Perfect information and heterogeneous values of landowners

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1 introduction

In this piece, I would like to talk about the extent of perfect information about the forest land and developed land rents for landowners, and the importance of heterogeneity of the values that landowners put on forest and development land.

2 Perfect information of the rents

Throughout the previous model, I assumed the landowners had perfect information on their land's rent for forestry or for housing development. By perfect information, I meant that the landowners knew the exact rent prices in the future. As a consequence, the selling price of the land was the discounted sum of the exact rent prices in the future.

In a stochastic model, I feel that such assumption is a bit too strong. So as you've mentioned, I think the more adequate assumption is to assume that the rent owner knows the model for the rent prices, but not the exact future prices. Knowing the model parameters of the rent prices, the landowners would know the expected rent prices in the future based on the rent price in the current time step she is in. This would make the selling price and the timing of land conversion from forest to housing complex based on the expected value of future rent prices (see model specification document dated 3/6/2020).

Given observed prices at time t, x_{rt} andx_{ft}, the expected future rent prices at time $t+\tau$ goes as follows (see model specification document for details of the parameters):

$$E(x_{f(t+\tau)}) = \lambda_f^\tau a_f^\tau x_{ft} + \sum_{i=1}^{\tau-1} \lambda_f^i a_f^i E(\epsilon_f)$$
$$E(x_{r(t+\tau)}) = \lambda_r^\tau a_r^\tau x_{rt} + \sum_{i=1}^{\tau-1} \lambda_r^i a_r^i E(\epsilon_r)$$

With large τ, the expected forest and development rent prices converge to mu_f and mu_r respectively.

$$\begin{aligned}
\lambda_f^\tau a_f^\tau x_{ft} + \sum_{i=1}^{\tau-1} \lambda_f^i a_f^i (1-\lambda_f a_f)\mu_f =& \frac{(1-\lambda_f a_f)\mu_f}{1-\lambda_f a_f} \\
=& \mu_f \\
\lambda_r^\tau a_r^\tau x_{rt} + \sum_{i=1}^{\tau-1} \lambda_r^i a_r^i (1-\lambda_r a_r)\mu_r =& \frac{(1-\lambda_r a_r)\mu_r}{1-\lambda_r a_r} \\
=& \mu_r
\end{aligned}$$

$\mu_f = \mu_r$ are equal since I assume the average of both rents are equal (depicting development frontier).

Setting current time t as 0, the expected rent prices graph as Figure 1. The function of expected rent price is strictly increasing or decreasing function depending on whether the current rent price is above or below the mean rent price respectively. (Proof in the Appendix 1.)

The conversion time t_j should be when the difference between the present value of expected forest land rent price starting from t_j and that of development rent is minimal and below 0.

$$z(t_j) = \sum_{i=t_j}^{\infty} (E(x_{fi}) + \theta_{fj} - (E(x_{ri}) + \theta_{rj}))\rho^i < 0 \quad (0 < \rho < 1) \tag{1}$$

$$min_{t_j} z(t_j) \tag{2}$$

As opposed to the previous model where conversion time was based on the actual future rent values and could assume number of different positive integer values, this revised model that uses the expected rent price can only assume 2 conversion time, either current time(t) or infinity (or terminal time). When x_{ft} is higher than x_{rt}, the conversion time is infinity because $E(x_{f(t+\tau)}) > E(x_{r(t+\tau)})$ for all τ. When x_{ft} is lower than x_{rt}, the conversion time is t. because $E(x_{r(t+\tau)}) > E(x_{f(t+\tau)})$. Therefore, not only can this model only have two conversion times, t_j is only determined by the current time step's rent

