Midterm 2 Version 1 Solution

April 3, 2020

Question 1

a.

$$\begin{aligned}
100 \div 2 &= 50, \textbf{Remainders } 0\\
50 \div 2 &= 25, \textbf{Remainders } 0\\
25 \div 2 &= 12, \textbf{Remainders } 1\\
12 \div 2 &= 6, \textbf{Remainders } 0\\
6 \div 2 &= 3, \textbf{Remainders } 0\\
3 \div 2 &= 1, \textbf{Remainders } 1\\
1 \div 2 &= 0, \textbf{Remainders } 1
\end{aligned}$$

Then, it follows from above that the binary representation of 100 is $(1100100)_2$.

b. The smallest number that can be expressed by an n-digit balanced ternary representation is

$$\sum_{i=0}^{n-1} d_i \cdot 3^i, \text{ where } d_i \in \{0, 1, 2\} \tag{1}$$

<u>Correct Solution:</u>

The smallest number that can be expressed by an n-digit balanced ternary representation is

$$-\left[\sum_{i=0}^{n-1} 3^i\right] \tag{2}$$

Notes:

- Realized professor is asking for an example of the smallest number.
- Learned a negative number could be expressed in in ternary or binary representation of numbers.

c.

$f(n) \in \Omega(n)$	True	$g(n) \in \Omega(n)$	False	$f(n) \in \mathcal{O}(g(n))$	False
$f(n) \in \Theta(g(n))$	False	$g(n) \in \Theta(\log_3 n)$	True	$f(n) + g(n) \in \Theta(f(n))$	True

Notes:

- $\forall g : \mathbb{N} \to \mathbb{R}^{\geq 0}$, and all numbers $a \in \mathbb{R}^{\geq 0}$, if $g \in \mathcal{O}(f)$, then $f + g \in \mathcal{O}(f)$
- $g \in \Theta(f) : \; g \in \mathcal{O}(f) \wedge g \in \Omega(f)$
  or
  $g \in \Theta(f) : \; \exists c_1, c_2, n_1 \in \mathbb{R}^+, \forall n \in \mathbb{N}, n \geq n_1 \Rightarrow c_1 g(n) \leq f(n) \leq c_2 g(n)$, where $f, g : \; \mathbb{N} \to \mathbb{R}^{\geq 0}$
- $g \in \Omega(f) : \; \exists c, n_o \in \mathbb{R}^+, \; \forall n \in \mathbb{N}, \; n \geq n_0 \Rightarrow g(n) \geq cf(n)$, where $f, g : \mathbb{N} \to \mathbb{R}^{\geq 0}$
- $g \in \mathcal{O}(f) : \; \exists c, n_o \in \mathbb{R}^+, \; \forall n \in \mathbb{N}, \; n \geq n_0 \Rightarrow g(n) \leq cf(n)$, where $f, g : \mathbb{N} \to \mathbb{R}^{\geq 0}$

d.

k	0	1	2
i_k	$3 = 3^1$	$9 = 3^2$	$81 = 3^4$

The value of i_k is

$$3^{2^k} \tag{1}$$

Notes:

- Realized we are only concerned with the lines **i = i * i** and **i = 3**

e. The number of iterations the function's loop will run is

$$\lceil \log_2 \log_3 n \rceil - 1 \tag{1}$$

Notes:

- The loop terminates when $3^{2^{(k+1)}} = i_{k+1} = i_k \cdot i_k \geq n$.
- $\forall x \in \mathbb{Z}$, $\forall y \in \mathbb{R}$, $\lfloor x + y \rfloor = x + \lfloor y \rfloor$
- Feel more confident there is no need to add an extra $\mathbf{+1}$. Done by playing with examples (i.e is $\lceil \log \log_3(82) \rceil - 1$ true? Would the loop run only once?)

Question 2

Question 3

Question 4