A Multi-state Markov Model to Infer the Latent Deterioration Process From the Maintenance Effect on Reliability Engineering of Ships

Target: Journal of applied statistics

ABSTRACT

Abstract

Maintenance optimization of naval ship equipment is crucial in terms of national defense. However, the mixed effect of the maintenance and the pure deterioration processes in the observed data hinders an exact comparison between candidate maintenance policies. That is, the observed data-annual failure counts of naval ships reflect counteracting actions between the maintenance and deterioration. The inference of the latent deteriorating process is needed in advance for choosing an optimal maintenance policy to be carried out. This study proposes a new framework for the separation of the true deterioration effect by predicting it from the current maintenance effect through the multi-state Markov model. Using an annual engine failure count of 99 ships in the Korean navy, we construct the framework consisting of imputation, transition matrix design, optimization, and validation. The hierarchical Gaussian process model is used for the imputation and the three-state Markov model is applied for the estimation of parameters in the deterioration and maintenance effect. To consider the natural (deterioration) and artificial (maintenance) effect respectively, the Bayesian HMM model with a categorical distribution is employed. Computational experiments under multiple settings showed the robustness of the estimated parameters, as well as an accurate recovery of the observed data, thereby confirming the credibility of our model. The framework could further be employed to establish a reliable maintenance system and to reduce an overall maintenance cost.

전면 수정 필요

Introduction

1 Introduction

A maintenance policy is crucial both in terms of the safety and efficiency in managing naval ships' equipment. Maintenance policy includes several controllable variables to be determined, such as an inspection frequency or an acceptable maintenance standard. Too long inspection interval or overly lenient standard for the repair would result in an unstable system, and the resulting failure costs will be markedly increased. On the other hand, strict maintenance with frequent inspections and excessively conservative standards would yield an excellent budget waste.

전면 수정 필요

Problem?
Method?
선행연구 관련해서 뭐가 novel한지??
왜 이 연구 필요한지?

Figure 1

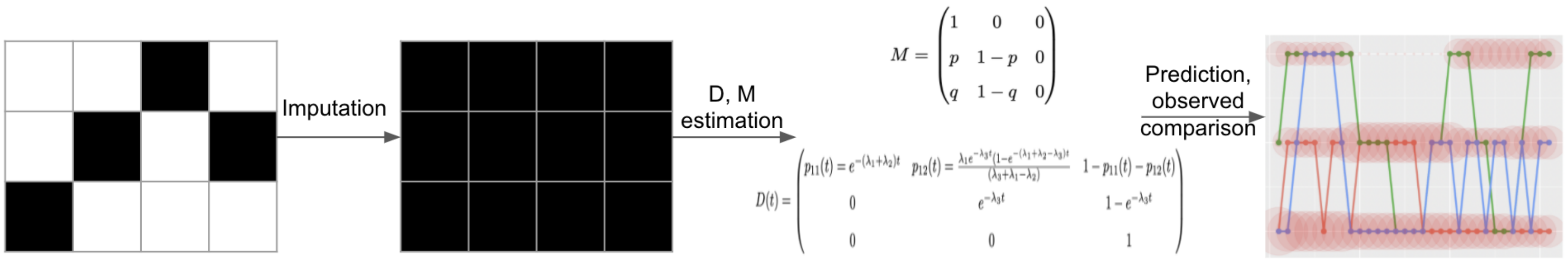


Figure 1: Process of the proposed model.

