Exercise 3.3 Find the Thévenin Equivalent for each network in Figure 3.4.

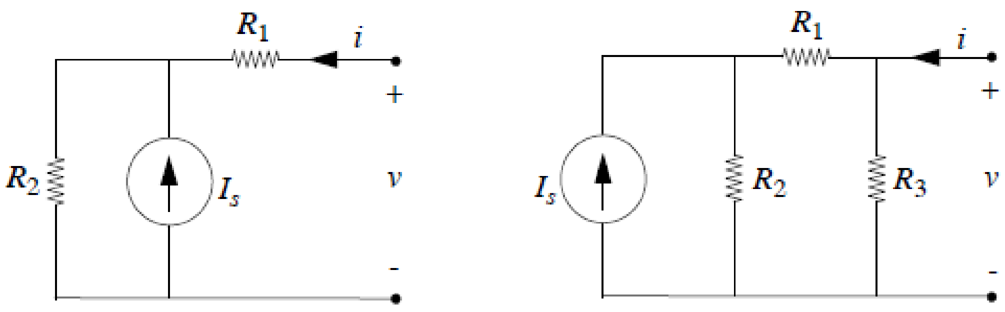

Figure 3.4:

Solution:

Left network:

$R_T = R_1 + R_2$ when I_S is made an open circuit.

$V_{OC} = I_S R_2$ since no current flows through R_1 in the open circuit case.

$R_T = R_3 || (R_1 + R_2)$ when I_S current source is made an open circuit.

Since $V_{OC} = R_3 \cdot$ (current through R_3) by Ohm's Law,

$$V_{OC} = \underbrace{\frac{I_S \cdot R_2}{R_1 + R_2 + R_3}}_{\text{current divider relation for fraction of } I_S \text{ that will flow through } R_1 \text{ and } R_3} \cdot R_3$$

ANS:: Left: $V_{OC} = I_S R_2, R_T = R_1 + R_2$, Right: $V_{OC} = \frac{I_S R_2 R_3}{R_1 + R_2 + R_3}, R_T = R_3 || (R_1 + R_2)$

Exercise 3.10 Find the Norton equivalent at the terminals marked xx in the circuit in Figure 3.12.

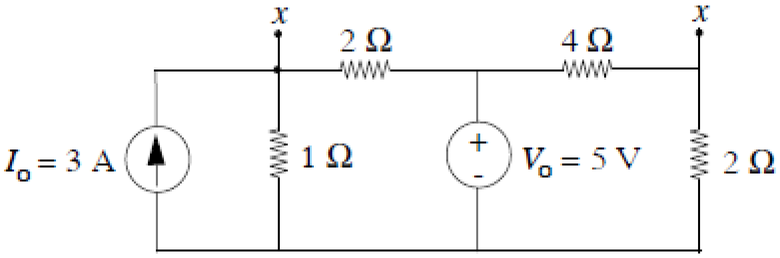

Figure 3.12:

Solution:

$$R_T = 2 \mid\mid 1 + 4 \mid\mid 2 = 2\Omega \quad \text{when both sources are "shut off"}$$

$$I_{SC} = \underbrace{1}_{\substack{\text{when}\\ \text{voltage}\\ \text{source}\\ \text{shut off}}} + \underbrace{0}_{\substack{\text{when}\\ \text{current}\\ \text{source}\\ \text{shut off}}} = 1\ A, \text{ by superposition}$$

ANS:: $R_T = 2\Omega$ and $I_{SC} == 1\ A$

Exercise 3.16 For the circuit shown in Figure 3.20, use superposition to find v in terms of the R's and source amplitudes.

