Exercise 10.4 In the circuit in Figure 10.6, the switch is closed at time $t = 0$ and opened at $t = 1$ second. Sketch $v_C(t)$ for all times.

Solution:

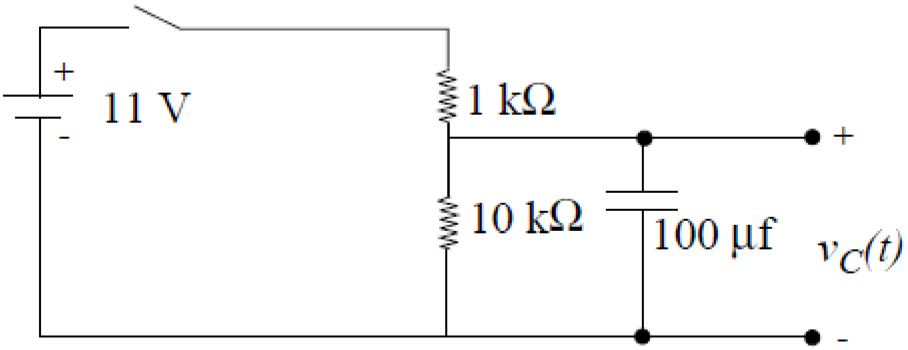


Figure 10.6:

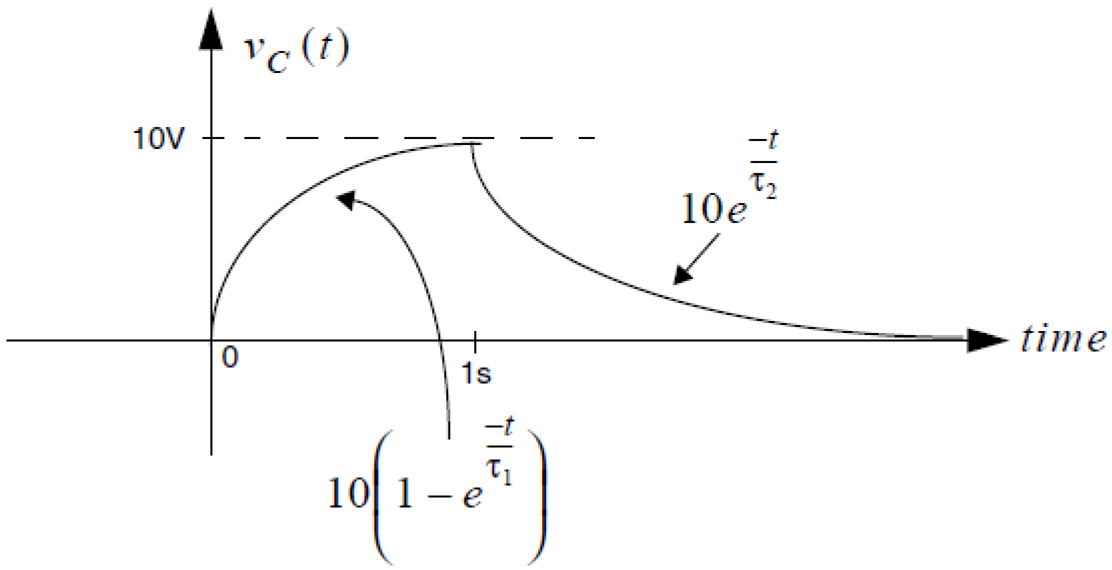


Figure 10.7:

Assume $v_C = 0$ for $t < 0$. When the switch is closed at $t = 0$, v_C rises from 0 to

$$11 \cdot \frac{10k}{10k + 1k} = 10 \text{ Volts} \quad \text{with} \;\; \tau_1 = [1k \,||\, 10k] \cdot C$$

$$\tau_1 = 9.09ms$$

When the switch is opened, v_C falls exponentially back to zero with $\tau_2 = 10k \cdot C = 1\ second$

Assuming $v_C = 0$ for $t < 0$, when the switch is closed at $t = 0$, v_C rises from 0 to 10V with $\tau_1 = \tau_1 = 9.09ms$; When the switch is opened, v_C falls exponentially back to zero with $\tau_2 = 1$ second.