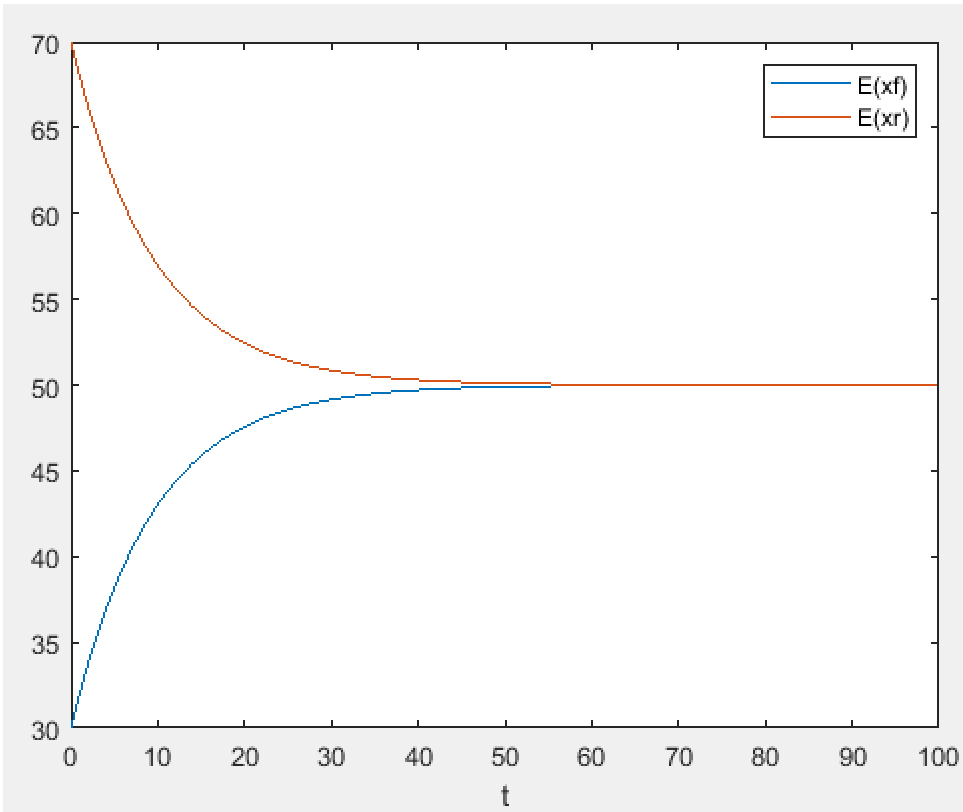

Figure 1: Expected rent prices $E(x_{fj})$ and $E(x_{rj})$. $\mu_f = \mu_r = 50$, $\lambda_f = \lambda_r = 0.9$, $a_f = a_r = 1$.

values. I think such characteristic of the model drastically undermines the value of discount rate and future rent prices in deciding land prices and conversion time.

One way I figured out to give more diversity in the conversion time is to have the individual heterogeneity among land owners on the values of forest and development complex.

3 Individual heterogeneity in land value

Now I consider how heterogeneous value the landowners put on their forest and development land differs things. Say a landowner's personal value on a forest land is θ_{fj} and on a development complex is θ_{rj}. Assuming the two parameters are time-invariant, the total value of forest and development for a landowner is

$$x_{fjt} = x_{ft} + \theta_{fj} \quad \theta_{fj} \sim N(0, \sigma_{fj}^2)$$
$$x_{rjt} = x_{rt} + \theta_{rj} \quad \theta_{rj} \sim N(0, \sigma_{rj}^2)$$

θ_{fj} and θ_{rj} are independent from the rent prices of the forest and development, signifying the landowner's personal value towards the type of land. With these parameters the total value of the land is based on the rent price and the individual values. The landowners know their personal value towards the types of the land, so the expected value of forest ($E(x_{fjt})$) and development ($E(x_{rjt})$) are expected rent prices plus the known θ_{fj} and θ_{rj} value.

$$E(x_{fjt}) = E(x_{fj}) + \theta_{fj}$$
$$E(x_{rjt}) = E(x_{rj}) + \theta_{rj}$$

Thus, $E(x_{fjt})$ and $E(x_{rjt})$converges to $\mu_f + \theta_{fj}$ and $\mu_r + \theta_{rj}$ respectively with increasing t.

