CSC343 Worksheet 12 Solution

July 1, 2020

1.
   - Keys
     - {id of molecule}
     - {x position, y position, z position}
   - Functional Dependencies
     1. id of molecule → x position, y position, z position, x velocity, y velocity, z velocity
     2. x position, y position, z position → id of molecule, x velocity, y velocity, z velocity

Notes:

- Function Dependencies
  - *Functional Dependency* is a relationship between two attributes typically between the key and other non-key attributes within a table.

    Example:

    SIN → Name, Address, Birthdate

    Example 2:

    ISBN → Title
- Key of Relations
  - One or more attributes $\{A_1, A_2, ..., A_n\}$ is a key for a relation R if
    1. Those attributes functionally determine all other attributes of the relation
    2. No proper subset of $\{A_1, A_2, ...A_n\}$ functionally determines all other attributes of R

       Example:

       Given relation

       $R =$ Movies1(title, year, length, genre, studioName, starName)

       i. {title, year, starName } form a key for the relation **Movies1**

ii. { year, starName } is not a key. Same star can be in multiple movies per year

- Superkeys
  * Means a a set of attributes that contains a key
  * Don't need to be minimal

Example:
Given relation

$R =$ Movies1(title, year, length, genre, studioName, starName)
· { title, year, starName } is a key and superkey
· { title, year, starName, title, year, length} is a superkey

References:

1) OpenTextBC, Chapter 11 Functional Dependencies, link

2. a) 1. $AB \rightarrow C$
   2. $AB \rightarrow D$
   3. $C \rightarrow A$
   4. $C \rightarrow B$
   5. $D \rightarrow B$
   6. $D \rightarrow C$
   7. $C \rightarrow D$
   8. $D \rightarrow A$

Second Attempt:
$\{A, B\}^+ = \{A, B, C, D\}$, so the following non-trivial FDs follows: $AB \rightarrow C$ and $AB \rightarrow D$.

$\{C\}^+ = \{D, A\}$, so the following non-trivial FDs follows $C \rightarrow D$ and $C \rightarrow A$.

$\{D\}^+ = \{A\}$, so the following non-trivial FDs follows: $D \rightarrow A$.

Notes:

- The Splitting / Combining Rule
  - Combining Rule
    * $A_1, A_2, \cdots, A_n \rightarrow B_i$ for $i = 1, 2, ..., m$
      to
      $A_1, A_2, \cdots A_n \rightarrow B_1, B_2, \cdots B_m$

Example:

Given

title year → length
title year → genre
title year → studioName

it's combined form is

title year → length genre studioName

- Splitting Rule
  -
  - $A_1, A_2, \cdots A_n \rightarrow B_1, B_2, \cdots B_m$
    to
    $A_1, A_2, \cdots, A_n \rightarrow B_i$ for $i = 1, 2, ..., m$

  Example:

  Given

  title year → length

  It's splitted form is

  title → length
  year → length

- Trivial Functional Dependencies
  - A functional dependency $FD : X \rightarrow Y$ is **trivial** if Y is a subset of X

  Exmaple:

  title year → title

  Example 2:

  title → title

- Non-trivial Functional Dependencies
  - is a case where some but not all of the attributes on the R.H.S of an FD are also on L.H.S

  Example:

  title year → title movieLength

- Can be simplified using **tirivial-dependency rule**
  * The FD $A_1A_2 \cdots A_n \rightarrow B_1B_2 \cdots B_m$ is equivalent to $A_1A_2 \cdots A_n \rightarrow C_1C_2 \cdots C_k$

    where C's are all those B's that are not in A's.

