

Worksheet 1 Review

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April 9, 2020

Question 1

- a. $A^c = \{1, 3, 4, 6\}$
- b. $A = U \setminus A$
- c. $A^c \cap B^c = \{x \mid x \in U, x \leq 0 \text{ and } x \geq 4\}$
 $A^c \cup B^c = \{x \mid x \in U, x < 1 \text{ and } x > 2\}$
 $(A \cap B)^c = \{x \mid x \in U, x < 1 \text{ and } x > 2\}$
 $(A \cup B)^c = \{x \mid x \in U, x \leq 0 \text{ and } x \geq 4\}$

Correct Solution:

$$A^c \cap B^c = \{x \mid x \in U, x \leq 0 \text{ or } x \geq 4\}$$
$$A^c \cup B^c = \{x \mid x \in U, x < 1 \text{ or } x > 2\}$$
$$(A \cap B)^c = \{x \mid x \in U, x < 1 \text{ or } x > 2\}$$
$$(A \cup B)^c = \{x \mid x \in U, x \leq 0 \text{ or } x \geq 4\}$$

It follows from above that $A^c \cap B^c = (A \cup B)^c$ and $A^c \cup B^c = (A \cap B)^c$

Question 2

- a. $T_0 = \{3, 6, 9, \dots\}$
 $T_1 = \{1, 4, 7, \dots\}$
 $T_2 = \{2, 5, 8, \dots\}$
 $T_3 = \{6, 12, 18, \dots\}$
- b. A partition of \mathbb{Z} is $\{T_0, T_1, T_2\}$.

All four sets can't be used because elements in T_3 overlaps with T_0 . A partition cannot have any elements in common.

Notes:

- **Definition of Partition:** Let A be a set. A (finite or infinite) collection of nonempty sets $\{A_1, A_2, A_3\}$ is called a **partition** of A when (1) A is the union of all of the A_i , and (2) the sets A_1, A_2, A_3, \dots do not have any element in common.

Question 3

a. All strings over the alphabet $\{0, 1\}$ of length three are

000, 100, 010, 001, 110, 101, 011, 111

b. $S_1 = \{aa, ab, ac, ba, bb, bc, ca, cb, cc\}$

$S_2 = \{a, b, c, aa, bb, cc, \dots\}$

$S_1 \cap S_2 = \{aa, bb, cc\}$

$S_1 \setminus S_2 = \{ab, ac, ba, bc, ca, cb\}$

c. $S_1 = (S_1 \cap S_2) \cup (S_1 \setminus S_2)$

Question 4

a.

	$\lfloor x \rfloor$	$\lceil x \rceil$
$\frac{25}{4}$	6	7
0.99	0	1
-2.01	-3.0	-2.0

Notes:

- floor of a negative number: ceiling but with negative sign
- ceiling of a negative number: floor but with negative sign

b. **Domain of the floor & ceiling function:** \mathbb{R}

Codomain of the floor & ceiling function: \mathbb{N}

c. The statement is false. Consider example $x = -0.5$ and $y = 0.5$.

Then, $\lfloor x + y \rfloor = 0$ and $\lfloor x \rfloor + \lfloor y \rfloor = -1 + 0 = -1$.

Question 5

a. $\sum_{k=1}^3 (k+1) = (1+1) + (2+1) + (3+1)$

$$\sum_{m=0}^1 \frac{1}{2^m} = \frac{1}{2^0}$$

$$\sum_{k=-1}^2 (k^2 + 3) = ((-1)^2 + 3) + (0^2 + 3) + (1^2 + 3) + (2^2 + 3)$$

$$\sum_{j=0}^4 (-1)^j \frac{j}{j+1} = (-1)^0 \cdot \frac{0}{0+1} + (-1) \cdot \frac{1}{1+1} + (-1)^2 \cdot \frac{2}{2+1} + (-1)^3 \cdot \frac{3}{3+1} + (-1)^4 \cdot \frac{4}{4+1}$$

$$\sum_{k=1}^5 (2 \cdot k) = (2 \cdot 1) + (2 \cdot 2) + (2 \cdot 3) + (2 \cdot 4) + (2 \cdot 5)$$

$$\prod_{i=2}^4 \frac{i(i+2)}{(i-1)(i+1)} = \left(\frac{0 \cdot (0+2)}{(0-1)(0+1)} \right) \left(\frac{1 \cdot (1+2)}{(1-1)(1+1)} \right) \left(\frac{2 \cdot (2+2)}{(2-1)(2+1)} \right) \left(\frac{3 \cdot (3+2)}{(3-1)(3+1)} \right) \left(\frac{4 \cdot (4+2)}{(4-1)(4+1)} \right)$$

b. $3 + 6 + 12 + 24 + 48 + 96 = \sum_{i=0}^6 3 \cdot 2^i$

$$\frac{1}{3} + \frac{4}{9} + \frac{9}{27} + \frac{16}{81} + \frac{25}{243} + \frac{36}{729} = \sum_{i=1}^6 \frac{i^2}{3^i}$$

$$0 + 1 - 2 + 3 - 4 + 5 = \sum_{i=0}^5 (-1)^{i+1} \cdot i$$

$$\left(\frac{1}{1+1} \right) \times \left(\frac{2}{2+1} \right) \times \left(\frac{3}{3+1} \right) \times \left(\frac{k}{k+1} \right) = \prod_{i=1}^k \left(\frac{i}{i+1} \right)$$

$$\left(\frac{1 \cdot 2}{3 \cdot 4} \right) \times \left(\frac{2 \cdot 3}{4 \cdot 5} \right) \times \left(\frac{3 \cdot 4}{5 \cdot 6} \right) = \prod_{i=1}^3 \frac{i \cdot (i+1)}{(i+2) \cdot (i+3)}$$

Question 6

a. $3 \cdot \sum_{i=1}^n (2i - 3) + \sum_{i=1}^n (4 - 5i)$

By the pulling of constant,

$$\begin{aligned} 3 \cdot \sum_{i=1}^n (2i - 3) + \sum_{i=1}^n (4 - 5i) &= \sum_{i=1}^n 3(2i - 3) + \sum_{i=1}^n (4 - 5i) \\ &= \sum_{i=1}^n (6i - 9) + \sum_{i=1}^n (4 - 5i) \end{aligned}$$

Then, by the separating sums

$$\begin{aligned}
\sum_{i=1}^n (6i - 9) + \sum_{i=1}^n (4 - 5i) &= \sum_{i=1}^n (6i - 9) + (4 - 5i) \\
&= \sum_{i=1}^n (i - 5)
\end{aligned}$$