다시 고화질로 예쁘게 만들기

Figure 2

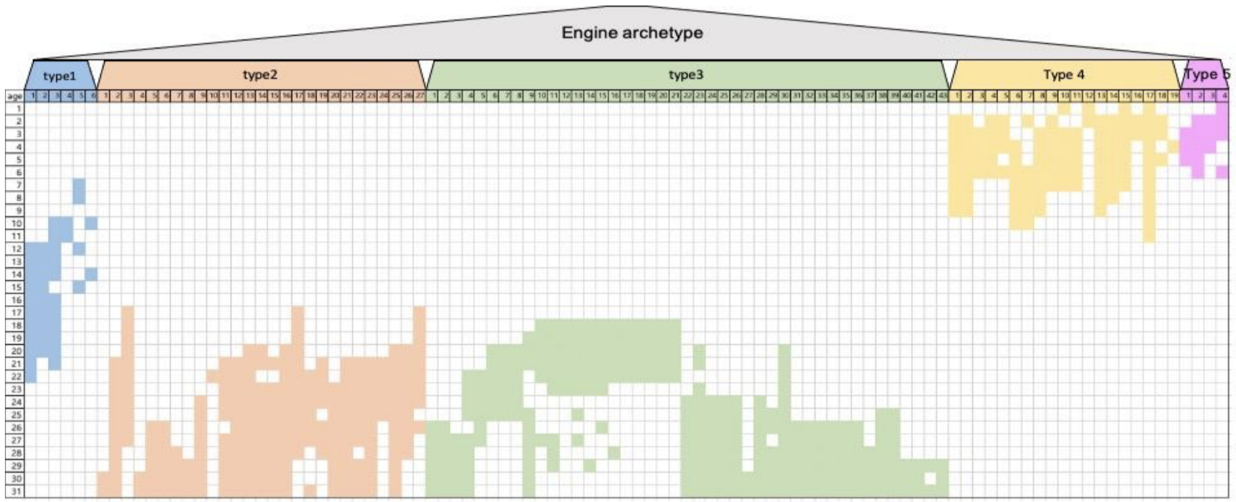
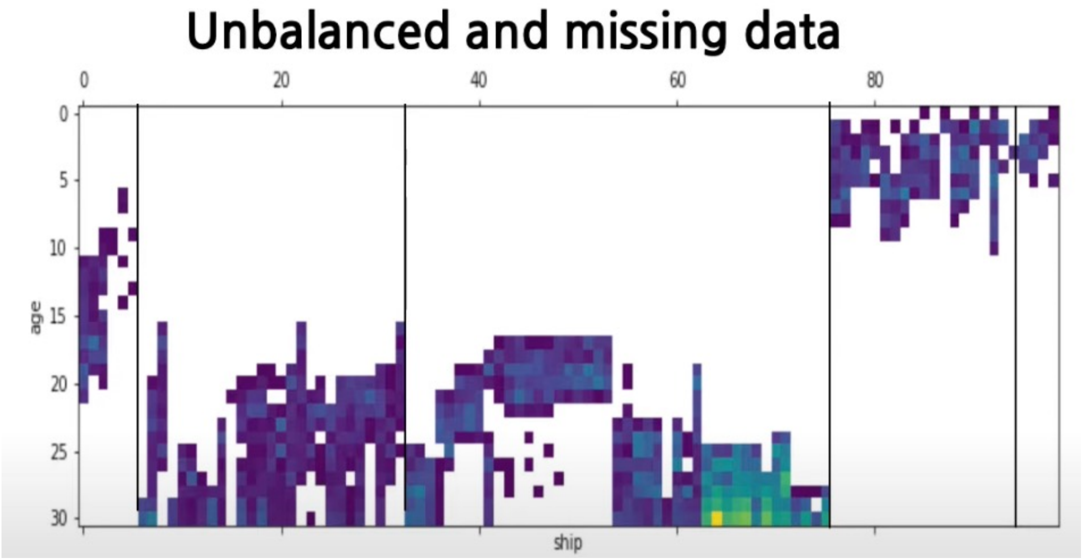


Figure 2: Overview of the failure counts from the 99 Korean naval ships.



variances of failure counts between each engine type look quite distinct. For example, while engine types 4 and 5 show a large variance, type 3 shows a small variance of the given data.

??? 분산, 평균이 어떻게 engine type마다 다른지 알 수 없음

데이터 값 (평균, 분산)을 알려줄 수 있는 다른 형태로 예쁘게 figure 만들기
Q. Data privacy 가능한 선에서 최대한 보여줄 수 있으면 좋을 것 같습니다. 실제 값이 아닌 대략적인 형태라도!

