

# Worksheet 13 Review

March 31, 2020

## Question 1

a. Since the loop starts from  $i = 0$  and ends at  $i = n - 1$ . The loop has

$$n - 1 - 0 + 1 = n \tag{1}$$

iterations.

Since each iteration runs 5 steps, the loop has total cost of

$$5 \cdot n = 5n \tag{2}$$

steps.

Because we know  $i = 0$  at line 2 has cost of 1, we can conclude that the algorithm has total cost of  $5n + 1$  steps.

### Correct Solution:

Because we know the loop starts from  $i = 0$  and ends at  $i = n - 1$  with  $i$  increasing by 5 per iteration, we can conclude the loop has

$$\left\lceil \frac{n}{5} \right\rceil \tag{3}$$

iterations.

Since each iteration takes constant time, the loop has runtime of  $\Theta(n)$

**Notes:**

- How does professor begin a proof after 'We will prove that...' or at the beginning of each case/parts?
- Noticed professor doesn't provide a detailed explanation for the number of iterations.
- Realized the goal of this problem is to determine the exact cost and runtime of each loop.

There are  $\lceil \frac{n}{5} \rceil$  iterations. ...

- b. Because we know the loop starts at  $i = 4$  and ends at  $i = n - 1$  with  $i$  increasing by 1 per iteration, we can conclude that the loop has

$$\lceil n - 14 + 1 \rceil = n - 4 \quad (1)$$

iterations.

Since each iteration takes a constant time, we can conclude that the loop has runtime of  $\mathcal{O}(n)$ .

**Correct Solution:**

Because we know the loop starts at  $i = 4$  and ends at  $i = n - 1$  with  $i$  increasing by 1 per iteration, we can conclude that the loop has **at most**

$$\lceil n - 14 + 1 \rceil = n - 4 \quad (1)$$

iterations.

Since each iteration takes a constant time, we can conclude that the loop has runtime of  $\Theta(n)$ .

**Notes:**

- Noticed professor doesn't count  $i = 4$  or  $i = 0$  in question 1.a to the total cost of algorithm. But in later parts of the question, the two are considered. I wonder if the line with constant runtime should always be accounted for, or if it can be ignored in certain circumstances. If latter, when can the constants be ignored?
- c. Since the loop starts at  $i = 0$  and ends at  $i = n - 1$  with  $i$  increasing by  $\frac{n}{10}$  per iteration, we can conclude that the loop has at most

$$\left\lceil \frac{(n - 1 - 0 + 1)}{\frac{n}{10}} \right\rceil = \left\lceil \frac{n \cdot 10}{n} \right\rceil \quad (1)$$

$$= 10 \quad (2)$$

iterations.

Since each iteration takes constant time, we can conclude that the loop has runtime of  $\Theta(1)$ .

- d. For the case where  $n^2 \leq 20$ , it's omitted because we are assuming the value of  $n$  is asymptotically large.

For the case where  $n^2 > 20$ , because we know the loop runs from  $i = 20$  to  $i = n^2 - 1$  with  $i$  increasing by 3 per iteration, we can conclude that the loop has at most

$$\left\lceil \frac{n^2 - 1 - 20 + 1}{3} \right\rceil = \left\lceil \frac{n^2 - 20}{3} \right\rceil \quad (1)$$

iterations.

Since the loop takes constant time per iteration, the loop has total runtime of  $\Theta(n^2)$ .

- e. Since we are considering asymptotic runtime or the case where  $n$  is very large, the case where  $n^2 \leq 20$  will be omitted.

For the case where  $n^2 > 20$ , because we know the first loop runs from  $i = 20$  and ends at  $i = n^2 - 1$  with  $i$  increasing by 3 per iteration, we can conclude that the first loop runs

$$\left\lceil \frac{n^2 - 1 - 20 + 1}{3} \right\rceil = \left\lceil \frac{n^2 - 20}{3} \right\rceil \quad (1)$$

iterations

For the second loop, because we know the loop runs from  $j = 0$  to  $j = n - 1$  with  $j$  increasing by 0.01 per iteration, we can conclude that the second loop runs at most

$$\left\lceil \frac{n - 1 - 0 + 1}{\frac{1}{100}} \right\rceil = \lceil 100 \cdot n \rceil \quad (2)$$

$$= 100 \cdot n \quad (3)$$

iterations

Since each iteration takes a constant time in both of the loops, the total runtime of the loops in this algorithm is

$$\Theta \left( \left\lceil \frac{n^2 - 20}{3} \right\rceil + 100 \cdot n \right) = \Theta(n^2) \quad (4)$$

**Question 2**

**Question 3**

**Question 4**