

# CSC236 Worksheet 8 Solution

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## Question 1

- Part 1 (Building  $L_1$  and  $L_2$ ):

$L_1$ :

$$Q = \{E, O\}$$

$$\Sigma = \{a, b\}$$

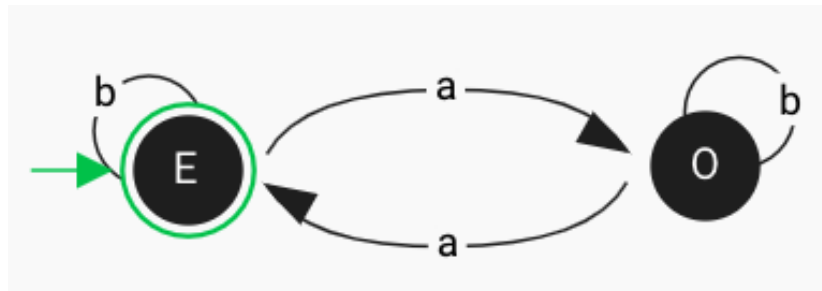
$$\delta =$$

	a	b
*E	O	E
O	E	O

$$q_0 = E$$

$$F = \{E\}$$

Draw Diagram



$L_2$ :

$$Q = \{0, 1, 2\}$$

$$\Sigma = \{a, b\}$$

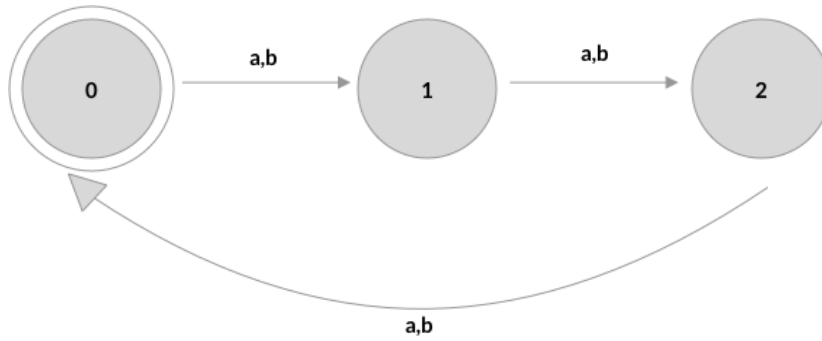
$$\delta =$$

	a	b
*0	1	1
1	2	2
2	0	0

$$q_0 = 0$$

$$F = \{0\}$$

Draw Diagram



Part 1 (Building  $L_1 \cap L_2$ ):

$$Q = \{(E, 0), (E, 1), (E, 2), (O, 0), (O, 1), (O, 2)\}$$

$$\Sigma = \{a, b\}$$

$$\delta =$$

	a	b
*(E,0)	1	1
(E,1)	2	2
(E,2)	0	0
(O,0)	1	1
(O,1)	2	2
(O,2)	0	0

$$q_0 = (E, 0)$$

$$F = \{(E, 0)\}$$

Correct Solution:

Part 1 (Building  $L_1$  and  $L_2$ ):

$L_1$ :

$$Q = \{E, O\}$$

$$\Sigma = \{a, b\}$$

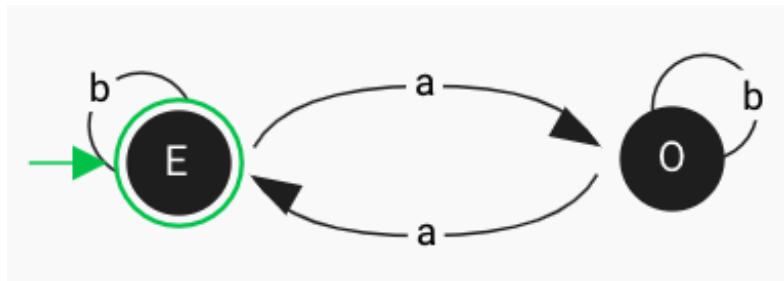
$$\delta =$$

	a	b
*E	O	E
O	E	O

$$q_0 = E$$

$$F = \{E\}$$

Draw Diagram



$L_2$ :

