

Learning to Simulate Complex Physics with Graph Networks

Alvaro Sanchez-Gonzalez¹, Jonathan Godwin¹, Tobias Pfaff¹, Rex Ying^{1,2}, Jure Leskovec², Peter W. Battaglia¹

DeepMind | Department of Computer Science, Stanford University



Index

Introduction

Related Work

GNS Model Framework

Experimental Methods

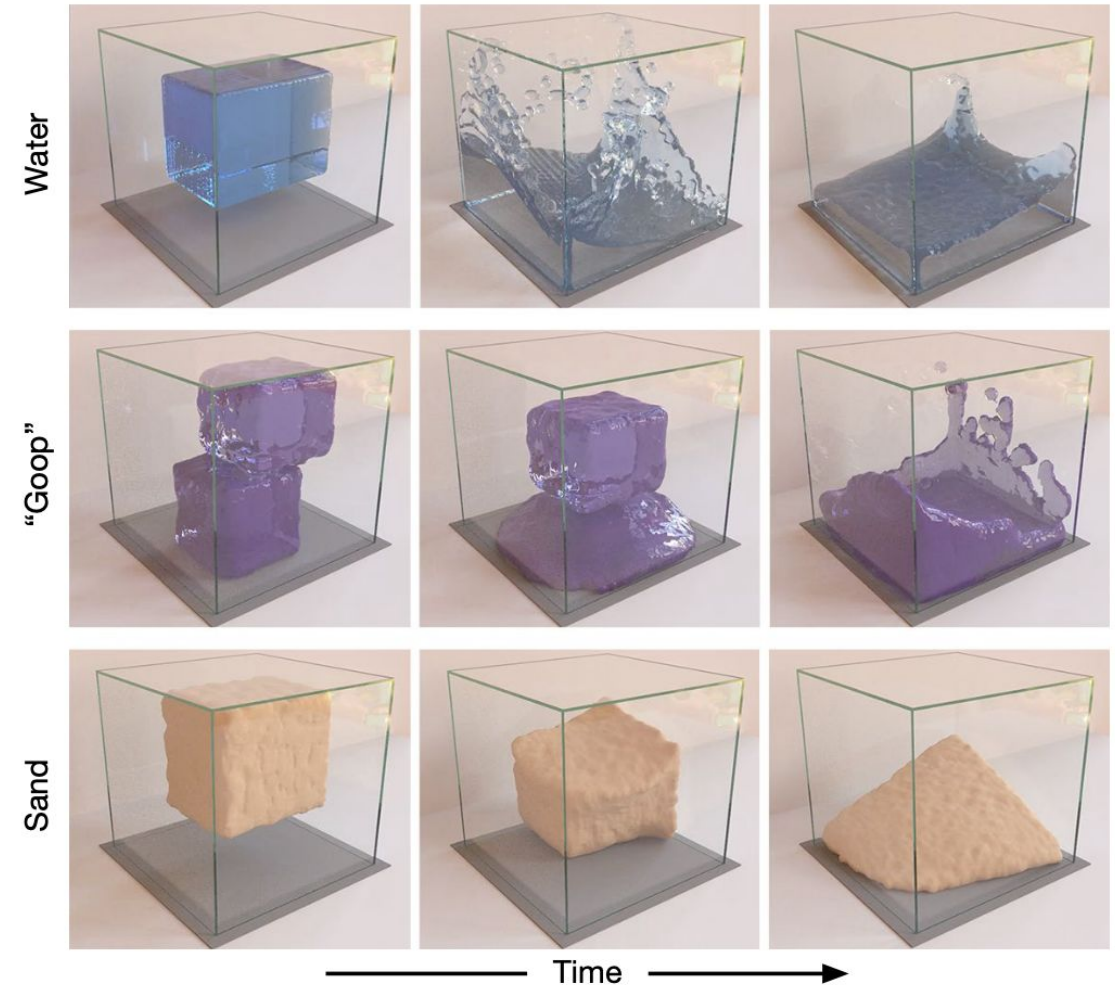
Training

Results



Introduction

- Traditional simulators can be **very expensive to create and use**. Building a simulator can entail years of engineering effort, and **often must trade off generality for accuracy in a narrow range of settings**.
 - The large state spaces and complex dynamics have been difficult for standard end-to-end learning approaches to overcome.
 - While previous learning simulation approaches **have been highly specialized for particular tasks**,
- We found our single GNS model performed well across dozens of experiments and was generally robust to hyperparameter choices.





Smoothed Particle Hydrodynamics (SPH)

- which evaluates pressure and viscosity forces around each particle, and updates particles' velocities and positions accordingly

Position-based Dynamics (PBD)

- incompressibility and collision dynamics involve re-solving pairwise distance constraints between particles, and directly predicting their position changes

Material Point Method (MPM)

- viable particle-based simulators have recently appeared, e.g., DiffTaichi (Hu et al., 2019), PhiFlow (Holl et al., 2020), and Jax-MD (Schoenholz & Cubuk, 2019), which can backpropagate gradients through the architecture.



Learning simulations from data

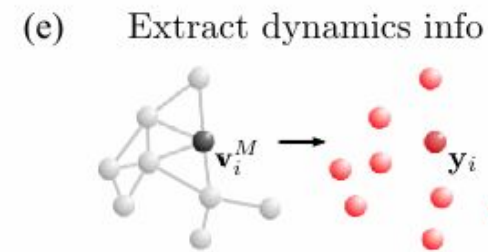
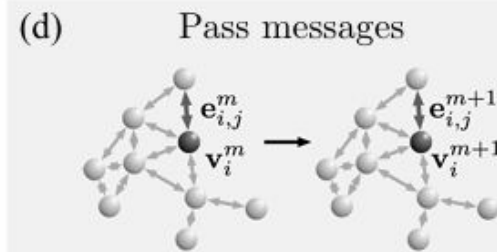
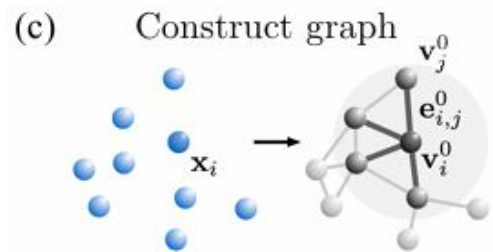
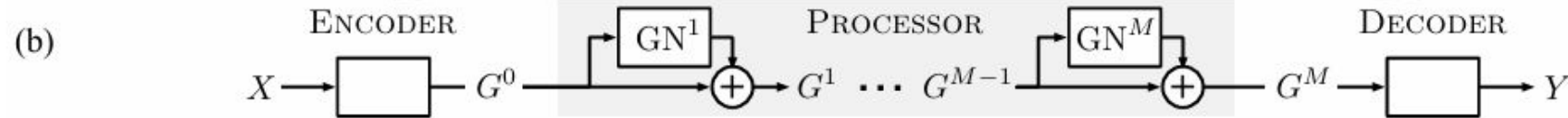
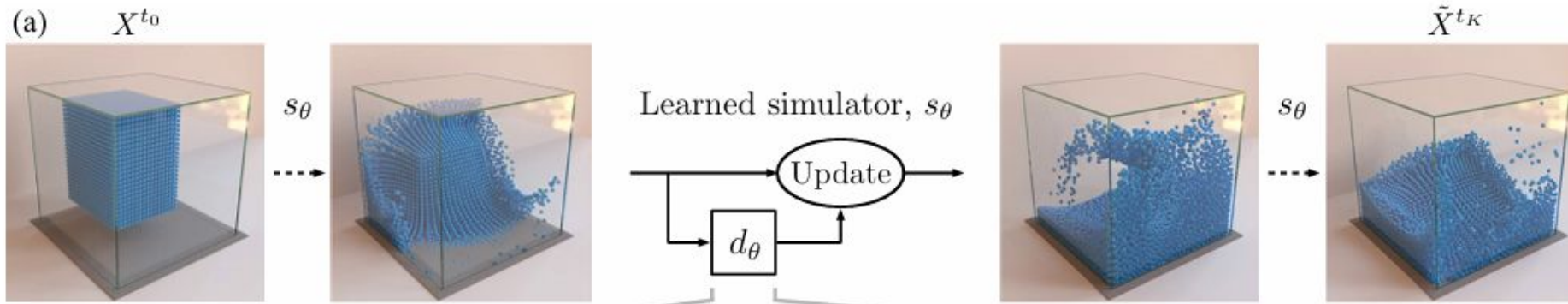
- Learning simulations from data (Grzeszczuk et al., 1998) has been an important area of study with applications in physics and graphics. Compared to engineered simulators, a learned simulator can be far more efficient for predicting complex phenomena (He et al., 2019); e.g., (Ladick`y et al., 2015; Wiewel et al., 2019) learn parts of a fluid simulator for faster prediction.

Graph Networks (GN)

- Graph Neural Network have recently proven effective at learning forward dynamics in various settings that involve interactions between many entities. Crucially, our GNS framework is a general approach to learning simulation, is simpler to implement, and is more accurate across fluid, rigid, and deformable material systems.