Exercise 10.7 For the current source shown in Figure 10.18, assume i_S consists of a single rectangular current pulse of amplitude I_0 amps and duration t_0 seconds.

a) Find the zero-state response to i_S.

b) Sketch the zero-state response for the cases:

   i) $t_0 >> RC$

   ii) $t_0 = RC$

   iii) $t_0 << RC$

c) Show that for $t_0 << RC$, (the case of a short pulse), the response for $t > t_0$ depends only on the area of the pulse $(I_0 t_0)$, and not on i_0 or t_0 separately.

Solution:

a) v: final value resulting from pulse $= I_0 \cdot R$

initial value $= 0$ (assumed zero state)

$\underline{0 < t < t_0 : v = I_0 \cdot R \left(1 - e^{-t/\tau}\right)}$; $\tau = RC$

When the pulse stops (at $t_0 = t$), exponential decay occurs in v, with the initial value $= I_0 \cdot R\ \left(1 - e^{-t_0/RC}\right)$ and final value $= 0$.

$\underline{t > t_0 : v = I_0 \cdot R \left(1 - e^{-t_0/RC}\right) e^{-(t-t_0)/RC}}$

b) i) $t_0 >> RC$

For $t_0 \gg RC$, v reaches max value since the pulse is sufficiently long.

ii) $t_0 = RC$

$t_0 = RC$: Here the pulse is not long enough for v to exponentially rise all the way to $I_0 \cdot R$. V only reaches 63% of its maximum before decaying.

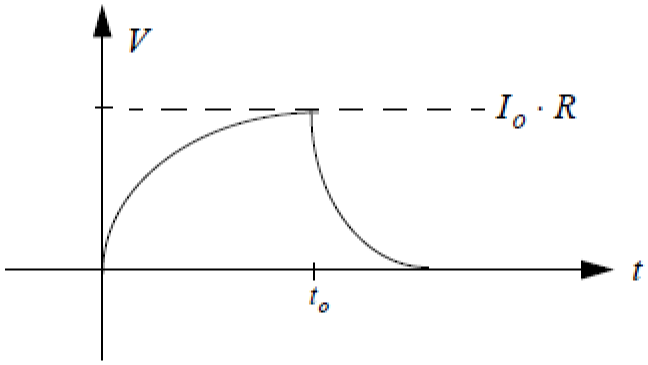


Figure 10.19:

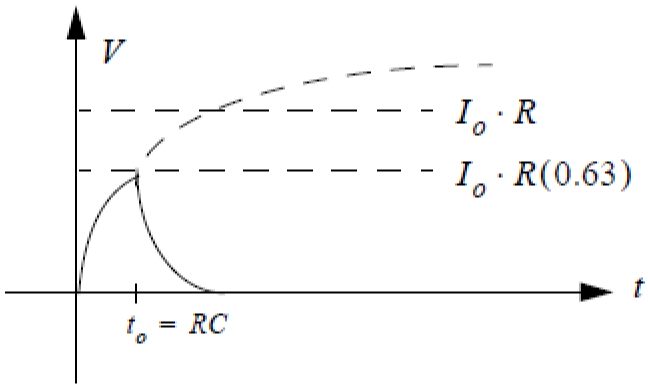


Figure 10.20:

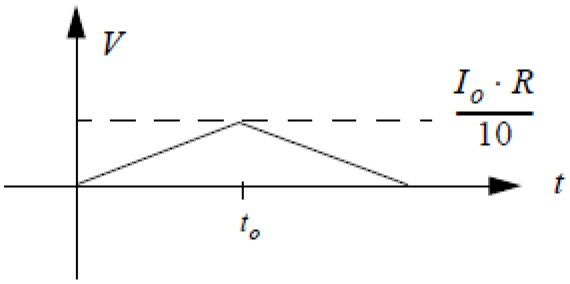


Figure 10.21:

iii) $t_0 << RC$

Here the exponential rise is very short, since the pulse is short

c) In case (iii), we see the output v for a constant pulse input is triangular, or ramped; nearly the integral of the input, i.e. proportional to the area under the input curve.

$i = v/R + C\,\frac{dv}{dt}$

$I_0 \cdot R = v + RC\,\frac{dv}{dt}$

$\frac{I_0}{C} = \frac{v}{RC} + \frac{dv}{dt}$

As RC becomes larger ($\gg t_0$), our equation can be approximated as

$$\frac{dv}{dt} = \frac{I_0}{C} \;\; \Rightarrow \;\; v = \int_0^{t_0} I_0/C$$

since $v/RC \to 0$ when RC is large.

