GPS Location Based Access Permission

Team 8

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To calculate distance, we referred here. If we assume that the Earth is a perfect sphere, the distance between two arbitrary points of the Earth is

$$d = R_E \cdot \arccos(\sin\phi_1 \cdot \sin\phi_2 + \cos\phi_1 \cdot \cos\phi_2 \cos\Delta\lambda) \tag{1}$$

, where ϕ is a latitude, λ is a longitude, R_E is radius of the Earth. However, if we approximate arccos with taylor expansion, we cannot acquire high accuracy easily as Figure 1. Thus, we calculated distance as follows.

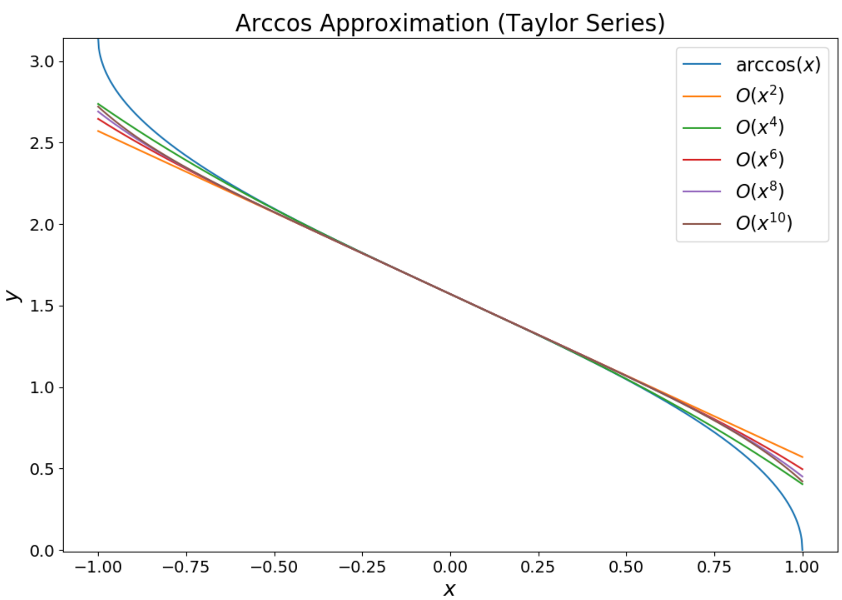

Figure 1: arccos approximation

First of all, we assumed the Earth is a perfect sphere with radius $R_E = 6378km$. In eq 1, let's define f as $f(\phi_1, \phi_2, \lambda_1, \lambda_2) = \sin\phi_1 \cdot \sin\phi_2 + \cos\phi_1 \cdot \cos\phi_2 \cos(\lambda_1 - \lambda_2)$. Since the length of arc l in circle is

$$l = R \cdot \theta \tag{2}$$

, f is $\cos\theta$, where θ is an angle between two points.

Now, with given two accuracies a_1, a_2, we can define maximum allowed angle θ_{max} as

$$\theta_{max} = \frac{(a_1 + a_2) * 0.001}{R_E} \tag{3}$$

. Finally, if $\theta_{max} \geq \theta$ or $\cos\theta_{max} \leq \cos\theta = f(\phi_1, \phi_2, \lambda_1, \lambda_2)$ holds, we can grant access permission between two points.

To calculate that, only we have to approximate is cos and sin. However, since we can reorder f as

$$\begin{aligned} f &= \sin\phi_1 \cdot \sin\phi_2 + \cos\phi_1 \cdot \cos\phi_2 \cos\Delta\lambda & (4)\\ &= \sin\phi_1 \cdot \sin\phi_2 + \cos\phi_1 \cdot \cos\phi_2 - \cos\phi_1 \cdot \cos\phi_2 + \cos\phi_1 \cdot \cos\phi_2 \cos\Delta\lambda & (5)\\ &= \cos(\phi_1 - \phi_2) - \cos\phi_1 \cdot \cos\phi_2(1 - \cos\Delta\lambda) & (6) \end{aligned}$$

, all we need is cos approximation. In experiment, we found that sin approximation is much more accurate than cos approximation when using talyor expansion. Thus we implemented cos function using

$$\sin(x) \approx \Sigma_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} x^{2n+1} \tag{7}$$

, and we adopted n as 7. Finally, we acquired highly approximated cosine function as Figure 2 with average relative error as 10^{-12}.

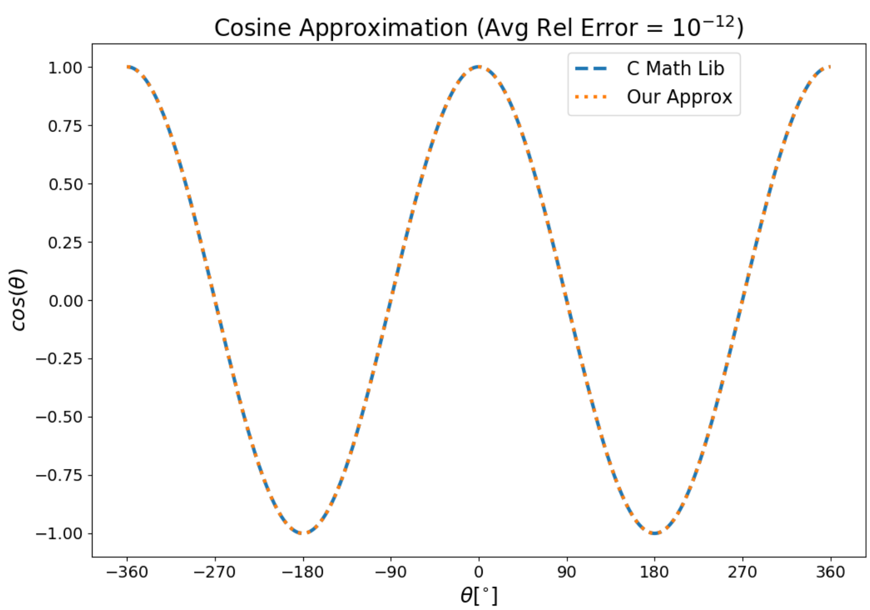

Figure 2: cos approximation result