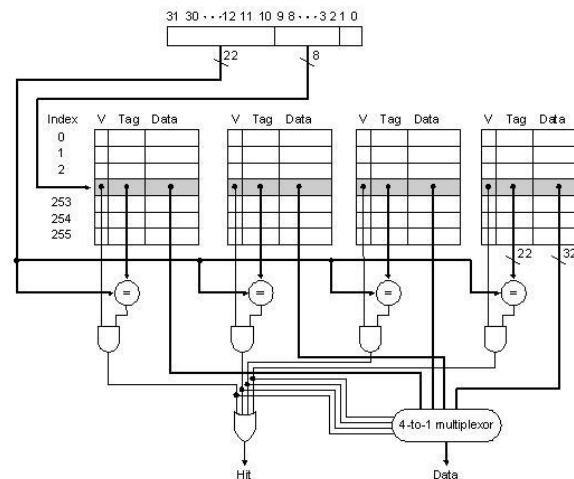


# Accessing The Outside World

## Cache Memories

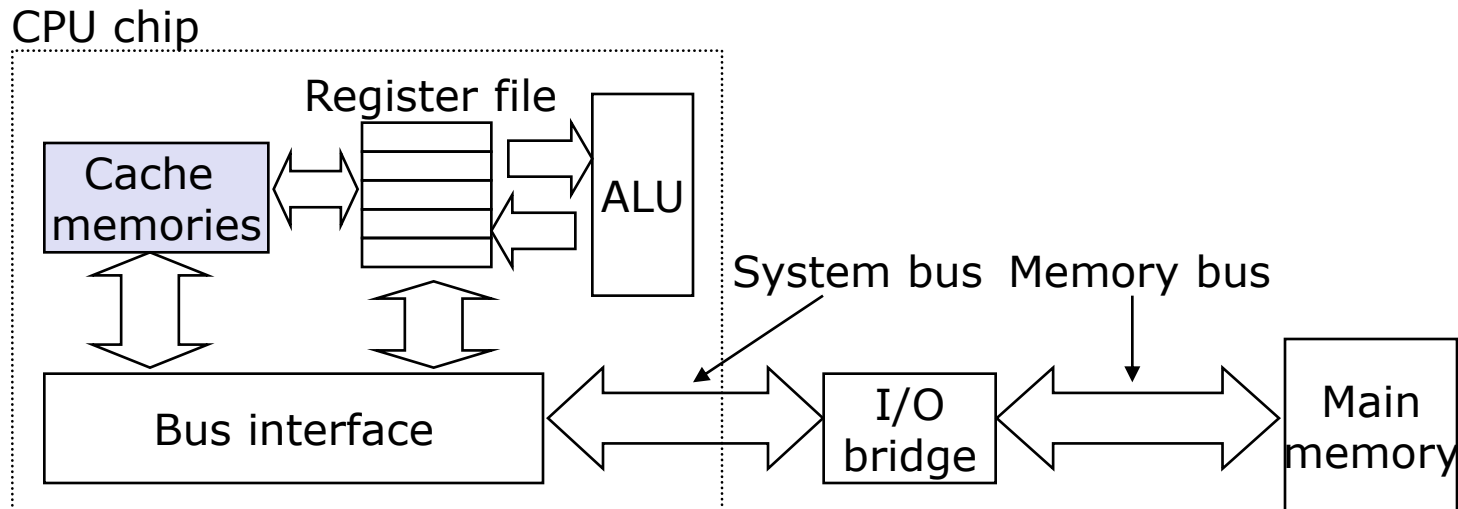


# Cache Memories

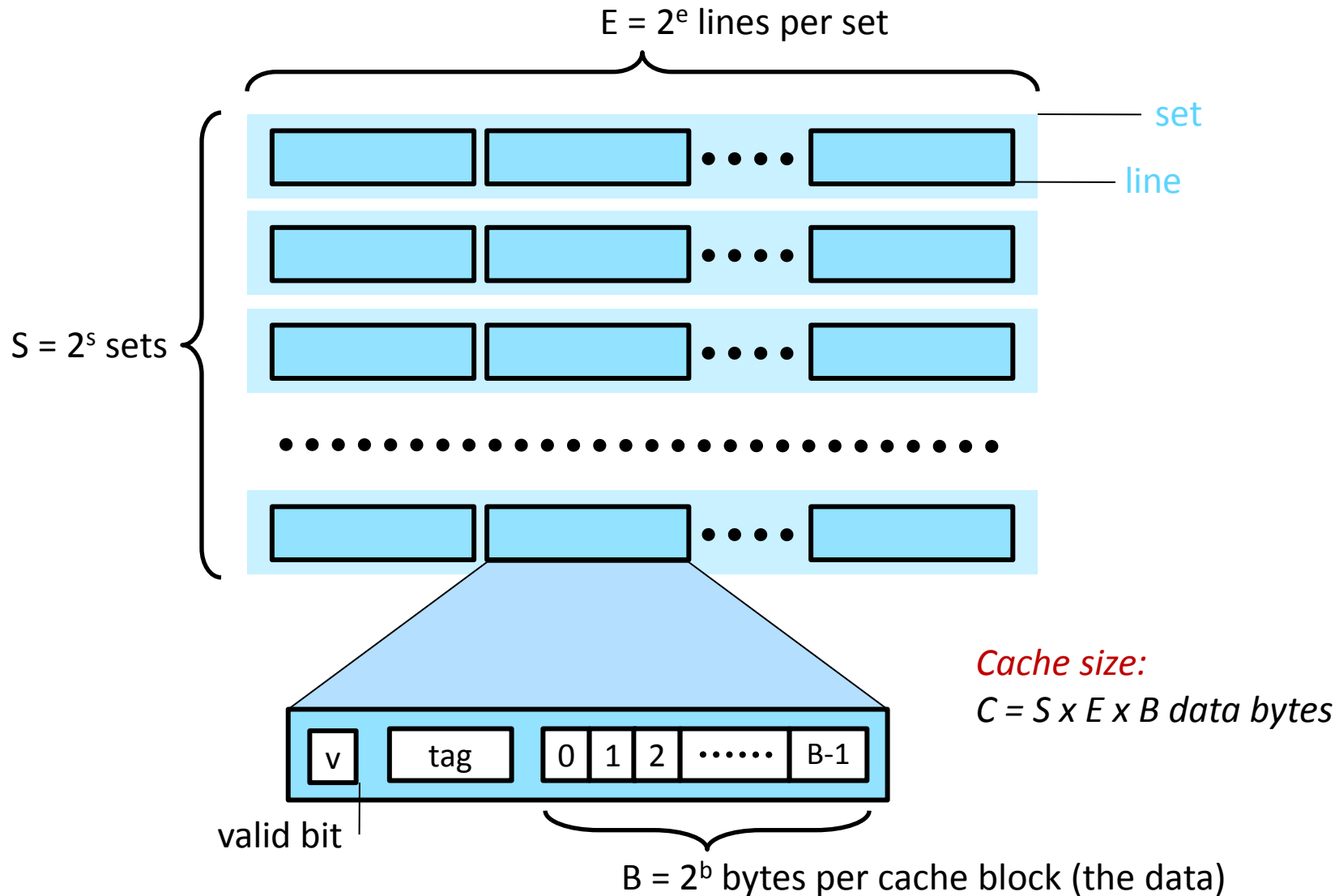
- Cache memory organization and operation
- Performance impact of caches
  - The memory mountain
  - Rearranging loops to improve spatial locality

