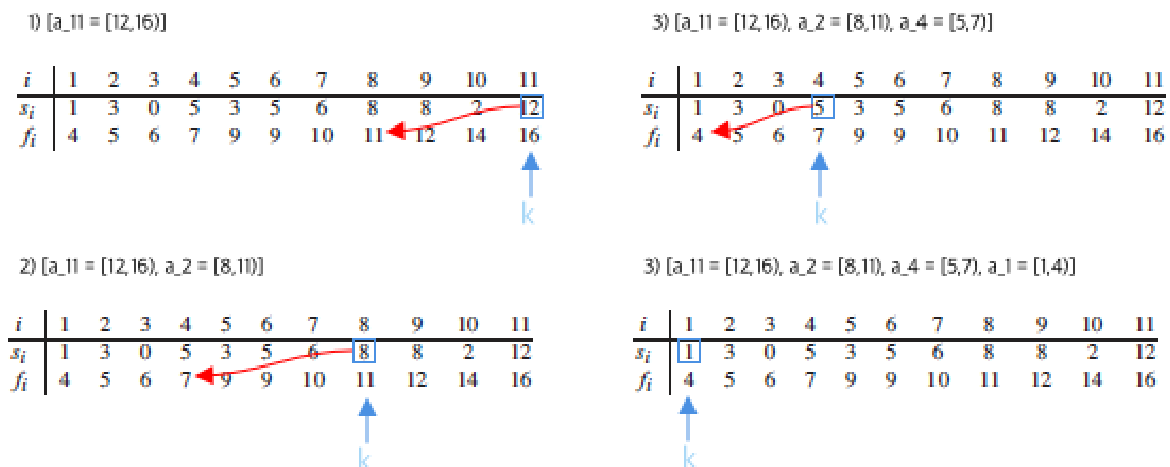
CSC373 Worksheet 2 Solution

July 25, 2020



i	1	2	3	4	5	6	7	8	9	10	11
s_i	1	3	0	5	3	5	6	8	8	2	12
f_i	4	5	6	7	9	9	10	11	12	14	16

1.

This approach is a greedy algorithm because algorithm

1) Has the greedy choice: selecting the last activity to start that is compatible with all previously selected activites
2) Has the greedy choice that is always part of optimal solution:

Claim:

Consider any nonempty subproblem S_k. Let a_m be an activity in S_k with the last activity to start that is compatible with all previously selected activities. Then a_m is included in some maximum-size subset of mutually compatible activities of S_k

Proof. Let A_k be a maximum-size subset of mutually compatible activities in S_k, and let a_j be the activity in A_k with the last activity to start that is compatible with all previously selected activities. □

Notes:

- Greedy Algorithm
  - Always makes the choice that looks best at the moment
    - Locally optimal solution leads to globally optimal solution
- Activity-selection Problem (Greedy algorithm using dynamic programming)
  - Goal: Selecting maximum size set of mutually compatible activities

    Example:

i	1	2	3	4	5	6	7	8	9	10	11
s_i	1	3	0	5	3	5	6	8	8	2	12
f_i	4	5	6	7	9	9	10	11	12	14	16

  - Suppose a set exists $S = \{a_1 = [s_1, f_1), a_2 = [s_2, f_2), ..., a_n = [s_n, f_n)\}$
    - a_i represents an i^{th} activity
    - s_i represents starting time
    - f_i represents finishing time
    - $0 \leq s_i < f_i < \infty$
    - $a_1, ..., a_n$ sorted in monotonically increasing order of finish time

      i.e.

      $f_1 \leq f_2 \leq f_3 \leq ... \leq f_{n-1} \leq f_n$
    - a_i and a_j are **compatible**, if intervals $[s_i, f_i)$ and $[s_j, f_j)$ don't overlap

      i.e

      $s_i \geq f_j$ and $s_j \geq f_i$
  - Steps
    1. Think about dynamic programming solution
       - Construct optimal solution using two subproblems

         S_{ij}**:** activities that start after activity a_i finishes and before activity a_j starts

         i.e.

