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MATH 3100
HW 9

Section 4.5 # 6, 8d

6) $F(x) = x^3$ for $0 < x \le 1$

a) $P(X \ge \frac{1}{2}) = 1 - P(X < \frac{1}{2})$
$= 1 - F(\frac{1}{2})$
$= 1 - (\frac{1}{2})^3$
$= 1 - \frac{1}{8}$
$= \boxed{7/8}$

b) $f(x) = \frac{d}{dx} F(x)$
$= \frac{d}{dx} x^3$
$= 3x^2$

c) $E(x) = \int_a^b x f(x)\, dx$
$\int_0^1 x\, 3x^2\, dx$
$\int_0^1 3x^3\, dx$
$\frac{3x^4}{4}\Big|_0^1$
$\boxed{3/4}$

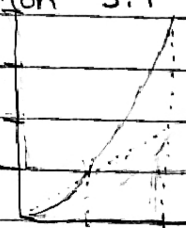
d) This means that for each y
$F_y(y) = P(Y \le x) = \begin{cases} 0 & x \le 0 \\ x & 0 < x < 1 \\ 1 & x \ge 1 \end{cases}$

However we know that x is the most right so...
$P(X \le x_i) = [P(Y \le x)]^3$
$= x^3 = F(x)$ ✓ □

8d) Lets say that L_i is ~~every individual component~~ the lifetime for every individual component in the system
Lets say that L_s is the lifetime for the whole system
Every component will be $P(L_i \le t) = 1 - e^{-\lambda_i t}$
However with mean time $\mu = \frac{1}{\lambda}$ it will be $1 - e^{-t/\mu_i}$
We have that 1 and 2 are parallel and 3 is in sequence
1&2 $1 - P(L_1 \le t) P(L_2 \le t)$
$1 - (1 - e^{-t/\mu_1})(1 - e^{-t/\mu_2})$
now add 3 to system as e^{-t/μ_3} so
$P(L_s > t) = [1 - (1 - e^{-t/\mu_1})(1 - e^{-t/\mu_2})][e^{-t/\mu_3}]$

Section 5.1

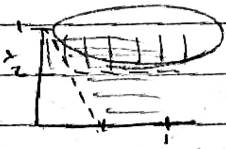
1b)



$P(Y<X^2) = \int_1^2 \int_x^{x^2} \frac{1}{2}\, dy\, dx = \int_1^2 \frac{x^2-x}{6}$

$\boxed{5/36}$

3) $P(Y \geq \frac{1}{2} \mid Y \geq 1-2x)$



$= \frac{P[(Y\geq \frac{1}{2}) \cap (Y \geq 1-2x)]}{P(Y \geq 1-2x)}$

$= \frac{\frac{1}{2} - (\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{4})}{1-\text{triangle}}$

$= \frac{\frac{1}{2}-\frac{1}{16}}{1-(1\times\frac{1}{2}\times\frac{1}{2})} = \frac{\frac{8}{16}-\frac{1}{16}}{1-\frac{1}{4}} = \frac{\frac{7}{16}}{\frac{3}{4}} = \frac{7}{16} \times \frac{4}{3} = \boxed{\frac{7}{12}}$