# Cache Memories

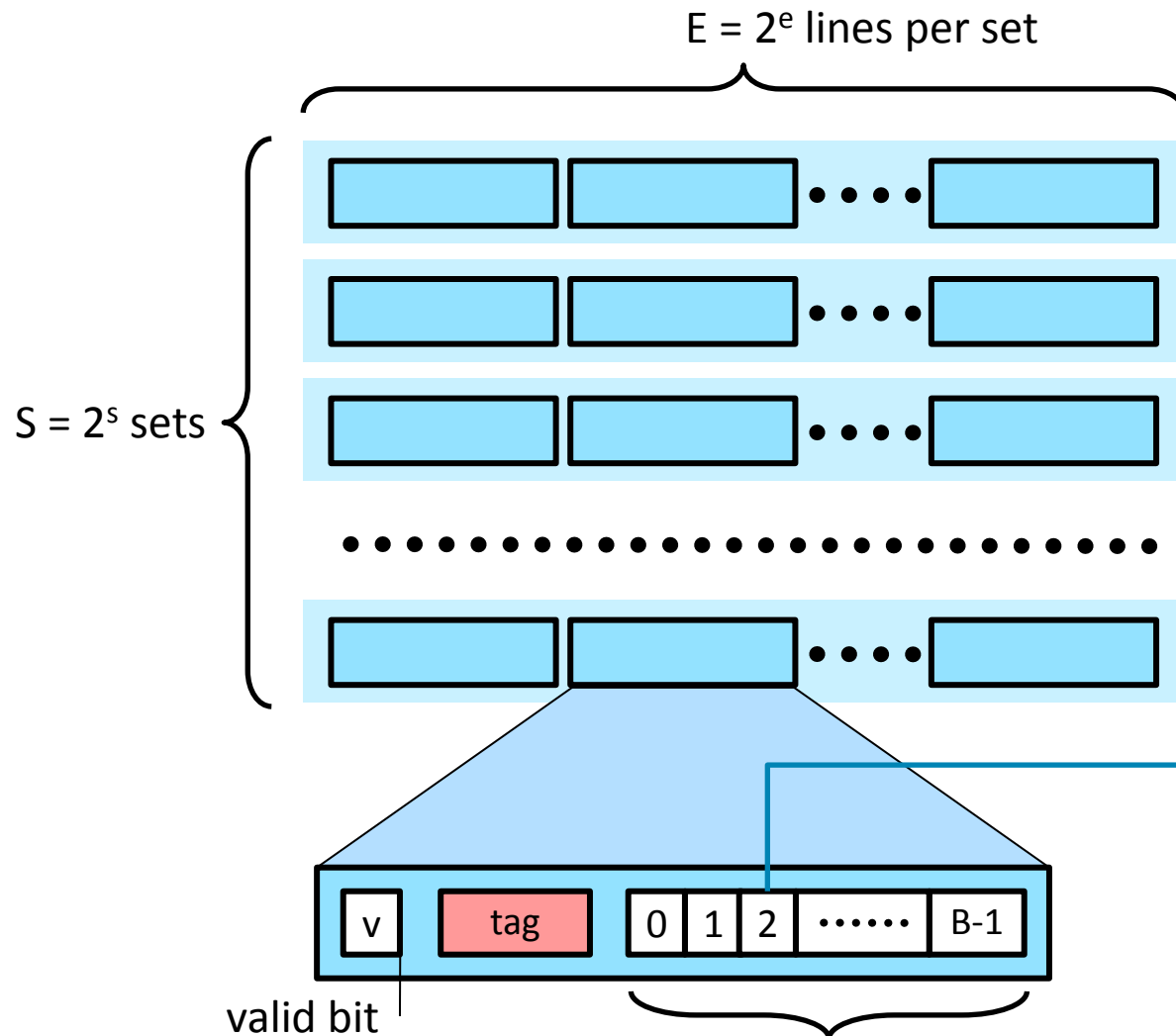
- **Cache memories** are small, fast SRAM-based memories managed automatically in hardware.
  - Hold frequently accessed blocks of main memory
- CPU looks first for data in caches (e.g., L1, L2, and L3), then in main memory.
- Typical system structure:



# General Cache Organization (S, E, B)

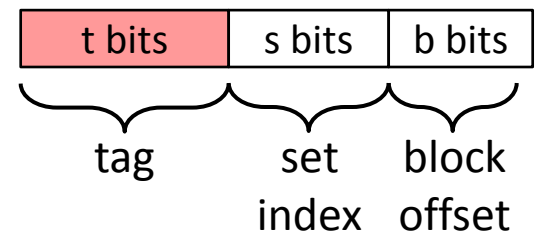


# Cache Read



- *Locate set*
- *Check if any line in set has matching tag*
- *Yes + line valid: hit*
- *Locate data starting at offset*

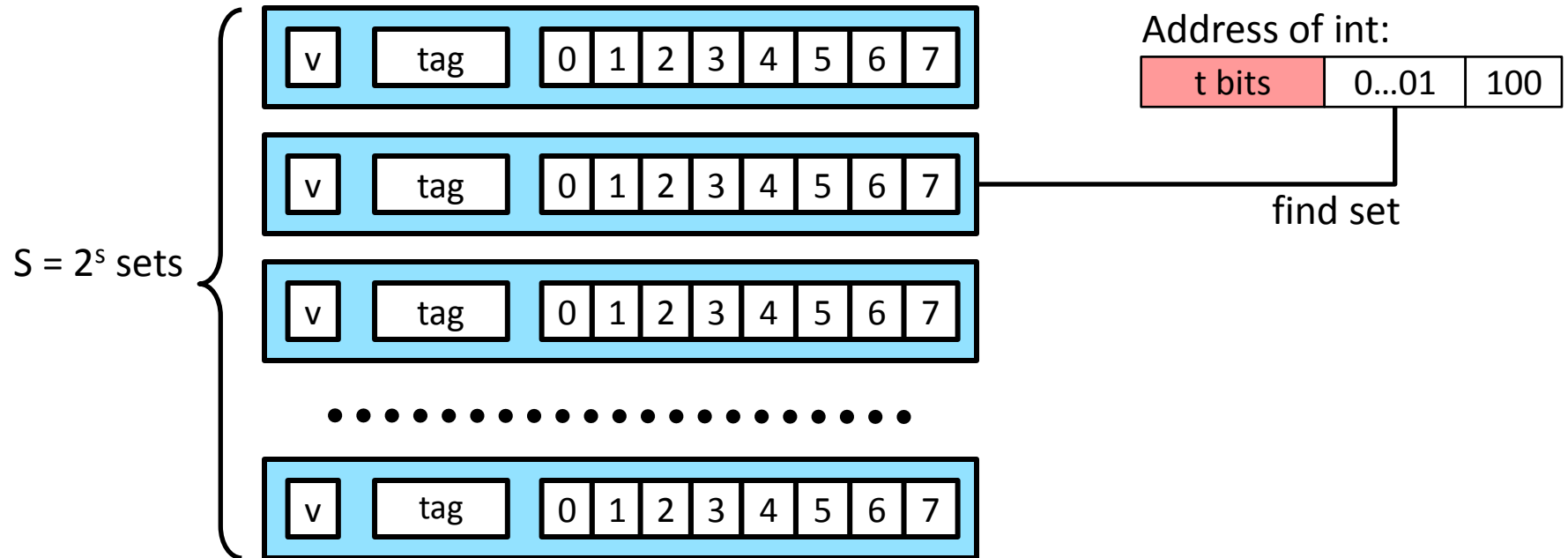
Address of word:



data begins at this offset

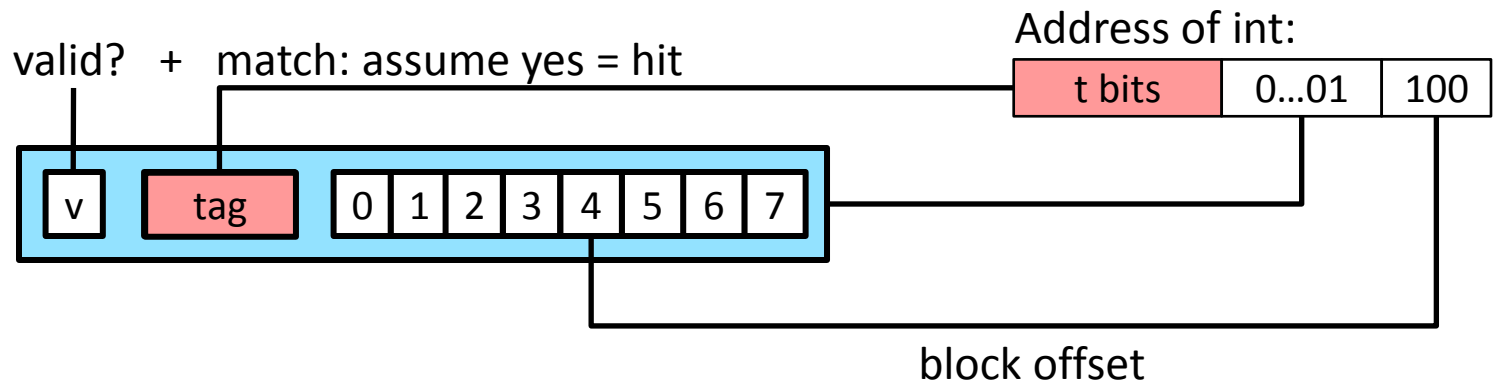
# Example: Direct Mapped Cache (E = 1)

