Statistical Tests of Model Specifications

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Outline

1. Introduction and Informal Tests
2. Review of Statistical Hypothesis Tests
3. Classical vs. Specification Tests
4. Goodness of Fit
5. Non-Nested Hypotheses
6. Nonlinear variables
7. Prediction Tests
    - Outlier Analysis
    - Market Segments

Appendix

1. Introduction

- Classical statistics:
    - Model specification based on a-priori knowledge and data analysis
    - Evaluation is done through specification testing
- Machine learning:
    - Increasing availability of big data makes machine learning attractive
    - An automated procedure for model specification and estimation
    - Consists of algorithms that can learn from data and achieve prediction capability

1. Introduction - Sequence of Tests

- Assume model structure is correct (e.g., Logit) and test the specification of the systematic utility function.
- For given specification of the utility function, test the model structure (e.g., Logit vs Nested Logit).

Specification Testing Strategy

- Based on a-priori consideration, generate a set of candidate or "reasonable" models. Then, use "informal tests", statistical tests, "goodness-of-fit", and prediction tests to select among them.
- Good fit does not necessarily mean adequate model.
- Cannot rely exclusively on "goodness-of-fit" to select among competing models

Informal Tests

- Signs and relative magnitudes of estimated coefficients must agree with *a priori* expectations
- Alternative-specific constants must be relatively small
- Key coefficient ratios must have realistic values
  - (e.g.) value of time, willingness to pay
- (BAL, pp. 157-160)

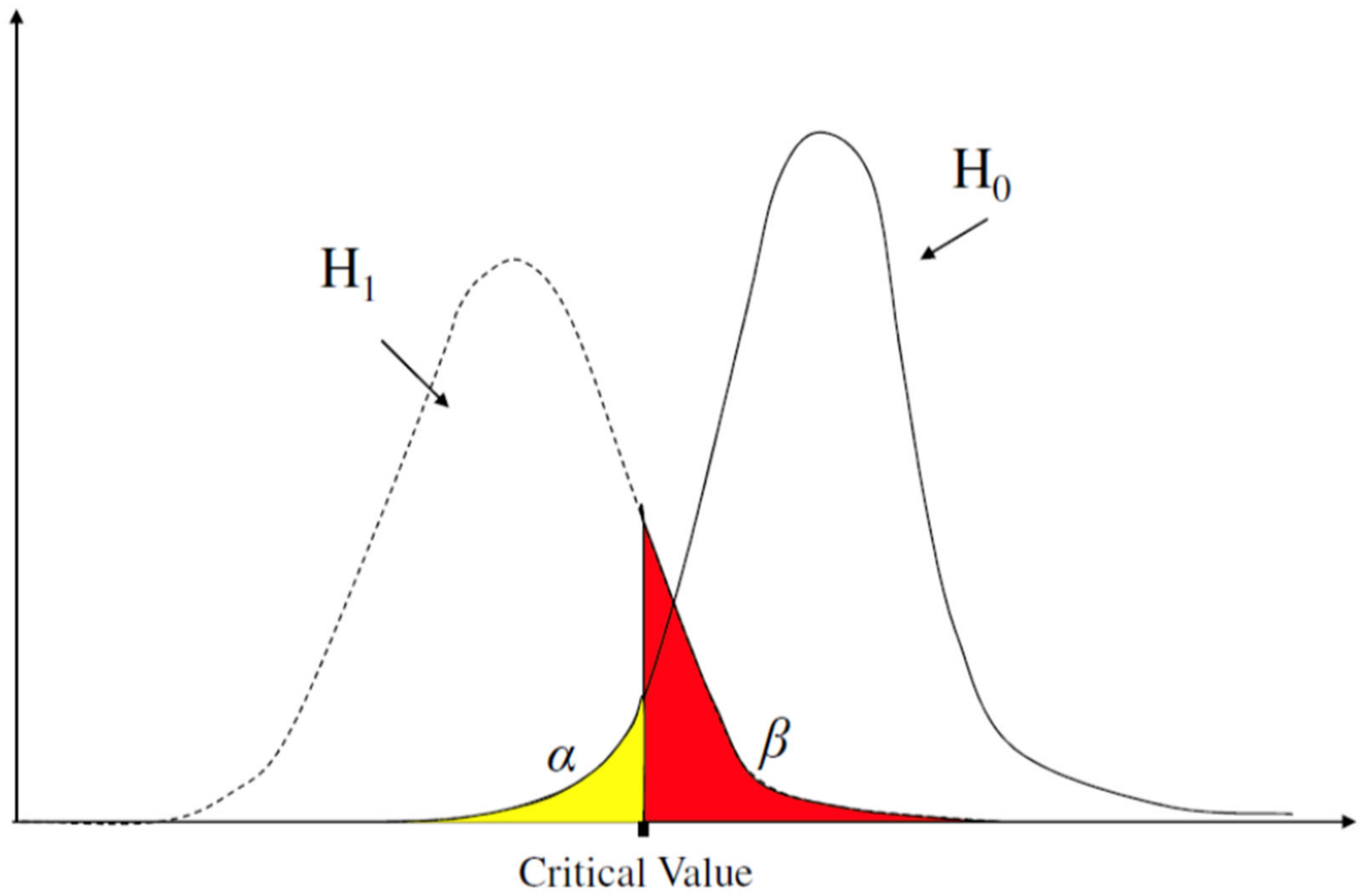
2. Review of Statistical Hypothesis Tests

- Would we always like α to be as small as possible? Why not 0.000001%?

