

Digital Logic Design

4190.201

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5. Combinational Logic Technologies

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Combinational Logic Technologies

- Standard gates
 - Gate packages
 - Cell libraries
- Regular logic
 - Multiplexers
 - Decoders
- Two-level programmable logic
 - PALs
 - PLAs
 - ROMs

Random Logic

- Transistors quickly integrated into logic gates (1960s)
- Catalog of common gates (1970s)
 - Texas Instruments Logic Data Book – the yellow bible
 - All common packages listed and characterized (delays, power)
 - Typical packages:
 - In 14-pin IC: 6-inverters, 4 NAND gates, 4 XOR gates
- Today, very few parts are still in use
 - Buffers survive and still evolve
 - Single gates are newly born
- However, parts libraries exist for chip design
 - Designers reuse already characterized logic gates on chips
 - Same reasons as before
 - Difference is that the parts don't exist in physical inventory – created as needed

History

- Standard parts (1960)
 - SSI and MSI
 - 7400 and 4000 series
- ROMs and PALs
 - ROMs, EPROMs and EEPROMs
 - PALs and GALs
 - PLDs
- ASICs
 - Gate arrays
 - Standard cells
- FPGAs

Technology Metrics

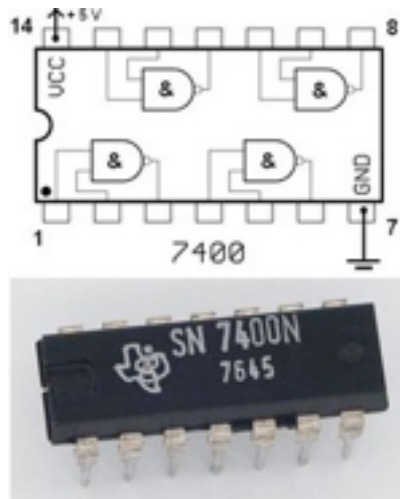
- Gate delay
- Degree of integration
- Power dissipation
- Noise margin
- Component cost
- Fan-out
- Driving capability
- Configurability

Random Logic

- Too hard to figure out exactly what gates to use
 - Map from logic to NAND/NOR networks
 - Determine minimum number of packages
 - Slight changes to logic function could decrease cost
- Changes to difficult to realize
 - Need to rewire parts
 - May need new parts
 - Design with spares (few extra inverters and gates on every board)

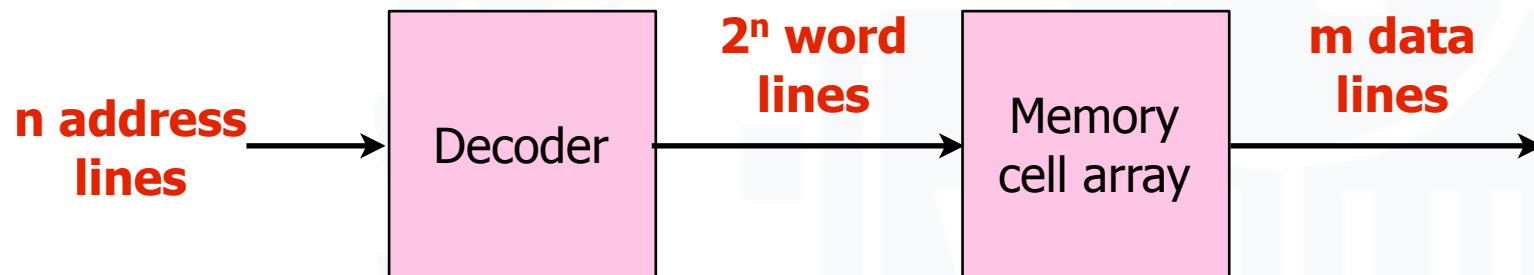
Fixed Logic

- Need to make design faster
- Need to make engineering changes easier to make
- Simpler for designers to understand and map to functionality
 - Harder to think in terms of specific gates
 - Better to think in terms of a large multi-purpose block



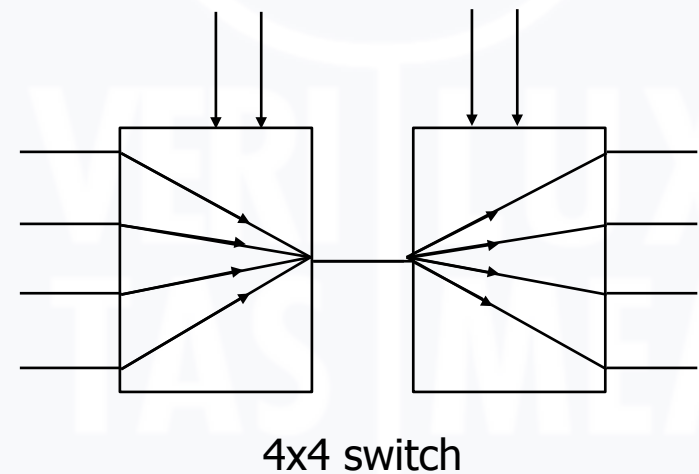
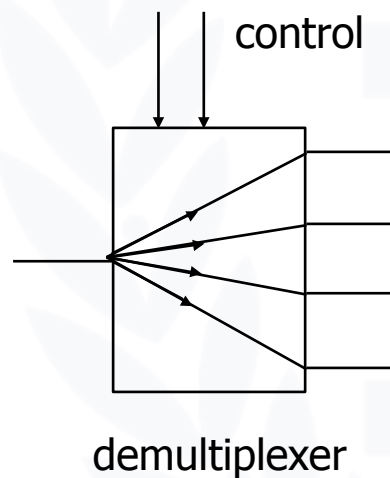
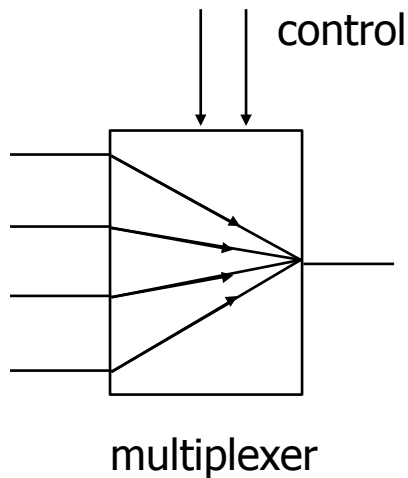
Look-up Table

- Read-only memory (ROM)



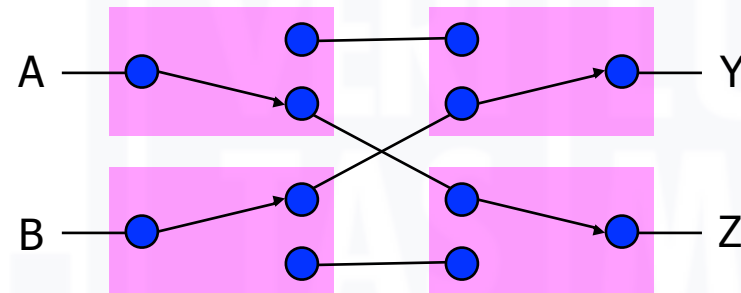
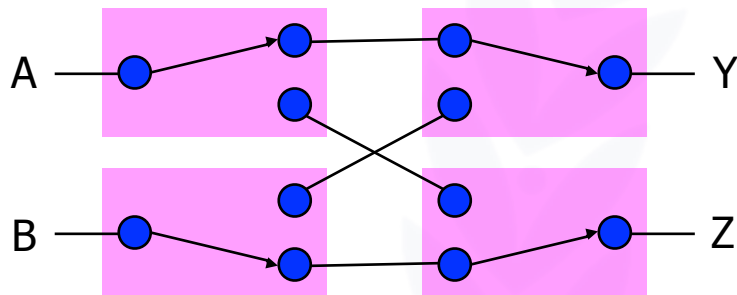
Making Connections

- Direct point-to-point connections between gates
 - Wires we've seen so far
- Route one of many inputs to a single output --- multiplexer
- Route a single input to one of many outputs --- demultiplexer



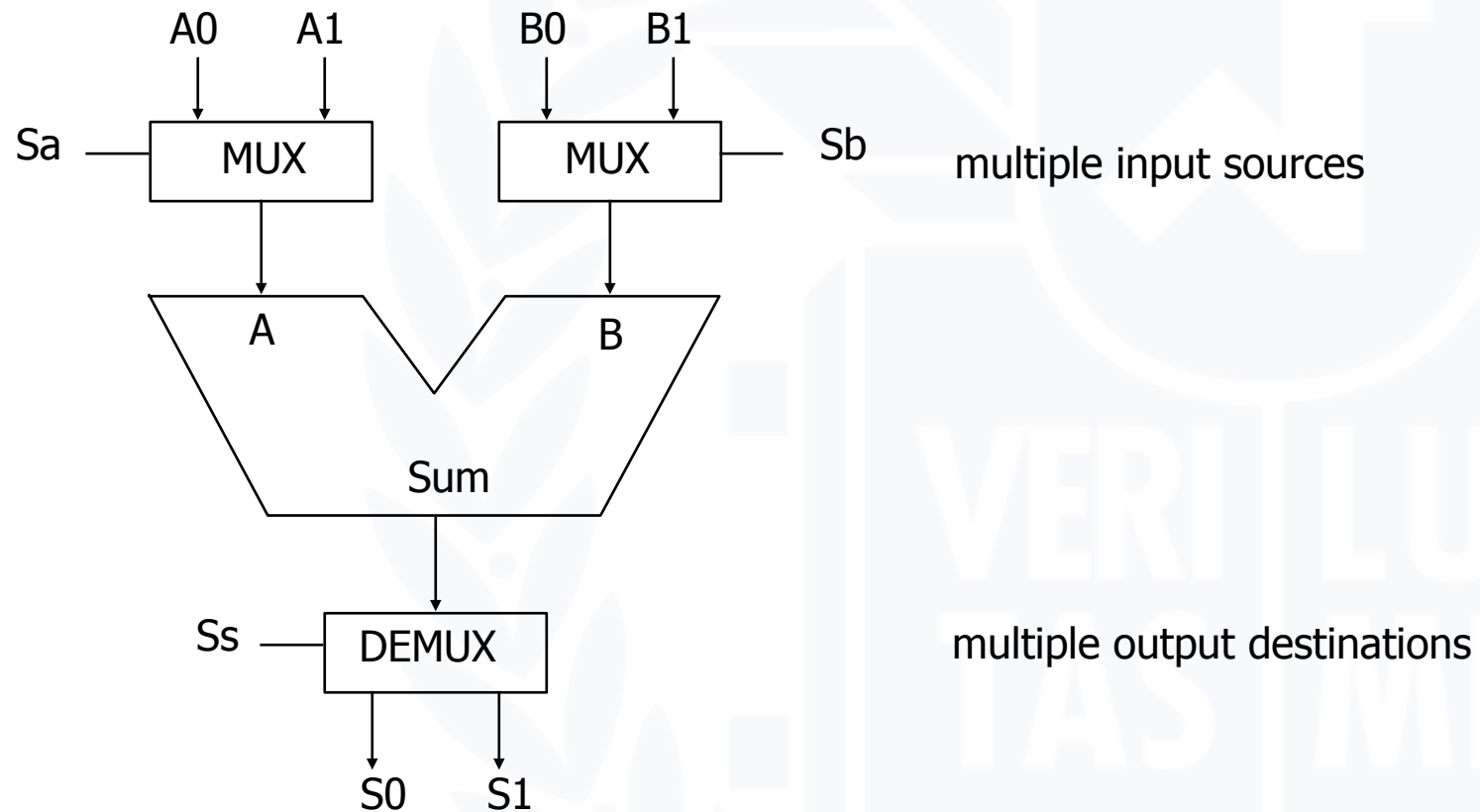
Mux and Demux

- Switch implementation of multiplexers and demultiplexers
 - Can be composed to make arbitrary size switching networks
 - Used to implement multiple-source/multiple-destination interconnections



Mux and Demux (cont'd)

- Uses of multiplexers/demultiplexers in multi-point connections



Multiplexers/Selectors

- Multiplexers/selectors: general concept
 - 2^n data inputs, n control inputs (called "selects"), 1 output
 - Used to connect 2^n points to a single point
 - Control signal pattern forms binary index of input connected to output

$$Z = A' I_0 + A I_1$$

A	Z
0	I_0
1	I_1

Functional form

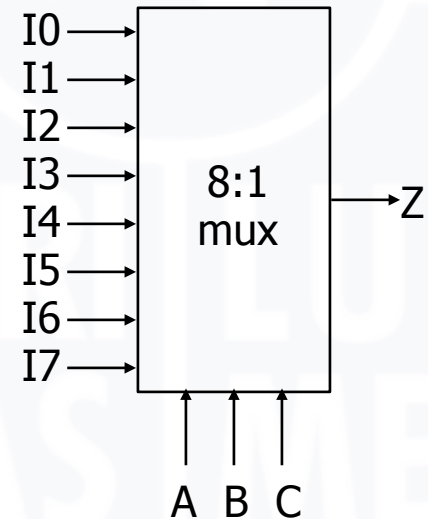
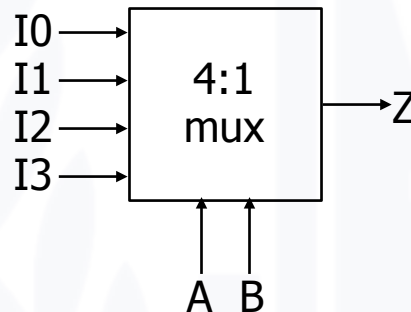
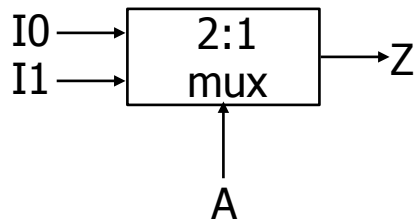
Logical form

Two alternative forms
for a 2:1 Mux truth table

I_1	I_0	A	Z
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	0
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

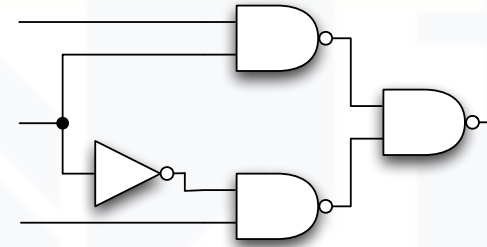
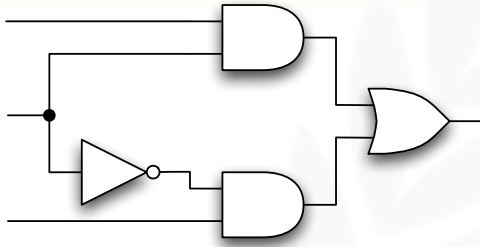
Multiplexers/Selectors (cont'd)

- 2:1 mux: $Z = A'I_0 + AI_1$
- 4:1 mux: $Z = A'B'I_0 + A'BI_1 + AB'I_2 + ABI_3$
- 8:1 mux: $Z = A'B'C'I_0 + A'B'CI_1 + A'BC'I_2 + A'BCI_3 + AB'C'I_4 + AB'CI_5 + ABC'I_6 + ABCI_7$
- In general: $Z = \sum_{k=0}^{2^n-1} (m_k I_k)$
 - In minterm shorthand form for a $2^n:1$ mux

