

Worksheet 4 Review 2

April 12, 2020

Question 1

- a. $\exists n \in \mathbb{N}, n > 3 \wedge n^2 - 1.5n \geq 5$
- b. The variable is existentially quantified
- c. Because the variable is existentially quantified, the variable's value should be a *concrete* natural number
- d. **Statement:** $\exists n \in \mathbb{N}, n > 3 \wedge n^2 - 1.5n \geq 5$

Proof. Let $n = 5$.

We will prove $n > 3 \wedge n^2 - 1.5n \geq 5$.

First, we need to prove $n > 3$.

The header tells us $n = 5$.

Using this fact, we can conclude $n > 3$.

Now, we need to show $n^2 - 1.5n \geq 5$.

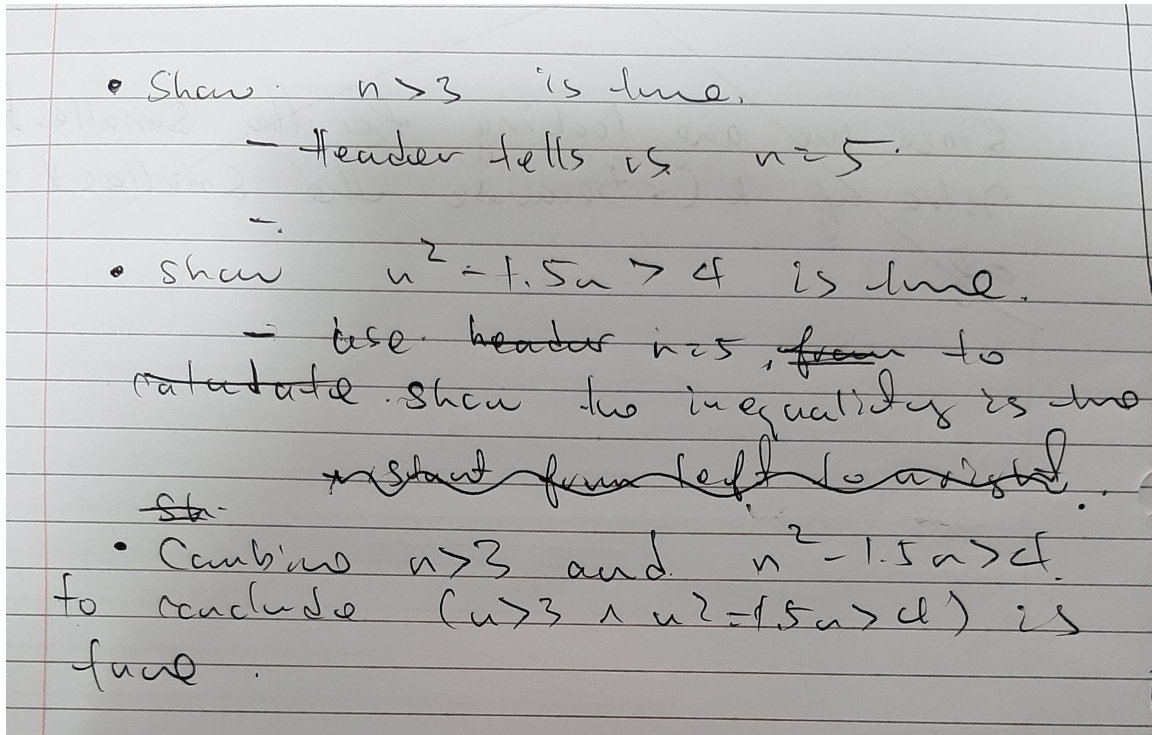
Using the fact $n = 5$, we can calculate

$$\begin{aligned} n^2 - 1.5n &= 25 - 7.5 & (1) \\ &= 17.5 & (2) \\ &\geq 5 & (3) \end{aligned}$$

Finally, since $n > 3$ and $n^2 - 1.5n \geq 5$ are true, we can conclude $n > 3 \wedge n^2 - 1.5n \geq 5$ are true. \square

Notes:

- Used the following pseudoproof used for this problem. Proof really feels smoother.



e. $\forall n \in \mathbb{N}, n \geq 3 \Rightarrow n^2 - 1.5n > 4$

f. The variable is universally quantified.

g. Because the variable is universally quantified, the variable's value should be an arbitrary natural number.

h. The assumption made is $n > 3$.

This conclusion is made by looking at the L.H.S of the \Rightarrow operator.

i. **Statement:** $\forall n \in \mathbb{N}, n > 3 \Rightarrow n^2 - 1.5n > 4$

Proof. Let $n \in \mathbb{N}$. Assume $n \geq 3$.

We will prove $n^2 - 1.5n > 4$.

Using the fact $n \geq 3$, we can conclude

$$n^2 - 1.5n \geq (3)^2 - 1.5(3) \quad (1)$$

$$= 9 - 4.5 \quad (2)$$

$$= 4.5 \quad (3)$$

$$> 4 \quad (4)$$

□

Question 2

a. $\forall n \in \mathbb{N}, n > 5 \Rightarrow 2 \mid n \vee 3 \mid n$

b. $\exists n \in \mathbb{N}, n > 5 \wedge (2 \nmid n) \wedge (3 \nmid n)$

Question 3