ANS:: (a) For $0 \leq t \leq t_0$, $v = RI_0\left(1 - e^{-t/RC}\right)$, and for $t > t_0$, $v = RI_0\left(1 - e^{-t_0/RC}\right)e^{-(t-t_0)/RC}$

Problem 10.1 Figure 10.55a illustrates an inverter $INV1$ driving another inverter $INV2$. The corresponding equivalent circuit for the inverter pair is illustrated in Figure 10.55b. A, B, and C represent logical values, and v_A, v_B, and v_C represent voltage levels. The equivalent circuit model for an inverter based on the SRC model of the MOSFET is depicted in Figure 10.56.

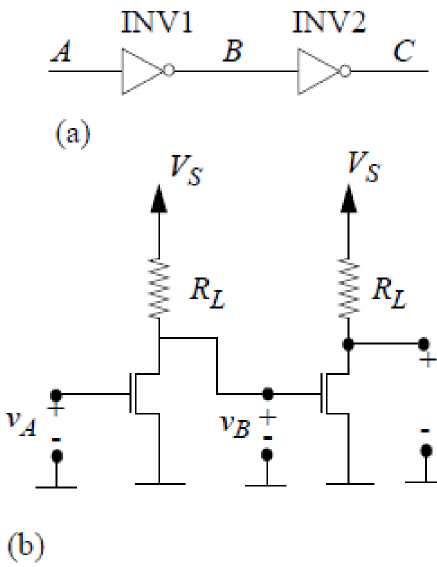


Figure 10.55:

a) Write expressions for the rise and fall times of $INV1$ for the circuit configuration shown in Figure 10.55. Assume that the inverters satisfy the static discipline with voltage thresholds $V_{IL} = V_{OL=V_L}$ and $V_{IH} = V_{OH} = V_H$.

   Hint: The rise time of $INV1$ is the time v_B requires to transition from the lowest voltage reached by v_B (given by the voltage divider action of R_L and R_{ON}) to V_H for a V_S to 0V step transition at the input v_A. Similarly, the fall time of $INV1$ is the time v_B requires to transition from the highest voltage reached by v_B (that is, V_S) to V_L for a 0V to V_S step transition at the input v_A.

b) What is the propagation delay t_{pd} of $INV1$ in the circuit configuration shown in Figure 10.55, for $R_{ON} = 1k$, $R_L = 10R_{ON}$, $C_{GS} = 1nF$, $V_S = 5V$, $V_L = 1V$, and $V_H = 3V$?

Solution:

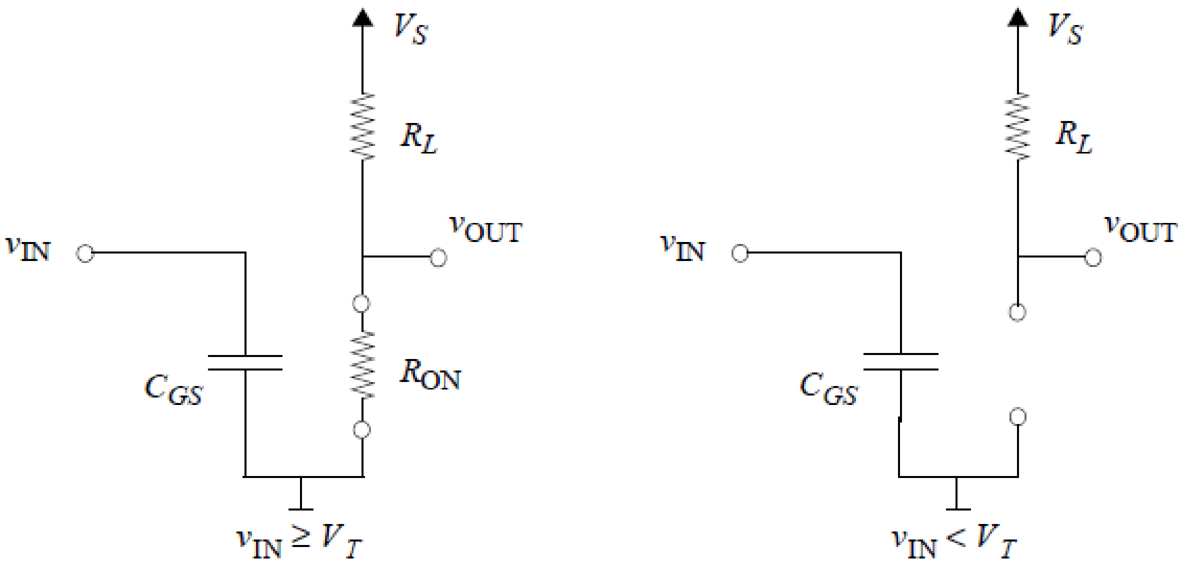


Figure 10.56:

a) For v_B going from low to high:

$$v_B = V_S + (V_S \frac{R_{ON}}{R_{ON}+R_L} - V_S)e^{-t/\tau}$$

$$t_{rise} = -\tau \ln \left(\frac{V_S - V_H}{V_S - V_S \frac{R_{ON}}{R_{ON}+R_L}} \right) \quad \tau = R_L C_{GS}$$

For v_B going from high to low:

$$v_B = V_S \frac{R_{ON}}{R_{ON}+R_L} + (V_S - V_S \frac{R_{ON}}{R_{ON}+R_L})\ e^{-t/\tau}$$

$$t_{fall} = -\tau \ln \left(\frac{V_L - V_S \frac{R_{ON}}{R_{ON}+R_L}}{V_S - V_S \frac{R_{ON}}{R_{ON}+R_L}} \right) \quad \tau = C_{GS} \frac{R_{ON} R_L}{R_{ON}+R_L}$$

$$t_{rise} = -\tau \ln \left(\frac{V_S - V_H}{V_S - V_S \frac{R_{ON}}{R_{ON}+R_L}} \right) \quad \tau = R_L C_{GS} \ t_{fall} = -\tau \ln \left(\frac{V_L - V_S \frac{R_{ON}}{R_{ON}+R_L}}{V_S - V_S \frac{R_{ON}}{R_{ON}+R_L}} \right)$$

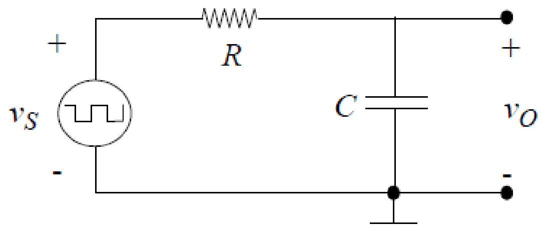
$$\tau = C_{GS} \frac{R_{ON} R_L}{R_{ON}+R_L}$$

b) $t_{pd} = t_{rise} = 8.2 \ \ \mu s$

ANS:: (a) $t_{rise} = -\tau \ln \left(\frac{V_S - V_H}{V_S - V_S \frac{R_{ON}}{R_{ON}+R_L}} \right) \quad \tau = R_L C_{GS}, \ t_{fall} = -\tau \ln \left(\frac{V_L - V_S \frac{R_{ON}}{R_{ON}+R_L}}{V_S - V_S \frac{R_{ON}}{R_{ON}+R_L}} \right) \quad \tau = C_{GS} \frac{R_{ON} R_L}{R_{ON}+R_L}$ (b) $t_{pd} = 8.2 \ \ \mu s$

Problem 10.10 As illustrated in Figure 10.80, a capacitor and resistor can be used to filter or smooth the waveforms we derived from a half-wave rectifier, to get something closer to a DC voltage at the output, for use in a power supply for example.

For simplicity, assume the voltage from source v_S is a square wave. Assume that at $t = 0$, $v_O = 0$, i.e., the circuit is at rest. Now assuming that R is small enough to make the circuit time constant much smaller than t_1 or t_2, calculate the voltage waveforms for each half cycle of the input wave. Find the average value of the output voltage v_O for



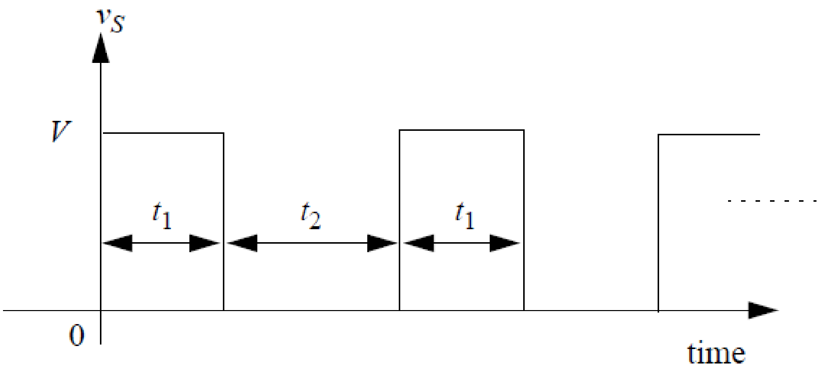


Figure 10.80:

$t_1 = t_2$. Sketch the waveforms carefully. For this choice of R, it should be clear that no useful smoothing has been accomplished.

