CSC236 Worksheet 5 Review

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Question 1

a. *Proof.* Define $P(k) : R(3^k) = k3^k$. Note that when $n = 3^k$, this is equivalent to $R(n) = n \log_3 n$. I will use simple induction to prove $P(k)$.

Base Case ($k = 0$):

Let $k = 0$.

Then,

$$
\begin{aligned}
R(3^k) &= 0 && \text{[By def., since } n = 3^0 = 1\text{]} \quad (1)\\
&= 0 \cdot 3^0 && (2)\\
&= k \cdot 3^k && (3)
\end{aligned}
$$

Thus, $P(k)$ is verified in this step.

Inductive Step:

Let $k \in \mathbb{N}$. Assume $P(k)$. That is, $R(3^k) = k \cdot 3^k$. I need to prove $P(k+1)$ follows. That is, $R(3^{k+1}) = (k+1) \cdot 3^{k+1}$

Starting from $R(3^{k+1})$, we have

$$
\begin{aligned}
R^(3^{k+1}) &= 3^{k+1} + 3R(\lceil 3^{k+1}/3 \rceil) && \text{[By def., since } 0 < k+1, \text{ and } 1 < 3^{k+1}\text{]} \quad (4)\\
&= 3^{k+1} + 3R(\lceil 3^k \rceil) && (5)\\
&= 3^{k+1} + 3R(3^k) && \text{[Since } \lceil 3^k \rceil = 3^k\text{]} \quad (6)\\
&= 3^{k+1} + 3(k \cdot 3^k) && \text{[By I.H]} \quad (7)\\
&= 3^{k+1} + (k \cdot 3^{k+1}) && (8)\\
&= (k+1) \cdot 3^{k+1} && (9)
\end{aligned}
$$

$\square$

b. **Rough Work:**

For convenience, define $P(k) : \bigwedge_{i=1}^{n=k} R(i) \leq R(k)$.

I will use complete induction to prove that $\forall n \in \mathbb{N}, n > 0 \Rightarrow P(n)$.

1. Inductive Step
2. Base Case $(n = 1)$
3. Base Case $(n = 2)$
4. Inductive Step