Problem Set 3 Solution

March 22, 2020

Question 1

1. Let $x \in \mathbb{R}$.

   Base Case (n = 0):

   Let $n = 0$.

   Then,

   $$a_0 = 0 \tag{1}$$

   Then it follows from above that the base case holds.

   Inductive Case ($n > 0$):

   Let $k \in \mathbb{N}$, and assume $a_n = x \prod_{i=0}^{n-1} a_i$.

   Then,

   $$\begin{aligned} x \prod_{i=0}^{n-1} a_i \cdot a_n &= x \prod_{i=0}^{n} a_i && (1) \\ &= a_{n+1} && (2) \end{aligned}$$

Then it follows from above that the recursive sequence of numbers is true for all natural numbers.

2. From the following table

String Length	Number of Even (Digit Sum)	Number of Odd (Digit Sum)	Total
1	2	1	3
2	5	4	9
3	14	13	27

we see that $E_n = \frac{3^n+1}{2}$ and $O_n = \frac{3^n-1}{2}$.

Question 2

Question 3

Question 4