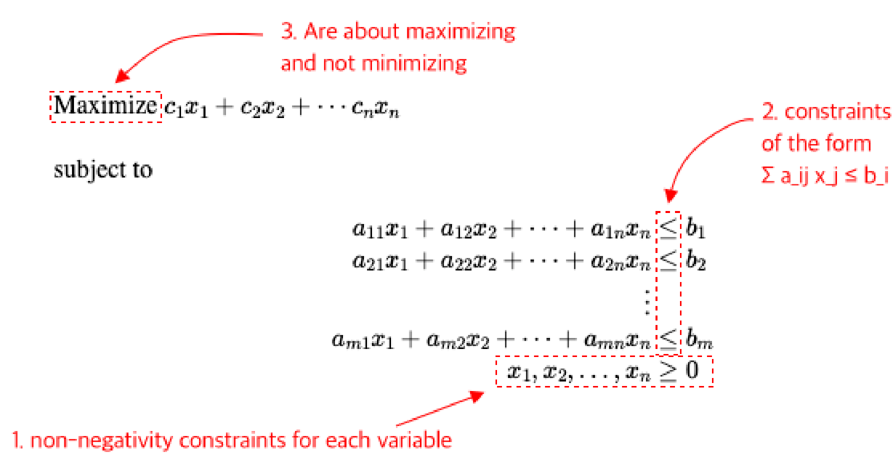
CSC373 Worksheet 6 Solution

August 12, 2020

1. **Notes:**

- **Linear Programming**
  - Is a method to achieve the best outcome (such as maximum profit or lowest cost) in a mathematical model whose requirements are represented by linear relationships. [1]
  - Is named to make it sound cool for government funding
    - Like dynamic programming
  - Applications
    - Microeconomics (maximize profits, minimize costs)
    - Company management
- **Standard Form**
  - Is a form of linear programming
  - Are about maximizing, not minimizing [2]
  - All have a positivity constraint for each variable [2]
  - All other constraints are all of the form "linear combination of variables $\leq$ constant". [2]



- **Converting Linear Programming to Standard Form**

1) The objective function might be a minimization rather than a maximization
   - Negate coefficients of the objective function

multiply by -1

$$\begin{array}{llcl}
\boxed{\text{minimize} \quad -2x_1 + 3x_2} & & & \\
\text{subject to} & & & \\
& x_1 + x_2 & = & 7 \\
& x_1 - 2x_2 & \leq & 4 \\
& x_1 & \geq & 0
\end{array}
\qquad
\begin{array}{llcl}
\boxed{\text{maximize} \quad 2x_1 - 3x_2} & & & \\
\text{subject to} & & & \\
& x_1 + x_2 & = & 7 \\
& x_1 - 2x_2 & \leq & 4 \\
& x_1 & \geq & 0
\end{array}$$

2) There might be variables without nonnegativity constraints
   - Replace each non-nonnegative variable x_i with x_i' and x_i''
   - Modify linear program

Replace x_i with x_i' and x_i"

$$\begin{array}{llcl}
\text{maximize} & 2x_1 - 3x_2 & & \\
\text{subject to} & & & \\
& x_1 + x_2 & = & 7 \\
& x_1 - 2x_2 & \leq & 4 \\
& x_1 \; \boxed{} & \geq & 0
\end{array}
\qquad
\begin{array}{llcl}
\text{maximize} & 2x_1 - 3x_2' + 3x_2'' & & \\
\text{subject to} & & & \\
& x_1 + x_2' - x_2'' & = & 7 \\
& x_1 - 2x_2' + 2x_2'' & \leq & 4 \\
& x_1, x_2', x_2'' & \geq & 0
\end{array}$$

x_2 is not nonnegative :(They are now nonnegative :)! Yayy!!

3) There might be **equality constraints**, which have an equal sign rather than a less-than-or-equal-to sign
   - Replace equality constraint $f(x_1, x_2, ..., x_n) = b$ with $f(x_1, x_2, ..., x_n) \leq b$ and $f(x_1, x_2, ..., x_n) \geq b$

Replace = with ≤ and ≥

$$\begin{array}{llcl}
\text{maximize} & 2x_1 - 3x_2' + 3x_2'' & & \\
\text{subject to} & & & \\
& x_1 + x_2' - x_2'' & \boxed{=} & 7 \\
& x_1 - 2x_2' + 2x_2'' & \leq & 4 \\
& x_1, x_2', x_2'' & \geq & 0
\end{array}
\qquad
\begin{array}{llcl}
\text{maximize} & 2x_1 - 3x_2' + 3x_2'' & & \\
\text{subject to} & & & \\
& x_1 + x_2' - x_2'' & \leq & 7 \\
& x_1 + x_2' - x_2'' & \geq & 7 \\
& x_1 - 2x_2' + 2x_2'' & \leq & 4 \\
& x_1, x_2', x_2'' & \geq & 0 .
\end{array}$$

4) There might be **inequality constraints**, but instead of having a less-than-or-equal-to-sign

Example:

References:

1) Wikipedia, Linear Programming, link

2) Instituto de Mathematicas, Standard form for Linear Programs, link