

CSC373 Worksheet 6 Solution

August 12, 2020

1. Rough Works:

1. Multiply objective function by - 1

Maximize

$$-2x_1 - 7x_2 - x_3$$

Subject to

$$x_1 - x_3 = 7$$

$$3x_1 + x_2 \geq 7$$

$$x_2 \geq 0$$

$$x_3 \leq 0$$

2. Replace non-nonnegative constraints x_1

Maximize

$$-2x'_1 + 2x''_1 - 7x_2 - x_3$$

Subject to

$$\begin{aligned}
 x'_1 - x''_1 - x_3 &= 7 \\
 3x'_1 - 3x''_1 + x_2 &\geq 7 \\
 x'_1, x''_1, x_2 &\geq 0 \\
 x_3 &\leq 0
 \end{aligned}$$

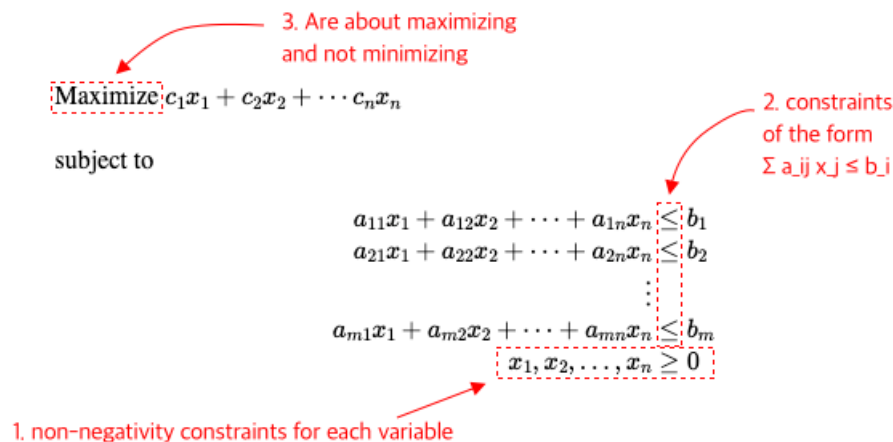
Notes:

- **Linear Programming**

- Is a method to achieve the best outcome (such as maximum profit or lowest cost) in a mathematical model whose requirements are represented by linear relationships. ^[1]
- Is named to make it sound cool for government funding
 - * Like dynamic programming
- Applications
 - * Microeconomics (maximize profits, minimize costs)
 - * Company management

- **Standard Form**

- Is a form of linear programming
- Are about maximizing, not minimizing ^[2]
- All have a positivity constraint for each variable ^[2]
- All other constraints are all of the form “linear combination of variables \leq constant”. ^[2]



- **Converting Linear Programming to Standard Form**

- 1) The objective function might be a minimization rather than a maximization

- Negate coefficients of the objective function

multiply by -1

<div style="border: 1px solid red; padding: 2px; display: inline-block;"> minimize $-2x_1 + 3x_2$ </div> subject to $\begin{aligned} x_1 + x_2 &= 7 \\ x_1 - 2x_2 &\leq 4 \\ x_1 &\geq 0 \end{aligned}$	<div style="border: 1px solid red; padding: 2px; display: inline-block;"> maximize $2x_1 - 3x_2$ </div> subject to $\begin{aligned} x_1 + x_2 &= 7 \\ x_1 - 2x_2 &\leq 4 \\ x_1 &\geq 0 \end{aligned}$
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- 2) There might be variables without nonnegativity constraints

- Replace each non-nonnegative variable x_i with x'_i and x''_i
- Modify linear program

Replace x_i with x'_i and x''_i

maximize $2x_1 - 3x_2$ subject to $\begin{aligned} x_1 + x_2 &= 7 \\ x_1 - 2x_2 &\leq 4 \\ x_1 &\geq 0 \end{aligned}$	maximize $2x_1 - 3x'_2 + 3x''_2$ subject to $\begin{aligned} x_1 + x'_2 - x''_2 &= 7 \\ x_1 - 2x'_2 + 2x''_2 &\leq 4 \\ x_1, x'_2, x''_2 &\geq 0 \end{aligned}$
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x₂ is not nonnegative :(
They are now nonnegative :) Yayy!!

- 3) There might be **equality constraints**, which have an equal sign rather than a less-than-or-equal-to sign

- Replace equality constraint $f(x_1, x_2, \dots, x_n) = b$ with $f(x_1, x_2, \dots, x_n) \leq b$ and $f(x_1, x_2, \dots, x_n) \geq b$

Multiply incorrect constraints by -1

maximize $2x_1 - 3x'_2 + 3x''_2$ subject to $\begin{aligned} x_1 + x'_2 - x''_2 &\leq 7 \\ x_1 + x'_2 - x''_2 &\geq 7 \\ x_1 - 2x'_2 + 2x''_2 &\leq 4 \\ x_1, x'_2, x''_2 &\geq 0 \end{aligned}$	maximize $2x_1 - 3x_2 + 3x_3$ subject to $\begin{aligned} x_1 + x_2 - x_3 &\leq 7 \\ -x_1 - x_2 + x_3 &\leq -7 \\ x_1 - 2x_2 + 2x_3 &\leq 4 \\ x_1, x_2, x_3 &\geq 0 \end{aligned}$
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- 4) There might be **inequality constraints**, but instead of having a less-than-or-equal-to sign

- Multiply incorrect inequality constraints by -1

maximize $2x_1 - 3x'_2 + 3x''_2$
 subject to
 $x_1 + x'_2 - x''_2 = 7$
 $x_1 - 2x'_2 + 2x''_2 \leq 4$
 $x_1, x'_2, x''_2 \geq 0$

maximize $2x_1 - 3x'_2 + 3x''_2$
 subject to
 $x_1 + x'_2 - x''_2 \leq 7$
 $x_1 + x'_2 - x''_2 \geq 7$
 $x_1 - 2x'_2 + 2x''_2 \leq 4$
 $x_1, x'_2, x''_2 \geq 0$

References:

- 1) Wikipedia, Linear Programming, [link](#)
- 2) Instituto de Matematicas, Standard form for Linear Programs, [link](#)