$$Q = \{0, 1, 2\}$$

$$\Sigma = \{a, b\}$$

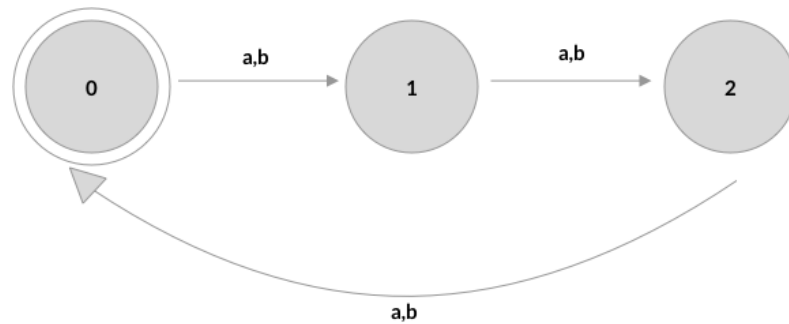
$$\delta =$$

	a	b
*0	1	1
1	2	2
2	0	0

$$q_0 = 0$$

$$F = \{0\}$$

Draw Diagram



Part 1 (Building  $L_1 \cap L_2$ ):

$$Q = \{(E, 0), (E, 1), (E, 2), (O, 0), (O, 1), (O, 2)\}$$

$$\Sigma = \{a, b\}$$

	a	b
$\delta =$		
$*(E, 0)$	$(O, 1)$	$(E, 1)$
$(E, 1)$	$(O, 2)$	$(E, 2)$
$(E, 2)$	$(O, 0)$	$(E, 0)$
$(O, 0)$	$(E, 1)$	$(O, 1)$
$(O, 1)$	$(E, 2)$	$(O, 2)$
$(O, 2)$	$(E, 0)$	$(O, 0)$

$$q_0 = (E, 0)$$

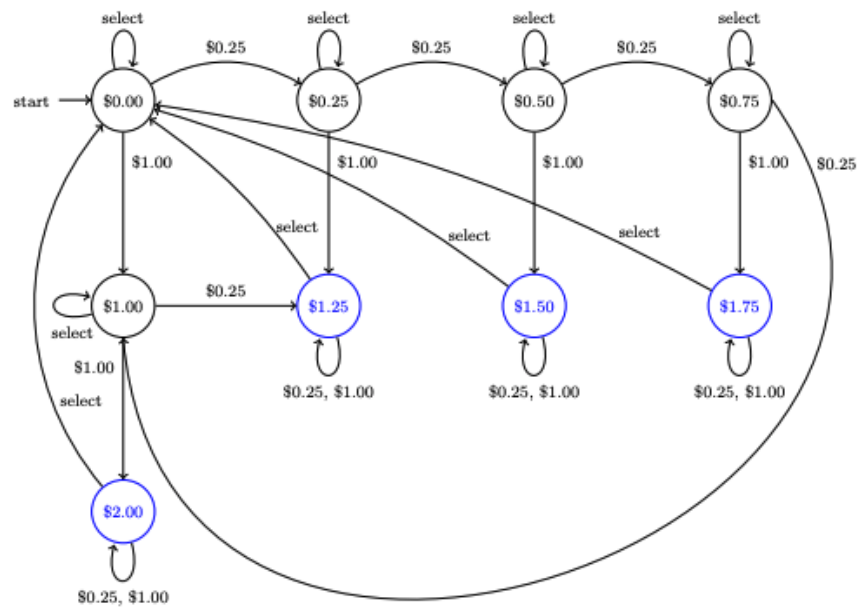
$$F = \{(E, 0)\}$$

Notes:

- **Deterministic Finite State Automaton (DFSA):** is a mathematical method of machine which, given any input string  $x$ , **accepts** or **rejects**  $x$ .

