Aggregate Forecasting and Microsimulation

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1.202 Demand Modeling

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Outline

- Aggregate Forecasting
- Methods
    - Average Individual
    - Sample Enumeration
    - Microsimulation
- Conclusion

Appendix: Demand Indicators

Aggregate Forecasting

- Analyze impact of changes in policy variables and/or demographics
- Make aggregate predictions for population p with N_p members
  - Using a disaggregate model, $P(i|x_n)$
  - x_n includes both individual characteristics and attributes
- If we knew the value of x_n for every member of the population, an aggregate prediction would be:

$$D_p(i) = \sum_{n=1}^{N_p} P\left(i \mid x_n\right)$$

Disaggregate Probabilities

Population	Alternatives				Total
	1	2	…	J	
1	$P(1\|x_1)$	$P(2\|x_1)$	…	$P(J\|x_1)$	1
2	$P(1\|x_2)$	$P(2\|x_2)$	…	$P(J\|x_2)$	1
⋮	⋮	⋮	⋮	⋮	⋮
N_p	$P(1\|x_{N_p})$	$P(2\|x_{N_p})$	…	$P(J\|x_{N_p})$	1
Total	$D_p(1)$	$D_p(2)$	…	$D_p(J)$	N_p

Aggregate shares by market segment

- **Aggregate share of alternative *i*** in population p with N_p members:

$$W_p(i) = \frac{D_p(i)}{N_p} = \frac{1}{N_p}\sum_{n=1}^{N_p} P\left(i \mid x_n\right)$$

- Aggregate share of alternative i **by market segment *g*** in population p with N_{pg} members:

$$W_g\left(i\right) = \frac{1}{N_{pg}}\sum_{n=1}^{N_{pg}} P\left(i \mid x_n\right)$$

- But we do not know x_n for everyone in population

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Appendix: Demand Indicators

1. Use an "Average" Individual

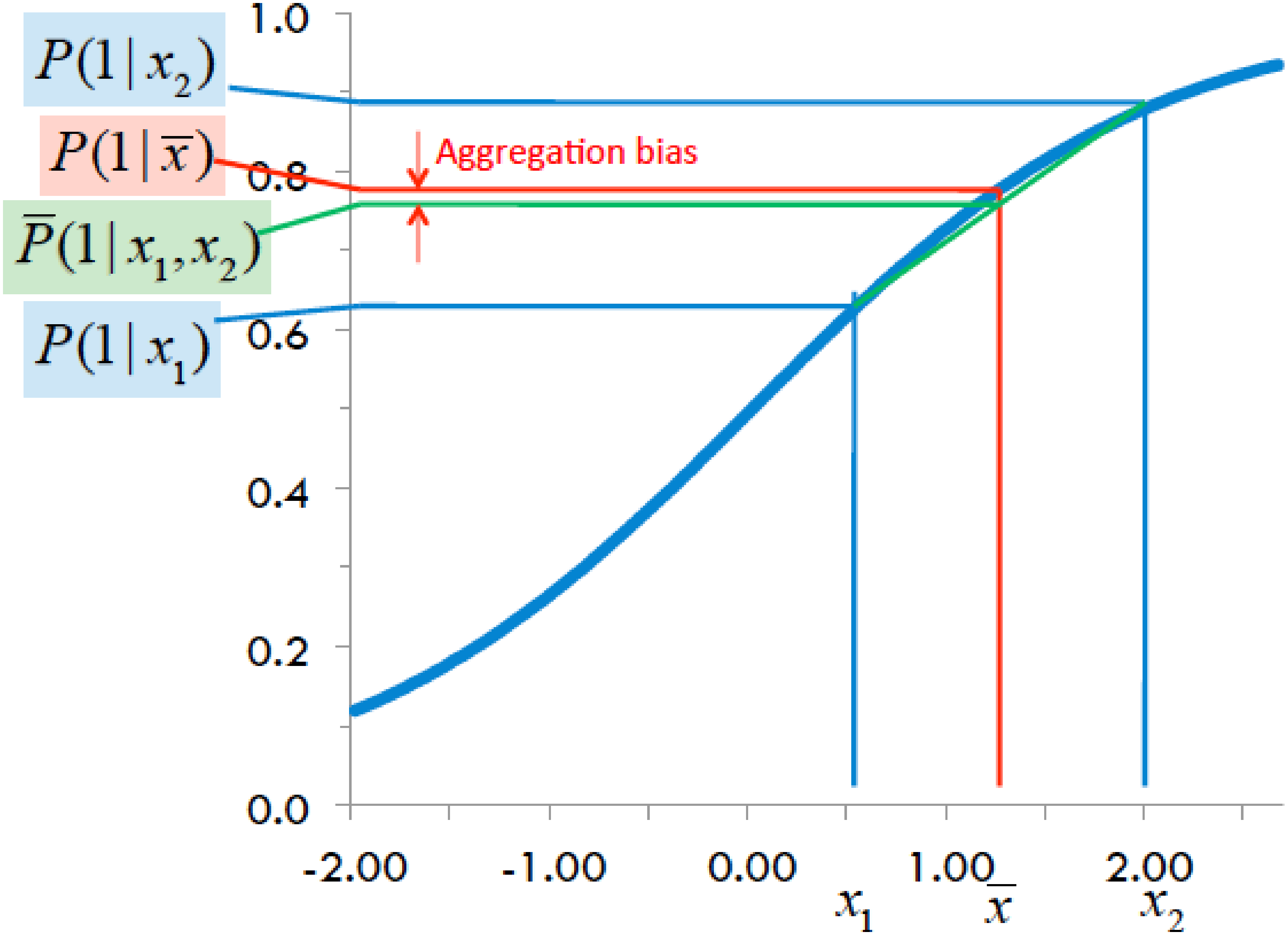
- $\bar{x}$ leads to bias in $W_p(i)$ unless population is homogenous.

