

Give CFGs for the following languages, and clearly explain how they work and the role of each nonterminal. Grammars can be very difficult to understand, and if the grader does not understand how your construction is intended to generate the language, then you will receive 0 points.

1. $\{w \mid w \in \{0, 1\}^* \text{ and } w \text{ is not a palindrome}\}$
2. $\{a^i b^j c^k \mid j = i + k\}$
3. $\{a^i b^j c^k d^l \mid i + l = j + k\}$

Solution:

(a) $\{w \mid w \in \{0, 1\}^* \text{ and } w \text{ is not a palindrome}\}$

$$\begin{aligned} \langle S \rangle &\rightarrow \langle F \rangle \langle S \rangle \langle F \rangle \mid \langle Q \rangle \\ \langle Q \rangle &\rightarrow 0 \langle T \rangle 1 \mid 1 \langle T \rangle 0 \\ \langle T \rangle &\rightarrow \langle F \rangle \langle T \rangle \langle F \rangle \mid \epsilon \\ \langle F \rangle &\rightarrow 1 \mid 0 \end{aligned}$$

$\langle S \rangle$ is a start state. $\langle F \rangle$ is for base state. $\langle Q \rangle$ is for making non-palindrome. $\langle T \rangle$ is for expansion.

(b) $\{a^i b^j c^k \mid j = i + k\}$

$$\begin{aligned} \langle S \rangle &\rightarrow \langle A \rangle \mid \langle C \rangle \mid \langle A \rangle \langle C \rangle \mid \epsilon \\ \langle A \rangle &\rightarrow a \langle A \rangle b \mid ab \\ \langle C \rangle &\rightarrow b \langle C \rangle c \mid bc \end{aligned}$$

$\langle S \rangle$ is a start state. $\langle A \rangle$ is for $k = 0$ and $j = i$. $\langle C \rangle$ is for $i = 0$ and $j = 1$. $\langle A \rangle \langle C \rangle$ is for $j = i + k$.

(c) $\{a^i b^j c^k d^l \mid i + l = j + k\}$

$$\begin{aligned} \langle S \rangle &\rightarrow \langle AB \rangle \mid \langle AC \rangle \mid \langle BD \rangle \mid \langle CD \rangle \mid \langle AB \rangle \langle CD \rangle \mid \epsilon \\ \langle AB \rangle &\rightarrow a \langle AB \rangle b \mid ab \\ \langle AC \rangle &\rightarrow a \langle AC \rangle c \mid ac \\ \langle BD \rangle &\rightarrow b \langle BD \rangle d \mid bd \\ \langle CD \rangle &\rightarrow c \langle CD \rangle d \mid cd \end{aligned}$$

$\langle S \rangle$ is a start state. $\langle AB \rangle$ is for $l=0, k=0$, and $i=j$. $\langle AC \rangle$ is for $l=0, j=0$, and $i=k$. $\langle BD \rangle$ is for $i=0$, $k=0$, and $j = 1$. $\langle CD \rangle$ is for $i=0, j=0$, and $k = 1$. $\langle AB \rangle \langle CD \rangle$ is for $i+l = j+k$.

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