CSC236 Worksheet 6 Review

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May 10, 2020

Question 1

- **Rough Work:**

  Assume that $\forall k \in \mathbb{N}$, $R(3^k) = k3^k$.

  I need to prove $R \in \Theta(n \lg n)$. That is, $R \in \mathcal{O}(n \lg n)$ and $R \in \Omega(n \lg n)$.

  I will do so in parts.

  1. Part 1 (Proving $R \in \mathcal{O}(n \lg n)$)

Let $n \in \mathbb{N}$. Define $n^* = 3^{\lceil \log_3 n \rceil}$. Then, we have

$$\lceil \log_3 n \rceil - 1 < \log_3 n \leq \lceil \log_3 n \rceil \tag{1}$$

I will also use the fact proved in week 7 tutorial exercise that R is non-decreasing.

Let $d = 6$. Then, $d \in \mathbb{R}^+$. Let $B = 3$. Then, $B \in \mathbb{R}^+$. Assume $n \geq B$.

I need to show $R(n) \leq dn \lg n$.

Starting from $R(n)$, we have

$$\begin{aligned}
R(n) &\leq R(n^*) && \text{[Since } n \leq n^* \text{ and } R \text{ is non-decreasing]} && (2)\\
&= n^* \log_3 n^* && \text{[By replacing } 3^k \text{ for } n^*\text{]} && (3)\\
&\leq 3n \log_3 3n && \text{[Since } n \leq n^* \Rightarrow 3n \leq 3n^*\text{]} && (4)\\
&= 3n(\log_3 n + 1) && && (5)\\
&= 3n(\log_3 n + \log_3 n) && \text{[Since } n \leq B = 3 \Rightarrow \log_3 n \leq 1\text{]} && (6)\\
&\leq 6n \log_3 n && && (7)\\
&\leq dn \log_3 n && \text{[Since } d = 6\text{]} && (8)\\
&\leq dn \lg n && && (9)
\end{aligned}$$

2. Part 2 (Proving $R \in \Omega(n \lg n)$)