- Applications of DFSA

1. Vending Machine



2. Protocol analysis
3. Text parsing
4. Video game character behavior

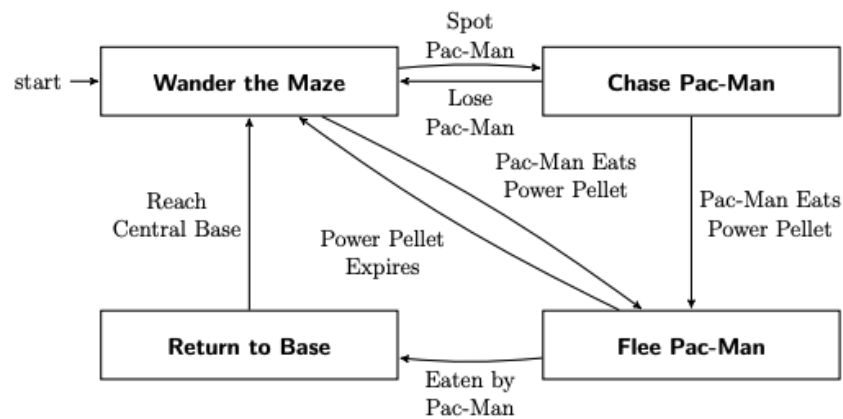
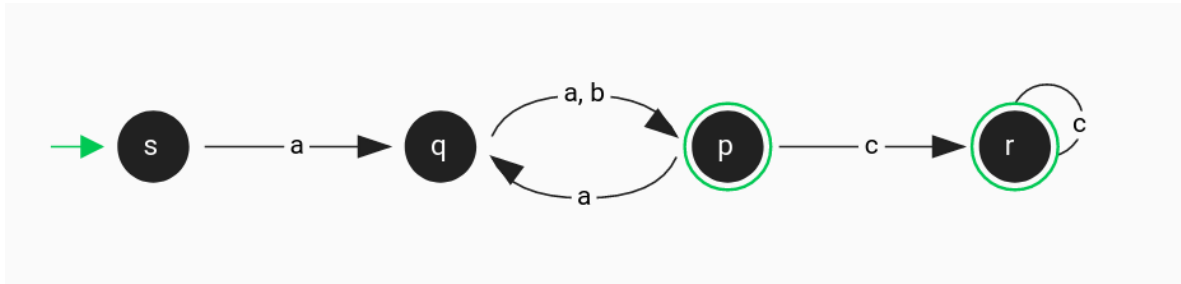


Figure 3: Behavior of a Pac-Man Ghost

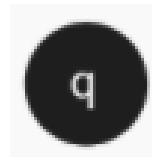
5. Security Analysis
6. CPU control units (\*\*)
7. Natural Language Processing (\*\*)
8. Speech Recognition (\*\*)

- Definitions and Syntax



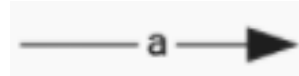
– DFSA  $M$  is a quintuple  $M = (Q, \Sigma, q_0, F, \delta)$ , where

- \*  $Q$  : a finite set of **states**.
  - Represents status of system
  - Is represented by a black circle, i.e. s,q

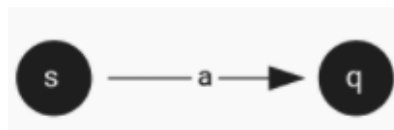


- i.e. automatic sliding door at walmart has two states: either close or open
- i.e. traffic light has three states: red, yellow, green

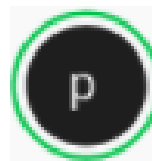
- \*  $\Sigma$  : a finite non-empty alphabet
  - is set of symbols in each transition, i.e. a, b, c



- \*  $q_0 \in Q$  : the start or initial state
- \*  $\delta : Q \times \sigma \rightarrow Q$  : a transition function
  - is a connection between two states.
  - is represented by an arrow



