Midterm 2 Version 1 Review

July 17, 2020

1. a) 1100100

   b) $-\sum_{i=0}^{n-1} 3^i$

   Notes:

   - Balanced Ternary
     - is a way of representing numbers
     - balanced ternary is in base 3, and has values 1,0 or -1

   $$\sum_{i=0}^{n-1} d_i \cdot 3^i \text{ where } d_i \in \{0, 1, -1\} \tag{1}$$

   c) i. $\underline{f(n) \in \Omega(n)}$

       True (since $n^2 + 10n + 2 \geq cn$)

      ii. $\underline{g(n) \in \Omega(n)}$

       False (Let $c = 100, n_0 = 100$. Then $100 \log_2 n < 100n$)

     iii. $\underline{f(n) \in \mathcal{O}(g(n))}$

       False ($f(n) = n^2 + 10n + 2$ grows faster than $g(n) = 100 \log_2 n$)

      iv. $\underline{f(n) \in \Theta(g(n))}$

       True (Set $c_1 = -1, c_2 = 1, n_1 = 100$. Then $c_1 f(n) \leq g(n) \leq c_2 f(n)$)

   Notes:

   - $g \in \Omega(f) : \exists c, n_o \in \mathbb{R}^+, \forall n \in \mathbb{N}, n \geq n_0 \Rightarrow g(n) \geq cf(n)$, where $f, g : \mathbb{N} \to \mathbb{R}^{\geq 0}$
   - $g \in \mathcal{O}(f) : \exists c, n_o \in \mathbb{R}^+, \forall n \in \mathbb{N}, n \geq n_0 \Rightarrow g(n) \leq cf(n)$, where $f, g : \mathbb{N} \to \mathbb{R}^{\geq 0}$
   - $g \in \Theta(f) : g \in \mathcal{O}(f) \land g \in \Omega(f)$
     or
     $g \in \Theta(f) : \exists c_1, c_2, n_1 \in \mathbb{R}^+, \forall n \in \mathbb{N}, n \geq n_1 \Rightarrow c_1 g(n) \leq f(n) \leq c_2 g(n)$, where $f, g : \mathbb{N} \to \mathbb{R}^{\geq 0}$