Exercise 7.6 Consider the MOSFET amplifier shown in Figure 7.6. Assume that the amplifier is operated under the saturation discipline. In its saturation region, the MOSFET is characterized by the equation

$$i_{DS} = \frac{K}{2}(v_{GS} - V_T)^2$$

where i_{DS} is the drain-to-source current when a voltage v_{GS} is applied across its gate-to-source terminals.

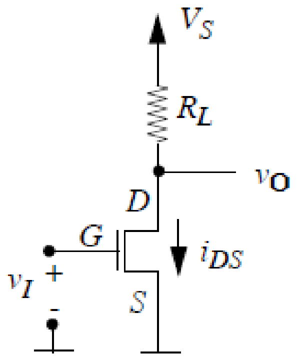


Figure 7.6:

a) Draw the equivalent circuit for the amplifier based on the SCS model of the MOSFET.

b) Write an expression relating v_O to i_{DS}.

c) Write an expression relating i_{DS} to v_I.

d) Write an expression relating v_O to v_I.

e) Suppose that an input voltage V_I results in an output voltage V_O. By what factor must V_I be increased (or decreased) so that the output voltage is doubled.

f) Suppose, again, that an input voltage V_I results in an output voltage V_O. Suppose, further, that we desire an output voltage that is $2V_O$. Assuming that both the input voltage and the MOSFET do not change, what are all the possible ways of accomplishing the desired doubling of the output voltage.

g) The power consumed by the MOSFET amplifier in Figure 7.6 is given by $V_S i_{DS}$, assuming that no current is draw out of the v_O terminal. Which of the alternatives for doubling V_O from parts (e) and (f) will result in the lowest power consumption.

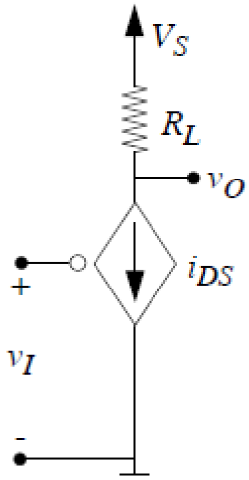


Figure 7.7:

Solution:

a) See Figure 7.7.

b) $v_O = V_S - R_L i_{DS}$

c) $i_{DS} = \frac{K}{2}(v_I - V_T)^2$ for $v_I \geq V_T$; $i_{DS} = 0$ otherwise

d) $v_O = V_S - \frac{R_L K}{2}(v_I - V_T)^2$ for $v_I \geq V_T$; $v_O = V_S$ otherwise

e) $V_O = V_S - \frac{R_L K}{2}(V_I - V_T)^2$

$2V_O = V_S - \frac{R_L K}{2}(NV_I - V_T)^2$

$V_S - 2V_O = \frac{R_L K}{2}(NV_I - V_T)^2$

$\frac{2}{R_L K}(V_S - 2V_O) = (NV_I - V_T)^2$

$NV_I - V_T = \sqrt{\frac{2}{R_L K}(V_S - 2V_O)}$

$N = \frac{\sqrt{\frac{2}{R_L K}(V_S - 2V_O)} + V_T}{V_I}$; $2V_O \leq V_S$

Scale V_I by factor N

f) $V_O = V_S - \frac{KR_L}{2}(V_I - V_T)^2$

This can be accomplished by changing V_S, R_L, or by changing both.

By changing R_L:

$2V_O = V_S - \frac{KR_L N_R}{2}(V_I - V_T)^2$

$N_R = \frac{2V_S - 4V_O}{KR_L(V_I - V_T)^2}$

Scale R_L by factor N_R. This will only work if $2V_O \leq V_S$

By changing V_S:

$2V_O = N_S V_S - \frac{KR_L}{2}(V_I - V_T)^2$

$N_S = \frac{2V_O + \frac{KR_L}{2}(V_I - V_T)^2}{V_S}$

Scale V_S by factor N_S

By changing V_S and R_L:

Scale V_S by factor X and scale R_L by factor Y where

$X = \frac{2V_O + \frac{KR_L Y}{2}(V_I - V_T)^2}{V_S}$. This will only work if $2V_O \leq XV_S$

g) The alternative from part e results in the lowest power consumption.

