

# CSC373 Worksheet 2 Solution

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## 1. Notes:

- Greedy Algorithm
  - Always makes the choice that looks best at the moment
    - \* Locally optimal solution leads to globally optimal solution
- Activity-selection Problem (Greedy algorithm using dynamic programming)
  - Goal: Selecting maximum size set of mutually compatible activities

### Example:

$i$	1	2	3	4	5	6	7	8	9	10	11
$s_i$	1	3	0	5	3	5	6	8	8	2	12
$f_i$	4	5	6	7	9	9	10	11	12	14	16

- Suppose a set exists  $S = \{a_1 = [s_1, f_1), a_2 = [s_2, f_2), \dots, a_n = [s_n, f_n)\}$ 
  - \*  $a_i$  represents an  $i^{th}$  activity
  - \*  $s_i$  represents starting time
  - \*  $f_i$  represents finishing time
  - \*  $0 \leq s_i < f_i < \infty$
  - \*  $a_1, \dots, a_n$  sorted in monotonically increasing order of finish time

i.e.

$$f_1 \leq f_2 \leq f_3 \leq \dots \leq f_{n-1} \leq f_n$$

- \*  $a_i$  and  $a_j$  are **compatible**, if intervals  $[s_i, f_i)$  and  $[s_j, f_j)$  don't overlap

i.e

$$s_i \geq f_j \text{ and } s_j \geq f_i$$

– Steps

1. Think about dynamic programming solution

\* Construct optimal solution using two subproblems

$S_{ij}$ : activities that start after activity  $a_i$  finishes and before activity  $a_j$  starts

i.e.

$$S_{19} = \{a_4 = [5, 7), a_6 = [5, 9), a_7 = [6, 10)\}$$

$A_{ij}$ : maximum set of mutually compatible activities in  $S_{ij}$  (including  $a_k$ )

- $A_{ik} = A_{ij} \cap S_{ik}$
- $A_{kj} = A_{ij} \cap S_{kj}$
- $A_{ij} = A_{ik} \cup \{a_k\} \cup A_{kj}$
- So,  $|A_{ij}| = |A_{ik}| + |A_{kj}| + 1$

\* Verify that optimal solution  $A_{ij}$  must include optimal solution to the two subproblems for  $S_{kj}$

Let  $A'_{kj}$  be another mutually compatible activities in  $S_{kj}$  where  $|A'_{kj}| > |A_{kj}|$ .

Then we could use  $A'_{kj}$  in a solution to subproblem of  $S_{ij}$

Then we have  $|A_{ik}| + |A'_{kj}| + 1 > |A_{ik}| + |A_{kj}| + 1 = |A_{ij}|$  mutually compatible activities

This contradicts assumption that  $A_{ij}$  is an optimal solution

\* Verify that optimal solution  $A_{ij}$  must include optimal solution to the two subproblems for  $S_{ik}$

The same applies for activities in  $S_{ik}$

2. Observe that only one choice - greedy choice, and that when we make the greedy choice, only one subproblem remains

\* Steps

1. Make a greedy choice

- Choose an activity that makes the most resource possible (intuition)
- Choose an activity that finishes the earliest (intuition)

2. Solve a subproblem: Find activities that start after  $a_1$  finishes

3. Verify that making greedy choices always arrive at optimal solution

**Theorem 16.1:**

Consider any non-empty subproblem  $S_k$ , and let  $a_m$  be an activity in  $S_k$  with the earliest finish time. Then  $a_m$  is included in some maximum-size subset of mutually compatible activities of  $S_k$

3. Develop recursive greedy solution
4. Convert the recursive algorithm into iterative one