CSC236 Worksheet 9 Solution

Hyungmo Gu

May 16, 2020

Question 1

a. I need to evalulate the reg. expressions for

$$L = \{x \in \Sigma \mid x \text{has even number of 1s or an odd number of 0s}\}$$

I will do so in parts.

<u>**Part 1 (Finding reg. expressions for even number of 1s):**</u>

In this part, I will find the reg. expressions for even number of 1's.

I will do so by finding patterns in series of small examples.

Starting with $L = \{x \in \Sigma \mid x \text{ has 0 number of 1s}\}$, it's reg. expressions is

$$0^* \tag{1}$$

Now for $L = \{x \in \Sigma \mid x \text{ has 2 number of 1s}\}$, it's reg. expressions is

$$0^*10^*10^* \tag{2}$$

Now for $L = \{x \in \Sigma \mid x \text{ has 4 number of 1s}\}$, it's reg. expressions is

$$0^*10^*10^*10^*10^* \tag{3}$$

From above, I see a pattern that

$$(0^*10^*1)(0^*10^*1)0^* \tag{4}$$

Using the pattern, I can conclude that the regular expression for even number of 1s is

$$(0^*10^*1)^*0^* \tag{5}$$

Part 2 (Finding reg. expressions for odd number of 0s):

In this part, I will find the reg. expressions for odd number of 0's.

I will do so by finding patterns in series of small examples.

Starting with $L = \{x \in \Sigma \mid x \text{ has 1 number of 0s}\}$, it's reg. expressions is

$$1^*01^* \tag{6}$$

Now for $L = \{x \in \Sigma \mid x \text{ has 3 number of 0s}\}$, it's reg. expressions is

$$1^*01^*01^*01^* \tag{7}$$

Now for $L = \{x \in \Sigma \mid x \text{ has 5 number of 0s}\}$, it's reg. expressions is

$$1^*01^*01^*01^*01^*01^* \tag{8}$$

From above, I see a pattern that

$$1^*(01^*)(01^*)(01^*)(01^*)(01^*) \tag{9}$$

Using the pattern, I can conclude that the regular expression for odd number of 0s is

$$1^*(01^*)^* \tag{10}$$

Thus, by combining the two parts with union, we have

$$(0^*10^*1)^*0^* + 1^*(01^*)^* \tag{11}$$

Notes:

- Regular Expression
  - Quick Guide

$$(0+1)((01)^*0) \tag{12}$$

The expression implies that

1. Starts with 0 **or** 1
   - indicated by (0 + 1)
2. Are then followed by **one or more repeatitions** of 01
   - indicated by $(01)^*$
3. Ends with 0
   - indicated by the final 0

- Examples
  1. $L = \{w \in \{a, b\}^* \mid w \text{ has an } a\}$

Answer:

$$(a+b)^*a(a+b)^* \tag{13}$$

- Means there is one or more repeatitions of a or b at front
- Means there is a in the middle

– Means there is zero or more repeatitions of a or b at end

2. $L = \{w \in \{a,b\}^* \mid w \text{ has at lest two } as\}$

Answer:

$$(a+b)^*a(a+b)^*a(a+b)^* \tag{14}$$

3. $L = \{w \in \{a,b\}^* \mid |w| \geq 2\}$

Answer:

$$(0+1)(0+1)(0+1)^* \tag{15}$$

In this example,

– Two characters are created (indicated by $(0+1)(0+1)$)
– And more :D!! (indicated by$(0+1)^*$)

b. I need to find the reg. expressions for $L = \{x \in \Sigma \mid x \text{ has at least one 1 and at least one 0}\}$. That is, regex expressions for $\{x \in \Sigma \mid x \text{ has at least one 1 followed by at least one 0}\}$ plus $\{x \in \Sigma \mid x \text{ has at least one 0 followed by at least one 1}\}$.

