

# Problem Set 1 Solution

March 14, 2020

## Question 1

- a.  $\forall t \in T, \text{Canadian}(t) \Rightarrow \neg \text{Stanley}(t)$
- b.  $\forall t \in T, \exists d \in D, \neg \text{Canadian}(t) \wedge \text{BelongsTo}(t, d)$
- c.  $\forall t \in T, \exists d \in D, \text{Stanley}(t) \wedge \text{BelongsTo}(t, d)$
- d.  $\forall t \in T, \exists d \in D, \text{BelongsTo}(t, d) \Rightarrow \forall d' \in D, d' \neq d \wedge \neg \text{BelongsTo}(t, d')$
- e.  $\forall t_1 \in T, \exists d \in D, \exists t_2 \in T, t_1 \neq t_2 \wedge (\text{BelongsTo}(t_1, d) \wedge \text{BelongsTo}(t_2, d)) \Rightarrow \forall t_3 \in T, t_3 \neq t_1 \wedge t_3 \neq t_2 \wedge \neg \text{BelongsTo}(t_3, d)$

## Question 2

- a.  $\forall x \in \mathbb{R}, f(-x) = f(x)$   
 $\forall x \in \mathbb{R}, -f(-x) = f(x)$
- b.  $\forall g, f : \mathbb{R} \rightarrow \mathbb{R}, \exists h : \mathbb{R} \rightarrow \mathbb{R}, \text{Odd}(f) \wedge \text{Odd}(g) \Rightarrow \text{Odd}(f) \times \text{Odd}(g) = \text{Even}(h)$
- c.  $f = 0$  is a solution, since  $-f(-x) = -(-0) = 0 = f(x)$  and  $f(-x) = -0 = 0 = f(x)$
- d.  $\forall f : \mathbb{R} \rightarrow \mathbb{R}, \exists f_1, f_2 : \mathbb{R} \rightarrow \mathbb{R}, \text{Odd}(f_1) \wedge \text{Even}(f_2) \wedge f = \text{Odd}(f_1) + \text{Even}(f_2)$

e. A solution is  $f = x^2 + x$  with  $f_1 = x^2$  and  $f_2 = x$ .

$f = x^2 + x$  is the summation  $\sum_{i=0}^{2n}$  with  $n = 1, a_0 = 0, a_1 = 1, a_2 = 1$ .  
 $f_1$  is odd since  $-f(-x) = -(-x) = x = f(x)$ , and  $f_2$  is even since  $f(-x) = (-x)^2 = x^2 = f(x)$

f. A solution is  $g_1(x) = \frac{2^x+2^{-x}}{2}$  and  $g_2(x) = \frac{2^x-2^{-x}}{2}$ .

$g_1 + g_2$  gives  $g$  since  $\frac{2^x+2^{-x}}{2} + \frac{2^x-2^{-x}}{2} = 2^x$ . Also,  $g_1(-x) = \frac{2^{-x}+2^{-(-x)}}{2} = \frac{2^{-x}+2^x}{2} = g_1(x)$  is even, and  $-g_2(-x) = -(\frac{2^{-x}-2^{-(-x)}}{2}) = \frac{-2^{-x}-2^x}{2} = g_2(x)$

### Question 3

a. One solution is  $\forall x \in \mathbb{N}, \exists y \in \mathbb{N}, x > y$ .

With the above as predicate, the first statement is true, because  $\forall x, x > 165$  is always greater than one.

Also, the second statement is false, because on the rhs,  $x = 1$  can be chosen, and  $1 \not> 1$

### Question 4