

TITLE

November 30, 2025

CONTENTS

References

1

REFERENCES

- [AP07] Jeffrey D. Achter and Rachel Pries. The integral monodromy of hyperelliptic and trielliptic curves. *Mathematische Annalen*, 338(1):187–206, 2007.
- [Arm92] Brumer Armand. The average rank of elliptic curves i. *Inventiones mathematicae*, 109(1):445–472, 1992.
- [Ayo23] Joseph Ayoub. Counterexamples to F. Morel’s conjecture on $\pi_0^{\mathfrak{D}^1}$. *Comptes Rendus. Mathématique*, 361:1087–1090, 2023.
- [Bal04] Paul Balmer. The spectrum of prime ideals in tensor triangulated categories, 2004.
- [BBD82] Alexander A. Beilinson, Joseph Bernstein, and Pierre Deligne. Analyse et topologie sur les espaces singuliers (i). *Astérisque*, 100, 1982.
- [BBDG18] Alexander A. Beilinson, Joseph Bernstein, Pierre Deligne, and Ofer Gabber. *Faisceaux pervers*, volume 4. Société mathématique de France Paris, 2018.
- [BBK⁺23] Barinder S. Banwait, Armand Brumer, Hyun Jong Kim, Zev Klagsbrun, Jacob Mayle, Padmavathi Srinivasan, and Isabel Vogt. Computing nonsurjective primes associated to galois representations of genus 2 curves. *LuCaNT: LMFDB, Computation, and Number Theory*, 796:129, 2023.
- [BC19] Tilman Bauer and Magnus Carlson. Tensor products of affine and formal abelian groups. *Documenta Mathematica*, 24:2525–2582, 2019.
- [BFK⁺17] Valentin Blomer, Étienne Fouvry, Emmanuel Kowalski, Philippe Michel, and Djordje Milićević. Some applications of smooth bilinear forms with kloosterman sums. *Proceedings of the Steklov Institute of Mathematics*, 296:18–29, 2017.
- [BGI71] Pierre Berthelot, Alexander Grothendieck, and Luc Illusie. *Théorie des Intersections et Théorème de Riemann-Roch (SGA6)*, volume 225 of *Lecture Notes in Mathematics*. Springer-Verlag, 1971.
- [BH12] Salman Baig and Chris Hall. Experimental data for goldfeld’s conjecture over function fields. *Experimental Mathematics*, 21(4):362–374, 2012.
- [BLGHT11] Tom Barnet-Lamb, David Geraghty, Michael Harris, and Richard Taylor. A family of Calabi–Yau varieties and potential automorphy ii. *Publications of the Research Institute for Mathematical Sciences*, 47(1):29–98, 2011.
- [Bor12] Armand Borel. *Linear algebraic groups*, volume 126 of *Graduate Texts in Mathematics*. Springer New York, 2012.
- [BS15a] Manjul Bhargava and Arul Shankar. Binary quartic forms having bounded invariants, and the boundedness of the average rank of elliptic curves. *Annals of Mathematics*, pages 191–242, 2015.
- [BS15b] Manjul Bhargava and Arul Shankar. Ternary cubic forms having bounded invariants, and the existence of a positive proportion of elliptic curves having rank 0. *Annals of Mathematics*, pages 587–621, 2015.