Direct mapped: One line per set  
Assume: cache block size 8 bytes



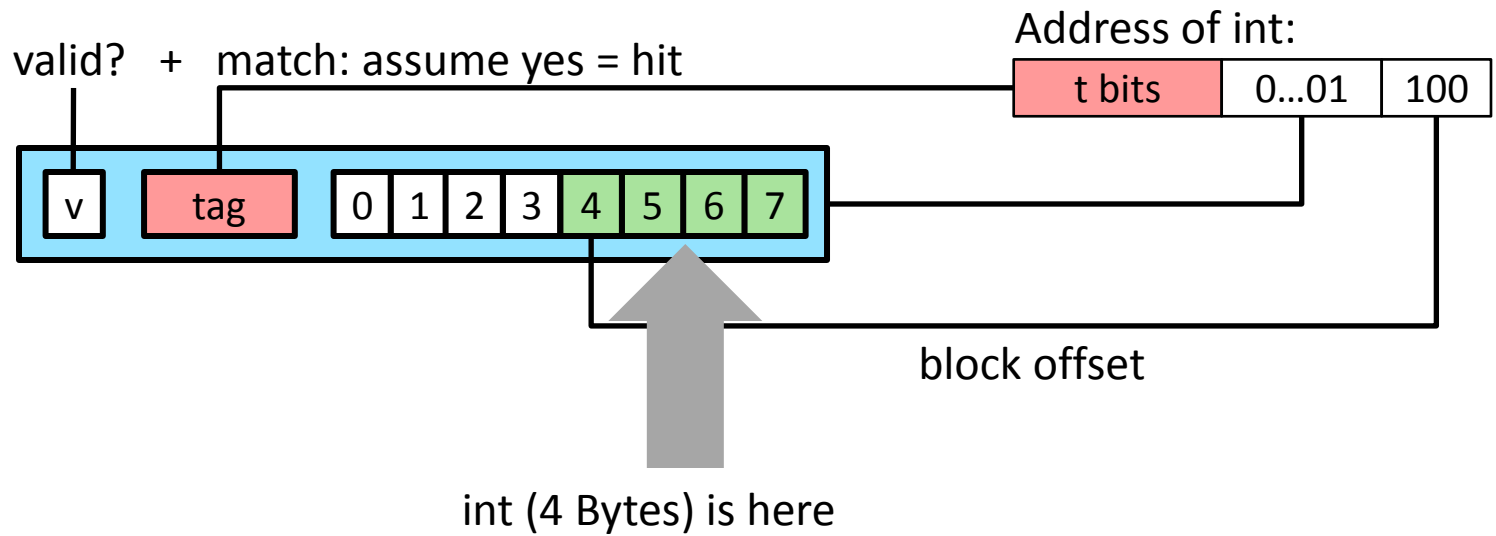
# Example: Direct Mapped Cache (E = 1)

Direct mapped: One line per set  
Assume: cache block size 8 bytes



# Example: Direct Mapped Cache (E = 1)

Direct mapped: One line per set  
Assume: cache block size 8 bytes



**No match:** old line is evicted and replaced



# Direct-Mapped Cache Simulation

t=1	s=2	b=1
x	xx	x

M=16 byte addresses, B=2 bytes/block,  
S=4 sets, E=1 Blocks/set

Address trace (reads, one byte per read):

0	[0000 <sub>2</sub> ],	miss
1	[0001 <sub>2</sub> ],	hit
7	[0111 <sub>2</sub> ],	miss
8	[1000 <sub>2</sub> ],	miss
0	[0000 <sub>2</sub> ]	miss

	v	Tag	Block
Set 0	1	0	M[0-1]
Set 1			
Set 2			
Set 3	1	0	M[6-7]

# A Higher Level Example

```
int sum_array_rows(double a[16][16])
{
    int i, j;
    double sum = 0;

    for (i = 0; i < 16; i++)
        for (j = 0; j < 16; j++)
            sum += a[i][j];
    return sum;
}
```

```
int sum_array_cols(double a[16][16])
{
    int i, j;
    double sum = 0;

    for (j = 0; j < 16; j++)
        for (i = 0; i < 16; i++)
            sum += a[i][j];
    return sum;
}
```

*Ignore the variables sum, i, j*

assume: cold (empty) cache,  
a[0][0] goes here



32 B = 4 doubles

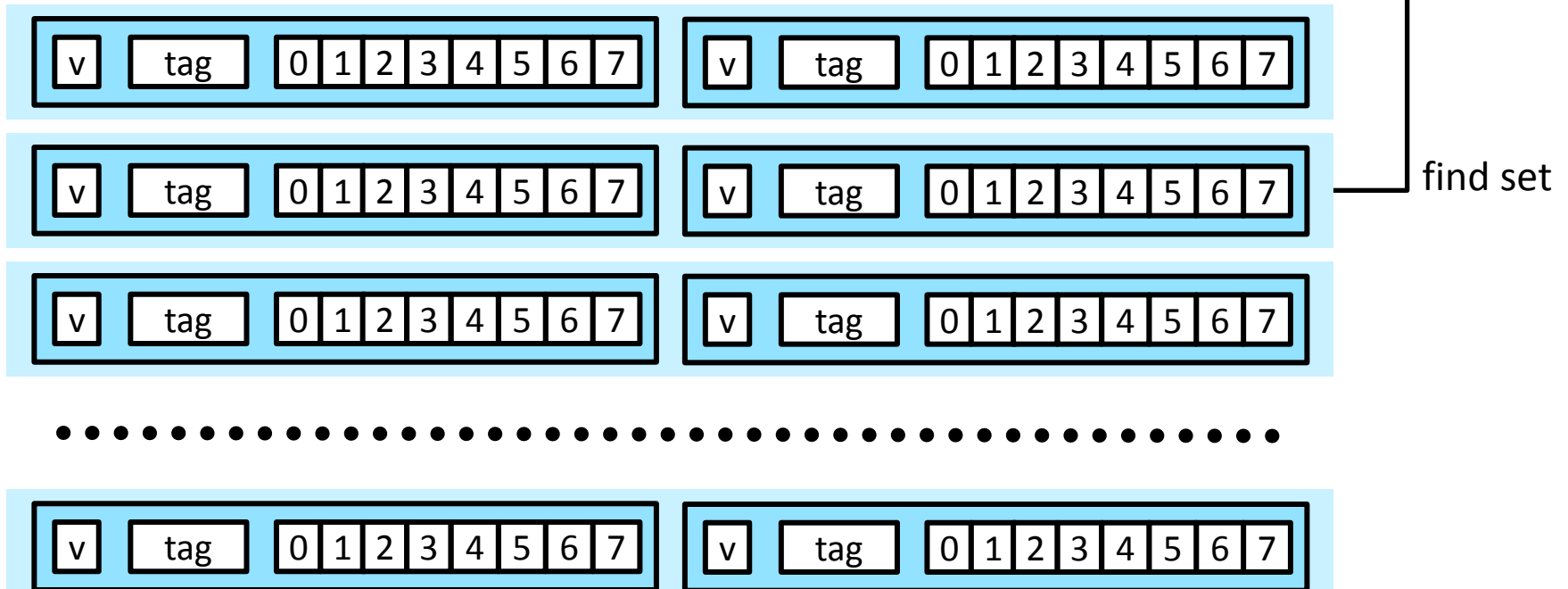
# E-way Set Associative Cache

E = 2: Two lines per set

Assume: cache block size 8 bytes

Address of short int:

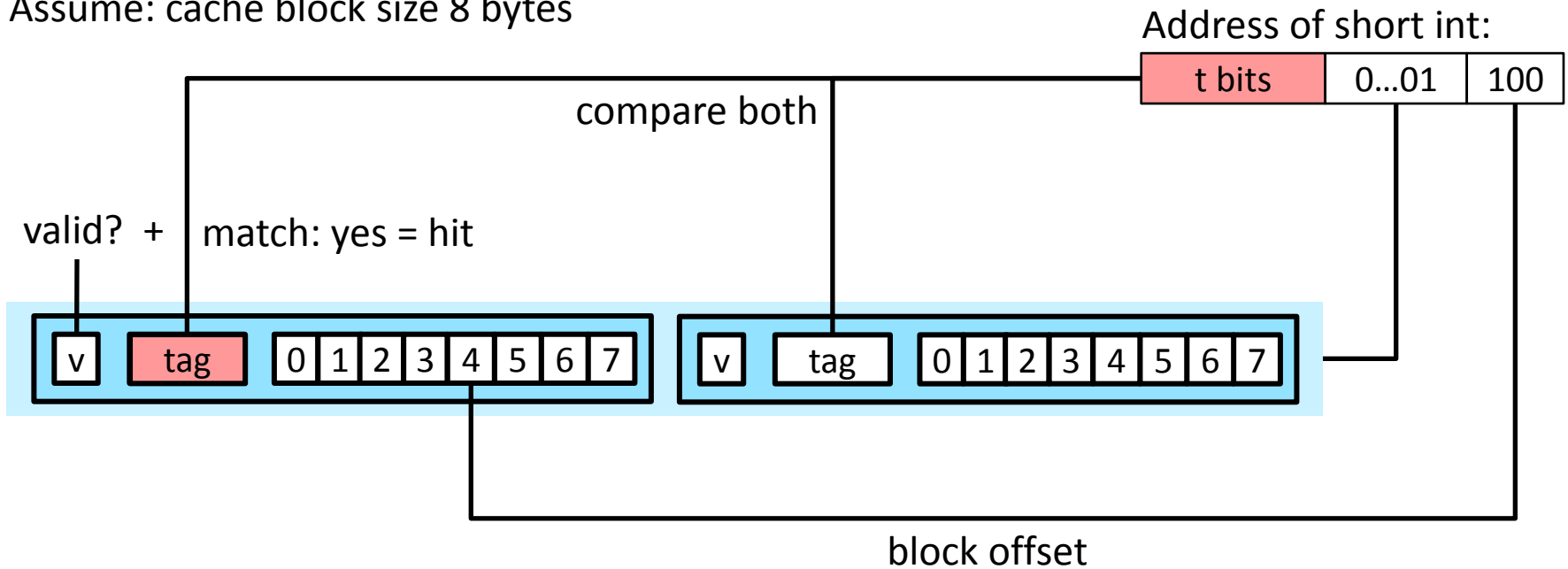
t bits	0...01	100
--------	--------	-----



