Homework 0

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Q1

Given the following predicates and their meanings

1. $P(x, y) : x > y$
2. $Q(x, y) : x \leq y$
3. $R(x) : x - 7 = 2$
4. $S(x) : x > 9$

If the universe of discourse is the real numbers, give the truth value (true or false) of each of the following propositions:

1. $(\exists x)R(x) =$ **True**
2. $(\forall y)[\neg S(y)] =$ **False**
3. $(\forall x)(\exists y)P(x, y) =$ **True**
4. $(\exists y)(\forall x)Q(x, y) =$ **True**
5. $(\forall x)(\forall y)[P(x, y) \vee Q(x, y)] =$ **False**
6. $(\exists x)S(x) \wedge \neg(\forall x)R(x) =$ **True**
7. $(\exists y)(\forall x)[S(y) \wedge Q(x, y)] =$ **True**
8. $(\forall x)(\forall y)[R(x) \wedge S(y) \rightarrow Q(x, y)] =$ **False**

Q2

Which of the following sentences has the logical form $(p \wedge q) \rightarrow r$

Option 3

1. If you don't attend the wedding, then Sam will be angry with you
2. Matt is happy and so are Sam and Fae
3. **If it rains and it snows then flooding will result**
4. Students will play football or students will play soccer; but they will not attend classes
5. Gene is smart and strong, additionally he is a good swimmer

Q3

Which of the following formulas represents the sentence, "If there are no fruit in the market then the farmers didn't plant fruit trees or the farmers didn't water the trees"

p means There are no fruit in the market

q means Farmers didn't plant fruit trees

r means Farmers didn't water the trees

Option 2

1. $\neg p \rightarrow q$
2. $p \rightarrow q \vee r$
3. $(p \rightarrow q) \vee \neg r$
4. $p \rightarrow q \vee \neg r$
5. $p \vee q \rightarrow \neg r$

Q4

Show $[p \wedge (p \rightarrow q)] \rightarrow q$ is a tautology

p	q	$p \rightarrow q$	$p \wedge (p \rightarrow q)$	$[p \wedge (p \rightarrow q)] \rightarrow q$
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

Since the final column with all operations is true $[p \wedge (p \rightarrow q)] \rightarrow q$ is a tautology

Q5

Argue that set A and set A' (the compliment of A) are disjoint

The complement of set A is defined as the set resulting from removing the set A from the Universe U. Let's say that we are looking at the example of the male sex and female sex of a class. Given the assumption that a class consists of male and female students, let us say that set A denotes all the male students. The complement of set A (A') is all the females in the class. There can be no overlaps between the sets thus proves our point that the set A (males) and set A' (females) are disjoint.

Q6

Which of the following is a one-to-one function?

Option 5

1. { (1,2), (2,3), (3,4), (4,5), (3,7), (2,2) }
2. x = 5
3. x=5, 10 < y < 25
4. { (1,2), (2,3), (3,4), (2,5), (3,7) }
5. **{ (1,2), (2,4), (3,6), (4,8) }**

Q7

Let $U = x : x$ is an integer and $2x10$

In each of the following cases, determine whether $A \subseteq B$, $B \subseteq A$, both or neither:

(i)	$A = \{x : x \text{ is odd}\}$	$B = \{x : x \text{ is a multiple of } 3\}$	Neither
(ii)	$A = \{x : x \text{ is even}\}$	$B = \{x : x^2 \text{ is even}\}$	$A \subseteq B$
(iii)	$A = \{x : x \text{ is even}\}$	$B = \{x : x \text{ is a power of } 2\}$	Neither
(iv)	$A = \{x : 2x + 1 > 7\}$	$B = \{x : x^2 > 20\}$	$A \subseteq B$
(v)	$A = \{x : \sqrt{x} \in \mathbb{Z}\}$	$B = \{x : x \text{ is a power of 2 or 3}\}$	$A \subseteq B$
(vi)	$A = \{x : \sqrt{x} \leq 2\}$	$B = \{x : x \text{ is a perfect square}\}$	Neither
(vii)	$A = \{x : x^2 - 3x + 2 = 0\}$	$B = \{x : x + 7 \text{ is a perfect square}\}$	$B \subseteq A$

Note : $\mathbb{Z}$ denotes the set of all integers