

CSC148 Worksheet 7 Solution

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Question 1

- Noticed that there are 11 students in total.
- Students should be grouped by year as closest as possible.

Notes:

- 형모 해낼 뚝 있쥬!
- 형모 화이팅!

Question 2

Name	Year	College	
Priya	3	Victoria	Group 1
Alain	2	New	
Zoe	3	Woodsworth	
Francesco	3	Victoria	
Mohammed	4	Woodsworth	Group 2
Xiaoyuan	5	New	
Rohit	2	New	
Yimin	3	Trinity	
Grace	5	Woodsworth	Group 3
Claire	1	Woodsworth	
Kai	1	Woodsworth	

Question 3

- First, we need to find the group as homogenous as possible in terms of year students are in.

The definition tells us group needs to be in 4, and the following table tell us there are 4 3rd year students.

Student Year	Number of Students
1	2
2	2
3	4
4	1
5	2

It follows from these facts that the group of 3rd year students best satisfy this criterion.

Next, we need to find the group as not homogenous as possible in terms of year students are in.

The same table tells us with 2 5th year students, 1 4th year students and 1 3rd year student, a group spanning 3 years can be created.

Since we know there can't be a group spanning 4 years, we can conclude the group of 3 years (2 5th year students, 1 4th year students and 1 3rd year student) best satisfy this criterion.

Correct Solution:

Group 1 is the most homogeneous.

Group 3 is the least homogeneous.

Question 4

- We will calculate the group score based on the code below. The code is also included in *worksheet_7_q4_solution.py*

```

1  def get_group_score(group):
2      """Evaluates the group score
3
4          Precondition: len(group) == 4
5      """
6      n = 4
7

```

```

8     max_year = get_max_year(group)
9     min_year = get_min_year(group)
10    similarity_list = []
11
12    i = 0
13
14    while i < 4:
15        j = 0
16        while j < 4:
17            # find the scaled distance
18            scaled_distance = 0
19            if max_year != min_year:
20                scaled_distance = abs(group[i]['year'] - group[j][
'year']) / float(max_year - min_year)
21
22            # find the similarity
23            similarity = 1 - scaled_distance
24
25            # add to list
26            similarity_list.append(similarity)
27
28            j += 1
29            i += 1
30
31    # find the average
32    average = float(sum(similarity_list))/len(similarity_list)
33    return average.as_integer_ratio()
34
35    def get_max_year(group):
36        """returns max value of year in group"""
37
38        max_value = -1
39
40        for student in group:
41            max_value = max(student['year'], max_value)
42
43        return max_value
44
45    def get_min_year(group):
46        """returns min value of year in group"""
47
48        min_value = 100 # this is impossible value
49
50        for student in group:
51            min_value = min(student['year'], min_value)
52
53        return min_value
54
55
56    if __name__ == '__main__':
57        group_1 = [{'name': 'Primya', 'year': 3},
58                   {'name': 'Zoe', 'year': 3},
59                   {'name': 'Francesco', 'year': 3},
60                   {'name': 'Yimin', 'year': 3}]

```

```

61     group_2 = [{'name': 'Primya', 'year': 5},
62                {'name': 'Zoe', 'year': 5},
63                {'name': 'Francesco', 'year': 4},
64                {'name': 'Yimin', 'year': 3}]
65
66
67
68     score_1 = get_group_score(group_1)
69     print(score_1)
70
71     score_2 = get_group_score(group_2)
72     print(score_2)
73
74
75

```

For the most homogeneous group, the group score is 1.

For the least homogeneous group, the group score is $\frac{9}{16}$.

Correct Solution:

We will calculate the group score **using** the code below. The code is also included in *worksheet_7_q4_solution.py*

```

1  def get_similarity_list(group):
2      """Evaluates the list of similarities""" # <- Correct solution
3      similarity_list = []
4
5      i = 0
6
7      while i < len(group): # <- correct solution
8          j = i + 1 # <- correct solution
9          while j < len(group): # <- correct solution
10             # find the scaled distance
11             scaled_distance = abs(group[i]['year'] - group[j]['year
12             ']) / 5.0 # <- correct solution
13
14             # find the similarity
15             similarity = 1 - scaled_distance
16
17             # add to list
18             similarity_list.append(similarity)
19
20             j += 1
21             i += 1
22
23     return similarity_list
24