# E-way Set Associative Cache

E = 2: Two lines per set

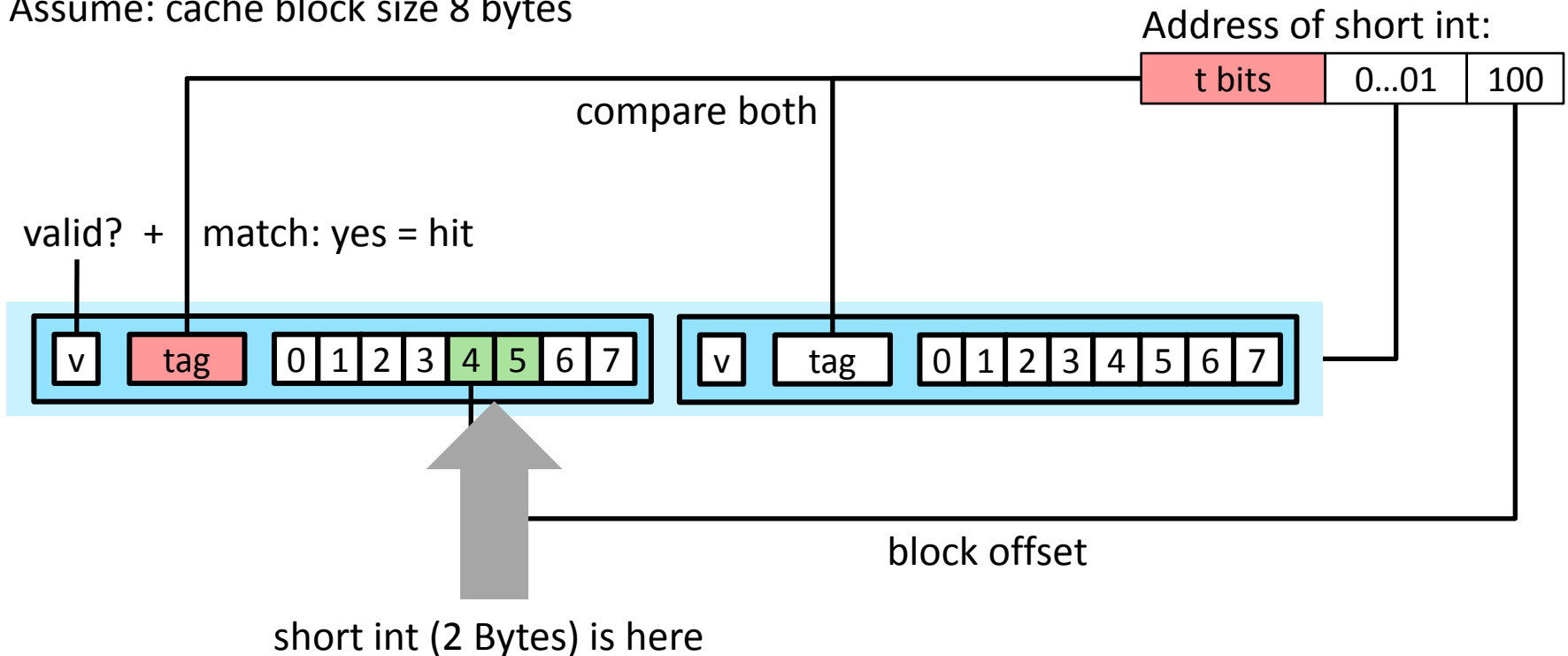
Assume: cache block size 8 bytes



# E-way Set Associative Cache

E = 2: Two lines per set

Assume: cache block size 8 bytes



## No match:

- One line in set is selected for eviction and replacement
- Replacement policies: random, least recently used (LRU), ...

# 2-Way Set Associative Cache Simulation

t=2	s=1	b=1
xx	x	x

M=16 byte addresses, B=2 bytes/block,  
S=2 sets, E=2 blocks/set

Address trace (reads, one byte per read):

0	[0000 <sub>2</sub> ],	miss
1	[0001 <sub>2</sub> ],	hit
7	[0111 <sub>2</sub> ],	miss
8	[1000 <sub>2</sub> ],	miss
0	[0000 <sub>2</sub> ]	hit

	v	Tag	Block
Set 0	1	00	M[0-1]
	1	10	M[8-9]
Set 1	1	01	M[6-7]
	0		

# A Higher Level Example

*Ignore the variables sum, i, j*

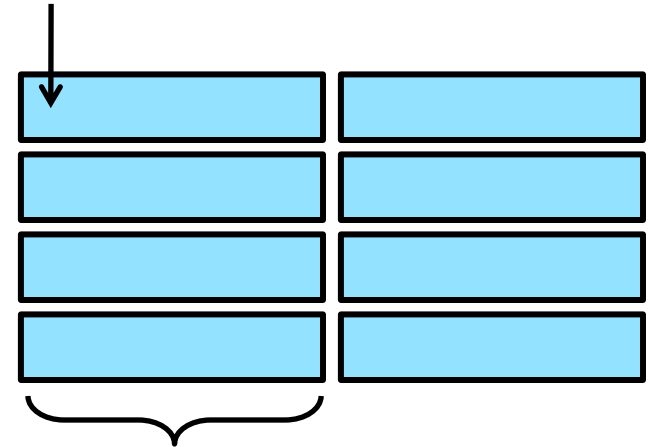
assume: cold (empty) cache,  
a[0][0] goes here

```
int sum_array_rows(double a[16][16])
{
    int i, j;
    double sum = 0;

    for (i = 0; i < 16; i++)
        for (j = 0; j < 16; j++)
            sum += a[i][j];
    return sum;
}
```

```
int sum_array_rows(double a[16][16])
{
    int i, j;
    double sum = 0;

    for (j = 0; j < 16; j++)
        for (i = 0; i < 16; i++)
            sum += a[i][j];
    return sum;
}
```



32 B = 4 doubles

# What about writes?

- Multiple copies of data exist:
  - L1, L2, Main Memory, Disk
- What to do on a write-hit?
  - **Write-through** (write immediately to memory)
  - **Write-back** (defer write to memory until replacement of line)
    - ▶ Need a dirty bit (line different from memory or not)
- What to do on a write-miss?
  - **Write-allocate** (load into cache, update line in cache)
    - ▶ Good if more writes to the location follow
  - **No-write-allocate** (writes immediately to memory)
- Typical
  - Write-through + No-write-allocate
  - **Write-back + Write-allocate**

