

# CSC343 Worksheet 12 Solution

July 2, 2020

1.
  - Keys
    - {id of molecule}
    - {x position, y position, z position}
  - Functional Dependencies
    - 1. id of molecule  $\rightarrow$  x position, y position, z position, x velocity, y velocity, z velocity
    - 2. x position, y position, z position  $\rightarrow$  id of molecule, x velocity, y velocity, z velocity

## Notes:

- Function Dependencies
  - *Functional Dependency* is a relationship between two attributes typically between the key and other non-key attributes within a table.

### Example:

SIN  $\rightarrow$  Name, Address, Birthdate

### Example 2:

ISBN  $\rightarrow$  Title

- Key of Relations
  - One or more attributes  $\{A_1, A_2, \dots, A_n\}$  is a key for a relation R if
    1. Those attributes functionally determine all other attributes of the relation
    2. No proper subset of  $\{A_1, A_2, \dots, A_n\}$  functionally determines all other attributes of R

### Example:

Given relation

$R = \text{Movies1}(\text{title, year, length, genre, studioName, starName})$

- i. {title, year, starName } form a key for the relation **Movies1**

- ii.  $\{ \text{year}, \text{starName} \}$  is not a key. Same star can be in multiple movies per year
- Superkeys
  - \* Means a set of attributes that contains a key
  - \* Don't need to be minimal

**Example:**

Given relation

 $R = \text{Movies1}(\text{title}, \text{year}, \text{length}, \text{genre}, \text{studioName}, \text{starName})$ 

- $\{ \text{title}, \text{year}, \text{starName} \}$  is a key and superkey
- $\{ \text{title}, \text{year}, \text{starName}, \text{title}, \text{year}, \text{length} \}$  is a superkey

**References:**

- 1) OpenTextBC, Chapter 11 Functional Dependencies, link
2. a)
  1.  $AB \rightarrow C$
  2.  $AB \rightarrow D$
  3.  $C \rightarrow A$
  4.  $C \rightarrow B$
  5.  $D \rightarrow B$
  6.  $D \rightarrow C$
  7.  $C \rightarrow D$
  8.  $D \rightarrow A$

**Second Attempt:**

$\{A, B\}^+ = \{A, B, C, D\}$ , so the following non-trivial FDs follows:  $AB \rightarrow C$  and  $AB \rightarrow D$ .

$\{C\}^+ = \{D, A\}$ , so the following non-trivial FDs follows  $C \rightarrow D$  and  $C \rightarrow A$ .

$\{D\}^+ = \{A\}$ , so the following non-trivial FDs follows:  $D \rightarrow A$ .

**Notes:**

- The Splitting / Combining Rule
  - Combining Rule
    - \*  $A_1, A_2, \dots, A_n \rightarrow B_i$  for  $i = 1, 2, \dots, m$   
to  
 $A_1, A_2, \dots, A_n \rightarrow B_1, B_2, \dots, B_m$

**Example:**

Given

title year  $\rightarrow$  length  
 title year  $\rightarrow$  genre  
 title year  $\rightarrow$  studioName

it's combined form is

title year  $\rightarrow$  length genre studioName

– Splitting Rule

\*

\*  $A_1, A_2, \dots, A_n \rightarrow B_1, B_2, \dots, B_m$

to

$A_1, A_2, \dots, A_n \rightarrow B_i$  for  $i = 1, 2, \dots, m$

**Example:**

Given

title year  $\rightarrow$  length

It's splitted form is

title  $\rightarrow$  length  
 year  $\rightarrow$  length

• Trivial Functional Dependencies

- A functional dependency  $FD : X \rightarrow Y$  is **trivial** if  $Y$  is a subset of  $X$

**Exmample:**

title year  $\rightarrow$  title

**Example 2:**

title  $\rightarrow$  title

• Non-trivial Functional Dependencies

- is a case where some but not all of the attributes on the R.H.S of an FD are also on L.H.S

**Example:**

title year  $\rightarrow$  title movieLength

- Can be simplified using **trivial-dependency rule**
  - \* The FD  $A_1A_2 \cdots A_n \rightarrow B_1B_2 \cdots B_m$  is equivalent to  $A_1A_2 \cdots A_n \rightarrow C_1C_2 \cdots C_k$

where  $C$ 's are all those  $B$ 's that are not in  $A$ 's.



Figure 3.3: The trivial-dependency rule

- Computing the Closure of Attributes
  - Closure of attribute set  $\{X\}$  is denoted as  $\{X\}^+$ .
  - The closure means a given set of attributes  $A$  satisfying FD, are a sets of all attributes  $B$  such that  $A \rightarrow B$

**Example:**

Given attributes  $A, B, C, D, E, F$  and FDs  $AB \rightarrow C$ ,  $BC \rightarrow AD$ ,  $D \rightarrow E$  and  $CF \rightarrow B$ , What is the closure of  $\{A, B\}$  or  $\{A, B\}^+$

1. Start with  $\{A, B\}$ .
2. Split  $BC \rightarrow AD$ 
  - \* We have  $BC \rightarrow A$  and  $BC \rightarrow D$
  - \* Since  $A$  is in  $\{A, B\}$ , this is not included
  - \* Since  $D$  is not in  $\{A, B\}$ , this IS included

So, we have  $\{A, B, D\}$

3. Since  $C$  in  $AB \rightarrow C$  is NOT in  $\{A, B, C, D\}$ ,  $C$  is included and we have  $\{A, B, C, D\}$
4. Since  $A$  in  $BC \rightarrow A$  is in  $\{A, B, C, D\}$ , this is skipped
5. Since  $E$  is not in  $D \rightarrow E$ ,  $E$  is included and we have  $\{A, B, C, D, E\}$  as our solution

- Why the Closure Algorithm Works
- Transitive Rule
  - Definition

If  $A_1A_2 \cdots A_n \rightarrow B_1B_2 \cdots B_m$  and  $B_1B_2 \cdots B_m \rightarrow C_1C_2 \cdots C_k$  hold in relation  $R$ ,  $A_1A_2 \cdots A_n \rightarrow C_1C_2 \cdots C_k$  also holds in  $R$ .