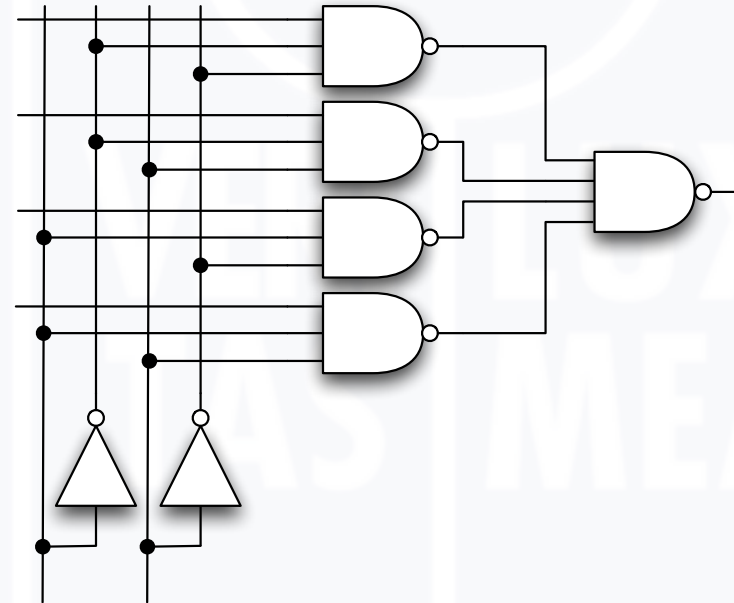
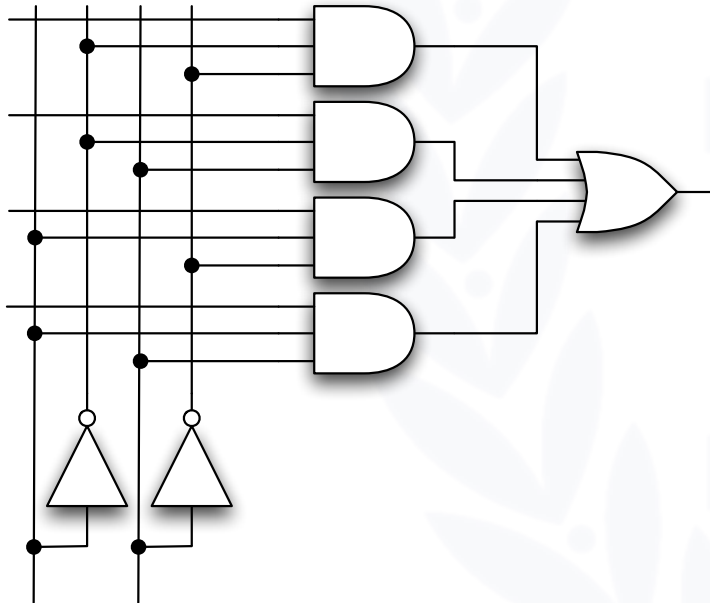


Gate Level Implementation of Muxes

2:1 mux

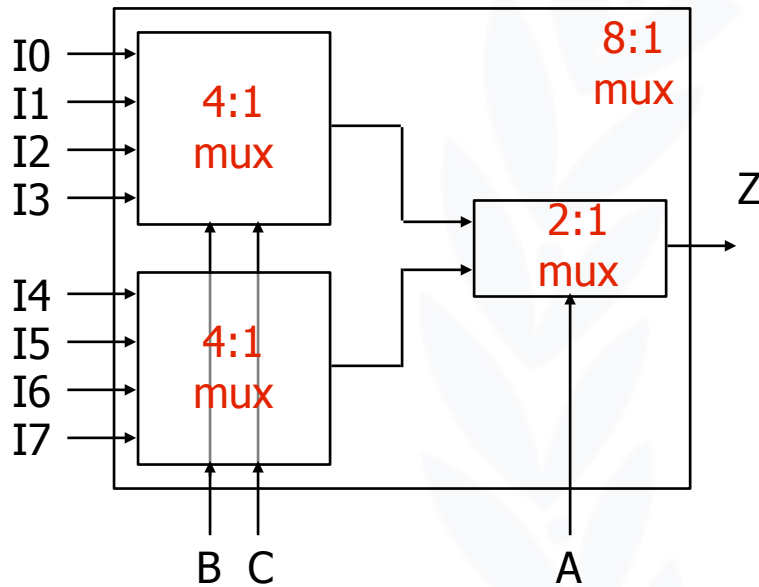


4:1 mux



Cascading Multiplexers

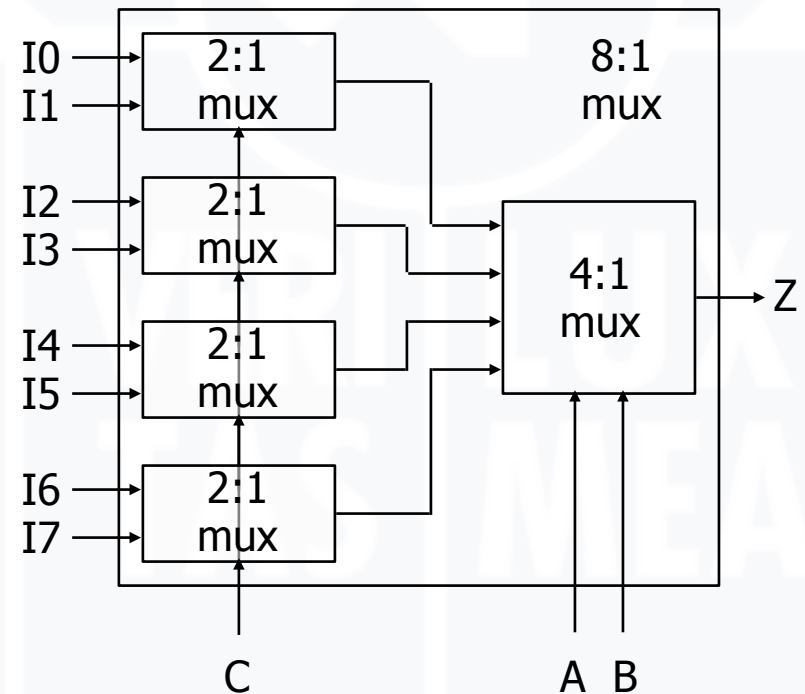
- Large multiplexers can be made by cascading smaller ones



Control signals B and C simultaneously choose one of I0, I1, I2, I3 and one of I4, I5, I6, I7

Control signal A chooses which of the upper or lower mux's output to gate to Z

Alternative implementation



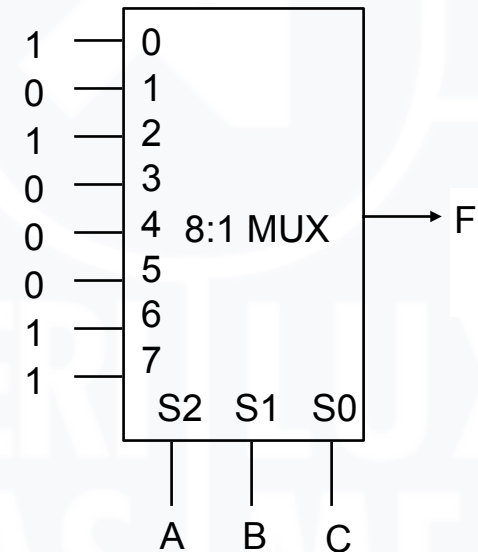
Multiplexers as General-Purpose Logic

- A $2^n:1$ multiplexer can implement any function of n variables
 - With the variables used as control inputs and
 - The data inputs tied to 0 or 1
 - In essence, a lookup table

- Example:

$$\begin{aligned} F(A,B,C) &= m_0 + m_2 + m_6 + m_7 \\ &= A'B'C' + A'BC' + ABC' + ABC \\ &= A'B'C'(1) + A'B'C(0) \\ &\quad + A'BC'(1) + A'BC(0) \\ &\quad + AB'C'(0) + AB'C(0) \\ &\quad + ABC'(1) + ABC(1) \end{aligned}$$

$$\begin{aligned} Z &= A'B'C'I_0 + A'B'CI_1 + A'BC'I_2 + A'BCI_3 + \\ &\quad AB'C'I_4 + AB'CI_5 + ABC'I_6 + ABCI_7 \end{aligned}$$



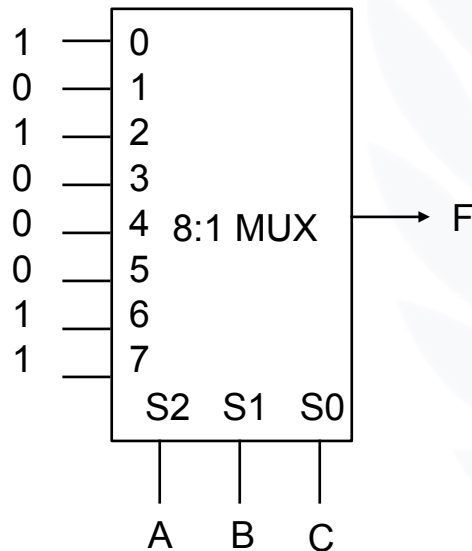
Multiplexers as General-Purpose Logic (cont'd)

- A $2^n-1:1$ multiplexer can implement any function of n variables

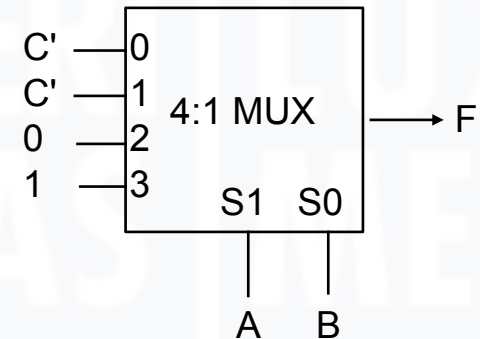
- With $n-1$ variables used as control inputs and
- The data inputs tied to the last variable or its complement

- Example:

$$\begin{aligned}
 F(A,B,C) &= m_0 + m_2 + m_6 + m_7 \\
 &= A'B'C' + A'BC' + ABC' + ABC \\
 &= A'B'(C') + A'B(C') + AB'(0) + AB(1)
 \end{aligned}$$

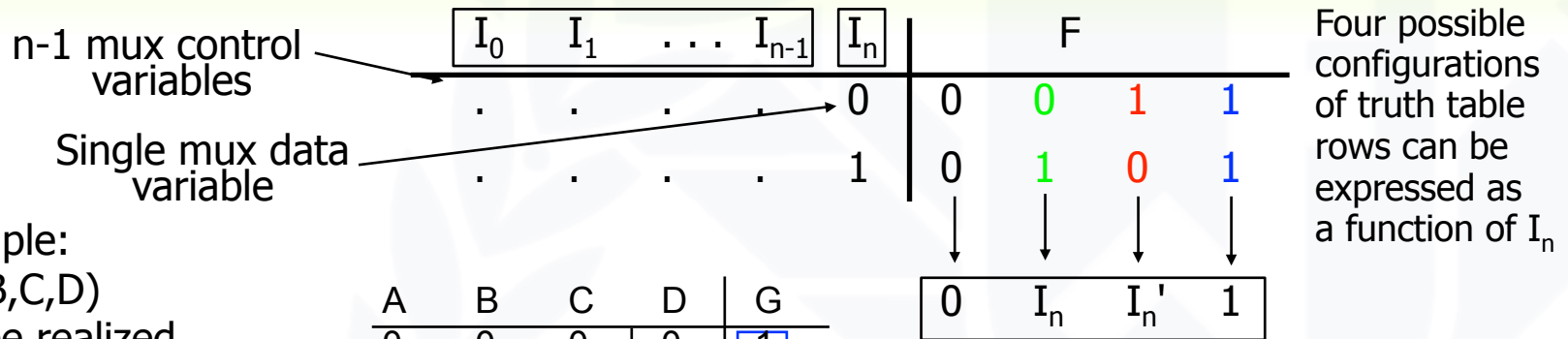


A	B	C	F
0	0	0	1
0	0	1	0
0	1	0	1
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	1



Multiplexers as General-Purpose Logic (cont'd)

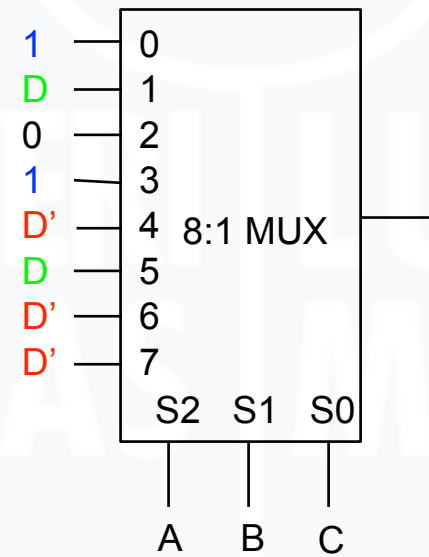
Generalization



Example:
 $G(A,B,C,D)$
 can be realized
 by an 8:1 MUX

Choose A,B,C as
 control variables

A	B	C	D	G
0	0	0	0	1
0	0	0	1	1
0	0	1	0	0
0	0	1	1	1
0	1	0	0	0
0	1	0	1	0
0	1	1	0	1
0	1	1	1	1
1	0	0	0	1
1	0	0	1	0
1	0	1	0	0
1	0	1	1	1
1	1	0	0	1
1	1	0	1	0
1	1	1	0	1
1	1	1	1	0



Activity

- Realize $F = B'CD' + ABC'$ with a 4:1 multiplexer and a minimum of other gates:

Demultiplexers/Decoders

- Decoders/demultiplexers: general concept
 - Single data input, n control inputs, 2^n outputs
 - Control inputs (called "selects" (S)) represent binary index of output to which the input is connected
 - Data input usually called "enable" (G)