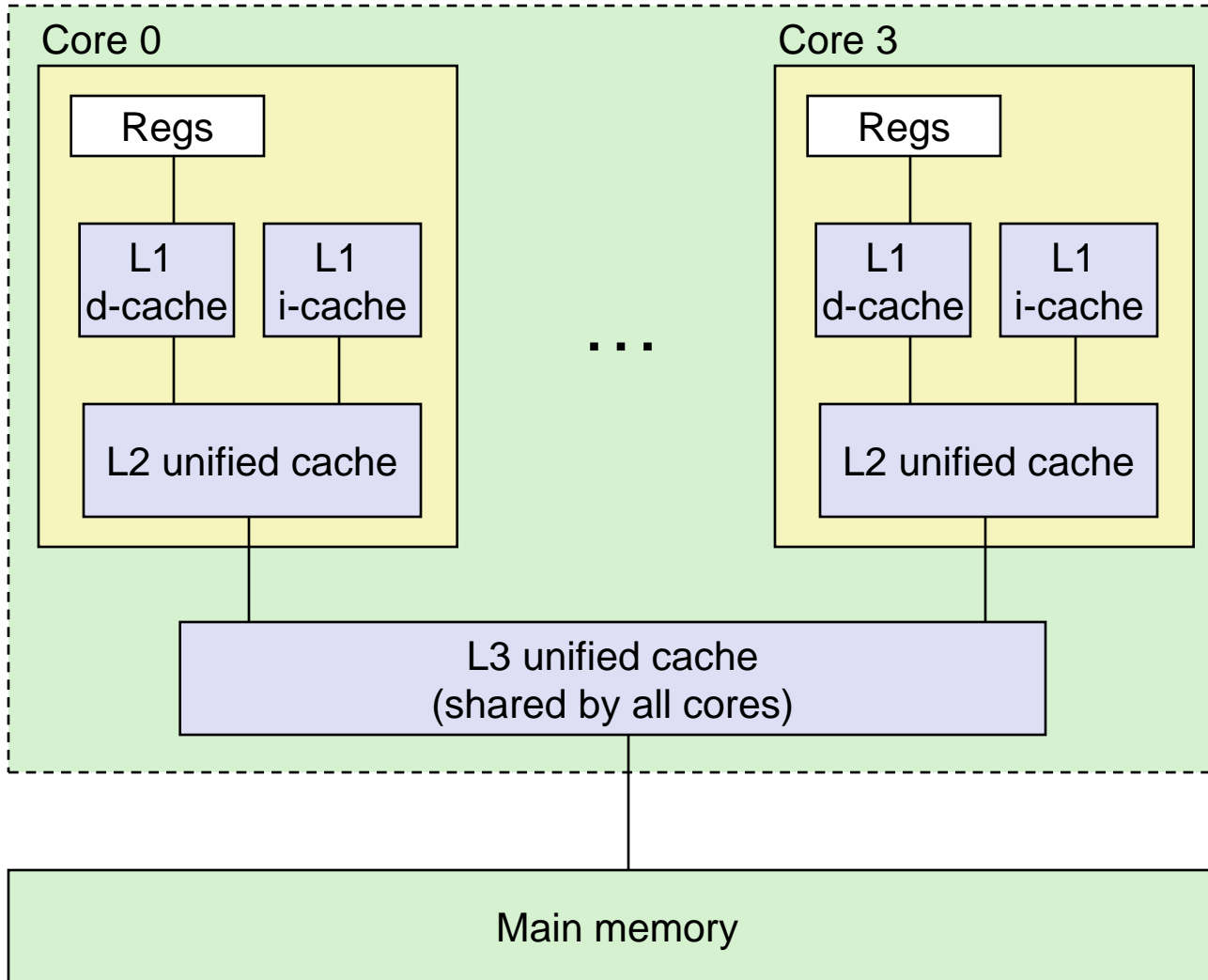


# A Common Framework for Memory Hierarchies

- Question 1: Where can a Block be Placed?  
One place (direct-mapped), a few places (set associative), or any place (fully associative)
- Question 2: How is a Block Found?  
Indexing (direct-mapped), limited search (set associative), full search (fully associative)
- Question 3: Which Block is Replaced on a Miss?  
Typically LRU or random
- Question 4: How are Writes Handled?  
Write-through or write-back

# Intel Core i7 Cache Hierarchy

Processor package



L1 i-cache and d-cache:  
32 KB, 8-way,  
Access: 4 cycles

L2 unified cache:  
256 KB, 8-way,  
Access: 11 cycles

L3 unified cache:  
8 MB, 16-way,  
Access: 30-40  
cycles

Block size: 64 bytes for  
all caches.

# Cache Performance Metrics

## ■ Miss Rate

- Fraction of memory references not found in cache (misses / accesses)  
= 1 – hit rate
- Typical numbers (in percentages):
  - ▶ 3-10% for L1
  - ▶ can be quite small (e.g., < 1%) for L2, depending on size, etc.

## ■ Hit Time

- Time to deliver a line in the cache to the processor
  - ▶ includes time to determine whether the line is in the cache
- Typical numbers:
  - ▶ 1-2 clock cycle for L1
  - ▶ 5-20 clock cycles for L2

## ■ Miss Penalty

- Additional time required because of a miss
  - ▶ typically 50-200 cycles for main memory (Trend: increasing!)

# Lets think about those numbers

- Huge difference between a hit and a miss
  - Could be 100x, if just L1 and main memory
- Would you believe 99% hits is twice as good as 97%?
  - Consider:  
cache hit time of 1 cycle  
miss penalty of 100 cycles
  - Average access time:  
97% hits:  $1 \text{ cycle} + 0.03 * 100 \text{ cycles} = \mathbf{4 \text{ cycles}}$   
99% hits:  $1 \text{ cycle} + 0.01 * 100 \text{ cycles} = \mathbf{2 \text{ cycles}}$
- This is why “miss rate” is used instead of “hit rate”

# Writing Cache Friendly Code

- Make the common case go fast
  - Focus on the inner loops of the core functions
- Minimize the misses in the inner loops
  - Repeated references to variables are good (temporal locality)
  - Stride-1 reference patterns are good (spatial locality)

Key point: Our qualitative notion of locality is quantified through our understanding of cache memories.

# Cache Memories

- Cache organization and operation
- Performance impact of caches
  - The memory mountain
  - Rearranging loops to improve spatial locality

# The Memory Mountain

- **Read throughput** (read bandwidth)
  - Number of bytes read from memory per second (MB/s)
- **Memory mountain:**  
Measured read throughput as a function of spatial and temporal locality.
  - Compact way to characterize memory system performance.

# Memory Mountain Test Function

```
/* The test function */
void test(int elems, int stride) {
    int i, result = 0;
    volatile int sink;

    for (i = 0; i < elems; i += stride)
        result += data[i];
    sink = result; /* So compiler doesn't optimize away the loop */
}

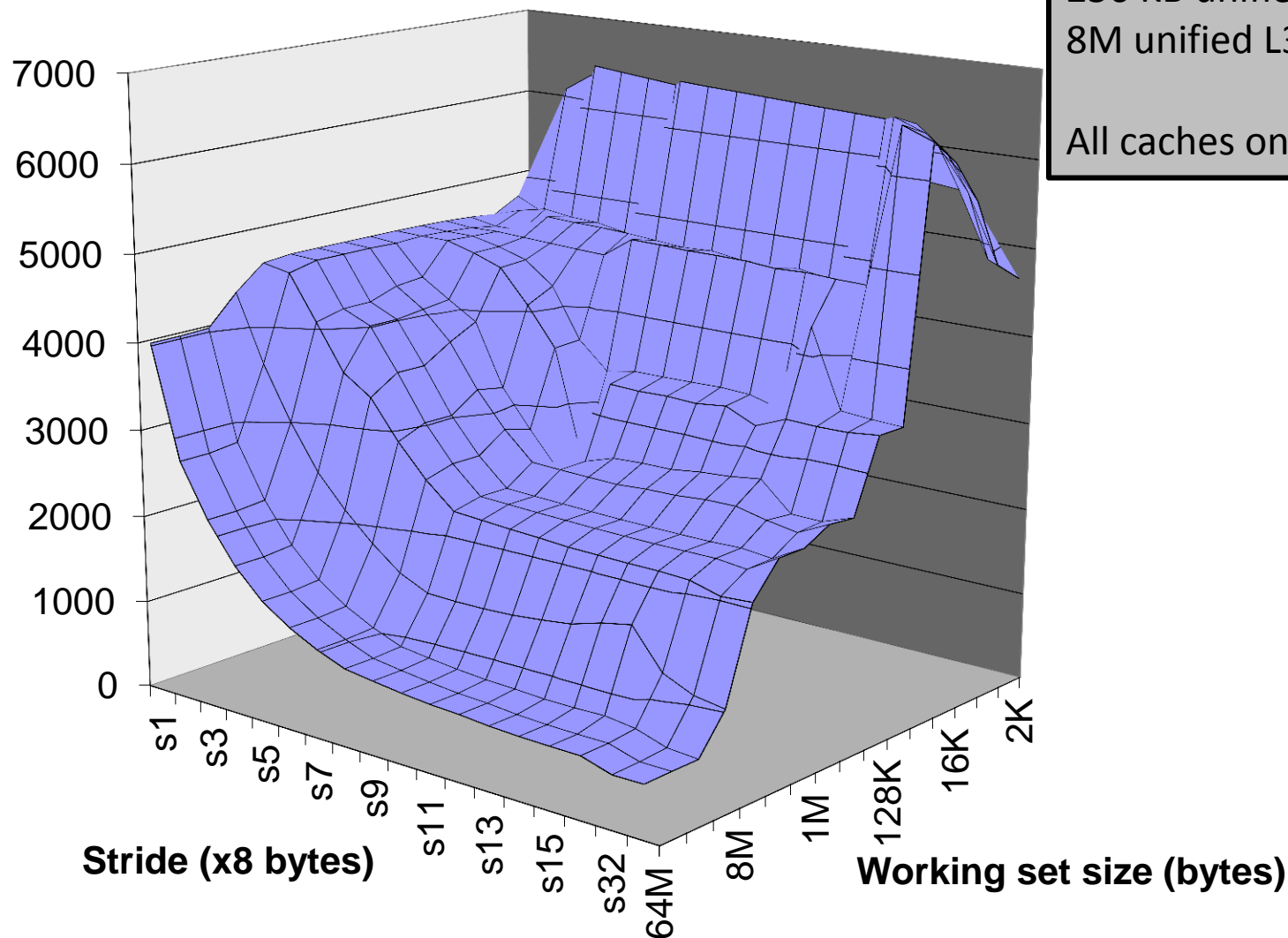
/* Run test(elems, stride) and return read throughput (MB/s) */
double run(int size, int stride, double Mhz)
{
    double cycles;
    int elems = size / sizeof(int);

    test(elems, stride); /* warm up the cache */
    cycles = fcyc2(test, elems, stride, 0); /* call test(elems, stride) */
    return (size / stride) / (cycles / Mhz); /* convert cycles to MB/s */
}
```



