HYUN SUK RYOO

Homework # 4

2.1 The Binomial Distribution

2.1.12

(a) We can assume from previous problems and page 7 that the probability of winning is $\frac{18}{38}$. This problem asks for what is the probability that she has to make exactly 8 bets before stopping. This means that what is the probability that she won exactly $\frac{4}{7}$ and wins on her 8^{th} trial.

Proof.
A = above situation ...

$$\begin{aligned}
P(A) &= \binom{7}{4} * \left(\frac{18}{38}\right)^4 * \left(\frac{20}{38}\right)^3 * \frac{18}{38} \\
&= \left(\frac{7!}{4! * 3!}\right) * \left(\frac{18}{38}\right)^4 * \left(\frac{20}{38}\right)^3 * \frac{18}{38} \\
&= \left(\frac{7 * 6 * 5}{3 * 2}\right) * \left(\frac{18}{38}\right)^4 * \left(\frac{20}{38}\right)^3 * \frac{18}{38} \\
&= 0.121689 \\
\therefore P(A) &= \mathbf{0.122}
\end{aligned}$$

□

(b) The condition that she must at least play 9 times implies that she hasn't one any, won once, twice, three times, or four times. We must calculate these probabilities...

Proof. A = above situation....

$$\begin{aligned}
P(A) = &\left(\frac{20}{38}\right)^8 + \binom{8}{1} * \left(\frac{20}{38}\right)^7 * \left(\frac{18}{38}\right) \\
&+ \binom{8}{2} * \left(\frac{20}{38}\right)^6 * \left(\frac{18}{38}\right)^2 + \binom{8}{3} * \left(\frac{20}{38}\right)^6 * \left(\frac{18}{38}\right)^3 + \binom{8}{4} * \left(\frac{20}{38}\right)^4 * \left(\frac{18}{38}\right)^4 \\
= &\left(\frac{20}{38}\right)^8 + \frac{8}{1} * \left(\frac{20}{38}\right)^7 * \left(\frac{18}{38}\right) \\
&+ \frac{8 * 7}{2} * \left(\frac{20}{38}\right)^6 * \left(\frac{18}{38}\right)^2 + \frac{8 * 7 * 6}{3 * 2} * \left(\frac{20}{38}\right)^5 * \left(\frac{18}{38}\right)^3 + \frac{8 * 7 * 6 * 5}{4 * 3 * 2} * \left(\frac{20}{38}\right)^4 * \left(\frac{18}{38}\right)^4 \\
= &\ 0.6926 \\
\therefore P(A) = &\ \mathbf{0.693}
\end{aligned}$$

□

2.1 An extra problem on the binomial distribution

Toss a fair coin 100 times. The expected number of Heads is 50. Use a calculator to compute the probability that there are exactly 50 heads. Discuss the answer. Does it feel intuitive?

Proof. We can solve this with binomial distribution...

$$\begin{aligned} P(Event) &= \binom{100}{50} * \left(\frac{1}{2}\right)^{50} * \left(\frac{1}{2}\right)^{50} \\ &= \binom{100}{50} * \left(\frac{1}{2^{100}}\right) \end{aligned}$$

We can use normal approximation to find this number.

$$\begin{aligned} \mu &= n * p = 100 * \frac{1}{2} = 50 \\ \sigma &= \sqrt{np * (1-p)} = \sqrt{100 * \frac{1}{4}} = 5 \\ P(a \leq \#H \leq b) &\simeq \phi\left(\frac{b + \frac{1}{2} - \mu}{\sigma}\right) - \phi\left(\frac{a - \frac{1}{2} - \mu}{\sigma}\right) \\ P(50 \leq \#H \leq 50) &\simeq \phi\left(\frac{50 + \frac{1}{2} - 50}{5}\right) - \phi\left(\frac{50 - \frac{1}{2} - 50}{5}\right) \\ P(50 \leq \#H \leq 50) &\simeq \phi(0.1) - \phi(-0.1) \\ P(50 \leq \#H \leq 50) &\simeq \phi(0.1) - (1 - \phi(0.1)) \\ P(50 \leq \#H \leq 50) &\simeq 0.5398 - (1 - (0.5398)) \\ P(50 \leq \#H \leq 50) &\simeq 0.0796 \\ \therefore P(\text{Exactly 50 heads}) &= \mathbf{0.08} \end{aligned}$$

This does not seem intuitive since the probability of getting heads is $\frac{1}{2}$, you would assume around 50 heads would show up. It seems intuitive that $P(50Heads)$ will be the most common, but it is actually only **8**%