- [BS15c] Bhargav Bhatt and Peter Scholze. The pro-étale topology for schemes. *Astérisque*, 360:99–201, 2015.
- [BSD65] Bryan John Birch and Peter Francis Swinnerton-Dyer. Notes on elliptic curves. ii. *Journal für die reine und angewandte Mathematik*, 218:79–108, 1965.
- [BSS18] Bhargav Bhatt, Christian Schnell, and Peter Scholze. Vanishing theorems for perverse sheaves on abelian varieties, revisited. *Selecta Mathematica*, 24:63–84, 2018.
- [Cho08] Utsav Choudhury. Homotopy theory of schemes and a^1 -fundamental groups. Master’s thesis, Università degli Studi di Padova, 2008.
- [CHT08] Laurent Clozel, Michael Harris, and Richard Taylor. Automorphy for some l -adic lifts of automorphic mod l galois representations. *Publications mathématiques*, 108:1–181, 2008.
- [DA73] Pierre Deligne and Michael Artin. *Théorie des Topos et Cohomologies Étale des Schémas. Séminaire de Géométrie Algébrique due Bois-Marie 1963-1964 (SGA 4)*. Lecture Notes in Mathematics. Springer Berlin, 1973.
- [DBG⁺77] Pierre Deligne, Jean-François Boutot, Alexander Grothendieck, Luc Illusie, and Jean-Louis Verdier. *Étale Cohomology. Séminaire de Géométrie Algébrique due Bois-Marie 1963-1964 (SGA 4 1/2)*. Lecture Notes in Mathematics. Springer-Verlag, 1977.
- [Del80] Pierre Deligne. La conjecture de Weil : II. *Publications Mathématiques de l’IHÉS*, 52:137–252, 1980.
- [Del89] Pierre Deligne. Le groupe fondamental de la droite projective moins trois points. In *Galois Groups over \mathbb{Q} : Proceedings of a Workshop Held March 23–27, 1987*, pages 79–297. Springer, 1989.
- [Die02] Luis V. Dieulefait. Explicit determination of the images of the Galois representations attached to abelian surfaces with $\text{End}(A) = \mathbb{Z}$. *Experiment. Math.*, 11(4):503–512 (2003), 2002.
- [DR04] Luis V. Dieulefait and Victor Rotger. The arithmetic of qm-abelian surfaces through their galois representations. *Journal of Algebra*, 281:124–143, 2004.
- [Dri89] Vladimir Gershonovich Drinfeld. Cohomology of compactified manifolds of modules of f -sheaves. *Journal of Soviet Mathematics*, 46(2):1789–1821, 1989.
- [Dru22] Anderi Eduardovich Druzhinin. Stable A^1 -connectivity over a base. *Journal für die reine und angewandte Mathematik (Crelles Journal)*, 2022(792):61–91, 2022.
- [DZ19] Alexander Dunn and Alexandru Zaharescu. Sums of Kloosterman sums over primes in an arithmetic progression. *The Quarterly Journal of Mathematics*, 70(1):319–342, 2019.
- [Eke07] Torsten Ekedahl. *On The Adic Formalism*, pages 197–218. Birkhäuser Boston, Boston, MA, 2007.
- [ELS20] Jordan S. Ellenberg, Wanlin Li, and Mark Shusterman. Nonvanishing of hyperelliptic zeta functions over finite fields. *Algebra & Number Theory*, 14(7):1895–1909, 8 2020.
- [EVW16] Jordan S. Ellenberg, Akshay Venkatesh, and Craig Westerland. Homology stability for Hurwitz spaces and the Cohen-Lenstra conjecture over function fields. *Annals of Mathematics*, 183:729–786, 2016.
- [FFK25] Arthur Forey, Javier Fresán, and Emmanuel Kowalski. Arithmetic fourier transforms over finite fields: generic vanishing, convolution, and equidistribution, 2025.
- [FK12] Sergey Finashin and Viatcheslav Kharlamov. Abundance of Real Lines on Real Projective Hypersurfaces. *International Mathematics Research Notices*, 2013(16):3639–3646, 06 2012.
- [FLR23] Tony Feng, Aaron Landesman, and Eric M. Rains. The geometric distribution of Selmer groups of elliptic curves over function fields. *Mathematische Annalen*, 387:615–687, 2023.
- [Fu15] Lei Fu. *Etale Cohomology Theory*, volume 14 of *Nankai Tracts in Mathematics*. World Scientific, 2015.
- [FvdG04] Carel Faber and Gerard van der Geer. Complete subvarieties of moduli spaces and the Prym map. *Journal für die reine und angewandte Mathematik*, 2004(573):117–137, 2004.
- [GL96] Ofer Gabber and François Loeser. Faisceaux pervers ℓ -adiques sur un tore. *Duke Math J.*, 83(3):501–606, 1996.
- [Gol06] Dorian Goldfeld. Conjectures on elliptic curves over quadratic fields. In *Number Theory Carbon-dale 1979: Proceedings of the Southern Illinois Number Theory Conference Carbondale, March 30 and 31, 1979*, pages 108–118. Springer, 2006.

