

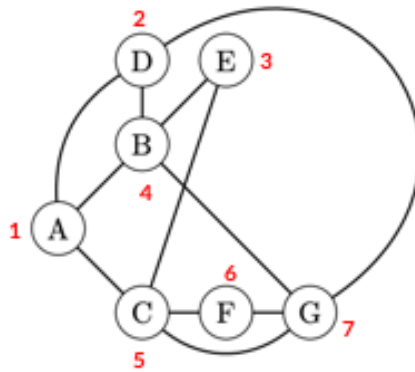
# Worksheet 19 Solution

Hyungmo Gu

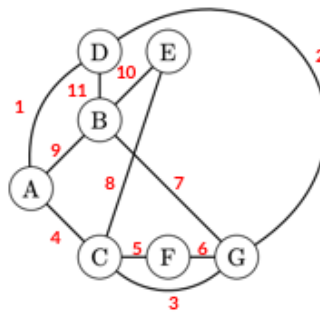
April 7, 2020

## Question 1

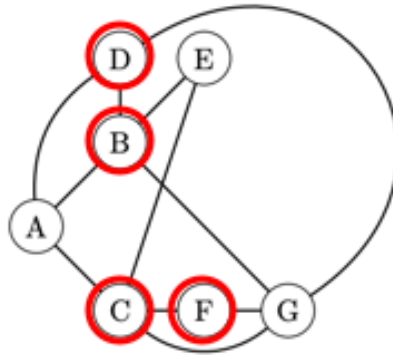
a. By the figure below, we can conclude there are 7 vertices.



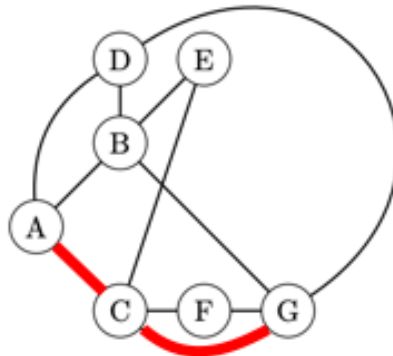
b. By the figure below, we can conclude there are 11 edges.



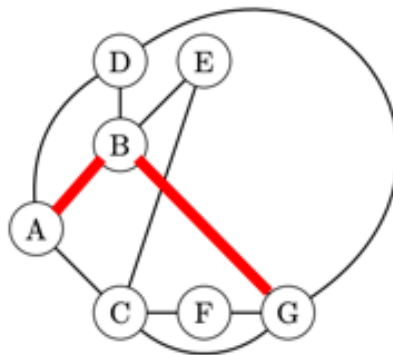
c. By the figure below, we can conclude there are 4 vertices adjacent to G.



d. By the figure below, we can conclude the distance between A and G is 2.

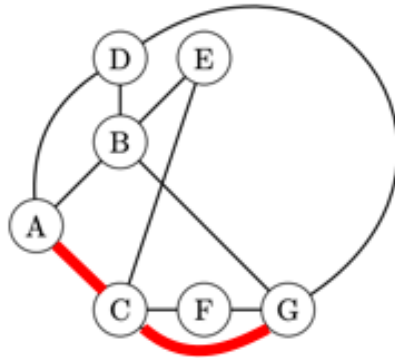


There are 2 shortest paths between A and G. One is the path from A to C to G as shown above, and the other is the path from A to B to G

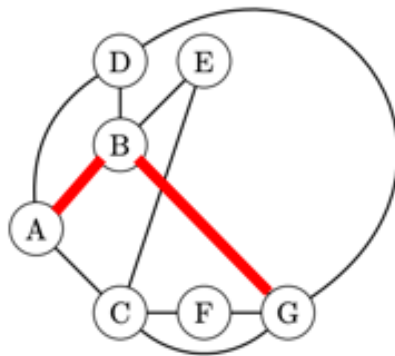


**Correct Solution:**

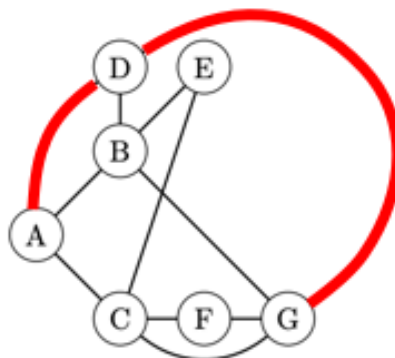
By the figure below, we can conclude the distance between A and G is 2.



