Worksheet 4 Review

March 22, 2020

Question 1

a. $\exists n \in \mathbb{N},\ n > 3 \wedge n^2 - 1.5n \geq 5$

b. The variable is existentially quantified

c. When introduced, the variable's value should be a **concrete natural number**.

d. Let $n = 5$.

Then $n > 3$, and

$$n^2 - 1.5n = 25 - 7.5 \tag{1}$$
$$= 17.5 \geq 5 \tag{2}$$

Then, it follows from above that the statement $\exists n \in \mathbb{N},\ n > 3 \wedge n^2 - 1.5n \geq 5$ is true.

e. $\forall n \in \mathbb{N}, n > 3 \Rightarrow n^2 - 1.5n > 4$

$\Rightarrow$ should be used, because it allows the scoping of the set $\mathbb{N}$.

f. Universally Quantified

g. The variable's value should be an arbitrary natural number.

h. The assumption made is $n > 3$. It is determined by seeing the lhs of $\Rightarrow$.

i. Let $n \in \mathbb{N}$. Assume $n > 3$.

Then,

$$n > 3 \tag{1}$$
$$(n - 0.75)^2 > (3 - 0.75)^2 \tag{2}$$
$$n^2 - 1.5n + 0.5625 > 5.0625 \tag{3}$$
$$n^2 - 1.5n > 4.5 \tag{4}$$
$$n^2 - 1.5n > 4 \tag{5}$$

Then, it follows from above that the statement $\forall n \in \mathbb{N},\ n > 3 \Rightarrow n^2 - 1.5n > 4$.

Question 2

a. $\forall n \in \mathbb{N},\ n > 5 \Rightarrow 2 \mid n \wedge 3 \mid n$

b. $\exists n \in \mathbb{N}, (n > 5) \wedge (2 \nmid n \vee 3 \nmid n)$

c. Let $n = 7$.

Then, $2 \nmid n \vee 3 \nmid n$.

Then, it follows from the negation that the statement $\forall n \in \mathbb{N},\ n > 5 \Rightarrow 2 \mid n \wedge 3 \mid n$ is false.

Question 3