

# Midterm 2 Version 1 Solution

April 3, 2020

## Question 1

a.

$$100 \div 2 = 50, \text{Remainders } 0$$

$$50 \div 2 = 25, \text{Remainders } 0$$

$$25 \div 2 = 12, \text{Remainders } 1$$

$$12 \div 2 = 6, \text{Remainders } 0$$

$$6 \div 2 = 3, \text{Remainders } 0$$

$$3 \div 2 = 1, \text{Remainders } 1$$

$$1 \div 2 = 0, \text{Remainders } 1$$

Then, it follows from above that the binary representation of 100 is  $(1100100)_2$ .

b. The smallest number that can be expressed by an  $n$ -digit balanced ternary representation is

$$\sum_{i=0}^{n-1} d_i \cdot 3^i, \text{ where } d_i \in \{0, 1, 2\} \quad (1)$$

**Correct Solution:**

The smallest number that can be expressed by an n-digit balanced ternary representation is

$$-\left[\sum_{i=0}^{n-1} 3^i\right] \quad (2)$$

**Notes:**

- Realized professor is asking for an example of the smallest number.
- Learned a negative number could be expressed in in ternary or binary representation of numbers.

c.

$f(n) \in \Omega(n)$	True	$g(n) \in \Omega(n)$	False	$f(n) \in \mathcal{O}(g(n))$	False
$f(n) \in \Theta(g(n))$	False	$g(n) \in \Theta(\log_3 n)$	True	$f(n) + g(n) \in \Theta(f(n))$	True

**Notes:**

- $\forall g : \mathbb{N} \rightarrow \mathbb{R}^{\geq 0}$ , and all numbers  $a \in \mathbb{R}^{\geq 0}$ , if  $g \in \mathcal{O}(f)$ , then  $f + g \in \mathcal{O}(f)$
- $g \in \Theta(f) : g \in \mathcal{O}(f) \wedge g \in \Omega(f)$   
or  
 $g \in \Theta(f) : \exists c_1, c_2, n_1 \in \mathbb{R}^+, \forall n \in \mathbb{N}, n \geq n_1 \Rightarrow c_1 g(n) \leq f(n) \leq c_2 g(n)$ , where  $f, g : \mathbb{N} \rightarrow \mathbb{R}^{\geq 0}$
- $g \in \Omega(f) : \exists c, n_o \in \mathbb{R}^+, \forall n \in \mathbb{N}, n \geq n_o \Rightarrow g(n) \geq cf(n)$ , where  $f, g : \mathbb{N} \rightarrow \mathbb{R}^{\geq 0}$
- $g \in \mathcal{O}(f) : \exists c, n_o \in \mathbb{R}^+, \forall n \in \mathbb{N}, n \geq n_o \Rightarrow g(n) \leq cf(n)$ , where  $f, g : \mathbb{N} \rightarrow \mathbb{R}^{\geq 0}$

d.

k	0	1	2
$i_k$	$3 = 3^1$	$9 = 3^2$	$81 = 3^4$

The value of  $i_k$  is

$$3^{2^k} \tag{1}$$

**Notes:**

- Realized we are only concerned with the lines **i = i \* i** and **i = 3**

e. The number of iterations the functions loop will run is

$$\lceil \log_2 \log_3 n \rceil - 1 \tag{1}$$

**Question 2**

**Question 3**

**Question 4**