### Example:

Given

title year  $\rightarrow$  studioName  
 studioName  $\rightarrow$  studioAddr

Transitive rule says the above is equal to the following

title year  $\rightarrow$  studioAddr

- Inference Rules
  - Is also called **Armstrong's Axioms**
  - Has 3 axioms
    1. *Reflexivity*
      - \* If  $\{B_1, B_2, \dots, B_n\} \subseteq \{A_1, A_2, \dots, A_n\}$  then  $A_1A_2 \cdots A_n \rightarrow B_1B_2 \cdots B_m$
      - \* also called **trivial FDs**
    2. *Augmentation*
      - \* If  $A_1A_2 \cdots A_n \rightarrow B_1B_2 \cdots B_m$  then  $A_1A_2 \cdots A_nC_1C_2 \cdots C_k \rightarrow B_1B_2 \cdots B_mC_1C_2 \cdots C_k$
      - \*  $C_1C_2 \cdots C_k$  are any set of attributes
    3. *Transitivity*
      - \* If  $A_1A_2 \cdots A_n \rightarrow B_1B_2 \cdots B_m$  and  $B_1B_2 \cdots B_m \rightarrow C_1C_2 \cdots C_k$  then  $A_1A_2 \cdots A_n \rightarrow C_1C_2 \cdots C_k$

b)  $A, B$  is the only key of  $R$ .

### Notes:

- Key of Attributes
  - **Definition:** A set of attributes  $\{A_1, A_2, \dots, A_n\}$  is a key for a relation  $R$  if
    1. Those attributes functionally determine all other attributes

2. No proper subset of  $\{A_1, A_2, \dots, A_n\}$  functionally determines all other attributes of  $R$ .

c) The superkeys that are not keys are:  $\{A, B, C\}$ ,  $\{A, B, D\}$ ,  $\{A, B, C, D\}$

3. i) a)  $\{A\}^+ = \{A, B, C, D\}$ , so we have  $A \rightarrow A$ ,  $A \rightarrow B$ ,  $A \rightarrow C$ ,  $A \rightarrow D$

$\{B\}^+ = \{C, D\}$ , so we have  $B \rightarrow C$  and  $B \rightarrow D$

b)  $\{A\}$  is the key of  $S$ .

c) The super keys that are not keys are:

$\{A, B\}$ ,  $\{A, C\}$ ,  $\{A, D\}$ ,  $\{A, B, C\}$ ,  $\{A, B, D\}$ ,  $\{A, B, C, D\}$

ii) a)  $\{A\}^+ = \{A\}$ , so this FD is trivial.

$\{B\}^+ = \{B\}$ , so this FD is trivial.

$\{C\}^+ = \{C\}$ , so this FD is trivial.

$\{D\}^+ = \{D\}$ , so this FD is trivial.

$\{A, B\}^+ = \{A, B, C, D\}$ , so we have  $AB \rightarrow A$ ,  $AB \rightarrow B$ ,  $AB \rightarrow C$ ,  $AB \rightarrow D$

$\{A, C\}^+ = \{A, C\}$ , so we have  $AC \rightarrow A$ ,  $AC \rightarrow C$

$\{A, D\}^+ = \{A, D, B\}$ , so we have  $AD \rightarrow A$ ,  $AD \rightarrow D$ ,  $AD \rightarrow B$

$\{B, C\}^+ = \{B, C, D, A\}$ , so we have  $BC \rightarrow A$ ,  $BC \rightarrow B$ ,  $BC \rightarrow C$ ,  $BC \rightarrow D$

$\{D, C\}^+ = \{D, C, A, B\}$ , so we have  $DC \rightarrow D$ ,  $DC \rightarrow C$ ,  $DC \rightarrow A$ ,  $DC \rightarrow B$

$\{A, B, C\}^+ = \{A, B, C, D\}$ , so we have  $ABC \rightarrow A$ ,  $ABC \rightarrow B$ ,  $ABC \rightarrow C$ ,  $ABC \rightarrow D$

$\{B, C, D\}^+ = \{B, C, D, A\}$ , so we have  $BCD \rightarrow A$ ,  $BCD \rightarrow B$ ,  $BCD \rightarrow C$ ,  $BCD \rightarrow D$

$\{C, D, A\}^+ = \{C, D, A, B\}$ , so we have  $CDA \rightarrow A$ ,  $CDA \rightarrow B$ ,  $CDA \rightarrow C$ ,  $CDA \rightarrow D$

$\{D, A, B\}^+ = \{D, A, B, C\}$ , so we have  $DAB \rightarrow A$ ,  $DAB \rightarrow B$ ,  $DAB \rightarrow C$ ,  $DAB \rightarrow D$

$\{D, A, B\}^+ = \{D, A, B, C\}$ , so we have  $DAB \rightarrow A$ ,  $DAB \rightarrow B$ ,  $DAB \rightarrow C$ ,  $DAB \rightarrow D$

$\{A, B, C, D\}^+ = \{A, B, C, D\}$ , so this FD is trivial.

b)  $\{A, B\}$ ,  $\{A, C\}$ ,  $\{B, C\}$ ,  $\{D, C\}$  are the keys of  $T$ .

c) The super keys that are not keys are:

$\{A, B, C\}$ ,  $\{A, B, D\}$ ,  $\{B, C, D\}$ ,  $\{A, D, C\}$ ,  $\{A, B, D\}$ ,  $\{A, B, C, D\}$

iii) a)  $\{A\}^+ = \{A, B, C, D\}$ , so we have  $A \rightarrow C$ ,  $A \rightarrow D$

$\{B\}^+ = \{A, B, C, D\}$ , so we have  $B \rightarrow A$ ,  $B \rightarrow D$

$\{C\}^+ = \{A, B, C, D\}$ , so we have  $C \rightarrow A$ ,  $C \rightarrow B$

$\{D\}^+ = \{A, B, C, D\}$ , so we have  $D \rightarrow B$ ,  $D \rightarrow C$

$\{A, B\}^+ = \{A, B, C, D\}$ , so we have  $AB \rightarrow C$ ,  $AB \rightarrow D$

$\{B, C\}^+ = \{A, B, C, D\}$ , so we have  $BC \rightarrow A$ ,  $BC \rightarrow D$

$\{B, D\}^+ = \{A, B, C, D\}$ , so we have  $BD \rightarrow A$ ,  $BD \rightarrow C$

$\{C, D\}^+ = \{A, B, C, D\}$ , so we have  $CD \rightarrow A$ ,  $CD \rightarrow B$

$\{C, D\}^+ = \{A, B, C, D\}$ , so we have  $CD \rightarrow A$ ,  $CD \rightarrow B$

$\{A, B, C\}^+ = \{A, B, C, D\}$ , so we have  $ABC \rightarrow D$

$\{B, C, D\}^+ = \{A, B, C, D\}$ , so we have  $BCD \rightarrow A$

$\{C, D, A\}^+ = \{A, B, C, D\}$ , so we have  $CDA \rightarrow B$

$\{D, A, B\}^+ = \{A, B, C, D\}$ , so we have  $DAB \rightarrow C$

**Correct Solution:**

$\{A\}^+ = \{A, B, C, D\}$ , so we have  $A \rightarrow C$ ,  $A \rightarrow D$

$\{B\}^+ = \{A, B, C, D\}$ , so we have  $B \rightarrow A$ ,  $B \rightarrow D$

$\{C\}^+ = \{A, B, C, D\}$ , so we have  $C \rightarrow A$ ,  $C \rightarrow B$

$\{D\}^+ = \{A, B, C, D\}$ , so we have  $D \rightarrow B$ ,  $D \rightarrow C$

$\{A, B\}^+ = \{A, B, C, D\}$ , so we have  $AB \rightarrow C$ ,  $AB \rightarrow D$

$\{A, C\}^+ = \{A, B, C, D\}$ , so we have  $AC \rightarrow B$ ,  $AC \rightarrow D$

$\{A, D\}^+ = \{A, B, C, D\}$ , so we have  $AD \rightarrow B$ ,  $AD \rightarrow C$

$\{B, C\}^+ = \{A, B, C, D\}$ , so we have  $BC \rightarrow A$ ,  $BC \rightarrow D$

$\{B, D\}^+ = \{A, B, C, D\}$ , so we have  $BD \rightarrow A$ ,  $BD \rightarrow C$

$\{C, D\}^+ = \{A, B, C, D\}$ , so we have  $CD \rightarrow A$ ,  $CD \rightarrow B$ 
 $\{A, B, C\}^+ = \{A, B, C, D\}$ , so we have  $ABC \rightarrow D$ 
 $\{B, C, D\}^+ = \{A, B, C, D\}$ , so we have  $BCD \rightarrow A$ 
 $\{C, D, A\}^+ = \{A, B, C, D\}$ , so we have  $CDA \rightarrow B$ 
 $\{D, A, B\}^+ = \{A, B, C, D\}$ , so we have  $DAB \rightarrow C$ 

b)  $\{A\}, \{B\}, \{C\}, \{D\}$  are the keys of  $U$ .

c) The super keys that are not keys are:

$\{A, B\}, \{A, C\}, \{A, D\}, \{B, C\}, \{B, D\}, \{C, D\}, \{A, B, C\}, \{B, C, D\}, \{C, D, A\}, \{D, A, B\}, \{A, B, C, D\}$

4. a) We need to show the closure of attributes  $\{A_1, A_2, \dots, A_n, C\}$  in  $FD$   $A_1, A_2, \dots, A_n, C \rightarrow B$  is  $\{A_1, A_2, \dots, A_n, C, B\}$ , that is  $\{A_1, A_2, \dots, A_n, C\}^+ = \{A_1, A_2, \dots, A_n, C, B\}$

Since we know  $\{A_1, A_2, \dots, A_n\}$  functionally determines  $B$ , we can conclude  $B$  can be added to  $\{A_1, A_2, \dots, A_n, C\}$ .

Thus, it follows from above that  $\{A_1, A_2, \dots, A_n, C\}^+ = \{A_1, A_2, \dots, A_n, C, B\}$ .

- b) Let  $A_1A_2 \dots A_n \rightarrow B$  is FD. That is,  $\{A_1A_2 \dots A_n\}^+ = \{A_1A_2 \dots A_n, B\}$

We need to show  $A_1A_2 \dots A_nC \rightarrow BC$  follows. That is,  $\{A_1, A_2, \dots, A_n, C\}^+ = \{A_1, A_2, \dots, A_n, C, B\}$

It follows from the combine and split rule that  $A_1A_2 \dots A_nC \rightarrow BC$  can be splitted into  $A_1A_2 \dots A_nC \rightarrow B$  and  $A_1A_2 \dots A_nC \rightarrow C$ .

