

CSC236 Midterm 2 Version 1 Solution

Hyungmo Gu

May 11, 2020

Question 1

- Let $n, q \in \mathbb{N}$. Let $r \in \{0, 1\}$

Assume $n > 2$, and $n = 2q + r$.

I need to find a closed form for $T(2q + r)$, using repeated substitution.

Starting from $T(n)$, we have

$$T(n) = n + T(n - 2) \quad [\text{By def. since } n > 2] \quad (1)$$

$$T(2q + r) = 2q + r + T(2q + r - 2) \quad [\text{By replacing } n \text{ for } 2q + r] \quad (2)$$

$$= 2q + r + T(2(q - 1) + r) \quad (3)$$

$$\vdots \quad (4)$$

$$= \sum_{i=0}^{i=q-1} (2(q - i) + r) + T(r) \quad [\text{After } q - 1 \text{ repeatitions}] \quad (5)$$

$$= 2 \sum_{i=0}^{i=q-1} (q - i) + \sum_{i=0}^{i=q-1} r + T(r) \quad (6)$$

$$= 2 \sum_{i=0}^{i=q-1} (q - i) + \sum_{i=0}^{i=q-1} r \quad [\text{Since } T(r) = 0] \quad (7)$$

$$= 2 \sum_{i'=1}^{i=q} i' + \sum_{i=0}^{i=q-1} r \quad (8)$$

$$= 2 \sum_{i'=1}^{i'=q} i' + \sum_{i=0}^{i=q-1} r \quad (9)$$

$$= 2 \sum_{i'=1}^{i'=q} i' + \sum_{i=0}^{i=q-1} r \quad (10)$$

$$= 2(q(q + 1))/2 + \sum_{i=0}^{i=q-1} r \quad [\text{By using } \sum_{i=1}^{i=n} i = (n(n + 1))/2] \quad (11)$$

$$= q(q + 1) + rq \quad (12)$$

$$= q(q + 1 + r) \quad (13)$$