□

2.2 Normal Approximation Method

ALL Z SCORE VALUES WERE TAKEN TO THE NEAREST 0.01

2.2.1

(a) 400 tosses of a fair coin...

$$\mu = n * p = 400 * \frac{1}{2} = 200$$
$$\sigma = \sqrt{np * (1-p)} = \sqrt{400 * \frac{1}{4}} = 10$$
$$P(a \leq \#H \leq b) \simeq \phi\left(\frac{b + \frac{1}{2} - \mu}{\sigma}\right) - \phi\left(\frac{a - \frac{1}{2} - \mu}{\sigma}\right)$$
$$P(190 \leq \#H \leq 210) \simeq \phi\left(\frac{210 + \frac{1}{2} - 200}{10}\right) - \phi\left(\frac{190 - \frac{1}{2} - 200}{10}\right)$$
$$P(190 \leq \#H \leq 210) \simeq \phi\left(\frac{10.5}{10}\right) - \phi\left(\frac{-10.5}{10}\right)$$
$$P(190 \leq \#H \leq 210) \simeq \phi(1.05) - \phi(-1.05)$$
$$P(190 \leq \#H \leq 210) \simeq \phi(1.05) - (1 - \phi(1.05))$$
$$P(190 \leq \#H \leq 210) \simeq 0.85314 - (1 - (0.85314))$$
$$P(190 \leq \#H \leq 210) \simeq 0.70628$$
$$\therefore P(190 \leq \#H \leq 210) = \mathbf{0.7062}$$

(b) Problem **C** Sorry for the wrong numbering...

$$\mu = n * p = 400 * \frac{1}{2} = 200$$
$$\sigma = \sqrt{np * (1-p)} = \sqrt{400 * \frac{1}{4}} = 10$$
$$P(a \leq \#H \leq b) \simeq \phi\left(\frac{b + \frac{1}{2} - \mu}{\sigma}\right) - \phi\left(\frac{a - \frac{1}{2} - \mu}{\sigma}\right)$$
$$P(200 \leq \#H \leq 200) \simeq \phi\left(\frac{200 + \frac{1}{2} - 200}{10}\right) - \phi\left(\frac{200 - \frac{1}{2} - 200}{10}\right)$$
$$P(200 \leq \#H \leq 200) \simeq \phi\left(\frac{0.05}{10}\right) - \phi\left(\frac{-0.05}{10}\right)$$
$$P(200 \leq \#H \leq 200) \simeq \phi(0.05) - \phi(-0.05)$$
$$P(200 \leq \#H \leq 200) \simeq \phi(0.05) - (1 - \phi(0.05))$$
$$P(200 \leq \#H \leq 200) \simeq 0.51994 - (1 - (0.51994))$$
$$P(200 \leq \#H \leq 200) \simeq 0.03988$$
$$\therefore P(\#H = 200) = \mathbf{0.03988}$$

(c) Problem **D** Sorry for the wrong numbering...

$$\mu = n * p = 400 * \frac{1}{2} = 200$$
$$\sigma = \sqrt{np * (1-p)} = \sqrt{400 * \frac{1}{4}} = 10$$
$$P(a \leq \#H \leq b) \simeq \phi\left(\frac{b + \frac{1}{2} - \mu}{\sigma}\right) - \phi\left(\frac{a - \frac{1}{2} - \mu}{\sigma}\right)$$
$$P(210 \leq \#H \leq 210) \simeq \phi\left(\frac{210 + \frac{1}{2} - 200}{10}\right) - \phi\left(\frac{210 - \frac{1}{2} - 200}{10}\right)$$
$$P(210 \leq \#H \leq 210) \simeq \phi\left(\frac{10.5}{10}\right) - \phi\left(\frac{9.5}{10}\right)$$
$$P(200 \leq \#H \leq 200) \simeq \phi(1.05) - \phi(0.95)$$
$$P(200 \leq \#H \leq 200) \simeq 0.85314 - 0.82894$$
$$P(200 \leq \#H \leq 200) \simeq 0.0242$$
$$\therefore P(\#H = 200) = \mathbf{0.0242}$$

2.2.2

(a) 400 tosses of a $P(heads) = 0.51$ coin...