So, we need to show  $A_1A_2 \dots A_nC \rightarrow B$  and  $A_1A_2 \dots A_nC \rightarrow C$  follows from the given.

We will do so in parts.

### 1. Part 1 (Showing $A_1A_2 \dots A_nC \rightarrow B$ ):

Here, we need to show  $A_1A_2 \dots A_nC \rightarrow B$  follows.

And indeed, this follows from the work of *augmenting left sides*. That is the solution to previous problem.



## 2. Part 2 (Showing $A_1A_2 \cdots A_nC \rightarrow C$ ):

Here, we need to show  $A_1A_2 \cdots A_nC \rightarrow C$  follows.

The definition of trivial FD tells us  $A_1A_2 \cdots A_n \rightarrow B_1B_2 \cdots B_m$  holds when  $\{B_1, B_2, \dots, B_m\} \subseteq \{A_1, A_2, \dots, A_n\}$

Since  $\{C\} \subseteq \{A_1, A_2 \cdots, A_n, C\}$ , we can conclude this FD follows trivially.

- c) Let  $A_1A_2 \cdots A_n \rightarrow B_1B_2 \cdots B_m$  and  $C_1C_2 \cdots C_k \rightarrow D$ , where  $B$  are each among the  $C$ 's.

We need to show  $A_1A_2 \cdots A_nE_1E_2 \cdots E_j \rightarrow D$  follows, where the  $E$ 's are all of those  $C$ 's not found among the  $B$ 's.

The transitive rule tells us if  $A_1A_2 \cdots A_n \rightarrow B_1B_2 \cdots B_m$  and  $B_1B_2 \cdots B_m \rightarrow C_1C_2 \cdots C_k$ , then  $A_1A_2 \cdots A_n \rightarrow C_1C_2 \cdots C_k$  also holds in  $R$ .

Since we know  $A_1A_2 \cdots A_n \rightarrow B_1B_2 \cdots B_m$  and  $C_1C_2 \cdots C_k \rightarrow D$  where  $B$ 's are each among the  $C$ 's, we can conclude from the transitive rule that  $A_1A_2 \cdots A_n \rightarrow D$ .

Then using **augmenting left sides** to all  $C$ 's not found among the  $B$ 's on  $A_1A_2 \cdots A_n \rightarrow D$ , we can conclude  $A_1A_2 \cdots A_nE_1E_2 \cdots E_j \rightarrow D$  follows.

- d) Assume  $FD$ 's  $A_1A_2 \cdots A_n \rightarrow B_1B_2 \cdots B_m$  and  $C_1C_2 \cdots C_k \rightarrow D_1D_2 \cdots D_j$  holds.

We need to show  $FD$   $A_1A_2 \cdots A_nC_1C_2 \cdots C_k \rightarrow B_1B_2 \cdots B_mD_1D_2 \cdots D_k$  follows.

Using the split / combine rule, we can conclude showing  $A_1A_2 \cdots A_nC_1C_2 \cdots C_k \rightarrow B_1B_2 \cdots B_mD_1D_2 \cdots D_k$  is the same as showing  $A_1A_2 \cdots A_nC_1C_2 \cdots C_k \rightarrow B_1B_2 \cdots B_m$  and  $A_1A_2 \cdots A_nC_1C_2 \cdots C_k \rightarrow D_1D_2 \cdots D_k$

So, we will prove the two, in parts

### 1. Part 1 (Showing $A_1A_2 \cdots A_nC_1C_2 \cdots C_k \rightarrow B_1B_2 \cdots B_m$ )

Here, we need to show  $A_1A_2 \cdots A_nC_1C_2 \cdots C_k \rightarrow B_1B_2 \cdots B_m$ .

The header of problem tells us  $A_1A_2 \cdots A_n \rightarrow B_1B_2 \cdots B_m$  holds.

Then by using **Augmenting Left Sides** rule to all  $C$ s not found among the  $A$ s,  $A_1A_2 \cdots A_nC_1C_2 \cdots C_k \rightarrow B_1B_2 \cdots B_m$  follows.

### 2. Part 2 (Showing $A_1A_2 \cdots A_nC_1C_2 \cdots C_k \rightarrow D_1D_2 \cdots D_k$ follows)

Here, we need to show  $A_1A_2 \cdots A_nC_1C_2 \cdots C_k \rightarrow D_1D_2 \cdots D_k$ .

The header of problem tells us  $C_1C_2 \cdots C_k \rightarrow D_1D_2 \cdots D_k$  holds.

Then by using **Augmenting Left Sides** rule to all  $A$ s not found among the  $C$ s,  $A_1A_2 \cdots A_nC_1C_2 \cdots C_k \rightarrow D_1D_2 \cdots D_k$  follows.

5. a) An example is

$A$  being movieID and  
 $B$  being movie length.

b) An example is

$A$  being movieID  
 $B$  being movieTitle  
 $C$  being movieLength

c) An example is

$A$  being movieTitle  
 $B$  being year  
 $C$  being length

6. Assume a relation has no attribute that is functionally determined by all the other attributes.

And, assume for the sake of contradiction that there is a relation with non-trivial FD  $X \rightarrow Y$ .

Then, it follows from the definition of non-trivial functional dependency that  $Y \not\subseteq X$ .

Then, we can conclude the attributes in  $Y$  is functionally determined by other attributes in  $X$ .

But this contradicts the assumption that no attribute is functionally determined by all other attributes.

7. Let  $X$  and  $Y$  be sets of attributes. Assume  $X \subseteq Y$ .

I need to show  $X^+ \subseteq Y^+$ .

I will do so in cases

1. **Case 1** ( $X = Y$ ):

Assume  $X = Y$ .