Binary Logit Example

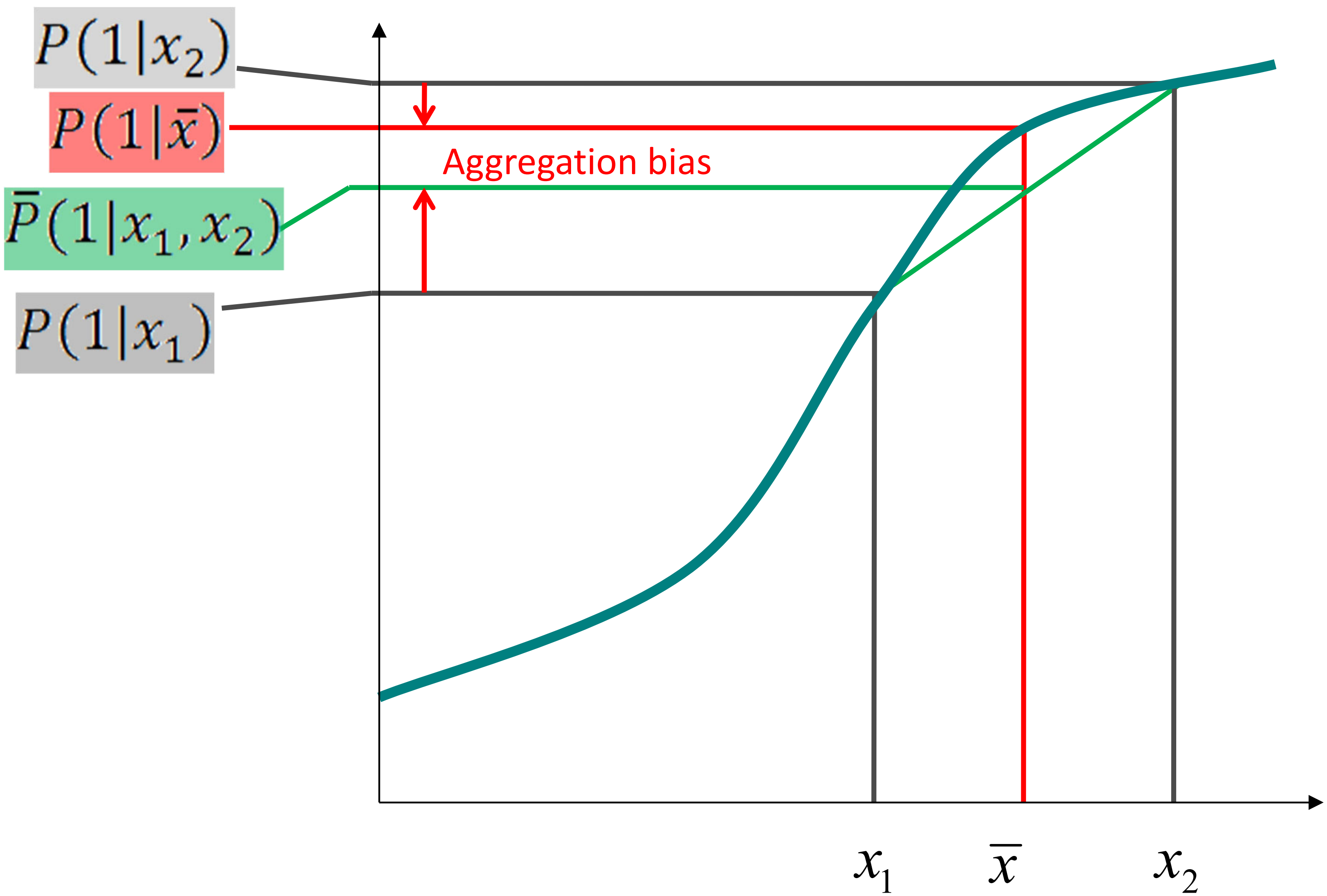
n	$\mathbf{x_n}$	$P(i \mid x_n) = \frac{1}{1+\exp(-x_n)}$
1	.5	.62
2	2	.88
Average	$\bar{x} = 1.25$	$\bar{P}(i \mid x_n) = 0.75$ $P(i \mid \bar{x}) = 0.78$