ANS:: (b) $v_O = V_S - R_L i_{DS}$ (c) $i_{DS} = \frac{K}{2}(v_I - V_T)^2$ for $v_I \geq V_T$; $i_{DS} = 0$ otherwise (d) $v_O = V_S - \frac{R_L K}{2}(v_I - V_T)^2$ for $v_I \geq V_T$; $v_O = V_S$ otherwise (e) $N = \frac{\sqrt{\frac{2}{R_L K}(V_S - 2V_O)} + V_T}{V_I}$
(g) e

Exercise 7.9 Consider the bipolar junction transistor (BJT) amplifier shown in Figure 7.10. Assume that the BJT is characterized by the large signal model from Exercise 7.8, and that the BJT operates in its active region. Assume further that $V_S = 5V$, $R_L = 10k$, $R_I = 500k$, and $\beta = 100$.

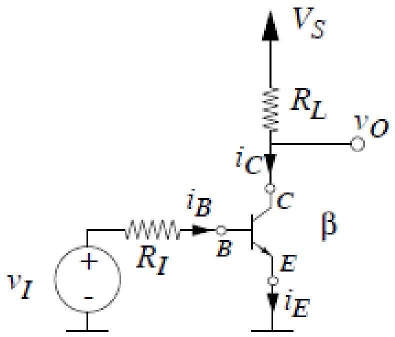


Figure 7.10:

a) Draw the equivalent circuit for the BJT amplifier based on the large signal BJT model from Exercise 7.8.

b) Write an expression relating v_O to i_C.

c) Write an expression relating i_C to v_I.

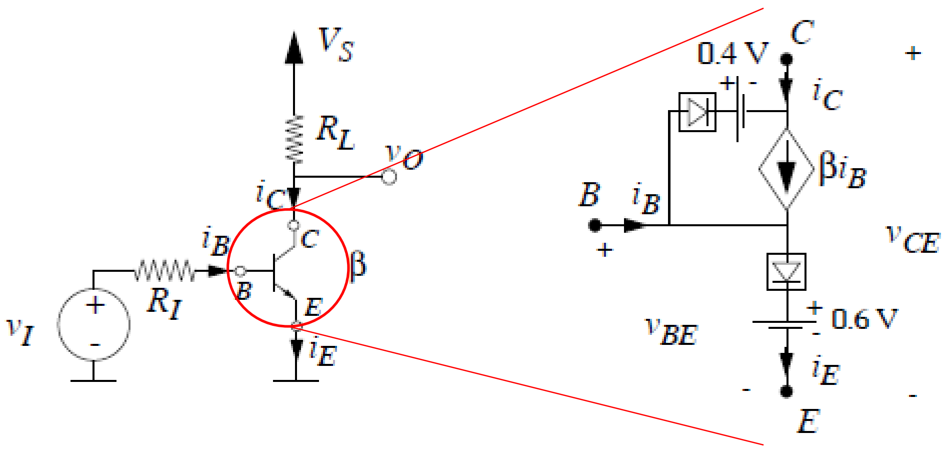
d) Write an expression relating i_E to i_B.

e) Write an expression relating v_O to v_I.

f) What is the value of v_O for an input voltage $v_I = 0.7$V? What are the corresponding values of i_B, i_C and i_E.

Solution:

a)



b)

$$v_O = V_S - i_C R_L$$

c)

$$i_C = \beta i_B = \beta \frac{v_I - 0.6}{R_I}$$

d)

$$i_E = i_B(\beta + 1)$$

e)

$$v_O = V_S - \frac{v_I - 0.6}{R_I} \beta R_L$$

Or, substituting known values

$$v_O = 6.2 - 2v_I$$

f) $v_O = 4.8V$, $i_B = 0.2\mu A$, $i_C = 20\mu A$, and $i_E = 20.2\mu A$.

ANS:: (b) $v_O = V_S - i_C R_L$ (c) $i_C = \beta \frac{v_I - 0.6}{R_I}$ (d) $i_E = i_B(\beta + 1)$ (e) $v_O = 6.2 - 2v_I$ (f) $v_O = 4.8V$, $i_B = 0.2\mu A$, $i_C = 20\mu A$, and $i_E = 20.2\mu A$.