1:2 Decoder:

$$O0 = G \cdot S'$$

$$O1 = G \cdot S$$

2:4 Decoder:

$$O0 = G \cdot S1' \cdot S0'$$

$$O1 = G \cdot S1' \cdot S0$$

$$O2 = G \cdot S1 \cdot S0'$$

$$O3 = G \cdot S1 \cdot S0$$

3:8 Decoder:

$$O0 = G \cdot S2' \cdot S1' \cdot S0'$$

$$O1 = G \cdot S2' \cdot S1' \cdot S0$$

$$O2 = G \cdot S2' \cdot S1 \cdot S0'$$

$$O3 = G \cdot S2' \cdot S1 \cdot S0$$

$$O4 = G \cdot S2 \cdot S1' \cdot S0'$$

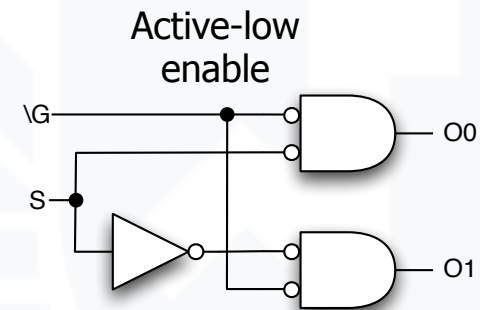
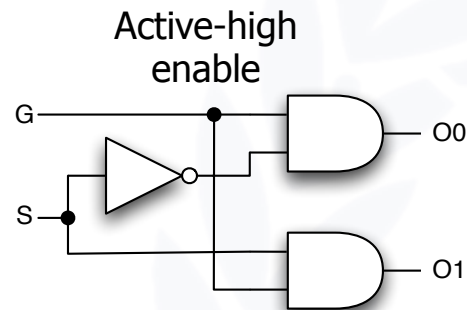
$$O5 = G \cdot S2 \cdot S1' \cdot S0$$

$$O6 = G \cdot S2 \cdot S1 \cdot S0'$$

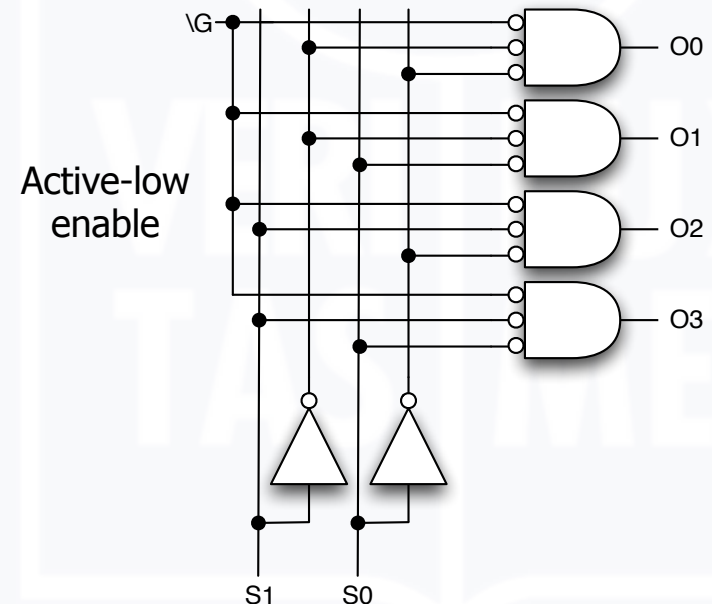
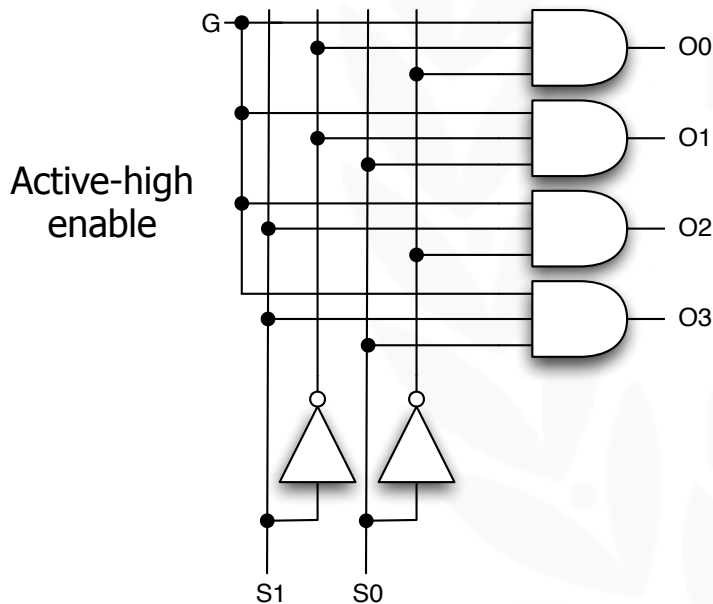
$$O7 = G \cdot S2 \cdot S1 \cdot S0$$

Gate Level Implementation of Demultiplexers

1:2 decoders

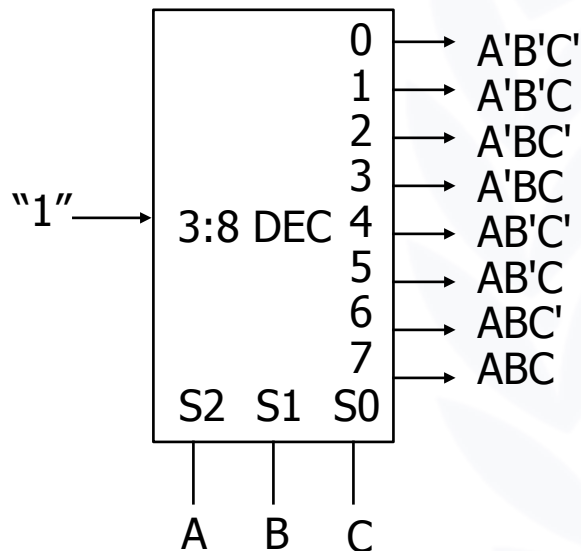


2:4 decoders



Demultiplexers as General-Purpose Logic

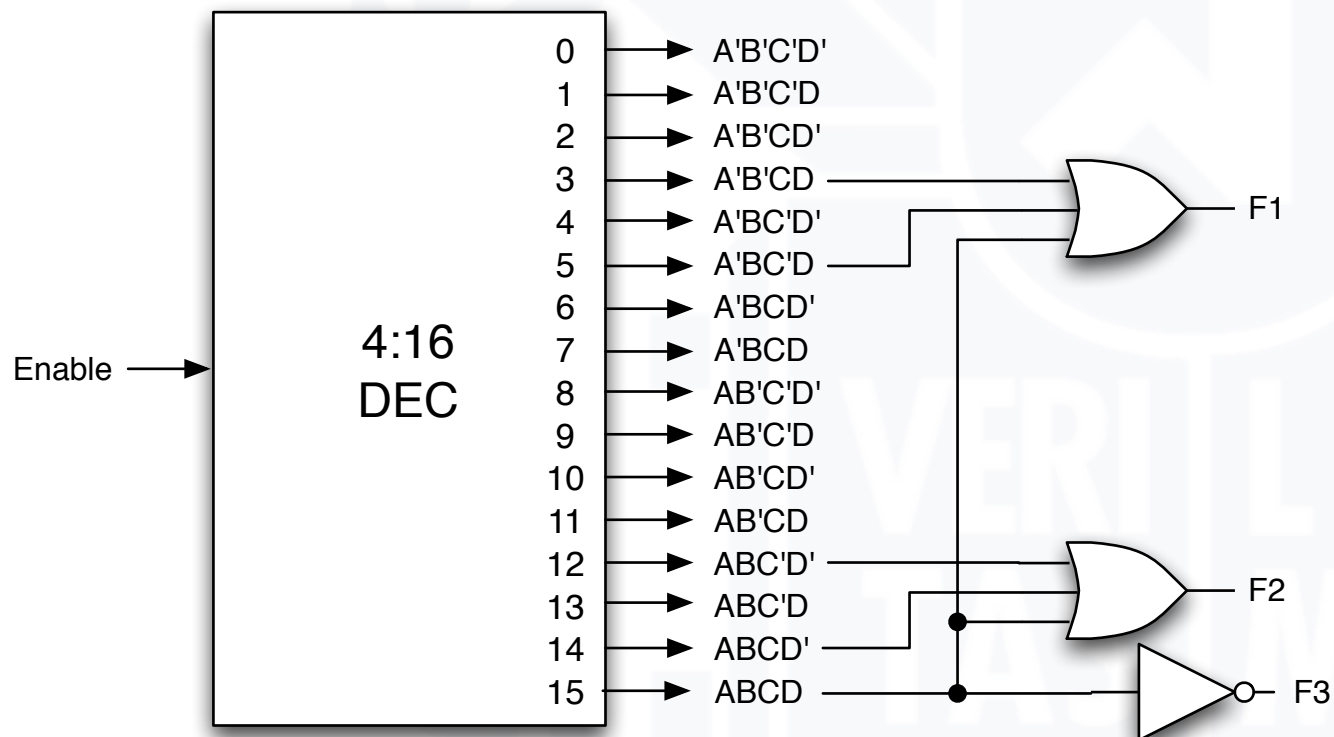
- A $n:2^n$ decoder can implement any function of n variables
 - With the variables used as control inputs
 - The enable inputs tied to 1 and
 - The appropriate minterms summed to form the function



Demultiplexer generates appropriate minterm based on control signals (it "decodes" control signals)

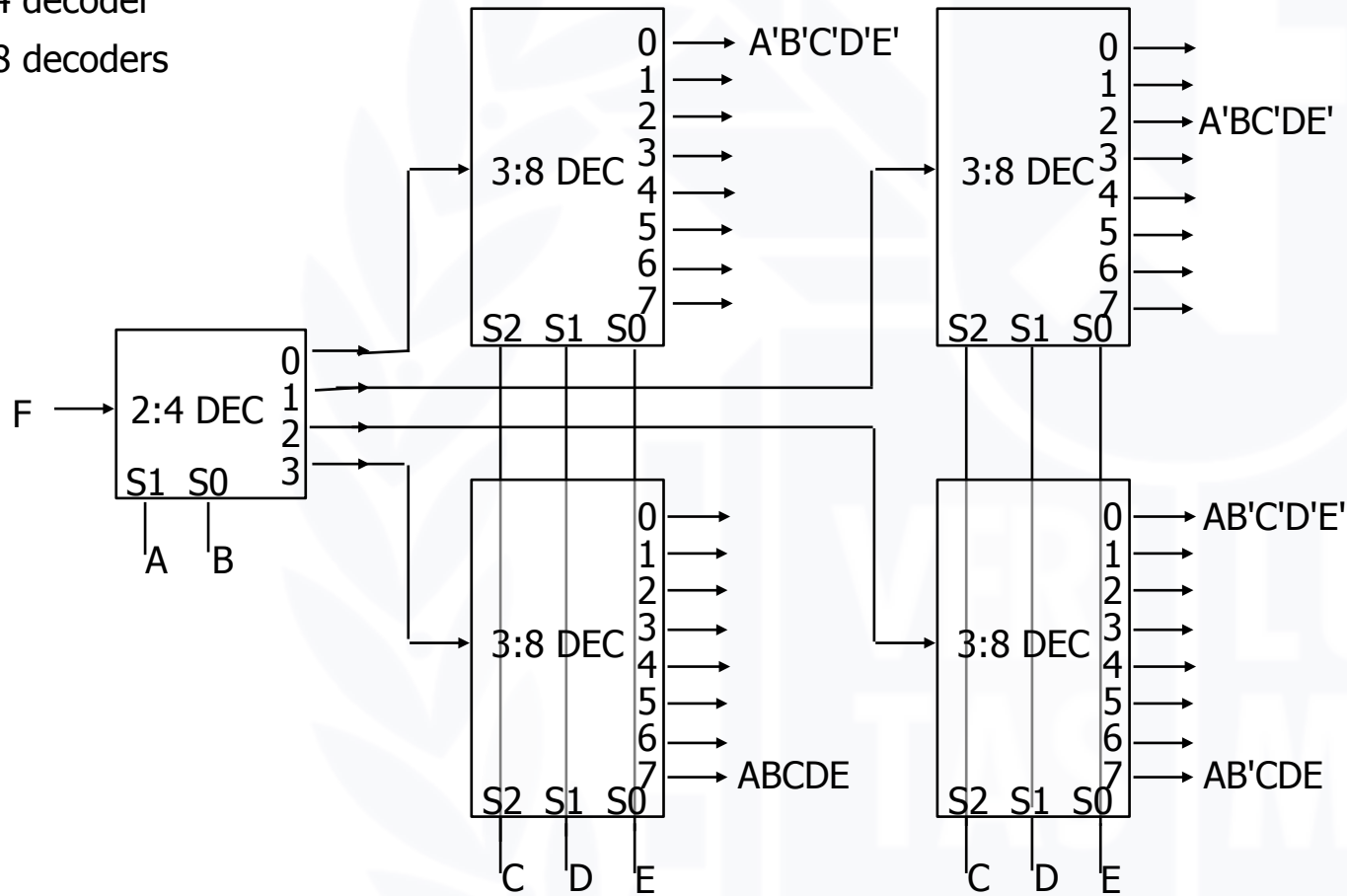
Demultiplexers as General-Purpose Logic (cont'd)

- $F1 = A'BC'D + A'B'CD + ABCD$
- $F2 = ABC'D' + ABC$
- $F3 = (A' + B' + C' + D')$



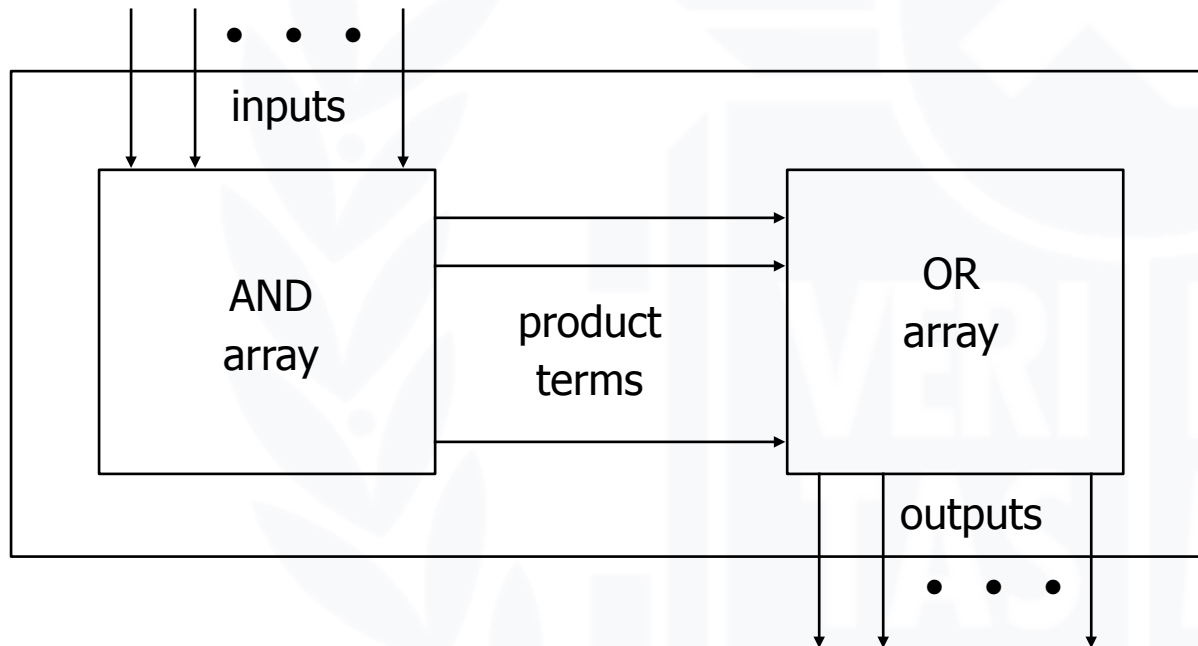
Cascading Decoders

- 5:32 decoder
- 1x2:4 decoder
- 4x3:8 decoders



Programmable Logic Arrays

- Pre-fabricated building block of many AND/OR gates
 - Actually NOR or NAND
 - "Personalized" by making/breaking connections among the gates
 - Programmable array block diagram for sum of products form



Enabling Concept

- Shared product terms among outputs

example:

$$\begin{aligned} F0 &= A + B' C' \\ F1 &= A C' + A B \\ F2 &= B' C' + A B \\ F3 &= B' C + A \end{aligned}$$

personality matrix

product term	inputs			outputs			
	A	B	C	F0	F1	F2	F3
AB	1	1	–	0	1	1	0
B'C	–	0	1	0	0	0	1
AC'	1	–	0	0	1	0	0
B'C'	–	0	0	1	0	1	0
A	1	–	–	1	0	0	1

input side:

1 = uncomplemented in term

0 = complemented in term

– = does not participate

output side:

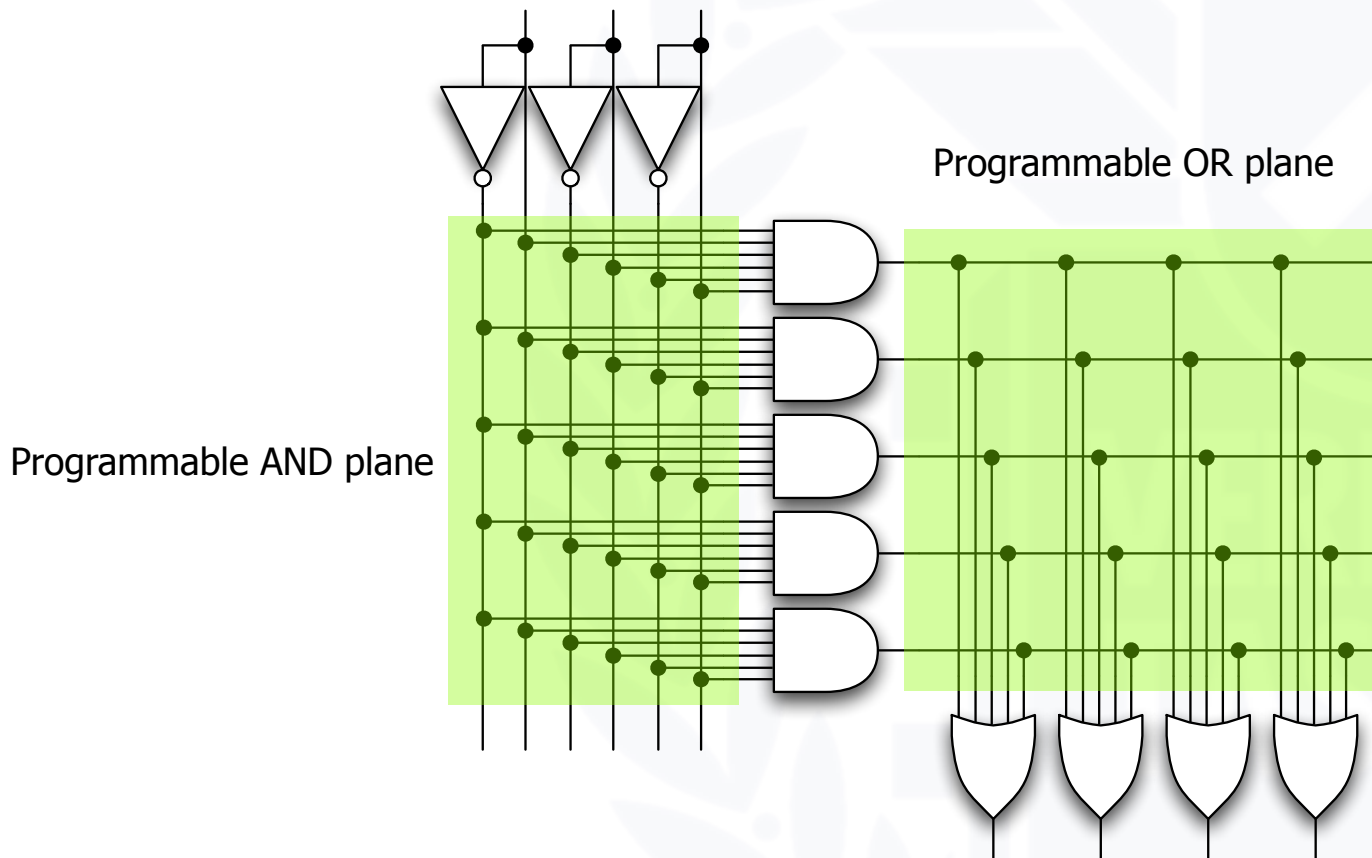
1 = term connected to output

0 = no connection to output

reuse of terms

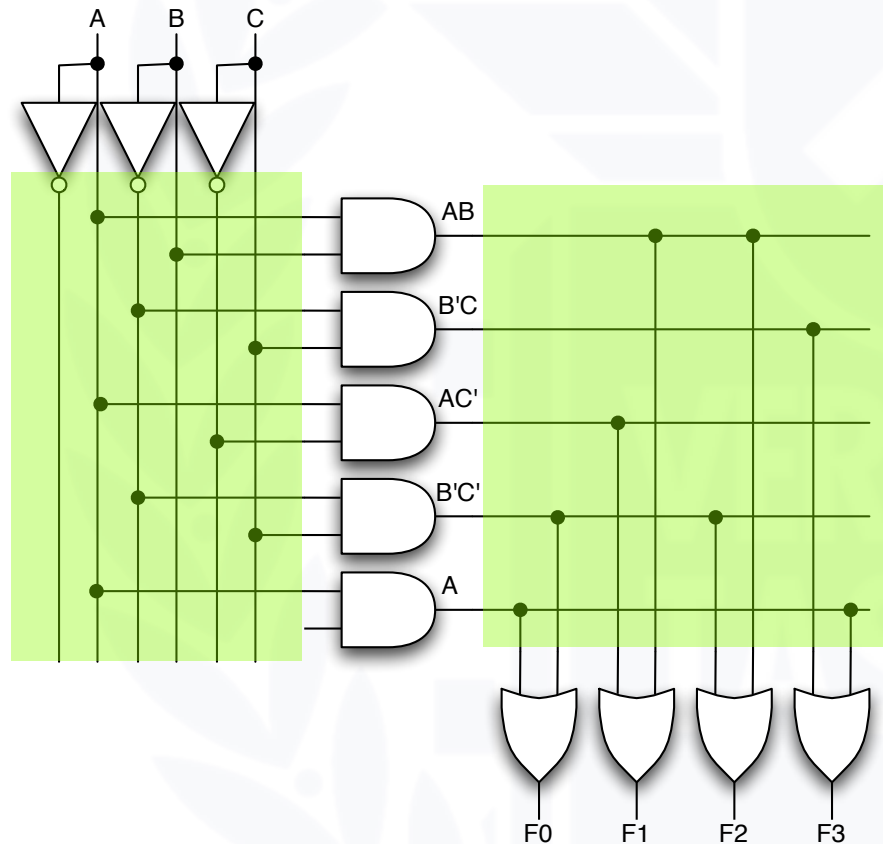
Before Programming

- All possible connections are available before "programming"
- In reality, all AND and OR gates are NANDs



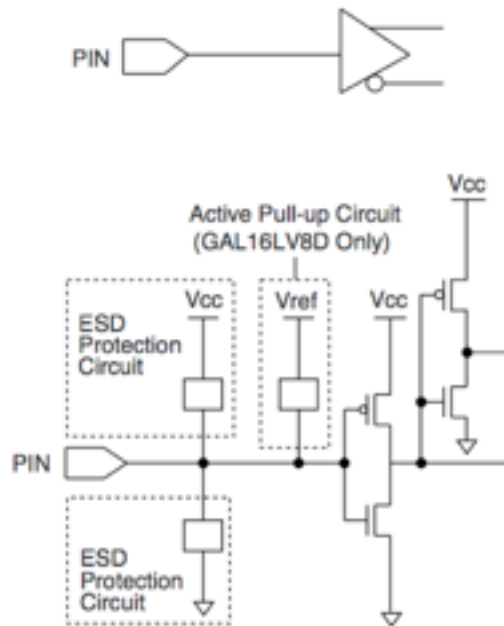
After Programming

- Unwanted connections are "blown"
- Fuse (normally connected, break unwanted ones)
- Anti-fuse (normally disconnected, make wanted connections)



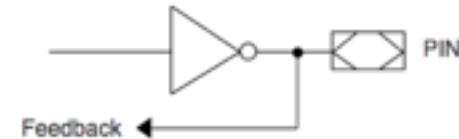
Input/Output Buffers

I/O buffer structures



Typ. Vref = Vcc

Typical Input

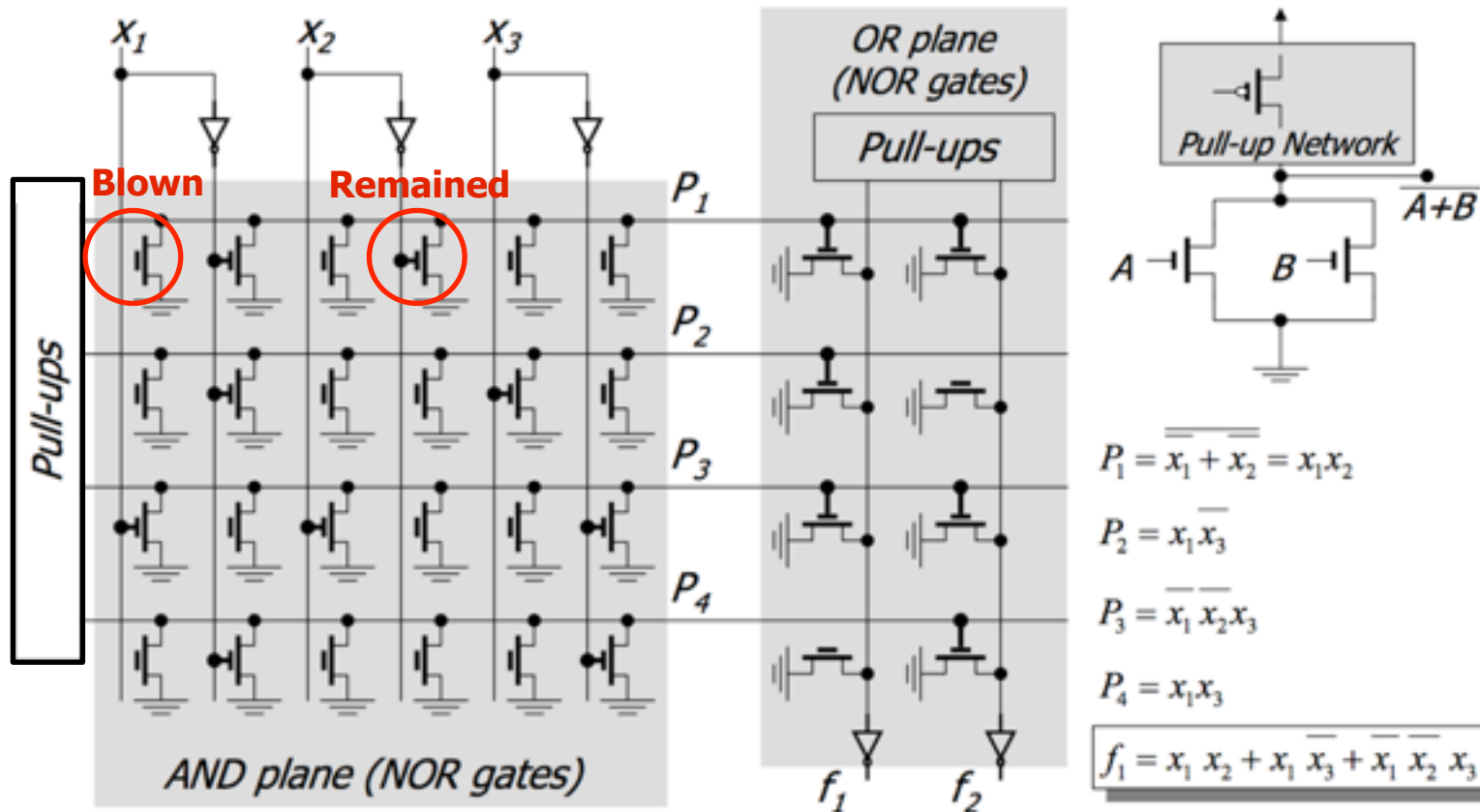


Typ. Vref = Vcc

Typical Output

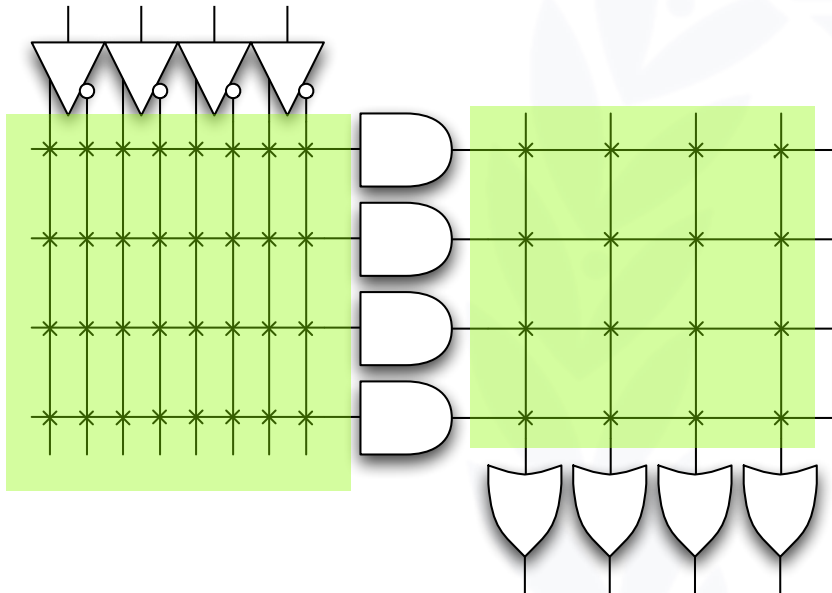
Internal Structure

- Internal AND-array and OR-array structures



Alternate Representation for High Fan-in Structures

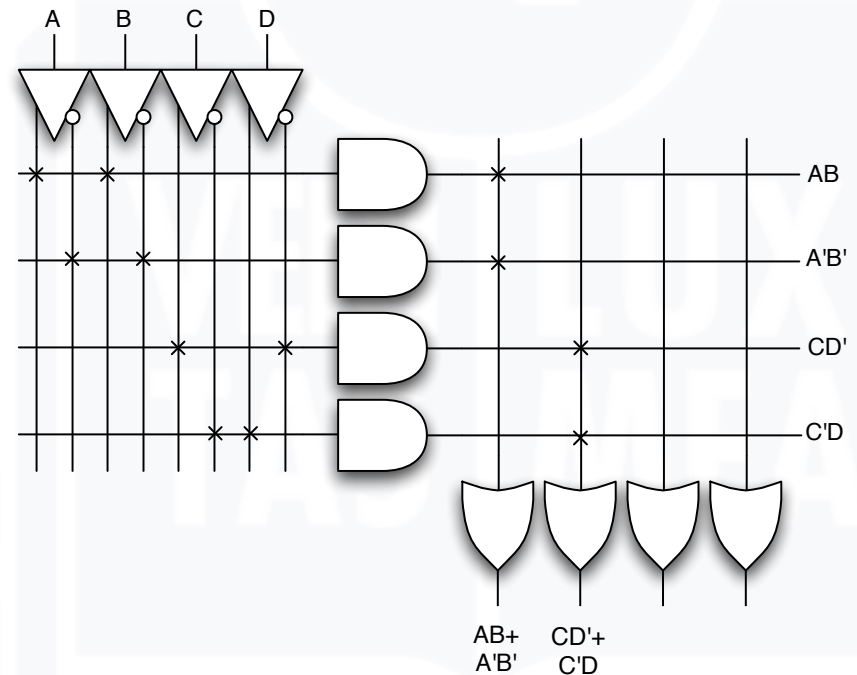
- Short-hand notation so we don't have to draw all the wires
- Signifies a connection is present and perpendicular signal is an input to gate