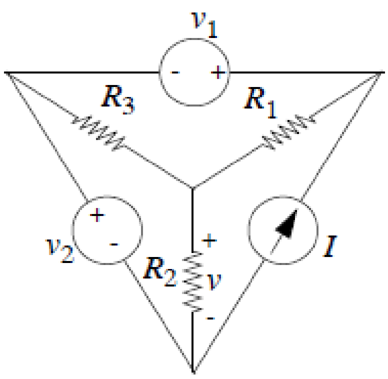

Figure 3.20:

Solution:

Redraw:

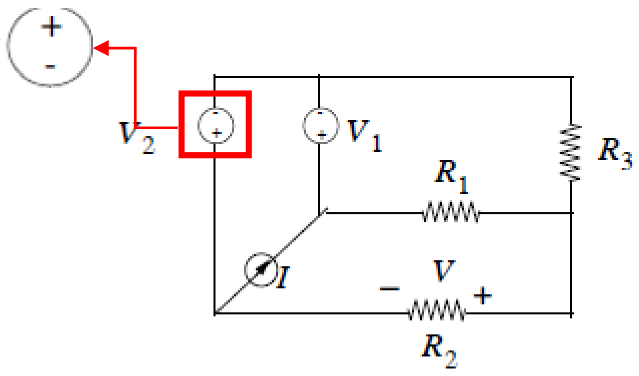

Figure 3.21:

Superposition:

1.

V_2, V_1 off; I on:

$V = 0$ since no current through R_2

2.

V_2 on; V_1 and I off:

$$V = \overset{+}{\boxed{-}} \underbrace{\frac{R_2}{R_2 + R_1 \,||\, R_3}}_{\text{voltage divider}} \cdot V_2$$

V_1 on; V_2 and I off:

$$V = \frac{R_2 \,||\, R_3}{R_1 + R_2 \,||\, R_3} \cdot V_1$$

Superposition:

$$V = V_1 \cdot \frac{R_2 \,||\, R_3}{R_1 + R_2 \,||\, R_3} \overset{+}{\boxed{-}} V_2 \cdot \frac{R_2}{R_2 + R_1 \,||\, R_3}$$

ANS:: $V = V_1 \cdot \frac{R_2 \,||\, R_3}{R_1 + R_2 \,||\, R_3} \overset{+}{\boxed{-}} V_2 \cdot \frac{R_2}{R_2 + R_1 \,||\, R_3}$

Exercise 3.25 Find the node potential E in Figure 3.39.

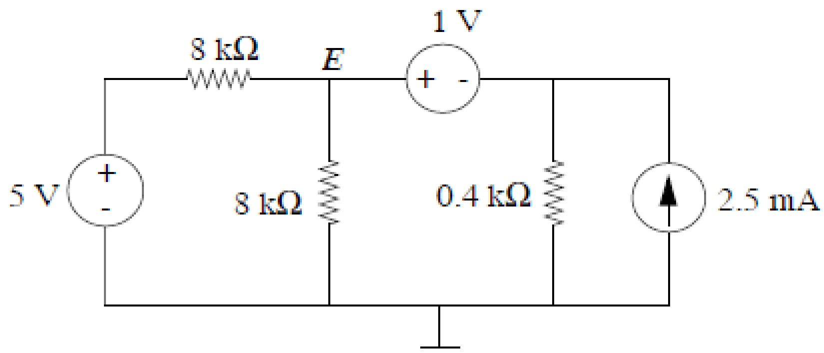

Figure 3.39:

Solution:

By superposition…

1) 5v on / 1v, 2.5mA off

$$E = 5 \times \frac{(8||0.4)}{8 + (8||0.4)} = \frac{5}{22}$$

2) 1v on / 5v, 2.5mA off

$$E = 1 \times \frac{(8||8)}{(8||8) + 0.4} = \frac{10}{11}$$

3) 2.5mA on / 1v, 5v off

$$E = 2.5 \times (8||8||0.4) = \frac{10}{11}$$

$$\therefore E = \frac{5}{22} + \frac{10}{11} + \frac{10}{11} = \frac{45}{22}$$

Problem 3.3 Find V_0 in Figure 3.49. Solve by (1) Node Method, (2) Superposition. All resistances are in Ohms.

