CSC343 Worksheet 12

June 30, 2020

1. **Exercise 3.1.2:** Consider a relation representing the present position of molecules in a clospd ront.ainer. The attributes are an ID for the molecule the x y and z coordinat.1 · s of the molecule, and its velocity in the x, y, and z di ensi;ns: What FD's would you expect to hold? What are the keys?

2. **Exercise 3.2.1:** Consider a relation with schema R(A, B, C, D) and FD's AB → C, C → D, and D → A.

   a) What are all the nontrivial FD's that follow from the given FD's? You should restrict yourself to FD's with single attributes on the right side.

   b) What are all the keys of R?

   c) What are all the superkeys for R that are not keys?

3. **Exercise 3.2.2:** Repeat Exercise 3.2.1 for the following schemas and sets of FD's:

   a) S(A, B, C, D) with FD's A→ B, B → C, and B →D.

   b) T(A, B, C, D) with FD's AB → C, BC → D, CD → A, and AD → B.

   c) U(A, B, C, D) with FD's A→ B, B → C, C → D, and D → A.

4. **Exercise 3.2.3:** Show that the following rules hold, by using the closure test of Section 3.2.4.

   a) Augmenting left sides. If $A_1A_2 \cdots A_n \to B$ is an FD, and C is another attribute, then $A_1A_2 \cdots A_nC \to B$ follows.

   b) Full augmentation. If $A_1A_2 \cdots A_n \to B$ is an FD, and C is another attribute, then $A_1A_2 \cdots A_nC \to BC$ follows. Note: from this rule, the "augmentation" rule mentioned in the box of Section 3.2.7 on "A Complete Set of Inference Rules" can easily be proved.

   c) Pseudotransitivity. Suppose FD's $A_1A_2 \cdots A_n \to B_1B_2 \cdots B_m$ and $C_1C_2 \cdots C_k \to D$ hold, and the B's are each among the C's. Then $A_1A_2 \cdots A_nE_1E_2 \cdots E_j \to D$ holds, where the E's are all those of the C's that are not found among the B's.

d) Addition If FD's $A_1A_2\cdots A_n \rightarrow B_1B_2\cdots B_m$ and $C_1C_2\cdots C_k \rightarrow D_1D_2\cdots D_j$ hold, then FD $A_1A_2\cdots A_nC_1C_2\cdots C_K \rightarrow B_1B_2\cdots B_mD_1D_2\cdots D_j$ also holds. In the above, we should remove one copy of any attribute that appears among both the A's and C's or among both the B's and D's.

5. **Exercise 3.2.4:** Show that each of the following are not valid rules about FD's by giving example relations that satisfy the given FD's (following the "if") but not the FD that allegedly follows (after the "then").

   a) If $A \rightarrow B$ then $B \rightarrow A$.
   b) If $AB \rightarrow C$ and $A \rightarrow C$, then $B \rightarrow C$.
   c) If $AB \rightarrow C$, then $A \rightarrow C$ or $B \rightarrow C$.

6. **Exercise 3.2.5:** Show that if a relation has no attribute that is functionally determined by all the other attributes, then the relation has no nontrivial FD's at all.

7. **Exercise 3.2.6:** Let X and Y be sets of attributes. Show that if $X \subseteq Y$, then $X^+ \subseteq Y^+$, where the closures are taken with respect to the same set of FD's.

8. **Exercise 3.2.7:** Prove that $(X^+)^+ = X^+$.

9. **Exercise 3.2.9:** Find all the minimal bases for the FD's and relation of Example 3.11.

10. **Exercise 3.2.10:** Suppose we have relation R(A,B,C,D,E), with some set of FD's, and we wish to project those FD's onto relation S(A, B, C). Give the FD's that hold in S if the FD's for Rare:

    a) $AB \rightarrow DE$, $C \rightarrow E$, $D \rightarrow C$, and $E \rightarrow A$.
    b) $A \rightarrow D$, $BD \rightarrow E$, $AC \rightarrow E$, and $DE \rightarrow B$.
    c) $AB \rightarrow D$, $AC \rightarrow E$, $BC \rightarrow D$, $D \rightarrow A$, and $E \rightarrow B$.
    d) $A \rightarrow B$, $B \rightarrow C$, $C \rightarrow D$, $D \rightarrow E$, and $E \rightarrow A$.

    In each case, it is sufficient to give a minimal basis for the full set of FD's of S.

11. **Exercise 3.3.1:** For each of the following relation schemas and Sf'tS of FD's:

    a) $R(A, B, C, D)$ with FD's $AB \rightarrow C$, $C \rightarrow D$, and $D \rightarrow A$.
    b) $R(A, B, C, D)$ with FD's $B \rightarrow C$ and $B \rightarrow D$.
    c) $R(A, B, C, D)$ with FD's $AB \rightarrow C$, $BC \rightarrow D, CD \rightarrow AandAD \rightarrow B$
    d) $R(A, B, C, D)$ with FD's ,$A \rightarrow B$, $B \rightarrow C$. $C \rightarrow D$, and $D \rightarrow A$.
    e) $R(A, B, C, D, E)$ with FD's $AB \rightarrow C$, $DE \rightarrow C$, and $B \rightarrow D$.
    f) $R(A, B, C, D, E)$ with FD's $AB \rightarrow C$, $C \rightarrow D$, $D \rightarrow B$, and $D \rightarrow E$.

do the following:

i) Indicate all the BCNF violations. Do not forget to consider FD's that are not in the given set, but follow from them. However, it is not necessary to give violations that have more than one attribute on the right side.

ii) Decompose the relations, as necessary, into collections of relations that are in BCNF.

12. **Exercise 3.3.2**: We mentioned in Section 3.3.4 that we would exercise our option to expand the right side of an FD that is a BCNF violation if possible. Consider a relation R whose schema is the set of attributes $\{A, B, C, D\}$ with FD's $A \rightarrow B$ and $A \rightarrow C$. Either is a BCNF violation, because the only key for R is $\{A, D\}$. Suppose we begin by decomposing R according to $A \rightarrow B$. Do we ultimately get the same result as if we first expand the BCNF violation to $A \rightarrow BC$? Why or why not?

13. **Exercise 3.3.3:** Let R be as in Exercise 3.3.2, but let the FD's be $A \rightarrow B$ and $B \rightarrow C$. Again compare decomposing using $A \rightarrow B$ first against decomposing by $A \rightarrow BC$ first.

14. **Exercise 3.3.4:** Suppose we have a relation schema $R(A, B, C)$ with FD $A \rightarrow B$. Suppose also that we decide to decompose this schema into $S(A, B)$ and $T(B, C)$. Give an example of an instance of relation R whose projection onto S and T and subsequent rejoining as in Section 3.4.1 does not yield the same relation instance. That is, $\pi_{A,B}(R) \bowtie \pi_{B,C}(R) \neq R$.