Solution:

$0 < t < t_1$: $v_O = V(1 - e^{-\frac{t}{RC}})$

$t_1 < t < t_1 + t_2$: $v_O = Ve^{-\frac{(t-t_1)}{RC}}$

The average value of v_O is $V/2$.

See Figure 10.81.

ANS:: $0 < t < t_1$: $v_O = V(1 - e^{-\frac{t}{RC}})$, $t_1 < t < t_1 + t_2$: $v_O = Ve^{-\frac{(t-t_1)}{RC}}$

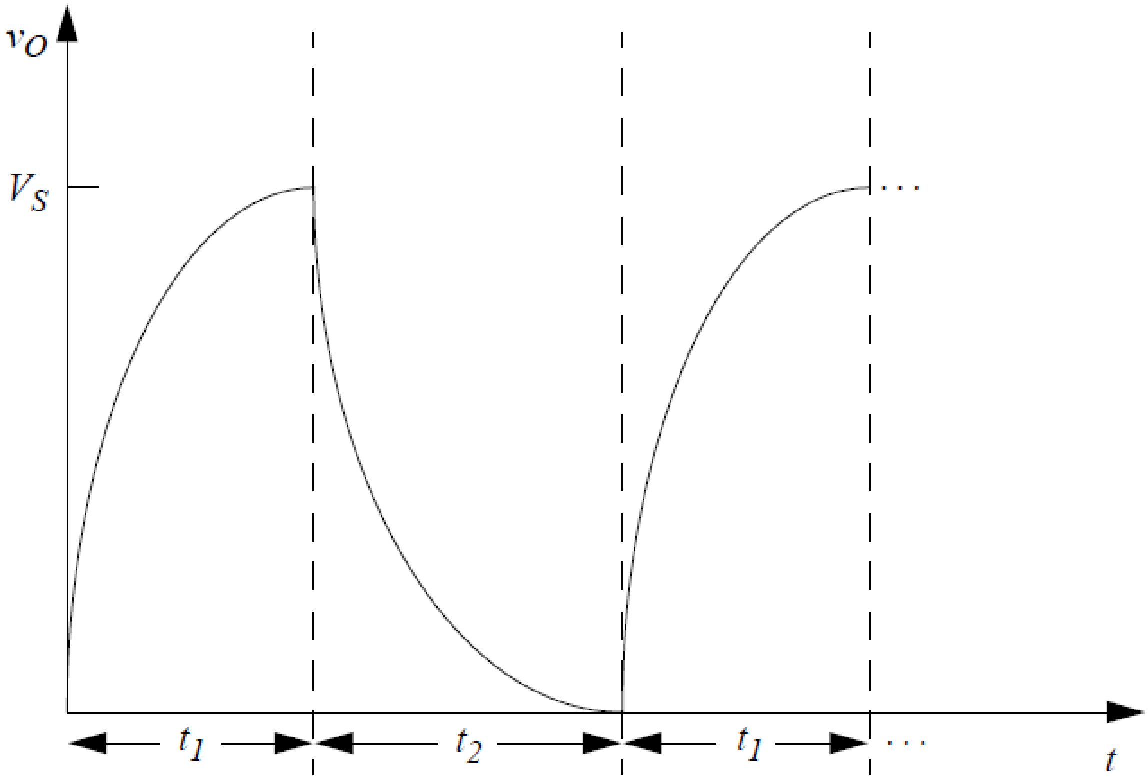


Figure 10.81:

Problem 10.20 In the circuit shown in Figure 10.107, the switch opens at $t = 0$. Sketch and label $i_L(t)$ and $v_L(t)$.

