HW #1 (CSE 4190.313)

Tuesday, March 31, 2020

Name: ____________________________ **ID No:** ____________________________

1. Suppose A is invertible and you exchange its i-th and j-th coulmns to reach B. Is the new matrix B invertible? Why? How would you find B^{-1} from A^{-1}?

2. True or false (with a counterexample if false and a reason if true):

   (a) A square matrix A with a column of zeros is not invertible.

   (b) If A^T is invertible then A is invertible.

3. Find the inverse of A

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ \frac{1}{4} & 1 & 0 & 0 \\ \frac{1}{3} & \frac{1}{3} & 1 & 0 \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 1 \end{bmatrix}$$

4. If A has column 1 + column 2 = column 3, show that A is not invertible:

   (a) Find a nonzero solution $\mathbf{x}$ to $A\mathbf{x} = \mathbf{0}$.

   (b) Explain why elimination keeps column 1 + column 2 = column 3.

   (c) Explain why there is no third pivot.

5. If A and B have nonzeros in the positions marked by $*$, which zeros are still zero in their factors L and U?

$$A = \begin{bmatrix} * & * & * & * \\ * & * & * & 0 \\ 0 & * & * & * \\ 0 & 0 & * & * \end{bmatrix}, \qquad B = \begin{bmatrix} * & * & * & 0 \\ * & * & 0 & * \\ * & 0 & * & * \\ 0 & * & * & * \end{bmatrix}$$

6. The less familiar form $A = LPU$ exchanges rows only at the end:

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 3 \\ 2 & 5 & 8 \end{bmatrix} \quad \rightarrow \quad L^{-1}A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 2 \\ 0 & 3 & 6 \end{bmatrix} = PU = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 0 & 3 & 6 \\ 0 & 0 & 2 \end{bmatrix}$$

What is L in this case?

7. Find the inverse of A

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ 2 & 3 & 1 & 0 \\ 2 & 3 & 4 & 1 \end{bmatrix}$$

8. (a) If P is any permutation matrix, find a nonzero vector $\mathbf{x}$ so that $(I - P)\mathbf{x} = \mathbf{0}$.

   (b) If P has 1s on the anitidiagonal from $(1, n)$ to $(n, 1)$, describe PAP.

9. (a) What matrix E has the same effect as these three steps? Subtract row 1 from row 2, subtract row 1 from row 3, then substract row 2 from row 3.

   (b) What single matrix L has the same effect as these three reverse steps? Add row 2 to row 3, add row 1 to row 3, then add row 1 to row 2.

10. (a) Explain why the inner product $\mathbf{x}^T\mathbf{y}$ of $\mathbf{x}$ and $\mathbf{y}$ equals the inner product of $P\mathbf{x}$ and $P\mathbf{y}$, where P is a permutation matrix.

(b) With $\mathbf{x}^T = (1, 2, 3)$ and $\mathbf{y}^T = (1, 4, 2)$, choose a 3×3 permutation matrix P to show that $(P\mathbf{x})^T\mathbf{y}$ is not always equal to $\mathbf{x}^T(P\mathbf{y})$.

11. (a) Suppose you solve $A\mathbf{x} = \mathbf{b}$ for three special right-hand sides $\mathbf{b}$:

$$A\mathbf{x}_1 = \mathbf{e}_1, \quad A\mathbf{x}_2 = \mathbf{e}_2, \quad A\mathbf{x}_3 = \mathbf{e}_3.$$

If the solutions $\mathbf{x}_1$, $\mathbf{x}_2$, $\mathbf{x}_3$ are the columns of a matrix X, what is AX?

(b) Find the inverses of

$$A_1 = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 2 & 0 \\ 0 & 3 & 0 & 0 \\ 4 & 0 & 0 & 0 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ 0 & 3 & 1 & 0 \\ 0 & 0 & 4 & 1 \end{bmatrix}, \quad A_3 = \begin{bmatrix} 1 & 2 & 0 & 0 \\ 3 & 4 & 0 & 0 \\ 0 & 0 & 5 & 6 \\ 0 & 0 & 7 & 8 \end{bmatrix}.$$

12. Write down the 5×5 finite-difference matrix equation ($h = \frac{1}{6}$) for

$$-\frac{d^2u}{dx^2} = f(x), \qquad \frac{du}{dx}(0) = \frac{du}{dx}(1) = 0.$$