

Worksheet 16 Review

April 2, 2020

Question 1

a. Let $k \in \mathbb{N}$.

Here, the minimum possible change occurs for the loop variable in a single iteration when $i = i + 1$.

The maximum possible change occurs for the loop variable in a single iteration when $i = i + 6$.

The exact upper bound of the variable after k iteration is

$$i_k \leq 6k \tag{1}$$

The exact lower bound of the variable after k iteration is

$$k \leq i_k \tag{2}$$

Using the fact that the termination occurs when $i_k = n$, we can calculate that for the upper bound, the loop terminates when

$$6k \geq n \quad (3)$$

$$k \geq \frac{n}{6} \quad (4)$$

Because we know $\frac{n}{6}$ may be a decimal, we can conclude the closest value at which the loop terminates is when

$$k = \left\lceil \frac{n}{6} \right\rceil \quad (5)$$

Using the same fact, we can calculate that for the lower bound, the loop terminates when

$$k \geq n \quad (6)$$

It follows from above that for the lower bound, the smallest value of k at which the loop termination occurs is when

$$k = n \quad (7)$$

Then, we can conclude the function has asymptotic lower bound of $\Omega(n)$, and asymptotic upper bound of $\mathcal{O}(n)$.

Then, since both Ω and \mathcal{O} have the same value, $\Theta(n)$ is also true.

Correct Solution:

Here, the minimum possible change occurs for the loop variable in a single iteration when $i = i + 1$.

The maximum possible change occurs for the loop variable in a single iteration when $i = i + 6$.

The exact upper bound of the variable after k iteration is

$$i_k \leq 6k \quad (8)$$

The exact lower bound of the variable after k iteration is

$$k \leq i_k \quad (9)$$

Using the fact that the termination occurs when $i_k = n$, we can calculate that for the upper bound, the loop terminates when

$$6k \geq n \quad (10)$$

$$k \geq \frac{n}{6} \quad (11)$$

Because we know $\frac{n}{6}$ may be a decimal, we can conclude the closest value at which the loop terminates is when

$$k = \left\lceil \frac{n}{6} \right\rceil + 1 \quad (12)$$

Using the same fact, we can calculate that for the lower bound, the loop terminates when

$$k \geq n \quad (13)$$

It follows from above that for the lower bound, the smallest value of k at which the loop termination occurs is when

$$k = n + 1 \tag{14}$$

Then, we can conclude the function has asymptotic lower bound of $\Omega(n)$, and asymptotic upper bound of $\mathcal{O}(n)$.

Since both Ω and \mathcal{O} have the same value, $\Theta(n)$ is also true.

Notes:

- Realized $+ 1$ required after thinking there are n iterations between $i = 0$ and $i = n - 1$ when $i = i + 1$.
- Realized $+ 1$ in $\frac{n}{6}$ is required after playing with the following example $[0,1,2,3,4,5]$ and $[0,1,2,3,4,5,6]$.
- Realized $\lceil \frac{5}{6} \rceil = 1$ and $\lceil \frac{6}{6} \rceil = 1$, and indeed $+ 1$ is required to reach loop termination.
- Perhaps the choice of ceiling and floor can be determined by playing with examples.

Question 2

Question 3