

Worksheet 8 Solution

March 17, 2020

Question 1

a. $P(n) : \forall n \in \mathbb{N}, n \leq 2^n$.

$$\forall k \in \mathbb{N}, P(0) \wedge P(k) \Rightarrow P(k+1)$$

Or, with P fully expanded,

$$\forall k \in \mathbb{N}, 0 \leq 2^0 \wedge k \leq 2^k \Rightarrow k+1 \leq 2^{k+1}$$

b. **Base Case:**

Let $n = 0$.

Then,

$$(0) \leq 2^0 \tag{1}$$

$$0 \leq 1 \tag{2}$$

Since, $n \leq 2^n$ is true for $n = 0$, the base case holds.

Inductive Case:

Let $k \in \mathbb{N}$, and assume that $P(k)$ is true.

Then,

$$2^{k+1} = 2^k + 2^k \tag{1}$$

$$\geq k + k \tag{2}$$

$$\tag{3}$$

Then,

$$2^{k+1} \geq k + k \tag{4}$$

$$\geq k + 1 \tag{5}$$

by the fact that $k \in \mathbb{N}$ and $k \geq 1$.

Then, it follows from proof by induction that the statement $k \leq 2^k$ is true.

Question 2

Question 3

Question 4