Exercise 7.10 In this exercise you will perform a large signal analysis of the BJT amplifier shown in Figure 7.10. Assume that the BJT is characterized by the large signal model from Exercise 7.8. Assume further that $V_S = 5V$, $R_L = 10k$, $R_I = 500k$, and $\beta = 100$.

a) Write an expression relating v_O to v_I.

b) What is the lowest value of the input voltage v_I for which the BJT operates in its active region? What are the corresponding values of i_B, i_C, and v_O?

c) What is the highest value of the input voltage v_I for which the BJT operates in its active region? What are the corresponding values of i_B, i_C, and v_O?

d) Sketch a graph of v_O versus v_I for the parameter values given above.

Solution:

a)

$$v_O = V_S - \frac{v_I - 0.6}{R_I}\beta R_L$$

Or, substituting known values

$$v_O = 6.2 - 2v_I$$

b)

$$v_I = 0.6V$$

The BJT goes into cutoff if v_I goes any lower.

The corresponding values of i_B, i_C, and v_O are as follows. $i_B = 0$, $i_C = 0$, and $v_O = 5V$.

c) As v_I increases, the BJT enters saturation when the collector diode gets forward biased. This happens when the base voltage is greater than the collector voltage by 0.4V. In other words, when $v_{CE} = v_{BE} - 0.4$, or when $v_{CE} = v_O$ falls to 0.2V. The corresponding value of v_I is obtained by solving

$$v_O = 0.2 = 6.2 - 2v_I$$

Solving, we get $v_I = 3V$. In other words, when v_I rises to 3V, the output falls to 0.2V, and the BJT goes into saturation.

The corresponding values of i_B, i_C, and v_O are as follows. $i_B = 24/5\mu A$, $i_C = 480\mu A$, and $v_O = 0.2V$.

d) A graph of v_O versus v_I is made up of three straightline segments.

In the first segment, v_O is at 5V for v_I ranging from 0V to 0.6V.

In the second segment, v_O decreases linearly from 5V to 0.2V as v_I increases from 0.6V to 3V. In other words, the second segment follows the equation

$$v_O = 0.2 = 6.2 - 2v_I$$

for $v_I = 0.6V$ to $v_I = 3V$.

In the third segment, v_O stays at 0.2V for v_I greater than 3V.

ANS:: (a) $v_O = 6.2 - 2v_I$ (b) $v_I = 0.6V$, $i_B = 0$, $i_C = 0$, and $v_O = 5V$. (c) $v_I = 3V$, $i_B = 24/5\mu A$, $i_C = 480\mu A$, and $v_O = 0.2V$.

Problem 7.1 Consider the MOSFET voltage divider circuit shown in Figure 7.11. Assume that both MOSFETs operate in the saturation region. Determine the output voltage V_O as a function of the supply voltage V_S, the gate voltages V_A and V_B, and the MOSFET geometries L_1, W_1 and L_2, W_2. Assume that the MOSFET threshold voltage is V_T, and remember, $K = K_n \frac{W}{L}$.

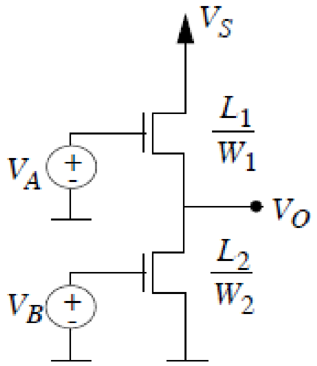


Figure 7.11:

Solution:

Since the current through both MOSFETs must be the same, V_O is forced to a value such that this is the case.

$$\frac{K_n W_2}{2L_2}(V_B - V_T)^2 = \frac{K_n W_1}{2L_1}(V_A - V_O - V_T)^2$$

$$V_O = V_A - V_T - \sqrt{\frac{W_2 L_1}{L_2 W_1}(V_B - V_T)^2}$$

ANS:: $V_O = V_A - V_T - \sqrt{\frac{W_2 L_1}{L_2 W_1}(V_B - V_T)^2}$

Problem 7.2

Solution:

a) When there is current going through R, the current is limited by two quantities: either $\frac{V_S}{R}$ or $\frac{K}{2}(v_{GS} - V_T)^2$, whichever is lower. If the limit is V_S/R, then the MOSFET is in the closed-switch region. If the limit is $\frac{K}{2}(v_{GS} - V_T)^2$, then the MOSFET is in the saturation region.

