CSC373 Worksheet 2 Solution

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1. **<u>Notes:</u>**

- Greedy Algorithm
    - Always makes the choice that looks best at the moment
        * Locally optimal solution leads to globally optimal solution
- Activity-selection Problem (Greedy algorithm using dynamic programming)
    - Goal: Selecting maximum size set of mutually compatible activities

    <u>Example:</u>

i	1	2	3	4	5	6	7	8	9	10	11
s_i	1	3	0	5	3	5	6	8	8	2	12
f_i	4	5	6	7	9	9	10	11	12	14	16

    - Suppose a set exists $S = \{a_1 = [s_1, f_1), a_2 = [s_2, f_2), ..., a_n = [s_n, f_n)\}$
        * a_i represents an ith activity
        * s_i represents starting time
        * f_i represents finishing time
        * $0 \le s_i < f_i < \infty$
        * $a_1, ..., a_n$ sorted in monotonically increasing order of finish time

            i.e.

            $$f_1 \le f_2 \le f_3 \le ... \le f_{n-1} \le f_n$$

        * a_i and a_j are **compatible**, if intervals $[s_i, f_i)$ and $[s_j, f_j)$ don't overlap

            i.e

            $$s_i \ge f_j \text{ and } s_j \ge f_i$$

– Steps

    1. Think about dynamic programming solution

       * S_{ij}: activities that start after activity a_i finishes and before activity a_j starts

       i.e.

$$S_{19} = \{a_4 = [5, 7), a_6 = [5, 9), a_7 = [6, 10)\}$$

       * A_{ij}: maximum set of mutually compatible activities in S_{ij} (including a_k)
  - $A_{ik} = A_{ij} \cap S_{ik}$
  - $A_{kj} = A_{ij} \cap S_{kj}$
  - $A_{ij} = A_{ik} \cup \{a_k\} \cup Akj$
  - So, $|A_{ij}|$

    2. Observe that only one choice - greedy choice, and that when we make the greedy choice, only one subproblem remains

    3. Develop recursive greedy solution

    4. Convert the recursive algorithm into iterative one