Study of Exponential Distribution and Comparison with Central Limit Theorem

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Overview:

In this study, we will investigate the exponential distribution in R by simulating 40 exponential number over a thousand times to investigat the distributions of the mean and variance. We will then compare the distribution with the Central Limit Theorem which will show that the distribution of averages of iid variables (properly normalised) becomes that of a standard normal as the sample size increases.

Simulations of a Exponential Distribution:

The exponential distribution is simulated in R using rexp(n, lambda) where lambda is the rate parameter which we will set it at 0.2 for all of the simulations here. In this study, we will investigate the distribution of mean and variance of 40 exponential numbers over a thousand times.

```r
rexp(40, 0.2)
```

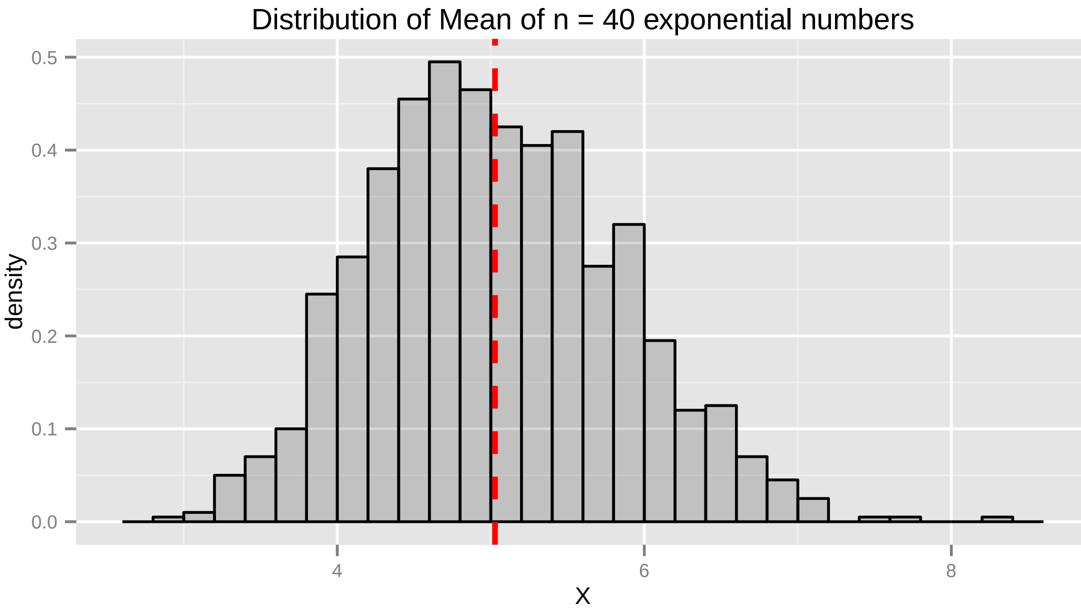
```
##  [1]  2.6272800  4.0679824  2.8323603 11.6767723  8.8191801  2.9080542
##  [7] 12.7376442  2.1711184  5.9229811 14.7014847  6.2239408 18.9114873
## [13]  3.2577490  3.8590943  1.5574598  6.8186312 11.0934631  5.1372332
## [19]  3.7019804  1.7208637 13.5733349  6.0588444  4.4093259 11.0919036
## [25]  1.8751483  9.7637612  5.1861778  1.1243914  6.3077169  7.3493274
## [31]  0.5658454  7.3258306  7.6986374  2.4842198  7.0377881  1.6275199
## [37]  1.0310851  7.2537141 13.0321657  0.1030723
```

Sample Mean versus Theoretical Mean:

The mean or expected value of an exponentially distributed random variable X with rate parameter lambda is given by

$$E[X] = \frac{1}{\lambda}$$

In our simulation, $\lambda = 0.2, E[X] = \frac{1}{0.2}$, the *Theoretical Mean* is **5**.



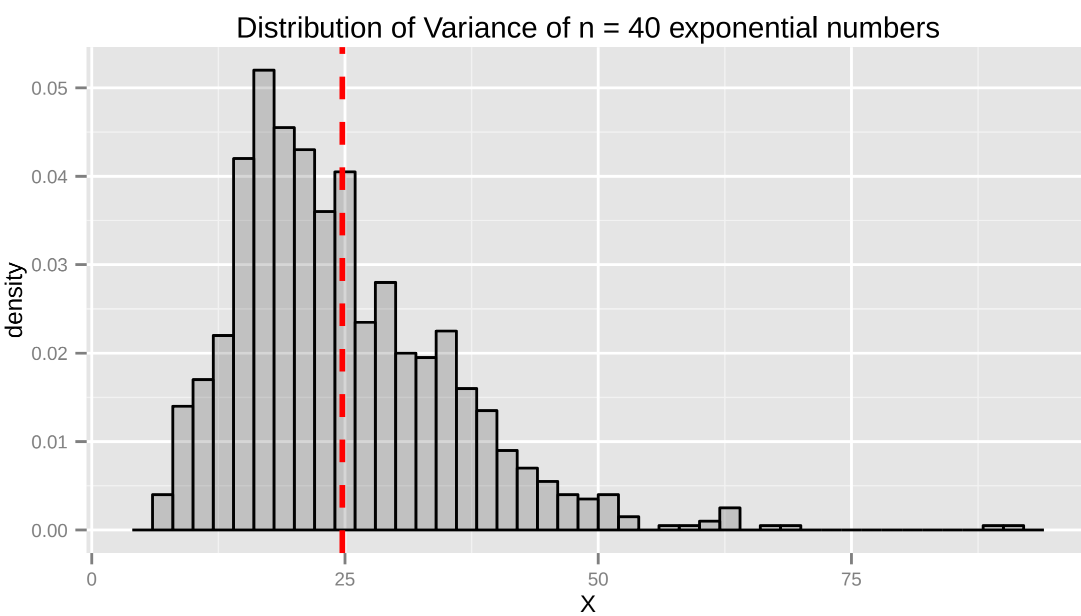
The *Sample Mean* is **5.0275**, which is very close to the *Theoretical Mean* of 5.