Notation for implementing

$$F0 = A B + A' B'$$

$$F1 = C D' + C' D$$

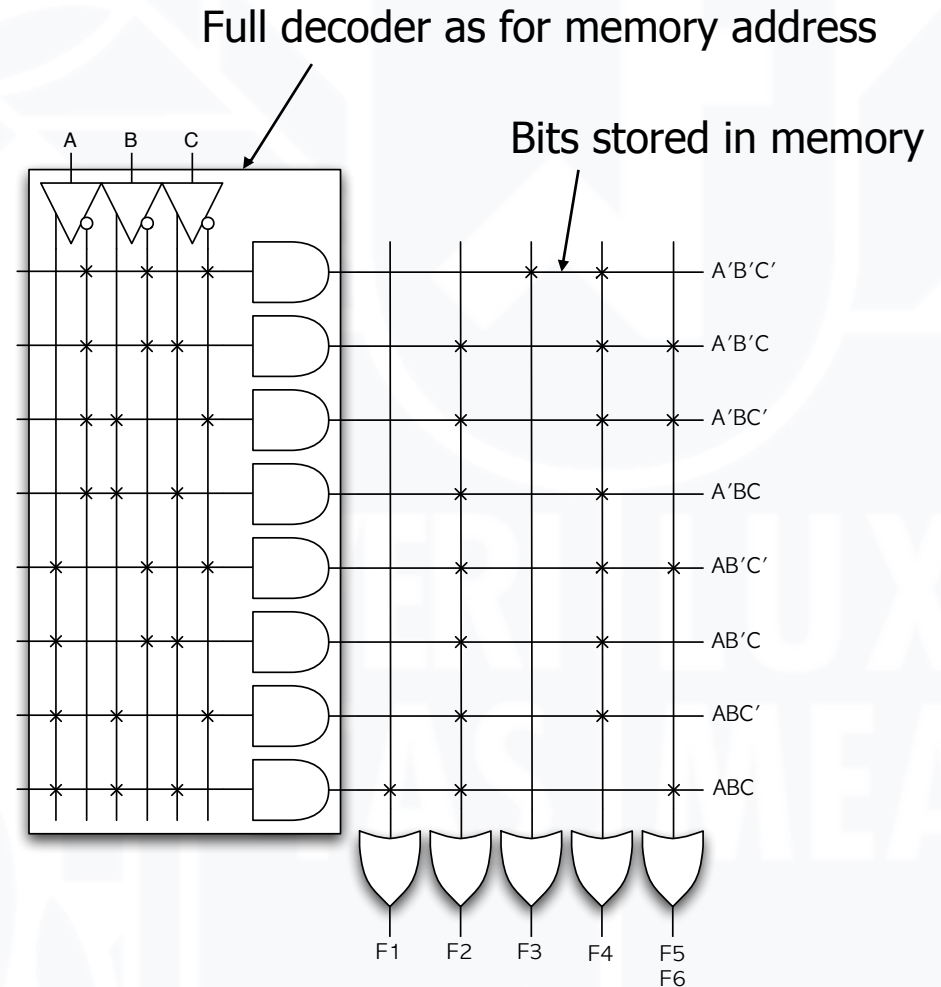


Programmable Logic Array Example

Multiple functions of A, B, C

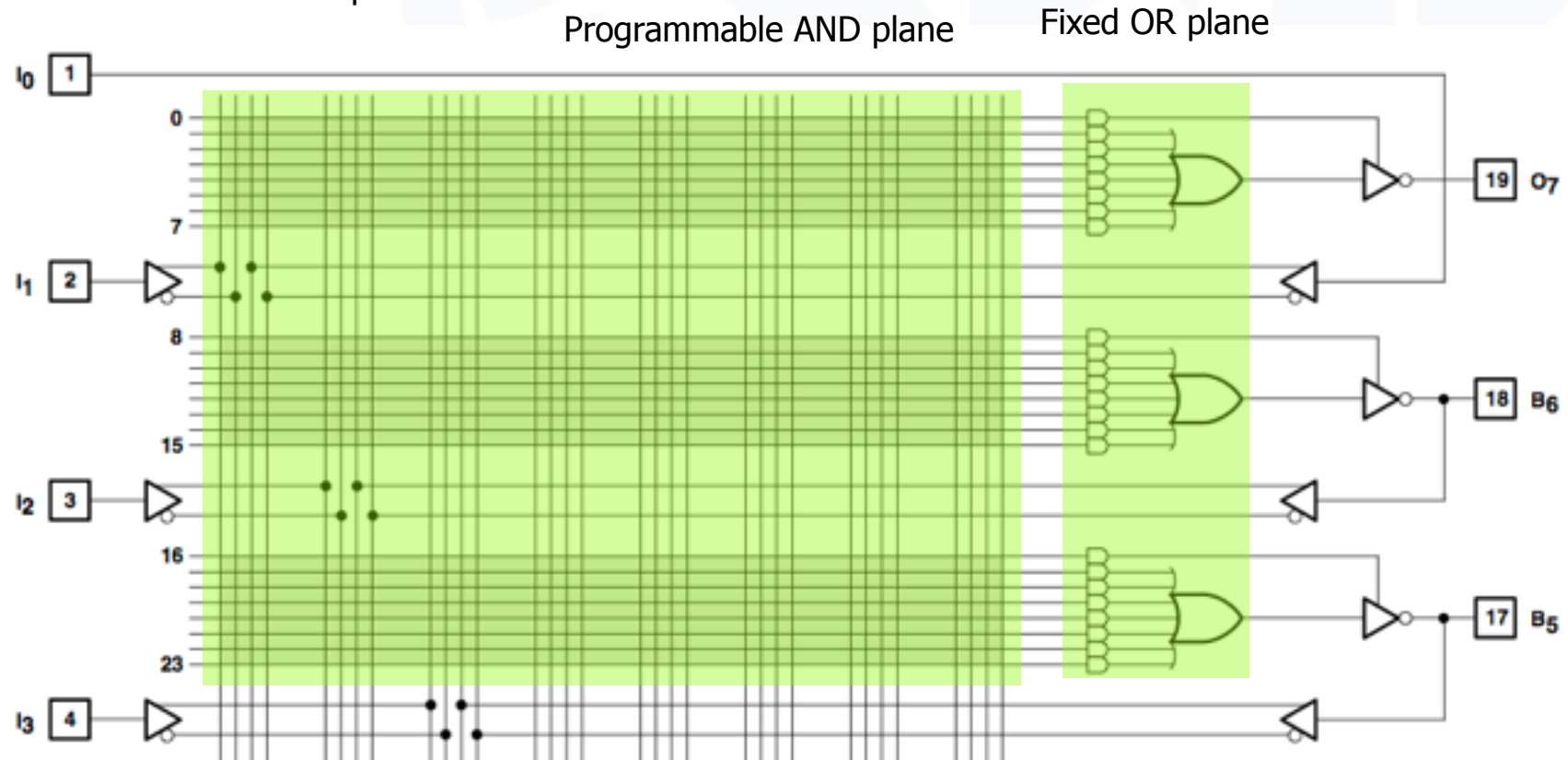
- $F1 = A B C$
- $F2 = A + B + C$
- $F3 = A' B' C'$
- $F4 = A' + B' + C'$
- $F5 = A \text{ xor } B \text{ xor } C$
- $F6 = A \text{ xnor } B \text{ xnor } C$

A	B	C	F1	F2	F3	F4	F5	F6
0	0	0	0	0	1	1	0	0
0	0	1	0	1	0	1	1	1
0	1	0	0	1	0	1	1	1
0	1	1	0	1	0	1	0	0
1	0	0	0	1	0	1	1	1
1	0	1	0	1	0	1	0	0
1	1	0	0	1	0	1	0	0
1	1	1	1	1	0	0	1	1



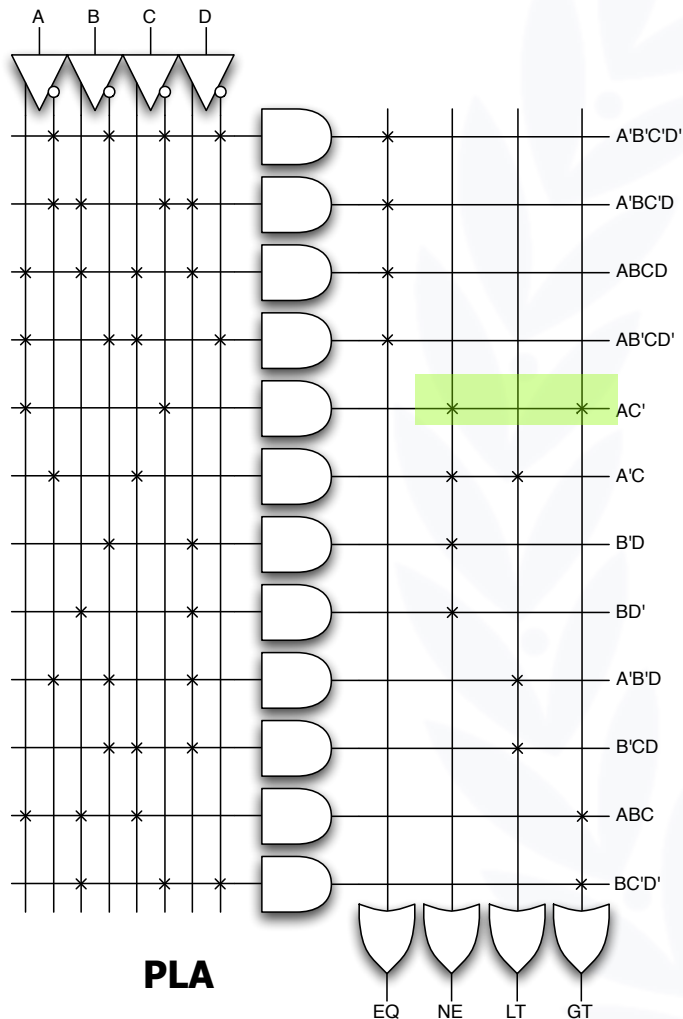
PALs and PLAs

- Programmable array logic (PAL)
 - Programmable AND plane
 - **Fixed OR plane - not more than xx product terms**
 - Innovation by Monolithic Memories
 - Faster and smaller OR plane

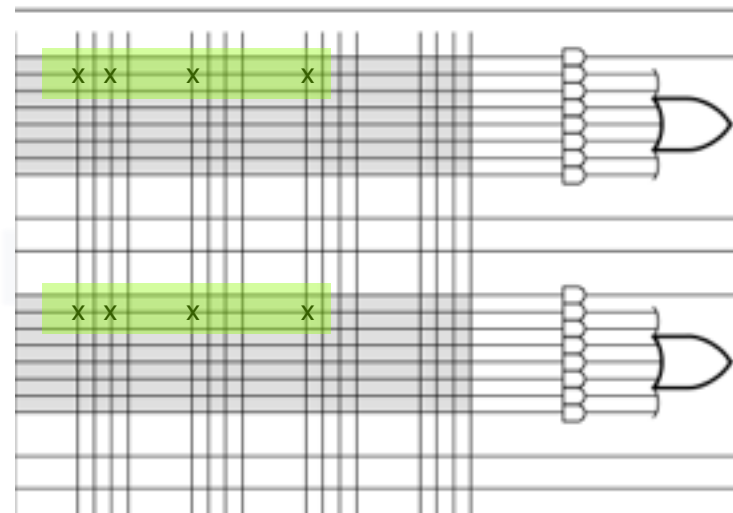


PLA and PAL

Product term sharing



PLA



PAL

PALs and PLAs: Design Example

- BCD to Gray code converter

A	B	C	D	W	X	Y	Z
0	0	0	0	0	0	0	0
0	0	0	1	0	0	0	1
0	0	1	0	0	0	1	1
0	0	1	1	0	0	1	0
0	1	0	0	0	1	1	0
0	1	0	1	1	1	1	0
0	1	1	0	1	0	1	0
0	1	1	1	1	0	1	1
1	0	0	0	1	0	0	1
1	0	0	1	1	0	0	0
1	0	1	—	—	—	—	—
1	1	—	—	—	—	—	—

Minimized functions:

$$W = A + BD + BC$$

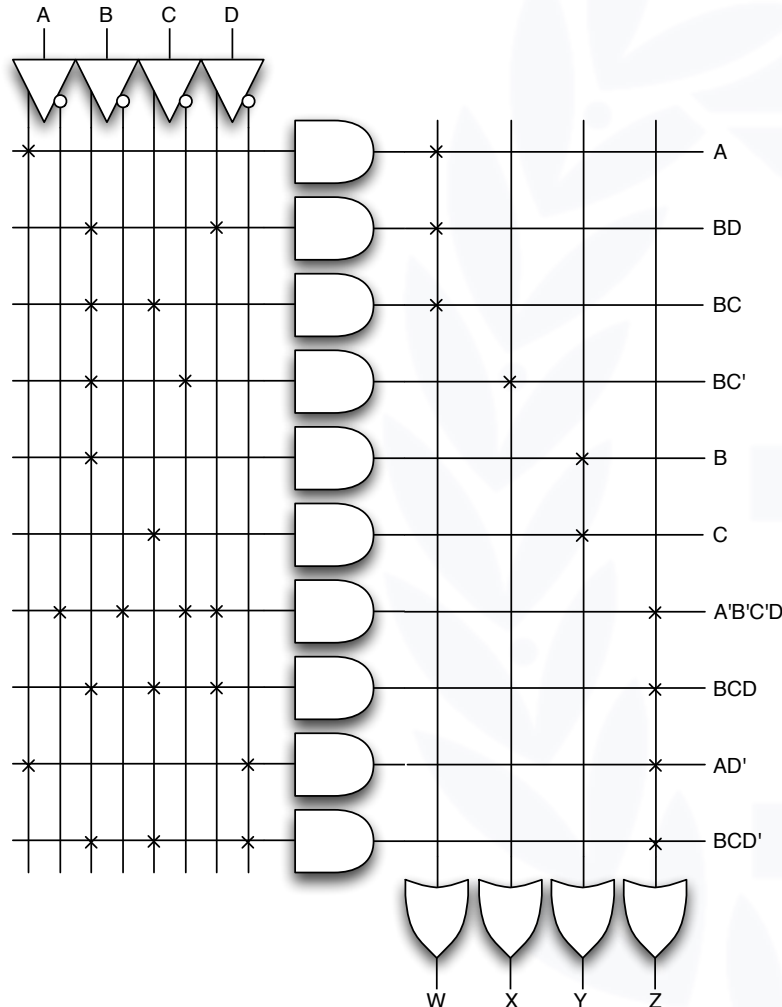
$$X = BC'$$

$$Y = B + C$$

$$Z = A'B'C'D + BCD + AD' + B'CD'$$

PALs and PLAs: Design Example (cont'd)

Code converter: programmed PLA



Minimized functions:

$$W = A + BD + BC$$

$$X = B C'$$

$$Y = B + C$$

$$Z = A'B'C'D + BCD + AD' + B'CD'$$

Not a particularly good candidate for PAL/PLA implementation since no terms are shared among outputs

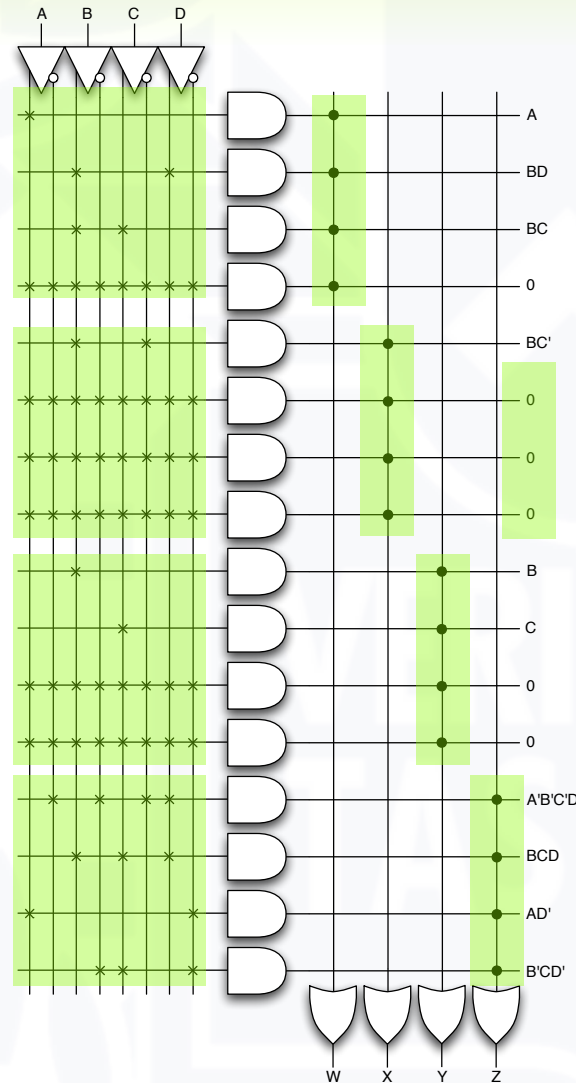
However, much more compact and regular implementation when compared with discrete AND and OR gates