There are 3 shortest paths between A and G. One is the path from A to C to G as shown above, and the other is the path from A to B to G

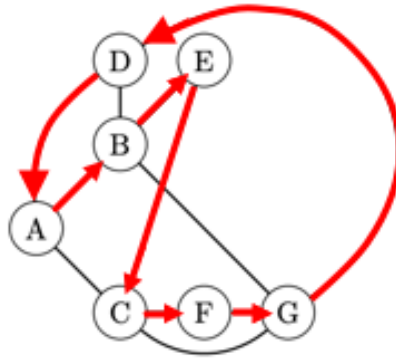


and the last one is from A to D to G



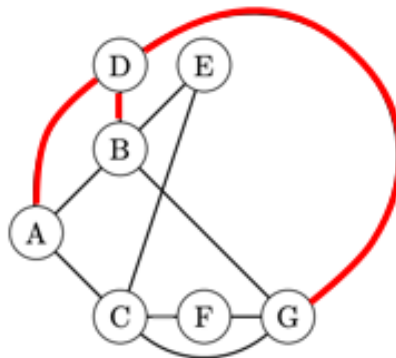
Notes:

- **Distance** is the number of edges in a shortest path.
- e. Path  $[C, F, G, D, A, B, E]$  is one example that goes through all vertices of the graph.



## Question 2

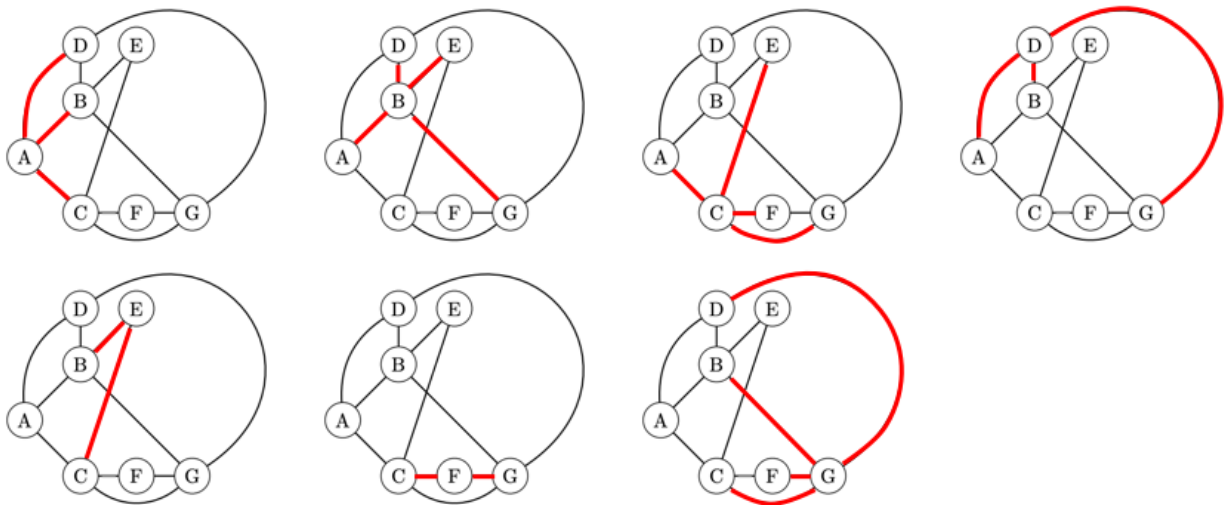
- a. By the figure below, we can conclude the degree of vertex D is 3.



- b. By the figure below, we can see

- vertex A has degree of 3
- vertex B has degree of 4
- vertex C has degree of 4
- vertex D has degree of 3
- vertex E has degree of 2
- vertex F has degree of 2
- vertex G has degree of 4

Using this fact, we can conclude the vertices with the largest degree are B, C and G.



c. **Statement:**  $\forall G = (V, E), (\forall v \in V, d(v) \leq 5) \Rightarrow |E| \leq \frac{5}{2}|V|$

*Proof.* Let  $G = (V, E)$  be a graph. Assume  $v \in V$  and  $d(v) \leq 5$ .

We need to show  $|E| \leq \frac{5}{2}|V|$ .

Because we know the number of edges is half of the summation of all degrees of  $v$  in  $V$ , we can write

$$|E| = \frac{1}{2} \cdot \sum_{v \in V} d(v) \quad (1)$$

Then, using the assumption  $d(v) \leq 5$ , we can conclude

$$|E| \leq \frac{1}{2} \cdot \sum_{v \in V} 5 \quad (2)$$

$$= \frac{5}{2}|V| \quad (3)$$

□

### Question 3