3.1. General Learnable Simulation



$$X^t \in \mathcal{X}$$

$$\mathbf{X}^{t_0:K} = (X^{t_0}, \dots, X^{t_K})$$

$$s: \mathcal{X} \rightarrow \mathcal{X}$$

$$d_\theta: \mathcal{X} \rightarrow \mathcal{Y}$$

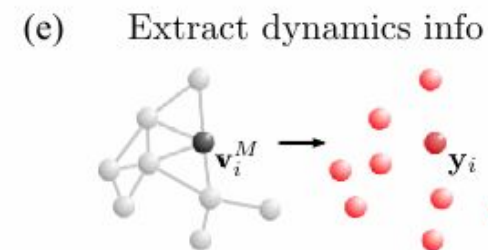
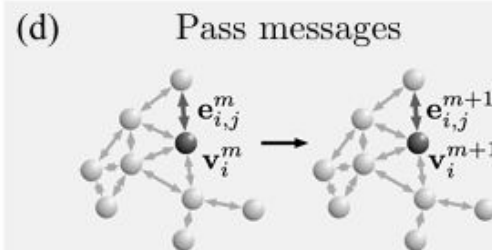
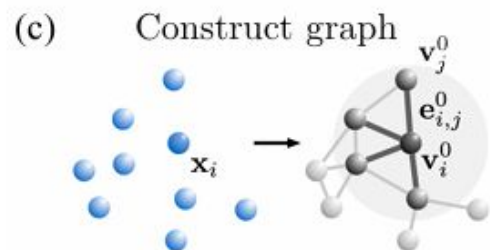
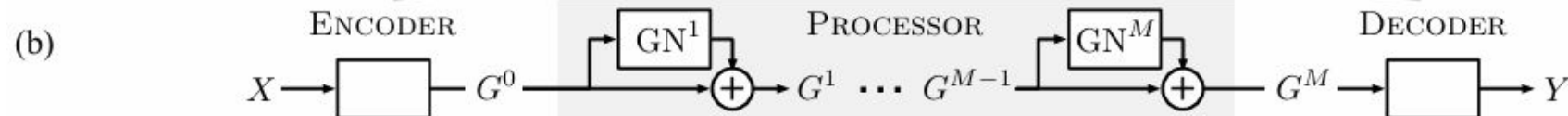
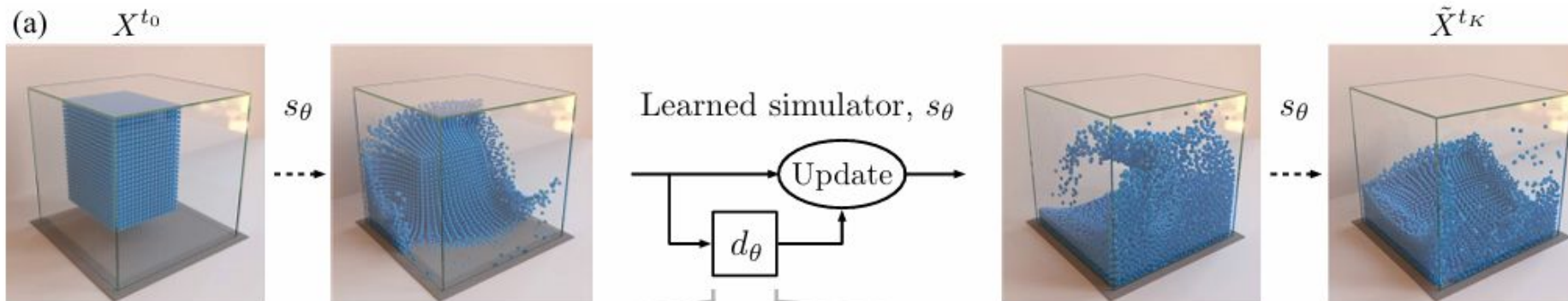
$$\tilde{\mathbf{X}}^{t_0:K} = (X^{t_0}, \tilde{X}^{t_1}, \dots, \tilde{X}^{t_K})$$

$$\tilde{X}^{t_{k+1}} = s(\tilde{X}^{t_k})$$

$$\tilde{X}^{t_{k+1}} = \text{Update}(\tilde{X}^{t_k}, d_\theta)$$



3.2. Simulation as Message-Passing on a Graph



The ENCODER : $\mathcal{X} \rightarrow \mathcal{G}$

The PROCESSOR : $\mathcal{G} \rightarrow \mathcal{G}$

The DECODER : $\mathcal{G} \rightarrow \mathcal{Y}$



4.1. Physical Domains

We explored how our GNS learns to simulate in datasets which contained three diverse, complex physical materials:

- water as a barely damped fluid, chaotic in nature;
- sand as a granular material with complex frictional behavior;
- and “goop” as a viscous, plastically deformable material.



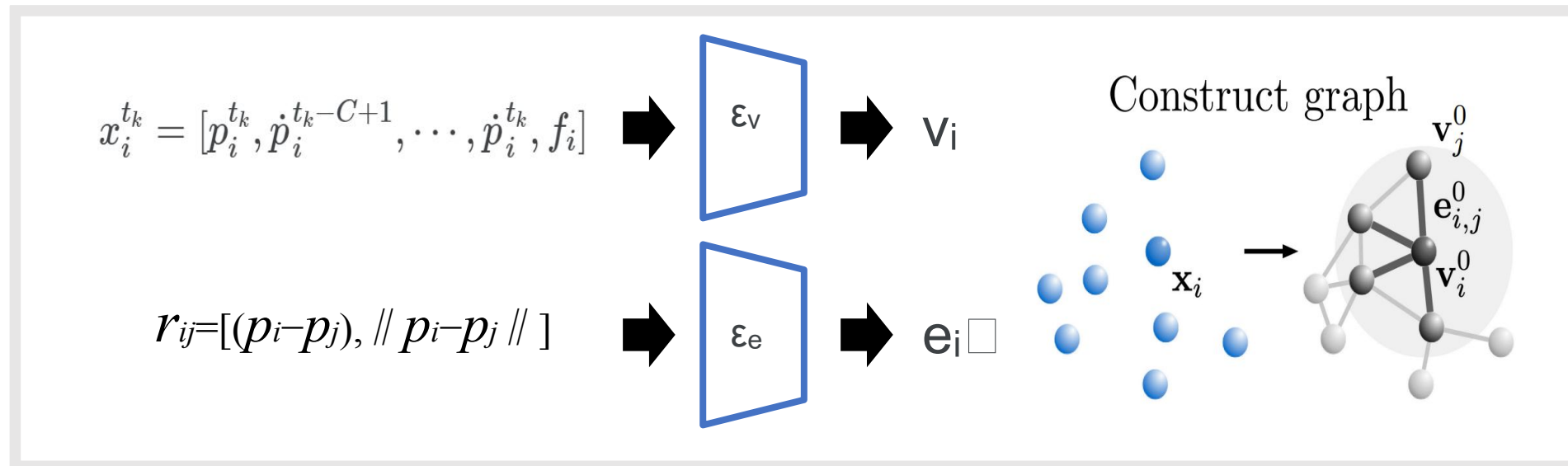
4.2. GNS Implementation Details

Input and output representations

Each particle's input state vector represents position, a sequence of $C = 5$ previous velocities, and features that capture static material properties (e.g., water, sand, goop, rigid, boundary particle)

ENCODER details

The ENCODER constructs the graph structure G_0 by assigning a node to each particle and adding edges between particles within a “connectivity radius”, R , which reflected local interactions of particles, and which was kept constant for all simulations of the same resolution. The ENCODER implements v and e as multilayer perceptrons (MLP), which encode node features and edge features into the latent vectors, v_i and e_{ij} , of size 128.



