ISyE323: Operations Research—Deterministic Models
Problem Set #5 Due: Friday, March 11, 11:59PM

Deliverables:

- Writeup (on in a file submit on Canvas) for all problems together as one pdf file. (In this write-up, include the answers to questions that are asked as part of problems where you are asked to implement a model in Julia/JuMP.)
- Pdf file displaying code and output when it is run for Problems 1-2 and 3-2 uploaded to Canvas. Please name the files as described in the problem.
- '*.ipnyb' file you created for Problems 1-2 and 3-2 uploaded to Canvas. Please name the files as described in the problem.

1 Cheese Production

CheeseCo processes dairy and non-dairy milks to make cheese products. Milk can be purchased for \$20/vat, and must first be pasteurized, which costs \$5 and uses 0.5 hours of labor per vat. The pasteurized milk can then be processed to produce cheese in two different ways. Method 1 yields 2 wheels of cheese, 9 small blocks of cheese, and 30 ounces of waste material. Method 1 costs \$3/vat of milk, and takes 1.2 hours of labor. Method 2 produces 3 wheels of cheese, 4 small blocks of cheese, and 40 ounces of waste material. Method 2 costs \$4/vat of milk, and takes 2 hours of labor. A cup of specialty cheese spread can be made using 12 ounces of waste material at a cost of \$3 per cup, and using 0.5 hours of labor. The sales price and maximum sales amount of each product are given in the table below. CheeseCo has at most 300 hours of labor available. (You may assume that excess cheeses or waste material, if any that cannot be sold due to sales limits, may be discarded at no cost.)

Products ($p \in P$)	Small blocks	Wheels	Waste Material	Specialty Spread
Sales Price (S_p)	\$3/block	\$10/wheel	\$1/oz	\$20/cup
Maximum Can Sell (M_p)	500 blocks	250 wheels	3000 oz	400 cups

1-1 Problem Formulate a linear program that can help CheeseCo determine how to maximize its profit.

1-2 Problem Solve the model you wrote in Problem 1-1 using the JuMP package in Julia. You should submit *three parts* to this solution:

1. A pdf file or HTML file displaying your code in Jupyter, together with the output that is obtained after it is run (you should look at the output to verify it completed correctly!). Name this file as 'hw5-q1-output'.

2. As part of your overall solution file (i.e., with the solutions to other problems in this homework set) you should include an answer for this problem that states an optimal solution and the optimal objective value.

3. Submit the actual "*.ipynb" implementation file, which can be obtained in Jupyter Notebook by selecting "File->Download As->Notebook". Name this file as 'hw5-q1.ipynb'.

2 Cheese Supply Chain

CheeseCo has dairy farms that supply their milk in Portage and Waupun. The Portage farm can deliver 400 lbs of milk per day, and the Waupun farm can deliver 650 lbs of milk per day. These production quantities are fixed and all milk is shipped to a processing plant each day. Milk can be shipped to one of two plants, in Columbus or Ripon. The Columbus plant can process up to 900 lbs per day at a cost of \$0.25 per lb, and the Ripon plant can process up to 500 lbs per day at a cost of \$0.30 per lb. Once processed, milk is shipped to Oshkosh, Fond du Lac, and Madison, and sold at \$3.50 per lb, \$2.50 per lb, and \$2.85 per lb (respectively) at the different cities. The maximum amount that can be sold in each city cities is 550, 600 , and 700 lbs, respectively. The cost per lb to transport milk between the cities is given in the table below:

	To				
From:	Columbus	Ripon	Oshkosh	Fon du Lac	Madison
Portage	0.35	0.50			
Waupun	0.30	0.30			
Columbus			0.60	0.50	0.25
Ripon			0.25	0.25	0.65

2-1 Problem Formulate a linear programming problem that will help CheeseCo determine how much milk to send between each pair of cities, and how much to process at the processing facilities, in order to maximize the daily net profit. You *do not* need to solve this model – just write the model.

3 Bags of Beans

I am very particular about the types of coffee beans I use. I like to get new bags of coffee beans from local roasters every week.[1] I'm trying to be efficient, so I only have $n = 6$ roasters who I purchase from. The acidity level of the beans from each roaster a_i is known. Coffee also gets better with age, so my enjoyment of each roaster's beans increases for each week I wait to purchase it. I've measured my "enjoyment function" for each roaster i as a function of the acidity a_i and the week in which I purchase it t as $E_i = b_i(a_i + t)^2 - c_i(a_i + t)$, where $t \in \{0, 1, 2, 3, 4\}$ and b_i, c_i are given constants. The data $a_i, b_i,$ and c_i are given in Table 1.

[1] This story is fictionalized for effect. I only buy coffee every *two* weeks. :-)

Table 1: Coffee Bean Properties

Roaster	a	b	c
1	20	2	10
2	5	3	15
3	4	5	20
4	20	3	5
5	10	1	25
6	15	4	15

3-1 Problem Formulate a linear program that maximizes the total enjoyment I obtain by purchasing and drinking coffee from each roaster for the next 6 weeks. Each coffee can only be purchased once (i.e., the 6 roasters must each be assigned to one of the next 6 weeks).
hint 1: The variables in your model should be $x_{it} = 1$ if I buy from roaster $i \in \{1, 2, 3, 4, 5, 6\}$ in week $t \in \{0, 1, 2, 3, 4, 5\}$, otherwise $x_{it} = 0$. This is an assignment problem.
hint 2: Create a *parameter* that contains the amount of enjoyment I obtain by purchasing beans from each roaster each week. You will compute the parameters in Julia before defining any constraints or objective of the model. Thus, even though it looks like there are nonlinear effects in this problem, this is a *linear* assignment problem. Your objective function will be linear!

3-2 Problem olve the model you wrote in Problem 3-1 using the JuMP package in Julia. You should submit *three parts* to this solution:

1. A pdf file or HTML file displaying your code in Jupyter, together with the output that is obtained after it is run (you should look at the output to verify it completed correctly!). Name this file as 'hw5-q3-output'.

2. As part of your overall solution file (i.e., with the solutions to other problems in this homework set) you should include an answer for this problem that states an optimal solution and the optimal objective value.

3. Submit the actual "*.ipynb" implementation file, which can be obtained in Jupyter Notebook by selecting "File->Download As->Notebook". Name this file as 'hw5-q3.ipynb'.

4 Taste of Madison

I have some friends visiting from out of town, and I want to take them on a tour of some of Madison's most iconic restaurants. We'll be driving to each location, so I'd like to try to minimize the total amount of time we spend in the car. Table 2 lists distances between restaurants that are convenient to drive between. (A

− in the table means that the locations are not convenient, so we would never travel from one to the other directly).

We will assume for this problem that all the connections are uni-directional. For example, it is possible to drive from the Great Dane to the Old Fashioned, but not vice-versa.

Table 2: **Minutes to drive between restaurants**

Restaurant	Great Dane	Old Fashioned	Red Sushi	Ian's at Garver	Weary Traveler	Alchemy
Short Stack	4	6	5	-	-	-
Great Dane		1	-	7	-	-
Old Fashioned			2	5.5	4	-
Red Sushi				-	5	-
Ian's at Garver					1	6
Weary Traveler						8

4-1 Problem Write a linear program (a shortest path problem) whose solution will identify the shortest path from Short Stack to Alchemy. (You do not need to solve this problem; just write the model.)