

# CSC148 Worksheet 11 Solution

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April 22, 2020

## Question 1

a. Here, the constant time means the running time of accessing and assigning element by index doesn't depend on the length of the list.

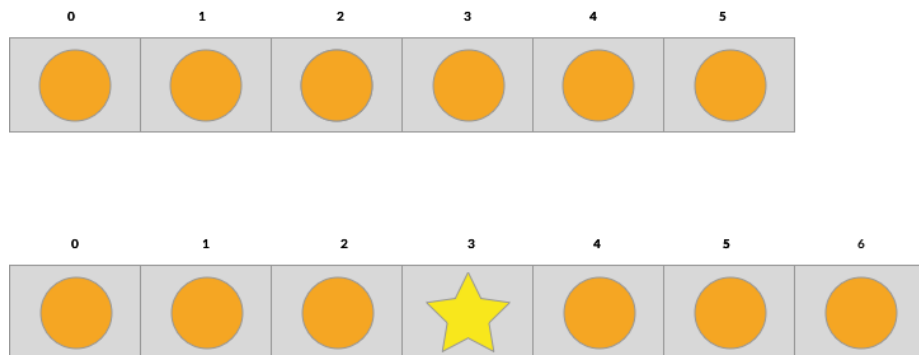
b.

$$n - i$$

many elements need to be shifted to right.

### Notes:

- The following example tells us



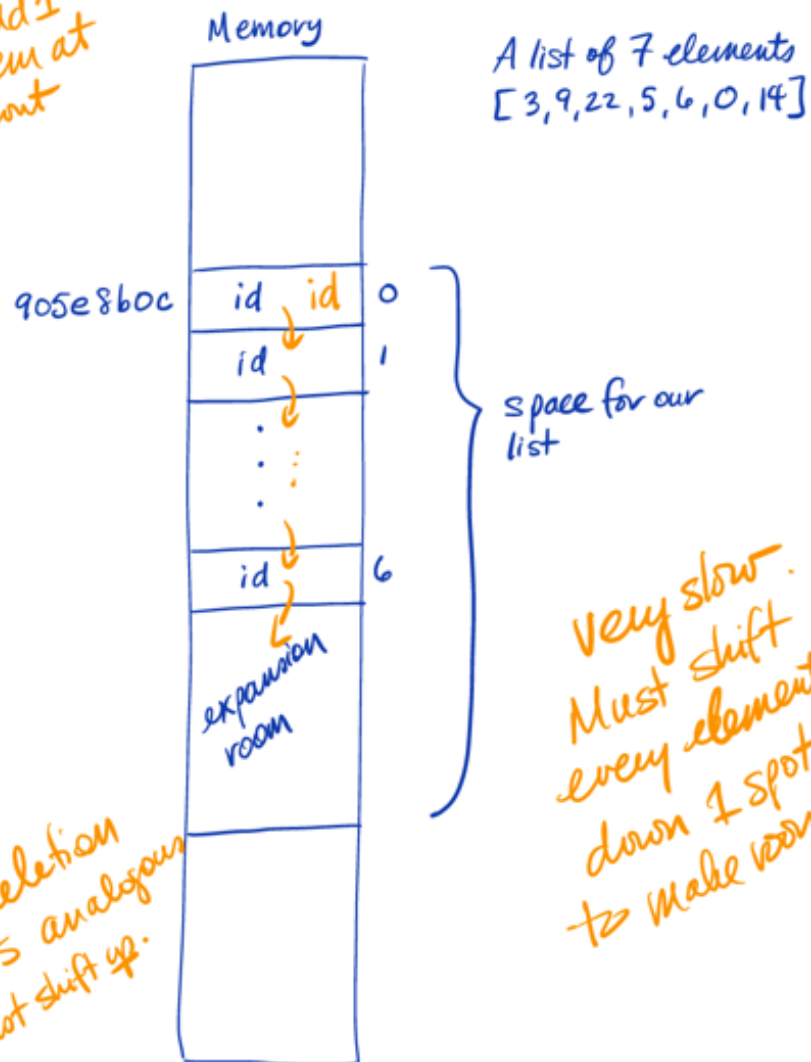
to position an element at index  $i = 3$  of the list,  $n - i = 6 - 3 = 3$  elements must be moved over.

Using this fact, we can generalize that to position an element at index  $i$  of the list,  $n - i$  many elements must be shifted.

- Learned that when items shifts, it shifts into the expansion room.