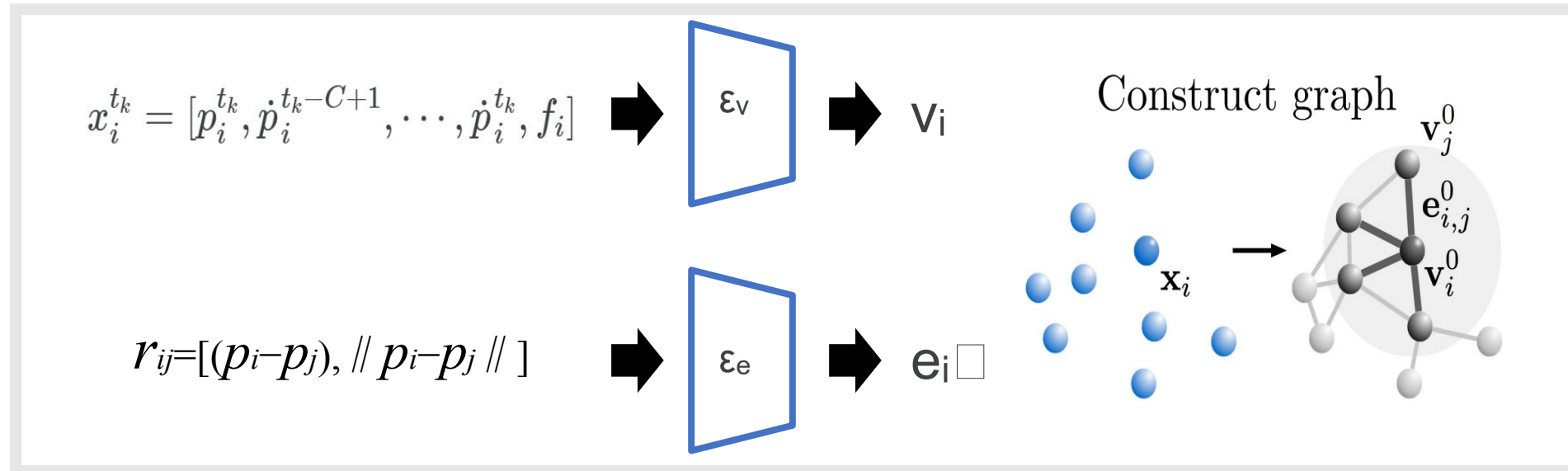


4.2. GNS Implementation Details

ENCODER details

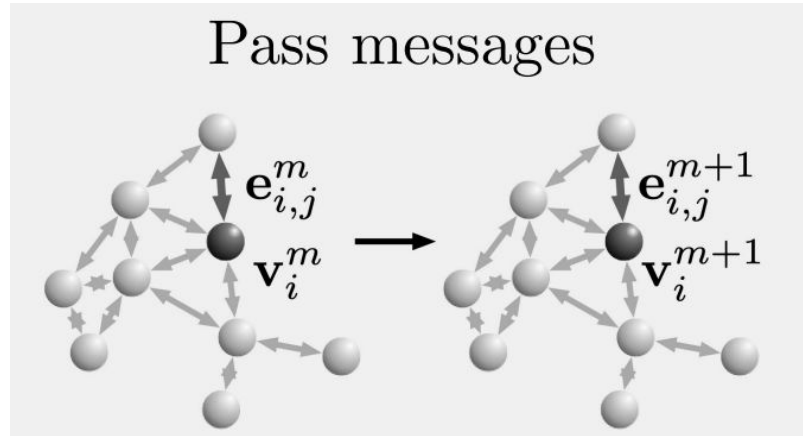
We tested two ENCODER variants, distinguished by whether they use absolute versus relative positional information.

The v was forced to ignore p_i information within x_i by masking it out. The e was provided with the relative positional displacement, and its magnitude², $r_{ij} = [(p_i - p_j), \|p_i - p_j\|]$.



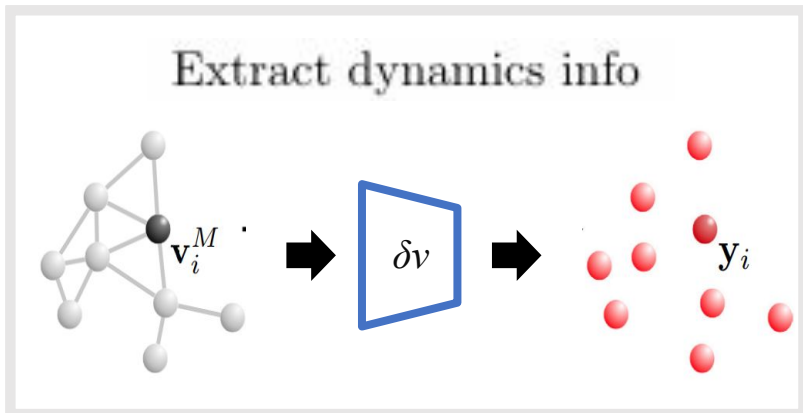


4.2. GNS Implementation Details



PROCESSOR details

We use GNs without global features or global updates (similar to an interaction network)³, and with a residual connections between the input and output latent node and edge attributes.



DECODER details.

Our decoder's learned function, v , is an MLP. After the DECODER, the future position and velocity are updated using an Euler integrator, so the y_i corresponds to accelerations, π_i , with 2D or 3D dimension, depending on the physical domain.



4.3. Training

Training noise

Modeling a complex and chaotic simulation system requires the model to mitigate error accumulation over long rollouts. Because we train our models on ground-truth one-step data, they are never presented with input data corrupted by this sort of accumulated noise.

Normalization

We normalize all input and target vectors elementwise to zero mean and unit variance, using statistics computed online during training.

Loss function and optimization procedures.

We randomly sampled particle state pairs $(\mathbf{x}_{tk_i}, \mathbf{x}_{tk+1_i})$ from training trajectories, calculated target accelerations $\ddot{\mathbf{p}}_{tk_i}$, and computed the L2 loss on the predicted per-particle accelerations, i.e.,

$$L(\mathbf{x}_{tk_i}, \mathbf{x}_{tk+1_i}; \theta) = \| d\theta(\mathbf{x}_{tk_i}) - \ddot{\mathbf{p}}_{tk_i} \|_2$$



4.4. Evaluation

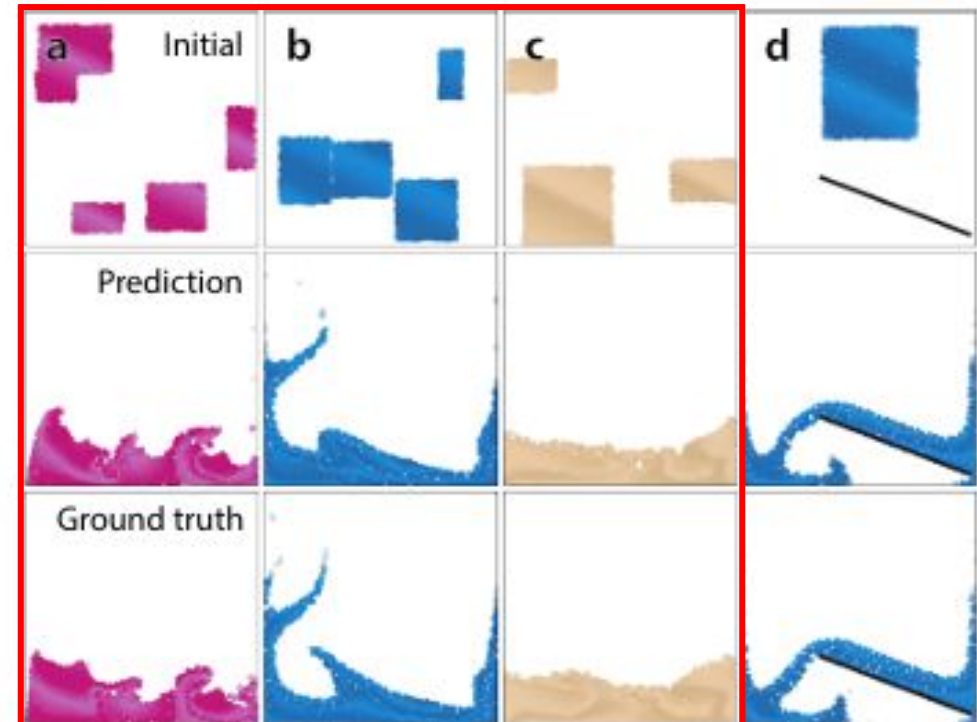
To report quantitative results, we evaluated our models after training converged by computing one-step and rollout metrics on held-out test trajectories, drawn from the same distribution of initial conditions used for training. We used particle-wise MSE as our main metric between ground truth and predicted data, both for rollout and one-step predictions, averaging across time, particle and spatial axes.



5.1. Simulating Complex Materials

Our GNSmodel was very effective at learning to simulate different complex materials. Table 1 shows the one-step and rollout accuracy, as MSE, for all experiments. For intuition about what these numbers mean, the edge length of the container was approximately 10, and Figure 3(a-c) shows rendered images of the rollouts of our model, compared to ground truth⁶.

Experimental domain	N	K	1-step ($\times 10^{-9}$)	Rollout ($\times 10^{-3}$)
WATER-3D (SPH)	13k	800	8.66	10.1
SAND-3D	20k	350	1.42	0.554
GOOP-3D	14k	300	1.32	0.618
WATER-3D-S (SPH)	5.8k	800	9.66	9.52
BOXBATH (PBD)	1k	150	54.5	4.2
WATER	1.9k	1000	2.82	17.4
SAND	2k	320	6.23	2.37
GOOP	1.9k	400	2.91	1.89
MULTIMATERIAL	2k	1000	1.81	16.9
FLUIDSHAKE	1.3k	2000	2.1	20.1
WATERDROP	1k	1000	1.52	7.01
WATERDROP-XL	7.1k	1000	1.23	14.9
WATERRAMPS	2.3k	600	4.91	11.6
SANDRAMPS	3.3k	400	2.77	2.07
RANDOMFLOOR	3.4k	600	2.77	6.72
CONTINUOUS	4.3k	400	2.06	1.06

