

# CSC373 Worksheet 6 Solution

August 12, 2020

## 1. Notes:

### • Linear Programming

- Is a method to achieve the best outcome (such as maximum profit or lowest cost) in a mathematical model whose requirements are represented by linear relationships. <sup>[1]</sup>
- Is named to make it sound cool for government funding
  - \* Like dynamic programming
- Applications
  - \* Microeconomics (maximize profits, minimize costs)
  - \* Company management

### • Standard Form

- Is a form of linear programming
- Are about maximizing, not minimizing <sup>[2]</sup>
- All have a positivity constraint for each variable <sup>[2]</sup>
- All other constraints are all of the form “linear combination of variables  $\leq$  constant”. <sup>[2]</sup>

The diagram shows a linear programming problem in standard form with three annotations:

- 3. Are about maximizing and not minimizing**: An arrow points to the word "Maximize" in the objective function.
- 2. constraints of the form  $\sum a_{ij} x_j \leq b_i$** : An arrow points to the right-hand side of the constraint equations.
- 1. non-negativity constraints for each variable**: An arrow points to the non-negativity constraint at the bottom.

The problem is written as:

$$\begin{aligned} &\text{Maximize } c_1x_1 + c_2x_2 + \cdots c_nx_n \\ &\text{subject to} \\ &\quad a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n \leq b_1 \\ &\quad a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n \leq b_2 \\ &\quad \vdots \\ &\quad a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n \leq b_m \\ &\quad x_1, x_2, \dots, x_n \geq 0 \end{aligned}$$

- **Converting Linear Programming to Standard Form**

- 1) Multiply inequality by -1 to get non-negative RHS <sup>[3]</sup>
- 2) Convert inequalities to equalities by adding or subtracting non-negative slack variables <sup>[3]</sup>
- 3) Resolve unrestrictive variables by writing the variable as the difference of two new non-negative variables <sup>[3]</sup>

**Example:**

**Minimize**

$$80x + 60y$$

**subject to**

$$x + y \geq 1$$

$$-.05x + .07y \leq 0$$

$$x, y \geq 0.$$

Introduce two new variables  $s_1, s_2 \geq 0$ .  
Convert inequalities to equalities

**Minimize**

$$80x + 60y$$

**subject to**

$$x + y - s_1 = 1$$

$$-.05x + .07y + s_2 = 0$$

$$x, y, s_1, s_2 \geq 0.$$

**References:**

- 1) Wikipedia, Linear Programming, [link](#)
- 2) Instituto de Matematicas, Standard form for Linear Programs, [link](#)