aggregation bias = 0.03

Aggregation Bias I



Aggregation Bias II



Average Individual

- A "representative individual"

- Reasonable if:
  - Variance in x_n is small,
  - or if applied to market segments
    - Called classification (BAL pages 138-140)
    - (e.g.) Low, medium and high incomes

- Other techniques described in BAL are not used frequently

2. Use Sample Enumeration

- To approximate aggregate shares, use a "representative" sample of the population
- The **share of alternative *i*** is estimated by:

$$\hat{W}(i) = \frac{1}{N_s} \sum_{n=1}^{N_s} \hat{P}(i \mid x_n)$$

  - N_s is the number of individuals in the sample
- The share of alternative *i* **in group *g*** is estimated by:

$$\hat{W}_g(i) = \frac{1}{N_{sg}} \sum_{n=1}^{N_{sg}} \hat{P}(i \mid x_n)$$

  - N_{sg} is the number of individuals in group *g* in the sample

Extension to Stratified Samples

- Different individuals (groups) have different weights
- Weight of each individual n is determined based on his/her segment g:

$$\omega_{g(n)} = \frac{N_{pg}}{N_{sg}} = \frac{\#\,\text{individuals in segment } g \text{ in the population}}{\#\,\text{individuals in segment } g \text{ in the sample}}$$

- Calculate weighted averages:

$$\hat{W}_p(i) = \sum_{n=1}^{N_s} \frac{\omega_{g(n)} \hat{P}(i \mid x_n)}{\sum_{n=1}^{N_s} \omega_{g(n)}}$$

- In random sample: $\omega_{g(n)} = 1$
- How to get weights? Iterative proportional fitting (IPF)

Example: Telephone Choice

- **Price Increase**: BM + \$0, SM + \$4, LF + \$6, EF + \$7 and MF + \$11.
- Calculate fitted probabilities for **Base Case** and for **Price Increase** (Python Biogeme)

$$\hat{P}\left(i \mid x_n, \text{Base Case}\right) \qquad \hat{P}\left(i \mid x_n, \text{Price Increase}\right)$$

- Calculate average probability by income group (equal weights)

$$\hat{W}_{lowInc}\left(i \mid \text{Base Case}\right) = \frac{1}{N_{lowInc}} \sum_{n \in lowInc} \hat{P}\left(i \mid x_n, \text{Base Case}\right)$$

$$\hat{W}_{lowInc}\left(i \mid \text{Price Increase}\right) = \frac{1}{N_{lowInc}} \sum_{n \in lowInc} \hat{P}\left(i \mid x_n, \text{Price Increase}\right)$$

Example: Telephone Choice (cont.)

	Base Case			Price Increase		
	lowINC	medINC	hiINC	lowINC	medINC	hiINC
BM	19%	14%	13%	34%	26%	23%
SM	30%	28%	23%	22%	21%	18%
LF	40%	43%	41%	34%	39%	37%
EF	0%	1%	2%	0%	1%	2%
MF	11%	14%	21%	10%	13%	19%

3. Use Microsimulation

- Sample enumeration requires keeping track of all probabilities (not always practical)
  - Large model systems with high dimensionality of decisions
  - Traffic simulators with sequences of decisions
- Microsimulation generates a realization for each individual in sample
  - For each person, make a draw from his/her probability distribution:

    $\hat{y}_{in} = 1$ if realization for n is alternative i, *0* otherwise
- Estimated shares are based on sums of realizations

$$\hat{W}_p(i) = \frac{1}{N_s} \sum_{n=1}^{N_s} \hat{y}_{in}$$

Microsimulation example

- Let $P_n(bike)=0.5$, $P_n(walk)=0.2$, $P_n(bus)=0.3$
- To make realization, draw a number $d_n \sim \mathrm{U}(0,1)$
  - if $d_n <= 0.5$ then realization for individual n is bike
  - If $0.5 < d_n <= 0.7$ then realization for individual n is walk
  - If $d_n > 0.7$ then realization for individual n is bus
- For example, $d_n = 0.52$ → *individual n walks*
- Repeat it many times and end up with 50% bike, 20% walk, 30% bus
- NOTE: Requires larger sample

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Conclusion

- Aggregation bias can be large if
  - Disaggregate model is nonlinear, and population is heterogeneous ($Var(x_n)$ is large)
- Average individual ignores aggregation bias
- Classification (average individual by market segment)
  - Reduces bias by forming homogeneous groups
  - Ignores aggregation bias within group
- Sample enumeration eliminates aggregation bias (sampling error)
- Microsimulation sums realizations instead of probabilities (requires a larger sample or a synthetic population)

Additional Readings

- Ben-Akiva, M. and S. Lerman (1985) Discrete Choice Analysis: Theory and Application to Travel Demand, MIT Press, Cambridge MA, USA, chapter 6 (pp. 131-153).

- Ben-Akiva, M., Bierlaire, M., McFadden D. and Walker, J. (2014), Discrete Choice Analysis, chapter 10, draft version, September 2014.

- Glerum, A., Stankovikj, L., Thémans, M., and Bierlaire, M. (2014). Forecasting the demand for electric vehicles: accounting for attitudes and perceptions, Transportation Science 48(4):483-499.

Appendix: Demand Indicators

- Willingness to Pay
- Elasticity

Willingness to Pay

- If the model contains a cost or price of alternatives
- The trade-off between any attribute and a money attribute is a willingness to pay
- It reflects the money value of one unit of an attribute
- Typical example in transportation: value of travel time

Willingness to Pay (cont.)

- c_{in} is the cost of alternative i for individual n
- x_{in} is the value of an attribute (e.g. travel time)
- $V_{in}(c_{in}, x_{in})$ is the value of the utility function
- Consider a change in the attribute value: $x'_{in} = x_{in} + \delta^x_{in}$
- δ^c_{in} is the additional cost that would achieve the same utility

$$V_{in}(c_{in} + \delta^c_{in}, x_{in} + \delta^x_{in}) = V_{in}(c_{in}, x_{in})$$

- The willingness to pay is the additional cost per unit of x

$$\delta^c_{in}/\delta^x_{in}$$

Example: Telephone Choice

- Costs for a given individual

Alternative	BM	SM	LF	EF	MF
Cost ($)	13.46	15.05	17.75	19.75	38.28

- Estimated Parameters

	β_{BM}	β_{SM}	β_{LF}	β_{EF}	β_{MF}	β_c
Value	-2.46	-1.74	-0.535	-0.737	0	-2.03

- Willingness to pay when switching from BM to LF

$$V_{LF} - V_{BM} = 0$$

$$\beta_{LF} + \beta_c \ln(\text{cost(BM)}+\delta) - \beta_{BM} - \beta_c \ln(\text{cost(BM)}) = 0$$

$$\ln\left(\frac{\text{cost(BM)}+\delta}{\text{cost(BM)}}\right) = \frac{\beta_{BM} - \beta_{LF}}{\beta_c}$$

δ= 13.46*(exp(2.46-0.535)/2.03)) -1) = 21.28

Elasticity for discrete choice models

- **Disaggregate elasticity:** The responsiveness of an individual's choice probability to a change in the value of some attribute.
- **Aggregate elasticity:** The responsiveness of some group of decision makers rather than that of any individual.

- Direct vs cross elasticity

	Point	Arc
Direct	$E_{ink}^{P_n(i)} = \frac{\partial P_n(i)}{\partial x_{ink}} \frac{x_{ink}}{P_n(i)}$	$\frac{\Delta P_n(i)}{\Delta x_{ink}} \frac{x_{ink}}{P_n(i)}$
Cross	$E_{jnk}^{P_n(i)} = \frac{\partial P_n(i)}{\partial x_{jnk}} \frac{x_{jnk}}{P_n(i)}$	$\frac{\Delta P_n(i)}{\Delta x_{jnk}} \frac{x_{jnk}}{P_n(i)}$

Example: Telephone Choice

- Price elasticity across the sample