$v_1 = 5V \quad v_2 = 3V, \quad R_1 = 2k, \quad R_2 = 3k, \quad L = 4mH$

Solution:

$\tau = \frac{L}{R_1 || R_2} = 3.33s.$

$i_L(0^-) = V_1/R_1 + V_2/R_2 = 2.5mA + 1mA = 3.5mA$

$i_L(t \to \infty) = V_1/R_1 = 2.5mA$

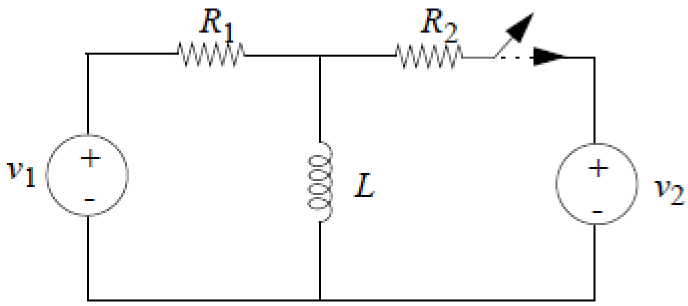


Figure 10.107:

$i_L(t) = 2.5 + e^{-t/\tau}[mA]$

$v_L(t) = L\frac{di_l}{dt} = -\frac{L}{\tau}e^{-\frac{t}{\tau}} = -2\, e^{-t/\tau}$

See Figure 10.108 and Figure 10.109.

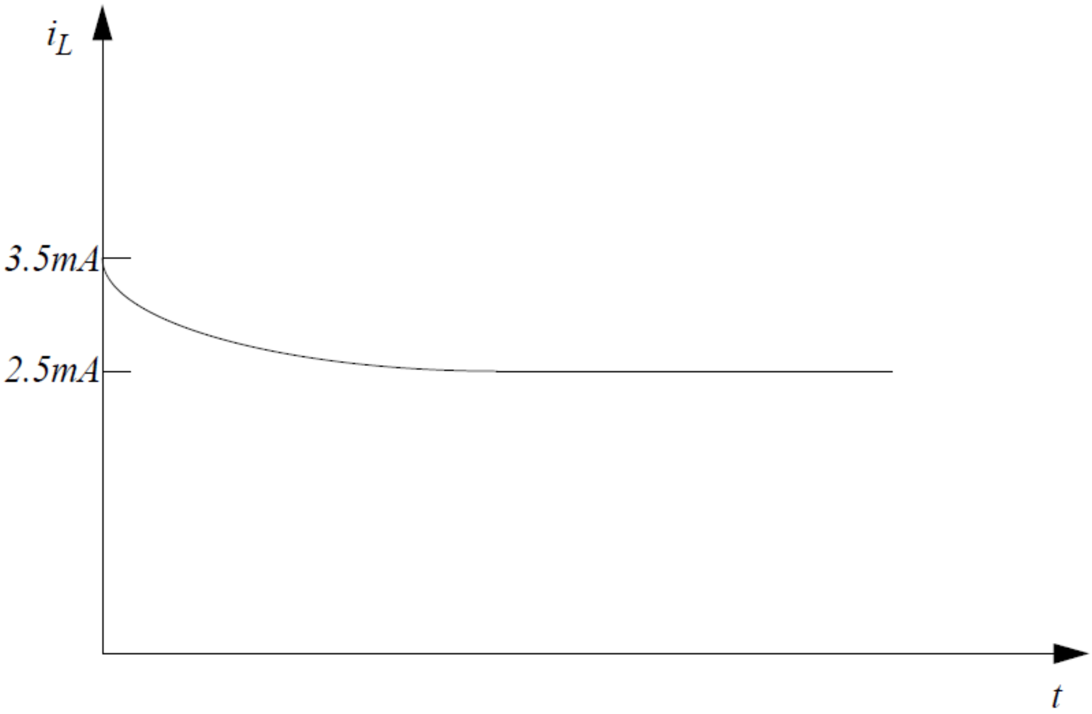


Figure 10.108:

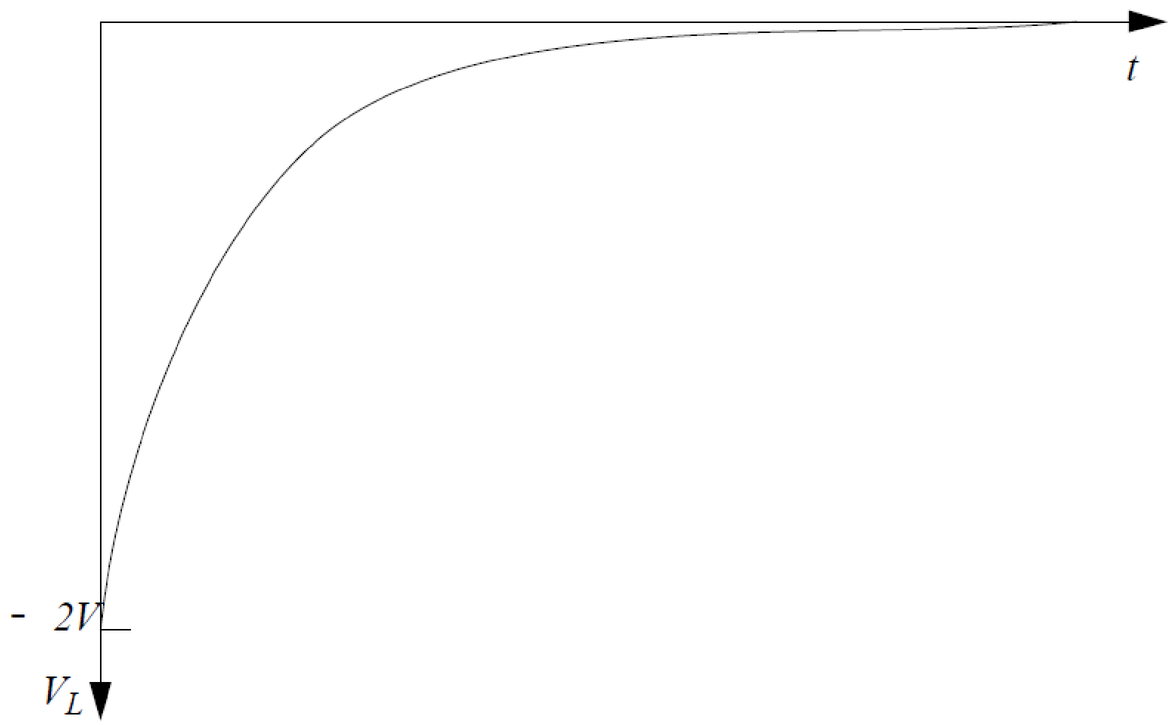


Figure 10.109:

Problem 10.22 The neon bulb in the circuit shown in Figure 10.111 has the following behavior: the bulb remains off and acts as an open circuit until the bulb voltage v reaches a threshold voltage $V_T = 65V$. Once v reaches V_T, a discharge occurs and the bulb acts like a simple resistor of value $R_N = 1k\Omega$; the discharge is maintained as long as the bulb current i remains above the value $I_S = 10mA$ needed to sustain the discharge (even if the voltage v drops below V_T). As soon as i drops below 10 mA, the bulb again becomes an open circuit.

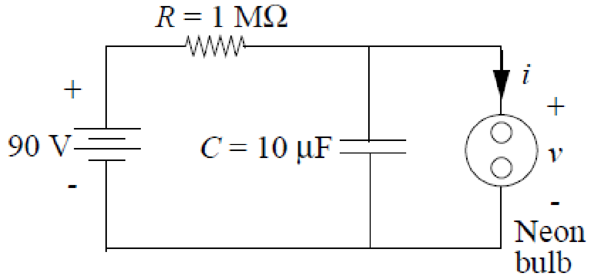


Figure 10.111:

a) Sketch and dimension $v(t)$ and $i(t)$, showing the first and second charging intervals.

b) Estimate the flashing rate.

Solution:

a) Charging ($v < 60V$):

$\tau_c = RC = (1M\Omega)(10\mu F) = 10s.$

$v_{charging} = 90(1 - e^{-t/\tau_c})$

Discharging ($i > 10mA$):

$\tau_d = R_{eq}C = \frac{1M\Omega \cdot 1k\Omega}{1M\Omega + 1k\Omega} 10\mu F = 10ms$

Note that when discharging v approaches $90\frac{1k\Omega}{1M\Omega + 1k\Omega} \cong 0$. Also note that $\tau_c \gg \tau_d$ so the charging time is much longer than the discharging time.

$v_{discharge} = 65e^{-t/\tau_d}$

The minimum v when discharging is $v_{min} = i_{min}/R = 10mA/1k\Omega = 10V$.

b) Since the discharge time is so small in comparison to the charge time, we will only consider the charge time.

After the first charging cycle, $v_{charging} = 90 + (10 - 90)e^{-t/\tau_c}$. The charging time, t_c is the amount of time it takes for $v_{charging}$ to reach 65 V.

$t_c = -\tau_c \ln\left(\frac{90-65}{80}\right) = 11.63s.$

Therefore the flashing rate is once every 11.63 s.

ANS:: (b) $1/11.63sec$