H_0: null hypothesis
H_1: alternate hypothesis

Truth \ Decision	Accept H_0	Reject H_0
H_0 True		Type I error probability α
H_0 False	Type II error probability β	

Distributions under H_0 and H_1



Probability of Type I and Type II Errors

- For given sample size (N), trade-off between α and β
  - Only way to reduce both Type I and Type II error probabilities is to increase N
- $\pi = 1 - \beta$ is the *power* of the test (probability of rejecting H_0 when H_0 is false)
- π can only be determined for a simple hypothesis
  - H_1 is usually a composite hypothesis

Example of t-test (BAL, pp. 24-25)

$$H_0 : \theta = q$$

$$H_1 : \theta \neq q$$

The test statistic

$$t_{\hat{\theta}} = \frac{\hat{\theta} - q}{s_{\hat{\theta}}}$$

Select α and compare against the critical value.

Classical Hypothesis Testing

- The classical hypothesis test fixes the probability of a Type I error (α) and maximizes the power ($\pi = 1 - \beta$)
- The likelihood ratio test has desirable properties

$$LR = L_0^* / L_1^*$$

  - Applicable when H_0 and H_1 are simple hypotheses and the likelihood can be specified under both hypotheses
- With a composite hypothesis ($H_1 : \theta \neq q$), a desirable test maximizes the power curve.
- For specification testing, H_0 is a set of restrictions on a model
  - H_1 implies an unrestricted model and H_0 a restricted model

Likelihood Ratio, Wald and Lagrange Multiplier Tests

For maximum likelihood estimation (MLE), three asymptotically equivalent tests can be used (more details in Appendix).

Test	Characteristics
Likelihood Ratio (LR)	Need to estimate both restricted and unrestricted models (is the easiest, if it is convenient to estimate both)
Wald	Need to estimate unrestricted model only (convenient if restriction is nonlinear)
Lagrange Multiplier (LM)	Need to estimate restricted model only (convenient if unrestricted model is complex, e.g., non IIA)

Example of Likelihood Ratio Test

Tablet Ownership	Education Low (k=1)	Medium (k=2)	High (k=3)	
Tablet (i=1)	10	100	90	200
No Tablet (i=2)	140	200	60	400
	150	300	150	600
	25%	50%	25%	

- Is tablet ownership independent of education level?

$$H_o: \ \pi_1 = \pi_2 = \pi_3$$

$$H_1: \quad \text{otherwise}$$

Example of Likelihood Ratio Test (cont.)

- Unrestricted: estimate separately for each group

$$\hat{\pi}_1 = \frac{10}{150} \approx 0.067 \qquad \hat{\pi}_2 = \frac{100}{300} \approx 0.33 \qquad \hat{\pi}_3 = \frac{90}{150} = 0.60$$

$$\begin{aligned} L_U(\hat{\pi}_1, \hat{\pi}_2, \hat{\pi}_3) &= \log[(0.067)^{10}(0.93)^{140}(0.33)^{100}\ldots \\ &\ldots(0.67)^{200}(0.60)^{90}(0.40)^{60}] \\ &= -328.71 \end{aligned}$$

- Restricted: estimate a common value across all education levels

$$\tilde{\pi} = \frac{200}{600} = \frac{1}{3}$$

$$\begin{aligned} L_R(\hat{\pi}_1, \hat{\pi}_2, \hat{\pi}_3) &= \log[(0.33)^{200}(0.67)^{400}] \\ &= -382.28 \end{aligned}$$

Example of Likelihood Ratio Test (cont.)

- Test Statistic: $-2\left[L_R(\hat{\pi}_1, \hat{\pi}_2, \hat{\pi}_3) - L_U(\hat{\pi}_1, \hat{\pi}_2, \hat{\pi}_3)\right] \sim \chi_2^2$

$$-2(-382.28 + 328.71) = 107.14$$

- *P-value*: < 0.0001
- H_0 can be rejected at any (reasonable) level of significance

3. Classical vs. Specification Tests

$$\text{Minimize expected cost of errors: } \sum_{\text{possible errors}} P(\text{error}) * (\text{Cost of error})$$

Error	Type I (incorrectly reject) (e.g., include irrelevant variables, loss of efficiency)	Type II (incorrectly accept) (e.g., omit relevant variables, bias)
Prior belief regarding hypothesis Conditional probability of error Cost of error	$P(H_0 \text{ true}) = \lambda$ $P(\text{reject} \mid H_0) = \alpha$ C_1	$P(H_1 \text{ true}) = 1 - \lambda$ $P(\text{accept} \mid H_1) = 1 - \pi = \beta$ C_2
Expected Cost:	$\lambda \alpha C_1$	$(1-\lambda)(1-\pi) C_2$
Classical test:	$\lambda \approx 1, C_1 \approx C_2 \Rightarrow$ choose small α *expect the null hypothesis to be true; assume true unless proven otherwise*	
Specification test:	$\lambda \approx 0.5, C_1 << C_2 \Rightarrow$ choose larger α *the null hypothesis represents a restricted model*	
No test:	$\lambda = 0, C_1 = 0, C_2 >> 0 \Rightarrow$ choose $\alpha=1$ *the null hypothesis represents an unacceptable model*	