- [GR04] Alexander Grothendieck and Michèle Raynaud. Revêtements étales et groupe fondamental (SGA 1). eprint arXiv math/0206203, 2004. Updated edition of the book of the same title published by Springer-Verlag in 1971 as volume 224 of the series Lecture Notes in Mathematics.
- [Gro77] Alexander Grothendieck. *Cohomologie l -adique et fonctions L* Séminaire de Géométrie Algébrique due Bois-Marie 1965-1966 (SGA 5), volume 589 of *Springer Lecture Notes*. Springer-Verlag, 1977. Avec la collaboration de I. Bucur, C. Houzel, L. Illusie, J.-P. Jouanolou, et J.-P. Serre.
- [GV72] Alexander Grothendieck and Jean-Louis Verdier. *Théorie des Topos et Cohomologie Étale des Schemas. Séminaire de Géométrie Algébrique du Bois-Marie 1963-1964 (SGA 4)*. Lecture Notes in mathematics. Springer-Verlag Berlin Heidelberg, 1 edition, 1972.
- [HB04] David Heath-Brown. The average analytic rank of elliptic curves. *Duke Math. J.*, 122:591–623, 2004.
- [HK25] Chris Hall and Hyun Jong Kim. Independence of ℓ (title to be determined). In progress, 2025.
- [HM73] Dale Husemoller and John Milnor. *Symmetric Bilinear Forms*, volume 73 of *Ergebnisse der Mathematik und ihrer Grenzgebiete. 2. Folge*. Springer Berlin Heidelberg, 1973.
- [HSBT05] Michael Harris, Nick Shepherd-Barron, and Richard Taylor. Ihara’s lemma and potential automorphy, 2005.
- [Hub97] Annette Huber. Mixed perverse sheaves for schemes over number fields. *Compositio Mathematica*, 108:107–121, 1997.
- [Ibu22] Tomoyoshi Ibukiyama. Supersingular loci of low dimensions and parahoric subgroups. *Osaka Journal of Mathematics*, 59:703–726, 2022.
- [Jor05] Andrei Jorza. The birch and swinnerton-dyer conjecture for abelian varieties over number fields, 2005.
- [Joy02] André Joyal. Quasi-categories and kan complexes. *Journal of Pure and Applied Algebra*, 175(1-3):207–222, 2002.
- [Kat90] Nicholas M. Katz. *Exponential sums and differential equations*, volume 124 of *Annals of Mathematics Studies*. Princeton University Press, 1990.
- [Kat96] Nicholas M. Katz. *Rigid Local Systems*, volume 139 of *annals of Mathematics Studies*. Princeton University Press, 1996.
- [Kat98] Nicholas M. Katz. *Gauss Sums, Kloosterman Sums, and Monodromy Groups*, volume 116 of *Annals of Mathematics Studies*. Princeton University Press, 1998.
- [Kat12] Nicholas M. Katz. *Convolution and Equidistribution Sato-Tate Theorems for Finite-Field Mellin Transforms*, volume 180 of *Annals of Mathematics Studies*. Princeton University Press, 2012.
- [Kim23] Hyun Jong Kim. `trouver`, 2023. GitHub repository: <https://github.com/hyunjongkimmath/trouver>.
- [Kim24] Hyun Jong Kim. *Cohen-Lenstra heuristics and vanishing of zeta functions for superelliptic curves over finite fields*. PhD thesis, University of Wisconsin-Madison, 2024.
- [KL85] Nicholas M. Katz and Gérard Laumon. Transformation de fourier et majoration de sommes exponentielles. *Publications Mathématiques de l’IHÉS*, 62:145–202, 1985.
- [KLSW23] Jesse Leo Kass, Marc Levine, Jake P. Solomon, and Kirsten Wickelgren. A quadratically enriched count of rational curves. arXiv 2307.01936, 2023.
- [KM23] Seoyoung Kim and M. Ram Murty. From the Birch and Swinnerton-Dyer conjecture to Nagao’s conjecture. *Mathematics of Computation*, 92(339):385–408, 2023.
- [KMS17] Emmanuel Kowalski, Philippe Michel, and Will Sawin. Bilinear forms with Kloosterman sums and applications. *Annals of Mathematics*, 186:413–500, 2017.
- [KP24] Hyun Jong Kim and Sun Woo Park. Global A^1 degrees of covering maps between modular curves, 2024.
- [Krä14] Thomas Krämer. Perverse sheaves on semiabelian varieties. *Rendiconti del Seminario Matematico della Università di Padova*, 132:83–102, 2014.
- [KS99] Nicholas M. Katz and Peter Sarnak. *Random matrices, Frobenius eigenvalues, and monodromy*, volume 45. American Mathematical Society, 1999.
- [KS22] Timo Keller and Michael Stoll. Exact verification of the strong bsd conjecture for some absolutely simple abelian surfaces. *Comptes Rendus Mathématique*, 360:483–489, 2022.

