

# Worksheet 16 Review

April 2, 2020

## Question 1

a. Let  $k \in \mathbb{N}$ .

Here, the minimum possible change occurs for the loop variable in a single iteration when  $i = i + 1$ .

The maximum possible change occurs for the loop variable in a single iteration when  $i = i + 6$ .

The exact upper bound of the variable after  $k$  iteration is

$$i_k \leq 6k \tag{1}$$

The exact lower bound of the variable after  $k$  iteration is

$$k \leq i_k \tag{2}$$

Using the fact that the termination occurs when  $i_k = n$ , we can calculate that for the upper bound, the loop terminates when

$$6k \geq n \tag{3}$$

$$k \geq \frac{n}{6} \tag{4}$$

Because we know  $\frac{n}{6}$  may be a decimal, we can conclude the closest value at which the loop terminates is when

$$k = \left\lceil \frac{n}{6} \right\rceil \tag{5}$$

Using the same fact, we can calculate that for the lower bound, the loop terminates when

$$k \geq n \tag{6}$$

It follows from above that for the lower bound, the smallest value of  $k$  at which the loop termination occurs is when

$$k = n \tag{7}$$

Then, we can conclude the function has asymptotic lower bound of  $\Omega(n)$ , and asymptotic upper bound of  $\mathcal{O}(n)$ .

Then, since both  $\Omega$  and  $\mathcal{O}$  have the same value,  $\Theta(n)$  is also true.

## Question 2

## Question 3