Introduction to Quantum computing (From a layperson to programmer in 30 steps)

This is one of the greatest books for beginners in quantum computing, in my opinion, especially for those who might feel intimidated by the abundance of complicated online resources. The book's Analogies section, which simplified complicated topics for laypeople, was its strongest point. The majority of the book's issues are resolved below with little to no prior knowledge required, and repeated reads of the book made nearly every topic easier.

Note:- This book is solved according to my understanding. please email me for any comments or feedback.

Book link
https://link.springer.com/book/10.1007/978-3-030-98339-0

Chapter 1

Basic intro

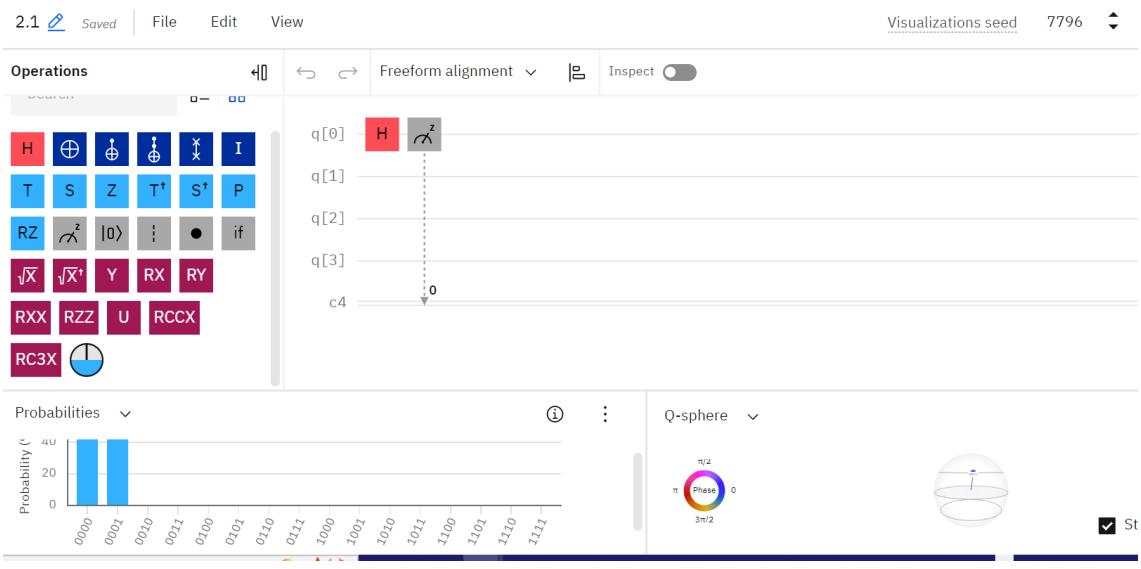
Code link

https://github.com/nithingovindugari/IntrotoQC

Chapter 2

2.1) Created IBM account

2.2)



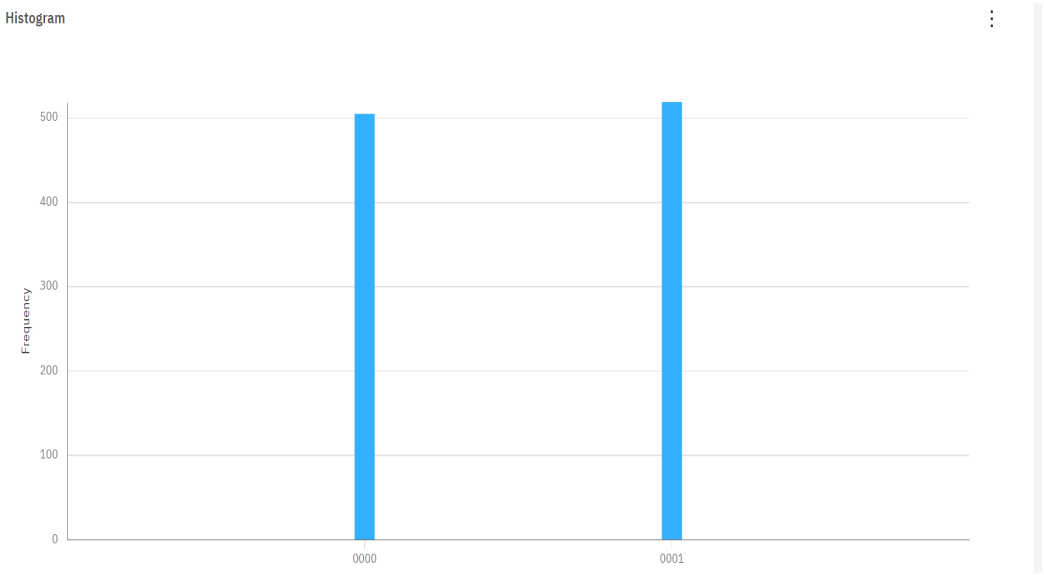
2.3)