- [KW13] Reinhardt Kiehl and Rainer Weissauer. *Weil Conjectures, Perverse Sheaves and ℓ -adic Fourier Transform*, volume 42 of *Ergebnisse der Mathematik und ihrer Grenzgebiete. 3. Folge / A Series of Modern Surveys in Mathematics*. Springer Berlin, Heidelberg, 2013.
- [KW15] Thomas Krämer and Rainer Weissauer. Vanishing theorems for constructible sheaves on abelian varieties. *J. Algebraic Geometry*, 24:531–568, 2015.
- [KW19] Jesse Leo Kass and Kirsten Wickelgren. The class of Eisenbud-Khimshiashvili-Levine is the local \mathbb{A}^1 – Brouwer degree. *Duke Mathematical Journal*, 168(3):429–469, 2019.
- [KW21] Jesse Leo Kass and Kirsten Wickelgren. An arithmetic count of the lines on a smooth cubic surface. *Compositio Mathematica*, 157(4):677–709, 2021.
- [Laf02] Laurent Lafforgue. Chtoucas de Drinfeld et correspondance de Langlands. *Inventiones mathematicae*, 147:1–241, 2002.
- [Lom17] Davide Lombardo. Galois representations attached to abelian varieties of cm type. *Bulletin de la Société mathématique de France*, 145(3):469–501, 2017.
- [Lur09] Jacob Lurie. *Higher topos theory*. Princeton University Press, 2009.
- [May99] Jon Peter May. *A Concise Course in Algebraic Topology*. Chicago Lectures in Mathematics. University of Chicago Press, 1999.
- [Mil80] James S. Milne. *Etale cohomology*. Number 33 in Princeton Mathematical Series. Princeton university press, 1980.
- [Mil07] James S. Milne. Quotients of Tannakian categories. *Theory and Applications of Categories*, 18(21):654–664, 2007.
- [Mil13] James S. Milne. Lie algebras, algebraic groups, and lie groups, 2013. Available at www.jmilne.org/math/.
- [Mil17] James S. Milne. *Algebraic Groups*, volume 170 of *Cambridge Studies in Advanced Mathematics*. Cambridge University Press, 2017.
- [Mit14] Howard H. Mitchell. The subgroups of the quaternary abelian linear group. *Trans. Amer. Math. Soc.*, 15(4):379–396, 1914.
- [Mor06] Fabien Morel. \mathbb{A}^1 -algebraic topology. In *International Congress of Mathematicians*, volume 2, pages 1035–1059, 2006.
- [Mor12] Fabien Morel. *\mathbb{A}^1 -Algebraic Topology over a field*. Lecture Notes in Mathematics. Springer Berlin, Heidelberg, 2012.
- [MV99] Fabien Morel and Vladimir Voevodsky. \mathbb{A}^1 -homotopy theory of schemes. *Publications Mathématiques de l’IHÉS*, 90:45–143, 1999.
- [Nag97] Koh-ichi Nagao. $Q(t)$ -rank of elliptic curves and certain limit coming from the local points. *Manuscripta mathematica*, 92(1):13–32, 1997.
- [nLa25a] nLab authors. geometric morphism. <https://ncatlab.org/nlab/show/geometric+morphism>, July 2025. Revision 61.
- [nLa25b] nLab authors. homotopy group of a spectrum. <https://ncatlab.org/nlab/show/homotopy+group+of+a+spectrum>, June 2025. Revision 7.
- [nLa25c] nLab authors. Introduction to Stable homotopy theory – 1-1. <https://ncatlab.org/nlab/show/Introduction+to+Stable+homotopy+theory+---+1-1>, June 2025. Revision 43.
- [nLa25d] nLab authors. model structure on topological sequential spectra. <https://ncatlab.org/nlab/show/model+structure+on+topological+sequential+spectra>, June 2025. Revision 61.
- [nLa25e] nLab authors. point of a topos. <https://ncatlab.org/nlab/show/point+of+a+topos>, July 2025. Revision 53.
- [nLa25f] nLab authors. sheafification. <https://ncatlab.org/nlab/show/sheafification>, September 2025. Revision 40.
- [nLa25g] nLab authors. stable homotopy category. <https://ncatlab.org/nlab/show/stable+homotopy+category>, June 2025. Revision 31.
- [OT14] Christian Okonek and Andrei Teleman. Intrinsic signs and lower bounds in real algebraic geometry. *Journal für die reine und angewandte Mathematik (Crelles Journal)*, 2014(688):219–241, 2014.
- [Poo18] Bjorn Poonen. Heuristics for the arithmetic of elliptic curves. In *Proceedings of the International Congress of Mathematicians: Rio de Janeiro 2018*, pages 399–414. World Scientific, 2018.

