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Homework # 2

1.3 Distributions

1.3.8

(a) $\mathbf{P(A \cup B) = 0.8}$ because of the proof shown...

Proof.

$$\begin{aligned} P(A \cup B) &= ? \\ &= P(A) + P(B) - P(AB) \\ &= 0.6 + 0.4 - 0.2 \\ \therefore P(A \cup B) &= 0.8 \end{aligned}$$

□

(b) $\mathbf{P(A^c) = 0.4}$ because of the proof shown...

Proof.

$$\begin{aligned} P(A^c) &= ? \\ &= 1 - P(A) \\ &= 1 - 0.6 \\ \therefore P(A^c) &= 0.4 \end{aligned}$$

□

(c) $\mathbf{P(B^c) = 0.6}$ because of the proof shown...

Proof.

$$\begin{aligned} P(B^c) &= ? \\ &= 1 - P(B) \\ &= 1 - 0.4 \\ \therefore P(B^c) &= 0.6 \end{aligned}$$

□

(d) $\mathbf{P(A^cB) = 0.2}$ because of the proof shown...

Proof. The probability of intersection of $AB = 0.2$. This means that 0.2 probability is shared by A and B. The question is asking for $P(A^cB)$. This is the space shared by A^c and B. We can safely assume that $B \in A^c$. Therefore, $P(A^cB) = P(B) - P(AB) = 0.4 - 0.2 = 0.2$ □

(e) $\mathbf{P(A \cup B^c) = 0.8}$ because of the proof shown...

Proof.

$$\begin{aligned} P(A \cup B^c) &= ? \\ &= P(A) + P(B^c) - P(AB^c) \\ &= P(A) + 1 - P(B) - P(AB^c) \end{aligned}$$

$P(AB^c) = 0.4$ due to the similar reasoning shown in part (d).

$$\begin{aligned} &= 0.6 + 1 - 0.4 - 0.4 \\ \therefore P(A \cup B^c) &= 0.8 \end{aligned}$$

□

(f) $\mathbf{P(A^c B^c) = 0.2}$ due to the proof shown below.

Proof. The only space that will be the intersection of A^c and B^c is space that is not in A nor B. Therefore the space is for not A and not B. The space that is not shared with A or B in any way is 0.2. □

1.3.11

(a) *Proof.*

$$\begin{aligned} A \cup B \cup C &= (A \cup B) \cup C \\ &= P(A \cup B) + P(C) - P((A \cup B) \cap C) \\ &= P(A \cup B) + P(C) - P(A \cap C) \cup P(B \cap C) \\ &= P(A \cup B) + P(C) - P(A \cap C) + P(B \cap C) - P(A \cap C \cap B \cap C) \\ &= P(A \cup B) + P(C) - P(AC) + P(BC) - P(ABC) \\ &= P(A) + P(B) - P(AB) + P(C) - (P(AC) + P(BC) - P(ABC)) \\ &= P(A) + P(B) + P(C) - P(AB) - P(AC) - P(BC) + P(ABC) \end{aligned}$$

□

1.4 Conditional Probability and Independence

1.4.2

This problem can be solved using conditional probability...
Proof.

$$\begin{aligned} P(AB) &= P(A|B)P(B) \\ \text{Where } A &= \text{ not defective} \\ \text{Where } B &= \text{ made by city} B \\ &= (0.99) * (\frac{1}{3}) \\ &= 0.33 \end{aligned}$$

□

Therefore, the probability that the bulb is B and defective is **0.33**

1.4.3

Given that ...
$A =$ rain today
$P(A) = 0.4$
$B =$ rain tomorrow
$P(B) = 0.5$
$P(AB) = 0.3$

Proof.

$$\begin{aligned} P(B|A) &= \frac{P(AB)}{P(A)} \\ &= \frac{0.3}{0.4} \\ &= 0.75 \end{aligned}$$

□

Therefore, given it rains today the probability that it will rain tomorrow is **0.75**

1.4.4

(a) Neither of the events occurs?