First, I need to find reg. expressions for

$$\{x \in \Sigma \mid x \text{ has at least one 1 followed by at least one 0}\} \tag{1}$$

Since the reg expressions for x with at least one 1 is $(0+1)^*1(0+1)^*$ and the reg expressions for x with at least one 0 is $(0+1)^*0(0+1)^*$, we have

$$(0+1)^*1(0+1)^*0(0+1)^* \tag{2}$$

Second, I need to find reg. expressions for

$$\{x \in \Sigma \mid x \text{ has at least one 0 followed by at least one 1}\} \tag{3}$$

Using the facts provided above, we have

$$(0+1)^*0(0+1)^*1(0+1)^* \tag{4}$$

Thus, by combining the two, we can conclude

$$(0+1)^*1(0+1)^*0(0+1)^* + (0+1)^*0(0+1)^*1(0+1)^* \tag{5}$$

c. I need to find reg. expressions for

$$\{x \in \Sigma \mid \text{every 1 in } x \text{ is immediately preceded and followed by a 0}\}$$

An example expresion of the above is

$$0^*0100^*0100^*0100^* \tag{1}$$

From above, we can see the following pattern

$$(0^*010)(0^*010)(0^*010)0^* \tag{2}$$

Thus, we have

$$(0^*010)^*0^* \tag{3}$$

Question 2

- **Negation:** $\exists r_1, r_2, r_3 \in \mathcal{R}\varepsilon$, $(r_1 r_2 \equiv r_2 r_1) \wedge (r_1 \not\equiv \varepsilon \not\equiv r_2) \wedge (r_1 \not\equiv \emptyset \not\equiv r_2) \wedge (r_1 \not\equiv r_2)$

  Let $r_1 = 1$ and $r_2 = 1^*$.

  Then, $r_1 r_2 \equiv r_2 r_1$ and $r_1 \not\equiv \varepsilon \not\equiv r_2$ and $r_1 \not\equiv \emptyset \not\equiv r_2$, but $r_1 \not\equiv r_2$.

  So, by counter-example, the statement is false.

  Notes:

  - Equivalence ($\equiv$) in Regular Expressions
    * Two regular expressions $\mathcal{R}$ and $\mathcal{S}$ are equivalent, written $R \equiv S$, if they denote the same language, i.e. $\mathcal{L}(\mathcal{R}) \equiv \mathcal{L}(\mathcal{S})$
      - Example, $(0^*1^*) \equiv (0+1)^*$
    * For all regular expressions $\mathcal{R}$, $\mathcal{S}$, $\mathcal{T}$, the following euivalences hold.
      - **Commutativity of union:** $(\mathcal{R}+\mathcal{S}) \equiv (\mathcal{S}+\mathcal{R})$
      - **Associativity of union:** $((\mathcal{R}+\mathcal{S})+\mathcal{T}) \equiv (\mathcal{R})+(\mathcal{S}+\mathcal{T})$
      - **Associativity of concatenation:** $((\mathcal{R}\mathcal{S})\mathcal{T}) \equiv (\mathcal{R}(\mathcal{S}\mathcal{T}))$
      - **Left distributivity:** $(\mathcal{R}(\mathcal{S}+\mathcal{T})) \equiv ((\mathcal{R}\mathcal{S})(\mathcal{R}\mathcal{T}))$
      - **Right distributivity:** $((\mathcal{S}+\mathcal{T})\mathcal{R}) \equiv ((\mathcal{S}\mathcal{R})(\mathcal{T}\mathcal{R}))$
      - **Identity for union:** $(\mathcal{R}+\emptyset) \equiv \mathcal{R}$
      - **Identity for concatenation:** $(\mathcal{R}\epsilon) \equiv R$ and $(\epsilon\mathcal{R}) \equiv \mathcal{R}$
      - **Annihilator for concatenation:** $(\emptyset\mathcal{R}) \equiv \emptyset$ and $(\mathcal{R}\emptyset) \equiv \emptyset$
      - **Idempotence of Kleene star:** $R^{**} \equiv R^*$