```

```

25 if __name__ == '__main__':
26     # Correct Solution
27     group_1 = [{'name': 'Primya', 'year': 3},
28               {'name': 'Alain', 'year': 2},
29               {'name': 'Zoe', 'year': 3},
30               {'name': 'Francesco', 'year': 3}]
31
32     # Correct Solution
33     group_2 = [{'name': 'Mohammed', 'year': 4},
34               {'name': 'Xiaoyuan', 'year': 5},
35               {'name': 'Rohit', 'year': 2},
36               {'name': 'Yimin', 'year': 3}]
37
38     # Correct Solution
39     group_3 = [{'name': 'Grace', 'year': 5},
40               {'name': 'Claire', 'year': 1},
41               {'name': 'Kai', 'year': 1}]
42
43     # Correct Solution
44     list_1 = get_similarity_list(group_1)
45     print(list_1)
46
47     # Correct Solution
48     list_2 = get_similarity_list(group_2)
49     print(list_2)
50
51     # Correct Solution
52     list_3 = get_similarity_list(group_3)
53     print(list_3)
54
55

```

We will calculate the score of each group in parts.

Part 1 (Score of Group 1):

We need to calculate the score of group 1.

The problem tells us the group score is

$$\text{Group Score} = \frac{\sum \text{similarities}}{\# \text{ of elements in list of similarities}} \quad (1)$$

and the above program tells us that the list of similarities are

$$[0.8, 1.0, 1.0, 0.8, 0.8, 1.0] \quad (2)$$

Then, using these facts, we can conclude that

$$\text{Group Score} = \frac{0.8 + 1.0 + 1.0 + 0.8 + 0.8 + 1.0}{6} \quad (3)$$

$$= \frac{\frac{4}{5} + 1 + 1 + \frac{4}{5} + \frac{4}{5} + 1}{6} \quad (4)$$

$$= \frac{\frac{4}{5} + \frac{5}{5} + \frac{5}{5} + \frac{4}{5} + \frac{4}{5} + \frac{5}{5}}{6} \quad (5)$$

$$= \frac{\frac{27}{5}}{6} \quad (6)$$

$$= \frac{27}{30} \quad (7)$$

$$= \frac{9}{10} \quad (8)$$

Part 2 (Score of Group 2):

We need to calculate the score of group 2.

The problem tells us the group score is

$$\text{Group Score} = \frac{\sum \text{similarities}}{\# \text{ of elements in list of similarities}} \quad (9)$$

and the above program tells us that the list of similarities are

$$[0.8, 0.6, 0.8, 0.4, 0.6, 0.8] \quad (10)$$

Then, we can conclude that the value of group score is

$$\frac{0.8 + 0.6 + 0.8 + 0.4 + 0.6 + 0.8}{6} = \frac{\frac{4}{5} + \frac{3}{5} + \frac{4}{5} + \frac{2}{5} + \frac{3}{5} + \frac{4}{5}}{6} \quad (11)$$

$$= \frac{\frac{20}{5}}{6} \quad (12)$$

$$= \frac{20}{30} \quad (13)$$

$$= \frac{2}{3} \quad (14)$$

Part 3 (Score of Group 3):

We need to calculate the score of group 3.

The problem tells us the group score is

$$\text{Group Score} = \frac{\sum \text{similarities}}{\# \text{ of elements in list of similarities}} \quad (15)$$

and the above program tells us that the list of similarities are

$$[0.2, 0.2, 1.0] \quad (16)$$

Then, we can conclude that the value of group score is

$$\frac{0.2 + 0.2 + 1.0}{3} = \frac{\frac{1}{5} + \frac{1}{5} + \frac{5}{5}}{3} \quad (17)$$

$$= \frac{\frac{7}{5}}{3} \quad (18)$$

$$= \frac{7}{15} \quad (19)$$

Notes:

- Realized that ‘group’ in question means ‘group 1’, ‘group 2’ and ‘group 3’ in question 2 :(.

Question 5

- We need to find the highest and the lowest possible score on this criterion.

We will do so in parts.

Part 1 (Calculating highest possible group score):

We need to find the highest possible group score on this criterion.

The definition of group score tells us that

$$\text{Group Score} = \frac{\sum_{i=0}^{n-1} \sum_{j=i+1}^{n-1} \left(1 - \frac{|student_year_i - student_year_j|}{5} \right)}{\# \text{ of elements in list of similarities}} \quad (1)$$

By observation, we can conclude that the highest possible group score happens when $|student_year_i - student_year_j| = 0$.