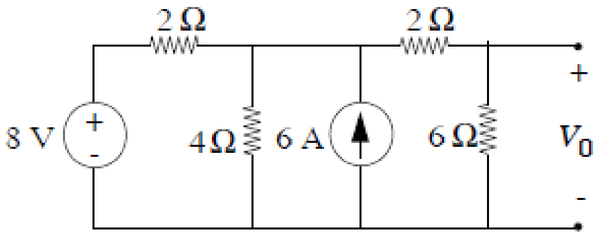

Figure 3.49:

Solution:

(1) Node Method

Label the nodes e_1 and e_2 as shown in Figure 3.50.

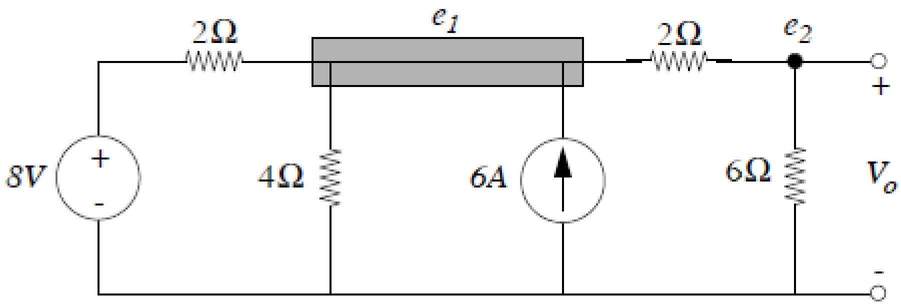

Figure 3.50:

By the node method, we obtain the following two equations:

$\frac{8\,V - e_1}{2\,Ohms} - \frac{e_1}{4\,Ohms} + 6\,A + \frac{e_2 - e_1}{2\,Ohms} = 0$

$\frac{e_1 - e_2}{2\,Ohms} - \frac{e_2}{6\,Ohms} = 0$

Thus, $V_0 = e_2 = 8.57\,V$

(2) Superposition

Find the voltage due to each source independently, as shown in Figure 3.51 and Figure 3.52.

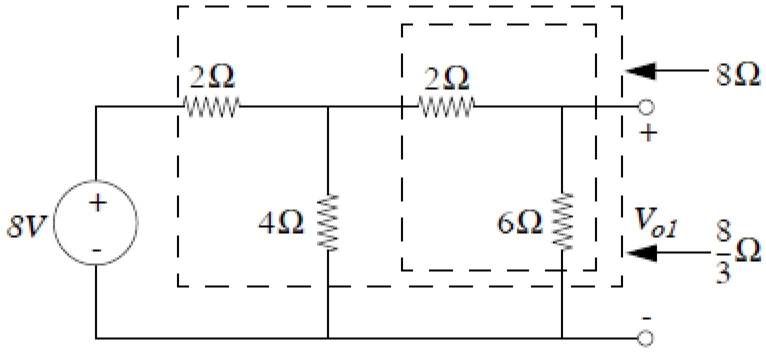

Figure 3.51:

$V_{01} = (8\ V)(\frac{\frac{8}{3}\ Ohms}{2+\frac{8}{3}\ Ohms}\frac{6\ Ohms}{8\ Ohms} = 3.43\ V$

$V_{02} = (6\ A)(\frac{\frac{4}{3}\ Ohms}{8+\frac{4}{3}\ Ohms}(6\ Ohms) = 5.14\ V$