$$\mu = n * p = 400 * 0.51 = 204$$
$$\sigma = \sqrt{np * (1-p)} = \sqrt{400 * 0.51 * 0.49} = 9.998$$
$$P(a \leq \#H \leq b) \simeq \phi\left(\frac{b + \frac{1}{2} - \mu}{\sigma}\right) - \phi\left(\frac{a - \frac{1}{2} - \mu}{\sigma}\right)$$
$$P(190 \leq \#H \leq 210) \simeq \phi\left(\frac{210 + \frac{1}{2} - 204}{9.998}\right) - \phi\left(\frac{190 - \frac{1}{2} - 204}{9.998}\right)$$
$$P(190 \leq \#H \leq 210) \simeq \phi(0.6501) - \phi(-1.4502)$$
$$P(190 \leq \#H \leq 210) \simeq \phi(0.6501) - (1 - \phi(1.4502))$$
$$P(190 \leq \#H \leq 210) \simeq 0.74215 - (1 - (0.92647))$$
$$P(190 \leq \#H \leq 210) \simeq 0.66862$$
$$\therefore P(190 \leq \#H \leq 210) = \mathbf{0.66862}$$

(b) Problem **C** Sorry for the wrong numbering...

$$\begin{aligned}
\mu &= n * p = 400 * 0.51 = 204 \\
\sigma &= \sqrt{np * (1-p)} = \sqrt{400 * 0.51 * 0.49} = 9.998 \\
P(a \leq \#H \leq b) &\simeq \phi\left(\frac{b + \frac{1}{2} - \mu}{\sigma}\right) - \phi\left(\frac{a - \frac{1}{2} - \mu}{\sigma}\right) \\
P(200 \leq \#H \leq 200) &\simeq \phi\left(\frac{200 + \frac{1}{2} - 204}{9.998}\right) - \phi\left(\frac{200 - \frac{1}{2} - 204}{9.998}\right) \\
P(200 \leq \#H \leq 200) &\simeq \phi(-0.3501) - \phi(-0.4501) \\
P(200 \leq \#H \leq 200) &\simeq (1 - \phi(0.3501) - (1 - \phi(0.4501)) \\
P(200 \leq \#H \leq 200) &\simeq (1 - 0.63683) - (1 - 0.67364) \\
P(200 \leq \#H \leq 200) &\simeq 0.03681 \\
\therefore P(\#H = 200) &= \mathbf{0.03681}
\end{aligned}$$

(c) Problem **D** Sorry for the wrong numbering...

$$\begin{aligned}
\mu &= n * p = 400 * 0.51 = 204 \\
\sigma &= \sqrt{np * (1-p)} = \sqrt{400 * 0.51 * 0.49} = 9.998 \\
P(a \leq \#H \leq b) &\simeq \phi\left(\frac{b + \frac{1}{2} - \mu}{\sigma}\right) - \phi\left(\frac{a - \frac{1}{2} - \mu}{\sigma}\right) \\
P(210 \leq \#H \leq 210) &\simeq \phi\left(\frac{210 + \frac{1}{2} - 204}{9.998}\right) - \phi\left(\frac{210 - \frac{1}{2} - 204}{9.998}\right) \\
P(210 \leq \#H \leq 210) &\simeq \phi(0.6501) - \phi(0.55011) \\
P(210 \leq \#H \leq 210) &\simeq (0.74215) - (0.70884) \\
P(210 \leq \#H \leq 210) &\simeq 0.03331 \\
\therefore P(\#H = 210) &= \mathbf{0.03331}
\end{aligned}$$

2.2.8

(a) We would use normal approximation method for this as well...

$$\begin{aligned}
\mu &= n * p = 600 * \frac{1}{6} = 100 \\
\sigma &= \sqrt{np * (1-p)} = \sqrt{600 * \frac{1}{6} * \frac{5}{6}} = 9.128709 \\
P(a \leq \#H \leq b) &\simeq \phi\left(\frac{b + \frac{1}{2} - \mu}{\sigma}\right) - \phi\left(\frac{a - \frac{1}{2} - \mu}{\sigma}\right) \\
P(100 \leq \#H \leq 100) &\simeq \phi\left(\frac{100 + \frac{1}{2} - 100}{9.128709}\right) - \phi\left(\frac{100 - \frac{1}{2} - 100}{9.128709}\right) \\
P(100 \leq \#H \leq 100) &\simeq \phi(0.05477) - \phi(-0.05477) \\
P(100 \leq \#H \leq 100) &\simeq \phi(0.05477) - (1 - \phi(0.05477)) \\
P(100 \leq \#H \leq 100) &\simeq (0.51994) - (1 - 0.51994) \\
P(100 \leq \#H \leq 100) &\simeq 0.03988 \\
\therefore P(\#6 = 100) &= \mathbf{0.03988}
\end{aligned}$$