Midterm 1 Version 2 Solution

March 19, 2020

Question 1

a. Since

$S_1 = \{1, 2, 3, 5, 7, 11, 13, 17, 19, 23, 29\}$, and $S_2 = \{1, 2, 3, 5, 6, 10, 15, 30\}$,

$S_1 \cap S_2 = \{1, 2, 3, 5\}$

b. See the table below

p	q	r	$\neg p$	$\neg p \Leftrightarrow q$	$(\neg p \Leftrightarrow q) \Rightarrow r$
T	T	T	F	F	T
T	T	F	F	F	T
T	F	T	F	T	T
F	T	T	T	T	F
T	F	F	F	T	F
F	F	T	T	F	T
F	F	F	T	F	T

Correct Solution:

p	q	r	$\neg p$	$\neg p \Leftrightarrow q$	$(\neg p \Leftrightarrow q) \Rightarrow r$
T	T	T	F	F	T
T	T	F	F	F	T
T	F	T	F	T	T
*F	*T	*T	*T	*T	*T
T	F	F	F	T	F
*F	*T	*F	*T	*T	*F
F	F	T	T	F	T
F	F	F	T	F	T

$*$ = Incorrect/missing solution

c. Let $x \in \mathbb{N}$. Assume $P(x)$.

We will prove that there is a natural number y such that the predicate $Q(x, y)$ is true.

Question 2

a. $\forall x \in P,\ Cat(x) \wedge Loves(x, x)$

b. $\forall x \in P,\ \exists y \in P,\ Cat(x) \wedge Cute(y) \wedge Loves(x, y)$

c. $\exists x \in P,\ Cat(x) \wedge Cute(x) \Rightarrow \forall y \in P, Cat(y) \wedge Cute(y)$

d. $\forall p_1, p_2 \in P, p_1 \neq p_2 \wedge Loves(p_1, p_2) \wedge Loves(p_2, p_1) \Rightarrow (Cat(p_1) \wedge \neg Cat(p_2)) \vee (\neg Cat(p_1) \wedge Cat(p_2))$

Question 3

Question 4