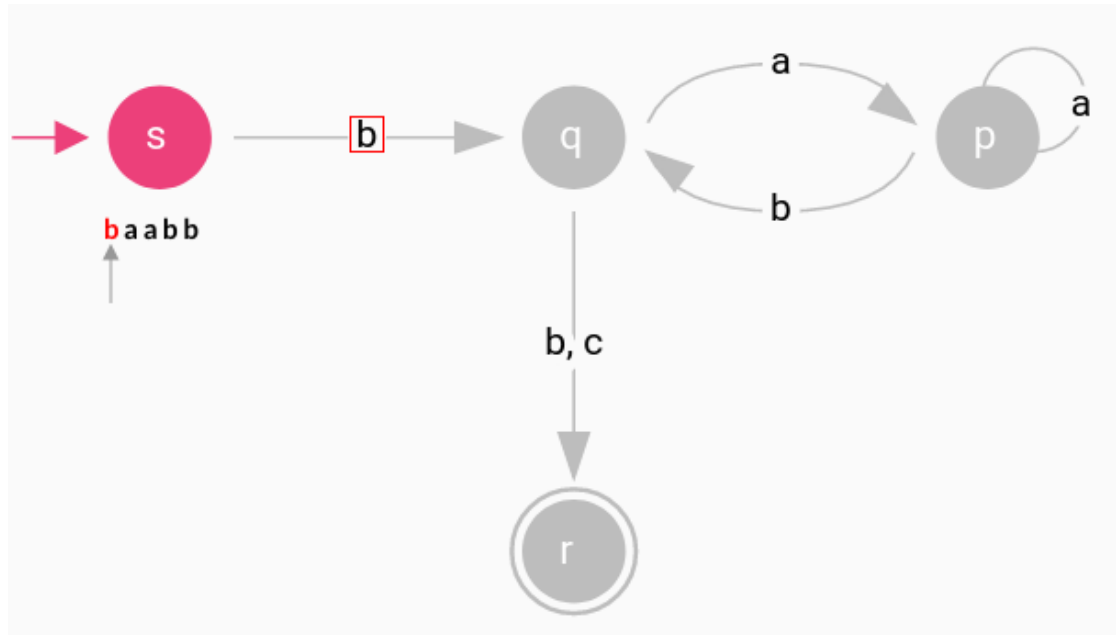
- \*  $F \subseteq Q$  : the set of accepting or final states
  - Is represented by a double circle



- Multiple accepting states may exists
- Purpose: When processing ends, the output is either *accept* or *reject*