open-switch region For $v_{GS} \le V_T$, the MOSFET is open, therefore $v_{OUT} = V_S$.

saturation region When v_{GS} begins to exceed V_T, the quantity $v_{GS} - V_T$ is still small, so the current is limited by $\frac{K}{2}(v_{GS} - V_T)^2$. This current determines the output voltage, which is given by $v_{OUT} = V_S - \frac{KR}{2}(v_{IN} - V_T)^2$.

closed-switch region i_{DS} increases until it reaches $\frac{V_S}{R}$ at some gate voltage V_{IN_T}. Now v_{DS} drops to zeros, and both i_{DS} and v_{DS} are no longer affected by the increase in v_{GS}.

In summary,

$$v_{OUT} = \begin{cases} V_S & 0 \le v_{IN} \le V_T \\ V_S - \frac{KR}{2}(v_{IN} - V_T)^2 & v_T \le v_{IN} \le V_{IN_T} \\ 0 & V_{IN_T} \le v_{IN} \le V_{IN_{MAX}} \end{cases}$$

b) The lowest value of v_{IN} for which $v_{OUT} = 0$ occurs when v_{IN} is at the *transition* between the saturation region and the closed-switch region. At this point, the saturation region current limit and the closed-switch region current limit are the same,

$$i_{DS} = \frac{V_S}{R} = \frac{K}{2}(V_{IN_T} - V_T)^2$$

Solving for V_{IN_T} we get

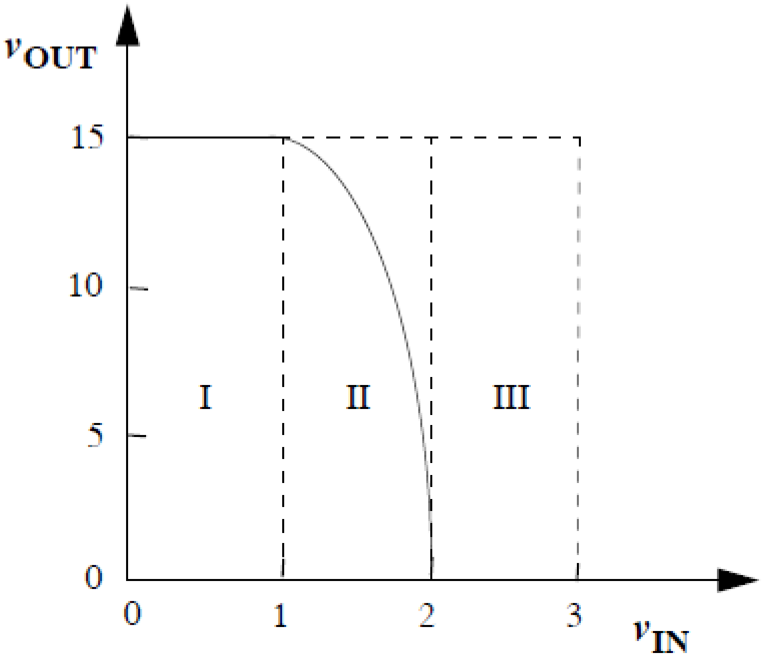
$$V_{IN_T} = \sqrt{\frac{2V_S}{KR}} + V_T$$

c) Combining the results of part (a) and (b), we obtain the following equations.

$$v_{OUT} = \begin{cases} 15 & 0 \le v_{IN} \le 1 \\ 15 - 15(v_{IN} - 1)^2 & 1 \le v_{IN} \le 2 \\ 0 & 2 \le v_{IN} \le 3 \end{cases}$$

The graph is shown in the figure.

d) Region I is the open switch region, where $v_{\mathrm{OUT}} = V_{\mathrm{S}} = 15$. Region II is the saturation region, where v_{OUT} drops according to $V_{\mathrm{S}} - \frac{KR}{2}(v_{\mathrm{IN}} - V_{\mathrm{T}})^2$. The MOSFET enters the closed-switch region when $v_{\mathrm{IN}} = V_{\mathrm{IN_T}} = 2$. In this region, $v_{\mathrm{OUT}} = 0$.