Example of Specification Test

- Choice of 5 telephone services (BM, SM, LF, EF, MF):
- Restricted model (only include cost and ASC)
  - $V_{BM} = \beta_{BM} + \beta_c \ ln(cost(BM))$
  - $V_{SM} = \beta_{SM} + \beta_c \ ln(cost(SM))$
  - $V_{LF} = \beta_{LF} + \beta_c \ ln(cost(LF))$
  - $V_{EF} = \beta_{EF} + \beta_c \ ln(cost(EF))$
  - $V_{MF} = \beta_c \ ln(cost(MF))$
- Unrestricted model (add the number of users per household)
  - $V_{BM} = \beta_{BM} + \beta_c \ ln(cost(BM)) + \beta_u \ users$
  - $V_{SM} = \beta_{SM} + \beta_c \ ln(cost(SM)) + \beta_u \ users$
  - $V_{LF} = \beta_{LF} + \beta_c \ ln(cost(LF)) + \beta_u \ users$
  - $V_{EF} = \beta_{EF} + \beta_c \ ln(cost(EF)) + \beta_u \ users$
  - $V_{MF} = \beta_c \ ln(cost(MF))$

Example of Specification Test (cont.)

Restricted model

	Estimate	Std Err	t-stat
β_{BM}	-2.46	0.313	-7.84
β_{SM}	-1.74	0.276	-6.28
β_{LF}	-0.535	0.208	-2.57
β_{EF}	-0.737	0.723	-1.02
β_{c}	-2.03	0.214	-9.47
$L(0)$	-560.25		
$L(\hat{\beta})$	-477.56		
# Obs	434		

Unrestricted model

	Estimate	Std Err	t-stat
β_{BM}	-2.12	0.455	-4.66
β_{SM}	-1.40	0.429	-3.28
β_{LF}	-0.212	0.381	-0.56
β_{EF}	-0.401	0.795	-0.50
β_{c}	-2.00	0.216	-9.26
β_{u}	-0.116	0.114	-1.02
$L(0)$	-560.25		
$L(\hat{\beta})$	-477.04		
# Obs	434		

Example of Specification Test (cont.)

H_0: $\beta_u = 0$ (restricted model)

H_1: otherwise (unrestricted model)

Likelihood Ratio Test:

- In the restricted case, $L_R(\hat{\beta}_{BM}, \hat{\beta}_{SM}, \hat{\beta}_{LF}, \hat{\beta}_{EF}, \hat{\beta}_c)$ = -477.56
- In the unrestricted case, $L_U(\hat{\beta}_{BM}, \hat{\beta}_{SM}, \hat{\beta}_{LF}, \hat{\beta}_{EF}, \hat{\beta}_c, \hat{\beta}_u)$= -477.04
- Test statistic: -2$[L_R - L_U] \sim \chi_1^2$; -2[-477.56-(-477.04)] = 1.04
- *P-value*: 0.31
- H_0 (the restricted model) can be rejected at any level of significance larger than 31%

4. Goodness of Fit (ρ^2 and $\bar{\rho}^2$)

- ρ^2 and $\bar{\rho}^2$ are analogous to R^2 and $\bar{R}^2$ in least squares.

$$\rho^2 \equiv 1 - \frac{L(\hat{\theta}_{MLE})}{L(\hat{\theta} = 0)}$$

- Equally likely model: *same probability for all alternatives*

$$\hat{\theta} = 0 \Rightarrow \rho^2 = 0$$

- Perfect fit: *probability of each observed choice is 1*

$$L(\hat{\theta}_{MLE}) = 0 \Rightarrow \rho^2 = 1$$

Goodness of Fit (ρ^2 and $\bar{\rho}^2$) (cont.)

- ρ^2 is a function of
  - The sample data
  - The dependent variable $P(i \mid C)$

    (i.e., the choice set C is important)
  - The number of parameters K

    (ρ^2 is monotonic in K)

<u>Remarks:</u>

1. ρ^2 should only be used to compare models when the above are the same.
2. Risk of over-fitting model to the data

Goodness of Fit (ρ^2 and $\bar{\rho}^2$) (cont.)

Adjusted ρ^2 ($\bar{\rho}^2$)

- Useful for comparing models with different numbers of parameters (i.e., different degrees of freedom)

$$\bar{\rho}^2 \equiv 1 - \frac{L(\hat{\theta}_{MLE}) - K}{L(\hat{\theta} = 0)}$$

- Decreasing $\bar{\rho}^2$ means that the model over-fits

5. Non-Nested Hypotheses
Nested vs. Non-Nested

- Nested hypotheses
  - Restricted model is special case of alternate model.
  - When H_0 is true, the alternate model simplifies to the restricted model.
- Non-nested hypotheses
  - Neither model is a special case of the other.

