Worksheet 14 Solution

March 25, 2020

Question 1

a. **Inner Loop:** n

Outer Loop: $n - 5$

Theta Expressions: $\Theta(n^2)$

Correct Solution:

Inner Loop: n

Outer Loop: $n \cdot \left\lceil \frac{n}{5} \right\rceil$

Theta Expressions: $\Theta(n^2)$

b. **Inner Loop:** $\frac{n}{3} + (n - 2)$

Outer Loop: $n - 4$

Theta Expressions: $\Theta(n^2)$

Correct Solution:

Inner Loop: $\lceil \log_3 n \rceil + \left\lceil \frac{n}{2} \right\rceil$

Outer Loop: $max(0, n - 4) \cdot \left[\lceil \log_3 n \rceil + \left\lceil \frac{n}{2} \right\rceil \right]$

Theta Expressions: $\Theta(n^2)$

c. **Inner Loop #2:** $\sum_{i=1}^{n} i = \dfrac{n(n+1)}{2}$

Inner Loop #1: $n \cdot \dfrac{n(n+1)}{2} = \dfrac{n^3+n^2}{2}$

Outer Loop: $\dfrac{n^3+n^2}{2} \cdot (n-4) = \dfrac{n^4-3n^3+4n^2}{2}$

Theta Expressions: $\Theta(n^4)$

Correct Solution:

Inner Loop #2: j

Inner Loop #1: $\sum_{j=1}^{n} j = \dfrac{n(n+1)}{2}$

Outer Loop: $\left\lceil \dfrac{n}{4} \right\rceil \cdot \dfrac{n(n+1)}{2}$

Theta Expressions: $\Theta(n^3)$

d. **Inner Loop:** 2^n

Outer Loop: $\displaystyle\sum_{i=0}^{\frac{n}{2}-1} 2^i = 2^{\frac{n}{2}-1}$

Theta Expressions: $\Theta(2^n)$

Correct Solution:

Inner Loop: i

Outer Loop: $\sum_{i=0}^{\log n-1} 2^i = \dfrac{1-2^{\log n-1+1}}{1-2} = 2^{\log n} - 1 = n-1$

Theta Expressions: $\Theta(n)$

Question 2

- **Inner Loop #2:** $j - i$

  Inner Loop #1: $\sum_{j=i}^{n-1}(j-i)$

  Outer Loop: $\sum_{i=0}^{n-1}\sum_{j=i}^{n-1}(j-i)$

  Calculation:

$$
\begin{aligned}
\sum_{i=0}^{n-1}\sum_{j=i}^{n-1}(j-i) &= \sum_{i=0}^{n-1-i}\sum_{j'=0}^{n-1}(j'+i-i) && (1)\\
&= \sum_{i=0}^{n-1}\sum_{j'=0}^{n-1-i} j' && (2)\\
&= \sum_{i=0}^{n-1}\frac{(n-1-i)(n-i)}{2} && (3)\\
&= \sum_{i=0}^{n-1}\frac{(n-1-i)(n-i)}{2} && (4)\\
&= \frac{1}{2}\left[\sum_{i=0}^{n-1} n^2 - \sum_{i=0}^{n-1} n - 2n\sum_{i=0}^{n-1} i + \sum_{i=0}^{n-1} i + \sum_{i=0}^{n-1} i^2\right] && (5)\\
&= \frac{1}{2}\left[\sum_{i=0}^{n-1} n^2 - \sum_{i=0}^{n-1} n - 2n\sum_{i=0}^{n-1} i + \sum_{i=0}^{n-1} i + \sum_{i=0}^{n-1} i^2\right] && (6)\\
&= \frac{1}{2}\left[\frac{(n^2-n)}{2} + \frac{2n^3-n^2-2n^2+n}{6}\right] && (7)\\
&= \frac{1}{2}\left[\frac{3n^2-3n}{6} + \frac{2n^3-3n^2+n}{6}\right] && (8)\\
&= \frac{1}{2}\left[\frac{2n^3-2n}{6}\right] && (9)\\
&= \frac{n^3-n}{6} && (10)
\end{aligned}
$$

  Theta Expressions: $\Theta(n^3)$

Correct Solution:

Inner Loop: i

Outer Loop: $\sum_{i=0}^{\log n-1} 2^i = \dfrac{1-2^{\log n-1+1}}{1-2} = 2^{\log n} - 1 = n - 1$

Theta Expressions: $\Theta(n)$

Note

- forgot that if starts at 0, has total of n + 1 many iterations.
- must be grouped in terms of variables before expanding
  $\frac{1}{2}\sum_{i=0}^{n-1}[n^2+n-2in-i+i^2]$

$$\text{NO:}\frac{1}{2}\left[\sum_{i=0}^{n-1} n^2 + \sum_{i=0}^{n-1} n - \sum_{i=0}^{n-1} 2in - \sum_{i=0}^{n-1} i + \sum_{i=0}^{n-1} i^2\right] \tag{11}$$

$$\text{YES:}\frac{1}{2}\left[\sum_{i=0}^{n-1}(n^2+n) + \sum_{i=0}^{n-1}(2n-1)i + \sum_{i=0}^{n-1} i^2\right] \tag{12}$$

- replace whole (j - i + 1) in $\sum_{j=i}^{n-1}(j-i+1)$ by setting $j' = j - i + 1$.
- the formula for arithematic sum starting at $i = 1$ is

$$\sum_{i=1}^{n} a_i = \left(\frac{n}{2}\right)(a_1 + a_n) \tag{13}$$

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