Idea) Real data / Imputed data를 섞어서 같이 보여줄 수 있으면 더 좋을 듯. Imputation 결과를 보여주기 위해

Imputation using GP

$$y_{t,j} \sim \mathcal{N}(\mu_{t,j}, \sigma^2_{k[j]}) \tag{1}$$

Mean $\mu_{t,j}$ is defined as an additive Gaussian process which is the sum of variables with normal distribution and Gaussian processes. μ is an overall mean of $\mu_{t,j}$, and $\theta_t^{age}, \theta_j^{ship}, and \theta_{k[j]}^{engine}$ follow a normal distribution with zero mean and variance $\sigma^2_{age}, \sigma^2_{ship}, and \sigma^2_{engine}$, respectively.

$$\begin{aligned} \mu_{t,j} &= \mu + \theta_t^{\ age} + \theta_j^{\ ship} + \theta_{k[j]}^{\ engine} + \gamma_{t,j} + \delta_{t,\ k[j]} \\ t &= 1, 2 \ldots T, \;\; j = 1, 2, \ldots N \end{aligned} \tag{2}$$

$$\begin{aligned} \theta_t^{\ age} &\sim \mathcal{N}(0, \sigma^2_{age}\ ) \\ \theta_j^{\ ship} &\sim \mathcal{N}(0, \sigma^2_{ship}\ ) \\ \theta_k^{\ engine} &\sim \mathcal{N}(0, \sigma^2_{engine}\ ) \end{aligned} \tag{3}$$

$$\begin{aligned} \gamma_j &\sim \mathcal{N}(\mathbf{0}, \boldsymbol{K}_{l^\gamma, \alpha^\gamma}) \\ \delta_k &\sim \mathcal{N}(\mathbf{0}, \boldsymbol{K}_{l^\delta, \alpha^\delta}) \end{aligned} \tag{4}$$

(위 GP 모델을 그대로 유지할 경우, estimated parameter 값 데이터 필요

Continuous time multi-state Markov model

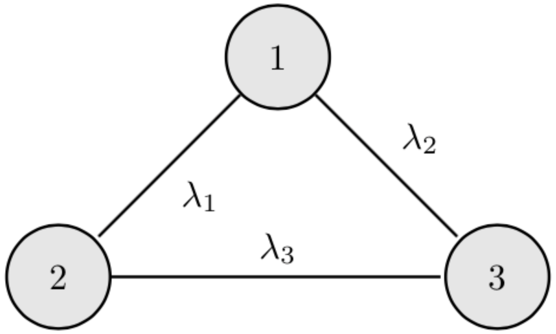


Figure 3: Deterioration rate between three states.

Table 1: Deterioration state of ship equipment according to the standardized annual failure counts.

State	Status Description	Annual failure counts
1	Normal	[-1.8304, -0.3340)
2	Near Failure	[-0.3340, -0.0703)
3	Failure	[-0.0703, 2.1273]

바로 이해하기 쉽게 예쁘게

Continuous time multi-state Markov model

between states follows a continuous-time Markov process, and transition probabilities only depend on a present state. For a time-homogeneous Markov chain, we can write the transition probability function from state i at time s to state j at time t as $p_{ij}(s,t) = Pr\{Y(t) = j|Y(s) = i\}$. The transition probabilities between all possible pairs (i,j) are represented by a $n \times n$ matrix called the transition probability matrix $P(s,t)$, where n is the number of possible condition states as shown in 9.

$$P(s,t) = \begin{pmatrix} p_{11}(s,t) & p_{12}(s,t) & p_{13}(s,t) \\ p_{21}(s,t) & p_{22}(s,t) & p_{23}(s,t) \\ p_{31}(s,t) & p_{32}(s,t) & p_{33}(s,t) \end{pmatrix} \tag{9}$$

To separate the effect of the deterioration and the maintenance, we first define a deterioration matrix $D(t)$, which is a function at the age t. Deterioration matrix $D(t)$ should be an upper-diagonal matrix reflecting the reality that engines continuously undergo degradation through the life cycle. Also, since the extent of the deterioration varies between different age ranges, it is constructed in a time-inhomogeneous manner. Older engines are prone to be readily weakened, and it's reasonable to design a more severe deterioration matrix $D(t)$ for them. Equation 11 shows the deterioration rate matrix $Q(t)$ for the three-state Markov model. This transition rate matrix requires three distinct parameters for each time t, and is also upper-diagonal, considering that the deterioration process can only occur in a forward direction.

