ISyE 323 Practice Implementation Quiz

November 4, 2021

1. You must complete the quiz independently. No communication with anyone else is allowed during the quiz.
2. You are allowed to access any materials from the course Canvas web page, and Julia + JuMP documentation. All other materials are not allowed.
3. You may ask Zach for clarifications during the (real) quiz.
4. Your screen must be visible to Zach as he proctors the (real) quiz.

1 MCNF with Node Capacities

In class, we saw the min cost network flow (MCNF) problem, which is characterized by the following data:

- A network with nodes N and arcs A
- For each arc $(i,j) \in A$, a capacity U_{ij}, lower bound L_{ij}, and unit cost c_{ij}
- For each node $i \in N$, a net supply b_i; depending on the value of b_i, node i is either a supply, demand, or transshipment node

Consider a modification of the MCNF problem, in which transshipment nodes have a capacity. Let $T \subset N$ be the set of transshipment nodes. We define the following additional parameters:

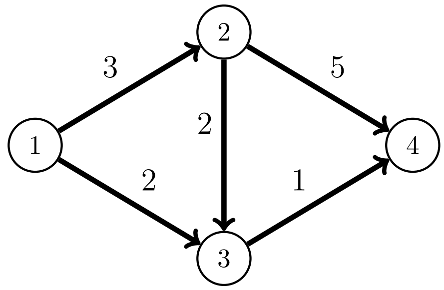
- u_i: The capacity of transshipment node $i \in T$ (note that u_i is lowercase to distinguish it from U_{ij}, the notation for an arc capacity)

The LP model for this modified problem is

$$
\begin{aligned}
\min \quad & \sum_{(i,j)\in A} c_{ij} x_{ij} & \\
\text{s.t.} \quad & \sum_{j:(i,j)\in A} x_{ij} - \sum_{j:(j,i)\in A} x_{ji} = b_i & \forall i \in N \\
& L_{ij} \le x_{ij} \le U_{ij} & \forall (i,j) \in A \\
& \sum_{j:(j,i)\in A} x_{ji} \le u_i & \forall i \in T
\end{aligned}
$$

where x_{ij} is the flow on arc $(i,j) \in A$.

1. Implement the above LP as a *general* model.
2. Consider the following network:



Node 1 supplies 10 units of flow and node 4 demands 10 units of flow. Nodes 2 and 3 are transshipment nodes, each of which has a capacity 6. Arcs are labeled with their unit costs; arcs have lower bounds 0 and infinite capacities.

Solve this specific instance using your general model; you should not write an explicit model, you should simply use the general model you have already written.

Upload a Jupyter notebook and a PDF of its output, and answer the remaining questions in Canvas.

2 Multiperiod Planning without Inventory

Consider a store that has zero storage space (meaning any available inventory at the end of a month must be thrown out) but does not need to meet all its demand on time except in the last month of a planning horizon (i.e., it's allowed to have orders be backlogged). You are interested in minimizing total cost to meet demand over the next 6 months. Consider the following data for the problem:

- c_t: Unit cost to produce a product in month t, $t = 1, \dots, 6$
- C: Unit cost per month the store is late in meeting demand
- D_t: Demand for the product in month t, $t = 1, \dots, 6$

Define the following decision variables:

- x_t: Number of units of product to produce in month t, $t = 1, \dots, 6$
- b_t: Number of units of product backlogged at the end of month t, $t = 1, \dots, 6$

The LP model for this problem is

$$
\begin{aligned}
\min \quad & \sum_{t=1}^{6} c_t x_t + C \sum_{t=1}^{6} b_t \\
\text{s.t.} \quad & x_1 + b_1 \geq D_1 \\
& x_t + b_t \geq b_{t-1} + D_t && \text{for } t = 2, \dots, 6 \\
& b_6 = 0 \\
& x_t, b_t \geq 0 && \text{for } t = 1, \dots, 6
\end{aligned}
$$

(In case you're wondering why the inventory balance constraints are now inequalities, it's to allow for the possibility of throwing out excess product at the end of a month.)

1. The data for a specific instance is the following:

t	c_t	D_t
1	12	100
2	14	132
3	21	137
4	22	153
5	15	144
6	13	116

, $C = 6$

You may use the following lines of code to get started:

```
nmonths = 6
cost = [12, 14, 21, 22, 15, 13]
backlogcost = 6
demand = [100, 132, 137, 153, 144, 116]
```

2. Implement the above LP to solve this specific instance.

Upload a Jupyter notebook and a PDF of its output, and answer the remaining questions in Canvas.