Worksheet 6 Solution

March 15, 2020

Question 1

a. $P(123) \wedge (\forall n \in \mathbb{N}, P(n) \Rightarrow n \leq 123)$

b. $isCD(x, y, d)$: $\exists x, y, d \in \mathbb{Z},\ d \mid x \wedge d \mid y$

$isGCD(x, y, d)$: $\exists x, y, d \in \mathbb{Z},\ (x = 0 \wedge y = 0 \wedge d = 0) \vee ((x \neq 0 \vee y \neq 0) \wedge isCD(x, y, d) \wedge \forall e \in \mathbb{Z},\ e > d \Rightarrow \neg isCD(x, y, e))$

c. Statement: $\forall x \in \mathbb{Z}^+, IsGCD(x, 0, x)$

For the value x, because we know $x \mid x$, and $\forall n \in \mathbb{Z}^+$ and $\forall d \in \mathbb{Z}, d \mid n \Rightarrow d \leq n$, x is the biggest divisor of x

For the value 0, because we know anything that divides 0 is 0, and $\exists k \in \mathbb{Z},\ 0 = k \times 0$, k can be chosen to be x.

Then, it follows from the definition of GCD that the statement $IsGCD(x, 0, x)$ is true.

Question 2

Question 3