Problem Set 1 Solution

March 14, 2020

Question 1

a. $\forall t \in T, Canadian(t) \Rightarrow \neg Stanley(t)$

b. $\forall t \in T,\ \exists d \in D,\ \neg Canadian(t) \wedge BelongsTo(t, d)$

c. $\forall t \in T,\ \exists d \in D,\ Stanley(t) \wedge BelongsTo(t, d)$

d. $\forall t \in T, \exists d \in D,\ BelongsTo(t, d) \Rightarrow \forall d' \in D, d' \neq d \wedge \neg BelongsTo(t, d')$

e. $\forall t_1 \in T, \exists d \in D, \exists t_2 \in T, t_1 \neq t_2 \wedge (BelongsTo(t_1, d) \wedge BelongsTo(t_2, d)) \Rightarrow \forall t_3 \in T, t_3 \neq t_1 \wedge t_3 \neq t_2 \wedge \neg BelongsTo(t_3, d)$

Question 2

a. $\forall x \in \mathbb{R}, f(-x) = f(x)$

$\forall x \in \mathbb{R}, -f(-x) = f(x)$

b. $\forall g, f : \mathbb{R} \to \mathbb{R},\ \exists h : \mathbb{R} \to \mathbb{R},\ Odd(f) \wedge Odd(g) \Rightarrow Odd(f) \times Odd(g) = Even(h)$

c. $f = 0$ is a solution, since $-f(-x) = -(-0) = 0 = f(x)$ and $f(-x) = -0 = 0 = f(x)$

d. $\forall f : \mathbb{R} \to \mathbb{R}, \exists f_1,_f 2 : \mathbb{R} \to \mathbb{R}, Odd(f_1) \wedge Even(f_2) \wedge f = Odd(f_1) + Even(f_2)$

e. A solution is $f = x^2 + x$ with $f_1 = x^2$ and $f_2 = x$.

$f = x^2 + x$ is the summation $\sum_{i=0}^{2n}$ with $n = 1, a_0 = 0, a_1 = 1, a_2 = 1$. f_1 is odd since $-f(-x) = -(-x) = x = f(x)$, and f_2 is even since $f(-x) = (-x)^2 = x^2 = f(x)$

f. A solution is $g_1(x) = \frac{2^x+2^{-x}}{2}$ and $g_2(x) = \frac{2^x-2^{-x}}{2}$.

$g_1 + g_2$ gives g since $\frac{2^x+2^{-x}}{2} + \frac{2^x-2^{-x}}{2} = 2^x$. Also, $g_1(-x) = \frac{2^{-x}+2^{-(-x)}}{2} = \frac{2^{-x}+2^x}{2} = g_1(x)$ is even, and $-g_2(-x) = -(\frac{2^{-x}-2^{-(-x)}}{2}) = -(\frac{-2^x+2^{-x}}{2})) = g_2(x)$

Question 3

Question 4