CSC236 Worksheet 6 Review

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Question 1

- **Rough Work:**

  Assume that $\forall k \in \mathbb{N}$, $R(3^k) = k3^k$.

  I need to prove $R \in \Theta(n \lg n)$. That is, $R \in \mathcal{O}(n \lg n)$ and $R \in \Omega(n \lg n)$.

  I will do so in parts.

  1. Part 1 (Proving $R \in \mathcal{O}(n \lg n)$)

     Let $n \in \mathbb{N}$. Define $n^* = 3^{\lceil \log_3 n \rceil}$. Then, we have

     $$\lceil \log_3 n \rceil - 1 < \log_3 n \leq \lceil \log_3 n \rceil \tag{1}$$

     I will also use the fact proved in week 7 tutorial exercise that R is non-decreasing.

  2. Part 2 (Proving $R \in \Omega(n \lg n)$)