6) a) A person arrives between 12:00 - 12:15

$f(x) = \frac{1}{15}$ for m where $0 \leq m \leq 15$

$f(x,y) = \frac{1}{15}$ for $0 \leq x \leq 15$, $0 \leq y \leq 15$

0 ; other wise

Some next $P(X-Y \geq 2) = P(X \geq 2+y)$

$= \int_0^{15} \int_{2+y}^{15} (\frac{1}{15})^2\, dx\, dy = \frac{165}{450}$

b) F = first arrives by 12:05

L = last arrives after 12:10

$F(FL) = 1 - F[(FL)^c] = 1 - F(F^c) - F(L^c) + F(F^c \cap L^c)$

$= 1 - (\frac{10}{15})^{10} - (\frac{10}{15})^{10} + (\frac{5}{15})^{10}$

$= 0.9653$

Section 5.2 # 4, 9, 16

4 a) $P(X \le x, Y \le y)$

$= \int_0^x \int_0^y 6e^{-2x-3y}\, dx\,dy$

$= \int_0^x 6e^{-2x} \int_0^y e^{-3y}\, dy$

$= \int_0^x 6e^{-2x} \left[-\frac{1}{3}e^{-3y}\right]_0^y$

$= \int_0^x 6e^{-2x} \left[-\frac{1}{3}e^{-3y} + \frac{1}{3}e^0\right]$

$= \int_0^x 6e^{-2x} \left[\frac{1}{3} - \frac{1}{3}e^{-3y}\right]$

$= \int_0^x 2e^{-2x}(1 - e^{-3y})\, dx$

$= 2(1-e^{-3y}) \int_0^x e^{-2x}\, dx$

$= 2(1-e^{-3y})\left[-\frac{1}{2}e^{-2x}\right]_0^x$

$= 2(1-e^{-3y})\left(-\frac{1}{2}e^{-2x} + \frac{1}{2}e^0\right)$

$= (1-e^{-3y})(1-e^{-2x})$

b) $f_X(x) = \int_0^\infty 6e^{-2x-3y}\, dy$

$= 6e^{-2x} \int_0^\infty e^{-3y}\, dy$

$= 6e^{-2x}\left[-\frac{1}{3}e^{-3y}\right]_0^\infty$

$= 6e^{-2x}\left(0 + \frac{1}{3}\right)$

$= 2e^{-2x}$

c) $f_Y(y) = \int_0^\infty 6e^{-2x-3y}\, dx$

$= 6e^{-3y} \int_0^\infty e^{-2x}$

$= 6e^{-3y}\left(-\frac{1}{2}e^{-2x}\right)\Big|_0^\infty$

$= 6e^{-3y}\left(\frac{1}{2}\right)$

$= 3e^{-3y}$

d) $f_X(x) \cdot f_Y(y) = 6e^{-3y}e^{-2x} = 6e^{-2x-3y}$ ✓

9) a) We know that $f_X(s) = \lambda e^{-\lambda s}$ and $f_Y(t) = \lambda e^{-\lambda t}$

$P(s < X < Y < t) = (e^{-\lambda s} - e^{-\lambda t})^2$

For Joint distribution, $f_{XY}(s,t)$

$f_{XY}(s,t) = \int_s^t \int_p^y \frac{\partial}{\partial}(e^{-\lambda s} - e^{-\lambda t})^2 \; d/dt \; d/ds$

$= 2\lambda^2 e^{-\lambda(s+t)}$

b) $\lambda e^{-\lambda s} \cdot \lambda e^{-\lambda t} \ne 2\lambda^2 e^{-\lambda(s+t)}$ So The are not independent

b) we know $z = y - x$

$P(X>s, z>t) = P(x>s, y>x+t)$

so we have

$= \int_{s}^{\infty}\int_{x+t}^{\infty} 2\lambda^2 e^{-\lambda(x+y)}\, dy\, dx$

$= \int_{s}^{\infty}\int_{t}^{\infty} 2\lambda^2 e^{-\lambda(2x+z)}\, dz\, dx$

$= \int_{s}^{\infty}\int_{t}^{\infty} f(x,z)\, dz\,dx$

to see if x and z are independent

we need to se if $f(x,z) = f(x) f(z)$

We know $(2\lambda^2 e^{-\lambda(2\lambda x})(\lambda e^{-\lambda z})$

$= 2\lambda^2 e^{-2\lambda x - \lambda z}$

There it is independent

c) Margirels

$f_x(x) = \int_{-\infty}^{\infty} 2\lambda^2 e^{-\lambda(2x+z)}\, dz$

$= 2\lambda e^{-2\lambda x}$

$f_z(z) = \int_{-\infty}^{\infty} 2\lambda^2 e^{-\lambda(2x+z)}\, dx$

$= \lambda e^{-\lambda x}$

16) $P(X_1 < X_2 < X_3) = \int_0^\infty \int_{x_1}^\infty \int_{x_2}^\infty \lambda_3 e^{-\lambda_3 x_3} \lambda_2 e^{-\lambda_2 x_2} \lambda_1 e^{-\lambda_1 x_1}$

take one by one

$\int_0^\infty \int_{x_1}^\infty \Big|_{x_2}^\infty -\frac{1}{\lambda_3} e^{-\lambda_3 x_3}$

$\int_0^\infty \int_{x_1}^\infty -\frac{1}{\lambda_3} e^{-\lambda_3 x_3} \lambda_2 e^{-\lambda_2 x_2} \dots$

$\vdots$

$= \frac{\lambda_2 \lambda_1}{(\lambda_2 + \lambda_3)(\lambda_1 + \lambda_2 + \lambda_3)}$