2nd draft

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1 simulation of GDP and annual returns of different components

Same notations: f for GDP value, f_f for forest annual return, f_r for development market annual return, and f_b for conservation organization's annual budget.

f is an AR(3) process.

$$f(t) = c_1 f(t-1) + c_2 f(t-2) + c_3 f(t-3) + e \quad e \sim N(0, s^2) \tag{1}$$

*In truth, I'm not really sure what order of AR I should use for f. I've asked for suggestions on forming f in my latest email.

f_f is an Markov process where next step is an weighted average of current state and GDP of next step.

$$f_f(t) = a_f f_f(t-1) + (1 - a_f)\gamma_f f(t) + e_f \quad e_f \sim N(0, \sigma_f^2) \tag{2}$$

a_f is the weight given to the previous state of f_f and γ_f is a scaling factor for f.

f_r and f_b is written similarly with e, γ, and weight with their own subscript. For simplicity, maybe we can put all the γ and weight terms equal for all 3 f_f, f_r, and f_b.

2 cost and clearing time

Let's say conservation happens through conservation organization buying the land offered at some cost at some time t ($c(t)$) and making it into a nature reserve.

Price of land offered (c) is derived from the land owner's expectation of future forest and development market return, $\hat{f}_f$ and $\hat{f}_r$ respectively.

$$c(t) = \sum_{i=t}^{t_j} \frac{\hat{f}_f(i)}{(1+\rho)^{i-t}} + \sum_{i=t_j}^{T} \frac{\hat{f}_r(i)}{(1+\rho)^{i-t}} \tag{3}$$

t_j is a time a forest is cleared for development. I omitted the individual variation of the annual returns in 2 different land use, ϵ_{fj} and ϵ_{rj}, which was used in the

last draft for simplicity. I'll add them later maybe.
ρ is a economic discount rate of the landowner. $\hat{f}_f$ and $\hat{f}_r$ is an AR(1) process .

$$\hat{f}_f(t+\tau) = \begin{cases} f_f(t), & \text{if } \tau = 0 \\ b_f f_f(t) + \sum_{i=0}^{\tau} e_{f,i}, & \tau > 0. \end{cases} \tag{4}$$

$\epsilon_{f,i}$ is $N(0,1)$. $\hat{f}_r$ is defined similarly with parameter b_r and $e_{r,i}$, which can equal b_f and e_f for simplicity.
Notice that I used same error variable for $\hat{f}_f$ and f_f.

With $\hat{f}_f$ defined, we can get expected value and variance of $\hat{f}_f(t+\tau)$.

$$E(\hat{f}_f(t+\tau)) = b_f f_f(t)$$
$$Var(\hat{f}_f(t+\tau)) = \tau$$

clearing time t_j is the first time forestry return is greater than development return. We can calculate the probability that $t+\tau$ is t_j.

$$\begin{aligned}
Pr(t_j = t+\tau) &= \prod_{i=0}^{\tau-1} Pr(\hat{f}_f(t+i) \geq \hat{f}_r(t+i)) Pr(\hat{f}_f(t+\tau) < \hat{f}_r(t+\tau)) \\
&= \prod_{i=0}^{\tau-1} (1 - Pr(\hat{f}_f(t+i) < \hat{f}_r(t+i))) Pr(\hat{f}_f(t+\tau) < \hat{f}_r(t+\tau)) \\
&= \prod_{i=0}^{\tau-1} (1 - Pr(\hat{f}_f(t+i) - \hat{f}_r(t+i) < 0)) Pr(\hat{f}_f(t+\tau) - \hat{f}_r(t+\tau) < 0) \\
&= \prod_{i=0}^{\tau-1} \left(1 - Pr\left(\frac{\hat{f}_f(t+i) - \hat{f}_r(t+i) - \mu}{\sigma\sqrt{i}} < \frac{-\mu}{\sigma\sqrt{i}}\right)\right) Pr\left(\frac{\hat{f}_f(t+\tau) - \hat{f}_r(t+\tau) - \mu}{\sigma\sqrt{\tau}} < \frac{-\mu}{\sigma\sqrt{\tau}}\right) \\
&= \prod_{i=0}^{\tau-1} \left(1 - \Phi\left(\frac{-\mu}{\sigma\sqrt{i}}\right)\right) \Phi\left(\frac{-\mu}{\sigma\sqrt{\tau}}\right)
\end{aligned}$$

in which $\mu = b_f f_f(t) - b_r f_r(t)$, $\sigma^2 = \sigma_f^2 + \sigma_r^2$.

Summary of assumptions from this section:
1) No variation among land parcels. For now.
2) 1 land offered at time t and offer goes away at next time step.
3) land cleared at the first instance forestry return is greater than development return.
4) Same error variable between f_f and $\hat{f}_f$ ($e_f \sim N(0, \sigma_f^2)$

3 ecological value of a parcel

Let's worry about this later...

4 Reference

Random walk with Gaussian steps:
https://math.stackexchange.com/questions/40224/probability-of-a-point-taken-from-a-certain-normal-distribution-will-be-greater
2 normal dist inequality:
https://www.math.ucla.edu/ caflisch/181.1.03f/Lect4-5.pdf