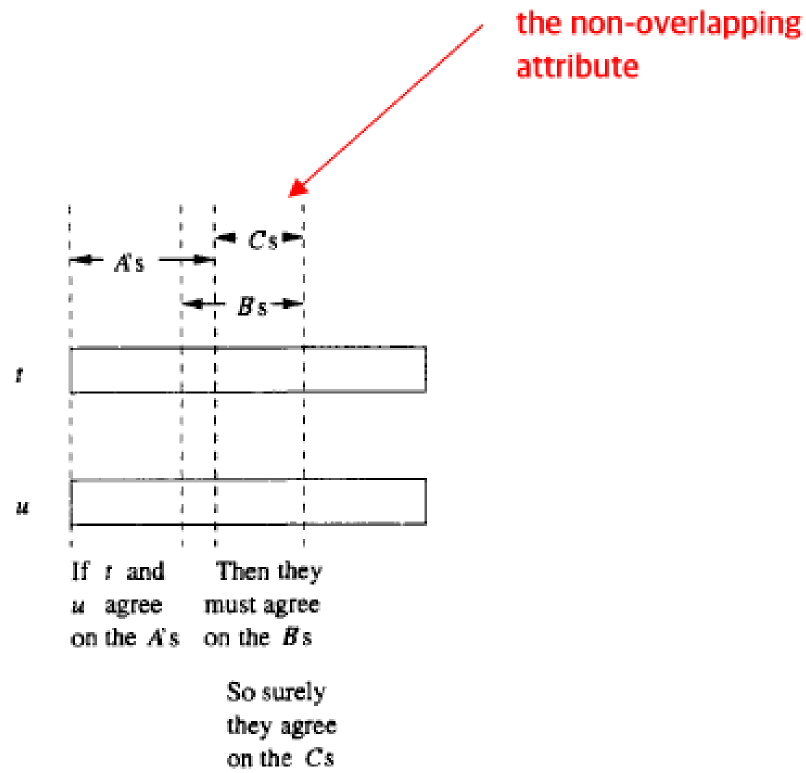

Figure 3.3: The trivial-dependency rule

- Computing the Clousre of Attributes
  - Closure of attribute set $\{X\}$ is denoted as $\{X\}^+$.
  - The closure means a given set of attributes A satisfying FD, are a sets of all attributes B such that $A \rightarrow B$

    Example:

    Given attributes A, B, C, D, E, F and FDs $AB \rightarrow C$, $BC \rightarrow AD$, $D \rightarrow E$ and $CF \rightarrow B$, What is the closure of $\{A, B\}$ or $\{A, B\}^+$

    1. Start with $\{A, B\}$.
    2. Split $BC \rightarrow AD$
       * We have $BC \rightarrow A$ and $BCtoD$
       * Since A is in $\{A, B\}$, this is not included
       * Since D is not in $\{A, B\}$, this IS included

       So, we have $\{A, B, D\}$

3. Since C in $AB \to C$ is NOT in $\{A, B, C, D\}$, C is included and we have $\{A, B, C, D\}$
4. Since A in $BC \to A$ is in $\{A, B, C, D\}$, this is skipped
5. Since E is not in $D \to E$, E is included and we have $\{A, B, C, D, E\}$ as our solution

- Why the Closure Algorithm Works
- Transitive Rule
  - Definition

    If $A_1A_2 \cdots A_n \to B_1B_2 \cdots B_m$ and $B_1B_2 \cdots B_m \to C_1C_2 \cdots C_k$ hold in relation R, $A_1A_2 \cdots A_n \to C_1C_2 \cdots C_k$ also holds in R.