  Example:

  model 1: $V_{in} = \gamma_i + \theta x_{in} + \varepsilon_{in}$

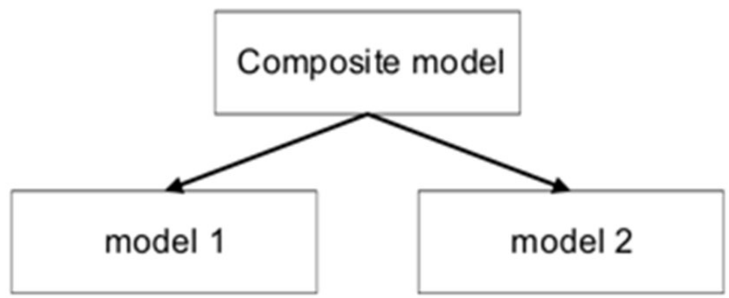
  model 2: $V_{in} = \gamma_i + \theta \ln x_{in} + \varepsilon_{in}$

Non-Nested Hypotheses

- Composite model
- $\bar{\rho}^2$ test
- J-test (see Appendix)

Non-Nested Hypotheses
Composite Model Test (Cox)

- Construct a composite model which has both models as special cases, e.g., $V_{in} = \gamma_i + \theta x_{in} + \mu \ln x_{in} + \varepsilon_{in}$



- *Use LR test twice* *(a)* H_0: *Model 1*
  H_1: *Composite*
  (b) H_0: *Model 2*
  H_1: *Composite*

Non-Nested Hypotheses

$\bar{\rho}^2$ test (Horowitz)

$$M_1 : U = g_1(X_1, \theta_1) + \varepsilon_1$$

$$M_2 : U = g_2(X_2, \theta_2) + \varepsilon_2$$

Estimate M_1 and M_2 to obtain $\bar{\rho}_1^2$, $\bar{\rho}_2^2$

$$P\left(\bar{\rho}_2^2 - \bar{\rho}_1^2 > z \mid M_1\right) \leq \Phi\left\{-\left[-2zL(0) + \left(K_2 - K_1\right)\right]^{1/2}\right\}, \quad z > 0$$

Where

$\bar{\rho}_l^2 =$ Adjusted Likelihood Ration Index of model $l = 1,2$

$K_l =$ Number of parameters of model l

$\Phi =$ The standard Normal CDF

Non-Nested Hypotheses

$\bar{\rho}^2$ test (Horowitz) (cont.)

- If all N observations have all J alternatives

$$P\left(\bar{\rho}_2^2 - \bar{\rho}_1^2 > z/M_1\right) \leq \Phi\left\{-\left[2Nz\ln J + \left(K_2 - K_1\right)\right]^{1/2}\right\}, \quad z > 0$$

- Example

Model	K	$\bar{\rho}^2$	N	J
M_1 (Table 7.4)	16	0.3089	1,136	3
M_2 (Table 7.3)	15	0.3102	1,136	3

$$P\left(\bar{\rho}_2^2 - \bar{\rho}_1^2 > 0.0013 \mid M_1\right) \leq 0.07$$

6. Nonlinear Specifications

- Nonlinear transformations of the independent variables
  - Discrete and qualitative variables
    - *L-1* dummy variables for variables with *L* categories
  - Continuous variables (no dummy variables)
    - Splines
    - Box-Cox
    - Power series
  - Interaction terms

Piecewise linear specification

- Specification:

$$V = \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4 + \ldots$$

where

$$x_1 = \begin{cases} x & \text{if } x < b_1 \\ b_1 & \text{otherwise} \end{cases}$$

$$x_2 = \begin{cases} 0 & \text{if } x < b_1 \\ x - b_1 & \text{if } b_1 \le x < b_2 \\ b_2 - b_1 & \text{otherwise} \end{cases}$$

$$x_3 = \begin{cases} 0 & \text{if } x < b_2 \\ x - b_2 & \text{if } b_2 \le x < b_3 \\ b_3 - b_2 & \text{otherwise} \end{cases}$$