Then, using this fact, we can calculate that

$$\text{Group Score} = \frac{\sum_{i=0}^{n-1} \sum_{j=i+1}^{n-1} \left(1 - \frac{|student_year_i - student_year_j|}{5} \right)}{\# \text{ of elements in list of similarities}} \quad (2)$$

$$= \frac{\sum_{i=0}^{n-1} \sum_{j=i+1}^{n-1} 1}{\# \text{ of elements in list of similarities}} \quad (3)$$

Then, because we know $\sum_{i=0}^{n-1} \sum_{j=i+1}^{n-1} 1 = \# \text{ of elements in list of similarities}$, we can conclude

$$= \frac{\# \text{ of elements in list of similarities}}{\# \text{ of elements in list of similarities}} \quad (4)$$

$$= 1 \quad (5)$$

Part 2 (Calculating lowest possible group score):

We need to find the lowest possible group score achievable on this criterion.

The definition of group score tells us that

$$\text{Group Score} = \frac{\sum_{i=0}^{n-1} \sum_{j=i+1}^{n-1} \left(1 - \frac{|student_year_i - student_year_j|}{5} \right)}{\# \text{ of elements in list of similarities}} \quad (6)$$

By observation, we can conclude that the lowest possible group score happens when $|student_year_i - student_year_j| = 5$.

Then, using this fact, we can calculate that

$$\text{Group Score} = \frac{\sum_{i=0}^{n-1} \sum_{j=i+1}^{n-1} \left(1 - \frac{|student_year_i - student_year_j|}{5} \right)}{\# \text{ of elements in list of similarities}} \quad (7)$$

$$= \frac{\sum_{i=0}^{n-1} \sum_{j=i+1}^{n-1} 0}{\# \text{ of elements in list of similarities}} \quad (8)$$

$$= \frac{0}{\# \text{ of elements in list of similarities}} \quad (9)$$

$$= 0 \quad (10)$$

Notes:

- 형모야. 차분히. 한결음 더.
- 문제 안풀린다고 주저 앞지마.
- 사랑하는 내 여보 향해 가는거야.
- 보고싶은 내 여보 향해.
- 형모야 화이팅 :)

Question 6

Name	Year	College		
Priya	3	Victoria	Group 1	0
Alain	2	New		
Zoe	3	Woodsworth		
Francesco	3	Victoria		
Mohammed	4	Woodsworth	Group 2	0
Xiaoyuan	5	New		
Rohit	2	New		
Yimin	3	Trinity		
Grace	5	Woodsworth	Group 3	1
Claire	1	Woodsworth		
Kai	1	Woodsworth		

Question 7

- We need to evaluate the total score for each group.

We will do so in parts.

Part 1 (Calculating the total score of Group 1):

We need to calculate the total score of group 1.

The answer from question 4 tells us group 1 has value of $\frac{9}{10}$, and for criterion a) and has 0 for criterion b).

Since we know criterion a) has weight of 0.8 and criterion has weight of 0.2, we can conclude the total score is

$$(0.8 \times \frac{9}{10}) + (0.2 \times 0) = 0.72 \quad (1)$$

$$\approx 1 \quad (2)$$

Part 2 (Calculating the total score of Group 2):

We need to calculate the total score of group 2.

The answer from question 4 tells us group 2 has value of $\frac{2}{3}$, and for criterion a) and has 0 for criterion b).

Since we know criterion a) has weight of 0.8 and criterion has weight of 0.2, we can conclude the total score is

$$(0.8 \times \frac{2}{3}) + (0.2 \times 0) = 0.5333333 \quad (3)$$

$$\approx 1 \quad (4)$$

Part 3 (Calculating the total score of Group 3):

We need to calculate the total score of group 3.

The answer from question 4 tells us group 3 has value of $\frac{7}{15}$, and for criterion a) and has 1 for criterion b).

Since we know criterion a) has weight of 0.8 and criterion has weight of 0.2, we can conclude the total score is

$$(0.8 \times \frac{7}{15}) + (0.2 \times 1) = 0.573333 \quad (5)$$

$$\approx 1 \quad (6)$$

Correct Solution:

We need to evaluate the total score for each group.

We will do so in parts.

Part 1 (Calculating the total score of Group 1):

We need to calculate the total score of group 1.

