Midterm 1 Version 2 Solution

March 19, 2020

Question 1

a. Since

$S_1 = \{1, 2, 3, 5, 7, 11, 13, 17, 19, 23, 29\}$, and $S_2 = \{1, 2, 3, 5, 6, 10, 15, 30\}$,

$S_1 \cap S_2 = \{1, 2, 3, 5\}$

b. See the table below

p	q	r	$\neg p$	$\neg p \Leftrightarrow q$	$(\neg p \Leftrightarrow q) \Rightarrow r$
T	T	T	F	F	T
T	T	F	F	F	T
T	F	T	F	T	T
F	T	T	T	T	F
T	F	F	F	T	F
F	F	T	T	F	T
F	F	F	T	F	T

Correct Solution:

p	q	r	$\neg p$	$\neg p \Leftrightarrow q$	$(\neg p \Leftrightarrow q) \Rightarrow r$
T	T	T	F	F	T
T	T	F	F	F	T
T	F	T	F	T	T
*F	*T	*T	*T	*T	*T
T	F	F	F	T	F
*F	*T	*F	*T	*T	*F
F	F	T	T	F	T
F	F	F	T	F	T

$*$ = Incorrect/missing solution

c. Let $x \in \mathbb{N}$. Assume $P(x)$.

We will prove that there is a natural number y such that the predicate $Q(x, y)$ is true.

Correct Solution:

Let $x \in \mathbb{N}$, and $y =$ _____. Assume $P(x)$.

We will prove that the predicate $Q(x, y)$ is true.

Question 2

a. $\forall x \in P,\ Cat(x) \wedge Loves(x, x)$

Correct Solution:

$\forall x \in P,\ Cat(x) \Rightarrow Loves(x, x)$

b. $\forall x \in P,\ \exists y \in P,\ Cat(x) \wedge Cute(y) \wedge Loves(x, y)$

Correct Solution:

$\forall x \in P,\ Cat(x) \wedge Cute(x) \Rightarrow (\forall y \in P, Cat(y) \Rightarrow Cute(y))$

c. $\exists x \in P,\ Cat(x) \wedge Cute(x) \Rightarrow \forall y \in P, Cat(y) \wedge Cute(y)$

d. $\forall p_1, p_2 \in P, p_1 \neq p_2 \wedge Loves(p_1, p_2) \wedge Loves(p_2, p_1) \Rightarrow (Cat(p_1) \wedge \neg Cat(p_2)) \vee (\neg Cat(p_1) \wedge Cat(p_2))$

Question 3

a. $\exists n \in \mathbb{N},\ n > 1 \Rightarrow \forall x \in \mathbb{R},\ \lfloor nx \rfloor = n \lfloor x \rfloor$

Correct Solution:

$\exists n \in \mathbb{N},\ n > 1 \wedge (\forall x \in \mathbb{R}^+,\ \lfloor nx \rfloor = n \lfloor x \rfloor)$

b. **Negation:** $\forall n \in \mathbb{N},\ n > 1 \wedge (\exists x \in \mathbb{R},\ \lfloor nx \rfloor \neq n\lfloor x \rfloor)$

Let $n = 2$, $x = 0.5$.

Then,

$$\begin{aligned} \lfloor nx \rfloor &= \lfloor 2(0.5) \rfloor && (1)\\ &= 1 && (2) \end{aligned}$$

And,

$$\begin{aligned} n\lfloor x \rfloor &= 2\lfloor 0.5 \rfloor && (3)\\ &= 2(0) && (4)\\ &= 0 && (5) \end{aligned}$$

Since $\lfloor nx \rfloor \neq n\lfloor x \rfloor$,the predicate logic is false.

Question 4