

CSC373 Worksheet 1 Solution

July 16, 2020

1. Notes:

- Strassen's method for matrix multiplication

- Reduces the time complexity of matrix multiplication from $O(n^3)$ to $O(n^{\log_2 7}) = O(n^{2.81})$
- Has four steps

- 1) Divide the input matrices A and B and output matrix C into $n/2 \times n/2$ submatrices

$$A = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix}, \quad B = \begin{pmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{pmatrix}, \quad C = \begin{pmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{pmatrix},$$

- 2) Create 10 matrices, S_1, S_2, \dots, S_{10} each of which is $n/2 \times n/2$ and is the sum or difference of two matrices created in step 1

$$S_1 = B_{12} - B_{22}$$

$$S_2 = A_{11} + A_{12}$$

$$S_3 = A_{21} + A_{22}$$

$$S_4 = B_{21} - B_{11}$$

$$S_5 = A_{11} + A_{22}$$

$$S_6 = B_{11} + B_{22}$$

$$S_7 = A_{12} - A_{22}$$

$$S_8 = B_{21} + B_{22}$$

$$S_9 = A_{11} - A_{21}$$

$$S_{10} = B_{11} + B_{12}$$

- 3) Recursively multiply $n/2 \times n/2$ matrices seven times to compute the following $n/2 \times n/2$ matrices

- Is not preferred in practical purposes

- 1) The constants used in Strassen's method are high and for a typical application Naive method works better.
- 2) For Sparse matrices, there are better methods especially designed for them.
- 3) The submatrices in recursion take extra space.
- 4) Because of the limited precision of computer arithmetic on noninteger values, larger errors accumulate in Strassen's algorithm than in Naive Method

References:

- 1) GeeksForGeeks, Divide and Conquer — Set 5 (Strassen's Matrix Multiplication), [link](#)
- Regular matrix multiplication
 -