$R_D(t) \leftarrow$

$Q_D(t)$

$$\cancel{Q(t)} = \begin{pmatrix} -(\lambda_{t,1} + \lambda_{t,2}) & \lambda_{t,1} & \lambda_{t,2} \\ 0 & -\lambda_{t,3} & \lambda_{t,3} \\ 0 & 0 & 0 \end{pmatrix} \tag{11}$$

As in equation 12, transition probabilities of the *deterioration matrix* $D(t)$ can be calculated solely with the given rate matrix $Q(t)$, which comes from the solution of Kolmogorov's forward and backward equation. Each element in $3 \times$

In the same way, we define a *maintenance matrix* M as in 14, which is a transition probability matrix with two parameters. The maintenance matrix M is multiplied to the state probability vector whenever the maintenance is performed. The construction of this maintenance matrix is based on several circumstantial assumptions as follows. First, maintenance interval is once a year. Second, an imperfect maintenance makes the transition probability from state 2 to be divided into state 1 and 2, which is p_{21} and $1 - p_{21}$ for certain probability. Likewise, the transition from state 3 could be parameterized with p_{31}, p_{32}, and $1 - p_{31} - p_{32}$. However, the preliminary experiment results showed that $1 - p_{31} - p_{32}$ term mostly converged to 0, we assumed it as 0 and replaced the probability p_{31}, p_{32} with p_{31} and $1 - p_{31}$, discarding the redundant parameter p_{32}.

$$M = \begin{pmatrix} 1 & 0 & 0 \\ p_{21} & 1 - p_{21} & 0 \\ p_{31} & 1 - p_{31} & 0 \end{pmatrix} \tag{14}$$

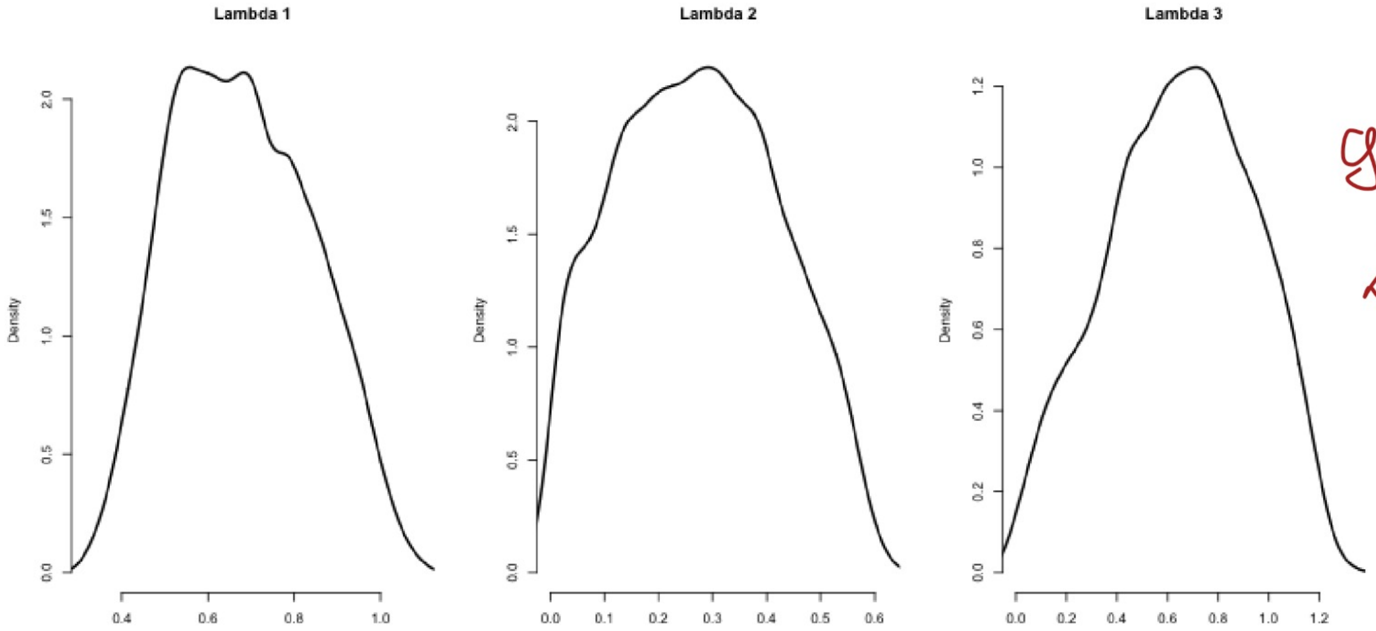
1. 각 matrix 이름을 겹치지 않게, 알아보기 쉽게 바꿀 것
2. 가장 중요한 modelling 부분인데, 잘 organized writing 하기