The formula for total score tells us

$$\text{Total Score} = (\text{Score of Criterion a} \times 80) + (\text{Score of Criterion b} \times 20) \quad (1)$$

Since we know group 1 has score of $\frac{9}{10}$ for criterion a) and score of 0 for criterion b), we can conclude the total score is

$$(\frac{9}{10} \times 80) + (0 \times 20) = 72 \quad (2)$$

Part 2 (Calculating the total score of Group 2):

We need to calculate the total score of group 2.

The formula for total score tells us

$$\text{Total Score} = (\text{Score of Criterion a} \times 80) + (\text{Score of Criterion b} \times 20) \quad (3)$$

Since we know group 2 has score of $\frac{2}{3}$ for criterion a) and score of 0 for criterion b), we can conclude the total score is

$$(\frac{2}{3} \times 80) + (0 \times 20) = 53.33333 \quad (1)$$

$$\approx 53 \quad (2)$$

Part 3 (Calculating the total score of Group 3):

We need to calculate the total score of group 3.

The formula for total score tells us

$$\text{Total Score} = (\text{Score of Criterion a} \times 80) + (\text{Score of Criterion b} \times 20) \quad (3)$$

Since we know group 3 has score of $\frac{7}{15}$ for criterion a) and score of 1 for criterion b), we can conclude the total score is

$$(\frac{7}{15} \times 80) + (1 \times 20) = 57.33333 \quad (1)$$

$$\approx 57 \quad (2)$$

Question 8

- We need to calculate the lowest and highest possible total score.

We will do so in parts.

Part 1 (Calculating the highest possible total score):

We need to calculate the highest possible total score.

The formula for total score tells us that

$$\text{Total Score} = (\text{Score of Criterion a} \times 80) + (\text{Score of Criterion b} \times 20) \quad (1)$$

By observation, we can conclude the maximum total score occurs when Score of Criterion a and Score of Criterion b are both at maximum.

Because we know the maximum possible score for criterion a and b are 1, we can conclude the maximum possible total score is

$$\text{Total Score} = (1 \times 80) + (1 \times 20) \quad (2)$$

$$= 100 \quad (3)$$

Part 2 (Calculating the lowest possible total score):

We need to calculate the lowest possible total score.

The formula for total score tells us that

$$\text{Total Score} = (\text{Score of Criterion a} \times 80) + (\text{Score of Criterion b} \times 20) \quad (4)$$

By observation, we can conclude the minimum total score occurs when Score of Criterion a and Score of Criterion b are both at minimum.

Because we know the minimum possible score for criterion a and b are 0, we can conclude the minimum possible total score is

$$\text{Total Score} = (0 \times 80) + (0 \times 20) \quad (5)$$

$$= 0 \quad (6)$$

Rough Work:

We need to calculate the lowest and highest possible total score.

We will do so in parts.

Part 1 (Calculating the highest possible total score):

We need to calculate the highest possible total score.

- State the formula

The formula for total score tells us that

$$\text{Total Score} = (\text{Score of Criterion a} \times 80) + (\text{Score of Criterion b} \times 20) \quad (7)$$

- Show the maximum occurs when score of criterion a and score of criterion b is maximum

By observation, we can conclude the maximum total score occurs when Score of Criterion a and Score of Criterion b are both at maximum.

- Calculate maximum possible total score using the fact 1 is the highest possible score for both criterion a and criterion b.

Because we know the maximum possible score for criterion a and b are 1, we can conclude the maximum possible total score is

$$\text{Total Score} = (1 \times 80) + (1 \times 20) \quad (8)$$

$$= 100 \quad (9)$$

Part 2 (Calculating the lowest possible total score):

We need to calculate the lowest possible total score.

- State the formula

The formula for total score tells us that

$$\text{Total Score} = (\text{Score of Criterion a} \times 80) + (\text{Score of Criterion b} \times 20) \quad (10)$$

- Show the minimum occurs when score of criterion a and score of criterion b is minimum

By observation, we can conclude the minimum total score occurs when Score of Criterion a and Score of Criterion b are both at minimum.

- Calculate minimum possible total score using the fact 0 is the lowest possible score for both criterion a and criterion b.

Because we know the minimum possible score for criterion a and b are 0, we can conclude the minimum possible total score is

$$\text{Total Score} = (0 \times 80) + (0 \times 20) \quad (11)$$

$$= 0 \quad (12)$$

Question 9

- We need to compute the average score across all the groups.