ANS:: (b)$V_{\mathrm{IN_T}} = \sqrt{\frac{2V_{\mathrm{S}}}{KR}} + V_{\mathrm{T}}$

Problem 7.14 Figure 7.28 shows a MOSFET amplifier driving a load resistor R_E. The MOSFET operates in saturation and is characterized by parameters K and V_T. Determine v_{OUT} versus v_{IN} for the circuit shown.

Solution:

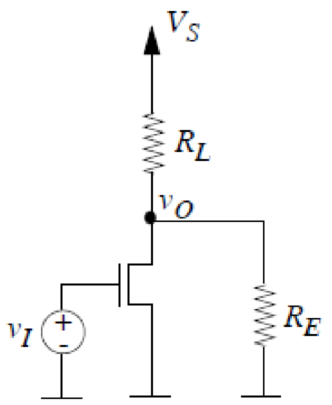


Figure 7.28:

First of all, assume that the circuit is in saturation. Call the three currents as follows: through resistor R_L: I_1, through the MOSFET: I_2, and through resistor R_E: I_3. All three of them point from higher voltage to lower, so therefore $I_1 = I_2 + I_3$. This is shown in Figure 7.29.

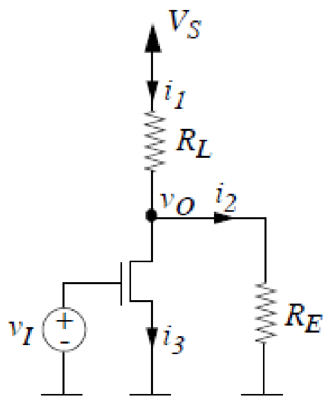


Figure 7.29:

The three currents can be determined in terms of v_{IN}, v_{OUT}, and MOSFET parameters:

$$v_{OUT} = I_3 R_E,$$

$$V_S - I_1 R_L - v_{OUT},$$

$$I_2 = \frac{K}{2}(v_{IN} - V_T)^2.$$

Substituting this into the KCL equation and solving for v_{OUT}, we get

$$v_{\mathrm{OUT}} = \frac{V_{\mathrm{S}} - \frac{KR_{\mathrm{L}}}{2}(v_{\mathrm{IN}} - V_{\mathrm{T}})^2}{1 + \frac{R_{\mathrm{L}}}{R_{\mathrm{E}}}} = \frac{2V_{\mathrm{S}}R_{\mathrm{E}} - KR_{\mathrm{E}}R_{\mathrm{L}}(v_{\mathrm{IN}} - V_{\mathrm{T}})^2}{2(R_{\mathrm{L}} + R_{\mathrm{E}})}.$$

However, this only applies for when the MOSFET is in saturation. We must find the range of v_{IN} for which this holds valid. The boundary between saturation and cutoff is merely $v_{\mathrm{IN}} \geq V_{\mathrm{T}}$. The boundary between saturation and triode can be found as follows.

$$\frac{2V_{\mathrm{S}}R_{\mathrm{E}} - R_{\mathrm{E}}R_{\mathrm{L}}K(v_{\mathrm{IN}} - V_{\mathrm{T}})^2}{2(R_{\mathrm{L}} + R_{\mathrm{E}})} \geq v_{\mathrm{IN}} - V_{\mathrm{T}}.$$

Solving this for v_{IN}, one gets the following boundary conditions for saturation:

$$V_{\mathrm{T}} \leq v_{\mathrm{IN}} \leq V_{\mathrm{T}} - \frac{R_{\mathrm{L}} + R_{\mathrm{E}}}{KR_{\mathrm{L}}R_{\mathrm{E}}} + \sqrt{\frac{1}{K^2}\left(\frac{1}{R_{\mathrm{L}}} + \frac{1}{R_{\mathrm{E}}}\right)^2 + \frac{2V_{\mathrm{S}}}{KR_{\mathrm{L}}}}.$$

For the cutoff region, we can find the output voltage through a simple voltage divider relation, since no current flows through the MOSFET:

$$v_{\mathrm{OUT}} = V_{\mathrm{S}}\frac{R_{\mathrm{E}}}{R_{\mathrm{E}} + R_{\mathrm{L}}}.$$

The voltage transfer characteristic for triode region will not be considered for this problem.