Continuous time multi-state Markov model

state 2 to be divided into state 1 and 2, which is p_{21} and $1 - p_{21}$ for certain probability. Likewise, the transition from state 3 could be parameterized with p_{31}, p_{32}, and $1 - p_{31} - p_{32}$. However, the preliminary experiment results showed that $1 - p_{31} - p_{32}$ term mostly converged to 0, we assumed it as 0 and replaced the probability p_{31}, p_{32} with p_{31} and $1 - p_{31}$, discarding the redundant parameter p_{32}.

Preliminary experiment 결과는 어디에? 데이터로 보여줘야 한다. (Supplementary 자료로)

Result



graph 합치기
with
색깔 구분

합치기

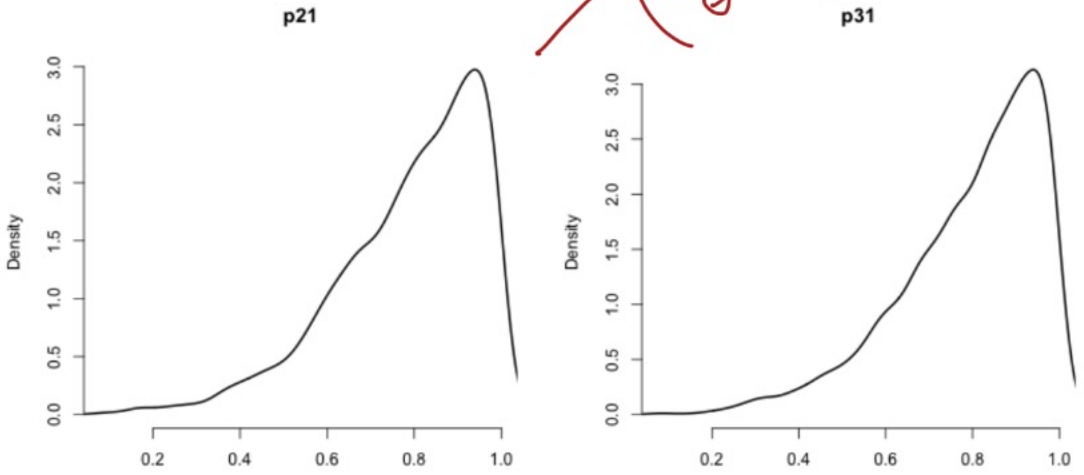
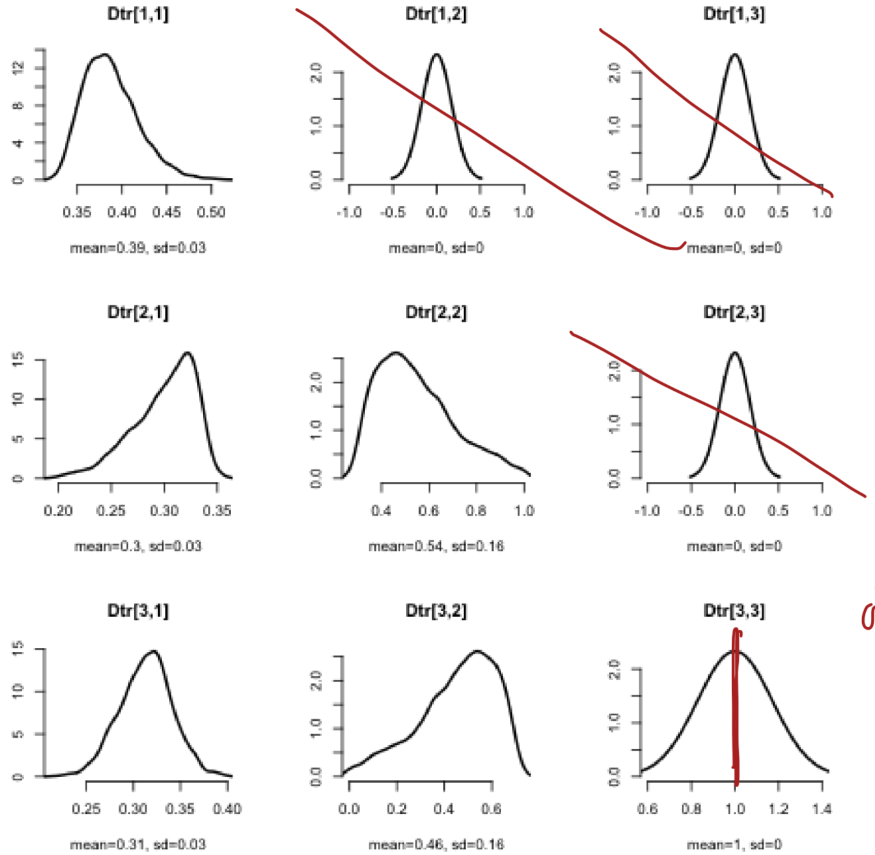


Figure 6: Resulting sampling results of the maintenance parameters: p_{21} and p_{31}.

Result



How 예쁘게…?

