Worksheet 7 Review 2

April 15, 2020

Question 1

a. In this case assume that $n \leq 1$.

We want to show $n \leq 1$.

Since the assumption tells us $n \leq 1$, we can conclude this is true.

b. **Pseudoproof:**

Let $a = d$ and $b = k$. Assume there exists $d \in \mathbb{N}$ where $(\exists k \in \mathbb{Z}, n = dk) \wedge d \neq 1 \wedge d \neq n$. Assume $n > 1$

We need to prove that $n \nmid a$, $n \nmid b$ and $n \mid ab$.

1. Show $n \nmid a$.

   First, we need to show $n \nmid a$.

   1. Show $n \geq d$.

The fact 2 tells us

$$\forall x, y \in \mathbb{N}, y \geq 1 \land x \mid y \Rightarrow 1 \leq x \leq y \tag{1}$$

and we know from headers that $d \mid n$, $n > 1$, and $n, d \in \mathbb{N}$.

Then, by using these facts, we can write

$$1 \leq d \leq n \tag{2}$$

2. Show that for n to divide d, $n = d$.

Now, the definition of divisibility tells us for n to divide d, there must be some $k_1 \in \mathbb{Z}$ such that d is equal to $k_1 \cdot n$.

Then, since we know $n \geq d$, by using these facts, we can conclude the definition of divisibility is satisfied when $k_1 = 1$, or when $n = d$.

3. Conclude $n \nmid a$.

Then, since we know from header that $n \neq d$, we can conclude $n \nmid d$.

First, we need to show $n \nmid a$.

The fact 2 tells us

$$\forall x, y \in \mathbb{N}, y \geq 1 \land x \mid y \Rightarrow 1 \leq x \leq y \tag{3}$$

and we know from headers that $d \mid n$, $n > 1$, and $n, d \in \mathbb{N}$.

Then, by using these facts, we can write

$$1 \leq d \leq n \tag{4}$$

Now, the definition of divisibility tells us for n to divide d, there must be some $k_1 \in \mathbb{Z}$ such that d is equal to $k_1 \cdot n$.

Then, since we know $n \geq d$, by using these facts, we can conclude the definition of divisibility is satisfied only when $k_1 = 1$, or when $n = d$.

Then, since we know from the header that $n \neq d$, we can conclude $n \nmid d$.

Then, since we know $d = a$ from the header, we can conclude $n \nmid a$.

2. Show $n \nmid b$
   - Show $k \mid n$
   - Show $k \geq 1$.

The header tells us $n > 1$ $d \geq 0$, and we know from assumption that $n = dk$.

Since the facts tell us $k \leq 0$ results in $n \leq 0$ and this cannot happen, we can conclude $k \geq 1$.

- Show $n \geq k$ using the fact $k \mid n$ and $k \geq 1$.

The fact 2 tells us

$$\forall x, y \in \mathbb{N}, y \geq 1 \wedge x \mid y \Rightarrow 1 \leq x \leq y \tag{5}$$

and we know from headers that $d \mid n$, $n > 1$, and $n, d \in \mathbb{N}$.

Then, by using these facts, we can write

$$1 \leq d \leq n \tag{6}$$

- Show that for n to divide k, $n = k$.

Now, the definition of divisibility tells us for n to divide k, there must be some $k_1 \in \mathbb{Z}$ such that d is equal to $k_1 \cdot n$.

Then, since we know $n \geq d$, by using these facts, we can conclude the definition of divisibility is satisfied when $k_1 = 1$, or when $n = d$.

- Conclude $n \nmid b$.

Then, since we know from header that $n \neq d$, we can conclude $n \nmid d$.

3. Show $n \mid ab$

Question 2

Question 3