$V_0 = V_{01} + V_{02} = 8.57\ V$

ANS:: 8.57 V

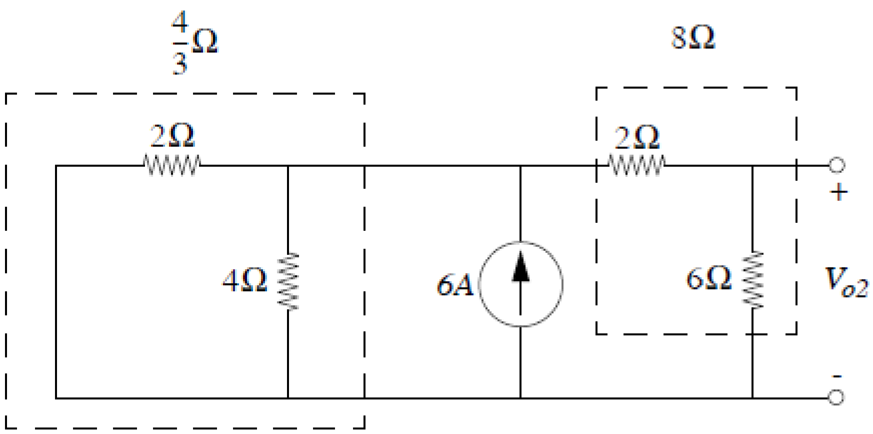

Figure 3.52:

Problem 3.8

a) Determine the equation relating i to v in Figure 3.63.

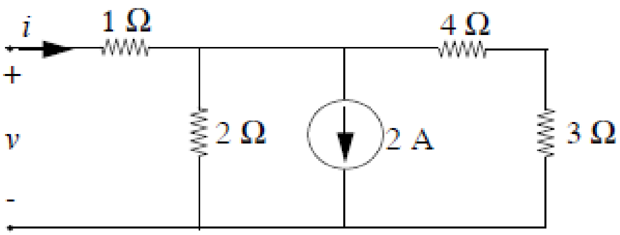

Figure 3.63:

b) Plot the i-v characteristic of the network.

c) Draw the Thévenin equivalent circuit.

d) Draw the Norton equivalent circuit.

Solution:

a) See Figure 3.64.

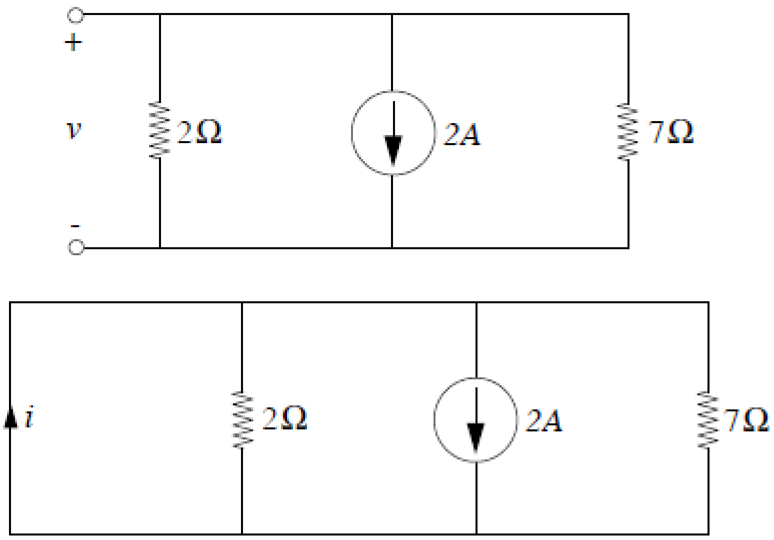

Figure 3.64:

In (i), $i = 0$, so $v = -(2\ A)\frac{(2\ Ohms)(7\ Ohms}{2+7\ Ohms} = -3.11\ V$.

In (ii), $v = 0$, so $i = (2\ A)\frac{\frac{14}{9}\ Ohms}{1+\frac{14}{9}Ohms} = 1.22\ A$.

Hence, by linearity, $v = (2.55\ Ohms)i - 3.11\ V$

b) See Figure 3.65.