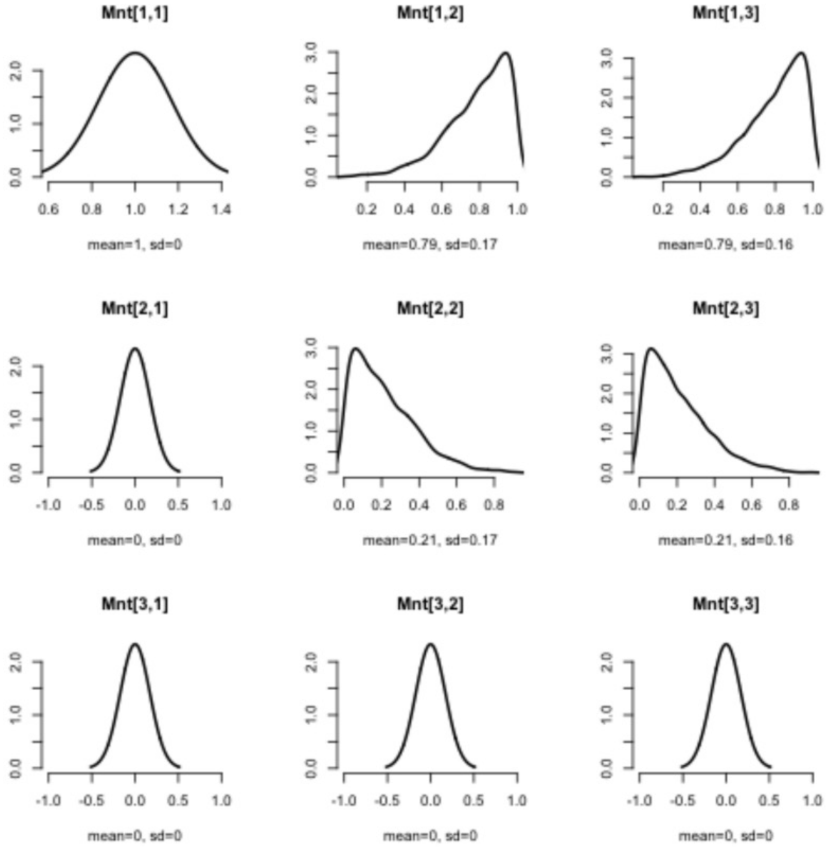


Figure 7: Estimated probability density of the maintenance matrix M.

다르게 표현할 수 있는 방법?
데이터를 어떻게 해석하고자 하는지에 따라서.

