Worksheet 17 Solution

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Question 1

a. We need to determine $|\mathcal{I}_n|$.

The problem tells that the values in inputs are either 1 or 0, and we know $\mathcal{I}_n$ represents all possible inputs of size n containing binary values.

After watching lecture videos, and reading notes, I do not yet understand the details of how to evaluate the $\mathcal{I}_n$, but from the pattern below

$$[0], [1], [1, 0], [0, 1], [1, 1], [0, 0], [0, 0, 0], [0, 0, 1], [0, 1, 0], [1, 0, 0], [1, 1, 0], [1, 0, 1], [0, 1, 1], [1, 1, 1]$$

we can see the inputs of size 1 have 2 different inputs, the inputs of size 2 have 4 different inputs, and the inputs of size 3 have 8 different inputs.

Using this pattern, I can make an educated guess that $|\mathcal{I}_n| = 2^n$.

Notes:

- The idea of average-case analysis is that some data structures and algorithms have poor worst-case performance but perform well in vast majority of others.
- Average-case analysis looks at running time on sets of inputs
- Average case: $AVG_{func}(n) = avg\{\text{runtime of func(x)} \mid x \in \mathcal{I}_n\}$
- Worst case: $WC_{func}(n) = max\{\text{runtime of func(x)} \mid x \in \mathcal{I}_n\}$

b.

n	i	Sets	$\lvert S_{n,i}\rvert$
2	0	$\{[0]\}$	1
2	0	$\{[0, 1], [0, 0]\}$	2
2	1	$\{[1, 0]\}$	1
3	0	$\{[0, 1, 1], [0, 0, 1], [0, 0, 0]\}$	3
3	1	$\{[1, 0, 1], [1, 0, 0]\}$	2
3	2	$\{[1, 1, 0]\}$	1

By the pattern outlined above, we can deduce that $|S_{n,i}| = n - i$.

Correct Solution:

n	i	Sets	$\lvert S_{n,i}\rvert$
1	0	$\{[0]\}$	1
2	0	$\{[0,1],[0,0]\}$	2
2	1	$\{[1,0]\}$	1
3	0	$\{[0,1,1],[0,0,1],[0,1,0],[0,0,0]\}$	4
3	1	$\{[1,0,1],[1,0,0]\}$	2
3	2	$\{[1,1,0]\}$	1

By the pattern outlined above, we can deduce that $|S_{n,i}| = 2^{n-i-1}$.

c. We will prove the statement informally using proof by cases.

Case 1 (when list doesn't have 0s):

The definition of $S_{n,i}$ tells us $0 \leq i \leq n$, $S_{n,i}$ contains all lists with 0 starting at ith position.

Using the fact, we can conclude $S_{n,n}$ is a set of lists containing 0 at n^{th} position.

Since i in a list starts at $i = 0$ and ends at $i = n-1$, there are no 0 in the list of a set $S_{n,n}$.

Then, we can conclude $S_{n,n}$ is one and the only that contains a list of only 1s.

Case 2 (when list has one or more 0s):

Since this list has 0 starting at i^{th} position, we can conclude this list exists in the set $S_{n,i}$.

Attempt 2:

We will prove the statement informally using proof by cases.

Case 1 (when list doesn't have 0s):

The definition of $S_{n,i}$ tells us $0 \leq i \leq n$, $S_{n,i}$ contains all lists with 0 starting at ith position.

Using the facts, we can conclude $S_{n,n}$ is a set of lists containing 0 at n^{th} position.

Since i in a list starts at $i = 0$ and ends at $i = n-1$, there are no 0 in the list of a set $S_{n,n}$.

Then, we can conclude $S_{n,n}$ is one and the only that contains a list of only 1s.

Case 2 (when list has one or more 0s):

The definition of $S_{n,i}$ tells us, the set $S_{n,i}$ contains all lists with 0 starting at i^{th} position.

Because we know this list has 0 starting at i^{th} position, using the fact, we can conclude this list exists in the set $S_{n,i}$.