CSC236 Worksheet 7 Solution

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Question 1

- **Rough Works:**

  – Find the value of k. And conclude the non-recursive cost of function.

  First, I need to find the value of k.

  > The definition tells us k is the non-recursive cost.
  >
  > Since the non-recursive part of call occurs when $len(s) < 2$ and it returns the input as output, it has cost of 1.

  – Find the value of b.

  Second, we need to find the value of b.

  > The definition tells us b is the number of almost-equal parts the input is divided into.
  >
  > Since the input s is divided into three roughly equal parts, we can conclude $b = 3$.

  – Find the value of a.

  Third, we need to find the value of a.

  > The definition tells us a is the number of recursive calls.
  >
  > Since the recursive calls in this problem are $r(s_1)$, $r(s_2)$ and $r(s_3)$, there are three of them, so $a = 3$.

- Find the value of f.

  Fourth, we need to find the value of f.

- Use master's theorem to evaluate asymptotic time complexity of function r.
- Compare its time complexity to using loop

Notes:

- **Divide and Conquer:** Partitions problem into b roughly equal subproblems, solve, and recombine:

$$T(n) = \begin{cases} k & \text{if } n \le B \\ a_1 T(\lceil n/b \rceil) + a_2 T(\lfloor n/b \rfloor) + f(n) & \text{if } n > B \end{cases} \tag{1}$$

$$T(n) = \begin{cases} k & \text{if } n \le B \\ aT(n/b) + f(n) & \text{if } n > B \end{cases} \tag{2}$$

where $b, k > 0$, $a_1, a_2 \ge 0$, and $a = a_1 + a_2 > 0$. $f(n)$ is the cost of slptting and recombining.

Note:

k : non-recursive cost, when $n < b$

b : number of almost-equal parts we divide problem into

a_1 : number of recursive calls to ceiling

a_2 : number of recursive calls to floor

a : number of recursive calls

f : cost of splittig and later recombining (should be n^d for master theorem)

- **Divide and Conquer Master Theorem:**

If $f \in \Theta(n^d)$, then

$$T(n) \in \begin{cases} \Theta(n^d) & \text{if } a \le b^d \\ \Theta(n^d \log_b n) & \text{if } a = b^d \\ \Theta(n^{\log_b a}) & \text{if } a > b^d \end{cases} \tag{3}$$

- The master theorem is for master method.
- The master method provides a cookbook method for solving recurrences of the form

$$T(n) = aT(n/b) + f(n) \tag{4}$$

where $a \geq 1$ and $b > 1$.