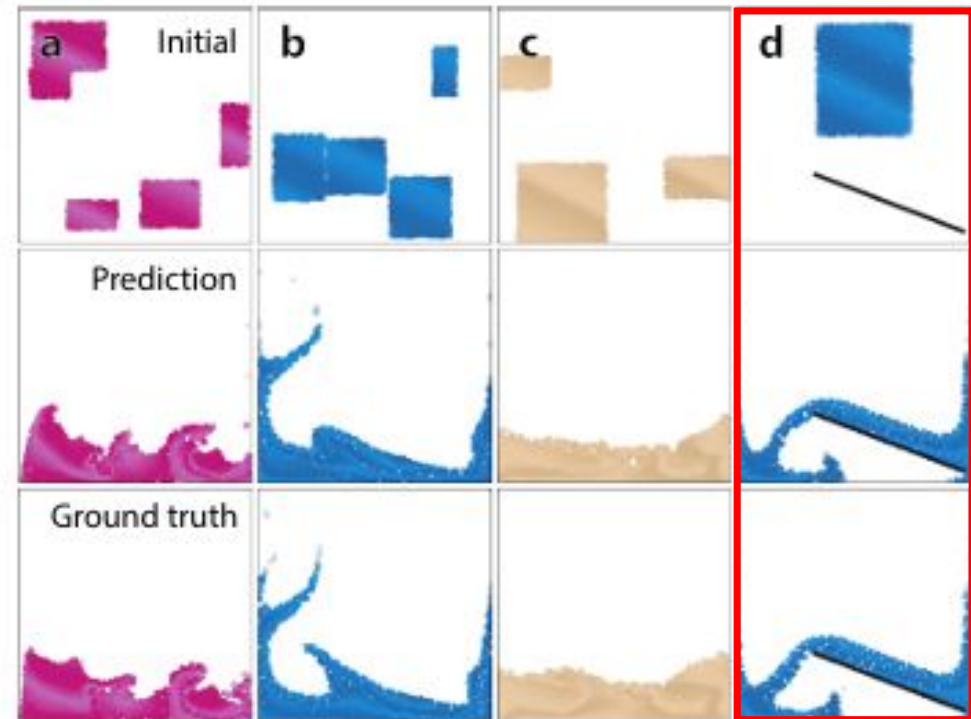




5.1. Simulating Complex Materials

Our GNSmodel was very effective at learning to simulate different complex materials. Table 1 shows the one-step and rollout accuracy, as MSE, for all experiments. For intuition about what these numbers mean, the edge length of the container was approximately 10, and Figure 3(a-c) shows rendered images of the rollouts of our model, compared to ground truth⁶.

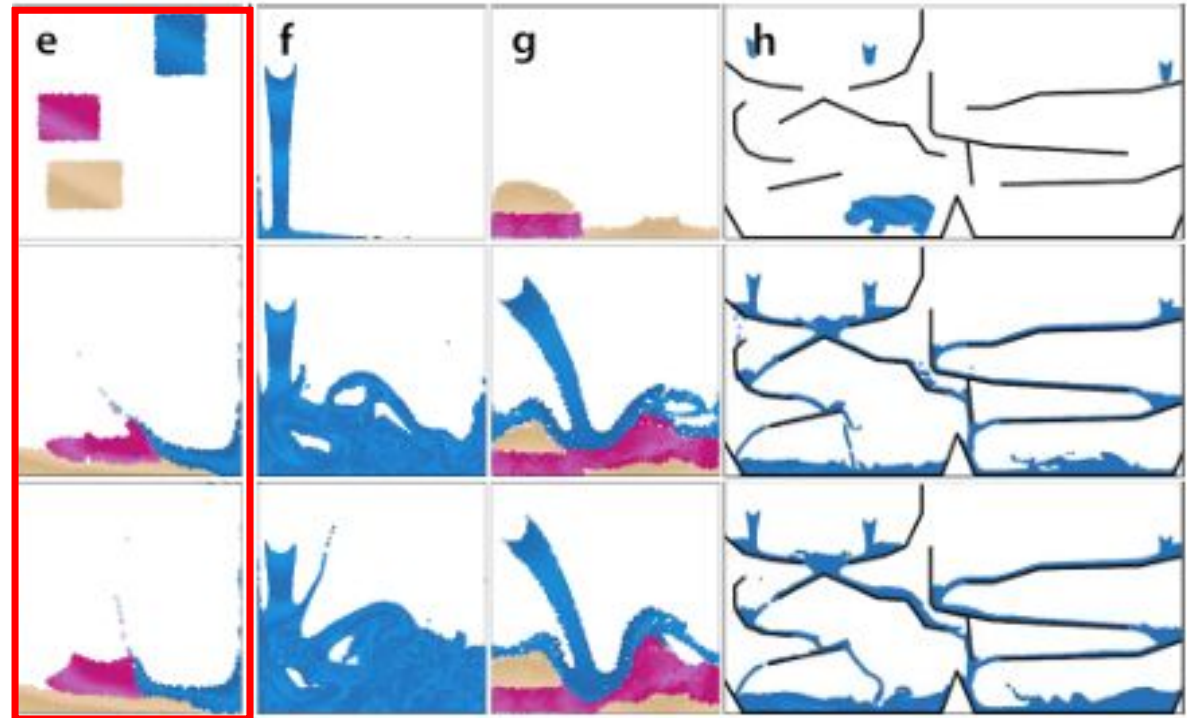
Experimental domain	N	K	1-step ($\times 10^{-9}$)	Rollout ($\times 10^{-3}$)
WATER-3D (SPH)	13k	800	8.66	10.1
SAND-3D	20k	350	1.42	0.554
GOOP-3D	14k	300	1.32	0.618
WATER-3D-S (SPH)	5.8k	800	9.66	9.52
BOXBATH (PBD)	1k	150	54.5	4.2
WATER	1.9k	1000	2.82	17.4
SAND	2k	320	6.23	2.37
GOOP	1.9k	400	2.91	1.89
MULTIMATERIAL	2k	1000	1.81	16.9
FLUIDSHAKE	1.3k	2000	2.1	20.1
WATERDROP	1k	1000	1.52	7.01
WATERDROP-XL	7.1k	1000	1.23	14.9
WATERRAMPS	2.3k	600	4.91	11.6
SANDRAMPS	3.3k	400	2.77	2.07
RANDOMFLOOR	3.4k	600	2.77	6.72
CONTINUOUS	4.3k	400	2.06	1.06





5.2. Multiple Interacting Materials & Generalization

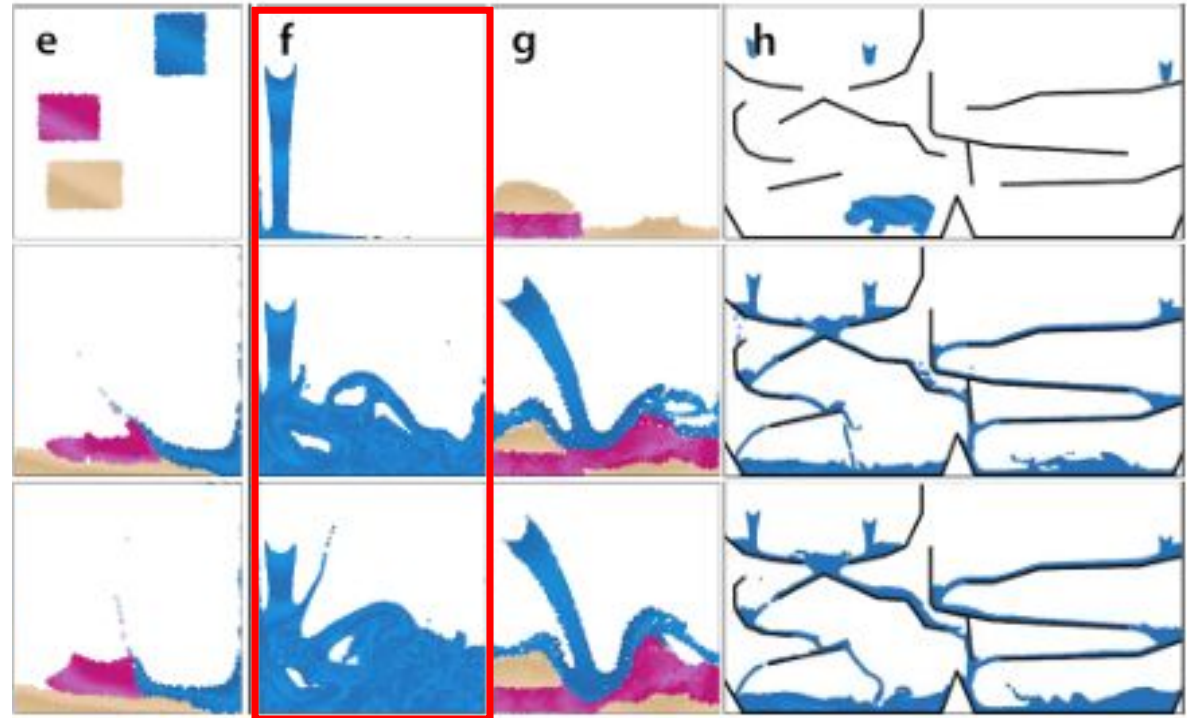
Experimental domain	N	K	1-step ($\times 10^{-9}$)	Rollout ($\times 10^{-3}$)
WATER-3D (SPH)	13k	800	8.66	10.1
SAND-3D	20k	350	1.42	0.554
GOOP-3D	14k	300	1.32	0.618
WATER-3D-S (SPH)	5.8k	800	9.66	9.52
BOXBATH (PBD)	1k	150	54.5	4.2
WATER	1.9k	1000	2.82	17.4
SAND	2k	320	6.23	2.37
GOOP	1.9k	400	2.91	1.89
MULTIMATERIAL	2k	1000	1.81	16.9
FLUIDSHAKE	1.3k	2000	2.1	20.1
WATERDROP	1k	1000	1.52	7.01
WATERDROP-XL	7.1k	1000	1.23	14.9
WATERRAMPS	2.3k	600	4.91	11.6
SANDRAMPS	3.3k	400	2.77	2.07
RANDOMFLOOR	3.4k	600	2.77	6.72
CONTINUOUS	4.3k	400	2.06	1.06





