

CSC373 Worksheet 6 Solution

August 12, 2020

1. Rough Works:

1. Multiply objective function by - 1

Maximize

$$-2x_1 - 7x_2 - x_3$$

Subject to

$$x_1 - x_3 = 7$$

$$3x_1 + x_2 \geq 7$$

$$x_2 \geq 0$$

$$x_3 \leq 0$$

2. Replace non-nonnegative constraints x_1

Maximize

$$-2x'_1 + 2x''_1 - 7x_2 - x_3$$

Subject to

$$\begin{aligned}
 x'_1 - x''_1 - x_3 &= 7 \\
 3x'_1 - 3x''_1 + x_2 &\geq 7 \\
 x'_1, x''_1, x_2 &\geq 0 \\
 x_3 &\leq 0
 \end{aligned}$$

3. Replace non-nonnegative constraints x_3

Maximize

$$-2x'_1 + 2x''_1 - 7x_2 - x'_3 + x''_3$$

Subject to

$$\begin{aligned}
 x'_1 - x''_1 - x'_3 + x''_3 &= 7 \\
 3x'_1 - 3x''_1 + x_2 &\geq 7 \\
 x'_1, x''_1, x_2, x'_3, x''_3 &\geq 0
 \end{aligned}$$

4. Replace equality constraints with \geq and \leq

Maximize

$$-2x'_1 + 2x''_1 - 7x_2 - x'_3 + x''_3$$

Subject to

$$\begin{aligned}
 x'_1 - x''_1 - x'_3 + x''_3 &\leq 7 \\
 x'_1 - x''_1 - x'_3 + x''_3 &\geq 7 \\
 3x'_1 - 3x''_1 + x_2 &\geq 7 \\
 x'_1, x''_1, x_2, x'_3, x''_3 &\geq 0
 \end{aligned}$$

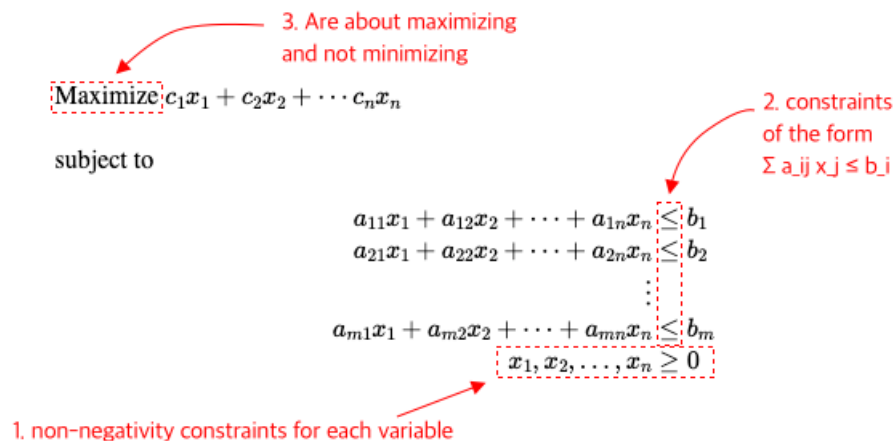
Notes:

- Linear Programming

- Is a method to achieve the best outcome (such as maximum profit or lowest cost) in a mathematical model whose requirements are represented by linear relationships. ^[1]
- Is named to make it sound cool for government funding
 - * Like dynamic programming
- Applications
 - * Microeconomics (maximize profits, minimize costs)
 - * Company management

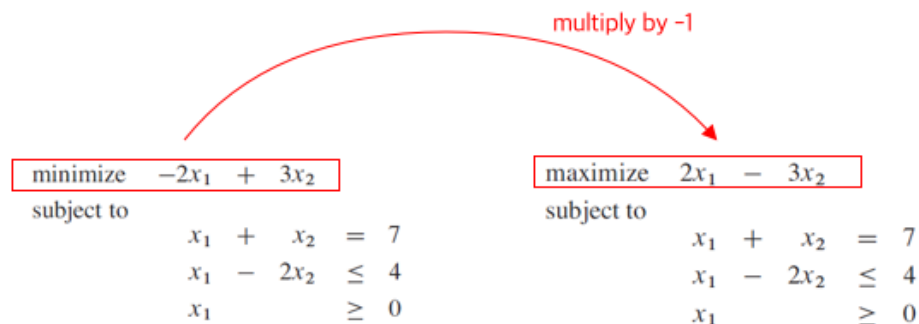
• Standard Form

- Is a form of linear programming
- Are about maximizing, not minimizing ^[2]
- All have a positivity constraint for each variable ^[2]
- All other constraints are all of the form “linear combination of variables \leq constant”. ^[2]



• Converting Linear Programming to Standard Form

- 1) The objective function might be a minimization rather than a maximization
 - Negate coefficients of the objective function



- 2) There might be variables without nonnegativity constraints
- Replace each non-nonnegative variable x_i with x'_i and x''_i
 - Modify linear program

maximize $2x_1 - 3x_2$
subject to
 $x_1 + x_2 = 7$
 $x_1 - 2x_2 \leq 4$
 $x_1 \geq 0$

maximize $2x_1 - 3x'_2 + 3x''_2$
subject to
 $x_1 + x'_2 - x''_2 = 7$
 $x_1 - 2x'_2 + 2x''_2 \leq 4$
 $x_1, x'_2, x''_2 \geq 0$

Replace x_i with x'_i and x''_i

x_2 is not nonnegative :(

They are now nonnegative :) Yayy!!

- 3) There might be **equality constraints**, which have an equal sign rather than a less-than-or-equal-to sign
- Replace equality constraint $f(x_1, x_2, \dots, x_n) = b$ with $f(x_1, x_2, \dots, x_n) \leq b$ and $f(x_1, x_2, \dots, x_n) \geq b$

maximize $2x_1 - 3x'_2 + 3x''_2$
subject to
 $x_1 + x'_2 - x''_2 \leq 7$
 $x_1 + x'_2 - x''_2 \geq 7$
 $x_1 - 2x'_2 + 2x''_2 \leq 4$
 $x_1, x'_2, x''_2 \geq 0$

maximize $2x_1 - 3x_2 + 3x_3$
subject to
 $x_1 + x_2 - x_3 \leq 7$
 $-x_1 - x_2 + x_3 \leq -7$
 $x_1 - 2x_2 + 2x_3 \leq 4$
 $x_1, x_2, x_3 \geq 0$

Multiply incorrect constraints by -1

- 4) There might be **inequality constraints**, but instead of having a less-than-or-equal-to sign
- Multiply incorrect inequality constraints by -1

maximize $2x_1 - 3x'_2 + 3x''_2$
subject to
 $x_1 + x'_2 - x''_2 = 7$
 $x_1 - 2x'_2 + 2x''_2 \leq 4$
 $x_1, x'_2, x''_2 \geq 0$

maximize $2x_1 - 3x'_2 + 3x''_2$
subject to
 $x_1 + x'_2 - x''_2 \leq 7$
 $x_1 + x'_2 - x''_2 \geq 7$
 $x_1 - 2x'_2 + 2x''_2 \leq 4$
 $x_1, x'_2, x''_2 \geq 0$

Replace = with \leq and \geq

References:

- 1) Wikipedia, Linear Programming, [link](#)
- 2) Instituto de Matematicas, Standard form for Linear Programs, [link](#)