ANS:: $v_{\mathrm{OUT}} = \frac{2V_{\mathrm{S}}R_{\mathrm{E}} - KR_{\mathrm{E}}R_{\mathrm{L}}(v_{\mathrm{IN}} - V_{\mathrm{T}})^2}{2(R_{\mathrm{L}} + R_{\mathrm{E}})}$

Problem 7.17 Determine v_O versus v_I for the circuit shown in Figure 7.34. Assume that the MOSFET operates in saturation and is characterized by the parameters K and V_T.

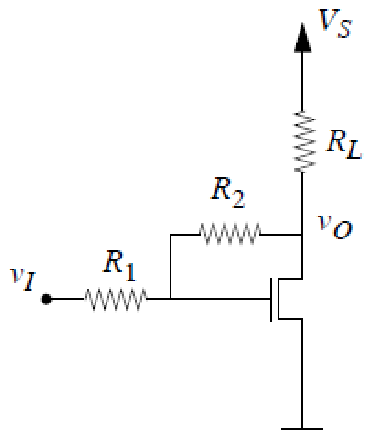


Figure 7.34:

Solution:

First of all, define v_G to be the gate voltage. Also, define three currents i_1, i_2, and i_3 to be the currents flowing through R_L, R_2, and the MOSFET, respectively. Define i_3 to be flowing towards ground, and let $i_1 + i_2 = i3$. This is shown in Figure 7.35.

The gate voltage can be found through a voltage divider rule since no current flows from between R_1 and R_2 to the gate.

$$v_G = \frac{R_2}{R_1 + R_2} v_{IN} + \frac{R_1}{R_1 + R_2} v_{OUT}$$

In cutoff, the output voltage and the input voltage are related by a voltage divider rule:

$$v_{OUT} = \frac{V_S(R_1 + R_2) + V_{IN} R_L}{R_1 + R_2 + R_L}$$

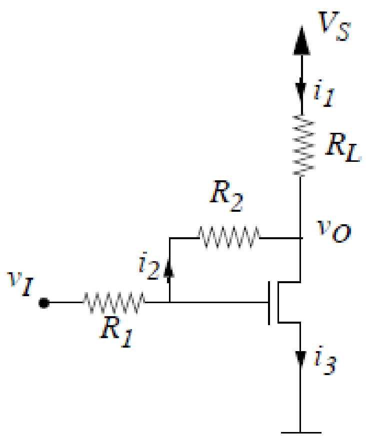


Figure 7.35:

In saturation, we have an extra current to worry about. We substitute into our original KCL equation to get

$$\frac{V_{\mathrm{S}} - v_{\mathrm{OUT}}}{R_{\mathrm{L}}} + \frac{v_{\mathrm{IN}} - v_{\mathrm{OUT}}}{R_{\mathrm{L}}} = \frac{K}{2}\left(\frac{R_2}{R_1 + R_2}v_{\mathrm{IN}} + \frac{R_1}{R_1 + R_2}v_{\mathrm{OUT}} - V_{\mathrm{T}}\right)^2$$

We can solve this for v_{OUT}, but it ends up being quite monstrous. Let $R_{\mathrm{T}} = R_1 + R_2$.

$$v_{\mathrm{OUT}} = \frac{R_2 R_{\mathrm{T}} V_{\mathrm{T}}}{R_1^2} - \frac{R_2 v_{\mathrm{IN}}}{R_1} - \frac{R_{\mathrm{T}}^2}{K R_{\mathrm{L}} R_1} - \frac{R_{\mathrm{T}}}{K R_1^2} + \frac{\sqrt{L + M + N}}{2K R_{\mathrm{L}} R_1},$$

with the following subexpressions:

$$L = R_{\mathrm{T}}^2 (R_{\mathrm{T}} + R_{\mathrm{L}})^2,$$

$$M = K^2 R_{\mathrm{L}}^2 R_{\mathrm{T}}^2 V_{\mathrm{T}} (R_1 - R_2)(2 v_{\mathrm{IN}} R_1 - V_{\mathrm{T}} R_{\mathrm{T}}),$$

$$N = 2K(V_{\mathrm{S}} R_{\mathrm{L}} R_1^2 R_{\mathrm{T}}^2 - V_{\mathrm{T}} R_1 R_2 R_{\mathrm{T}}^2 (R_1 + R_2 + R_{\mathrm{L}}) + v_{\mathrm{IN}} R_{\mathrm{L}} R_1 R_{\mathrm{T}}^2 (R_{\mathrm{L}} + R_2)).$$