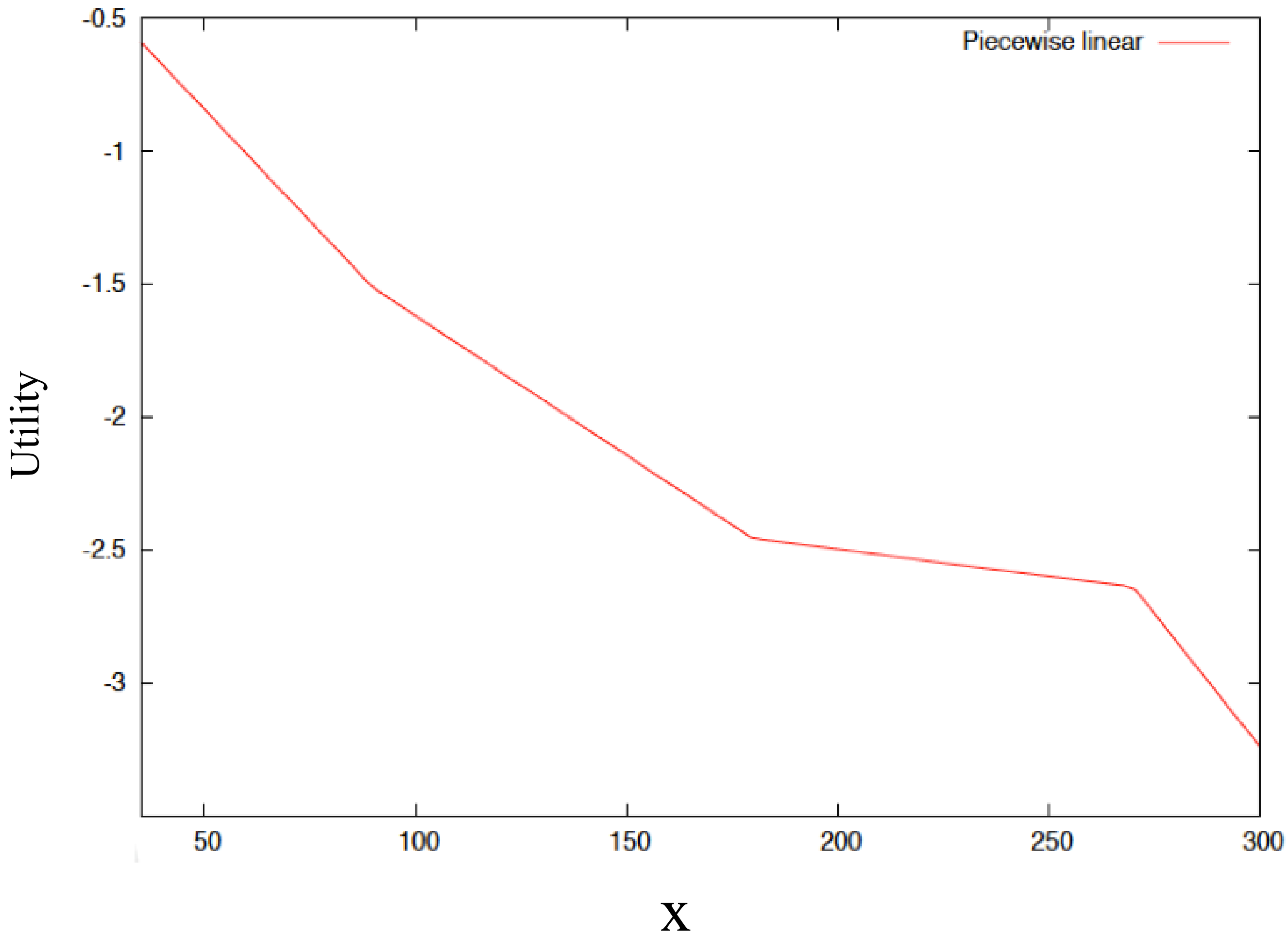
$$x_4 = \begin{cases} 0 & \text{if } x < b_3 \\ x - b_3 & \text{otherwise} \end{cases}$$

Piecewise linear specification (cont.)

- Examples: $b_1 = 90$, $b_2 = 180$, $b_3 = 270$

x	x_1	x_2	x_3	x_4
40	40	0	0	0
100	90	10	0	0
200	90	90	20	0
300	90	90	90	30

Piecewise linear specification (cont.)



Box-Cox Transforms

- $V = \beta x(\lambda) + \cdot\cdot\cdot$

  where

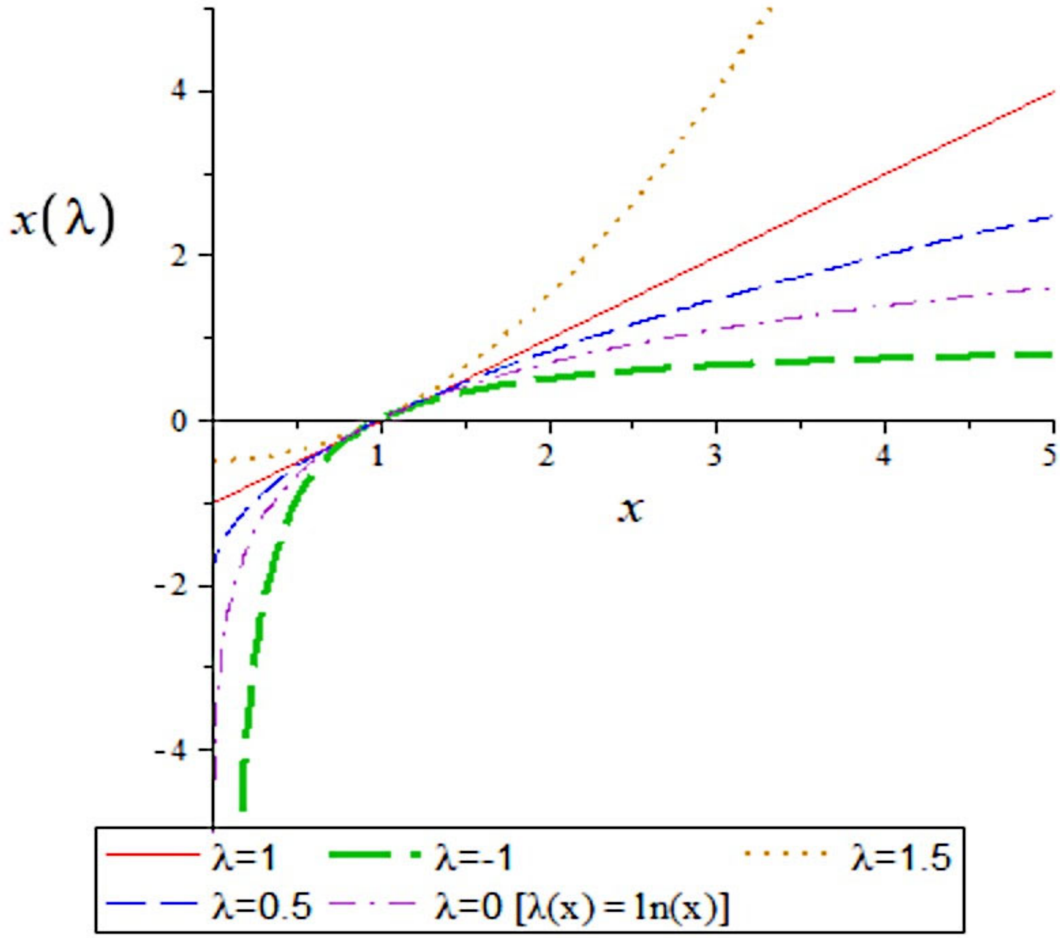
  $$x(\lambda) = \begin{cases} (x^{\lambda} - 1) / \lambda & \text{if } \lambda \neq 0 \\ \ln x & \text{if } \lambda = 0 \end{cases}$$

