CSC236 Worksheet 5 Solution

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Question 1

a. *Proof.* For convenience, define $H(k) : R(3^k) = 3^k k$. Notice that when $n = 3^k$, $3^k k$ is the same as $n \log_3 n$. I will use simple induction to prove $\forall k \in \mathbb{N},\ H(k)$.

Base Case ($k = 0$):

Let $k = 0$.

Then,

$$\begin{aligned} 3^k \cdot k &= 3^0 \cdot 0 && (1)\\ &= 0 && (2)\\ &= R(n) && \text{[By def.]} \quad (3) \end{aligned}$$

Thus, $H(0)$ is verified.

Inductive Step:

Let $k \in \mathbb{N}$. Assume $H(k)$. That is $R(3^k) = 3^k k$.

I will show that $H(k+1)$ follows. That is, $R(3^{k+1}) = (k+1)3^{k+1}$.

The definition of $R(3^{k+1})$ tells us,

$$\begin{aligned} R(3^{k+1}) &= 3^{k+1} + 3R(\lceil 3^{k+1}/3 \rceil) && (4)\\ &= 3^{k+1} + 3R(3^k) && (5) \end{aligned}$$

Then, using this fact, we can conclude

$$R(3^{k+1}) = 3^{k+1} + 3 \cdot 3^k k \qquad \text{[By I.H]} \tag{6}$$
$$= 3^{k+1} + 3^{k+1} k \tag{7}$$
$$= 3^{k+1}(k+1) \tag{8}$$

$\square$