Sample Variance versus Theoretical Variance:

The variance of X is given by

$$Var[X] = \frac{1}{\lambda^2}$$

In our simulation, $\lambda = 0.2, Var[X] = \frac{1}{0.2^2}$, the *Theoretical Variance* is **25**.



The *Sample Variance* is **24.7386**, which is very close to the *Theoretical Variance* of 25.

Distribution:

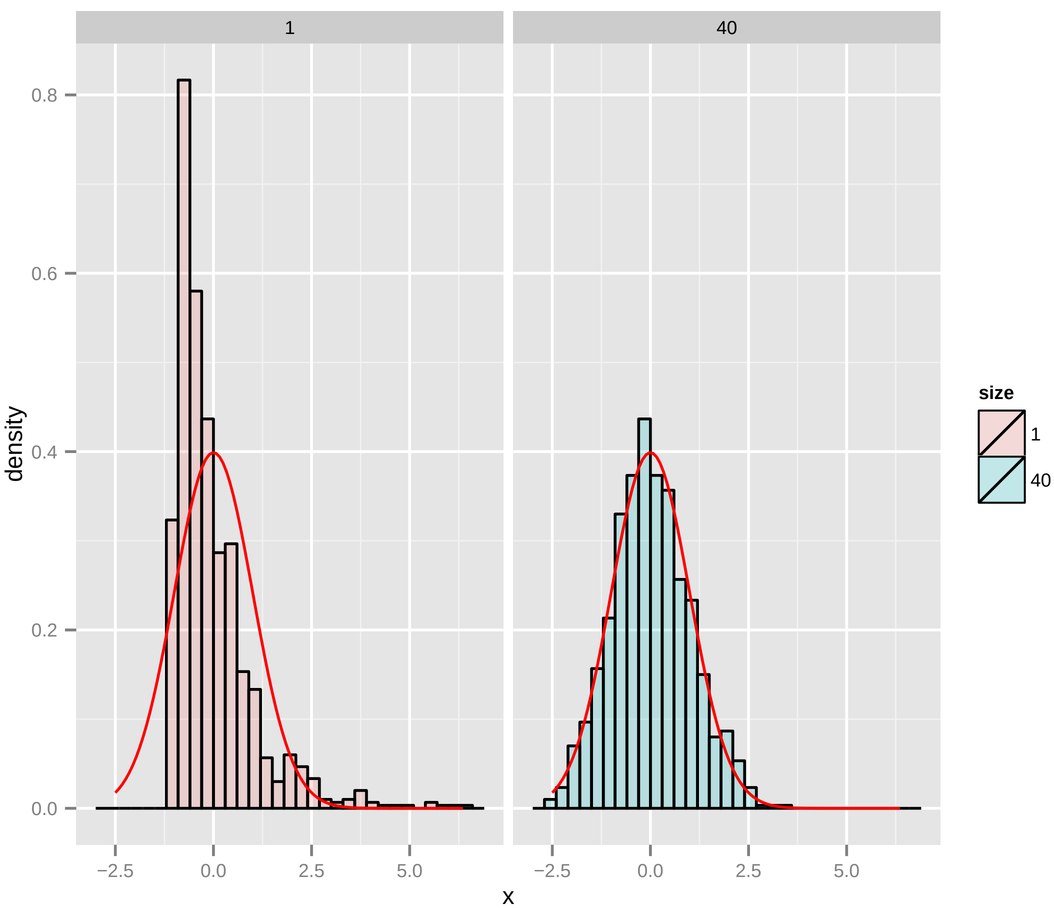
Central Limit Theorem:

- states that the distribution of averages of iid variables (properly normalised) becomes that of a standard normal as the sample size increases.

In our simulation,

- We will simulate a standard normal random exponential variable n.
- $\mu = E[X] = \frac{1}{\lambda} = 5$
- $Var(X) = \frac{1}{\lambda^2} = 25$
- Standard Error $= \sqrt{\frac{25}{n}} = \frac{5}{\sqrt{n}}$
- Let's simulate n exponential numbers, take their mean, subtract off 5, and divide by $\frac{5}{\sqrt{n}}$ and repeat this over for 1000 simulations.
- We will perform the simulations for $n = 1$ and 40

Results of our simulation of exponential distribution:



The graph shows that the distribution of the mean of iid variables (properly normalised) becomes that of a standard normal as the sample size increases.