## Updates at the front of our list

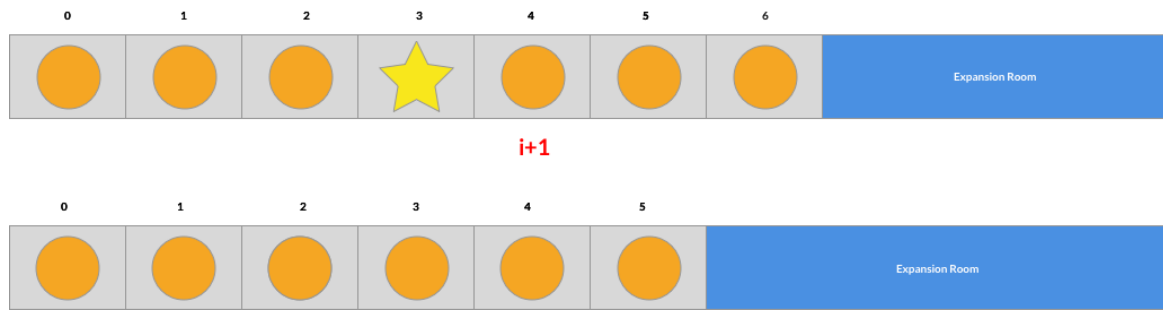
Add 1  
item at  
front



- c. Because we know the list size stays as is when an element is removed, we can conclude 0 many list elements must be moved.

### Correct Solution:

The following example tells us



when an element at index  $i = 3$  is removed from the list  $n - (i + 1) = 7 - (3 + 1) = 3$  many elements must be moved.

Using this fact, we can generalize that when an element is removed,  $n - (i + 1) = n - i - 1$  many elements must be shifted to left.

- d. i. A solution is *LIST.remove(...)*.

The answer to question 1.d tells us when an element is removed,  $n - i$  must be shifted to left.

Using this fact, we can write a list of smaller size needs to shift elements less.

Then, it follows from this fact that  $n = 100$  works faster than  $n = 1,000,000$ .

- ii. A solution is *LIST.append(...)*

The definition of append tells us that upon call, an element is added to the end of a list, it takes a constant time to add an element as long as the expansion room is not filled.

It follows from this fact that  $n = 100$  and  $n = 1,000,000$  takes roughly the same amount of time.

- e. The definition of *Queue* tells us *Queue* is FIFO. That is, the first element inserted is the first to come out.

Since we know the front of *Queue* is the front of the list, we can conclude *LIST.insert(...)* and *LIST.pop(0)* are used to support *QUEUE.enqueue* and *QUEUE.dequeue*, respectively.

Since we know *LIST.insert(...)* requires shifting of elements by  $n - i = n - 0 = n$  and *LIST.pop(0)* requires shifting of  $n - i - 1 = n - 1$  many elements, we can conclude both *QUEUE.enqueue* and *QUEUE.dequeue* takes longer time as size increases.

### Correct Solution:

The definition of *Queue* tells us *Queue* is FIFO. That is, the first element inserted is the first to come out.

Since we know the front of *Queue* is the front of the list, we can conclude *LIST.append(...)* and *LIST.pop(0)* are used to support *QUEUE.enqueue* and *QUEUE.dequeue*, respectively.

Since we know *LIST.append(...)* requires the shifting of elements by  $n - i = n - n = 0$  and *LIST.pop(0)* requires shifting of  $n - i - 1 = n - 1$  many elements, we can conclude *QUEUE.dequeue* takes longer time as size increases.

### Notes:

- Learned that the **front of queue** is where *QUEUE.dequeue* occurs.



## Question 2

a. Stack 1:

$$\begin{aligned}\text{Number of steps for } s.\text{push}(1) + \text{Number of steps for } s.\text{pop}() &= 1 + 1 \\ &= 2\end{aligned}$$

Stack 2:

$$\begin{aligned}\text{Number of steps for } s.\text{push}(1) + \text{Number of steps for } s.\text{pop}() &= (n + 1) + (n + 1) \\ &= 2n + 1\end{aligned}$$

b. Stack 1:

We need to determine the total number of steps taken in the code using *Stack 1*.

First, we need to find the number of steps taken per iteration.

The code tells us there is only  $s.push(i)$  in for loop.

Since we know the *push* operation in *Stack 1* takes 1 step, we can conclude 1 step is taken per iteration.

Finally, we need to determine the total number of steps taken.

Because we know the loop performs 5 iterations and each iteration takes 1 step, we can conclude the code takes total of

$$5 \cdot 1 = 5 \tag{1}$$

steps.

### **Stack 2:**

We need to determine the total number of steps taken in the code using *Stack 2*.

First, we need to find the number of steps taken per iteration.

The code tells us there is only  $s.push(i)$  in for loop.

Since we know the list in  $s$  starts empty, and the size of list grows to  $i$  at  $i^{th}$  iteration, we can conclude the size of list  $n$  is  $i$ .

Since we know the *push* operation in *Stack 2* takes  $n + 1$  step, and since we know  $n = i$ , we can conclude  $i + 1$  step is taken per iteration.

