# Worksheet 9 Solution

March 18, 2020

## Question 1

a. Every set $S$ of size 0 has $\frac{0(0-1)}{2} = 0$ subsets of size 2

b. Let $n = 0$, and S be an arbitrary set. Assume $S$ has size 0.

   Then, $S$ only has empty subsets by the fact that $S$ has size 0.

   Since empty subset has size 0, there are 0 subsets with size 2.

c. **Section 1:**

   Every set of size $k$ has $\frac{k(k-1)}{2}$ subsets of size 2.

   **Section 2:**

   Every set of sie $k+1$ has $\frac{(k+1)k}{2}$ subsets of size 2.

   **Section 3.1:**

   Because we know

| Index | Set | # of subsets of size 2 containing last element |
|---|---|---|
| 2 | $\{s_1, s_2\}$ | has 1 subset containing $s_2$ |
| 3 | $\{s_1, s_2, s_3\}$ | has 2 subsets containing $s_3$ |
| 4 | $\{s_1, s_2, s_3, s_4\}$ | has 3 subsets containing $s_4$ |

, we can deduce from above that the number of subsets of size 2 containing $s_{k+1}$ is $k$.

**Section 3.2:**

P(n): $\forall n \in \mathbb{N}$, every set of size $n$ has $\frac{n(n-1)}{2}$ subsets of size 2

Let $k \in \mathbb{N}$, and assume P(k).

Then, the number of subsets of $S$ of size 2 that don't contain $s_{k+1}$ is $\frac{k(k-1)}{2}$.

**Section 3.3:**

$$\frac{k(k-1)}{2} + k = \frac{k(k-1)}{2} + \frac{2k}{2} \tag{1}$$

$$= \frac{k}{2}\left[(k-1) + 2\right] \tag{2}$$

$$= \frac{k(k+1)}{2} \tag{3}$$

Then, it follows from the proof of induction that the statement $\forall n \in \mathbb{N}$, every set of size $n$ has $\frac{n(n-1)}{2}$ is true.

# Question 2

# Question 3