# The Memory Mountain

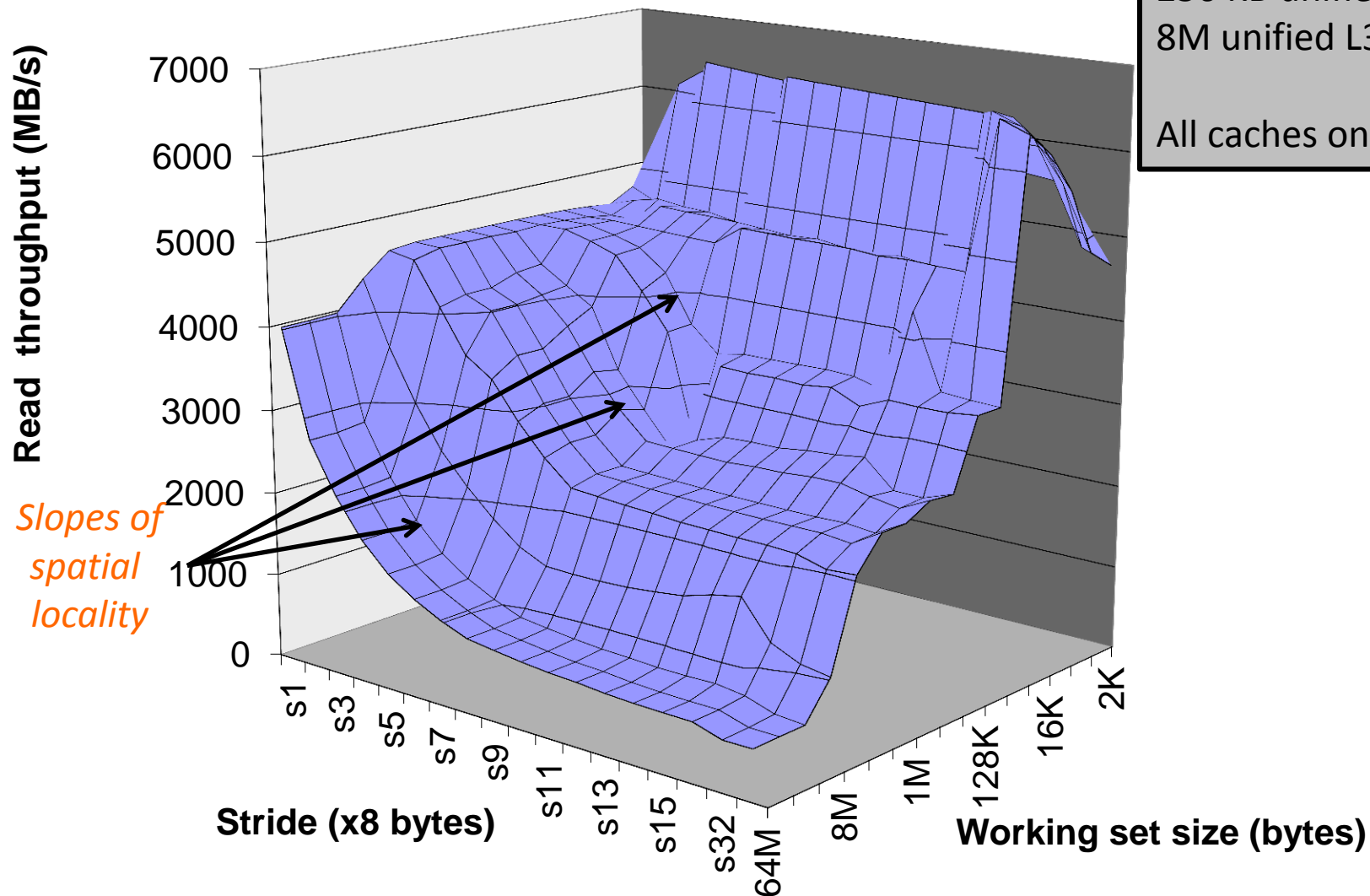
Read throughput (MB/s)



Intel Core i7  
32 KB L1 i-cache  
32 KB L1 d-cache  
256 KB unified L2 cache  
8M unified L3 cache

All caches on-chip

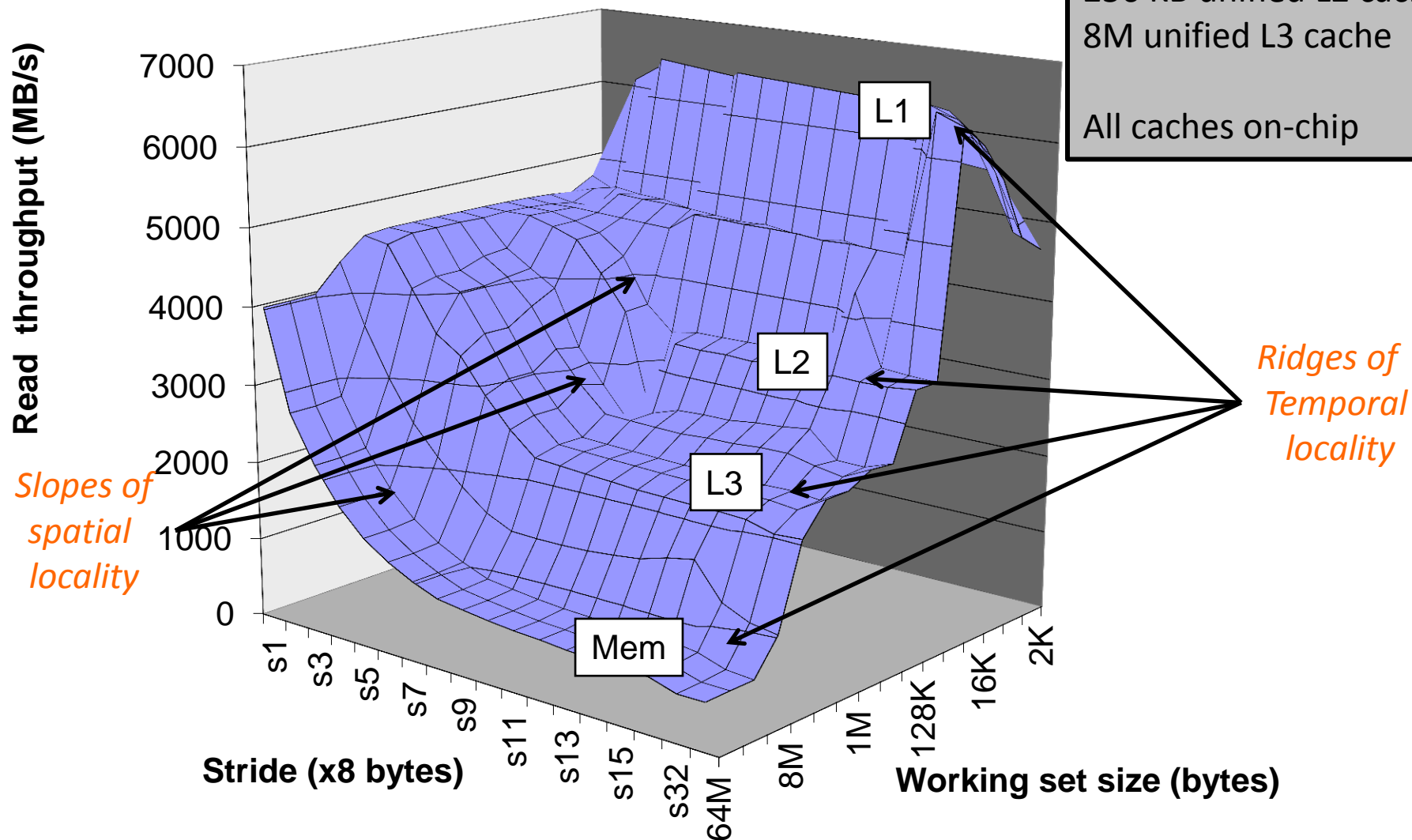
# The Memory Mountain



Intel Core i7  
32 KB L1 i-cache  
32 KB L1 d-cache  
256 KB unified L2 cache  
8M unified L3 cache

All caches on-chip

# The Memory Mountain



# Cache Memories

- Cache organization and operation
- Performance impact of caches
  - The memory mountain
  - Rearranging loops to improve spatial locality