Figure 2 illustrates how t_j value can be diversified with parcel value heterogeneity. Figure 2 shows a case when the current forest value is higher than the current development value ($x_{fj0} > x_{rj0}$), but as time passes, the converging expected forest rent is lower than that of expected development rent ($\theta_{fj} < \theta_{rj}$). Top panel of Figure 2 shows the expected value of forest and development value over time with observed x_{fj0} and x_{rj0}. Bottom panel of Figure 2 is the value of z with respect to different t_j values. You can see that the necessary conditions are met when $t_j = 7$ and this is also when $E(x_{fj})$ becomes lower than $E(x_{rj})$. It turns out when $x_{fj0} > x_{rj0}$ and $\theta_{fj} < \theta_{rj}$ the conversion time that meets the necessary condition are always equal to the time when the $E(x_{fj})$ becomes lower than $E(x_{rj})$ (Proof in appendix 2.). Therefore, by varying the parameters $\lambda_f, \lambda_r, a_f, a_r, \theta_{fj}, and \theta rj$, t_j can now assume many different values. Discount rate affects the magnitude of the minimum value of the z in the necessary condition, but does it does not affect t_j itself. This fact becomes clear in the proof.

One of the problem is that the necessary condition for t_j, (1) and (2), will not guarantee that t_j will maximize c_j.Figure 3 showcases a situation when $\theta_{rj} \leq \theta_{fj}$ and $x_{fj0} \leq x_{rj0}$. In this situation t_j that meets the necessary conditions may not maximize the profit. We can see that the $t_j = 0$ satisfies the necessary conditions. However, if you look at the bottom panel, you can see that, if the land is converted at $t = 0$, the landowner would miss all of the large profit that he could gain from forest in the future after time step 1, which is the area under the curve after $t = 1$. Therefore, the correct t_j value is ∞. We see that selecting t_j based on the necessary conditions (1) and (2) alone will result in incorrect clearing time (which is what I have been doing all throughout with the previous model).

This makes me wonder what the sufficient condition is for c_j (term specified in model specification) to be a global maximum. If θ_{rj} is higher than θ_{fj},

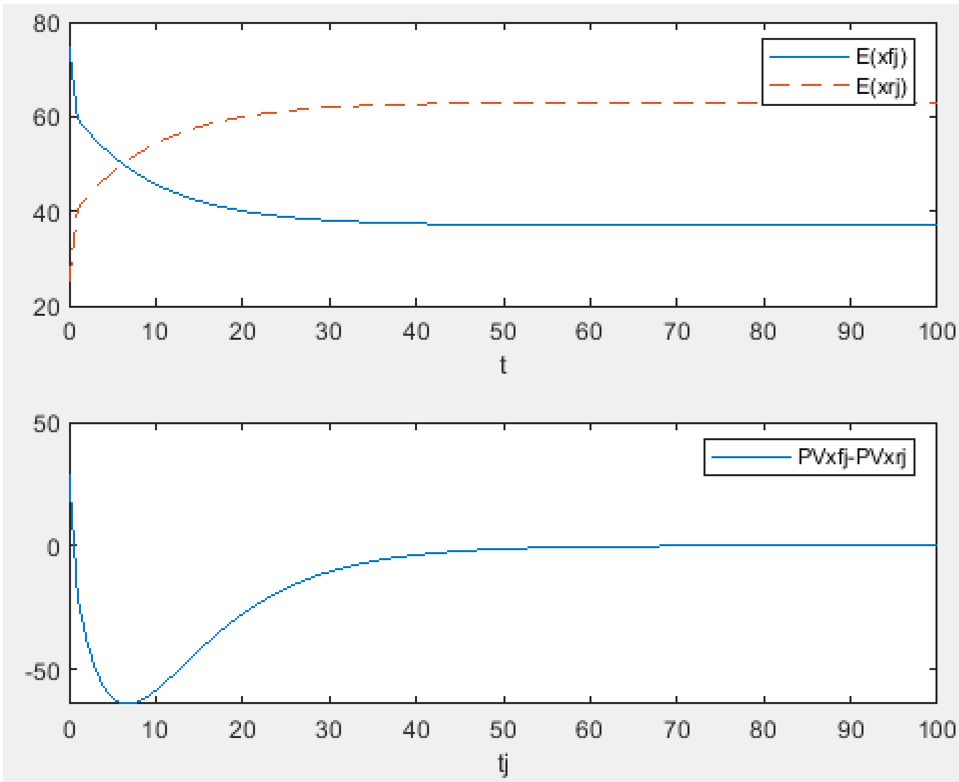

Figure 2: (top)Expected forest value and expected development value given the observed forest and development value at time 0. (bottom) Present value of forest land subtracted by the present value of development value after the t_j $(z(t_j) = \sum_{i=t_j}^{\infty} E(x_{fi})\rho^i - E(x_{ri})\rho^i)$. $x_{fj0} = 75, x_{rj0} = 25, \theta_{rj} = 13, \theta_{fj} = -13, \delta = 0.9, \lambda_f = \lambda_r = 0.9, a_f = a_r = 1, \mu_f = \mu_r = 50$