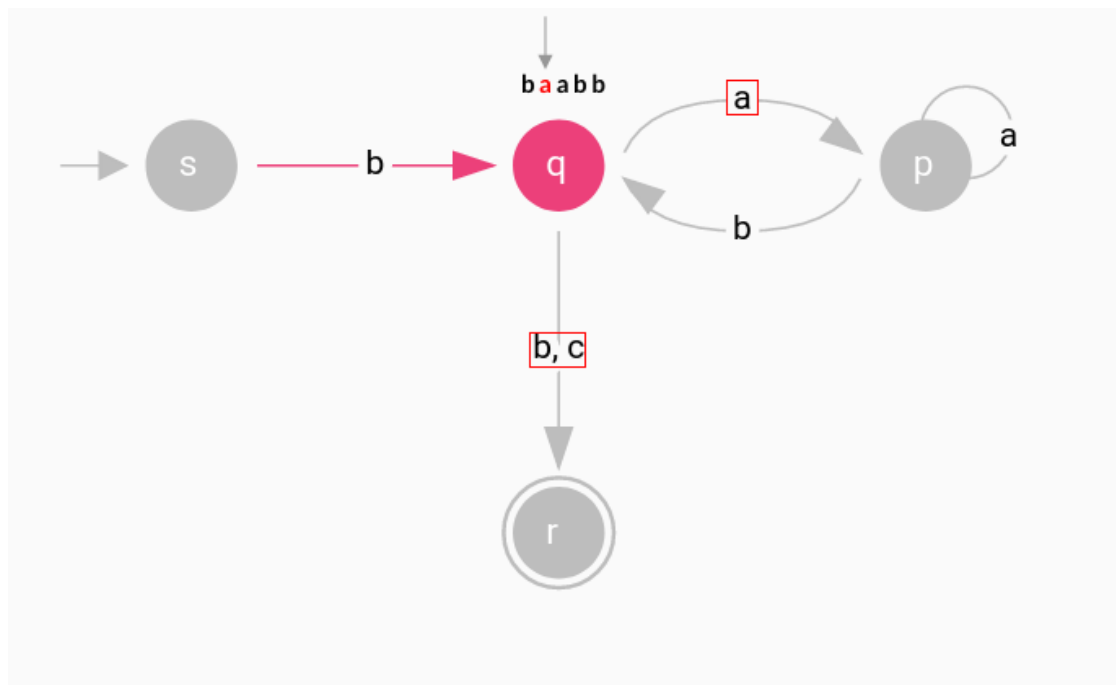
- Simple Example

– Step 1



1. First symbol of the input **baabb** is **b** and the current state is *s*.
2. Ask, is there any exiting transition from *s* that contains the symbol **b**?
3. The answer is yes, so move to *q*

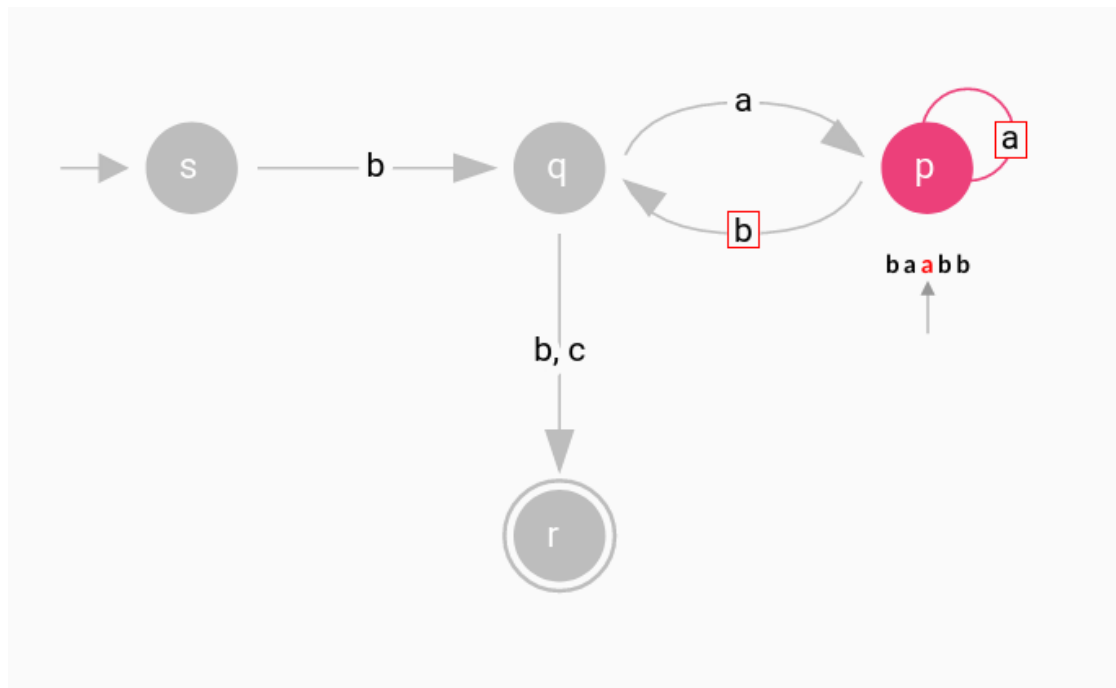
– Step 2



1. Next symbol of the input **baabb** is **a** and the current state is *q*.
2. Ask, is there any exiting transition from *q* that contains the symbol **a** or **b,c**?

