

Problem Set 1 Solution

March 15, 2020

Question 1

- a. $\forall t \in T, \text{Canadian}(t) \Rightarrow \neg \text{Stanley}(t)$
- b. $\forall t \in T, \exists d \in D, \neg \text{Canadian}(t) \wedge \text{BelongsTo}(t, d)$
- c. $\forall t \in T, \exists d \in D, \text{Stanley}(t) \wedge \text{BelongsTo}(t, d)$
- d. $\forall t \in T, \exists d \in D, \text{BelongsTo}(t, d) \Rightarrow \forall d' \in D, d' \neq d \wedge \neg \text{BelongsTo}(t, d')$
- e. $\forall t_1 \in T, \exists d \in D, \exists t_2 \in T, t_1 \neq t_2 \wedge (\text{BelongsTo}(t_1, d) \wedge \text{BelongsTo}(t_2, d)) \Rightarrow \forall t_3 \in T, t_3 \neq t_1 \wedge t_3 \neq t_2 \wedge \neg \text{BelongsTo}(t_3, d)$

Question 2

- a. $\forall x \in \mathbb{R}, f(-x) = f(x)$
 $\forall x \in \mathbb{R}, -f(-x) = f(x)$
- b. $\forall g, f : \mathbb{R} \rightarrow \mathbb{R}, \exists h : \mathbb{R} \rightarrow \mathbb{R}, \text{Odd}(f) \wedge \text{Odd}(g) \Rightarrow \text{Odd}(f) \times \text{Odd}(g) = \text{Even}(h)$
- c. $f = 0$ is a solution, since $-f(-x) = -(-0) = 0 = f(x)$ and $f(-x) = -0 = 0 = f(x)$
- d. $\forall f : \mathbb{R} \rightarrow \mathbb{R}, \exists f_1, f_2 : \mathbb{R} \rightarrow \mathbb{R}, \text{Odd}(f_1) \wedge \text{Even}(f_2) \wedge f = \text{Odd}(f_1) + \text{Even}(f_2)$

e. A solution is $f = x^2 + x$ with $f_1 = x^2$ and $f_2 = x$.

$f = x^2 + x$ is the summation $\sum_{i=0}^{2n}$ with $n = 1, a_0 = 0, a_1 = 1, a_2 = 1$.
 f_1 is odd since $-f(-x) = -(-x)^2 = -x^2 = -f(x)$, and f_2 is even since $f(-x) = (-x)^2 = x^2 = f(x)$

f. A solution is $g_1(x) = \frac{2^x + 2^{-x}}{2}$ and $g_2(x) = \frac{2^x - 2^{-x}}{2}$.

$g_1 + g_2$ gives g since $\frac{2^x + 2^{-x}}{2} + \frac{2^x - 2^{-x}}{2} = 2^x$. Also, $g_1(-x) = \frac{2^{-x} + 2^{-(-x)}}{2} = \frac{2^{-x} + 2^x}{2} = g_1(x)$ is even, and $-g_2(-x) = -(\frac{2^{-x} - 2^{-(-x)}}{2}) = \frac{-2^{-x} + 2^x}{2} = g_2(x)$

Question 3

a. One solution is $\forall x \in \mathbb{N}, \exists y \in \mathbb{N}, x > y$.

With the above as predicate, the first statement is true, because $\forall x, x > 165$ is always greater than one.

Also, the second statement is false, because on the rhs, $x = 1$ can be chosen, and $1 \not> 1$

b. One solution is $\forall x \in \mathbb{N}, \exists y \in \mathbb{N}, x > y$.

With the above as predicate, the first statement is true, because $\forall x, x > 165$ is always greater than one.

Also, the second statement is false, because on the rhs, $x = 1$ can be chosen, and $1 \not> 1$

c. One solution is $x \in \mathbb{R}, y \in \mathbb{N}, Q(x) : x \notin \mathbb{R}, P(x, y) : x \geq y, S = \mathbb{R}, T = \mathbb{R}$.

With the above, the second statement is vacuous truth because we know by choosing an arbitrary real number for y , $x = y - 2$ can be chosen that makes the predicate P false.

Also with the above, the first statement is false because we know by choosing $y = x$, lhs of the statement becomes true, and because x is always a real number, the rhs of the statement is false

Question 4

a. $\exists n_0 \in \mathbb{N}, \forall n \in \mathbb{N}, n \leq n_0 \wedge \neg \text{Prime}(n)$

- b. $\forall n_0 \in \mathbb{N}, \exists n, a \in \mathbb{N}, n > n_0 \wedge \text{prime}(n) \wedge \text{prime}(n + a) \wedge (\forall b \in \{x \mid x \in \mathbb{N}, n < x < n + a\}, \neg \text{prime}(b))$