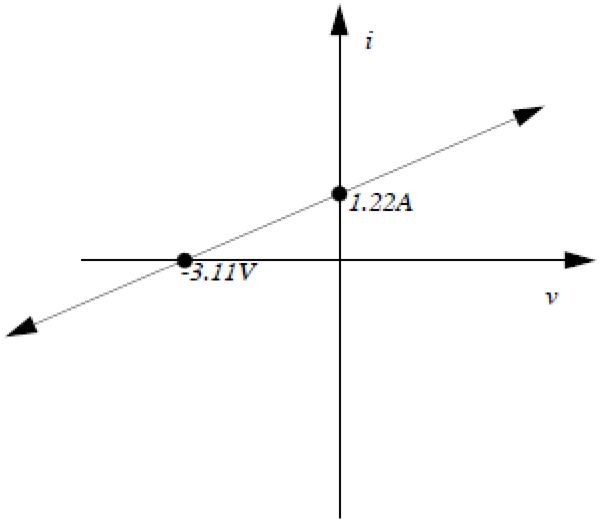

Figure 3.65:

c) See Figure 3.66.

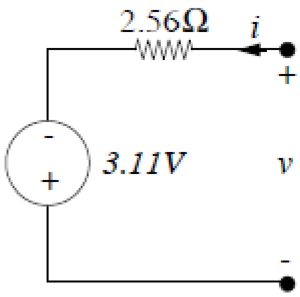

Figure 3.66:

d) See Figure 3.67.

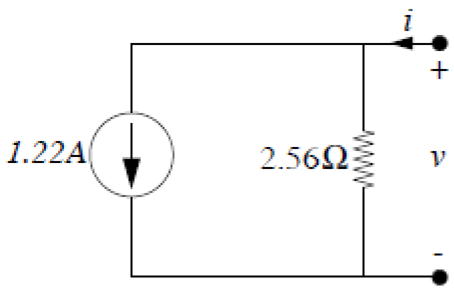

Figure 3.67:

ANS:: (a) $v = (2.55 Ohms)i - 3.11V$

Problem 3.13

a) Find the Thévenin equivalent for the network in Figure 3.80 at the terminals CB. The current source is a *controlled source*. The current flowing through the current source is βI_1, where β is some constant. (We will discuss controlled sources in more detail in the later chapters.)

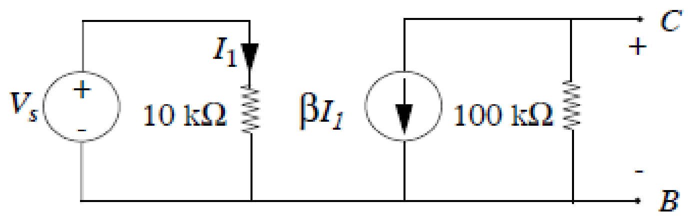

Figure 3.80:

b) Now suppose you connect a load resistor across the output of your equivalent circuit as shown in Figure 3.81. Find the value of R_L which will provide the maximum power transfer to the load.

Solution:

a) $R_{TH} = 100kOhms$

$v_T = v_{OC} = (100\ kOhms)(-\beta \frac{V_S}{10\ kOhms}) = -10\beta V_S$

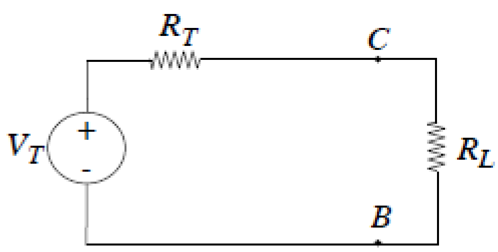

Figure 3.81:

b) $P = I^2 R = R_L(\frac{V_T}{R_T + RL})^2 = V_T^2 R_L (R_T + R_L)^{-2}$

To maximize P, we write P as a function of R_L and set its derivative with respect to R_L equal to zero. So,

$P'(R_L) = V_T^2[(R_T + R_L)^{-2} - 2R_L(R_T + R_L)^{-3}] = 0$

$\Rightarrow R_L = R_T$

ANS:: (a) $R_{TH} = 100k\Omega$, $v_T = -10\beta V_S$ (b) $R_L = R_T$