Nithin Reddy Govindugari(nithinreddy1747@gmail.com)

Details

23m 57.8s
Total completion time

ibmq_manila
Backend



Nithin Reddy Govindugari(nithinreddy1747@gmail.com)

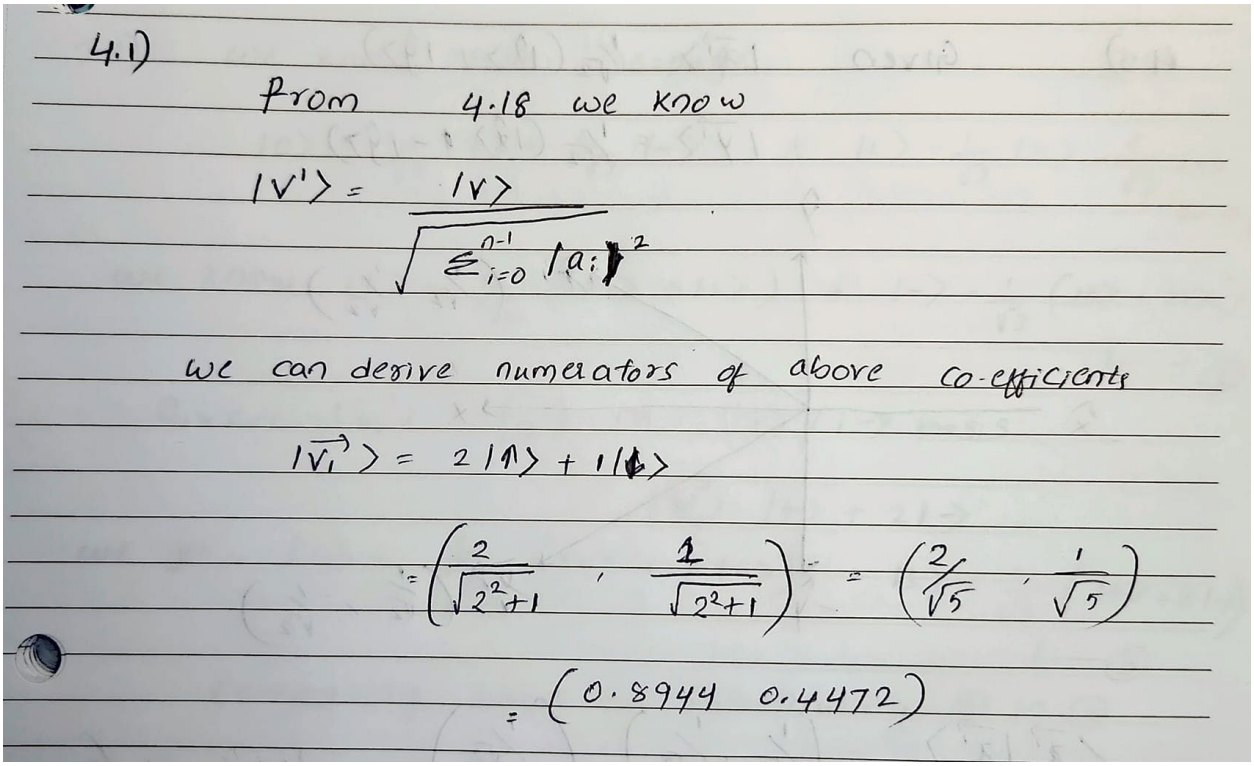
Chapter 3

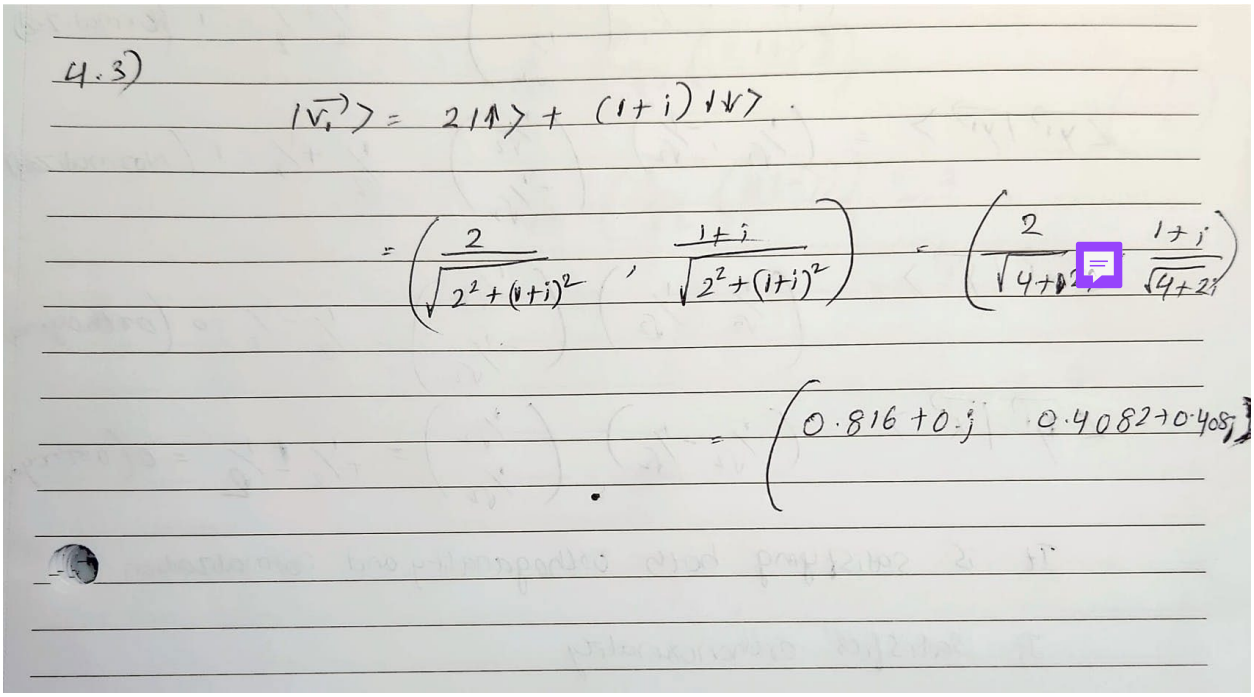
3.1) Had a google collab account already