  where $x > 0$

- If $x < 0$, let α such that $x + \alpha > 0$ and

  $$x(\lambda,\alpha) = \begin{cases} ((x + \alpha)^{\lambda} - 1\ ) / \lambda & \text{if } \lambda \neq 0 \\ \ln\,(x + \alpha) & \text{if } \lambda = 0 \end{cases}$$

Box-Cox Transforms (cont.)



Power Series

$$V = \beta_1 x + \beta_2 x^2 + \beta_3 x^3 + \ldots$$

- In practice, these terms could be correlated
- Difficult to interpret
- Risk of over-fitting
- Unexpected consequences possible outside the range of observed (survey) data

7. Prediction Tests- Outlier Analysis

- Apply the model to the sample
  - Examine observation with lowest predicted probability for the observed choice
  - Test model sensitivity to outliers
    - Small probabilities have significant impact on estimates
- Potential causes of low probability
  - Coding or measurement error in the data
  - Model misspecification
  - Unexplainable variation in choice behavior

Prediction Tests - Outlier Analysis (cont.)

- Coding or measurement errors in the data
  - Look for signs of data errors
  - Correct or remove the observation
- Model misspecification
  - Seek clues on missing variables for the observation
  - Keep the observation and improve the model
- Unexplainable variation in choice behavior
  - Keep the observation
  - Avoid overfitting of the model to the data

Prediction Tests - Market Segments

- Compare predicted vs. observed shares per segment
- Let N_g be the number of sampled individuals in segment g
- Observed share for alternative i and segment g

$$W_g(i) = \frac{1}{N_g} \sum_{n=1}^{N_g} y_{in}$$

- Predicted share for alternative i and segment g

$$\hat{W}_g(i) = \frac{1}{N_g} \sum_{n=1}^{N_g} \hat{P}_n(i)$$

Prediction Tests - Market Segments (cont.)

Note:

- With a full set of alternative-specific constants

$$\sum_{n=1}^{N} y_{in} = \sum_{n=1}^{N} \hat{P}_n(i)$$

Additional Readings

- Ben-Akiva, M., Bierlaire, M., McFadden D. and Walker, J. (2015), 'Specification testing' in *Discrete Choice Analysis,* chapter 6, draft version, September 2015.