5.2. Multiple Interacting Materials & Generalization

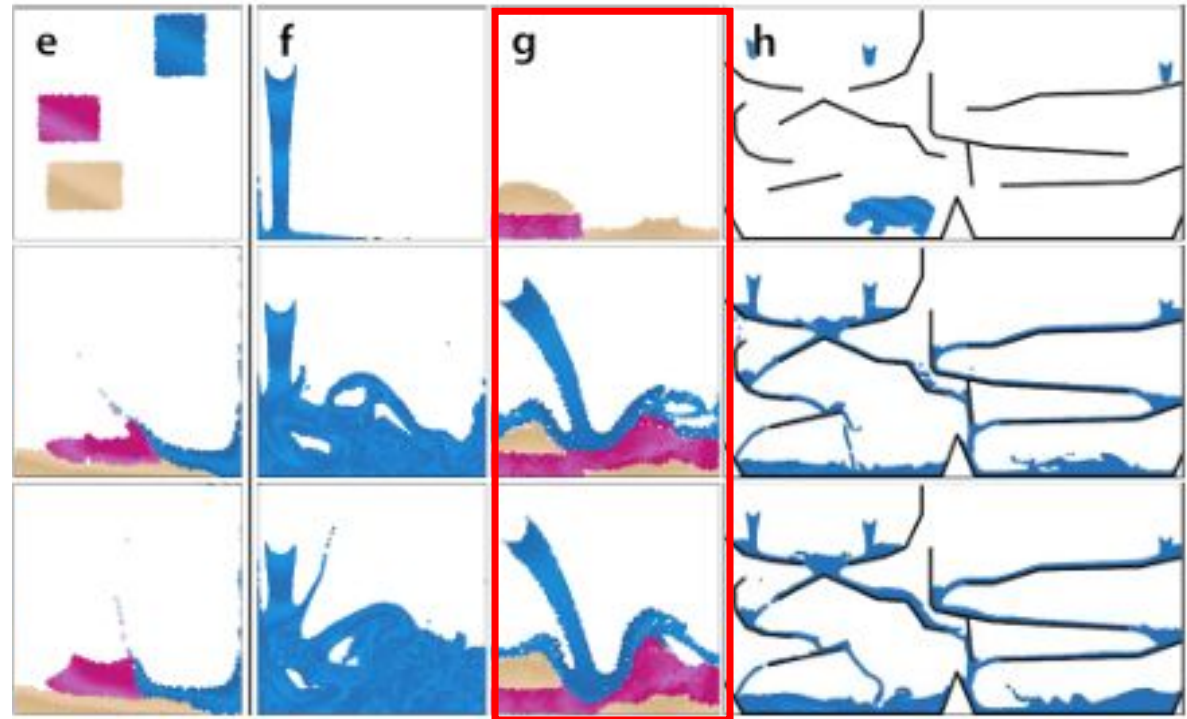
Experimental domain	N	K	1-step ($\times 10^{-9}$)	Rollout ($\times 10^{-3}$)
WATER-3D (SPH)	13k	800	8.66	10.1
SAND-3D	20k	350	1.42	0.554
GOOP-3D	14k	300	1.32	0.618
WATER-3D-S (SPH)	5.8k	800	9.66	9.52
BOXBATH (PBD)	1k	150	54.5	4.2
WATER	1.9k	1000	2.82	17.4
SAND	2k	320	6.23	2.37
GOOP	1.9k	400	2.91	1.89
MULTIMATERIAL	2k	1000	1.81	16.9
FLUIDSHAKE	1.3k	2000	2.1	20.1
WATERDROP	1k	1000	1.52	7.01
WATERDROP-XL	7.1k	1000	1.23	14.9
WATERRAMPS	2.3k	600	4.91	11.6
SANDRAMPS	3.3k	400	2.77	2.07
RANDOMFLOOR	3.4k	600	2.77	6.72
CONTINUOUS	4.3k	400	2.06	1.06





5.2. Multiple Interacting Materials & Generalization

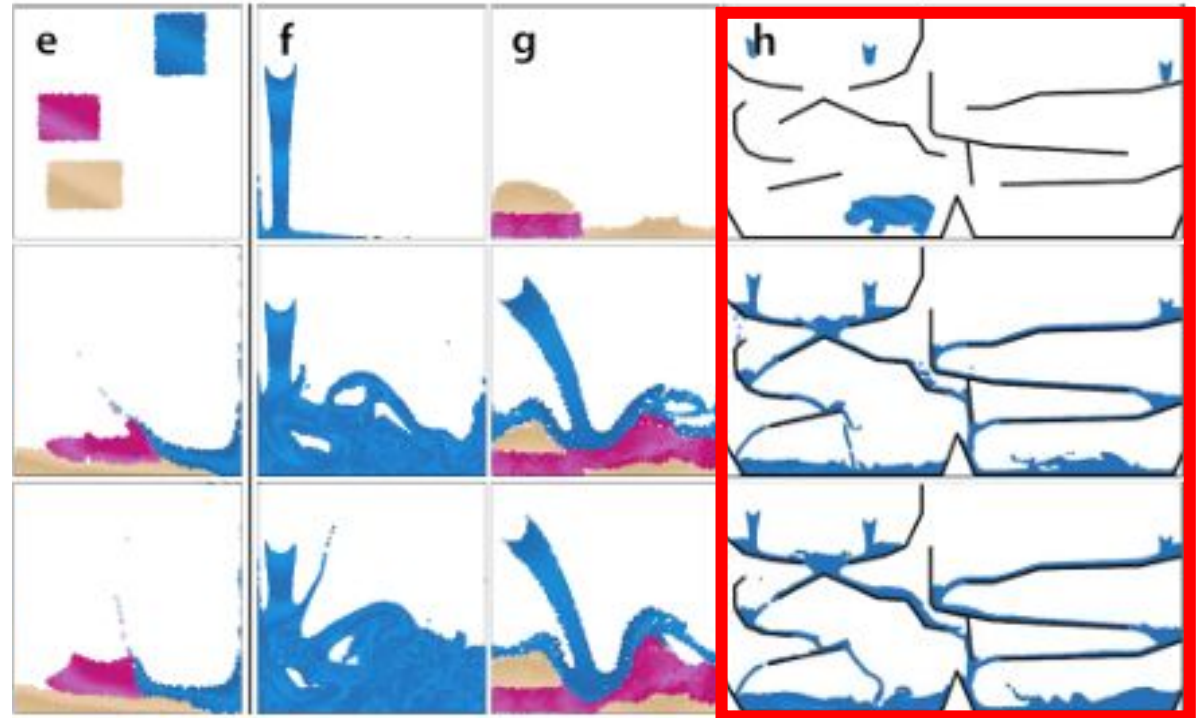
Experimental domain	N	K	1-step ($\times 10^{-9}$)	Rollout ($\times 10^{-3}$)
WATER-3D (SPH)	13k	800	8.66	10.1
SAND-3D	20k	350	1.42	0.554
GOOP-3D	14k	300	1.32	0.618
WATER-3D-S (SPH)	5.8k	800	9.66	9.52
BOXBATH (PBD)	1k	150	54.5	4.2
WATER	1.9k	1000	2.82	17.4
SAND	2k	320	6.23	2.37
GOOP	1.9k	400	2.91	1.89
MULTIMATERIAL	2k	1000	1.81	16.9
FLUIDSHAKE	1.3k	2000	2.1	20.1
WATERDROP	1k	1000	1.52	7.01
WATERDROP-XL	7.1k	1000	1.23	14.9
WATERRAMPS	2.3k	600	4.91	11.6
SANDRAMPS	3.3k	400	2.77	2.07
RANDOMFLOOR	3.4k	600	2.77	6.72
CONTINUOUS	4.3k	400	2.06	1.06





5.2. Multiple Interacting Materials & Generalization

Experimental domain	N	K	1-step ($\times 10^{-9}$)	Rollout ($\times 10^{-3}$)
WATER-3D (SPH)	13k	800	8.66	10.1
SAND-3D	20k	350	1.42	0.554
GOOP-3D	14k	300	1.32	0.618
WATER-3D-S (SPH)	5.8k	800	9.66	9.52
BOXBATH (PBD)	1k	150	54.5	4.2
WATER	1.9k	1000	2.82	17.4
SAND	2k	320	6.23	2.37
GOOP	1.9k	400	2.91	1.89
MULTIMATERIAL	2k	1000	1.81	16.9
FLUIDSHAKE	1.3k	2000	2.1	20.1
WATERDROP	1k	1000	1.52	7.01
WATERDROP-XL	7.1k	1000	1.23	14.9
WATERRAMPS	2.3k	600	4.91	11.6
SANDRAMPS	3.3k	400	2.77	2.07
RANDOMFLOOR	3.4k	600	2.77	6.72
CONTINUOUS	4.3k	400	2.06	1.06