$$\begin{aligned} P(A) &= 0.1 \\ P(B) &= 0.3 \end{aligned}$$

because they are independent events we can use the following equation

$$\begin{aligned} P(A^c B^c) &= P(A^c) * P(B^c) \\ &= 0.9 * 0.7 \\ &= 0.63 \end{aligned}$$

Therefore, neither of the events occurring is **0.63**

(b) At least one of the events occurs?

$$\begin{aligned} P(A \cup B) &= P(A) + P(B) - P(A)P(B) \\ &= 0.1 + 0.3 - 0.03 \\ &= 0.4 - 0.03 \\ &= 0.37 \end{aligned}$$

Therefore, the probability of at least one of the events happening is **0.37**

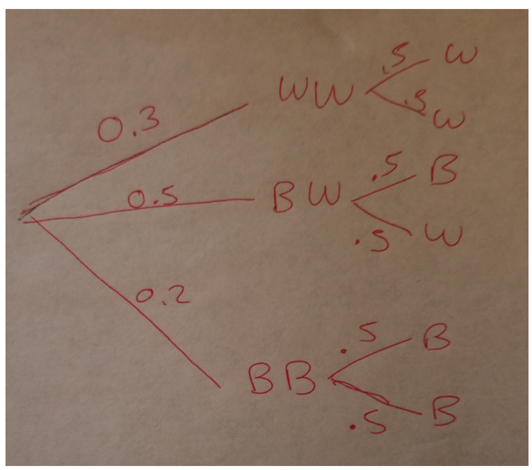
(c) exactly one of the events occurs?
This means the probability of A happening with B not happening or B happening with A not happening.

$$\begin{aligned} P(A^c \cap B) \cup P(A \cap B^c) &= ? \\ &= P(A^cB) + P(AB^c) \\ &= 0.9 * 0.3 + 0.1 * 0.7 \\ &= 0.34 \end{aligned}$$

Therefore, the probability of exactly one of the events occurring is **0.34**

1.4.8

$P(WW) = 0.3$ = Both sides being white
$P(BW) = 0.5$ = One side being white and one side being black
$P(BB) = 0.2$ = Both sides being black
Assuming when placing the card both sides have an equally likely chance of being face up, we can show the following



We should look at the probability of choosing a white card with the top side black is $0.5 * 0.5$.
The probability that the top is black is $0.5 * 0.5 + 0.2 * 0.5 + 0.2 * 0.5$
The probability then is $\frac{0.25}{0.25+0.1+0.1} = \frac{0.25}{0.45} = \frac{5}{9}$
Therefore, the probability of this occurring is $\mathbf{\frac{5}{9}}$

1.5 Special Problem (Bayes' Rule)

Problem
A dashboard warning light is supposed to flash red if a cars oil pressure is too low. On a certain model, the probability of the light flashing when it should is 0.99; 2% of the time, though, it flashes for no apparent reason. If there is a 10% chance that the oil pressure really is low, what is the probability that a driver needs to be concerned if the warning light goes on?

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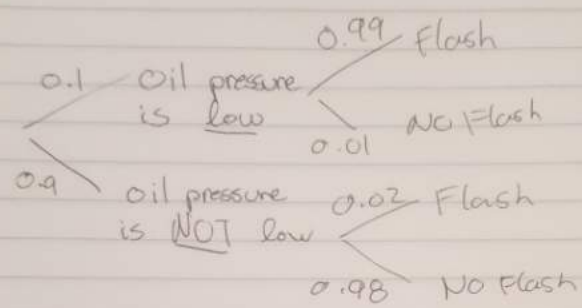
Solution
Picture of work shown on the next page!!! Sorry for the inconvenience.

Bayes' Rule.



Bayes Rule.

$$P(B_i|A) = \frac{P(A|B_i)P(B)}{P(A|B_1)P(B_1) + \cdots + P(A|B_n)P(B_n)} \quad (i = 1 \ldots n)$$

$$\frac{0.1 \times 0.99}{(0.1 \times 0.99) + (0.9 \times 0.02)}$$

← oil low & Flash

← all Flash

$= 0.8461$

$\approx \boxed{0.85}$