- [PR12] BJORN POONEN and ERIC RAINS. Random maximal isotropic subspaces and selmer groups. *Journal of the American Mathematical Society*, 25(1):245–269, 2012.
- [Pri24] Rachel Pries. The torelli locus and newton polygons, 2 2024. Lecture Notes for the 2024 Arizona Winter School.
- [Pri25] Rachel Pries. Some cases of oort’s conjecture about newton polygons of curves. *Nagoya Mathematical Journal*, 257:93–103, 2025.
- [PW21] Sabrina Pauli and Kirsten Wickelgren. Applications to \mathbb{A}^1 -enumerative geometry of the \mathbb{A}^1 -degree. *Research in the Mathematical Sciences*, 8(24):24–29, 2021.
- [Ros02] Michael Rosen. *Number Theory in Function Fields*, volume 210 of *Graduate Texts in Mathematics*. Springer-Verlag New York, 2002.
- [RS98] Michael Rosen and Joseph H. Silverman. On the rank of an elliptic surface. *Inventiones mathematicae*, 133:43–67, 1998.
- [Rud87] Walter Rudin. *Real and Complex Analysis*. Mathematics Series. McGraw-Hill Book Company, 3 edition, 1987.
- [Saw24] Will Sawin. General multiple dirichlet series from perverse sheaves. *Journal of Number Theory*, 262:408–453, 2024.
- [Ser72] Jean-Pierre Serre. Propriétés galoisiennes des points d’ordre fini des courbes elliptiques. *Invent. math*, 15:259–331, 1972.
- [Ser00] Jean-Pierre Serre. Lettre à marie-france vignéras du 10/2/1986. *Oeuvres–Collected Papers*, 4:38–55, 2000.
- [SFFK23] Will Sawin, Arthur Forey, Javier Fresán, and Emmanuel Kowalski. Quantitative sheaf theory. *Journal of the American Mathematical Society*, 36(3):653–726, 2023.
- [Sil89] Joseph H. Silverman. Elliptic curves of bounded degree and height. *Proceedings of the American Mathematical Society*, 105(3):540–545, 1989.
- [Sil09] Joseph H. Silverman. *The arithmetic of elliptic curves*, volume 106 of *Graduate Texts in Mathematics*. Springer-Verlag, 2 edition, 2009.
- [ST21] Will Sawin and Jacob Tsimerman. Bounds for the stalks of perverse sheaves in characteristic p and a conjecture of shende and tsimerman. *Inventiones mathematicae*, 224(1):1–32, 2021.
- [Sta25] The Stacks project authors. The stacks project. <https://stacks.math.columbia.edu>, 2025.
- [Tat65] John T. Tate. Algebraic cycles and poles of zeta functions. In *Arithmetical Algebraic Geometry (Proc. Conf. Purdue Univ., 1963)*, pages 93–110. Harper & Row, 1965. Also in *Collected works of John Tate* (2 vols.), Amer. Math. Soc. (2016), vol. 2.
- [Tat66] John Tate. On the conjectures of Birch and Swinnerton-Dyer and a geometric analog. In *Séminaire Bourbaki : années 1964/65 1965/66, exposés 277-312*, number 9 in *Astérisque*, pages 415–440. Société mathématique de France, 1966. talk:306.
- [Tay08] Richard Taylor. Automorphy for some l -adic lifts of automorphic mod l galois representations. ii. *Publications mathématiques*, 108:183–239, 2008.
- [Voe98] Vladimir Voevodsky. \mathbb{A}^1 -homotopy theory. In *Proceedings of the international congress of mathematicians*, volume 1, pages 579–604. Berlin, 1998.
- [Wei94] Charles A. Weibel. *An Introduction to Homological Algebra*, volume 38 of *Cambridge Studies in Advanced Mathematics*. Cambridge University Press, 1994. First paperback edition 1995 Reprinted 1997.
- [Wik25] Wikipedia contributors. Frobenius endomorphisms#frobenius for schemes — Wikipedia, the free encyclopedia, 2025. [Online; accessed 08-July-2025].
- [You06] Matthew Young. Low-lying zeros of families of elliptic curves. *Journal of the American Mathematical Society*, 19(1):205–250, 2006.
- [Yu97] Jiu-Kang Yu. Toward a proof of the Cohen-Lenstra conjecture in the function field case. preprint, 1997.