

# Midterm 1 Version 2 Solution

March 19, 2020

## Question 1

a. Since

$$S_1 = \{1, 2, 3, 5, 7, 11, 13, 17, 19, 23, 29\}, \text{ and } S_2 = \{1, 2, 3, 5, 6, 10, 15, 30\},$$

$$S_1 \cap S_2 = \{1, 2, 3, 5\}$$

b. See the table below

$p$	$q$	$r$	$\neg p$	$\neg p \Leftrightarrow q$	$(\neg p \Leftrightarrow q) \Rightarrow r$
T	T	T	F	F	T
T	T	F	F	F	T
T	F	T	F	T	T
F	T	T	T	T	F
T	F	F	F	T	F
F	F	T	T	F	T
F	F	F	T	F	T

**Correct Solution:**

$p$	$q$	$r$	$\neg p$	$\neg p \Leftrightarrow q$	$(\neg p \Leftrightarrow q) \Rightarrow r$
T	T	T	F	F	T
T	T	F	F	F	T
T	F	T	F	T	T
*F	*T	*T	*T	*T	*T
T	F	F	F	T	F
*F	*T	*F	*T	*T	*F
F	F	T	T	F	T
F	F	F	T	F	T

\* = Incorrect/missing solution

- c. Let  $x \in \mathbb{N}$ . Assume  $P(x)$ .

We will prove that there is a natural number  $y$  such that the predicate  $Q(x, y)$  is true.

**Correct Solution:**

Let  $x \in \mathbb{N}$ , and  $y = \underline{\hspace{2cm}}$ . Assume  $P(x)$ .

We will prove that the predicate  $Q(x, y)$  is true.

## Question 2

- a.  $\forall x \in P, Cat(x) \wedge Loves(x, x)$

**Correct Solution:**

$\forall x \in P, Cat(x) \Rightarrow Loves(x, x)$

- b.  $\forall x \in P, \exists y \in P, Cat(x) \wedge Cute(y) \wedge Loves(x, y)$

**Correct Solution:**

$\forall x \in P, Cat(x) \wedge Cute(x) \Rightarrow (\forall y \in P, Cat(y) \Rightarrow Cute(y))$

- c.  $\exists x \in P, Cat(x) \wedge Cute(x) \Rightarrow \forall y \in P, Cat(y) \wedge Cute(y)$

- d.  $\forall p_1, p_2 \in P, p_1 \neq p_2 \wedge Loves(p_1, p_2) \wedge Loves(p_2, p_1) \Rightarrow (Cat(p_1) \wedge \neg Cat(p_2)) \vee (\neg Cat(p_1) \wedge Cat(p_2))$

## Question 3

- a.  $\exists n \in \mathbb{N}, n > 1 \Rightarrow \forall x \in \mathbb{R}, \lfloor nx \rfloor = n \lfloor x \rfloor$

**Correct Solution:**

$\exists n \in \mathbb{N}, n > 1 \wedge (\forall x \in \mathbb{R}^+, \lfloor nx \rfloor = n \lfloor x \rfloor)$

b. **Negation:**  $\forall n \in \mathbb{N}, n > 1 \wedge (\exists x \in \mathbb{R}, \lfloor nx \rfloor \neq n \lfloor x \rfloor)$

Let  $n = 2, x = 0.5$ .

Then,

$$\lfloor nx \rfloor = \lfloor 2(0.5) \rfloor \quad (1)$$

$$= 1 \quad (2)$$

And,

$$n \lfloor x \rfloor = 2 \lfloor 0.5 \rfloor \quad (3)$$

$$= 2(0) \quad (4)$$

$$= 0 \quad (5)$$

Since  $\lfloor nx \rfloor \neq n \lfloor x \rfloor$ , the predicate logic is false.

**Correction, First Case ( $n \leq 1$ ):**

Assume  $n \leq 1$ . Then, the first case of the negation is true.

**Correction, Second Case ( $\exists x \in \mathbb{R}, n \lfloor x \rfloor \neq \lfloor nx \rfloor$ ):**

Let  $n \in \mathbb{N}, x = \frac{1}{n}$ . Assume  $n > 1$ .

Then,

$$n \lfloor x \rfloor = n \lfloor \frac{1}{n} \rfloor \quad (1)$$

$$= 0 \quad (2)$$

And,

$$\lfloor nx \rfloor = \lfloor \frac{1}{n} \rfloor \tag{3}$$

$$= \lfloor 1 \rfloor \tag{4}$$

$$= 1 \tag{5}$$

Since  $n\lfloor x \rfloor \neq \lfloor nx \rfloor$ , the second case of negation is also false.

Then, it follows from the negation that the statement is false.

## Question 4