CSC373 Worksheet 6 Solution

August 12, 2020

1. 1. Multiply objective function by - 1

> Maximize
>
> $$-2x_1 - 7x_2 - x_3$$
>
> Subject to
>
> $$x_1 - x_3 = 7$$
> $$3x_1 + x_2 \geq 7$$
> $$x_2 \geq 0$$
> $$x_3 \leq 0$$

2. Replace non-nonnegative constraints x_1

> Maximize
>
> $$-2x_1' + 2x_1'' - 7x_2 - x_3$$
>
> Subject to
>
> $$x_1' - x_1'' - x_3 = 7$$
> $$3x_1' - 3x_1'' + x_2 \geq 7$$
> $$x_1', x_1'', x_2 \geq 0$$
> $$x_3 \leq 0$$

3. Replace non-nonnegative constraints x_3

Maximize

$$-2x_1' + 2x_1'' - 7x_2 - x_3' + x_3''$$

Subject to

$$x_1' - x_1'' - x_3' + x_3'' = 7$$
$$3x_1' - 3x_1'' + x_2 \geq 7$$
$$x_1', x_1'', x_2, x_3', x_3'' \geq 0$$

4. Replace equality constraints with $\geq$ and $\leq$

Maximize

$$-2x_1' + 2x_1'' - 7x_2 - x_3' + x_3''$$

Subject to

$$x_1' - x_1'' - x_3' + x_3'' \leq 7$$
$$x_1' - x_1'' - x_3' + x_3'' \geq 7$$
$$3x_1' - 3x_1'' + x_2 \geq 7$$
$$x_1', x_1'', x_2, x_3', x_3'' \geq 0$$

5. Correct greater-than-or-equal-to inequality constraints

Maximize

$$-2x_1' + 2x_1'' - 7x_2 - x_3' + x_3''$$

Subject to

$$x_1' - x_1'' - x_3' + x_3'' \leq 7$$
$$-x_1' + x_1'' + x_3' - x_3'' \leq -7$$
$$-3x_1' + 3x_1'' - x_2 \leq 7$$
$$x_1', x_1'', x_2, x_3', x_3'' \geq 0$$

Notes:

- **Linear Programming**
  - Is a method to achieve the best outcome (such as maximum profit or lowest cost) in a mathematical model whose requirements are represented by linear relationships. [1]
  - Is named to make it sound cool for government funding
    * Like dynamic programming
  - Applications
    * Microeconomics (maximize profits, minimize costs)
    * Company management
- **Standard Form**
  - Is a form of linear programming
  - Are about <u>maximizing</u>, not minimizing [2]
  - All have a positivity constraint for each variable [2]
  - All other constraints are all of the form "linear combination of variables $\leq$ constant". [2]
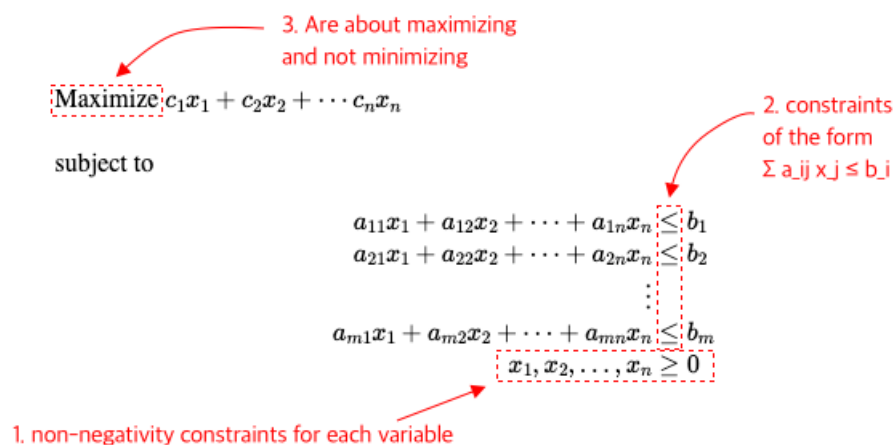
- **Converting Linear Programming to Standard Form**

