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Homework # 5

3.1 RANDOM VARIABLES INTRODUCTION

3.1.2

(a)

Joint Distribution for sampling with replacement...

	$x=1$	$x=2$	$x=3$	$x=4$
$y=1$	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$
$y=2$	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$
$y=3$	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$
$y=4$	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$

There are 4 cases where 1 is less than or equal to the y values, 3 cases for 2, 2 for 3 and 1 for 4. This means the possibilities are $4+3+2+1$

$\therefore P(x \leq y) = \frac{10}{16}$

(b)

Joint Distribution for sampling without replacement...

	$x=1$	$x=2$	$x=3$	$x=4$
$y=1$	N/A	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$
$y=2$	$\frac{1}{16}$	N/A	$\frac{1}{16}$	$\frac{1}{16}$
$y=3$	$\frac{1}{16}$	$\frac{1}{16}$	N/A	$\frac{1}{16}$
$y=4$	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$	N/A

There are 3 cases where 1 is less than or equal to the y values, 2 cases for 2, 1 for 3 and 0 for 4. This means the possibilities are $3+2+1+0$

$\therefore P(x \leq y) = \frac{6}{12}$

3.1.3

(a) The range of the sum of rolling a die twice is between 2 and 12. This is because the lowest number possible will be 1 occurring two times, and the highest number possible will be 6 occurring two times.

(b)

Distribution of S...

S	2	3	4	5	6	7	8	9	10	11	12
Probability	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$
Possibilities	1,1	1,2 2,1	1,3 2,2 3,1	1,4 2,3 3,2 4,1	1,5 2,4, 3,3 4,2 5,1	1,6 2,7 3,4 4,3 5,2 6,1	2,6 3,5 4,4 5,3 6,2	3,6 4,5 5,4 6,3	4,6 5,5 6,4	5,6 6,5	6,6

3.1.4

(a)

Joint Distribution for (X_1, X_2)...

	$x_1 = 1$	$x_2 = 2$	$x_3 = 3$	$x_4 = 4$	$x_5 = 5$	$x_6 = 6$
$x_2 = 1$	$\frac{1}{36}$	$\frac{1}{36}$	$\frac{1}{36}$	$\frac{1}{36}$	$\frac{1}{36}$	$\frac{1}{36}$
$x_2 = 2$	$\frac{1}{36}$	$\frac{1}{36}$	$\frac{1}{36}$	$\frac{1}{36}$	$\frac{1}{36}$	$\frac{1}{36}$
$x_2 = 3$	$\frac{1}{36}$	$\frac{1}{36}$	$\frac{1}{36}$	$\frac{1}{36}$	$\frac{1}{36}$	$\frac{1}{36}$
$x_2 = 4$	$\frac{1}{36}$	$\frac{1}{36}$	$\frac{1}{36}$	$\frac{1}{36}$	$\frac{1}{36}$	$\frac{1}{36}$
$x_2 = 5$	$\frac{1}{36}$	$\frac{1}{36}$	$\frac{1}{36}$	$\frac{1}{36}$	$\frac{1}{36}$	$\frac{1}{36}$
$x_2 = 6$	$\frac{1}{36}$	$\frac{1}{36}$	$\frac{1}{36}$	$\frac{1}{36}$	$\frac{1}{36}$	$\frac{1}{36}$

(b)

Joint Distribution for (Y_1, Y_2)...

	$y_1 = 1$	$y_2 = 2$	$y_3 = 3$	$y_4 = 4$	$y_5 = 5$	$y_6 = 6$
$y_2 = 1$	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{2}{36}$	$\frac{2}{36}$	$\frac{2}{36}$	$\frac{2}{36}$
$y_2 = 2$	0	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{2}{36}$	$\frac{2}{36}$	$\frac{2}{36}$
$y_2 = 3$	0	0	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{2}{36}$	$\frac{2}{36}$
$y_2 = 4$	0	0	0	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{2}{36}$
$y_2 = 5$	0	0	0	0	$\frac{1}{36}$	$\frac{2}{36}$
$y_2 = 6$	0	0	0	0	0	$\frac{1}{36}$

3.1.15

(a) This is problem c...