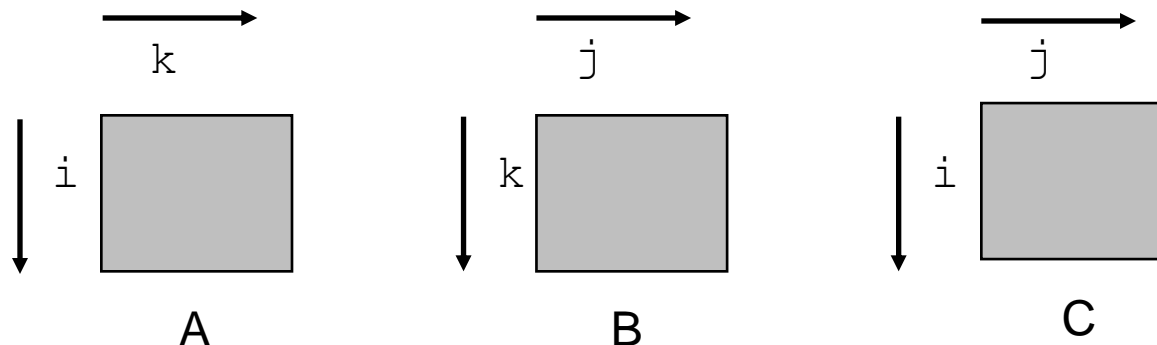
# Miss Rate Analysis for Matrix Multiply

## ■ Assume:

- Line size = 32B (big enough for four 64-bit words)
- Matrix dimension (N) is very large
  - ▶ Approximate  $1/N$  as 0.0
- Cache is not even big enough to hold multiple rows

## ■ Analysis Method:

- Look at access pattern of inner loop



# Matrix Multiplication Example

## ■ Description:

- Multiply  $N \times N$  matrices
- $O(N^3)$  total operations
- $N$  reads per source element
- $N$  values summed per destination
  - ▶ but may be able to hold in register

```
/* ijk */
for (i=0; i<n; i++) {
    for (j=0; j<n; j++) {
        sum = 0.0;
        for (k=0; k<n; k++)
            sum += a[i][k] * b[k][j];
        c[i][j] = sum;
    }
}
```

*Variable `sum`  
held in register*

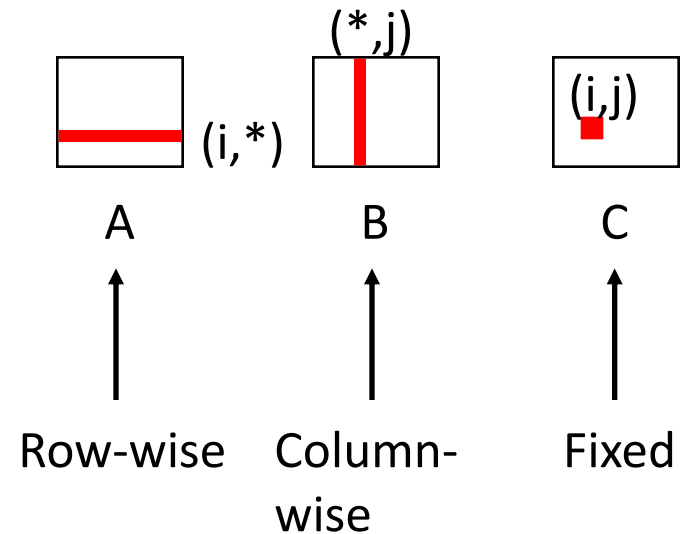
# Layout of C Arrays in Memory (review)

- C arrays allocated in row-major order
  - each row in contiguous memory locations
- Stepping through columns in one row:
  - `for (i = 0; i < N; i++)`  
    `sum += a[0][i];`
  - accesses successive elements
  - if block size (B) > 4 bytes, exploit spatial locality
    - ▶ compulsory miss rate = 4 bytes / B
- Stepping through rows in one column:
  - `for (i = 0; i < n; i++)`  
    `sum += a[i][0];`
  - accesses distant elements
  - no spatial locality!
    - ▶ compulsory miss rate = 1 (i.e. 100%)

# Matrix Multiplication (ijk)

```
/* ijk */
for (i=0; i<n; i++) {
    for (j=0; j<n; j++) {
        sum = 0.0;
        for (k=0; k<n; k++)
            sum += a[i][k] * b[k][j];
        c[i][j] = sum;
    }
}
```

Inner loop:



Misses per inner loop iteration:

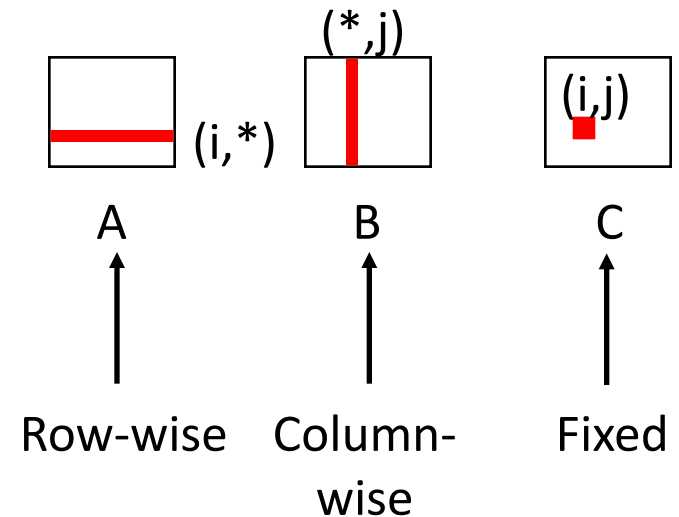
<u>A</u>	<u>B</u>	<u>C</u>
0.25	1.0	0.0



# Matrix Multiplication (jik)

```
/* jik */
for (j=0; j<n; j++) {
    for (i=0; i<n; i++) {
        sum = 0.0;
        for (k=0; k<n; k++)
            sum += a[i][k] * b[k][j];
        c[i][j] = sum
    }
}
```

Inner loop:



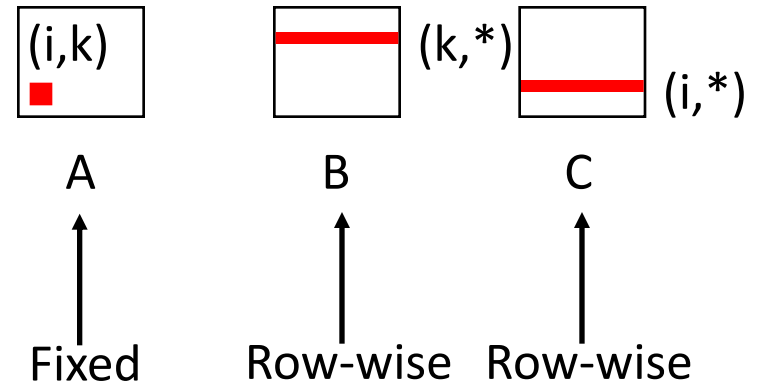
Misses per inner loop iteration:

<u>A</u>	<u>B</u>	<u>C</u>
0.25	1.0	0.0

# Matrix Multiplication (kij)

```
/* kij */  
for (k=0; k<n; k++) {  
    for (i=0; i<n; i++) {  
        r = a[i][k];  
        for (j=0; j<n; j++)  
            c[i][j] += r * b[k][j];  
    }  
}
```

Inner loop:



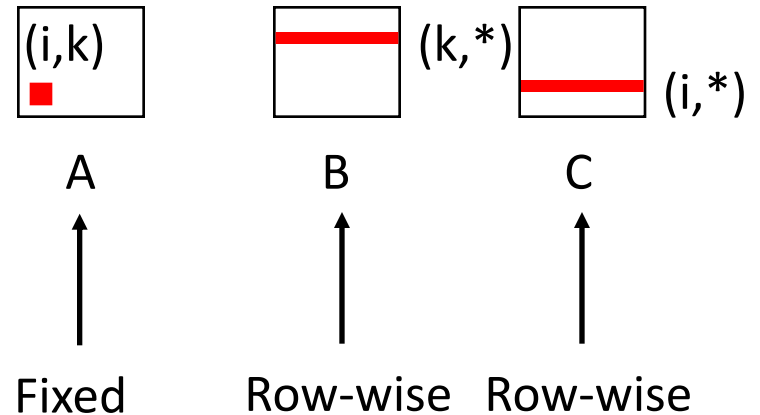
Misses per inner loop iteration:

<u>A</u>	<u>B</u>	<u>C</u>
0.0	0.25	0.25

# Matrix Multiplication (ikj)

```
/* ikj */
for (i=0; i<n; i++) {
    for (k=0; k<n; k++) {
        r = a[i][k];
        for (j=0; j<n; j++)
            c[i][j] += r * b[k][j];
    }
}
```