1) The objective function might be a minimization rather than a maximization
   - Negate coefficients of the objective function

multiply by –1

$$\text{minimize} \quad -2x_1 + 3x_2$$
subject to
$$
\begin{aligned}
x_1 + x_2 &= 7 \\
x_1 - 2x_2 &\le 4 \\
x_1 &\ge 0
\end{aligned}
$$

$$\text{maximize} \quad 2x_1 - 3x_2$$
subject to
$$
\begin{aligned}
x_1 + x_2 &= 7 \\
x_1 - 2x_2 &\le 4 \\
x_1 &\ge 0
\end{aligned}
$$

2) There might be variables without nonnegativity constraints
   - Replace each non-nonnegative variable x_i with x_i' and x_i''
   - Modify linear program

Replace x_i with x_i' and x_i"

$$\text{maximize} \quad 2x_1 - 3x_2$$
subject to
$$
\begin{aligned}
x_1 + x_2 &= 7 \\
x_1 - 2x_2 &\le 4 \\
x_1 \quad\quad &\ge 0
\end{aligned}
$$

$$\text{maximize} \quad 2x_1 - 3x_2' + 3x_2''$$
subject to
$$
\begin{aligned}
x_1 + x_2' - x_2'' &= 7 \\
x_1 - 2x_2' + 2x_2'' &\le 4 \\
x_1, x_2', x_2'' &\ge 0
\end{aligned}
$$

x_2 is not nonnegative :(

They are now nonnegative :)! Yayy!!

3) There might be **equality constraints**, which have an equal sign rather than a less-than-or-equal-to sign
   - Replace equality constraint $f(x_1, x_2, ..., x_n) = b$ with $f(x_1, x_2, ..., x_n) \le b$ and $f(x_1, x_2, ..., x_n) \ge b$

Multiply incorrect constraints by –1

$$\text{maximize} \quad 2x_1 - 3x_2' + 3x_2''$$
subject to
$$
\begin{aligned}
x_1 + x_2' - x_2'' &\le 7 \\
x_1 + x_2' - x_2'' &\ge 7 \\
x_1 - 2x_2' + 2x_2'' &\le 4 \\
x_1, x_2', x_2'' &\ge 0
\end{aligned}
$$

$$\text{maximize} \quad 2x_1 - 3x_2 + 3x_3$$
subject to
$$
\begin{aligned}
x_1 + x_2 - x_3 &\le 7 \\
-x_1 - x_2 + x_3 &\le -7 \\
x_1 - 2x_2 + 2x_3 &\le 4 \\
x_1, x_2, x_3 &\ge 0
\end{aligned}
$$

4) There might be **inequality constraints**, but instead of having a less-than-or-equal-to-sign
   - Multiply incorrect inequality constraints by -1

Replace = with ≤ and ≥

$$
\begin{array}{llllll}
\text{maximize} & 2x_1 & - & 3x_2' & + & 3x_2'' \\
\text{subject to} \\
& x_1 & + & x_2' & - & x_2'' & = & 7 \\
& x_1 & - & 2x_2' & + & 2x_2'' & \leq & 4 \\
& x_1, x_2', x_2'' & & & & & \geq & 0
\end{array}
$$

$$
\begin{array}{llllll}
\text{maximize} & 2x_1 & - & 3x_2' & + & 3x_2'' \\
\text{subject to} \\
& x_1 & + & x_2' & - & x_2'' & \leq & 7 \\
& x_1 & + & x_2' & - & x_2'' & \geq & 7 \\
& x_1 & - & 2x_2' & + & 2x_2'' & \leq & 4 \\
& x_1, x_2', x_2'' & & & & & \geq & 0 \ .
\end{array}
$$

References:

1) Wikipedia, Linear Programming, link

2) Instituto de Mathematicas, Standard form for Linear Programs, link

2. ### Notes:

   - **Converting Linear Programs into Slack Form**