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Math 3100 HW 7

Section 3.3 #12, 13, 17, 19

12) $E(X)=10 \quad \sigma=5$

a) smallest upper bound for $P(x\geq 20)$

$P(x\geq 20) \leq ?$

$P(|x-10| \geq 10) \leq \frac{Var(x)}{a^2} = \frac{25}{100} = \frac{1}{4}$

b) Let us check.

$E(x) = 10 = np$

$Var(X) = np(1-p)$

$5 = 10(1-p)$

$\frac{1}{2} = 1-p$

$p = \frac{1}{2}$

Yes, X could be binomial random variable.

13) mean 100 and SD= 10

a) $P(x\geq 130) \leq ?$

$P(|X-100| \geq 3\cdot 10) \leq \frac{10^2}{30^2} = \frac{1}{9}$

1 million $\times \frac{1}{9} = 111,111.11 \approx 111,112$

b) If symmetric it would be exactly $\frac{1}{2}$

$\frac{1}{2}\cdot\frac{1}{9} = \frac{1}{18}$

1 million $\cdot \frac{1}{18} = 55,556$.

c) $P(x\geq 130) = 1-\Phi\left(\frac{x-100}{10}\right) = 1-\Phi\left(\frac{130-100}{10}\right)$

$1-\Phi(3) \times 1mil$

1,300

17) a) $E(X) = -1\left(\frac{1}{4}\right)+0\left(\frac{1}{4}\right)+1\left(\frac{1}{2}\right) = -\frac{1}{4}+\frac{1}{2} = \frac{1}{4}$

$Var(X) = E(X^2)-[E(X)]^2$

$\frac{1}{4}+\frac{1}{2}$

$\frac{3}{4} - \left(\frac{1}{4}\right)^2 = \frac{3}{4}-\frac{1}{16} = \frac{12}{16}-\frac{1}{16} = \frac{11}{16}$

$SD = \sqrt{Var(x)} = \frac{\sqrt{11}}{4}$

Expectation of 25 times $= 25\cdot\frac{1}{4} = 6.25$

STD of 25 times $= 25^{\frac{1}{2}}\cdot\frac{\sqrt{11}}{4} = \frac{5\sqrt{11}}{4}$

continue next page.

$P(S<0) = P(S \le -1)$ since the sum is a integer.

$P\left(-\infty \le \frac{S_n - n\mu}{\sigma\sqrt{n}} \le -1\right)$

$\phi\left(\frac{b + \frac{1}{2} - \mu}{\sigma}\right)$

$\phi\left[\frac{-1 + \frac{1}{2} - 6.25}{\frac{5\sqrt{11}}{4}}\right] - 0$

$= \phi[-1.628]$

$= 1 - \phi[1.628]$

$\approx 1 - \phi[1.63]$

$\approx 1 - 0.9484$

≈ 0.0516

≈ 0.05

b) $P(S=0)$

Now the equation will be

$P\left(0 \le \frac{S_n - n\mu}{\sigma\sqrt{n}} \le 0\right)$

$\phi\left[\frac{b + \frac{1}{2} - \mu}{\sigma}\right] - \phi\left[\frac{a - \frac{1}{2} - \mu}{\sigma}\right]$

$\phi\left[\frac{\frac{1}{2} - 6.25}{\frac{5\sqrt{11}}{4}}\right] \left[\frac{-\frac{1}{2} - 6.25}{\frac{5\sqrt{11}}{4}}\right]$

$[-1.3869] - [-1.62816]$

$1 - \phi[1.3869] - [1 - \phi(1.628)]$

$1 - \phi[1.39] - [1 - \phi(1.63)]$

$1 - 0.9177 - (1 - 0.9484))$

$= 0.03$

c) $P(S>0) = 1 - [P(S=0) + P(S<0)]$

$1 - [0.05 + 0.03]$

$= 0.92$

1a) 30 people

$E(x) = 30 \cdot 150 = 4500$

$SD = 55 \cdot \sqrt{30} = 55\sqrt{30}$

$P(5000 \le x < \infty)$

$\cancel{\Phi(\infty)} - \Phi\left(\frac{5000 - \frac{1}{2} - \mu}{\sigma}\right)$

$1 - \Phi(1.658)$

$\approx 1 - \Phi(1.66)$

$\approx 1 - 0.9515$

≈ 0.0485

Section 4.1 4, 12

4) $f(x) = cx^2(1-x)^2$ for $0<x<1$

Density curve = area under curve is 1. $0<x<1$

$\int_0^1 cx^2(1-x)^2 = 1$

$c\int_0^1 x^2(1-x)^2 = 1$

$c\int_0^1 x^2(1-2x+x^2) = 1$

$c\int_0^1 x^2 - 2x^3 + x^4 = 1$

$c\left[\frac{x^3}{3} - \frac{2}{4}x^4 + \frac{x^5}{5}\right]_0^1 = 1$

$c\left[\frac{1}{3} - \frac{2}{4} + \frac{1}{5} - 0\right) = 1$

$\frac{c}{30} = 1$

$c = 30$

b) $E(x) = \int_{-\infty}^{\infty} x f(u)\, du$

$= \int_0^1 x\, cx^2(1-x)^2$

$\int_0^1 cx^3(1-2x+x^2)$

$c\int_0^1 x^3 - 2x^4 + x^5$

$= c\left[\frac{x^4}{4} - \frac{2x^5}{5} + \frac{x^6}{6}\right]_0^1 =$

$= 30\left[\frac{1}{4} - \frac{2}{5} + \frac{1}{6}\right]$

$30\left(\frac{15 - 24 + 10}{60}\right) = 30\left(\frac{25-24}{60}\right) = \boxed{\frac{1}{2}}$