    Example:

    Given

    title year $\to$ studioName
    studioName $\to$ studioAddr

    Transitive rule says the above is equal to the following

    title year $\to$ studioAddr
- Inference Rules
  - Is allso called **Armstrong's Axioms**
  - Has 3 axioms

1. *Reflexivity*
   * If $\{B_1, B_2, ..., B_n\} \subseteq \{A_1, A_2, ..., A_n\}$ then $A_1A_2 \cdots A_n \to B_1B_2 \cdots B_m$
   * also called **trivial FDs**
2. *Augmentation*
   * If $A_1A_2 \cdots A_n \to B_1B_2 \cdots B_m$ then $A_1A_2 \cdots A_nC_1C_2 \cdots C_k \to B_1B_2 \cdots B_mC_1C_2 \cdots C_k$
   * $C_1C_2 \cdots C_k$ are any set of attributes
3. *Transitivity*
   * If $A_1A_2 \cdots A_n \to B_1B_2 \cdots B_m$ and $B_1B_2 \cdots B_m \to C_1C_2 \cdots C_k$ then $A_1A - 2 \cdots A_n \to C_1C_2 \cdots C_k$

b) A, B is the only key of R.

Notes:

- Key of Attributes
  - **Definition:** A set of attributes $\{A_1, A_2, \cdots, A_n\}$ is a key for a relation R if

1. Those attributes functionally determine all other attributes

2. No proper subset of $\{A_1, A_2, ..., A_n\}$ functionally determines all other attributes of R.

c) The superkeys that are not keys are: $\{A, B, C\}$, $\{A, B, D\}$, $\{A, B, C, D\}$

3. i) a) $\{A\}^+ = \{A, B, C, D\}$, so we have $A \to A$, $A \to B$, $A \to C$, $A \to D$

$\{B\}^+ = \{C, D\}$, so we have $B \to C$ and $B \to D$

b) $\{A\}$ is the key of S.

c) The super keys that are not keys are:

$\{A, B\}$, $\{A, C\}$, $\{A, D\}$, $\{A, B, C\}$, $\{A, B, D\}$, $\{A, B, C, D\}$

ii) a) $\{A\}^+ = \{A\}$, so this FD is trivial.

$\{B\}^+ = \{B\}$, so this FD is trivial.

$\{C\}^+ = \{C\}$, so this FD is trivial.

$\{D\}^+ = \{D\}$, so this FD is trivial.

$\{A, B\}^+ = \{A, B, C, D\}$, so we have $AB \to A$, $AB \to B$, $AB \to C$, $AB \to D$

$\{A, C\}^+ = \{A, C\}$, so we have $AC \to A$, $AC \to C$

$\{A, D\}^+ = \{A, D, B\}$, so we have $AD \to A$, $AD \to D$, $AD \to B$

$\{B, C\}^+ = \{B, C, D, A\}$, so we have $BC \to A$, $BC \to B$, $BC \to C$, $BC \to D$

$\{D, C\}^+ = \{D, C, A, B\}$, so we have $DC \to D$, $DC \to C$, $DC \to A$, $DC \to B$

$\{A, B, C\}^+ = \{A, B, C, D\}$, so we have $ABC \to A$, $ABC \to B$, $ABC \to C$, $ABC \to D$

$\{B, C, D\}^+ = \{B, C, D, A\}$, so we have $BCD \to A$, $BCD \to B$, $BCD \to C$, $BCD \to D$

$\{C, D, A\}^+ = \{C, D, A, B\}$, so we have $CDA \to A$, $CDA \to B$, $CDA \to C$, $CDA \to D$

$\{D, A, B\}^+ = \{D, A, B, C\}$, so we have $DAB \to A$, $DAB \to B$, $DAB \to C$, $DAB \to D$

$\{D, A, B\}^+ = \{D, A, B, C\}$, so we have $DAB \to A$, $DAB \to B$, $DAB \to C$, $DAB \to D$

$\{A, B, C, D\}^+ = \{A, B, C, D\}$, so this FD is trivial.

b) $\{A, B\}$, $\{A, C\}$, $\{B, C\}$, $\{D, C\}$ are the keys of T.

c) The super keys that are not keys are:

$\{A, B, C\}$, $\{A, B, D\}$, $\{B, C, D\}$, $\{A, D, C\}$, $\{A, B, D\}$, $\{A, B, C, D\}$

iii) a) $\{A\}^+ = \{A, B, C, D\}$, so we have $A \to C$, $A \to D$

$\{B\}^+ = \{A, B, C, D\}$, so we have $B \to A$, $B \to D$

$\{C\}^+ = \{A, B, C, D\}$, so we have $C \to A$, $C \to B$

$\{D\}^+ = \{A, B, C, D\}$, so we have $D \to B$, $D \to C$

$\{A, B\}^+ = \{A, B, C, D\}$, so we have $AB \to C$, $AB \to D$

$\{B, C\}^+ = \{A, B, C, D\}$, so we have $BC \to A$, $BC \to D$

$\{B, D\}^+ = \{A, B, C, D\}$, so we have $BD \to A$, $BD \to C$

$\{C, D\}^+ = \{A, B, C, D\}$, so we have $CD \to A$, $CD \to B$

$\{C, D\}^+ = \{A, B, C, D\}$, so we have $CD \to A$, $CD \to B$

$\{A, B, C\}^+ = \{A, B, C, D\}$, so we have $ABC \to D$

$\{B, C, D\}^+ = \{A, B, C, D\}$, so we have $BCD \to A$

$\{C, D, A\}^+ = \{A, B, C, D\}$, so we have $CDA \to B$

$\{D, A, B\}^+ = \{A, B, C, D\}$, so we have $DAB \to C$

Correct Solution:

$\{A\}^+ = \{A, B, C, D\}$, so we have $A \to C$, $A \to D$

$\{B\}^+ = \{A, B, C, D\}$, so we have $B \to A$, $B \to D$

$\{C\}^+ = \{A, B, C, D\}$, so we have $C \to A$, $C \to B$

$\{D\}^+ = \{A, B, C, D\}$, so we have $D \to B$, $D \to C$

$\{A, B\}^+ = \{A, B, C, D\}$, so we have $AB \to C$, $AB \to D$

$\{A, C\}^+ = \{A, B, C, D\}$, so we have $AC \to B$, $AC \to D$

$\{A, D\}^+ = \{A, B, C, D\}$, so we have $AD \to B$, $AD \to C$

$\{B, C\}^+ = \{A, B, C, D\}$, so we have $BC \to A$, $BC \to D$

$\{B, D\}^+ = \{A, B, C, D\}$, so we have $BD \to A$, $BD \to C$

$\{C, D\}^+ = \{A, B, C, D\}$, so we have $CD \to A$, $CD \to B$

$\{A, B, C\}^+ = \{A, B, C, D\}$, so we have $ABC \to D$

$\{B, C, D\}^+ = \{A, B, C, D\}$, so we have $BCD \to A$

$\{C, D, A\}^+ = \{A, B, C, D\}$, so we have $CDA \to B$

$\{D, A, B\}^+ = \{A, B, C, D\}$, so we have $DAB \to C$

b) $\{A\}, \{B\}, \{C\}, \{D\}$ are the keys of U.

c) The super keys that are not keys are:

$\{A, B\}$,$\{A, C\}$,$\{A, D\}$,$\{B, C\}$, $\{B, D\}$,$\{C, D\}$, $\{A, B, C\}$, $\{B, C, D\}$, $\{C, D, A\}$, $\{D, A, B\}$. $\{A, B, C, D\}$

4. a) We need to show the closure of attributes $\{A_1, A_2, \cdots, A_n, C\}$ in FD $A_1, A_2, \cdots, A_n, C \to B$ is $\{A_1, A_2, \cdots, A_n, C, B\}$, that is $\{A_1, A_2, \cdots, A_n, C\}^+ = \{A_1, A_2, \cdots, A_n, C, B\}$

Since we know $\{A_1, A_2, \cdots, A_n\}$ functionally determines B, we can conclude B can be added to $\{A_1, A_2, \cdots, A_n, C\}$.

