

Worksheet 4 Solution

March 13, 2020

Question 1

- a. $\exists n \in \mathbb{N}, (n > 3) \wedge (n^2 - 1.5n \geq 5)$
- b. The variable is existentially quantified
- c. Concrete natural number
- d. Let $n = 5$.

Then,

$$(5)^2 - 1.5(5) \tag{1}$$

Then,

$$25 - 7.5 \tag{2}$$

Then,

$$17.5 \tag{3}$$

which is greater than 5. So, the statement is True

- e. $\forall n \in \mathbb{N}, n > 3 \Rightarrow n^2 - 1.5n > 4$

Here \Rightarrow should be used because $n > 3$ is a given, and we are using it to show that the statement $n^2 - 1.5n > 4$ is True

- f. The variable is universally quantified
- g. In this proof the variable must be **arbitrary** natural number
- h. The assumption made is that the any natural number greater than 3 satisfies the statement $n^2 - 1.5n > 4$.

This assumption is made since the predicate logic is the proof of an implication

- i. Let $n \in \mathbb{N}$ be an arbitrary number of \mathbb{N} , and assume $n > 3$. Then,

$$n^2 > 3n \tag{1}$$

$$n^2 - 1.5n > 3n - 1.5n \tag{2}$$

$$n^2 - 1.5n > 1.5n \tag{3}$$

Because we know that $n > 3$, we can conclude

$$n^2 - 1.5n > 1.5(3) \tag{4}$$

$$n^2 - 1.5n > 4.5 \tag{5}$$

It follows that the statement $\forall n \in \mathbb{N}, n > 3 \Rightarrow n^2 - 1.5n > 4$ is true.

Question 2

- a. $\forall n \in \mathbb{N}, n > 5 \Rightarrow 2 \mid n \vee 3 \mid n$
- b. $\exists n \in \mathbb{N}, (n > 5) \wedge (2 \nmid n \vee 3 \nmid n)$
- c. Let $n = 7$.

Since 7 is a prime number, 7 is not divisible by both 2 and 3.

It follows from the above that the original statement $\forall n \in \mathbb{N}, n > 5 \Rightarrow 2 \mid n \vee 3 \mid n$ is False

Question 3

- a. Let x be an arbitrary number of \mathbb{R} . Let $y = 165 - x + 1$
- b. Let $y = 166$. Let x be an arbitrary number of \mathbb{N}
- c. Let $x = 164 - y$. Let y be an arbitrary number of \mathbb{R} . Then,

$$x + y > 165 \tag{6}$$

$$(164 - y) + y > 165 \tag{7}$$

$$164 > 165 \tag{8}$$

It follows from the above that $\exists y \in \mathbb{R}, \forall x \in \mathbb{R}, x + y > 165$ is False