Worksheet 3 Solution

March 12, 2020

Question 1

Part 1

a) $Correct(my\_prog) \wedge Python(my\_prog)$

b) $\exists x \in P,\ \neg Correct(x) \Rightarrow Python(x)$

c) $\forall x \in P,\ \neg Python(x) \Rightarrow Correct(x)$

d) $\forall x \in P,\ \neg Correct(x) \Rightarrow Correct(x)$

e) A program is written in Python and is correct

f) All programs are not written in Python and is correct

g) It is not true that all programs written in python is correct

h) All programs that are not written in python is correct, and all correctly running programs are not written in python

Question 2

a) All program written in Python is correct, or all program written in Python is not correct

b) $(\exists x \in P, Python(x) \Rightarrow Correct(x)) \Rightarrow (\forall y \in P, Python(x) \Rightarrow Correct(x))$

c) The first statement considers two different natural numbers, where as the second uses the same number

The first statement is True (with $x_1 = 5$ and $x_2 = 35$), but the second statement is False (165 cannot be in multiples of 7)

Question 3

a) $Odd(x) :\ \exists n \in \mathbb{Z},\ 2 \mid (n+1)$

b) $(\forall m \in \mathbb{Z},\ Odd(m)) \land (\forall n \in \mathbb{Z},\ Odd(n)) \Rightarrow Odd(mn)$

c) $\forall m, n \in \mathbb{Z}, \exists k, l \in \mathbb{Z}, ((m+1) = 2k) \land ((n+1) = 2l) \Rightarrow \exists o \in \mathbb{Z}, (mn+1) = 2o$

d) $\forall m, n \in \mathbb{Z}, \exists k \in \mathbb{Z}, ((mn+1) = 2k) \Rightarrow \exists l, o \in \mathbb{Z}, ((m+1) = 2l) \land ((n+1) = 2o)$

Question 4

a)

$$\neg((a \land b) \Leftrightarrow c) \tag{1}$$

Expanding definition of iff:

$$\neg(((a \land b) \land c) \lor (\neg(a \land b) \land \neg c)) \tag{2}$$

Using $\neg(p \lor q) \Rightarrow \neg p \land \neg q$:

$$\neg((a \land b) \land c) \land \neg(\neg(a \land b) \land \neg c) \tag{3}$$

Using $\neg(p \land q) \Rightarrow \neg p \lor \neg q$:

$$(\neg(a \land b) \lor \neg c) \land ((a \land b) \lor c) \tag{4}$$

b) Using rule #6 and #7:

$$\exists x, y \in S,\ \forall z \in S,\ \neg(P(x,y) \land Q(x,z)) \tag{1}$$

Using rule #2:

$$\exists x, y \in S, \forall z \in S, \neg P(x,y) \vee \neg Q(x,z) \tag{2}$$

c) Using rule #4:

$$((\exists x \in S,\ P(x)) \wedge \neg(\exists y \in S,\ Q(y))) \tag{1}$$

Using rule #6:

$$((\exists x \in S,\ P(x)) \wedge (\forall y \in S,\ \neg Q(y)) \tag{2}$$

Question 5

- A solution is $U = \mathbb{R}$, $P(x) : x < 1$ and $Q(x) : x > 3$. By choosing $x = 2$ on rhs of statement, this statement doesn't hold.