PALs and PLAs: Design Example (cont'd)

- Code converter: programmed PAL

4 product terms
per each OR gate

Programmable AND plane
Product terms are not
shared

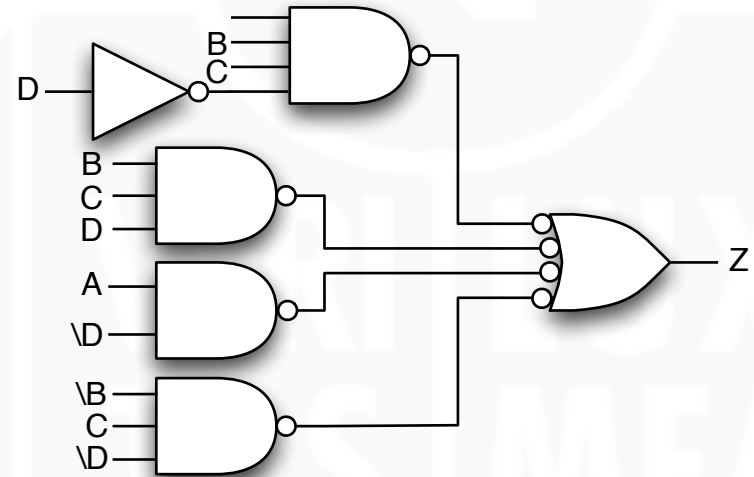
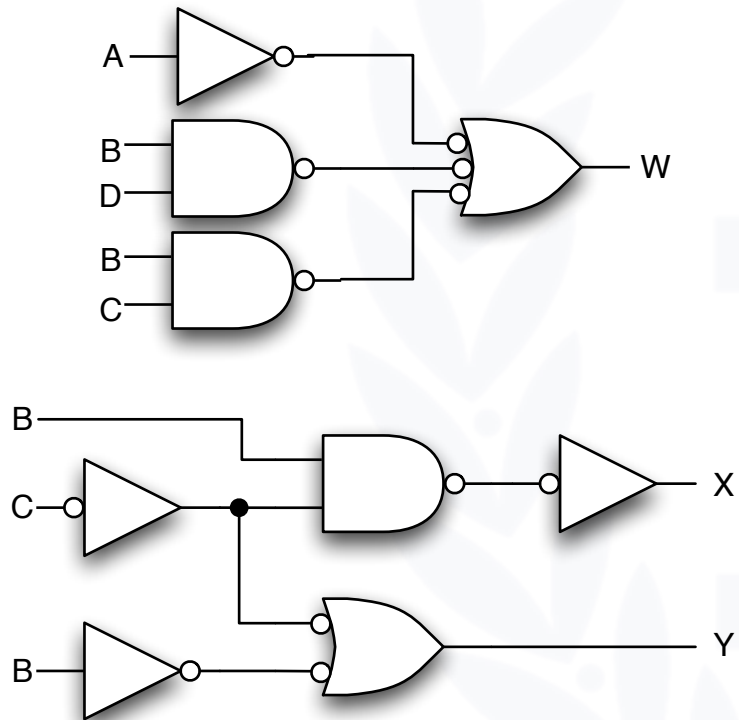


Fixed OR plane
4 input OR

Have to make 0 for
unused OR input

PALs and PLAs: Design Example (cont'd)

- Code converter: NAND gate implementation
 - Loss of regularity, harder to understand
 - Harder to make changes



PALs and PLAs: Another Design Example

Magnitude comparator

A	B	C	D	EQ	NE	LT	GT
0	0	0	0	1	0	0	0
0	0	0	1	0	1	1	0
0	0	1	0	0	1	1	0
0	0	1	1	0	1	1	0
0	1	0	0	0	1	0	1
0	1	0	1	1	0	0	0
0	1	1	0	0	1	1	0
0	1	1	1	0	1	1	0
1	0	0	0	0	1	0	1
1	0	0	1	0	1	0	1
1	0	1	0	1	0	0	0
1	0	1	1	0	1	1	0
1	1	0	0	0	1	0	1
1	1	0	1	0	1	0	1
1	1	1	0	0	1	0	1
1	1	1	1	1	0	0	0

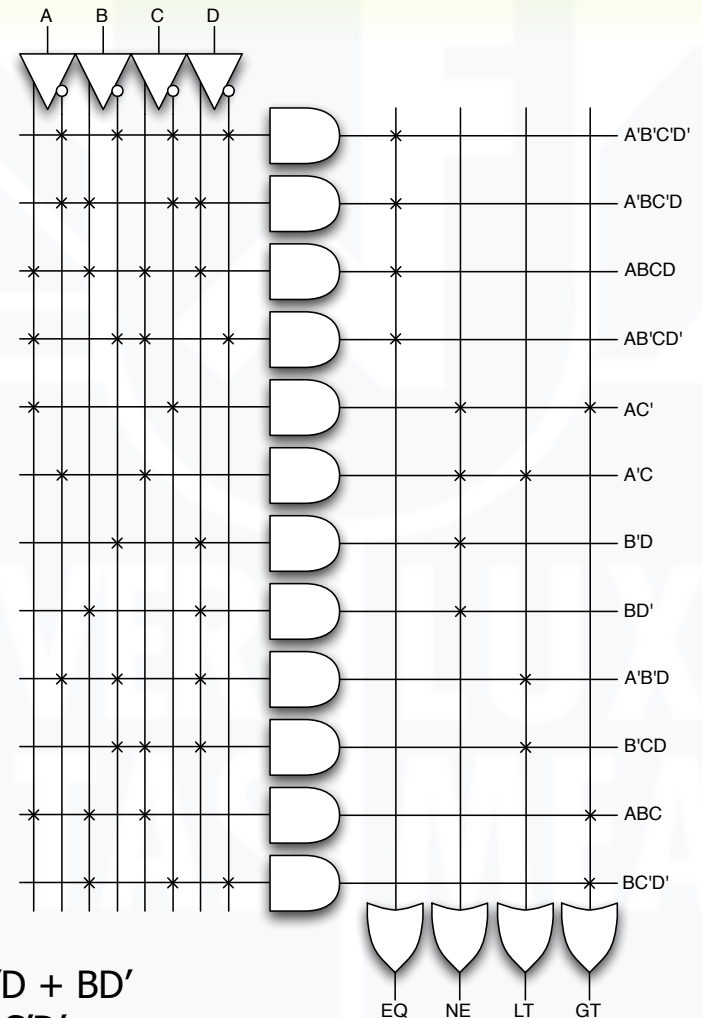
Minimized functions:

$$EQ = A'B'C'D' + A'BC'D + ABCD + AB'CD'$$

$$LT = A'C + A'B'D + B'CD$$

$$NE = AC' + A'C + B'D + BD'$$

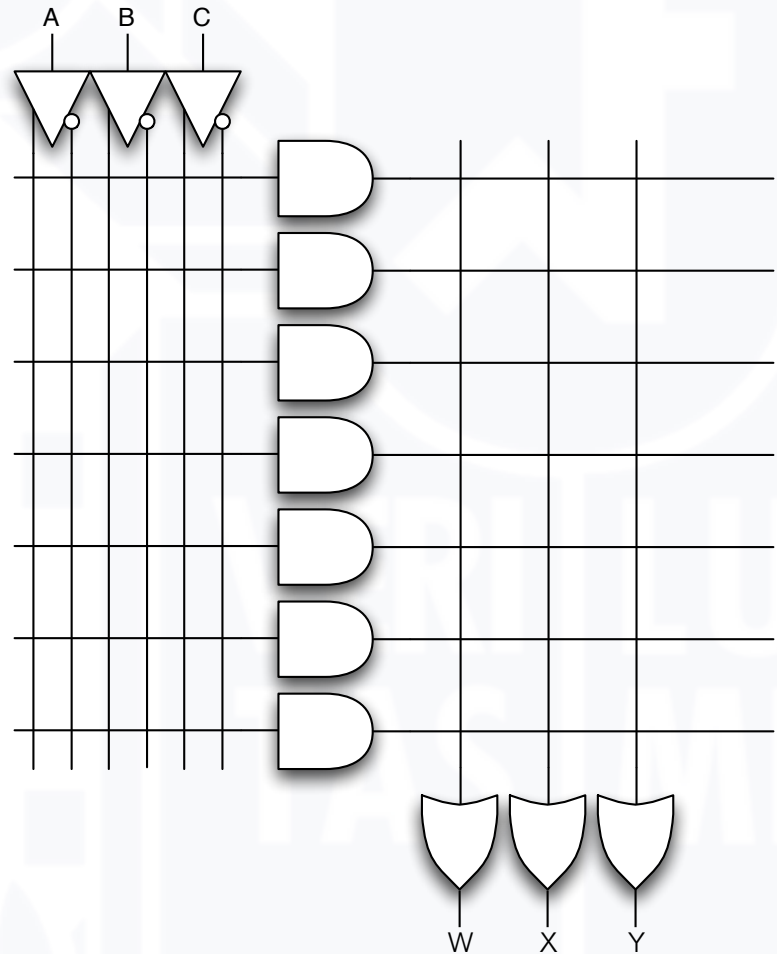
$$GT = AC' + ABC + BC'D'$$



Activity

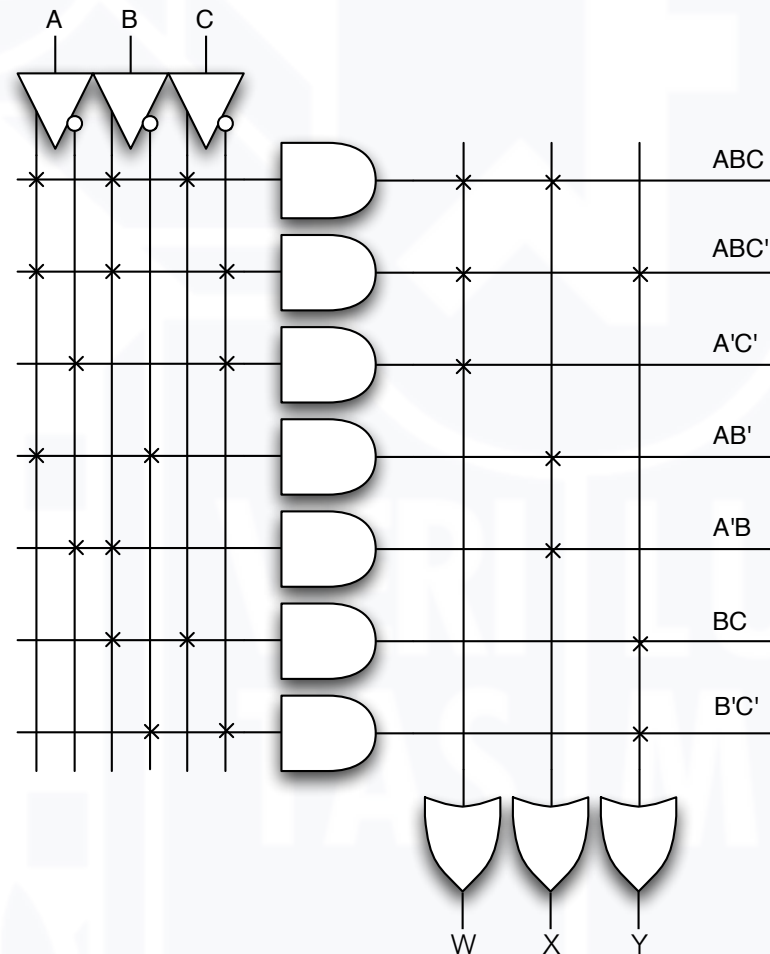
Map the following functions to the PLA below:

- $W = AB + A'C' + BC'$
- $X = ABC + AB' + A'B$
- $Y = ABC' + BC + B'C'$



Activity (cont'd)

- 9 terms won't fit in a 7 term PLA
 - Can apply consensus theorem to W to simplify to:
 $W = AB + A'C'$
- 8 terms won't fit in a 7 term PLA
 - Observe that $AB = ABC + ABC'$
 - Can rewrite W to reuse terms:
 $W = ABC + ABC' + A'C'$
- Now it fits
 - $W = ABC + ABC' + A'C'$
 - $X = ABC + AB' + A'B$
 - $Y = ABC' + BC + B'C'$
- This is called technology mapping
 - Manipulating logic functions so that they can use available resources

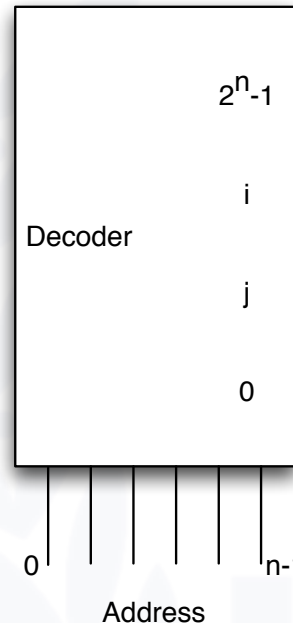


Read-Only Memories

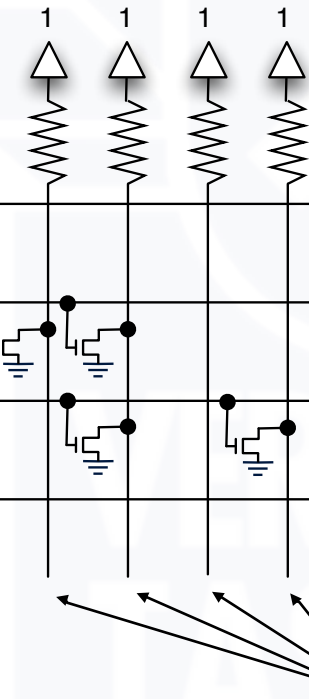
- Two dimensional array of 1s and 0s
 - Entry (row) is called a "word"
 - Width of row = word-size
 - Index is called an "address"
 - Address is input
 - Selected word is output

Fixed AND plane
Generating all minterms

Internal organization



Programmable OR plane



Word lines (only one is active – decoder is just right for this)

word[i] = 0011

word[j] = 1010

Bit lines (normally pulled to 1 through resistor – selectively connected to 0 by word line controlled switches)

ROMs and Combinational Logic

- Combinational logic implementation (two-level canonical form) using a ROM

$$F0 = A' B' C + A B' C' + A B' C$$

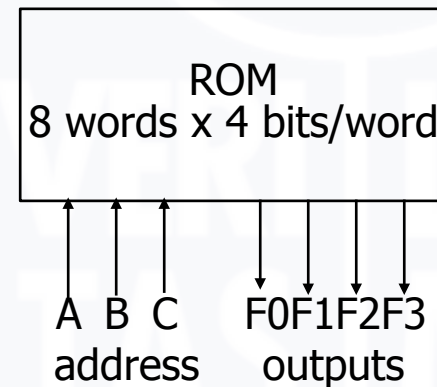
$$F1 = A' B' C + A' B C' + A B C$$

$$F2 = A' B' C' + A' B' C + A B' C'$$

$$F3 = A' B C + A B' C' + A B C'$$

A	B	C	F0	F1	F2	F3
0	0	0	0	0	1	0
0	0	1	1	1	1	0
0	1	0	0	1	0	0
0	1	1	0	0	0	1
1	0	0	1	0	1	1
1	0	1	1	0	0	0
1	1	0	0	0	0	1
1	1	1	0	1	0	0

