Worksheet 10 Review

March 28, 2020

Question 1

a.

$$(165)_8 = 5 \cdot 8^0 + 6 \cdot 8^1 + 1 \cdot 8^2 \tag{1}$$
$$= 5 + 48 + 64 \tag{2}$$
$$= 53 + 64 \tag{3}$$
$$= 117 \tag{4}$$

b.

$$(B4)_{16} = 4 \cdot 16^0 + 11 \cdot 16^1 \tag{1}$$
$$= 4 + (11 \cdot 16) \tag{2}$$
$$= 4 + 176 \tag{3}$$
$$= 180 \tag{4}$$

Question 2

a.

$$357 \div 2 = 178, \text{remainder } \mathbf{1},$$
$$178 \div 2 = 89, \text{remainder } \mathbf{0},$$
$$89 \div 2 = 44, \text{remainder } \mathbf{1},$$
$$44 \div 2 = 22, \text{remainder } \mathbf{0},$$
$$22 \div 2 = 11, \text{remainder } \mathbf{0},$$
$$11 \div 2 = 5, \text{remainder } \mathbf{1},$$
$$5 \div 2 = 2, \text{remainder } \mathbf{1},$$
$$2 \div 2 = 1, \text{remainder } \mathbf{0},$$
$$1 \div 2 = 0, \text{remainder } \mathbf{1}$$

Combining it together, the binary representation of 357 is $(101100101)_2$

b.

$$1 \cdot 2^0 + 0 \cdot 2^1 + 1 \cdot 2^2 = \frac{1 + 0 + 4}{8^0} = 5$$
$$0 \cdot 2^3 + 0 \cdot 2^4 + 1 \cdot 2^5 = \frac{0 + 0 + 32}{8^1} = 4$$
$$1 \cdot 2^6 + 0 \cdot 2^7 + 1 \cdot 2^8 = \frac{64 + 0 + 256}{8^2} = 5$$

Combining it together, the octal representation of $(101100101)_2$ is $(545)_8$.

c.

$$357 \div 16 = 22, \text{remainder } \mathbf{5},$$
$$22 \div 16 = 1, \text{remainder } \mathbf{5},$$
$$1 \div 16 = 0, \text{remainder } \mathbf{1}$$

Combining it together, the hexadecimal representation of 357 is $(155)_{16}$.

Correct Solution:

$$357 \div 16 = 22, \text{remainder } \mathbf{5},$$
$$22 \div 16 = 1, \text{remainder } \mathbf{6},$$
$$1 \div 16 = 0, \text{remainder } \mathbf{1}$$

Combining it together, the hexadecimal representation of 357 is $(165)_{16}$.

Question 3

a.

$$0.375 \times 2 = 0.75 + \mathbf{0} \tag{1}$$
$$0.75 \times 2 = 0.5 + \mathbf{1} \tag{2}$$
$$0.5 \times 2 = 0 + \mathbf{1} \tag{3}$$

Combining the above, the binary representation of 0.375 is $(0.011)_2$.

Notes:

- **Converting decimal to binary**

$$0.8125 \times 2 = 0.625 + \mathbf{1} \tag{4}$$
$$0.625 \times 2 = 0.25 + \mathbf{1} \tag{5}$$
$$0.25 \times 2 = 0.5 + \mathbf{0} \tag{6}$$
$$0.5 \times 2 = 0 + \mathbf{1} \tag{7}$$

Binaries read *top to bottom*

b.

$$0.1 \times 2 = 0.2 + \mathbf{0} \tag{1}$$
$$0.2 \times 2 = 0.4 + \mathbf{0} \tag{2}$$
$$0.4 \times 2 = 0.8 + \mathbf{0} \tag{3}$$
$$0.8 \times 2 = 0.6 + \mathbf{1} \tag{4}$$
$$0.6 \times 2 = 0.2 + \mathbf{1} \tag{5}$$
$$0.2 \times 2 = 0.4 + \mathbf{0} \tag{6}$$
$$0.4 \times 2 = 0.8 + \mathbf{0} \tag{7}$$
$$0.8 \times 2 = 0.6 + \mathbf{1} \tag{8}$$

Combining the above, the binary representation of 0.1 is $(0.0\overline{0011})_2$.

Question 4

a.

$$\sum_{i=0}^{\infty} \left(\frac{1}{2} \right) = \frac{\frac{1}{2}}{1 - \frac{1}{2}} \tag{1}$$
$$= \frac{\frac{1}{2}}{\frac{1}{2}} \tag{2}$$
$$= 1 \tag{3}$$

b. Since 1st 1 repeats every 4 decimal places, and 2nd 1 repeats every 5 decimal places, we have

$$(0.0\overline{0011})_2 = \sum_{i=1}^{\infty} \left(\frac{1}{2}\right)^{4i} + \sum_{i=1}^{\infty} \left(\frac{1}{2}\right)^{4i+1} \tag{1}$$

$$= \sum_{i=1}^{\infty} \frac{1}{16} + \frac{1}{2} \cdot \sum_{i=1}^{\infty} \frac{1}{16} \tag{2}$$

$$= \frac{1}{15} + \frac{1}{30} \tag{3}$$

$$= \frac{3}{30} \tag{4}$$

$$= \frac{1}{10} \tag{5}$$