Result



UNIVERSITY OF ULSAN

MIR²L

Medical
Imaging
Intelligent
Reality
Lab

Surface mesh



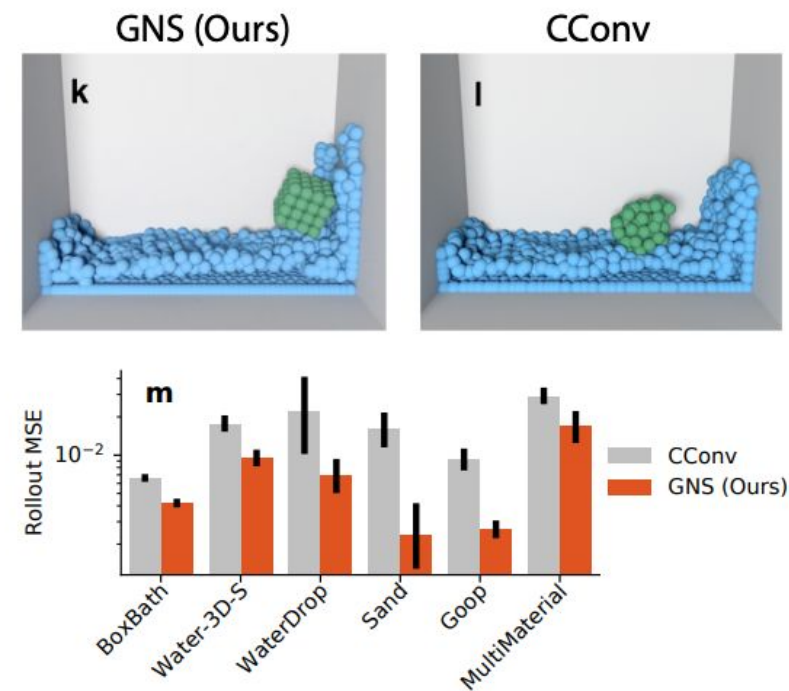
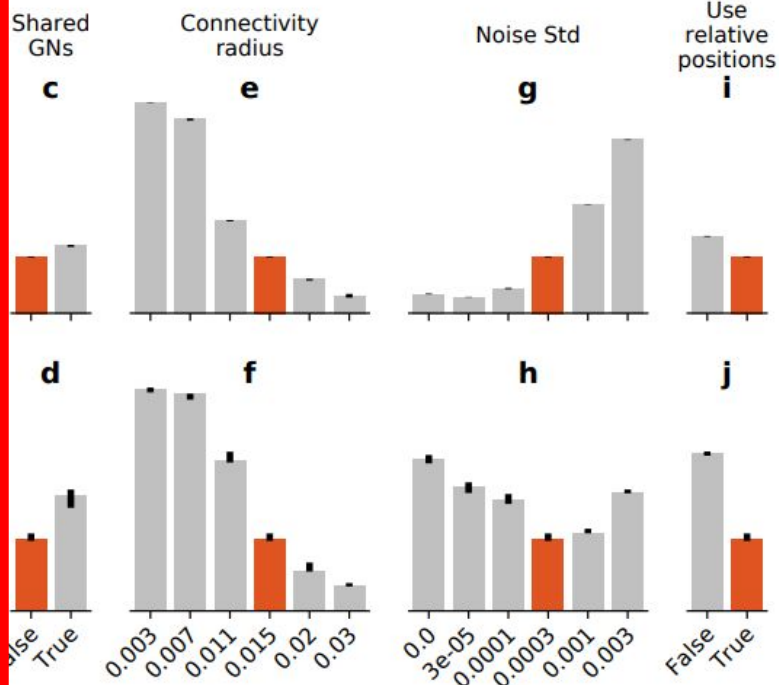
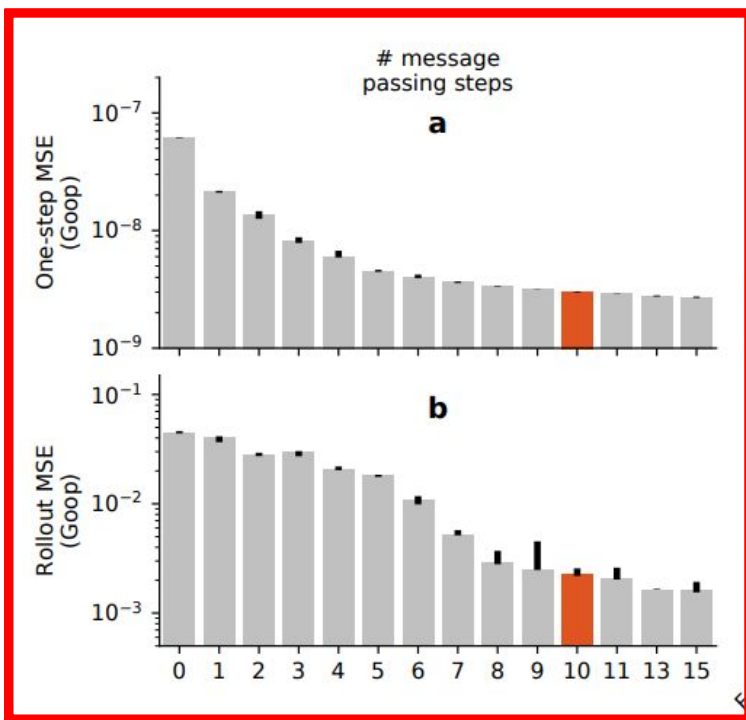
Underlying particles



Particle representation



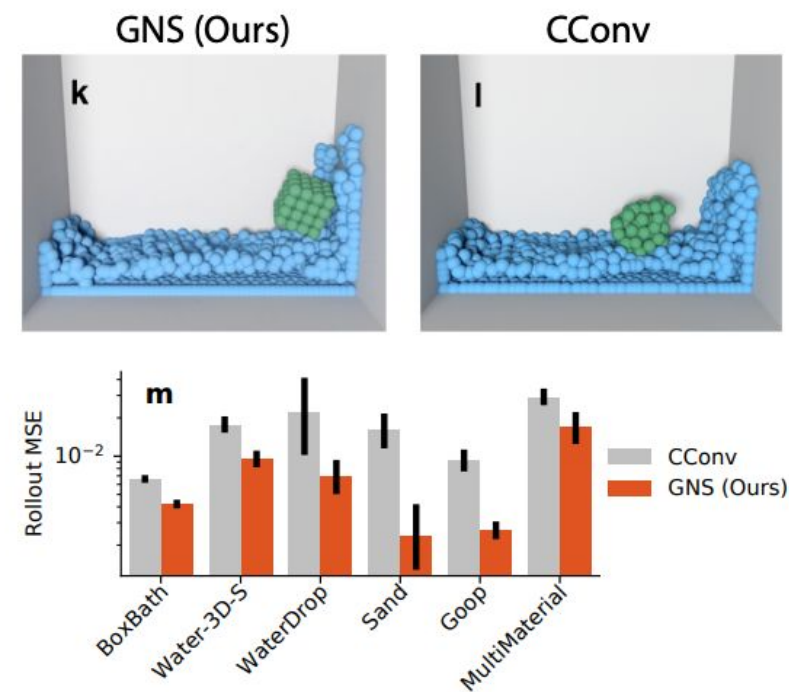
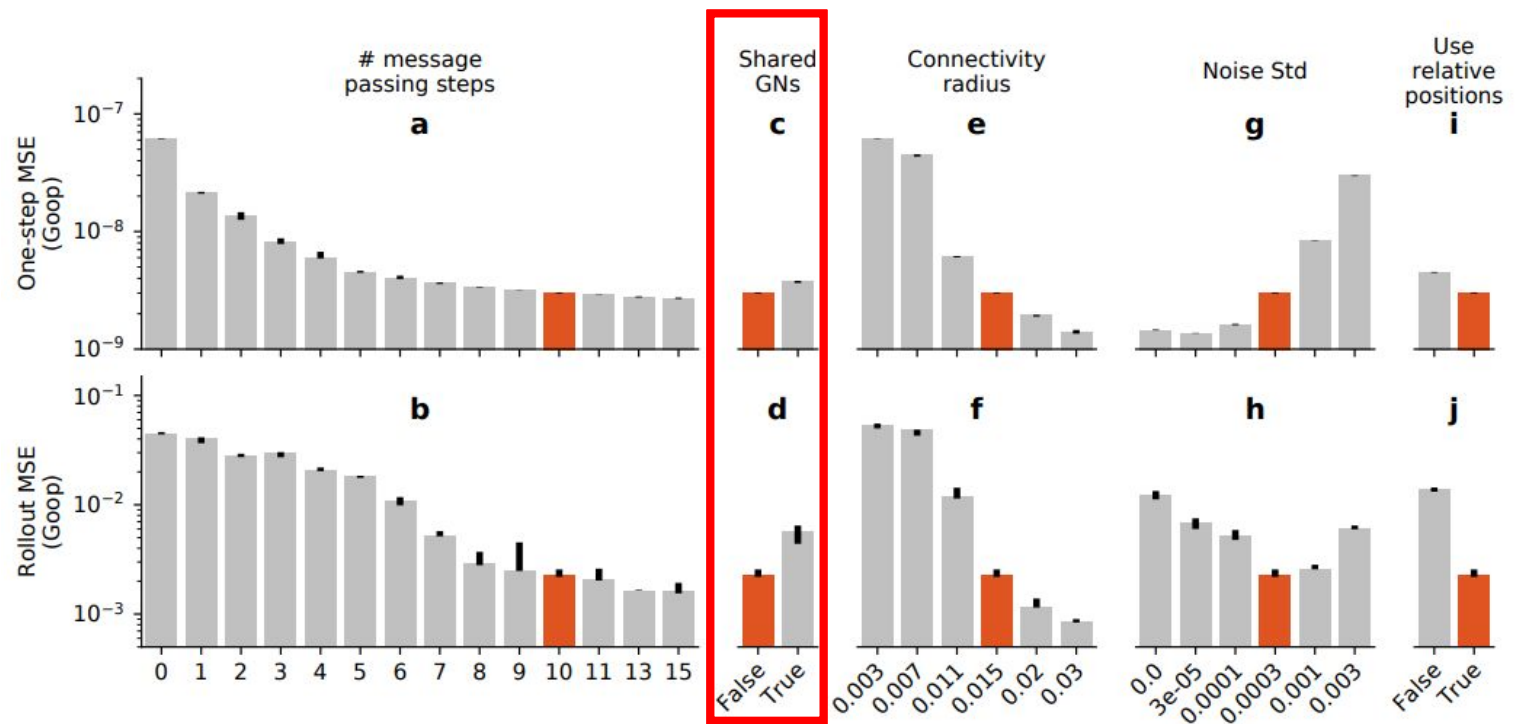
5.4. Key Architectural Choices