3. The answer is yes, and it's **a**. So move to  $p$

– Step 3

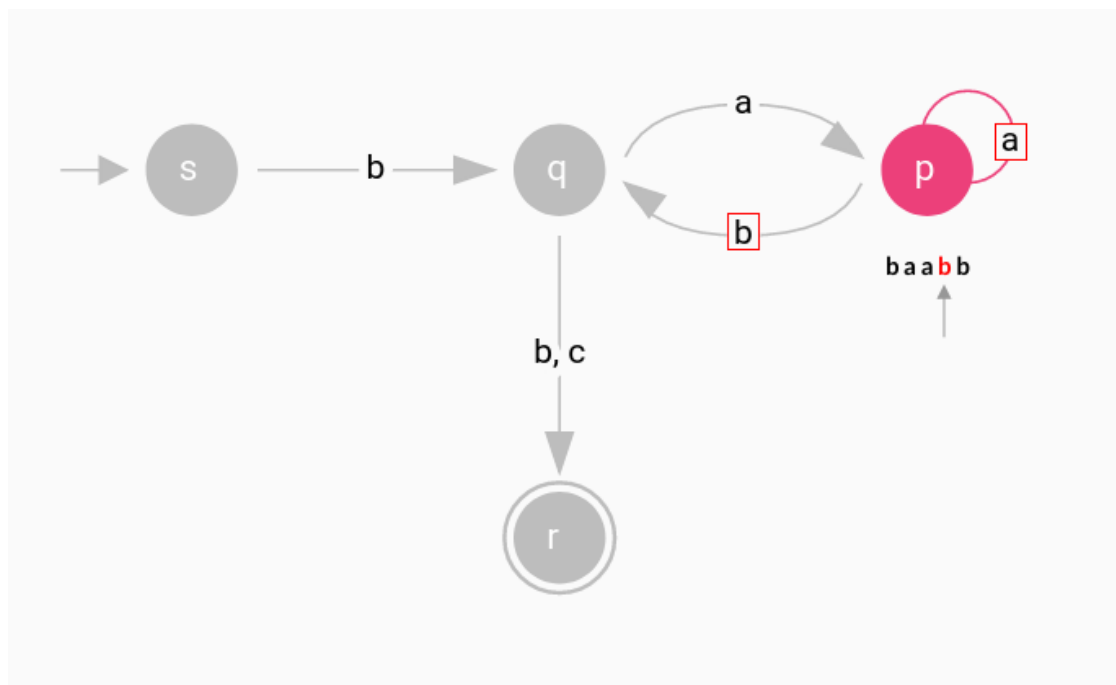


1. Next symbol of the input **baabb** is **a** and the current state is  $p$ .

2. Ask, is there any exiting transition from  $p$  that contains the symbol **a** or **b**?

3. The answer is yes, and it's **a**. So move to  $p$

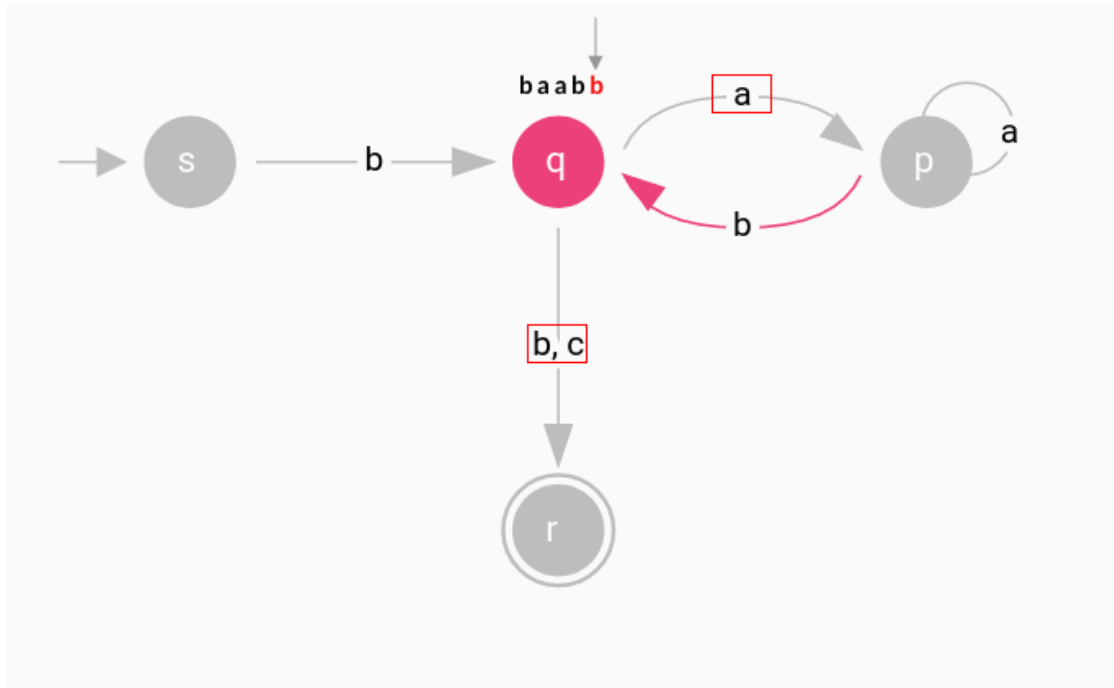
– Step 4





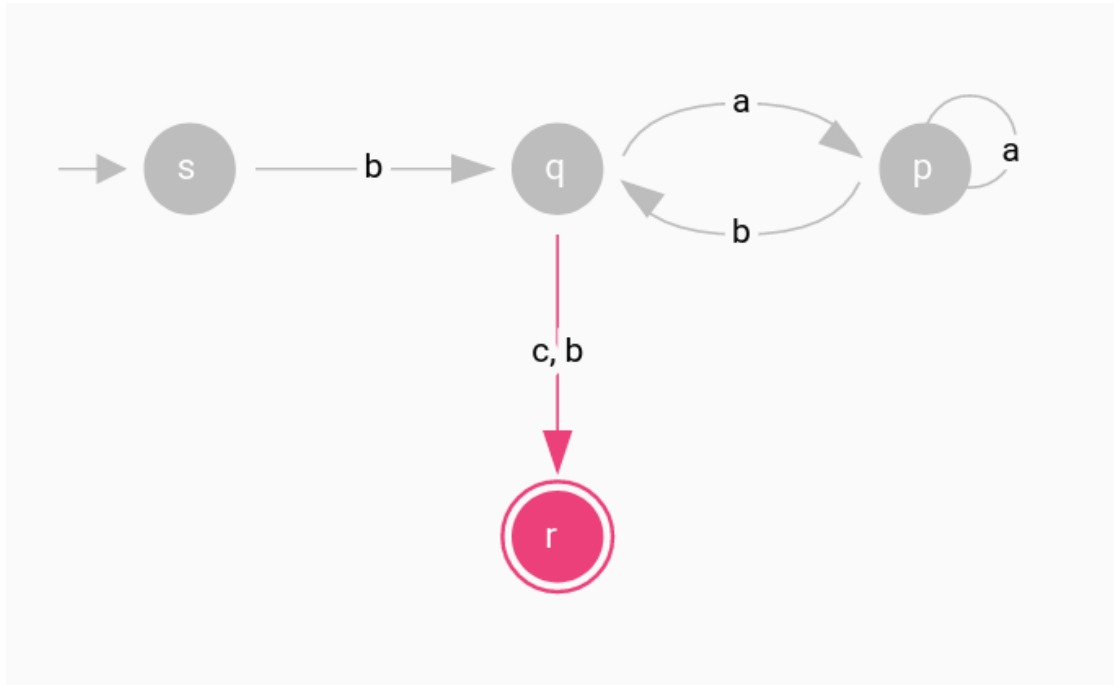
1. Next symbol of the input **baabb** is **b** and the current state is  $p$ .
2. Ask, is there any exiting transition from  $p$  that contains the symbol **a** or **b**?
3. The answer is yes, and it's **b**. So move to  $q$

– Step 5



1. Next symbol of the input **baabb** is **b** and the current state is  $q$ .
2. Ask, is there any exiting transition from  $q$  that contains the symbol **a** or **b,c**?
3. The answer is yes, and it's **b**. So move to  $r$

– Step 6



1. Next symbol of the input **baabb** is **b** and the current state is *r*.
2. Ask, if it satisfies the accepting or final state (i.e., has the end of string been reached?). If so, the output is accept. Otherwise, it's reject.