Result

Figure 9

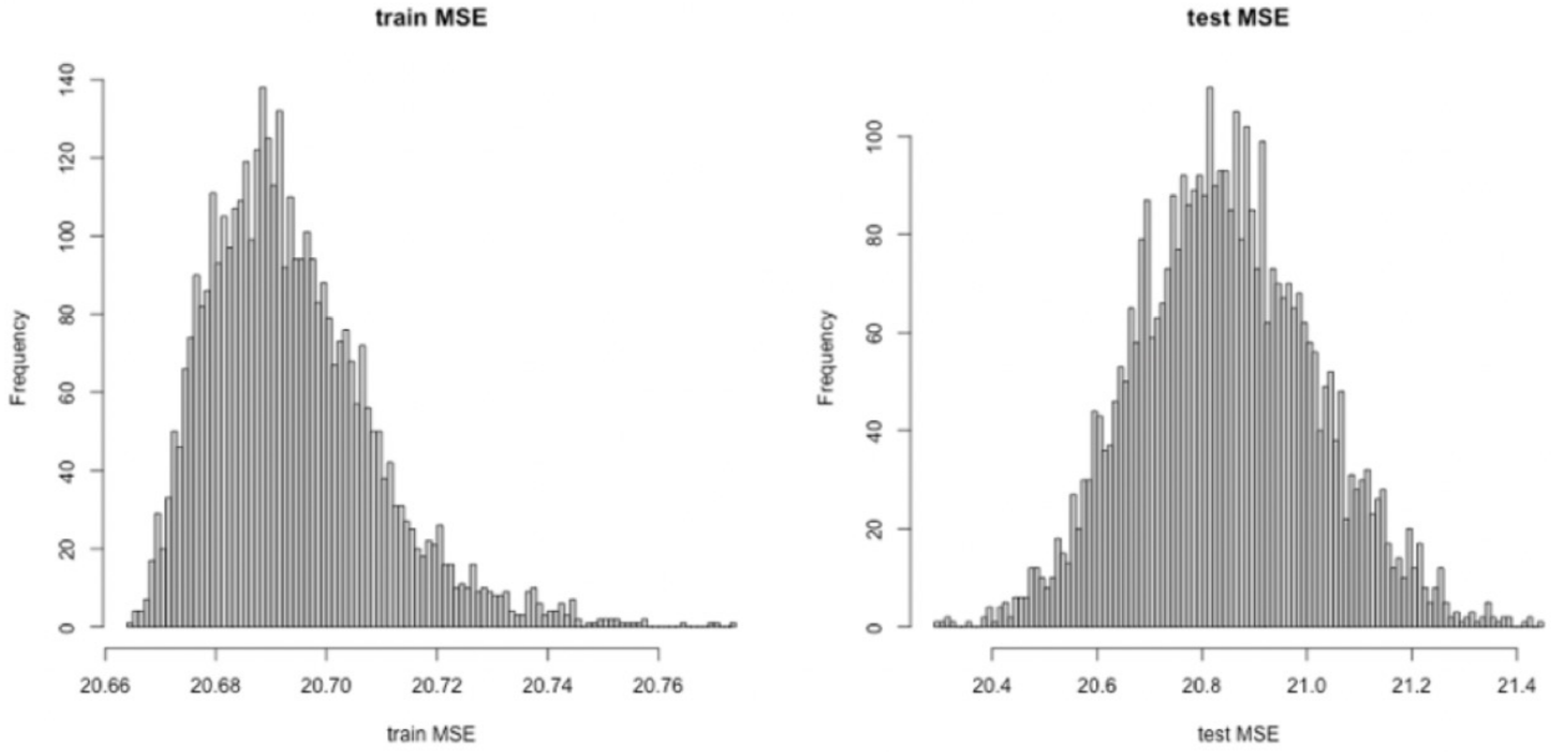


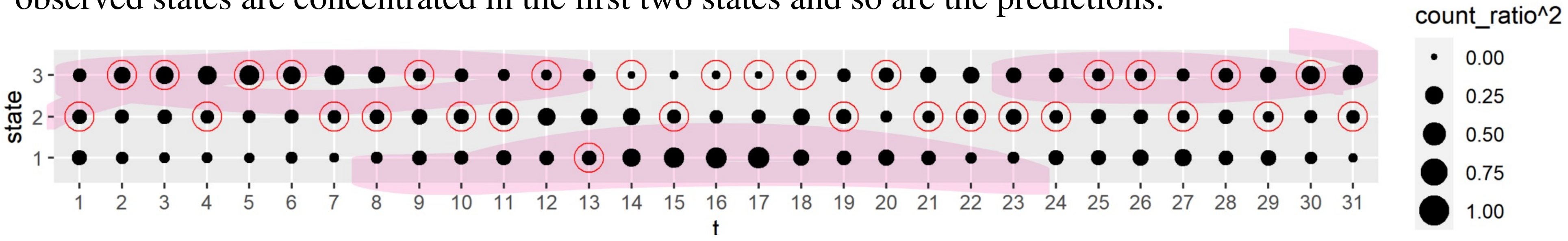
Figure 9: Histogram of the training and test sets MSE values.

add the average value of MSE (20.7 and 20.9, respectively) in red vertical lines.

Result

5.2 Predicted states

The main purpose of the model is to predict the deteriorating state of each engine so that management could be calculated based on their predictions. Figure 8 shows the observed ratio of the 99 naval ship's deterioration states, representing their observed probabilities with the size of the black circles. Red circles indicate the predicted states for each time. Predictions fit well with the observed state from three out of the sample ship's engines in most cases. For example, between years 10 and 20, the flat area which corresponds to the flat area of a bathtub in previous studies, the observed states are concentrated in the first two states and so are the predictions.