Inner loop:



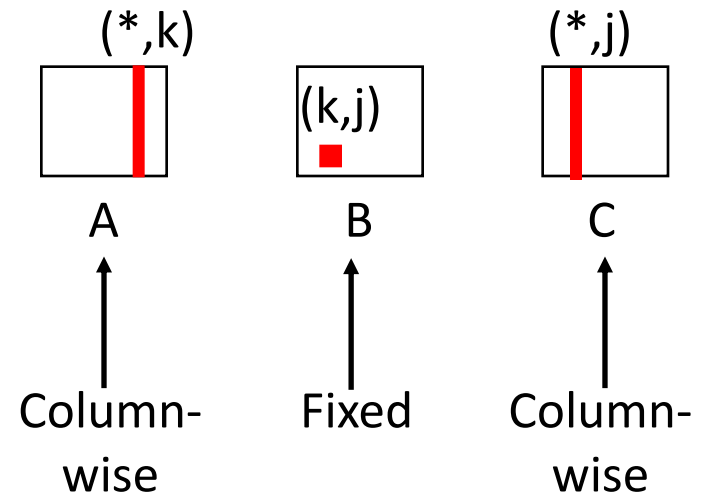
Misses per inner loop iteration:

<u>A</u>	<u>B</u>	<u>C</u>
0.0	0.25	0.25

# Matrix Multiplication (jki)

```
/* jki */  
for (j=0; j<n; j++) {  
    for (k=0; k<n; k++) {  
        r = b[k][j];  
        for (i=0; i<n; i++)  
            c[i][j] += a[i][k] * r;  
    }  
}
```

Inner loop:



Misses per inner loop iteration:

A  
1.0

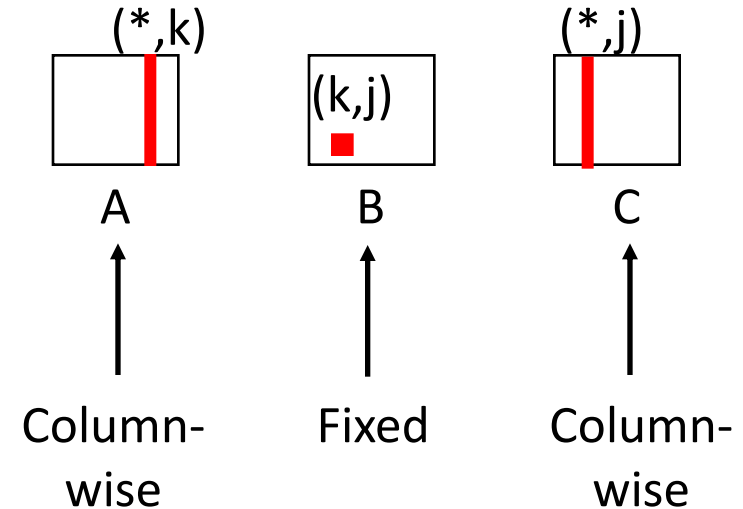
B  
0.0

C  
1.0

# Matrix Multiplication (kji)

```
/* kji */  
for (k=0; k<n; k++) {  
    for (j=0; j<n; j++) {  
        r = b[k][j];  
        for (i=0; i<n; i++)  
            c[i][j] += a[i][k] * r;  
    }  
}
```

Inner loop:



Misses per inner loop iteration:

A  
1.0

B  
0.0

C  
1.0

# Summary of Matrix Multiplication

```
for (i=0; i<n; i++) {  
    for (j=0; j<n; j++) {  
        sum = 0.0;  
        for (k=0; k<n; k++)  
            sum += a[i][k] * b[k][j];  
        c[i][j] = sum;  
    }  
}
```

ijk (& jik):

- 2 loads, 0 stores
- misses/iter = 1.25

```
for (k=0; k<n; k++) {  
    for (i=0; i<n; i++) {  
        r = a[i][k];  
        for (j=0; j<n; j++)  
            c[i][j] += r * b[k][j];  
    }  
}
```

kij (& ikj):

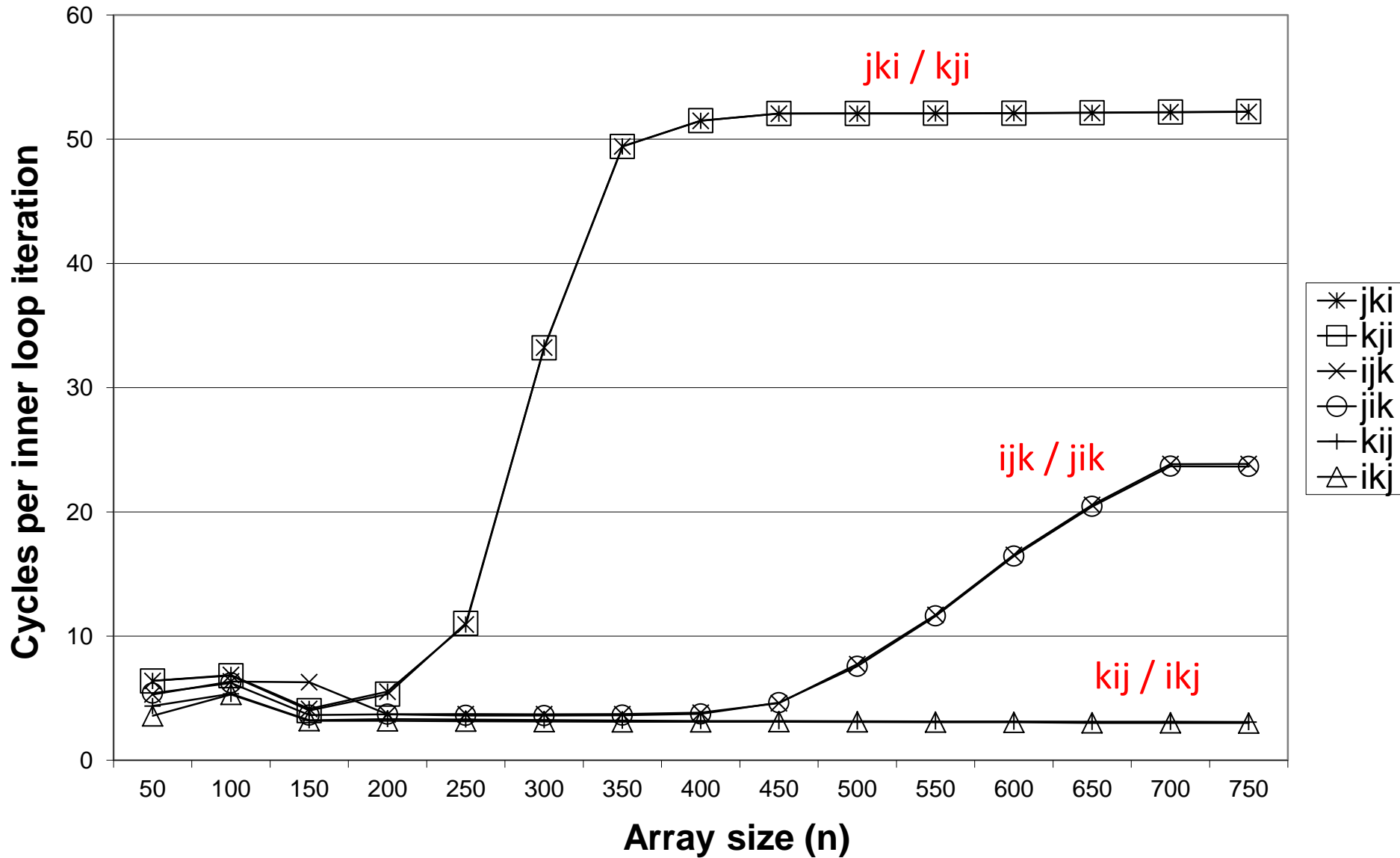
- 2 loads, 1 store
- misses/iter = 0.5

```
for (j=0; j<n; j++) {  
    for (k=0; k<n; k++) {  
        r = b[k][j];  
        for (i=0; i<n; i++)  
            c[i][j] += a[i][k] * r;  
    }  
}
```

jki (& kji):

- 2 loads, 1 store
- misses/iter = 2.0

# Core i7 Matrix Multiply Performance



# Summary

- Memory hierarchies are an optimization resulting from a perfect match between memory technology and two types of program locality
  - Temporal locality
  - Spatial locality
- The goal is to provide a “virtual” memory technology (an illusion) that has an access time of the highest-level memory with the size and cost of the lowest-level memory
- Cache memory is an instance of a memory hierarchy
  - exploits both temporal and spatial localities
  - direct-mapped caches are simple and fast but have higher miss rates
  - set-associative caches have lower miss rates but are complex and slow
  - multilevel caches are becoming increasingly popular
- Programmer can optimize for cache performance
  - How data structure are organized
  - How data are accessed (nested loop structure)