

Worksheet 5 Solution

March 15, 2020

Question 1

- $\forall n, p \in \mathbb{N}, \text{Odd}(n) \wedge \text{Odd}(p) \Rightarrow \text{Odd}(n \times p)$

Let $n, p \in \mathbb{Z}$, and assume n, p are odd numbers.

Then, $\exists k, m \in \mathbb{Z}, n = 2k - 1, p = 2m - 1$ by the definition of odd numbers

Then,

$$n \times p = (2k - 1)(2m - 1) \tag{1}$$

$$= 2k2m - 2k - 2m + 1 \tag{2}$$

$$= (2k2m - 2k - 2m + 2) - 1 \tag{3}$$

$$= 2(2km - k - m + 1) - 1 \tag{4}$$

$$= 2l - 1 \tag{5}$$

where $l = 2km - k - m + 1$.

Since $l \in \mathbb{Z}$, it follows from the definition of odd number that the product of two odd numbers is odd.

Question 2

- a. $\forall m, n \in \mathbb{Z}, \text{Even}(m) \wedge \text{Odd}(n) \Rightarrow m^2 - n^2 = m + n$
- b. The flaw is in the same value k . This implies that the statement is true only when n is 1 less than m . This doesn't mean it's true for all even and odd numbers.

Question 3

- a. $\text{Dom}(f, g) : \forall n \in \mathbb{N}, g(n) \leq f(n)$, where $f, g : \mathbb{N} \rightarrow \mathbb{R}^{\geq 0}$
- b. Let $n \in \mathbb{R}^{\geq 0}$, $f(n) = 3n$ and $g(n) = n$.

Then,

$$g(n) = n \leq n + n + n \tag{6}$$

$$\leq 3n \tag{7}$$

$$\leq f(n) \tag{8}$$

Then, it follows from the definition that f dominates g .

- c. Predicate Logic: $\exists n \in \mathbb{N}, g(n) > f(n)$.

Let $n = 1$.

Then,

$$g(1) = (1) + 165 = 166 > 1 \tag{9}$$

$$> f(1) \tag{10}$$

Then, it follows from negation of the definition that f does not dominate g .

d. Predicate Logic: $\exists x \in \mathbb{R}^{\geq 0}, \exists n \in \mathbb{N}, g(n) > f(n)$

Let $x = 1$ and $n = 1$.

Then,

$$g(1) = (1) + 1 = 2 > 1 \tag{11}$$

$$> f(1) \tag{12}$$

Then, it follows from negation of the definition that f does not dominate g .

Question 4

a. Let $x \in \mathbb{R}^{\geq 0}$, $\epsilon = x - \lfloor x \rfloor$, and assume $x \geq 4$.

Then,

$$(\lfloor x \rfloor)^2 = (x - \epsilon)^2 \tag{1}$$

by the fact that ϵ can be rewritten as $\lfloor x \rfloor = x - \epsilon$.

Then,

$$(x - \epsilon)^2 = x^2 - 2x\epsilon + \epsilon^2 \tag{2}$$

$$\geq x^2 \tag{3}$$

$$\geq \frac{1}{2}x^2 \tag{4}$$

Conclusion can be made from the above fact that ϵ is sufficiently small and $\epsilon^2 - 2x\epsilon \geq 0$