then (1) and (2) are the sufficient conditions for maximum c_j. However, when $\theta_{rj} \leq \theta_{fj}$ and $x_{ft0} \leq x_{rt0}$, we need more than (1) and (2) to guarantee maximum c_j. Under such condition, the z is negative for certain values of t_j and positive for others. If the sum of negative values of z are bigger in magnitude than the sum of positive values of z, then $t_j = 0$. Otherwise, $t_j = \infty$.

The inequality between θ_{rj} and θ_{fj} is critical in this model because when $\theta_{rj} > \theta_{fj}$ the conversion time can be many different values based on the parameters of the autoregression model, but when $\theta_{fj} \geq \theta_{rj}$, t_j can only be either 0 or ∞.

4 conclusion

I think the added heterogeneity is important going forward with expected rent values because it allows the conversion time to assume more than 2 values (now or never).

5 Appendix

1. Prove that $E(x_{ft})$ is a monotonically decreasing function when $x_{f0} > \mu_f$. The proof that $E(x_{ft})$ monotonically decreases when $x_{f0} > \mu_f$ follows similar logic, and the proof for $E(x_{rt})$ is exactly the same.

$$E(x_{f0}) > \mu_f \tag{3}$$

$$(1-\lambda_f a_f)E(x_{f0}) + \lambda_f a_f E(x_{f0}) > (1-\lambda_f a_f)\mu_f + \lambda_f a_f E(x_{f0}) \tag{4}$$

$$E(x_{f0}) > (1-\lambda_f a_f)\mu_f + E(\sqrt{1-\lambda_f^2 a_f^2}\epsilon_f) + \lambda_f a_f E(x_{f0}) \tag{5}$$

$$E(x_{f0}) > E(x_{f1}) \tag{6}$$

$E(x_{f1}) > \mu_f$ because

$$\begin{aligned} E(x_{f1}) &= E(\epsilon_f) + \lambda_f a_f \\ &= (1-\lambda_f a_f)\mu_f + \lambda_f a_f E(x_{f0}) \\ &= \mu_f + \lambda_f a_f (E(x_{f0}) - \mu_f) \end{aligned}$$

Generalizing this, we can repeat the process from (1) to (4) to show that $E(x_{ft}) > E(x_{f(t+1)})$ and all $E(x_{ft}) > \mu_f$

2. $x_{f0} > x_{r0}$ and $\theta_{fj} < \theta_{rj}$. t_j minimizes $\sum_{i=t_j}^{\infty}(E(x_{fji}) - E(x_{rji}))\rho^i$, and t' is the first time step when $E(x_{rjt}) < E(x_{fjt})$. Show that $t_j = t'$.

$$\sum_{i=t_j}^{\infty}(E(x_{fji}) - E(x_{rji}))\rho^i < \sum_{i=t_j+1}^{\infty}(E(x_{fji}) - E(x_{rji}))\rho^i$$

$$E(x_{fjt_j}) - E(x_{rjt_j}) < 0 \tag{7}$$

$$\sum_{i=t_j}^{\infty}(E(x_{fji}) - E(x_{rji}))\rho^i < \sum_{i=t_j-1}^{\infty}(E(x_{fji}) - E(x_{rji}))\rho^i$$

$$0 < E(x_{fj(t_j-1)}) - E(x_{rj(t_j-1)}) \tag{8}$$

Because t' is the first time step when the expected forest value becomes less than expected development value,

$$\begin{aligned} E(x_{rjt'}) &< E(x_{fjt'}) \\ E(x_{rjt'}) - E(x_{fjt'}) &< 0 \qquad (9) \\ &\mathit{and} \\ E(x_{rjt'-1}) &> E(x_{fjt'-1}) \\ E(x_{rj(t'-1)}) - E(x_{fj(t'-1)}) &> 0 \qquad (10) \end{aligned}$$