The formula for average total score tells us

$$\text{Average Total Score} = \frac{\sum_i \text{Total Score of Group } i}{\# \text{ of Groups}} \quad (1)$$

Using the fact there are 3 groups with the total score of 72 for group 1, 53 for group 2, and 57 for group 3, we can calculate

$$\text{Average Total Score} = \frac{72 + 53 + 57}{3} \quad (2)$$

$$= \frac{182}{3} \quad (3)$$

$$\approx 60.667 \quad (4)$$

Rough Work:

We need to compute the average score across all the groups.

- State the formula of average total score

The formula for average total score tells us

$$\text{Average Total Score} = \frac{\sum_i \text{Total Score of Group } i}{\# \text{ of Groups}} \quad (5)$$

- Show the average total score using the fact group 1 has total score of 72, group 2 has total score of 53 and group 3 has total score of 57.

Using the fact there are 3 groups with the total score of 72 for group 1, 53 for group 2, and 57 for group 3, we can calculate

$$\text{Average Total Score} = \frac{72 + 53 + 57}{3} \quad (6)$$

$$= \frac{182}{3} \quad (7)$$

$$\approx 60.667 \quad (8)$$

Question 10

Rough Work:

Consider the following 3 groups.

Group 1		
Name	Year	College
Priya	3	Victoria
Zoe	3	Woodsworth
Francesco	3	Victoria
Yimin	3	Trinity

Group 2		
Name	Year	College
Alain	2	New
Rohit	2	New
Claire	1	Woodsworth
Kai	1	Woodsworth

Group 3		
Name	Year	College
Mohammed	4	Woodsworth
Xiaoyuan	5	New
Grace	5	Woodsworth

We need to show the 3 groups result in the better average total score than 60.667.

- Show the score of criterion a for each group, using the same steps from question 4

First, we need to calculate the score of criterion a for each group.

We will do so in parts.

1. Part 1 (Calculating the score of criterion a for group 1)

We need to calculate the score of criterion a for group 1.

The formula of calculating the score of criterion a tells us

$$\text{Score of Criterion a} = \frac{\sum \text{similarities}}{\# \text{ of elements in list of similarities}} \quad (9)$$

, and we know from the code we know from the code in question 4 that group 1 has the following list of similarities

$$[1, 1, 1, 1, 1, 1] \quad (10)$$

Using these facts, we can conclude

$$\text{Score of Criterion a} = \frac{1 + 1 + 1 + 1 + 1 + 1}{\# \text{ of elements in list of similarities}} \quad (11)$$

$$= \frac{1 + 1 + 1 + 1 + 1 + 1}{6} \quad (12)$$

$$= \frac{6}{6} \quad (13)$$

$$= 1 \quad (14)$$

2. Part 2 (Calculating the score of criterion a for group 2)

We need to calculate the score of criterion a for group 2.

The formula of calculating the score of criterion a tells us

$$\text{Score of Criterion a} = \frac{\sum \text{similarities}}{\# \text{ of elements in list of similarities}} \quad (15)$$

, and we know from the code we know from the code in question 4 that group 2 has the following list of similarities

$$[1, 0.8, 0.8, 0.8, 0.8, 1] \quad (16)$$

Using these facts, we can conclude

$$\text{Score of Criterion a} = \frac{1 + 0.8 + 0.8 + 0.8 + 0.8 + 1}{\# \text{ of elements in list of similarities}} \quad (17)$$

$$= \frac{\frac{5}{5} + \frac{4}{5} + \frac{4}{5} + \frac{4}{5} + \frac{4}{5} + \frac{5}{5}}{6} \quad (18)$$

$$= \frac{26}{30} \quad (19)$$

3. Part 3 (Calculating the score of criterion a for group 3)

We need to calculate the score of criterion a for group 3.

The formula of calculating the score of criterion a tells us

$$\text{Score of Criterion a} = \frac{\sum \text{similarities}}{\# \text{ of elements in list of similarities}} \quad (20)$$

, and we know from the code we know from the code in question 4 that group 3 has the following list of similarities

$$[0.8, 0.8, 1] \quad (21)$$

Using these facts, we can conclude

$$\text{Score of Criterion a} = \frac{0.8 + 0.8 + 1}{\# \text{ of elements in list of similarities}} \quad (22)$$

$$= \frac{\frac{4}{5} + \frac{4}{5} + \frac{5}{5}}{3} \quad (23)$$

$$= \frac{13}{15} \quad (24)$$

- Show the total score of each group
- Show the average total score of the 3 groups is higher than 60.667