

# Midterm 2 Version 1 Review

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1. a) 1100100

b)  $-\sum_{i=0}^{n-1} 3^i$

## Notes:

- Balanced Ternary
  - is a way of representing numbers
  - balanced ternary is in base 3, and has values 1,0 or -1

$$\sum_{i=0}^{n-1} d_i \cdot 3^i \text{ where } d_i \in \{0, 1, -1\} \quad (1)$$

c) i.  $f(n) \in \Omega(n)$

True (since  $n^2 + 10n + 2 \geq cn$ )

ii.  $g(n) \in \Omega(n)$

False (Let  $c = 100, n_0 = 100$ . Then  $100 \log_2 n < 100n$ )

iii.  $f(n) \in \mathcal{O}(g(n))$

False ( $f(n) = n^2 + 10n + 2$  grows faster than  $g(n) = 100 \log_2 n$ )

iv.  $f(n) \in \Theta(g(n))$

True (Set  $c_1 = -1, c_2 = 1, n_1 = 100$ . Then  $c_1 f(n) \leq g(n) \leq c_2 f(n)$ )

v.  $g(n) \in \Theta(\log_3 n)$

True (set  $c_1 = -1, c_2 = 1, n_1 = 2$ . Then  $c_1 g(n) \leq \log_3 n \leq c_2 g(n)$ )

vi.  $g(n) \in \Theta(\log_3 n)$

False (set  $c_1 = -1, c_2 = 1, n_1 = 2$ . Then  $c_1 g(n) \leq \log_3 n \leq c_2 g(n)$ )

vii.  $f(n) + g(n) \in \Theta(f(n))$

True (set  $c_1 = -2, c_2 = 2, n_1 = 1$ . Then  $c_1(f(n) + g(n)) \leq f(n) \leq c_2(f(n) + g(n))$ )

**Notes:**

- $g \in \Omega(f) : \exists c, n_o \in \mathbb{R}^+, \forall n \in \mathbb{N}, n \geq n_o \Rightarrow g(n) \geq cf(n)$ , where  $f, g : \mathbb{N} \rightarrow \mathbb{R}^{\geq 0}$
  - $g \in \mathcal{O}(f) : \exists c, n_o \in \mathbb{R}^+, \forall n \in \mathbb{N}, n \geq n_o \Rightarrow g(n) \leq cf(n)$ , where  $f, g : \mathbb{N} \rightarrow \mathbb{R}^{\geq 0}$
  - $g \in \Theta(f) : g \in \mathcal{O}(f) \wedge g \in \Omega(f)$
- or
- $g \in \Theta(f) : \exists c_1, c_2, n_1 \in \mathbb{R}^+, \forall n \in \mathbb{N}, n \geq n_1 \Rightarrow c_1 g(n) \leq f(n) \leq c_2 g(n)$ , where  $f, g : \mathbb{N} \rightarrow \mathbb{R}^{\geq 0}$