Proof. Looking at the possibilities of this probability, we can see that it is a simple summation problem. For when $X = 1$ there is no way that this possibility will hold. However for $X = 2$ there will be 1 way, which is $Y = 1$. This patter of $ways = x - 1$ will equate to be $1 + 2 + 3...n - 1$. The number of conditions that satisfy this rule over the total (n^2) will be the following.

$$\frac{1+2+3...+n-1}{n^2}$$

$$\frac{\frac{(n-1)*n}{2}}{n^2}$$

$$\frac{n-1}{2n}$$

$$\therefore P = \frac{\mathbf{n-1}}{\mathbf{2n}}$$

□

(b) Problem e...

Proof.

We can start by looking at probabilities for when $k =$ a certain number.

$$\begin{aligned}
k = 1 &= (1,1)(1,2)...(1,n)... \\
&= (2,1)(3,1)...(n,1) \\
&= n + n - 1 \\
k = 2 &= (2,2)(2,3)...(2,n)... \\
&= (3,2)(4,2)....(n,2) \\
&= n - 1 + n - 2 \\
k = n - 1 &= (n-1, n-1)(n-1, n) \\
&= (n, n-1) \\
&= 3 \\
k = n &= (n,n) \\
&= 1
\end{aligned}$$

The general rule is $2(n - k + 1) - 1$.

The number of possibilities is n^2. Therefore, the probability will then become $\frac{\mathbf{2(n-k+1)-1}}{\mathbf{n^2}}$

□

(c) Problem f ...

Proof.

Let us visualize.

$$\begin{aligned}
&k = 1 = N/A = 0 \\
&k = 2 = (1,1) = 1 \\
&k = 3 = (1,2),(2,1) = 2 \\
&k = n - 1 = ... = k - 1 - 1 \\
&k = n = ... = k - 1
\end{aligned}$$

The general rule is $k - 1$.

The number of possibilities is n^2. Therefore, the probability will then become $\frac{\mathbf{k-1}}{\mathbf{n^2}}$

□

3.1.23

(a) *Proof.*

The condition of $x \leq t$ will be a subset of the condition $y \leq t$ because of the condition of $x \geq y$. We can say that $\{(x,y)|x \leq t\} \subseteq \{(x,y)|y \leq t\}$. This means that $\mathbf{P}(\mathbf{x} \leq \mathbf{t}) \leq \mathbf{P}(\mathbf{y} \leq \mathbf{t})$ Since the number of events occurring is greater for the second condition.

□

3.2 Expectations

3.2.3

(a) The total outcome space has $6 * 6 * 6 = 216$ combinations. There are 6 possibilities for all three places of (a, b, c).
Let's look at the possibilities of sixes. ...

$$\begin{aligned}
P(3six) &= \frac{1}{216} \\
&= (6,6,6) \\
P(2six) &= \frac{15}{216} \\
&= ((5 \text{ possibilities}), 6, 6), (6, (5 \text{ possibilities}), 6), (6, 6, (5 \text{ possibilities})) \\
P(1six) &= \frac{75}{216} \\
&= (6, 1, (5 \text{ possibilities}))...(6, 5, (5 \text{ possibilities}))(6, (5 \text{ possibilities}), 1)...(6, (5 \text{ possibliities}), 5)... \\
P(0six) &= \frac{125}{216} \\
&= 5 * 5 * 5 \\
Expectation &= 0 * \frac{125}{216} + 1 * \frac{75}{216} + 2 * \frac{15}{216} + 3 * \frac{1}{216} \\
&= \mathbf{\frac{1}{2}}
\end{aligned}$$

(b) The total outcome space has $6 * 6 * 6 = 216$ combinations. There are 6 possibilities for all three places of (a, b, c).
Let's look at the possibilities of sixes. ...

$$\begin{aligned}
P(3 \text{ odd}) &= \frac{27}{216} \\
P(2 \text{ odd}) &= \frac{81}{216} \\
P(1 \text{ odd}) &= \frac{81}{216} \\
P(0 \text{ odd}) &= \frac{27}{216} \\
Expectation &= 0 * \frac{27}{216} + 1 * \frac{81}{216} + 2 * \frac{81}{216} + 3 * \frac{27}{216} \\
&= \mathbf{1.5}
\end{aligned}$$

3.2.4

(a) For this problem we will be able to use the Markov's Inequality...

$$\begin{aligned}
If X \geq 0, then P(X \geq a) &\leq \frac{E(X)}{a} \text{ for every } a > 0 \\
P(X > 8) &\leq \frac{2}{8} \\
P(X > 8) &\leq \frac{1}{4}
\end{aligned}$$

We can conclude that at least $\frac{1}{4}$ of the 100 numbers is 25 and at least are greater than 8.