

Worksheet 10 Review

March 28, 2020

Question 1

a.

$$(165)_8 = 5 \cdot 8^0 + 6 \cdot 8^1 + 1 \cdot 8^2 \quad (1)$$

$$= 5 + 48 + 64 \quad (2)$$

$$= 53 + 64 \quad (3)$$

$$= 117 \quad (4)$$

b.

$$(B4)_{16} = 4 \cdot 16^0 + 11 \cdot 16^1 \quad (1)$$

$$= 4 + (11 \cdot 16) \quad (2)$$

$$= 4 + 176 \quad (3)$$

$$= 180 \quad (4)$$

Question 2

a.

$$\begin{aligned}357 \div 2 &= 178, \text{ remainder } \mathbf{1}, \\178 \div 2 &= 89, \text{ remainder } \mathbf{0}, \\89 \div 2 &= 44, \text{ remainder } \mathbf{1}, \\44 \div 2 &= 22, \text{ remainder } \mathbf{0}, \\22 \div 2 &= 11, \text{ remainder } \mathbf{0}, \\11 \div 2 &= 5, \text{ remainder } \mathbf{1}, \\5 \div 2 &= 2, \text{ remainder } \mathbf{1}, \\2 \div 2 &= 1, \text{ remainder } \mathbf{0}, \\1 \div 2 &= 0, \text{ remainder } \mathbf{1}\end{aligned}$$

Combining it together, the binary representation of 357 is $(101100101)_2$

b.

$$\begin{aligned}1 \cdot 2^0 + 0 \cdot 2^1 + 1 \cdot 2^2 &= \frac{1 + 0 + 4}{8^0} = 5 \\0 \cdot 2^3 + 0 \cdot 2^4 + 1 \cdot 2^5 &= \frac{0 + 0 + 32}{8^1} = 4 \\1 \cdot 2^6 + 0 \cdot 2^7 + 1 \cdot 2^8 &= \frac{64 + 0 + 256}{8^2} = 5\end{aligned}$$

Combining it together, the octal representation of $(101100101)_2$ is $(545)_8$.

c.

$$\begin{aligned}357 \div 16 &= 22, \text{ remainder } \mathbf{5}, \\22 \div 16 &= 1, \text{ remainder } \mathbf{5}, \\1 \div 16 &= 0, \text{ remainder } \mathbf{1},\end{aligned}$$

Combining it together, the hexadecimal representation of 357 is $(155)_{16}$.

Correct Solution:

$$357 \div 16 = 22, \text{ remainder } \mathbf{5},$$

$$22 \div 16 = 1, \text{ remainder } \mathbf{6},$$

$$1 \div 16 = 0, \text{ remainder } \mathbf{1},$$

Combining it together, the hexadecimal representation of 357 is $(\mathbf{165})_{16}$.

Question 3

a.

$$0.375 \times 2 = 0.75 + \mathbf{0} \quad (1)$$

$$0.75 \times 2 = 0.5 + \mathbf{1} \quad (2)$$

$$0.5 \times 2 = 0 + \mathbf{1} \quad (3)$$

Combining the above, the binary representation of 0.375 is $(0.011)_2$.

Notes:

- Converting decimal to binary

$$0.8125 \times 2 = 0.625 + \mathbf{1} \quad (4)$$

$$0.625 \times 2 = 0.25 + \mathbf{1} \quad (5)$$

$$0.25 \times 2 = 0.5 + \mathbf{0} \quad (6)$$

$$0.5 \times 2 = 0 + \mathbf{1} \quad (7)$$

Binaries read *top to bottom*

b.

$$0.1 \times 2 = 0.2 + \mathbf{0} \quad (1)$$

$$0.2 \times 2 = 0.4 + \mathbf{0} \quad (2)$$

$$0.4 \times 2 = 0.8 + \mathbf{0} \quad (3)$$

$$0.8 \times 2 = 0.6 + \mathbf{1} \quad (4)$$

$$0.6 \times 2 = 0.2 + \mathbf{1} \quad (5)$$

$$0.2 \times 2 = 0.4 + \mathbf{0} \quad (6)$$

$$0.4 \times 2 = 0.8 + \mathbf{0} \quad (7)$$

$$0.8 \times 2 = 0.6 + \mathbf{1} \quad (8)$$

Combining the above, the binary representation of 0.1 is $(0.0\overline{0011})_2$.

Question 4

a.

$$\sum_{i=0}^{\infty} \left(\frac{1}{2} \right) = \frac{\frac{1}{2}}{1 - \frac{1}{2}} \quad (1)$$

$$= \frac{\frac{1}{2}}{\frac{1}{2}} \quad (2)$$

$$= 1 \quad (3)$$