- Formal Languages

- is a subset of all possible words  $\Sigma^*$  formed by symbols of alphabet  $\Sigma$ .

- \*  $\Sigma^*$  is set of all possible strings over the alphabet  $\Sigma$ .

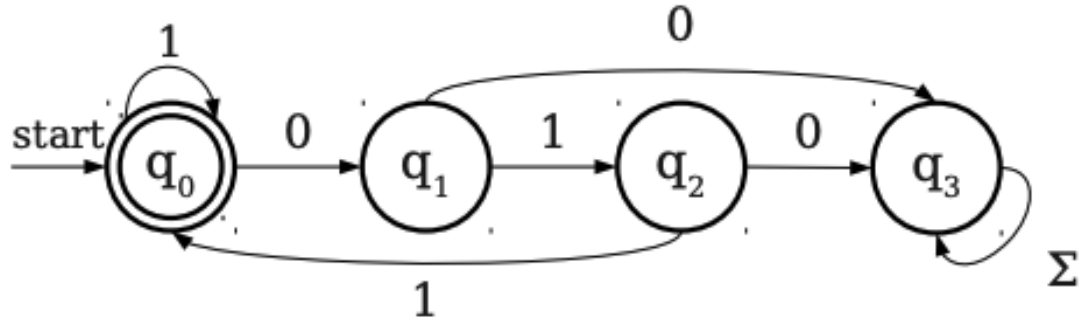
- \* i.e.  $\Sigma = \{a, b\}$ ,  $\Sigma^* = \{a, b, aa, ab, ba, bb, aaa, aab, \dots\}$

- Example

1.  $L = \{w \mid w \text{ has at most seventeen 0's}\}$
2.  $L = \{w \mid w \text{ has equal number of 0's and 1's}\}$
3.  $L = \{x \in \{a, b\}^* \mid \text{the number of a's in } x \text{ is even}\}$ 
  - \*  $*$  in  $\{a, b\}^*$  means all possible combinations
  - \* i.e.  $\{a, b, aa, ab, ba, bb, aaa, baa, aba, \dots\}$

- Tabular DFAs

- Example



$$\delta =$$

	0	1
<sup>*</sup> q <sub>0</sub>	q <sub>1</sub>	q <sub>0</sub>
q <sub>1</sub>	q <sub>3</sub>	q <sub>2</sub>
q <sub>2</sub>	q <sub>3</sub>	q <sub>0</sub>
q <sub>3</sub>	q <sub>3</sub>	q <sub>3</sub>

Note: \* means it's an accepting state

## Question 2

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### Rough Works:

1. Prove that  $M_1$  accepts  $L_1$

First, define  $\Sigma^*$  as the smallest set such that

- (a)  $\epsilon \in \Sigma^*$
- (b)  $s \in \Sigma^* \Rightarrow sa \in \Sigma^* \wedge sb \in \Sigma^*$

I will prove that  $M_1$  accepts  $L_1$ .

Define  $P(s)$  as:

$$P(s) : \delta^*(E, s) = \begin{cases} E & \text{if } s \text{ has an even number of } as \\ O & \text{if } s \text{ has an odd number of } as \end{cases} \quad (1)$$

I will prove  $\forall s \in \Sigma^*, P(s)$  by structural induction.

1. Basis Case
2. Inductive Step
  1. Case  $sa$
  2. Case  $sb$  (Let's first start with this)

2. Prove that  $M_2$  accepts  $L_2$
3. Prove that  $M_{1 \wedge 2}$  accepts  $L_1 \cap L_2$