The boundaries for which the device is in saturation can be found by evaluating $v_{\mathrm{G}} \geq V_{\mathrm{T}}$ and $v_{\mathrm{OUT}} \geq v_{\mathrm{G}} - v_{\mathrm{T}}$. This evaluation is even more complicated than the previous equation, since v_{G} is given in terms of v_{OUT}, and needs to be put in terms of v_{IN}. In terms of both v_{IN} and v_{OUT}, the boundary conditions are derived much more easily.

Between saturation and cutoff:

$$\frac{R_2}{R_1 + R_2}v_{\mathrm{IN}} + \frac{R_1}{R_1 + R_2}v_{\mathrm{OUT}} \geq V_{\mathrm{T}}$$

Between saturation and triode:

$$v_{\mathrm{OUT}} \geq v_{\mathrm{IN}} - \frac{R_1 + R_2}{R_2}v_{\mathrm{T}}$$

Problem 7.19 Consider the compound three terminal device formed by connecting two BJTs in the configuration shown in Figure 7.37. The three terminals are labeled C', B' and E'. The two BJTs are identical, each with $\beta = 100$. Assume that each of the BJTs operates in the active region.

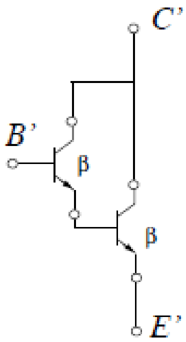


Figure 7.37:

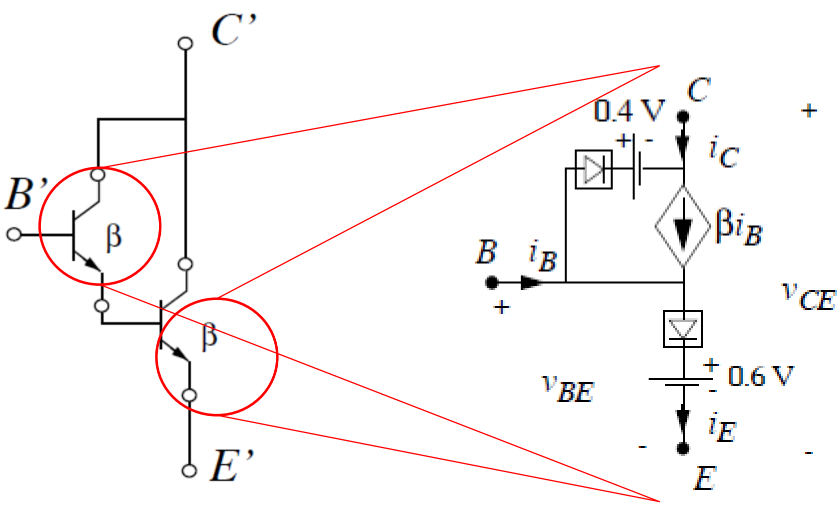
a) Draw the active-region equivalent circuit of the compound BJT by replacing each of the BJTs by the piecewise linear model shown in Exercise 7.8. Clearly label the C', B' and E' terminals.

b) In the configuration shown, the compound device behaves like a BJT. Determine the value of the current gain β' for this compound BJT.

c) When the base current $i_{B'} > 0$, determine the voltage between the B' and E' terminals.

Solution:

a)



b) The current gain of the new device is given by

$$\beta' = (\beta + 2)\beta$$

c) When the base current $i_{B'} > 0$, both transistors are in their active region. In this situation, the voltage between the B' and E' terminals is 1.2V.

ANS:: (b) $\beta' = (\beta + 2)\beta$ (c) 1.2V