I need to show  $X^+ \subseteq Y^+$  follows.

The header tells us  $X = Y$ .

Using this fact,  $X^+ = Y^+$  is true.

Then it follows from above that  $X^+ \subseteq Y^+$  is also true.

## 2. Case 2 ( $X \subset Y$ )

Assume  $X \subset Y$ .

I need to show  $X^+ \subseteq Y^+$  follows.

Since the attributes in  $X$  is in  $Y$ , we can conclude the attributes in  $X^+$  is also in  $Y^+$ .

And, since  $Y$  has attributes not in  $X$ , we can conclude  $Y^+$  may contain attributes not in  $X^+$ .

Thus, we can conclude  $X^+ \subseteq Y^+$ .

## 8. 1. Only one solution will be included for now :)

The following

1.  $A \rightarrow C$
2.  $B \rightarrow A$
3.  $B \rightarrow C$
4.  $C \rightarrow A$
5.  $C \rightarrow B$
6.  $AB \rightarrow C$
7.  $AC \rightarrow B$
8.  $BC \rightarrow A$
9.  $A \rightarrow BC$
10.  $A \rightarrow A$

can be simplified to

1.  $A \rightarrow C$
2.  $B \rightarrow A$
3.  $B \rightarrow C$
4.  $C \rightarrow A$
5.  $C \rightarrow B$
6.  $A \rightarrow C$  **B removed from here!!**
7.  $AC \rightarrow B$
8.  $BC \rightarrow A$
9.  $A \rightarrow BC$
10.  $A \rightarrow A$

since **augmenting left sides** rule tells us  $AB \rightarrow C$  can be attained by adding  $B$  to L.H.S of  $A \rightarrow C$ .

Then, the following

1.  $A \rightarrow C$
2.  $B \rightarrow A$
3.  $B \rightarrow C$
4.  $C \rightarrow A$
5.  $C \rightarrow B$
6.  $A \rightarrow C$
7.  $AC \rightarrow B$
8.  $BC \rightarrow A$
9.  $A \rightarrow BC$
10.  $A \rightarrow A$

can be simplified to

1.  $A \rightarrow C$
2.  $B \rightarrow A$
3.  $B \rightarrow C$
4.  $C \rightarrow A$
5.  $C \rightarrow B$
6.  $AC \rightarrow B$
7.  $BC \rightarrow A$
8.  $A \rightarrow BC$
9.  $A \rightarrow A$

by removing redundant  $A \rightarrow C$ .

Then, the following

1.  $A \rightarrow C$
2.  $B \rightarrow A$
3.  $B \rightarrow C$
4.  $C \rightarrow A$
5.  $C \rightarrow B$
6.  $AC \rightarrow B$
7.  $BC \rightarrow A$
8.  $A \rightarrow BC$
9.  $A \rightarrow A$

can be simplified to

1.  $A \rightarrow C$
2.  $B \rightarrow A$
3.  $B \rightarrow C$
4.  $C \rightarrow A$
5.  $C \rightarrow B$

6.  $AC \rightarrow B$
7.  $BC \rightarrow A$
8.  $A \rightarrow B$  Splitted from  $A \rightarrow BC$
9.  $A \rightarrow C$  Splitted from  $A \rightarrow BC$
10.  $A \rightarrow A$

by using **splitting rule** on  $A \rightarrow BC$ .

Then, the following

1.  $A \rightarrow C$
2.  $B \rightarrow A$
3.  $B \rightarrow C$
4.  $C \rightarrow A$
5.  $C \rightarrow B$
6.  $AC \rightarrow B$
7.  $BC \rightarrow A$
8.  $A \rightarrow B$
9.  $A \rightarrow C$
10.  $A \rightarrow A$

can be simplified to

1.  $A \rightarrow C$
2.  $B \rightarrow A$
3.  $B \rightarrow C$
4.  $C \rightarrow A$
5.  $C \rightarrow B$
6.  $AC \rightarrow B$
7.  $BC \rightarrow A$
8.  $A \rightarrow B$
9.  $A \rightarrow A$

by removing redundant  $A \rightarrow C$ .

Then, the following

1.  $A \rightarrow C$
2.  $B \rightarrow A$
3.  $B \rightarrow C$
4.  $C \rightarrow A$
5.  $C \rightarrow B$
6.  $AC \rightarrow B$
7.  $BC \rightarrow A$

$$8. A \rightarrow B$$

$$9. A \rightarrow A$$

can be simplified to

$$1. A \rightarrow C$$

$$2. B \rightarrow A$$

$$3. B \rightarrow C$$

$$4. C \rightarrow A$$

$$5. C \rightarrow B$$

$$6. A \rightarrow B \text{ } C \text{ removed here!!}$$

$$7. BC \rightarrow A$$

$$8. A \rightarrow B$$

$$9. A \rightarrow A$$

since **augmenting left sides** tells us  $AC \rightarrow B$  can be attained by adding  $C$  to  $A \rightarrow B$ .

Then, the following

$$1. A \rightarrow C$$

$$2. B \rightarrow A$$

$$3. B \rightarrow C$$

$$4. C \rightarrow A$$

$$5. C \rightarrow B$$

$$6. A \rightarrow B$$

$$7. BC \rightarrow A$$

$$8. A \rightarrow B$$

$$9. A \rightarrow A$$

can be simplified to

$$1. A \rightarrow C$$

$$2. B \rightarrow A$$

$$3. B \rightarrow C$$

$$4. C \rightarrow A$$

$$5. C \rightarrow B$$

$$6. A \rightarrow B$$

$$7. BC \rightarrow A$$

$$8. A \rightarrow A$$

by removing redundant  $A \rightarrow B$ .

