Model specification

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1 Question Statement

What are some good strategies of land acquisition in the face of varying land values and budget over time?

2 model

We have 3 autoregressive processes that depict income in general economy (x), rent from forest lands(x_f), and rent from developed lands (x_r). x is a general AR(1) process, and x_f and x_r is a modified AR(1) process where their current value is not only dependent on their previous value but also on current x value.

$$x_t = \lambda x_{(t-1)} + \sqrt{1-a^2}\epsilon_t + \mu(1-a), \quad \epsilon_t \sim N(0,\sigma^2), \quad 0 < \lambda < 1 \tag{1}$$

$$x_{ft} = \lambda_f(a_f x_{f(t-1)} + (1-a_f)(x_t - \mu)) + \sqrt{1-\lambda_f^2 a_f^2}\epsilon_{ft} + \mu_f(1-\lambda_f a_f) \tag{2}$$
$$\epsilon_{ft} \sim N(0,\sigma_f^2)$$
$$x_{rt} = \lambda_r(a_r x_{r(t-1)} + (1-a_r)(x_t - \mu)) + \sqrt{1-\lambda_r^2 a_r^2}\epsilon_{rt} + \mu_r(1-\lambda_r a_r) \tag{3}$$
$$\epsilon_{rt} \sim N(0,\sigma_r^2)$$
$$0 < a_f, a_r, \lambda_f, \lambda_r < 1$$

It follows that $E(x_{ft}) = \mu_f, Var(x_{ft}) = \sigma_f$ and analogous for x and x_r. In order to depict the development frontier, the average rent price for forest and development complex is equal (μ_f and μ_r)

Rent price x_f and x_r are homogeneous across land parcels. However, the total value of a forest and developed land (x_{fj} and x_{rj}) is a sum of rent price and the landowner's heterogeneous value towards the land.

$$x_{fjt} = x_{ft} + \theta_{fj} \quad \theta_{fj} \sim N(0, \sigma_{fj}^2) \tag{4}$$

$$x_{rjt} = x_{rt} + \theta_{rj} \quad \theta_{rj} \sim N(0, \sigma_{rj}^2) \tag{5}$$

θ_{fj} and θ_{rj} are time-invariant parameters independent from the rent prices of the forest and development, signifying the landowner's personal value towards the type of land.

A landowner j chooses to maximize the expected value from the land by converting the forest at t_j

$$max_{t_j} c_j = \sum_{i=0}^{t_j-1} (E(x_{fi}) + \theta_{fj})\rho^i + \sum_{i=t_j}^{\infty} (E(x_{ri}) + \theta_{rj})\rho^i \quad t_j \geq 1 \tag{6}$$

where ρ is a discount factor for economic goods ($0 < \rho < 1$). The assumption here is that there is no cost of conversion into development complex and the land owners are risk-neutral.

The necessary condition for t_j to be maximizing the objective is when, after t_j, present value of forest land is less than that of development land and when that difference is maximal.

$$z(t_j) = \sum_{i=t_j}^{\infty} (E(x_{fi}) + \theta_{fj}) - (E(x_{ri}) + \theta_{rj}))\rho^i < 0 \tag{7}$$

$$min_{t_j} z(t_j) \tag{8}$$

The landowner will sell the land to a conservation agent at c_j.

The conservation agent can buy the land with money accrued from donation. The amount of donation that the agent receives is denoted as x_b, and it has same autoregressive equation as x_f and x_r.

$$x_{bt} = \lambda_b(a_b x_{b(t-1)} + (1 - a_b)(xt - \mu)) + \epsilon_{bt} \tag{9}$$
$$\epsilon_{bt} \sim N(\mu_b(1 - \lambda_b a_b), \sigma_b^2(1 - \lambda_b^2 a_b^2))$$

In each time step, the conservation agent decides whether to buy a forest parcel with cost of c_j. The decision parameter d_t is a binary variable that is 1 if the decision is to buy at time t, and 0 otherwise. The donation that the conservation agent receives and is unspent at that time step is put in a bank account f with interest rate of $1/\rho$.

$$f(t) = \frac{1}{\rho} f(t-1) + x_{bt} - c_t$$
$$c_t = \begin{cases} c_j & d_t = 1 \\ 0 & d_t = 0 \end{cases}$$

Every forest land parcel has a fixed ecological benefit of b. The conservation benefit of a parcel is a discounted ecological benefit from the time step the parcel is expected to be converted

$$B_j = \sum_{i=t_j}^{\infty} b\delta^i$$

δ is a discount factor applied to ecological goods ($0 < \delta < 1$), and it is equal to the discount rate for the economic good ρ by default.

3 simulation

The values of x, x_r, x_f, and x_b are simulated to time step $T + 700$ and the first 200 values are discarded to get the values that have converged to the stationary process. The simulation goes on for time step T, where in every time step, a forest land becomes available for sale for a conservation agent, and it can choose to purchase the land. The conservation agent decides to purchase according to a specified strategy throughout the simulation, and the total conservation benefit of purchased parcels are tallied at the end of time step T.