(7) and (8) is equivalent to (9) and (10), so $t_j = t'$

$Q.E.D$

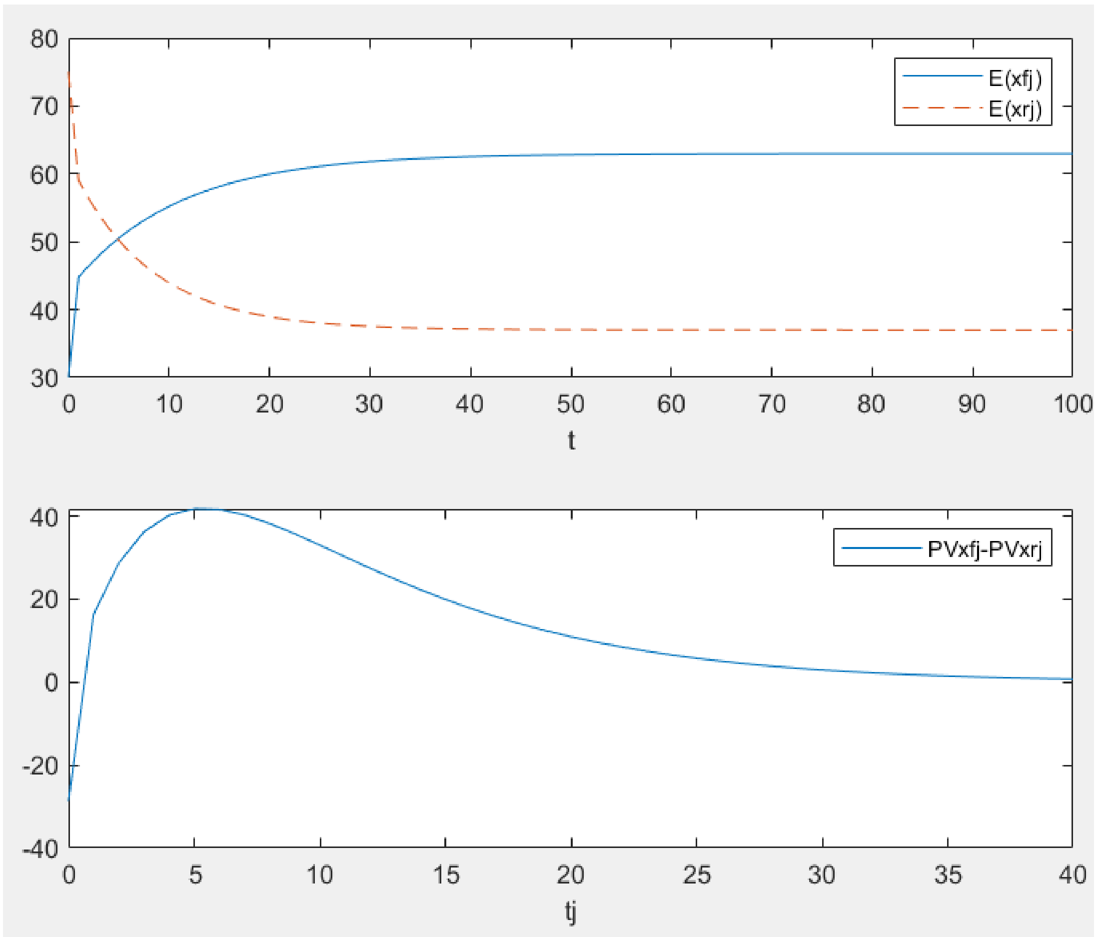

Figure 3: Same type of figure as Figure 2, but $\theta_{fj} < \theta_{rj}$ and $x_{rj0} < x_{fj0}$. $x_{fj0} = 75, x_{rj0} = 30, \theta_{rj} = 13, \theta_{fj} = -13, \delta = 0.87, \lambda_f = 0.91, \lambda_r = 0.88, a_f = a_r = 1$