Then, the following

$$1. A \rightarrow C$$

2.  $B \rightarrow A$
3.  $B \rightarrow C$
4.  $C \rightarrow A$
5.  $C \rightarrow B$
6.  $A \rightarrow B$
7.  $BC \rightarrow A$
8.  $A \rightarrow A$

can be simplified to

1.  $A \rightarrow C$
2.  $B \rightarrow A$
3.  $B \rightarrow C$
4.  $C \rightarrow A$
5.  $C \rightarrow B$
6.  $A \rightarrow B$
7.  $BC \rightarrow A$

since  $A \rightarrow A$  can be attained by using **transitivity** rule on  $A \rightarrow C$  and  $C \rightarrow A$ .

Then, the following

1.  $A \rightarrow C$
2.  $B \rightarrow A$
3.  $B \rightarrow C$
4.  $C \rightarrow A$
5.  $C \rightarrow B$
6.  $A \rightarrow B$
7.  $BC \rightarrow A$

can be simplified to

1.  $A \rightarrow C$
2.  $B \rightarrow A$
3.  $B \rightarrow C$
4.  $C \rightarrow A$
5.  $C \rightarrow B$
6.  $A \rightarrow B$
7.  $B \rightarrow A$  C removed here!!

since **augmenting let sides** rule tells us  $BC \rightarrow A$  can be attained by adding  $C$  to L.H.S of  $B \rightarrow A$ .

Then, the following

1.  $A \rightarrow C$

2.  $B \rightarrow A$
3.  $B \rightarrow C$
4.  $C \rightarrow A$
5.  $C \rightarrow B$
6.  $A \rightarrow B$
7.  $B \rightarrow A$

can be simplified to

1.  $A \rightarrow C$
2.  $B \rightarrow A$
3.  $B \rightarrow C$
4.  $C \rightarrow A$
5.  $C \rightarrow B$
6.  $A \rightarrow B$

by removing redundant  $B \rightarrow A$ .

Then, the following

1.  $A \rightarrow C$
2.  $B \rightarrow A$
3.  $B \rightarrow C$
4.  $C \rightarrow A$
5.  $C \rightarrow B$
6.  $A \rightarrow B$

can be simplified to

1.  $A \rightarrow C$
2.  $B \rightarrow A$
3.  $C \rightarrow A$
4.  $C \rightarrow B$
5.  $A \rightarrow B$

since **transitivity** rule tells us  $B \rightarrow C$  can be attained by using  $B \rightarrow A$  and  $A \rightarrow C$ .

Then, the following

1.  $A \rightarrow C$
2.  $B \rightarrow A$
3.  $C \rightarrow A$
4.  $C \rightarrow B$
5.  $A \rightarrow B$

can be simplified to



1.  $A \rightarrow C$
2.  $B \rightarrow A$
3.  $C \rightarrow A$
4.  $C \rightarrow B$

since **transitivity** rule tells us  $A \rightarrow B$  can be attained by using  $A \rightarrow C$  and  $C \rightarrow B$ .

### Rough Works:

1. Add attributes from  $A^+$  to L.H.S of  $A_1A_2 \cdots A_n \rightarrow A^+$ .
2. Show that the R.H.S is still  $A^+$ .

### Notes:

- Closure (Definition)
  - Suppose  $A = \{A_1, A_2, \dots, A_n\}$  is a set of attributes of R and S is a set of FD'.

The closure of  $A$  under the set  $S$ , denoted by  $A^+$ , is the set of attributes  $B$  such that any relation that satisfies all the FD's in S is also satisfies  $A_1A_2 \cdots A_n \rightarrow A^+$ .

  - In other words  $A_1 \cdots A_n \rightarrow A^+$  follows from the FD's of S.
- I wish the definition is a little more clear :(

### 9. Notes:

- Basis
  - Is the set of FD's that represent the full set of FD's of a relation
- Finding minimal bases for FD's
  - A minimal basis for a relation satisfies three conditions
    1. All the FD's in  $B$  have singleton right sides.
    2. If any  $FD$  is removed from  $B$ , the result is no longer a basis
    3. If for any  $FD$  in  $B$  we remove one or more attributes from the left side of  $F$ , the result is no longer a basis
  - Steps
    1. Get rid of redundant attributes
    - \*
    2. Get rid of redundant dependencies
- Example

The following

1.  $A \rightarrow B$
2.  $ABCD \rightarrow E$
3.  $EF \rightarrow G$
4.  $EF \rightarrow H$
5.  $ACDF \rightarrow E$
6.  $ACDF \rightarrow G$

can be simplified to

1.  $A \rightarrow B$
2.  $ACD \rightarrow E$  **B removed here!!**
3.  $EF \rightarrow G$
4.  $EF \rightarrow H$
5.  $ACDF \rightarrow E$
6.  $ACDF \rightarrow G$

since by **augmentation rule**,  $A \rightarrow B$  can be re-written as  $ACD \rightarrow BCD$ . And by **trivial rule**,  $ACD \rightarrow BCD$  can be re-written as  $ACB \rightarrow ABCD$ , which then can be used to get  $E$  from  $ABCD \rightarrow E$ .

Second, the following

1.  $A \rightarrow B$
2.  $ACD \rightarrow E$
3.  $EF \rightarrow G$
4.  $EF \rightarrow H$
5.  $ACDF \rightarrow E$
6.  $ACDF \rightarrow G$

can be simplified to

1.  $A \rightarrow B$
2.  $ACD \rightarrow E$
3.  $EF \rightarrow G$
4.  $EF \rightarrow H$
5.  $ACD \rightarrow E$  **F Removed here!!**
6.  $ACDF \rightarrow G$

since **augmenting left side** rule tells us  $ACDF \rightarrow E$  can be attained by adding  $F$  to  $ACD$  in  $ACD \rightarrow E$ .