Finally, we need to determine the total number of steps taken.

Because we know the loop starts at  $i = 0$ , ends at  $i = 4$  with each iteration taking  $i + 1$  steps, we can conclude the code has total of

$$\sum_{i=0}^4 (i + 1) = \sum_{i'=1}^5 i' \tag{1}$$

$$= \frac{5(5 + 1)}{2} \tag{2}$$

$$= \frac{30}{2} \tag{3}$$

$$= 15 \tag{4}$$

steps.

c. **Stack 1:**

We need to determine the total number of steps taken in the code using *Stack 1*.

First, we need to find the number of steps taken per iteration.

The code tells us there is only  $s.push(i)$  in for loop.

Since we know the *push* operation in *Stack 1* takes 1 step, we can conclude 1 step is taken per iteration.

Finally, we need to determine the total number of steps taken.

Because we know the loop performs  $(k - 1) - 0 + 1 = k$  iterations and each iteration takes 1 step, we can conclude the code takes total of

$$k \cdot 1 = k \tag{5}$$

steps.

**Stack 2:**

We need to determine the total number of steps taken in the code using *Stack 2*.

First, we need to find the number of steps taken per iteration.

The code tells us there is only  $s.push(i)$  in for loop.

Since we know the list in  $s$  starts empty, and the size of list grows to  $i$  at  $i^{th}$  iteration, we can conclude the size of list  $n$  is  $i$ .

Since we know the *push* operation in *Stack 2* takes  $n + 1$  step, and since we know  $n = i$ , we can conclude  $i + 1$  step is taken per iteration.

Finally, we need to determine the total number of steps taken.

Because we know the loop starts at  $i = 0$ , ends at  $i = k - 1$  with each iteration taking  $i + 1$  steps, we can conclude the code has total of

$$\sum_{i=0}^{k-1} (i + 1) = \sum_{i'=1}^k i' \tag{1}$$

$$= \frac{k(k + 1)}{2} \tag{2}$$

steps.

### Correct Solution:

#### Stack 1:

We need to determine the total number of steps taken in the code using *Stack 1*.

First, we need to find the number of steps taken per iteration.

The code tells us there is only  $s.push(i)$  in for loop.

Since we know the *push* operation in *Stack 1* takes 1 step, we can conclude 1 step is taken per iteration.

Finally, we need to determine the total number of steps taken.

Because we know the loop performs  $(k - 1) - 0 + 1 = k$  iterations and each iteration takes 1 step, we can conclude the code takes total of

$$k \cdot 1 = k \tag{1}$$

steps.

#### Stack 2:

We need to determine the total number of steps taken in the code using *Stack 2*.

First, we need to find the number of steps taken per iteration.

The code tells us there is only  $s.push(i)$  in for loop.

Since we know the list in  $s$  starts empty, and the size of list grows to  $i$  at  $i^{th}$  iteration, we can conclude the size of list  $n$  is  $i$ .

Since we know the *push* operation in *Stack 2* takes  $n + 1$  step, and since we know  $n = i$ , we can conclude  $i + 1$  step is taken per iteration.

Finally, we need to determine the total number of steps taken.

Because we know the loop starts at  $i = 0$ , ends at  $i = k - 1$  with each iteration taking  $i + 1$  steps, we can conclude the code has total of

$$\sum_{i=0}^{k-1} (i+1) = \sum_{i'=1}^k i' \quad (1)$$

$$= \frac{k(k+1)}{2} \quad (2)$$

steps.

d. **Stack 1:**

We need to determine the total number of steps taken in the code using *Stack 1*.

First, we need to determine the number of steps taken by  $s2.push(s1.pop())$ .

The problem tells us both  $s2.push(...)$  and  $s1.pop()$  takes 1 step.

Using this fact, we can conclude  $s2.push(s1.pop())$  takes 1 step.

Second, we need to determine the total number of steps taken by loop 1.

The code tells us that for loop 1, an element popped from  $s1$  is inserted to  $s2$ , and this is repeated until no elements are left in  $s1$ .

Since we know  $s1$  starts with  $n$  many elements, and its size decreases by 1 until its length is 0, we can write that with  $k$  representing  $k^{th}$  iteration in loop 1, the terminating condition is reached when

$$n - k \leq 0 \quad (1)$$

$$n \leq k \quad (2)$$

Since we are looking for the smallest value of  $k$  (because it represents the number of iterations), we can conclude loop 1 has

$$\lceil n \rceil = n \quad (3)$$

many iterations.

Since we know each iteration in loop 1 takes 1 step, we can conclude loop 1 takes total of



$$n \cdot 1 = n \tag{4}$$

steps.

Third, we need to show that the second loop takes total of 0 steps.