형광펜이 Observed / 동그라미가 Estimated 인데, 예측 accuracy 가 많이 떨어지지 않나요..?

1. Prediction 효율 높일 수 있는 방법?
2. Graph 다른 방식으로 제시 (Heatmap?)

SBC

앞 챕터들이 먼저 개선된 뒤에 다시 해보기

Question

Q1. Bayesian 인데 prior를 줄 수 있을까요?

> been introduced in [2] and [3], a hierarchical model with its advanced flexibility could increase the model's accuracy. Second, prior knowledge of the engines (deterioration and operation patterns) could be reflected. Priors could greatly affect the performance of the model if properly understood in the context of the entire Bayesian analysis, from inference to prediction to model evaluation [16].

Q2. Imbalanced missing data 를 imputation하는 것이 reliable한가?
Imputation 값이 대부분이다보니, 이 데이터가 왜 reliable 한지에 대한 설명 필요

Q3. 왜 missing 값이 많은 것이죠?

Q4. https://github.com/hyunjimoon/defense-reliability 에 있는 데이터 & 코드로 논문의 내용을 바로 구현 가능한가요?
Public으로 되어있던데 데이터가 공개되어 있는 것 같아요

Q5. 우리의 방법을 적용할 만한 다른 데이터가 있을지? (optional)

Q6. Journal of applied statistics 가 좋아보이는데 다른 의견이 있으신지?