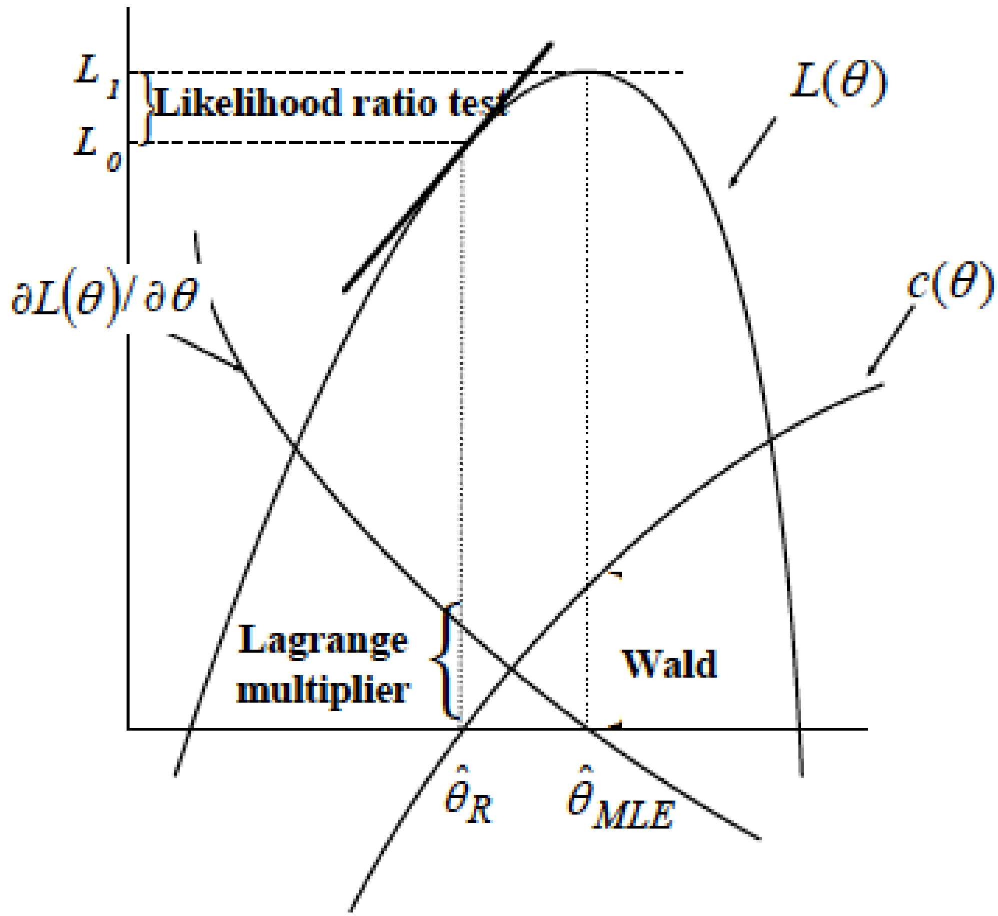
- Ben-Akiva, M. and Lerman, S. (1985), *Discrete Choice Analysis: Theory and Application to Travel Demand*, MIT Press, Cambridge MA, USA, chapter 7 (pp. 154-216).

Appendix

- A. Likelihood Ratio, Wald and Lagrange Multiplier Tests
- B. J-test for non-nested hypothesis
- C. Constrained estimation
- D. IIA test

A. Likelihood Ratio, Wald and Lagrange Multiplier Tests

- Graphical representation of the three tests
- H_0: c(θ)=0
- g restrictions (linear or nonlinear)



Likelihood Ratio, Wald and Lagrange Multiplier Tests (cont.)

Test	Test statistic
Likelihood Ratio (LR)	$-2[L_R(\hat{\theta}) - L_U(\hat{\theta})] \sim \chi^2_g$
Wald	$c(\hat{\theta}_U)'(\mathrm{Var}[c(\hat{\theta}_U)])^{-1} c(\hat{\theta}_U) \sim \chi^2_g$
Lagrange Multiplier (LM)	$\left(\dfrac{\partial L_U(\hat{\theta}_R)}{\partial \hat{\theta}_R}\right)' \left[I(\hat{\theta}_R)\right]^{-1} \left(\dfrac{\partial L_U(\hat{\theta}_R)}{\partial \hat{\theta}_R}\right) \sim \chi^2_g$

Where:

- g is the number of restrictions
- $\mathbf{I}(\theta)$ is the information matrix

B. Non-Nested Hypotheses

J-Test (Davidson-MacKinnon)

$$M_1 : U = g_1(X_1, \theta_1) + \varepsilon_1$$

$$M_2 : U = g_2(X_2, \theta_2) + \varepsilon_2$$

- Estimate M_2 to obtain $\hat{\theta}_2$
- Consider the model obtained by convex combination:

$$U = (1-\lambda)g_1(X_1, \theta_1) + \lambda\ g_2(X_2, \hat{\theta}_2) + \varepsilon$$

- Note that λ and θ_1 are estimated, not θ_2
- If M_1 is true, the true value of λ is zero
- Perform a t-test of H_0: $\lambda = 0$

C. Constrained Estimation

- Incorporate prior information into the estimation procedures
- (BAL, pp. 179-182)

D. IIA Test

- Test whether IIA is valid
- If not, search for a better model specification.
    - Find alternatives with missing or misspecified variables
    - Point toward an acceptable nested logit structure

Alternative IIA Tests

- **Method 1:** Estimate a model with a subset of the choice set and reject IIA if the parameter estimates differ from the full choice set estimates.
  - Tests:
    - Informal test
    - Hausman-McFadden test
- **Method 2:** Estimate the full choice set model and use a Lagrange Multiplier test
  - McFadden's omitted variables test

Intuition behind IIA Tests

- If IIA holds, a logit model with a full choice set C_n :

$$P(i \mid C_n) = \frac{\exp(\beta' x_{in})}{\sum_{j \in C_n} \exp(\beta' x_{jn})}$$

and a logit model with a subset choice set $\tilde{C}_n$

$$P(i \mid \tilde{C}_n \subseteq C_n) = \frac{\exp(\beta' x_{in})}{\sum_{j \in \tilde{C}_n} \exp(\beta' x_{jn})}$$

should yield similar estimates since, under IIA, exclusion of alternative does not affect the consistency of the estimators.