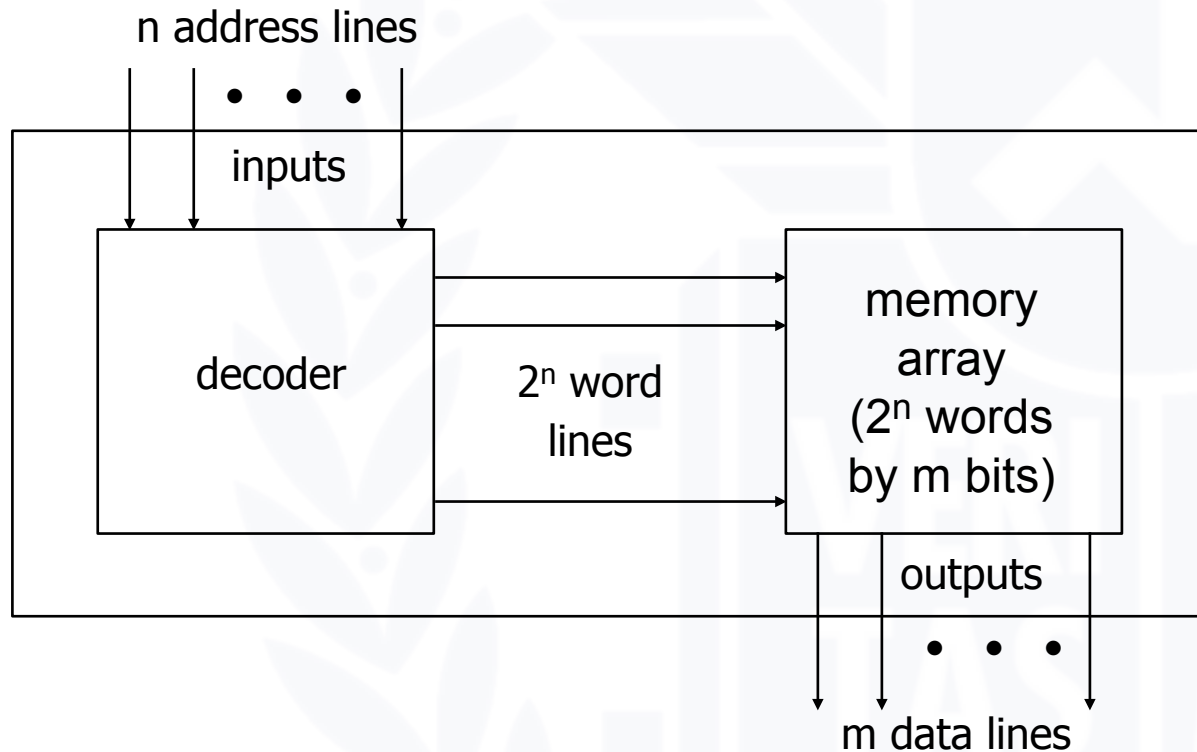
Truth table



Block diagram

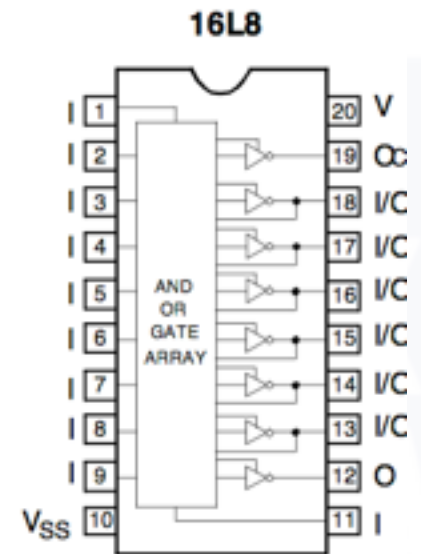
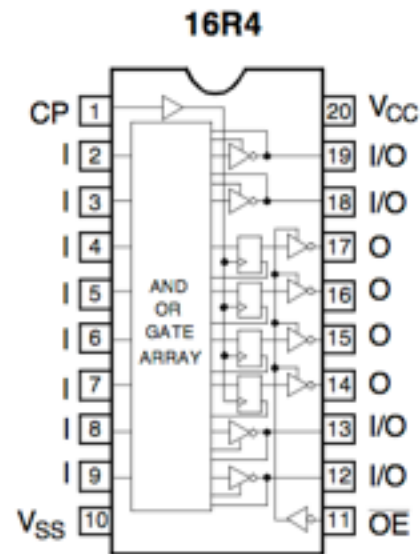
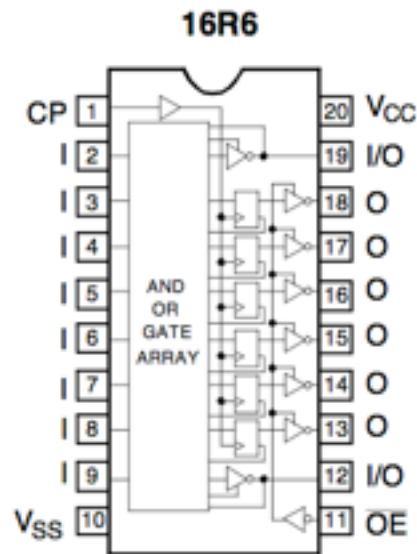
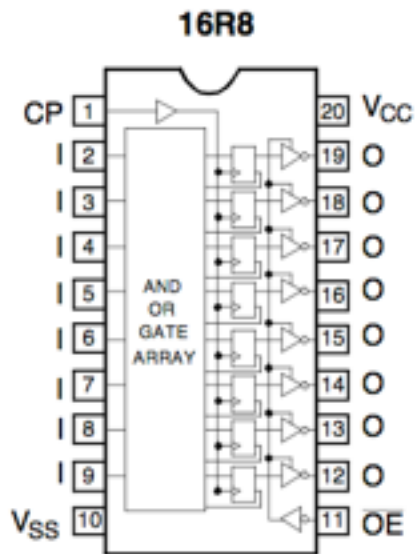
ROM structure

- Similar to a PLA structure but with a fully decoded AND array
 - Completely flexible OR array (unlike PAL)



Types of PALs

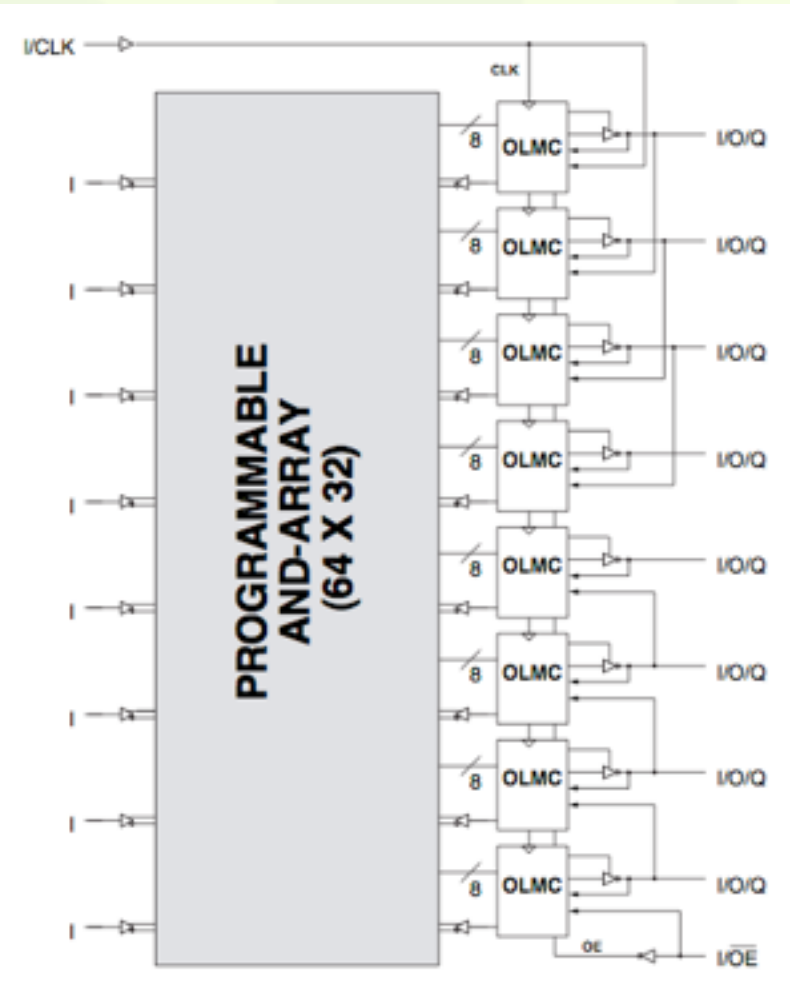
- Typical PAL types
 - Number of inputs
 - Number of OR-plane inputs
 - Number of outputs
 - Registered output



www.cypress.com

GAL

- GAL (Generic Array Logic)
 - High-speed EECMOS
 - 2.5 ns maximum clock input to output data



www.latticesemi.com

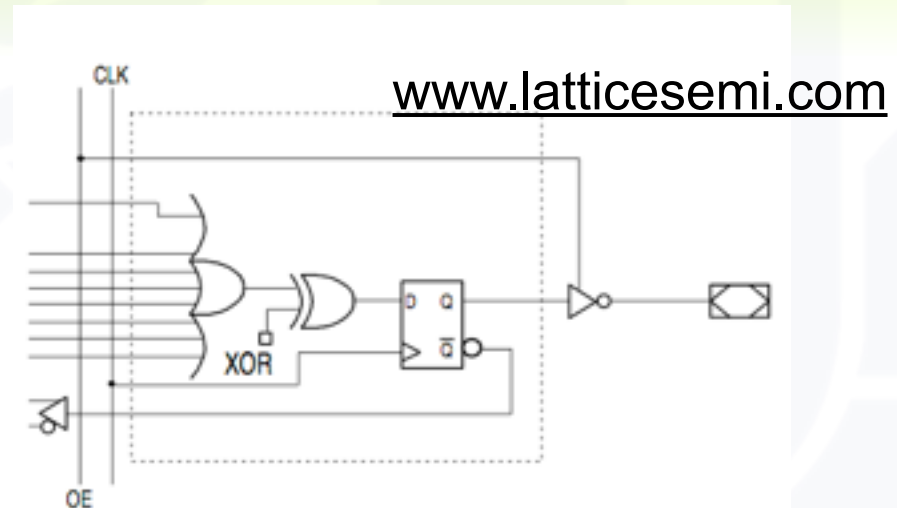
Electrical Characteristics

- 3.3 and 5 V compatible
- 3 to 15 ns input to output delay

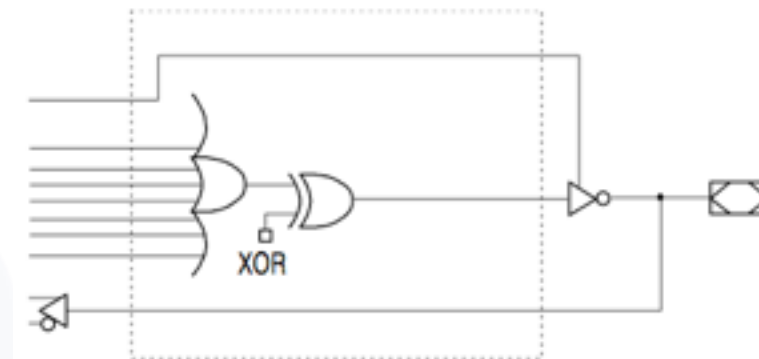
SYMBOL	PARAMETER	CONDITION	MIN.	TYP. ³	MAX.	UNITS
V_{IL}	Input Low Voltage		$V_{SS} - 0.5$	—	0.8	V
V_{IH}	Input High Voltage		2.0	—	$V_{CC} + 1$	V
I_{IL}¹	Input or I/O Low Leakage Current	$0V \leq V_{IN} \leq V_{IL} (MAX.)$	—	—	-100	μA
I_{IH}	Input or I/O High Leakage Current	$3.5V \leq V_{IN} \leq V_{CC}$	—	—	10	μA
V_{OL}	Output Low Voltage	$I_{OL} = MAX. \quad V_{in} = V_{IL} \text{ or } V_{IH}$	—	—	0.5	V
V_{OH}	Output High Voltage	$I_{OH} = MAX. \quad V_{in} = V_{IL} \text{ or } V_{IH}$	2.4	—	—	V
I_{OL}	Low Level Output Current		—	—	12	mA
I_{OH}	High Level Output Current		—	—	-2	mA
I_{OS}²	Output Short Circuit Current	$V_{CC} = 5V \quad V_{OUT} = 0.5V \quad T_A = 25^\circ C$	-30	—	-150	mA
I_{CC}	Operating Power Supply Current	$V_{IL} = 0.5V \quad V_{IH} = 3.0V$ $f_{toggle} = 15MHz$ Outputs Open	L-7/-10	—	75	130 mA

GAL

- Configurable to registered- and combinatorial outputs
- Cover both L- and R- type PALs



Registered configuration



Combinatorial configuration

ROM vs. PLA

- ROM approach advantageous when
 - Design time is short (no need to minimize output functions)
 - Most input combinations are needed (e.g., code converters)
 - Little sharing of product terms among output functions
- ROM problems
 - Size doubles for each additional input
 - Generates all the minterms
 - Can't exploit don't cares
- PLA approach advantageous when
 - Design tools are available for multi-output minimization
 - There are relatively few unique minterm combinations
 - Many minterms are shared among the output functions
- PAL problems
 - Constrained fan-ins on OR plane

Regular Logic Structures for Two-Level Logic

- ROM – full AND plane, general OR plane
 - Cheap (high-volume component)
 - Can implement any function of n inputs
 - Medium speed
- PAL – programmable AND plane, fixed OR plane
 - Intermediate cost
 - Can implement functions limited by number of terms
 - High speed (only one programmable plane that is much smaller than ROM's decoder)
- PLA – programmable AND and OR planes
 - Most expensive (most complex in design, need more sophisticated tools)
 - Can implement any function up to a product term limit
 - Slow (two programmable planes)

Regular Logic Structures for Multi-Level Logic

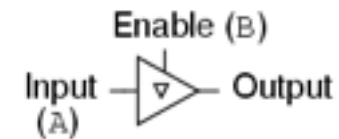
- Difficult to devise a regular structure for arbitrary connections between a large set of different types of gates
 - Efficiency/speed concerns for such a structure
 - In 467 you'll learn about field programmable gate arrays (FPGAs) that are just such programmable multi-level structures
 - Programmable multiplexers for wiring
 - Lookup tables for logic functions (programming fills in the table)
 - Multi-purpose cells (utilization is the big issue)
- Use multiple levels of PALs/PLAs/ROMs
 - Output intermediate result
 - Make it an input to be used in further logic

Non-Logic Gates

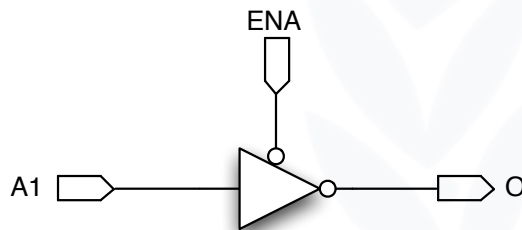
- Tri-state output
 - Tri-State is a TM, and Three-State is the real name
 - Neither 0 nor 1, high impedance
 - Floating
 - Use of symbol Z