         $S_{19} = \{a_4 = [5, 7), a_6 = [5, 9), a_7 = [6, 10)\}$

         A_{ij}**:** maximum set of mutually compatible activities in S_{ij} (including a_k)
         - $A_{ik} = A_{ij} \cap S_{ik}$
         - $A_{kj} = A_{ij} \cap S_{kj}$
         - $A_{ij} = A_{ik} \cup \{a_k\} \cup Akj$

· So, $|A_{ij}| = |A_{ik}| + |A_{kj}| + 1$

* Verify that optimal solution A_{ij} must include optimal solution to the two subproblems for S_{kj}

Let A'_{kj} be another mutually compatible activities in S_{kj} where $|A'_{kj}| > |A_{kj}|$.

Then we could use A'_{kj} in a solution to subproblem of S_{ij}

Then we have $|A_{ik}| + |A'_{kj}| + 1 > |A_{jk}| + |A_{kj}| + 1 = |A_{ij}|$ mutually compatible activites

This contradicts assumption that A_{ij} is an optimal solution

* Verify that optimal solution A_{ij} must include optimal solution to the two subproblems for S_{ik}

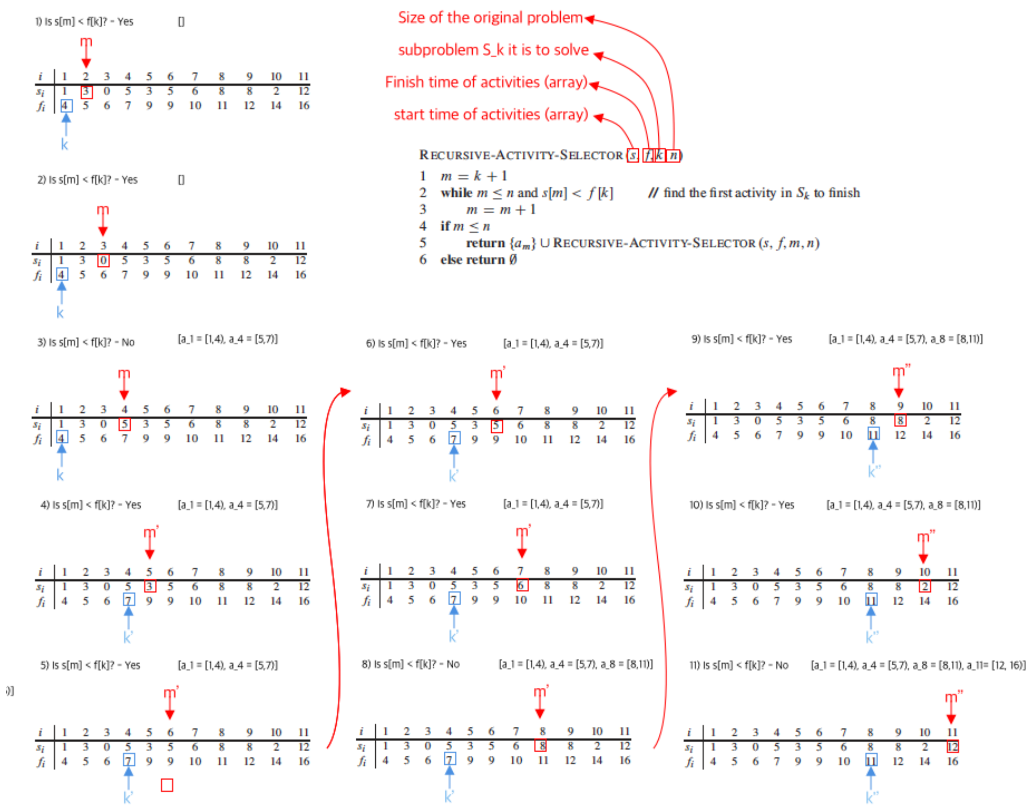
The same applies for activities in S_{ik}

2. Observe that only one choice - greedy choice, and that when we make the greedy choice, only one subproblem remains
   * Steps
     1. Make a greedy choice
        · Choose an activity that makes the most resource possible (intuition)
        · Choose an acitivty that finishes the earliest (intuition)
     2. Solve a subproblem: Find activities that start after a_1 finishes
     3. Verify that making greedy choices always arrive at optimal solution

Theorem 16.1 (Page 418):

Consider any non-empty subproble S_k, and let a_m be an activity in S_k with the earliest finish time. Then a_m is included in some maximum-size subset of mutually compatible activities of S_k

3. Develop recursive greedy solution



4. Convert the recursive algorithm into iterative one