Thus, it follows from above that $\{A_1, A_2, \cdots, A_n, C\}^+ = \{A_1, A_2, \cdots, A_n, C, B\}$.

b) Let $A_1A_2 \cdots A_n \to B$ is FD. That is, $\{A_1A_2 \cdots A_n\}^+ = \{A_1A_2 \cdots A_n, B\}$

We need to show $A_1A_2 \cdots A_nC \to BC$ follows. That is, $\{A_1, A_2, \cdots, A_n, C\}^+ = \{A_1, A_2, \cdots, A_n, C, B\}$

It follows from the combine and split rule that $A_1A_2 \cdots A_nC \to BC$ can be spliited into $A_1A_2 \cdots A_nC \to B$ and $A_1A_2 \cdots, A_nC \to C$.

So, we need to show $A_1A_2 \cdots A_nC \to B$ and $A_1A_2 \cdots, A_nC \to C$ follows from the given.

We will do so in parts.

1. **Part 1 (Showing $A_1A_2 \cdots A_nC \to B$):**

   Here, we need to show $A_1A_2 \cdots A_nC \to B$ follows.

   And indeed, this follows from the work of *augmenting left sides*. That is the solution to previous problem.

2. **Part 2 (Showing $A_1A_2 \cdots A_nC \to C$):**

   Here, we need to show $A_1A_2 \cdots A_nC \to C$ follows.

   The definition of trivial FD tells us $A_1A_2 \cdots A_n \to B_1B_2 \cdots B_m$ holds when $\{B_1, B_2, \cdots, B_m\} \subseteq \{A_1, A_2, \cdots, A_n\}$

   Since $\{C\} \subseteq \{A_1, A_2 \cdots, A_n, C\}$, we can conclude this FD follows trivially.

c) Let $A_1A_2 \cdots A_n \to B_1B_2 \cdots B_m$ and $C_1C_2 \cdots C_k \to D$, where B are each among the C's.

We neeed to show $A_1A_2 \cdots A_nE_1E_2 \cdots E_j \to D$ follows, where the E's are all of those C's not found among the B's.

The transitive rule tells us if $A_1A_2 \cdots A_n \to B_1B_2 \cdots B_m$ and $B_1B_2 \cdots B_m \to C_1C_2 \cdots C_k$, then $A_1A_2 \cdots A_n \to C_1C_2 \cdots C_k$ also holds in R.

Since we know $A_1A_2 \cdots A_n \to B_1B_2 \cdots B_m$ and $C_1C_2 \cdots C_k \to D$ where B's are each among the C's, we can conclude from the transitive rule that $A_1A_2 \cdots A_n \to D$.

Then using **augmenting left sides** to all C's not found among the B's on $A_1A_2 \cdots A_n \to D$, we can conclude $A_1A_2 \cdots A_nE_1E_2 \cdots E_j \to D$ follows.

d) **Rough Work:**

Assume FD's $A_1A_2 \cdots A_n \to B_1B_2 \cdots B_m$ and $C_1C_2 \ldots C_k \to D_1D_2 \cdots D_j$ holds.

We need to show FD $A_1A_2 \cdots A_nC_1C_2 \cdots C_k \to B_1B_2 \cdots B_mD_1D_2 \cdots D_k$ follows.

1. Using split combine rule, split $A_1A_2 \cdots A_nC_1C_2 \cdots C_k \to B_1B_2 \cdots B_mD_1D_2 \cdots D_k$ into $A_1A_2 \cdots A_nC_1C_2 \cdots C_k \to B_1B_2 \cdots B_m$ and $A_1A_2 \cdots A_nC_1C_2 \cdots C_k \to D_1D_2 \cdots D_k$

   Then, in parts, show:

   1. Show $A_1A_2 \cdots A_nC_1C_2 \cdots C_k \to B_1B_2 \cdots B_m$ follows
   2. Show $A_1A_2 \cdots A_nC_1C_2 \cdots C_k \to D_1D_2 \cdots D_k$ follows