Then, the following

1.  $A \rightarrow B$
2.  $ACD \rightarrow E$
3.  $EF \rightarrow G$

4.  $EF \rightarrow H$
5.  $ACD \rightarrow E$
6.  $ACDF \rightarrow G$

can be simplified to

1.  $A \rightarrow B$
2.  $ACD \rightarrow E$
3.  $EF \rightarrow G$
4.  $EF \rightarrow H$
5.  $ACDF \rightarrow G$

by removing redundant  $ACD \rightarrow E$ .

Then, the following

1.  $A \rightarrow B$
2.  $ACD \rightarrow E$
3.  $EF \rightarrow G$
4.  $EF \rightarrow H$
5.  $ACD \rightarrow E$
6.  $ACDF \rightarrow G$

can be simplified to

1.  $A \rightarrow B$
2.  $ACD \rightarrow E$
3.  $EF \rightarrow G$
4.  $EF \rightarrow H$

since **augmentation** rule tells us  $ACDF \rightarrow G$  can be re-written to get  $ACDF \rightarrow EF$  and then use **transitivity rule** on  $EF \rightarrow G$  to get  $ACDF \rightarrow G$ .

10. a) • Finding subsets  
 $X_1 = \{A\}$ ,  $X_2 = \{B\}$ ,  $X_3 = \{C\}$ ,  $X_4 = \{A, B\}$ ,  $X_5 = \{A, C\}$ ,  $X_6 = \{B, C\}$ ,  
 $X_7 = \{A, B, C\}$ ,  $X_8 = \{\}$
- Finding  $X_i^+$
1.  $X_1^+ = \{A\}$
  2.  $X_2^+ = \{B\}$
  3.  $X_3^+ = \{C, E, A\}$
  4.  $X_4^+ = \{A, B, C, D, E\}$
  5.  $X_5^+ = \{A, C, E\}$
  6.  $X_6^+ = \{A, B, C, D, E\}$
  7.  $X_7^+ = \{A, B, C, D, E\}$
  8.  $X_8^+ = \{\}$

- Putting all nontirival FD's in  $T$   
 $T = \{C \rightarrow E, C \rightarrow A, AB \rightarrow C, AB \rightarrow D, AB \rightarrow E, AC \rightarrow E, BC \rightarrow A, BC \rightarrow D, BC \rightarrow E, ABC \rightarrow D, ABC \rightarrow E\}$
- Finding minimal basis for the FD of  $S$   
 $T_{\text{minimal}} = \{C \rightarrow E, C \rightarrow A, B \rightarrow C, B \rightarrow D, B \rightarrow E, B \rightarrow A\}$

### Notes:

- Projecting Functional Dependency
  - Remember that  $\pi$  is equivalent to SQL's SELECT of columns
  - Answers the question to "given a relation  $R$  and a set of FD's  $S$ , what FD's hold if we project  $R$  by  $R_1 = \Pi_L(R)$ ?"
  - The new set  $S'$ 
    1. Follows from  $S$
    2. Involves only attributes of  $R_1$



- Algorithm for Projecting a set of Functional Dependencies
  - Inputs and Outputs
    - \* Input
      - **R**: The original relation
      - **R1**: The projection of  $R$
      - **S**: The set of FD's that hold in  $R$
    - \* Output
      - **T**: The set of  $FD$ 's that hold in  $R_1$
  - Steps
    1. Initialize  $T = \{\}$ .
    2. Construct a set of all subsets of attributes of  $R_1$  called  $X$
    3. Compute  $X_i^+$  for all members of  $X$  under  $S$ .
      - \*  $X_i^+$  may consist of attributes that are not in  $R_1$
    4. Add to  $T$  all nontirival FD's  $X \rightarrow A$  such that  $A$  is both in  $X_i^+$  and an attributes of  $R_1$
    5. Now,  $T$  is a basis for the  $FD$ 's that hold in  $R_1$  but may not be a minimal basis. Modify  $T$  as follows.
      - a) If there is an  $FD$  in  $F$  in  $T$  that follows from the other  $FD$ 's in  $T$ , remove  $F$
      - b) Let  $Y \rightarrow B$  be an  $FD$  in  $T$ , with at least two attributes in  $Y$ . Remove one attribute from  $Y$  and call it  $Z$ . If  $Z \rightarrow B$  follows from the  $FD$ 's in  $T$ , then replace  $Z \rightarrow B$  with  $Y \rightarrow B$ .

## – Example

Consider  $R(A, B, C, D)$  has FD's  $A \rightarrow B$ ,  $B \rightarrow C$ , and  $C \rightarrow D$ .

$R_1(A, C, D)$  is a projection of  $R$ . Find FD's for  $R_1$

1. Initialize  $T = \{\}$ .

\*  $T = \{\}$

2. Construct a set of all subsets of attributes of  $R_1$  called  $X$

\* There are 8 subsets

$X_1 = \{A\}$ ,  $X_2 = \{C\}$ ,  $X_3 = \{D\}$ ,  $X_4 = \{A, C\}$ ,  $X_5 = \{A, D\}$ ,  $X_6 = \{C, D\}$ ,  $X_7 = \{A, C, D\}$ ,  $X_8 = \{\}$

