

CSC373 Worksheet 3

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Source: link

1. **CLRS 15.2-1:** Find an optimal parenthesization of a matrix-chain product whose sequence of dimension is $\langle 5, 10, 3, 12, 5, 50, 6 \rangle$
2. **CLRS 15.2-2:** Give a recursive algorithm $MATRIX-CHAIN-MULTIPLY(A, s, i, j)$ that actually performs the optimal matrix-chain multiplication, given the sequence of matrices $\langle A_1, A_3, \dots, A_n \rangle$, the s table computed by $MATRIX-CHAIN-ORDER$, and the indices i and j . (The initial call would be $MATRIX-CHAIN-MULTIPLY(A, s, 1, n)$).
3. **CLRS 15.2-5:** Let $R(i, j)$ be the number of times that table entry $m[i, j]$ is referenced while computing other table entries in call of $MATRIX-CHAIN-ORDER$. Show that the total number of references for the entire table is

$$\sum_{i=1}^n \sum_{j=1}^n R(i, j) = \frac{n^3 - n}{3}$$

4. **CLRS 15.3-2:** Draw the recursion tree for $MERGE-SORT$ procedure from Section 2.3.1 on an array of 16 elements. Explain why memoization fails to speed up a good divide-and-conquer algorithm such as $MERGE-SORT$
5. **CLRS 15.3-3:** Consider a variant of the matrix-chain multiplication problem in which the goal is to parenthesize the sequence of matrices so as to maximize, rather than minimize the number of scalar multiplications. Does this problem exhibit optimal substructure?
6. **CLRS 15.3-4:** As stated, in dynamic programming we first solve the subproblems and then choose which of them to use in an optimal solution to the problem. Professor Capulet claims that we do not always need to solve all the subproblems in order to find an optimal solution. She suggests that we can find an optimal solution to the matrix chain multiplication problem by always choosing the matrix A_k at which to split the subproduct $A_i A_{i+1} \dots A_j$

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