

CSC373 Worksheet 5 Solution

August 8, 2020

1. Rough Works:

Assume that a flow network $G = (V, E)$ violates the assumption that the network contains a path $s \rightsquigarrow v \rightsquigarrow t$ for all vertices $v \in V$. Let u be a vertex for which there is no path $s \rightsquigarrow u \rightsquigarrow t$.

I must show such that there is no flow at vertex u . That is, there exists a maximum flow f in G such that $f(u, v) = f(v, u) = 0$ for all vertices $v \in V$.

Assume for the sake of contradiction that there is a vertex u with flow f . That is, there exists some vertices $v \in V$ such that $f(u, v) > 0$ or $f(v, u) > 0$.

I see that three cases follows, and I will prove each separately.

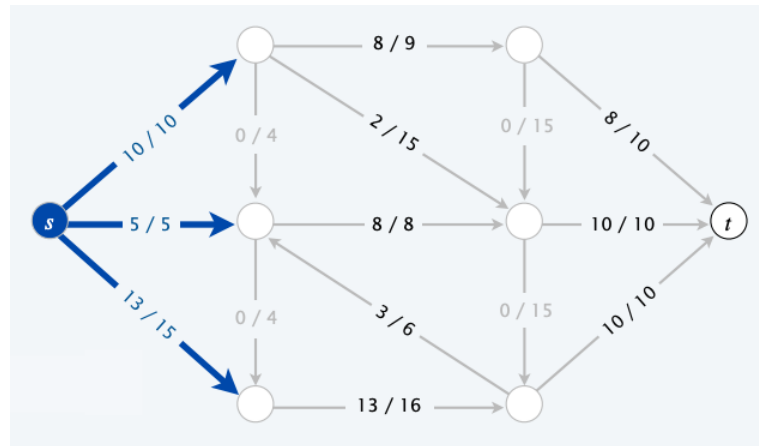
1. **Cases 1:** $f(u, v) > 0$ and $f(v, u) = 0$
2. **Cases 2:** $f(u, v) = 0$ and $f(v, u) > 0$
3. **Cases 3:** $f(u, v) > 0$ and $f(v, u) > 0$
 - Write that path $s \rightsquigarrow v \rightsquigarrow t$ exists
 - Write that this results in contradiction to the header that a vertex u has no path $s \rightsquigarrow u \rightsquigarrow t$.
 - Conclude that $f(u, v) = 0$ and $f(v, u) = 0$

Notes

• **Maximum Flow:**

- Finds a flow of maximum value ^[1]

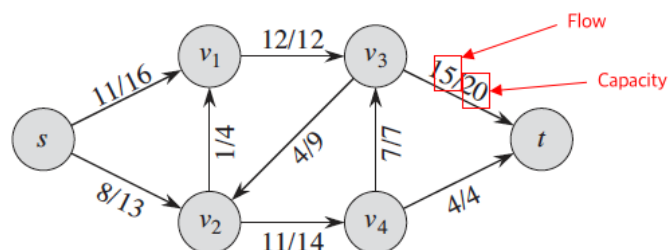
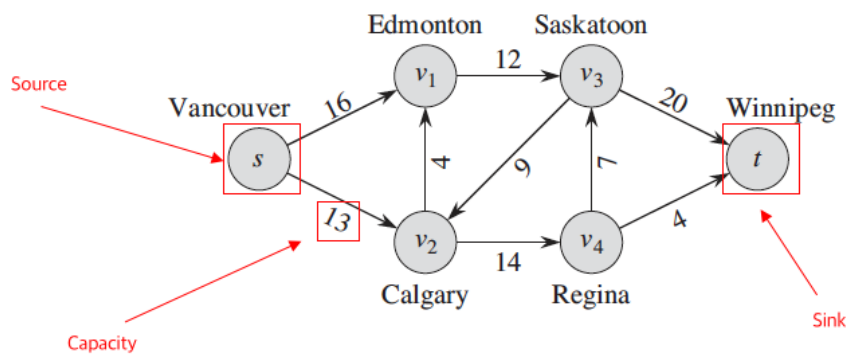
Example



Here, the maximum flow is $10 + 5 + 13 = 28$

• **Flow Network:**

- $G = (V, E)$ is a directed graph in which each edge $(u, v) \in E$ has a nonnegative capacity $c(u, v) \geq 0$.
- Two vertices must exist: **source** s and **sink** t
- **path** from source s to vertex v to sink t is represented by $s \rightsquigarrow v \rightsquigarrow t$



• **Capacity:**

- Is a non-negative function $f : V \times V \rightarrow \mathbb{R}_{\geq 0}$

- Has **capacity constraint** where for all $u, v \in V$ $0 \leq f(u, v) \leq c(u, v)$
 - * Means flow cannot be above capacity constraint

• **Flow:**

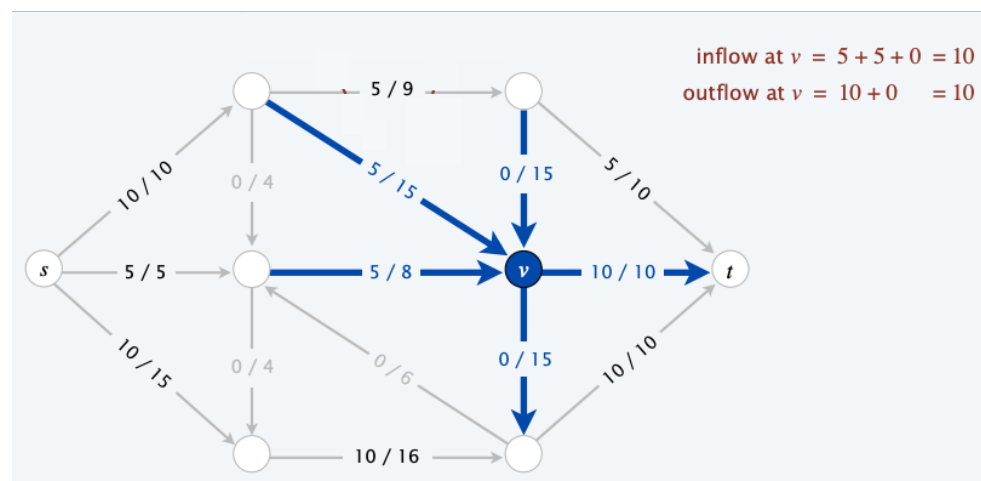
- Is a real valued function $f : V \times V \rightarrow \mathbb{R}$ in G
- Satisfies **capacity constraint** (i.e for all $u, v \in V$, $0 \leq f(u, v) \leq c(u, v)$)
- Satisfies **flow conservation**

For all $u \in V - \{s, t\}$, we require

$$\sum_{v \in V} f(v, u) = \sum_{v \in V} f(u, v) \quad (1)$$

Means flow into vertex u is the same as flow going out of vertex u . ^[1]

Example:



References

- 1) Princeton University, Network Flow 1, link