The code tells us  $s2$  starts a stack of size 0.

Since loop 2 terminates when the size of  $s2$  is 0, we can conclude loop 2 has 0 iterations.

Using this fact, we can conclude loop 2 has total of 0 steps.

Finally, we need to determine the total number of steps taken in this code.

Since we know loop 1 takes total of  $n$  steps and loop 2 takes total of 0 step, we can conclude the total number of steps taken by the code is

$$n + 0 = n \tag{5}$$

steps.

### **Rough Work:**

We need to determine the total number of steps taken in the code using *Stack 1*.

1. Determine the number of steps taken by  $s2.push(s1.pop())$

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The problem tells us both  $s2.push(...)$  and  $s1.pop()$  takes 1 step.

Using this fact, we can conclude  $s2.push(s1.pop())$  takes 1 step.

2. Determine the total number of steps taken by loop 1.

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The code tells us that for loop 1, an element popped from  $s1$  is inserted to  $s2$ , and this is repeated until no elements are left in  $s1$ .

Since we know  $s1$  starts with  $n$  many elements, and its size decreases by 1 until its length is 0, we can write that with  $k$  representing  $k^{th}$  iteration in loop 1, the terminating condition is reached when

$$n - k \leq 0 \tag{6}$$

$$n \leq k \tag{7}$$

Since we are looking for the smallest value of  $k$  (because it represents the number of iterations), we can conclude loop 1 has

$$\lceil n \rceil = n \tag{8}$$

many iterations.

Since we know each iteration in loop 1 takes 1 step, we can conclude loop 1 takes total of

$$n \cdot 1 = n \tag{9}$$

steps.

3. Show that the second loop takes total of 0 steps.

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The code tells us  $s2$  starts a stack of size 0.

Since loop 2 terminates when the size of  $s2$  is 0, we can conclude loop 2 has 0 iterations.

Using this fact, we can conclude loop 2 has total of 0 steps.

4. Calculate the total number of steps.

Finally, we need to determine the total number of steps taken in this code.

Since we know loop 1 takes total of  $n$  steps and loop 2 takes total of 0 step, we can conclude the total number of steps taken by the code is

$$n + 0 = n \tag{10}$$

steps.

**Stack 1:**

We need to determine the total number of steps taken in the code using *Stack 1*.

First, we need to determine the number of steps taken by  $s2.push(s1.pop())$ .

The problem tells us both  $s2.push(...)$  and  $s1.pop()$  takes 1 step.

Using this fact, we can conclude  $s2.push(s1.pop())$  takes 1 step.

Second, we need to determine the total number of steps taken by loop 1.

The code tells us that for loop 1, an element popped from  $s1$  is inserted to  $s2$ , and this is repeated until no elements are left in  $s1$ .

Since we know  $s1$  starts with  $n$  many elements, and its size decreases by 1 until its length is 0, we can write that with  $k$  representing  $k^{th}$  iteration in loop 1, the terminating condition is reached when

$$n - k \leq 0 \quad (11)$$

$$n \leq k \quad (12)$$

Since we are looking for the smallest value of  $k$  (because it represents the number of iterations), we can conclude loop 1 has

$$\lceil n \rceil = n \quad (13)$$

many iterations.

Since we know each iteration in loop 1 takes 1 step, we can conclude loop 1 takes total of

$$n \cdot 1 = n \quad (14)$$

steps.

Third, we need to show that the second loop takes total of 0 steps.

The code tells us  $s2$  starts a stack of size 0.

Since loop 2 terminates when the size of  $s2$  is 0, we can conclude loop 2 has 0 iterations.

## Stack 2:

### Rough Work:

We need to determine the total number of steps taken in the code using *Stack 2*.

1. Determine the number of steps taken by  $s2.push(s1.pop())$  in loop 1.

First, we need to determine the number of steps taken by  $s2.push(s1.pop())$  in loop 1.

1. State that  $s1.pop()$  takes  $n_1 + 1$  steps and  $s2.push(...)$  takes  $n_2 + 1$ .
  2. Show that  $s1.pop()$  takes  $(n - k) + 1$  steps and  $s2.push(...)$  takes  $k + 1$ .
  3. Show that  $s2.push(s1.pop())$  takes  $i + 2$  steps.
2. Determine the total number of steps taken by loop 1.
    1. State that for loop 1, an element popped from  $s1$  is inserted to  $s2$ , and this is repeated until no elements are left in  $s1$ .
    2. Show loop 1 takes  $n$  iterations.
    3. Show loop 1 takes total of  $\frac{(n + 2)(n + 3)}{2} - 3$  steps.
  3. Show that the total number of steps taken by loop 2 is 0.
  4. Conclude by calculating the total number of steps taken in this code.