Informal Test

- Do estimated coefficients differ between the two estimations?
- Check for a scale effect, where $\hat{\beta}'_C$ are an approximate shrinking towards zero of $\hat{\beta}'_{\tilde{C}}$
  - It will be shown with an example during the nested logit lecture

Hausman-McFadden Test

- Requires estimating two logit models:
  - With the full choice set (C)
  - With a subset of the alternatives ($\tilde{C}$)
- Remarks
  - ASC of excluded alternatives cannot be estimated
  - Observations with choice of excluded alternatives must be omitted

Hausman-McFadden Test (cont.)

- Hypotheses:
  - H_0 : IIA holds
  - H_1 : IIA doesn't hold
- Test statistic

$$\left(\hat{\beta}_{\tilde{C}} - \hat{\beta}_C\right)' \left(\Sigma_{\hat{\beta}_{\tilde{C}}} - \Sigma_{\hat{\beta}_C}\right)^{-1} \left(\hat{\beta}_{\tilde{C}} - \hat{\beta}_C\right) \sim \chi^2_{\tilde{K}}$$

- Remarks
  - $\Sigma_{\hat{\beta}_{\tilde{C}}}, \Sigma_{\hat{\beta}_C}$ are the variance-covariance matrices of the (*common*) estimated coefficients of the two models.
  - $\tilde{K}$ is the number of coefficients in the subset model

McFadden's Omitted Variables Test

Intuition:

- If the ratio of probabilities for alternatives i and j depends on the attributes of alternative m (in violation of IIA), then the attributes of alternative m will enter significantly the utility of alternative i or j.

- IIA is rejected if cross-alternative variables are significant.

McFadden's Omitted Variables Test (cont.)

Step 1:

- Estimate the full choice set model and calculate fitted systematic utilities $\hat{V}_{in}$ and choice probabilities $\hat{P}(i / C_n)$ for all N observations.

$$\hat{V}_{in} = \hat{\beta}' x_{in}$$

$$\hat{P}(i \mid C_n) = \frac{\exp\left(\hat{\beta}' x_{in}\right)}{\sum_{j \in C_n} \exp\left(\hat{\beta}' x_{jn}\right)}$$

$$\forall i \in C_n \qquad n = 1,.., N$$

McFadden's Omitted Variables Test (cont.)

Step 2:

- For a given $\tilde{C}_n \subseteq C_n$, calculate auxiliary variables, as follows:

$$z_{in}^{\tilde{C}_n} = \begin{cases} \hat{V}_{in} - \hat{V}_{\tilde{C}_n} & if\ i \in \tilde{C}_n \\ 0 & if\ i \notin \tilde{C}_n \end{cases} \quad \begin{matrix} n = 1,\ldots,N \\ \forall i \in C_n \end{matrix}$$

where

$$\hat{V}_{\tilde{C}_n} = \frac{\sum_{j \in \tilde{C}_n} \hat{V}_{jn} \hat{P}\left(j \mid C_n\right)}{\sum_{j \in \tilde{C}_n} \hat{P}\left(j \mid C_n\right)} \quad n = 1,\ldots N$$

McFadden's Omitted Variables Test (cont.)

Step 3:

- Re-estimate the full choice set model with auxiliary variables

$$P(i \mid C_n) = \frac{\exp\left(\beta' x_{in} + \gamma^{\tilde{C}} z_{in}^{\tilde{C}}\right)}{\sum_{j \in C_n} \exp\left(\beta' x_{jn} + \gamma^{\tilde{C}} z_{jn}^{\tilde{C}}\right)}$$

- And perform a likelihood ratio test

$$\text{Test:} \quad \text{H}_0 : \gamma^{\tilde{C}} = 0$$

$$\text{H}_1 : \gamma^{\tilde{C}} \neq 0$$

$$\text{Reject H}_0 \Rightarrow \text{Reject IIA}$$

Example: Telephone Data

- Consider $\tilde{C}_1$ = {BM, SM} and $\tilde{C}_2$ = {LF,EF,MF}
- Estimation results with 2 auxiliary variables

Variable number	Parameter name	Param. estimate	Robust std. error	t-stat	p-value
1	ASC_BM	-0.184	0.233	-0.79	0.43
2	ASC_EF	1.07	0.833	1.28	0.20
3	ASC_LF	0.801	0.166	4.82	0.00
4	ASC_MF	1.83	0.279	6.56	0.00
5	B_COST	-1.26	0.228	-5.50	0.00
6	B_IIA_f	1.83	0.538	3.41	0.00
7	B_IIA_m	0.832	0.334	2.49	0.01

The Test

- L(Model with auxiliary variables) = −460.747
- L(Base Model) = −477.557
- The test : H_0 : B_IIA_m = B_IIA_f = 0
- The test statistic:

$$-2(L_R - L_U) = -2(-477.557 + 460.747) = 33.620$$

- Since χ^2 with 2 df at 95% confidence level = 5.991, we reject H_0 and conclude that IIA does not hold for the full choice set model.

Remarks

- One could test IIA with Logit Mixture or MEV as the unrestricted model and Logit as the restricted model.
- Such tests would work if the general model is estimable. The tests presented above require logit estimation.
- If IIA fails the tests described in this lecture do not provide an alternative model that can be used.
- The McFadden’s omitted variables test can be used to test against the Logit Mixture Model.