Tristate buffer symbol



- In general, output enable is active low



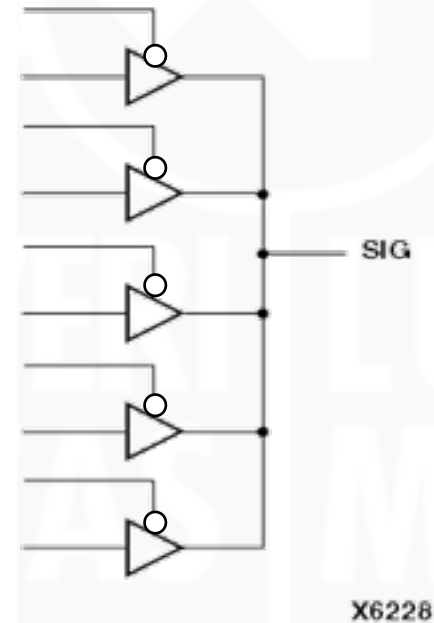
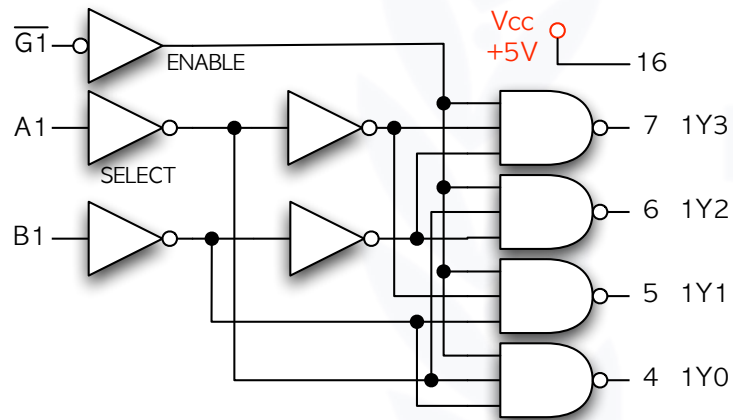
Truth table

A	B	Output
0	0	High-Z
0	1	0
1	0	High-Z
1	1	1

- More than two outputs can share the same signal line in a time-division multiplexing manner

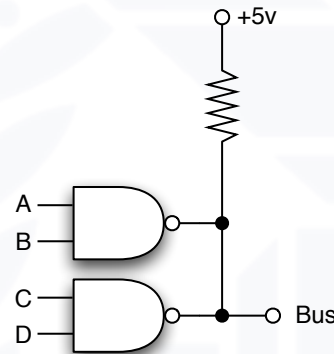
Non-Logic Gate

- Tri-state gate multiplexer
- Combination of a decoder and tri-state gates

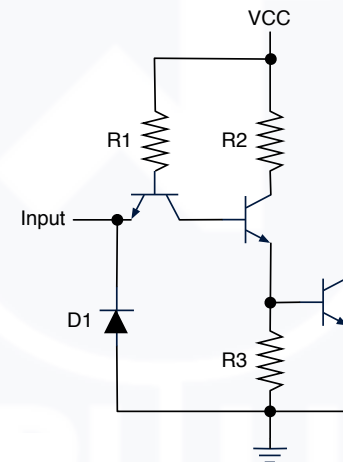


Open-Collector Output

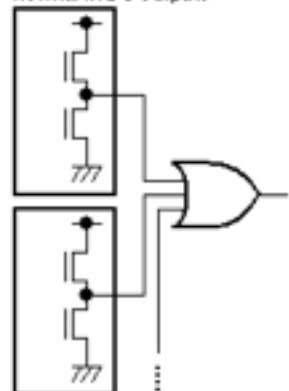
- Bipolar open-collector gate
- CMOS open-drain gate
- No pull-up transistor
 - Use of a pull-up resistor
- Wired logic



Inverter circuit with open-collector output

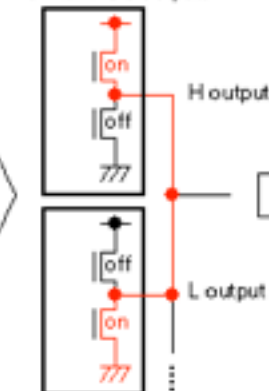


ORing normal MOS outputs



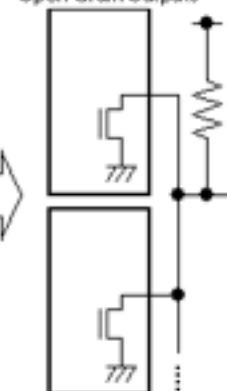
H is output if either is H.

Integrating normal MOS outputs



Short-circuit between power supply and GND

Wired ORing open-drain outputs



L is output if either is L (on).

Combinational Logic Technology Summary

- Random logic
 - Single gates or in groups
 - Conversion to NAND-NAND and NOR-NOR networks
 - Transition from simple gates to more complex gate building blocks
 - Reduced gate count, fan-ins, potentially faster
 - More levels, harder to design
- Time response in combinational networks
 - Gate delays and timing waveforms
 - Hazards/glitches (what they are and why they happen)
- Regular logic
 - Multiplexers/decoders
 - ROMs
 - PLAs/PALs
 - Advantages/disadvantages of each

Logic Design Practice in a Lab

- Design Entry
 - Truth table
 - Boolean equation
 - Schematic diagram
 - Random logic and fixed logic (built-in library)
 - Programmable logic (GAL): currently, combinational only
- Logic simulation
 - Xilinx ISE 9.2
- Implementation
 - TTL 74 series
 - PALCE16V8 (GAL16V8)
 - Other passive components

Digital Logic Design

4190.201

2014 Spring Semester

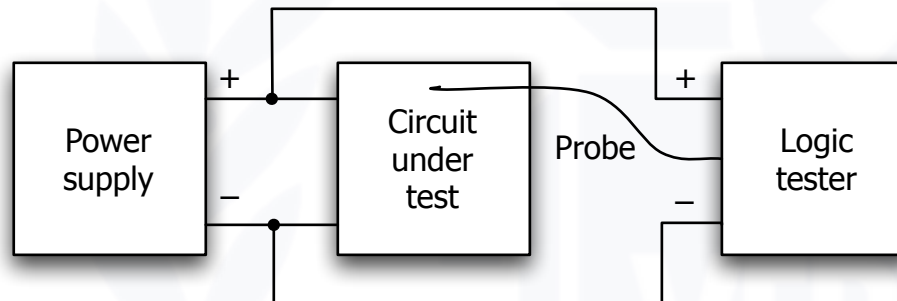
Supplementary Slide for the lab

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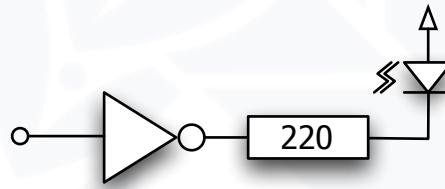
Logic Tester

- Detect high and low
 - Share the same power supply
 - Share the reference voltage, i.e., GND

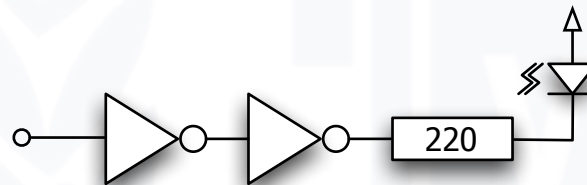


Logic Tester

- Detect high
 - Should have no loading effect

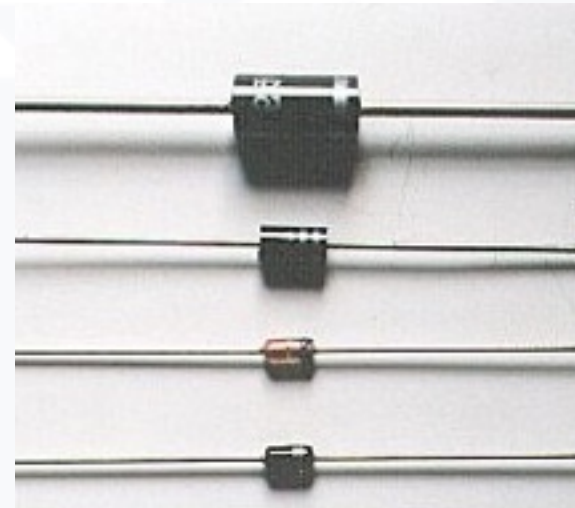
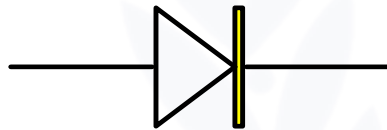
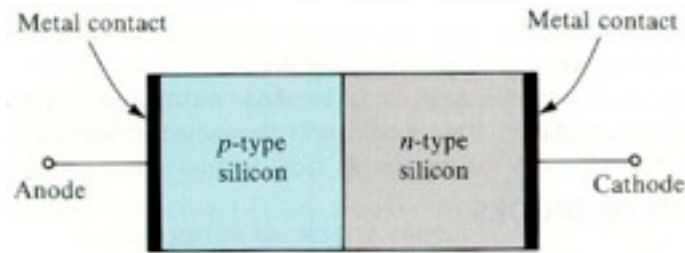


- Detect low
 - Should have no loading effect



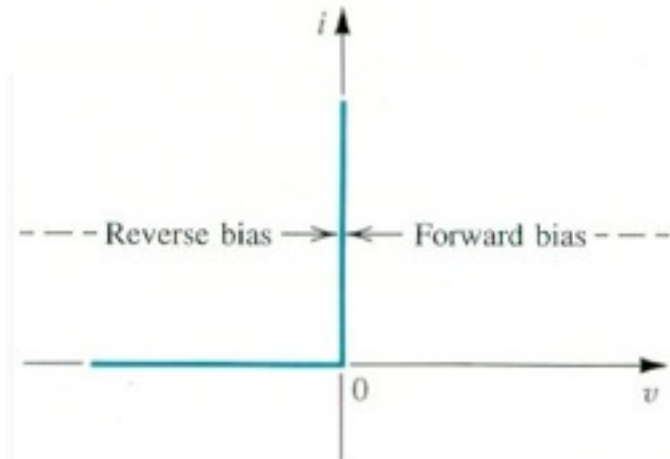
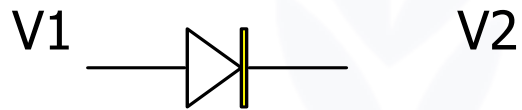
Diode (1)

- P-N junction diode
 - 1N4148



Diode (2)

- Ideal diode
 - Forward bias: $V_1 > V_2$
 - Reverse bias: $V_1 < V_2$



Diode (4)

- Real diode
 - Break down
- Silicon Diode
- Germanium Diode

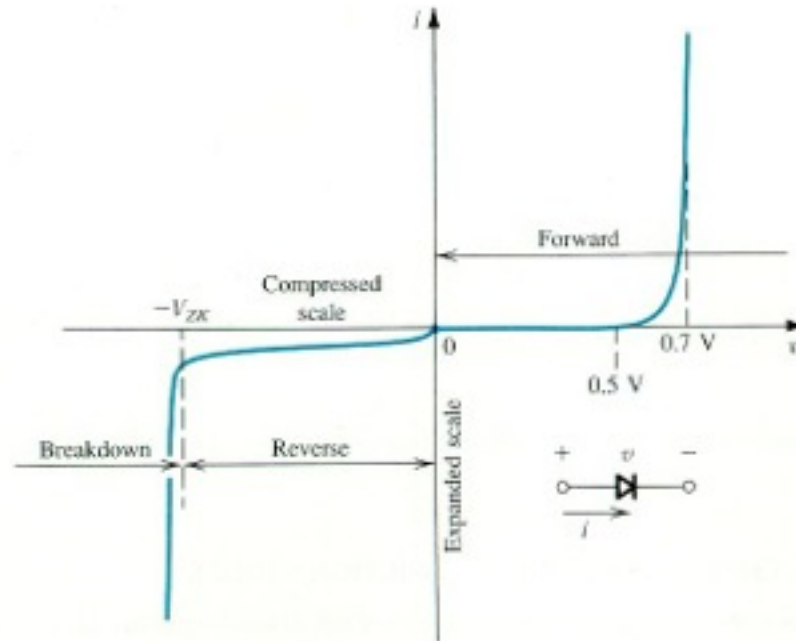
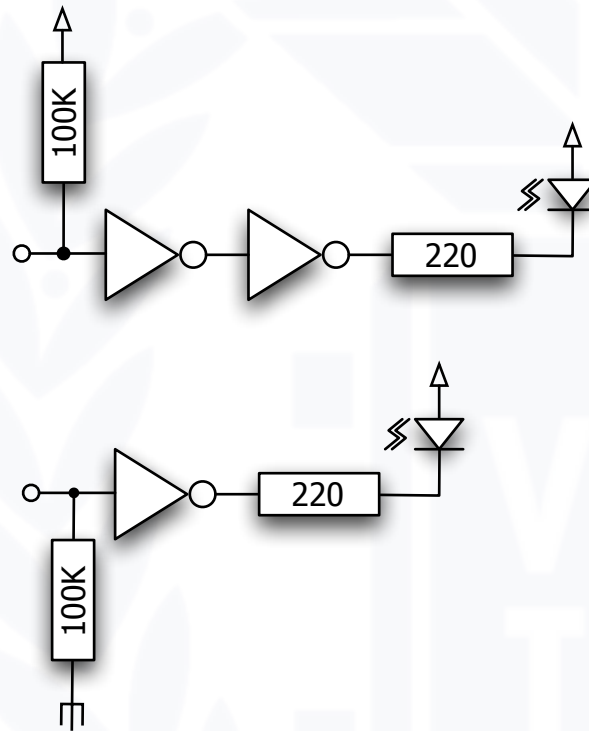


Fig. 3.8 The diode i - v relationship with some scales expanded and others compressed in order to reveal details.

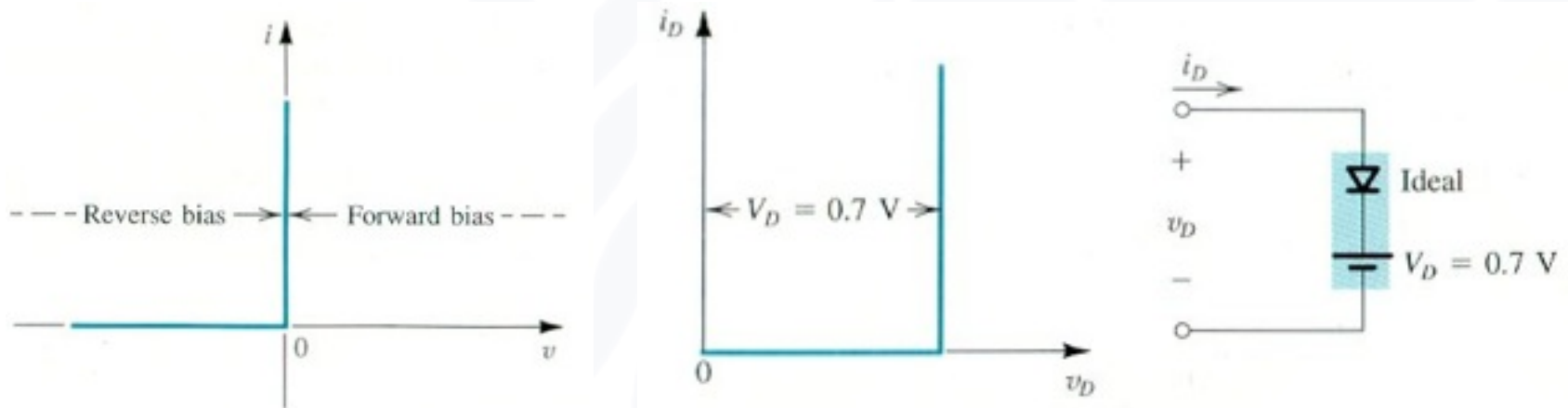
Logic Tester

- Prevent from floating
 - Can we combine these together?



Diode (6)

- Diode in digital circuits
 - Silicon Diode: switching and rectification
 - Germanium Diode: detection



Logic Tester

- Final design

