

# CSC236 Midterm 2 Version 1 Solution

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## Question 1

- Rough Works:

Let  $n, q \in \mathbb{N}$ . Let  $r \in \{0, 1\}$

Assume  $n > 2$ .

I need to find a closed form for  $T(2q + r)$ , using repeated substitution.

1. Find  $T(2q + r)$  in closed form

Starting from  $T(n)$ , we have

$$T(n) = n + T(n - 2) \quad [\text{By def. since } n > 2]$$

(1)

$$T(2q + r) = 2q + r + T(2q + r - 2) \quad [\text{By replacing } n \text{ for } 2q + r]$$

(2)

$$= 2q + r + T(2(q - 1) + r) \quad (3)$$

$$\vdots \quad (4)$$

$$= \sum_{i=0}^{q-1} (2(q - i) + r) + T(r) \quad [\text{After } q - 1 \text{ repeatitions}]$$

(5)

$$= 2 \sum_{i=0}^{q-1} (q - i) + \sum_{i=0}^{q-1} r + T(r) \quad (6)$$

$$= 2 \sum_{i=0}^{q-1} (q - i) + \sum_{i=0}^{q-1} r \quad [\text{Since } T(r) = 0]$$

(7)

$$= 2 \sum_{i'=1}^q i' + \sum_{i=0}^{q-1} r \quad (8)$$

$$= 2 \sum_{i'=1}^q i' + \sum_{i=0}^{q-1} r \quad (9)$$

$$= 2 \sum_{i'=1}^q i' + \sum_{i=0}^{q-1} r \quad (10)$$

$$= 2(q(q + 1))/2 + \sum_{i=0}^{q-1} r \quad [\text{By using } \sum_{i=1}^n i = (n(n + 1))/2]$$

(11)

$$= q(q + 1) + rq \quad (12)$$

$$= q(q + 1 + r) \quad (13)$$