3. Compute  $X_i^+$  for all members of  $X$  under  $S$ .

\*  $X_1 = \{A\}$

$X_1^+ = \{A, B, C, D\}$

\*  $X_2 = \{C\}$

$X_2^+ = \{C, D\}$

\*  $X_3 = \{D\}$

$X_3^+ = \{D\}$

\*  $X_4 = \{A, C\}$

$X_4^+ = \{A, B, C, D\}$

\*  $X_5 = \{A, D\}$

$X_5^+ = \{A, B, C, D\}$

\*  $X_6 = \{C, D\}$

$X_6^+ = \{C, D\}$

\*  $X_7 = \{A, C, D\}$

$X_7^+ = \{A, B, C, D\}$

\*  $X_8 = \{\}$

$X_8^+ = \{\}$

4. Add to  $T$  all nontrivial FD's  $X \rightarrow A$  such that  $A$  is both in  $X_i^+$  and an attributes of  $R_1$

\*  $T = \{A \rightarrow C, A \rightarrow D, C \rightarrow D, AC \rightarrow D, AD \rightarrow C\}$

5. Now,  $T$  is a basis for the FD's that hold in  $R_1$  but may not be a minimal basis. Modify  $T$  as follows.

\*  $T = \{C \rightarrow D, A \rightarrow D, A \rightarrow C\}$

- b)
- Finding subsets  
 $X_1 = \{A\}$ ,  $X_2 = \{B\}$ ,  $X_3 = \{C\}$ ,  $X_4 = \{A, B\}$ ,  $X_5 = \{A, C\}$ ,  $X_6 = \{B, C\}$ ,  
 $X_7 = \{A, B, C\}$ ,  $X_8 = \{\}$
  - Finding  $X_i^+$ 
    - $X_1^+ = \{A, D\}$
    - $X_2^+ = \{B\}$
    - $X_3^+ = \{C\}$
    - $X_4^+ = \{A, B, D, E\}$
    - $X_5^+ = \{A, B, C, D, E\}$
    - $X_6^+ = \{B, C\}$
    - $X_7^+ = \{A, B, C, D, E\}$
    - $X_8^+ = \{\}$
  - Putting all nontirival FD's in  $T$   
 $T = \{A \rightarrow D, AB \rightarrow D, AB \rightarrow E, AC \rightarrow B, AC \rightarrow D, AC \rightarrow E, ABC \rightarrow D, ABC \rightarrow E\}$
  - Finding minimal basis for the FD of  $S$   
 $T_{\text{minimal}} = \{A \rightarrow D, AB \rightarrow E, AC \rightarrow B, AC \rightarrow E\}$
- c)
- Finding subsets  
 $X_1 = \{A\}$ ,  $X_2 = \{B\}$ ,  $X_3 = \{C\}$ ,  $X_4 = \{A, B\}$ ,  $X_5 = \{A, C\}$ ,  $X_6 = \{B, C\}$ ,  
 $X_7 = \{A, B, C\}$ ,  $X_8 = \{\}$
  - Finding  $X_i^+$ 
    - $X_1^+ = \{A\}$
    - $X_2^+ = \{B\}$
    - $X_3^+ = \{C\}$
    - $X_4^+ = \{A, B, D\}$
    - $X_5^+ = \{A, B, C, D, E\}$
    - $X_6^+ = \{A, B, C, D, E\}$
    - $X_7^+ = \{A, B, C, D, E\}$
    - $X_8^+ = \{\}$
  - Putting all nontirival FD's in  $T$   
 $T = \{AB \rightarrow D, AC \rightarrow B, AC \rightarrow D, AC \rightarrow E, BC \rightarrow A, BC \rightarrow D, BC \rightarrow E, ABC \rightarrow D, ABC \rightarrow E\}$
  - Finding minimal basis for the FD of  $S$   
 $T_{\text{minimal}} = \{A \rightarrow D, AC \rightarrow B, C \rightarrow A, C \rightarrow B, C \rightarrow E\}$
- d)
- Finding subsets  
 $X_1 = \{A\}$ ,  $X_2 = \{B\}$ ,  $X_3 = \{C\}$ ,  $X_4 = \{A, B\}$ ,  $X_5 = \{A, C\}$ ,  $X_6 = \{B, C\}$ ,  
 $X_7 = \{A, B, C\}$ ,  $X_8 = \{\}$
  - Finding  $X_i^+$ 
    - $X_1^+ = \{A, B, C, D, E\}$
    - $X_2^+ = \{A, B, C, D, E\}$

3.  $X_3^+ = \{A, B, C, D, E\}$
  4.  $X_4^+ = \{A, B, C, D, E\}$
  5.  $X_5^+ = \{A, B, C, D, E\}$
  6.  $X_6^+ = \{A, B, C, D, E\}$
  7.  $X_7^+ = \{A, B, C, D, E\}$
  8.  $X_8^+ = \{\}$
- Putting all nontirival FD's in  $T$   
 $T = \{A \rightarrow B, A \rightarrow C, A \rightarrow D, A \rightarrow E, B \rightarrow A, B \rightarrow C, B \rightarrow D, B \rightarrow E, C \rightarrow A, C \rightarrow B, C \rightarrow D, C \rightarrow E, AB \rightarrow C, AB \rightarrow D, AB \rightarrow E, AC \rightarrow B, AC \rightarrow D, AC \rightarrow E, BC \rightarrow A, BC \rightarrow D, BC \rightarrow E, ABC \rightarrow D, ABC \rightarrow E\}$
  - Finding minimal basis for the FD of  $S$   
 $T_{\text{minimal}} = \{A \rightarrow B, A \rightarrow C, A \rightarrow D, A \rightarrow E, B \rightarrow A, B \rightarrow C, B \rightarrow D, B \rightarrow E, C \rightarrow A, C \rightarrow B, C \rightarrow D, C \rightarrow E\}$

e) a) **Notes:**

- Anomalies
  - means “Something that you don’t expect” in layman’s terms