CSC236 Worksheet 3 Review

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May 8, 2020

Question 2

- *Proof.* Define $P(e) : S_1(e) = 3(s_2(e) - 1)$

  I will use structural induction to prove $\forall e \in \varepsilon,\ P(e)$.

  Basis:

  Let $\{x, y, z\} \in \varepsilon$.

  In this step, there are following cases to consider: $e = x$, $e = y$, and $e = z$.

  In each of the cases, we have $s_1(e) = 0$ and $s_2(e) = 1$.

  Thus,

$$
\begin{align}
s_1(e) = 0 &= 3(0) \tag{1}\\
&= 3(1 - 1) \tag{2}\\
&= 3(s_2(e) - 1) \tag{3}
\end{align}
$$

  So, , $P(e)$ holds.

  Inductive Step:

  Let $e_1, e_2 \in \varepsilon$. Assume $H(e) : P(e_1)$ and $P(e_2)$. That is, $s_1(e_1) = 3(s_2(e_1) - 1)$ and $s_2(e_2) = 3(s_2(e_2) - 1)$.

  I need to prove all possible combinations of e_1 and e_2 satisfy the statement. That is $P((e_1 + e_2))$ and $P((e_1 - e_2))$.

In each of the combination, the total number of variables of e is the sum of the number of variables in e_1 and e_2, and the total number of parenthesis and operators in e is the sum of operators and parenthesis in e_1 and e_2 plus 3.

Then, using these facts, we have

$$
\begin{aligned}
s_2(e) &= s_2(e_1) + s_2(e_2) && (4)\\
s_1(e) &= s_1(e_1) + s_1(e_2) + 3 && (5)
\end{aligned}
$$

Thus, we can calculate

$$
\begin{aligned}
s_1(e) &= s_1(e_1) + s_1(e_2) + 3 && \text{[By 5]} && (6)\\
&= 3(s_2(e_1) - 1) + 3(s_2(e_2) - 1) + 3 && \text{[By I.H]} && (7)\\
&= 3(s_2(e_1) + s_2(e_2)) - 6 + 3 && \text{[By I.H]} && (8)\\
&= 3s_2(e) - 3 && \text{[By 4]} && (9)\\
&= 3(s_2(e) - 1) && && (10)
\end{aligned}
$$

$\square$

Correct Solution:

Define $P(e) : S_1(e) = 3(s_2(e) - 1)$

I will use structural induction to prove $\forall e \in \varepsilon,\ P(e)$.

Basis:

Let $e \in \{x, y, z\}$.

Then, $s_1(e) = 0$ and $s_2(e) = 1$.

Thus, we have,

$$
\begin{aligned}
s_1(e) = 0 &= 3(0) && (11)\\
&= 3(1 - 1) && (12)\\
&= 3(s_2(e) - 1) && (13)
\end{aligned}
$$

So, $P(e)$ holds.

Inductive Step:

Let $e_1, e_2 \in \varepsilon$. Assume $H(e) : P(e_1)$ and $P(e_2)$. That is, $s_1(e_1) = 3(s_2(e_1) - 1)$ and $s_2(e_2) = 3(s_2(e_2) - 1)$.

I will prove $P(e)$ holds for any e that can be constructed from e_1 and e_2. There are two cases: $e = e_1 + e_2$ and $e = e_1 - e_2$.

In each cases, we have

$$s_2(e) = s_2(e_1) + s_2(e_2) \tag{14}$$
$$s_1(e) = s_1(e_1) + s_1(e_2) + 3 \tag{15}$$

Thus, we can calculate

$$\begin{aligned}
s_1(e) &= s_1(e_1) + s_1(e_2) + 3 && \text{[By 5]} && (16)\\
&= 3(s_2(e_1) - 1) + 3(s_2(e_2) - 1) + 3 && \text{[By I.H]} && (17)\\
&= 3(s_2(e_1) + s_2(e_2)) - 6 + 3 && \text{[By I.H]} && (18)\\
&= 3s_2(e) - 3 && \text{[By 4]} && (19)\\
&= 3(s_2(e) - 1) && && (20)
\end{aligned}$$

So, $P(e)$ follows from $H(e)$.