Monte Carlo Method

- Birthday problem
- Neutron shielding problem

Birthday problem

- *What* is the probability of that at least two of a group of N persons have the same birthday ?
- ***Assumption: each of the 365 days of the year is equally likely to be someone's birthday.***
- **Theoretical result:**

Birthday problem - theoretical results

	The probability that the Nth person's birthday is different from the first N-1 persons birthday, each birthday of whom are different from each other.	The probability that there is no common birthday among N persons
1st person	$p_1 = \frac{365}{365}$	
2nd person	$p_2 = \frac{364}{365}$	$p_1 p_2$
3rd person	$p_3 = \frac{363}{365}$	$p_1 p_2 p_3$
4th person	$p_4 = \frac{362}{365}$	$p_1 p_2 p_3 p_4$
$\vdots$	$\vdots$	$\vdots$
Nth person	$p_N = \frac{365-(N-1)}{365}$	$p_1 p_2 \cdots p_N$

The probability the Nth person provide a match is the following

$$p = 1 - p_1 p_2 \cdots p_n = 1 - (\frac{365}{365})(\frac{364}{365}) \cdots [\frac{365-(N-1)}{365}]$$

Birthday problem - Simulation

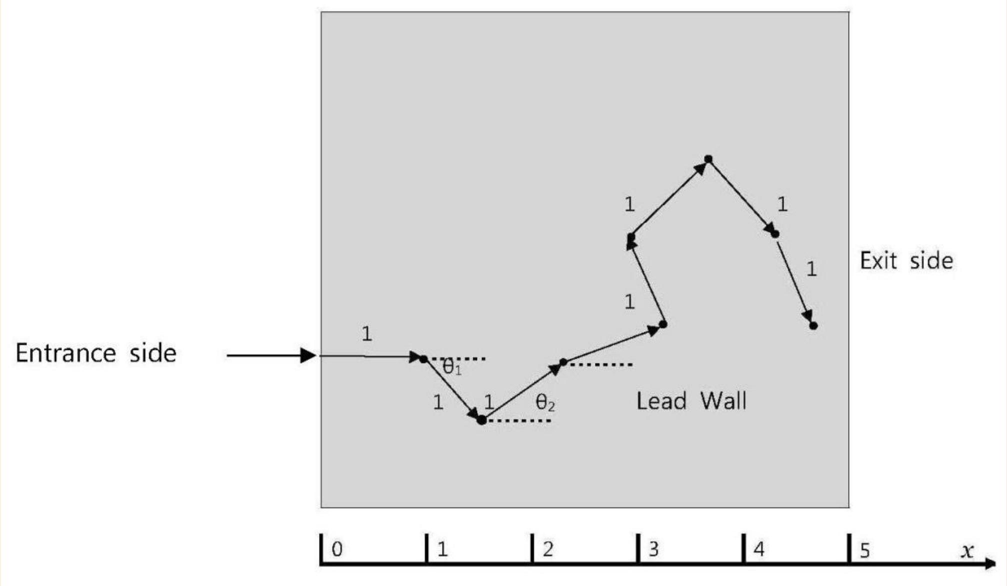
- Generate a large number *Ng* of groups. Eg. 1,000 groups
- Each group has N members represented by its member's birthday chosen *randomly* from 1 up to 365.
- Each day is equally likely to be someone's birthday. (equally distributed random number generator)
- Count the number *Nm* of groups with at least one match in the group.
- The probability that there is a birthday match in a group with N members will be *Nm/Ng*.
- Compare the probability with the theoretical results by filling out the blanks on the left table.
- Excel functions: randbetween(i,t), if, countif, count, sum

N	Theoretical	Monte Carlo simulation
5	0.027	
10	0.117	
15		
20		
25		
30		
35		
40		
45		
50	0.970	
55		
80		
100		
183		

Neutron shielding

- Assumptions
  1. Each neutron enters the lead wall at a right angle to the wall and travel a unit distance.
  2. Each neutron collides with a lead atom and rebounds in a random direction, and again it travels a unit distance before colliding with an other lead atom. And so on…
  3. After eight collisions, all the neutron's energy is spent out.
  4. The lead wall is 5 unit thick in the *x*-direction and for all practical purpose infinitely thick in the *y*-direction.

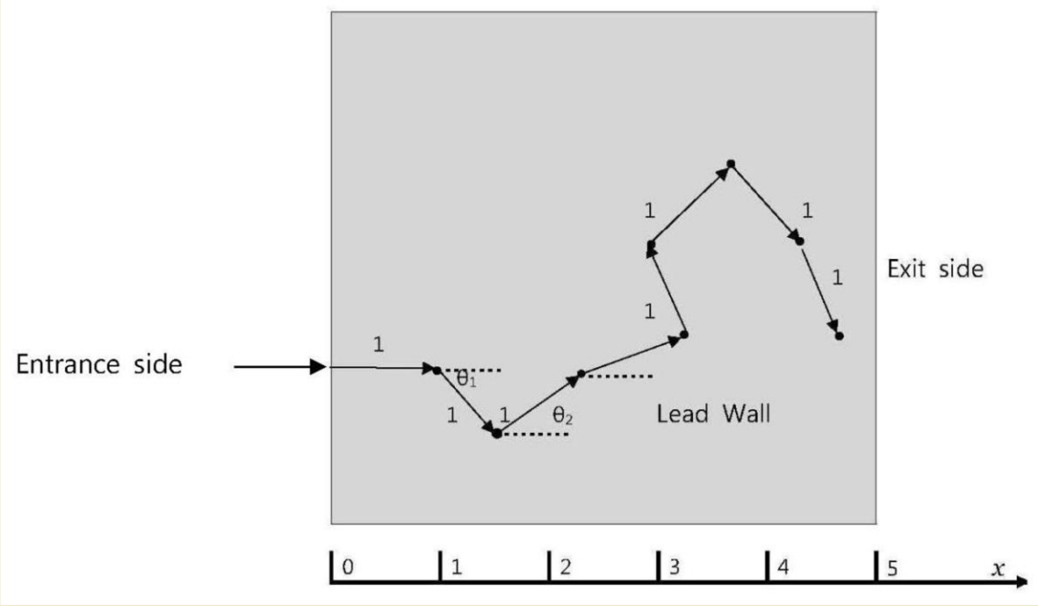
- ***What percentage of neutrons can be expected to emerge from the other side of the lead wall?***



Neutron shielding (2)

- The x-coordinate of the neutron is denoted by x, which is the distance from the initial surface where the neutron enters.
- After the first collision of a neutron with a lead atom, the x-coordinate of the entered neutron will be $x = 1 + \cos\theta_1$, where θ_1 is the redirected angle measured with respect to the horizontal after collision.
  In general, $0 \le \theta_i \le 2\pi$.
- The third collision occurs at $x = 1 + \cos\theta_1 + \cos\theta_2$.
- If $x \ge 5$, then the neutron has exited.
- For a Monte Carlo simulation, we can use random angles θ_i in the interval $(0, 2\pi)$.

- ***About 1.8% out of 16,384 neutrons entered the wall randomly may exit the wall.***



Numerical Integration via Monte Carlo method