the number of message-passing steps



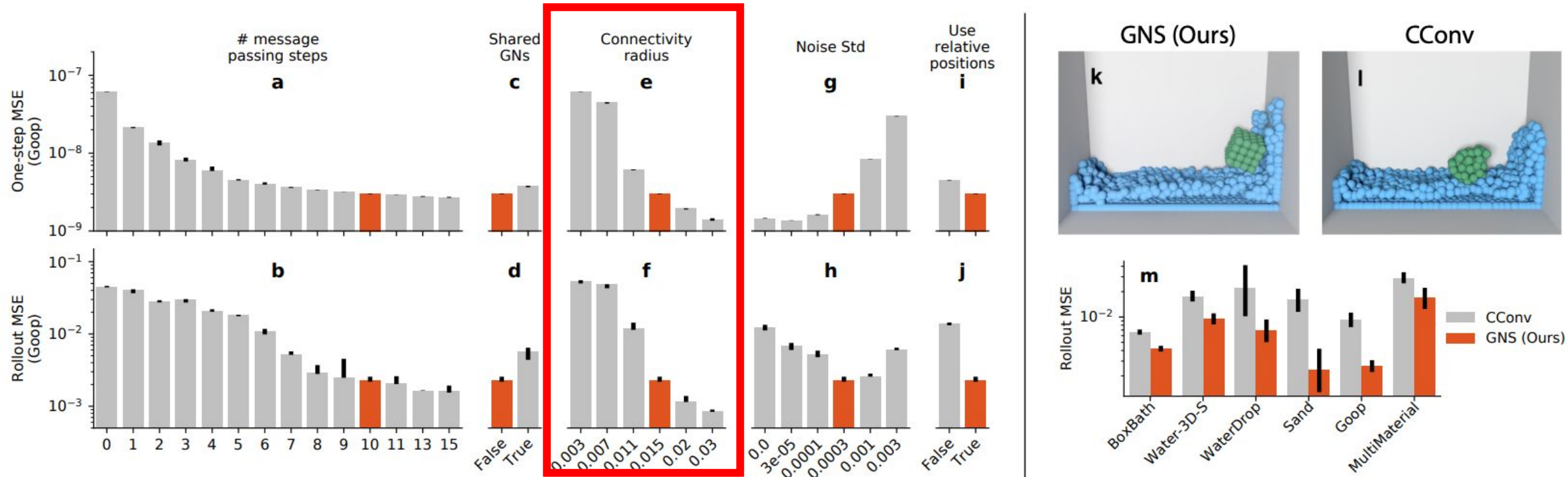
5.4. Key Architectural Choices



shared vs. unshared PROCESSOR GN parameters



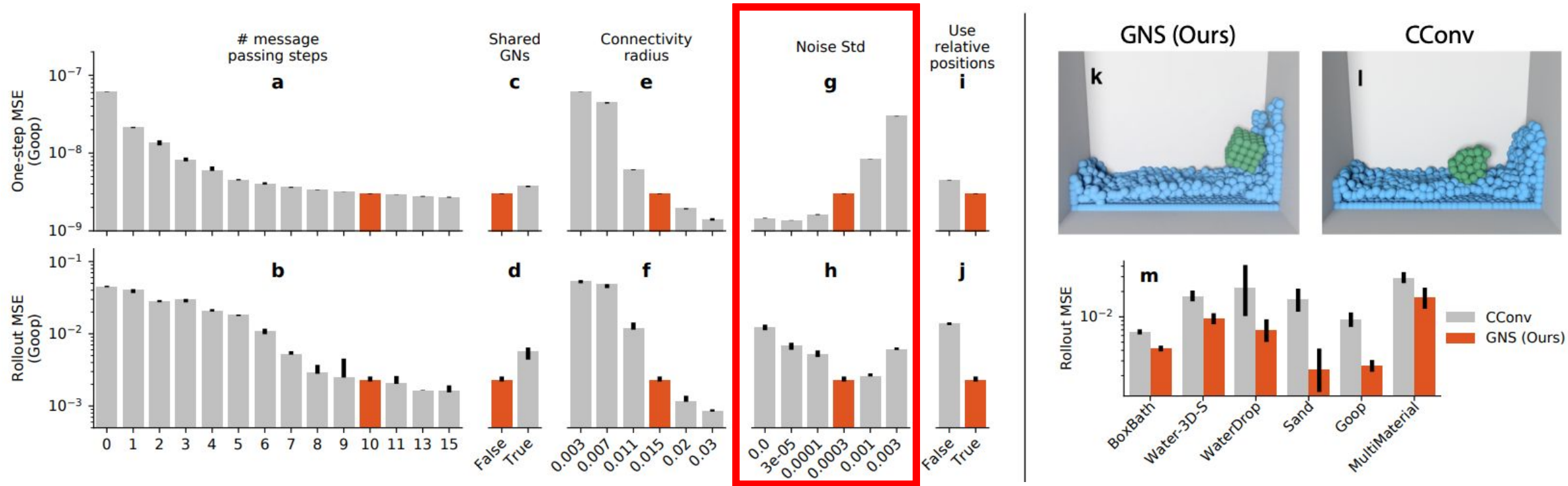
5.4. Key Architectural Choices



the connectivity radius



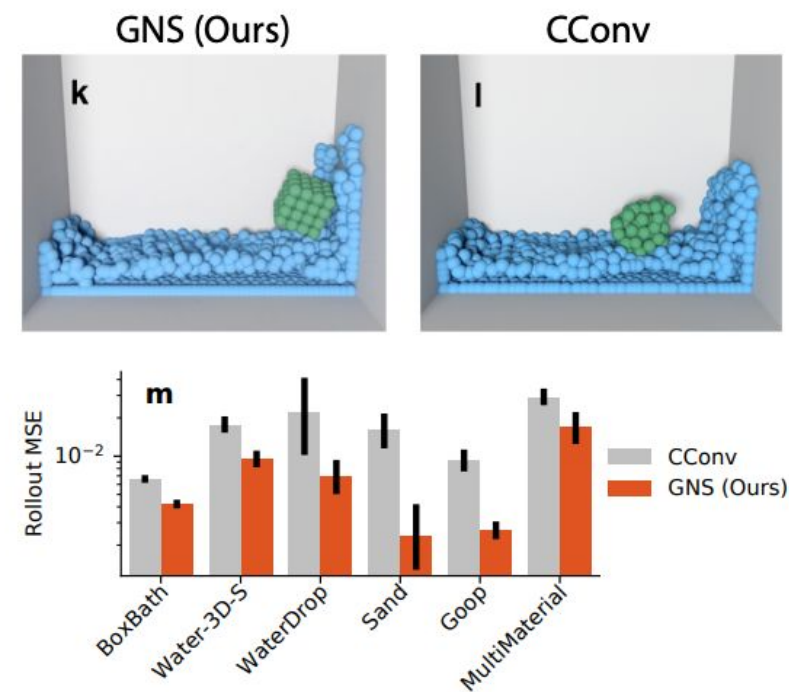
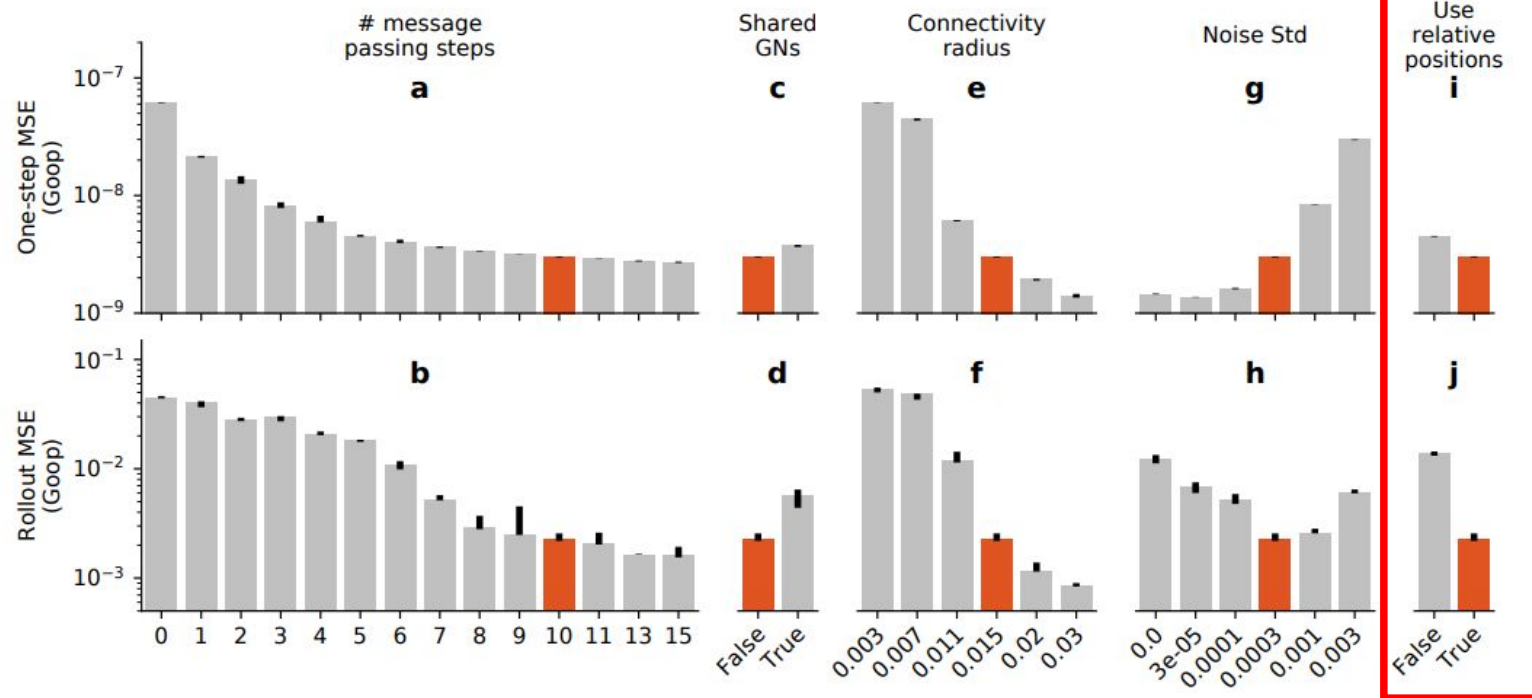
5.4. Key Architectural Choices



the scale of noise added
to the inputs during training



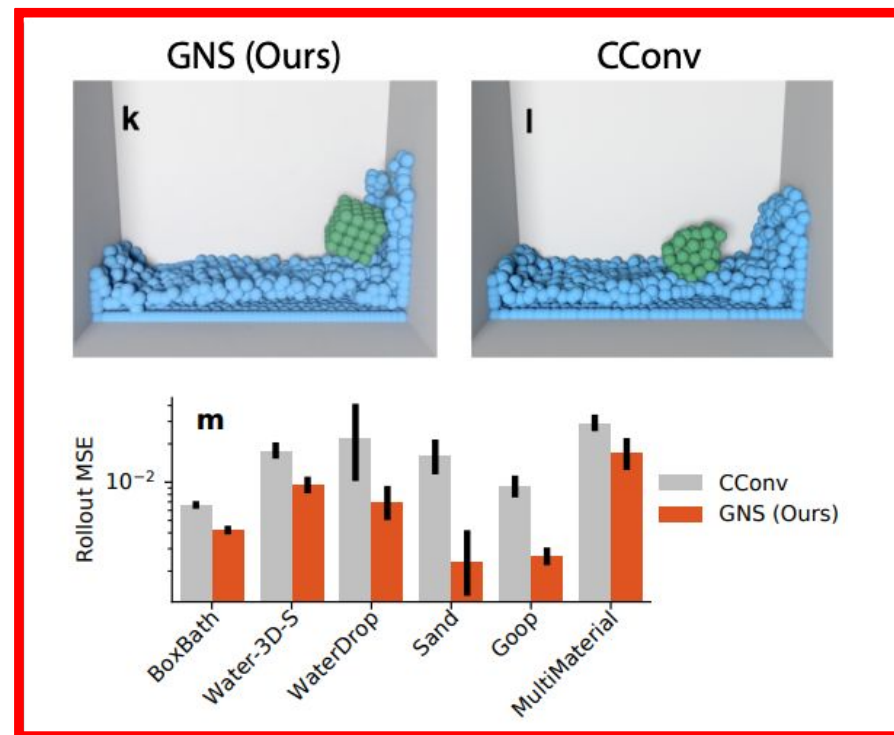
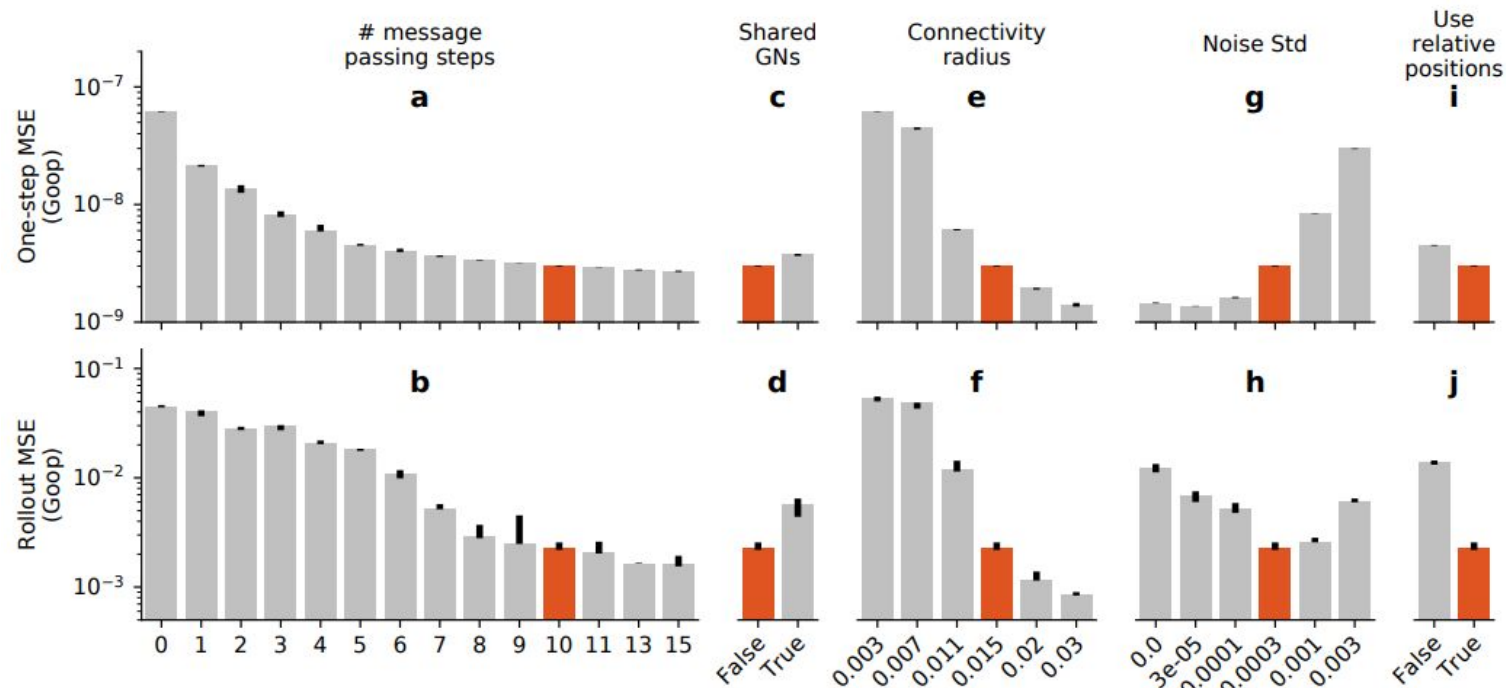
5.4. Key Architectural Choices



relative vs. absolute ENCODER



5.4. Key Architectural Choices



CConv vs. GNS



Conclusion



UNIVERSITY OF ULSAN



Medical
Imaging
Intelligent
Reality
Lab

- We presented a powerful machine learning framework for learning to simulate complex systems, based on particle based representations of physics and learned message passing on graphs.
- Our experimental results show our single GNS architecture can learn to simulate the dynamics of fluids, rigid solids, and deformable materials, interacting with one another, using tens of thousands of particles over thousands time steps.
- More broadly, this work is a key advance toward more sophisticated generative models, and furnishes the modern AI toolkit with a greater capacity for physical reasoning.



UNIVERSITY OF ULSAN



ASAN
Medical Center

Thank you

MIR²L | Medical
Imaging
Intelligent
Reality
Lab