Test 1 Version 2 Review

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## Question 1

- **Rough Works:**

Define $P(n) : f(n) = 3^n$.

I will use complete induction to prove that $\forall n \in \mathbb{N}, P(n)$.

1. Inductive Step

   Let $n \in \mathbb{N}$. Assume $H(n) : \bigwedge_{i=0}^{n-1} P(i)$. I will show $P(n)$ follows.

2. Base Case ($n = 0$)

   Let $n = 0$.

   Then,

   $$\begin{aligned} f(n) &= 1 && \text{[By def.]} && (1) \\ &\leq 3^0 && && (2) \\ &= 3^n && && (3) \end{aligned}$$

   Thus, $P(n)$ follows in this step.

3. Base Case ($n = 1$)

Let $n = 1$.

Then,

$$\begin{aligned} f(n) &= 1 && \text{[By def., since } n = 1\text{]} \quad (4) \\ &\leq 3^1 && \quad (5) \\ &= 3^n && \quad (6) \end{aligned}$$

Thus, $P(n)$ follows in this step.

4. Base Case ($n = 2$)

Let $n = 2$.

Then,

$$\begin{aligned} f(n) &= 9 && \text{[By def., since } n = 2\text{]} \quad (7) \\ &\leq 3^2 && \quad (8) \\ &= 3^n && \quad (9) \end{aligned}$$

Thus, $P(n)$ follows in this step.

5. Base Case ($n = 3$)

Let $n = 3$.

Then,

$$f(n) = f(n-1) + 3f(n-2) + 9f(n-3) \quad \text{[By def., since } n = 2\text{]} \tag{10}$$

$$= 9 + 3 \cdot 3 + 9 \cdot 1 \quad \text{[By def., since } n-1 = 2,\ n-2 = 1,\ n-3 = 0\text{]} \tag{11}$$

$$= 3^2 + 3^2 + 3^2 \tag{12}$$

$$= 3^3 \tag{13}$$

$= 3^n \leq 3^n$

Thus, $P(n)$ follows in this step.

6. Case $(n > 3)$

Let $n > 3$.

Then, since $0 \leq n-3 < n-2 < n-1 < n$, $P(n-3)$, $P(n-2)$, $P(n-1)$ holds by induction hypothesis. That is, $P(n-3) \leq 3^{n-3}$, $P(n-2) \leq 3^{n-2}$, $P(n-1) \leq 3^{n-1}$.

Thus,

$$\begin{aligned} f(n) &= f(n-1) + 3f(n-2) + 9f(n-3) && \text{[By def., since } n > 2\text{]} \quad (14) \\ &\leq 3^{n-1} + 3 \cdot 3^{n-2} + 9 \cdot 3^{n-3} && \text{[By header]} \quad (15) \\ &= 3^{n-1} + 3^{n-1} + 3^{n-1} && (16) \\ &= 3^n && (17) \end{aligned}$$

So, $P(n)$ follows from $H(n)$ in this step.