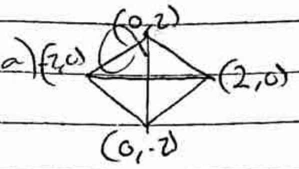
c) $E(x^2) = \int_0^1 x^2 cx^2(1-x)^2$

$\int_0^1 cx^4(1-2x+x^2)$

$c\int_0^1 x^4 - 2x^5 + x^6 \rightarrow 30\left[\frac{x^5}{5} - \frac{2}{6}x^6 + \frac{x^7}{7}\right]_0^1$

$2/7 = E(x^2)$

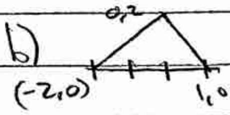
$Var(x) = E(x^2) - E(X)^2 = \frac{2}{7} - \left(\frac{1}{2}\right)^2 = \boxed{\frac{1}{28}}$

12) a) 

total area $= 2 \times (4 \cdot 2 \cdot \frac{1}{2}) = 8.$

function icreasing $= x+2$
but we multiply by 2 since height is 

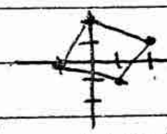
$$f(x) = \begin{cases} 2\frac{(x+2)}{8} & -2 < x < 0 \\ 2\frac{(2-x)}{8} & 0 < x < 2 \end{cases}$$

b) 

~~total area $= 3 \times 2 \times \frac{1}{2} = 3$~~

function incresing at $x+2$ until 0 and $2-2x$ until 1

$$f(x) = \begin{cases} \frac{x+2}{3} & -2 < x < 0 \\ \frac{2-2x}{3} & 0 < x < 1 \end{cases}$$

c) 

total area $= \sqrt{5} \cdot \sqrt{5} = 5$
All sides are $\sqrt{5}$

For $-1 < x < 0$

top curve $= 2x+2$
bottom curve $= -\frac{1}{2}x - \frac{1}{2}$ $\Big\rangle - = \frac{5}{2}x + \frac{5}{2}$ / total area

$\left(\frac{5}{2}x + \frac{5}{2}\right)\frac{1}{5} = \frac{1}{2}x + \frac{1}{2}$

For $0 < x < 1$

top curve $= 2 - \frac{1}{2}x$
bottom curve $= -\frac{1}{2}x - \frac{1}{2}$ $\Big\rangle - = \frac{5}{2}$ / total area

$\frac{1}{2}$

For $1 < x < 2$

top curve $= 2 - \frac{1}{2}x$
bottom curve $= 2x - 3$ $\Big\rangle - = 5 - \frac{5}{2}x$ / total

$(5 - \frac{5}{2}x)\frac{1}{5} = 1 - \frac{1}{2}x$

$$f(x) \begin{cases} \frac{1}{2}x + \frac{1}{2} & -1 < x < 0 \\ \frac{1}{2} & 0 < x < 1 \\ 1 - \frac{1}{2}x & 1 < x < 2 \end{cases}$$

Section 4.2 #5

5) $\lambda = 1$ / per second

$f(t) = \lambda e^{-\lambda t}$

$= e^{-t}$

a) $\int_0^2 f(t)\,dt = \int_0^2 e^{-t}\,dt = -e^{-t}\Big|_0^2 = -e^{-2} - (-1)$

$= 1 + -e^{-2} = 1 - 0.1353 = 0.86466 = 0.86$

b) P(at least 4 calls) = 1 − P(at most 3 calls)

$= 1 - e^{-\lambda t}\frac{(\lambda t)^{r-1}}{(r-1)!}\lambda$

$= 1 - e^{-t}\left(\frac{t^{r-1}}{(r-1)!}\right) \qquad t = 5$

$= 1 - e^{-5}\left(\frac{5^{r-1}}{(r-1)!}\right)$

$= 1 - e^{-5}\left[\frac{5^0}{0!} + \frac{5^1}{1!} + \frac{5^2}{2!} + \frac{5^3}{3!}\right]$

$= 1 - e^{-5}\left[1 + 5 + \frac{25}{2} + \frac{125}{6}\right]$

$= 0.73497$

≈ 0.735

c) rate = 1 per second

4 calls arrive.

$\frac{\text{number of calls}}{\text{rate}} = \frac{4}{1} = 4$ seconds

Section 3.5

6) $\lambda = 30$ drops/min = 5 drops/10 seconds.

$P(N_\mu = 0) = e^{-\mu} \qquad \mu = 5$

$= e^{-5}$

$= 0.00674$

9a) $e^{-\mu}\mu^k/k!$

$P(X=1)\cdot P(X=2)$ (Independent.

$e^{-1}1^1/1! \cdot e^{-2}2^2/2!$

$e^{-1} \cdot e^{-2}\,2$

$2e^{-3}$

$0.0995741 3$

10b) $Var(3X+5)$

$Var(3x) + Var(5)$

$9\,Var(x) + 0$

9λ