3.2 & 3.3 & 3.4)

```
import numpy as np
from sklearn.preprocessing import normalize

#problem3
v1 = np.array([[np.sqrt(3)],[1]])
v2 = np.array([[1],[1]])
inner_result = np.vdot(v1,v2)
print(inner_result)
print(v1.shape)


v3 = np.array([[1j],[1]])
v4 = np.array([[-1j],[1]])
inner_result1 = np.vdot(v3,v4)
print(inner_result1)

v5 = np.array([[1+2j],[1]])
v6 = np.array([[1-1j],[1]])
inner_result2 = np.vdot(v5,v6)
print(inner_result2)
```

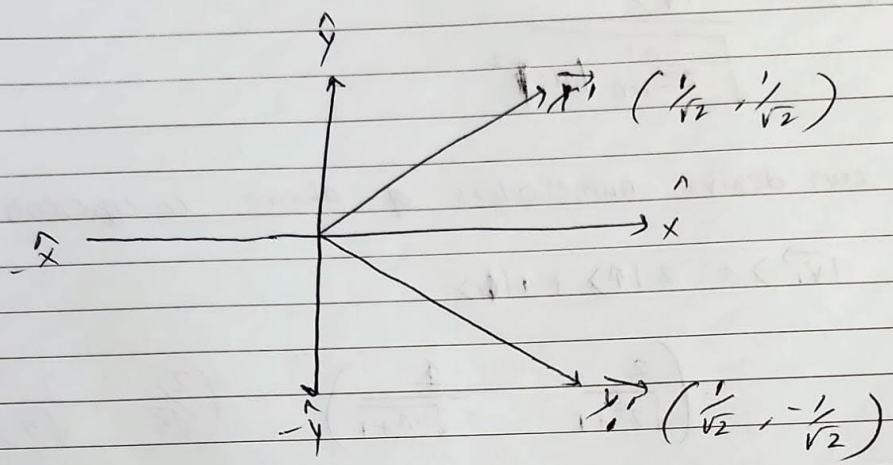
Chapter 4


4.1)
From 4.18 we know
$|V'\rangle = \frac{|V\rangle}{\sqrt{\sum_{i=0}^{n-1} |a_i|^2}}$
we can derive numerators of above co-efficients
$|\vec{v_1}\rangle = 2|\uparrow\rangle + 1|\downarrow\rangle$
$= \left(\frac{2}{\sqrt{2^2+1}}, \frac{1}{\sqrt{2^2+1}}\right) = \left(\frac{2}{\sqrt{5}}, \frac{1}{\sqrt{5}}\right)$
$= (0.8944 \quad 0.4472)$


4.3)
$|\vec{v_1}\rangle = 2|\uparrow\rangle + (1+i)|\downarrow\rangle$
$= \left(\frac{2}{\sqrt{2^2+(1+i)^2}}, \frac{1+i}{\sqrt{2^2+(1+i)^2}}\right)$
$= (0.816 + 0.j \quad 0.4082 + 0.408j)$

4.5) Given $|\vec{x'}\rangle = \frac{1}{\sqrt{2}}(|\hat{x}\rangle + |\hat{y}\rangle)$

$|\vec{y'}\rangle = \frac{1}{\sqrt{2}}(|\hat{x}\rangle - |\hat{y}\rangle)$



$\langle \vec{x'} | \vec{x'} \rangle = \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)\begin{pmatrix}\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}}\end{pmatrix} = \frac{1}{2} + \frac{1}{2} = 1$ (Normalized)

$\langle \vec{y'} | \vec{y'} \rangle = \left(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)\begin{pmatrix}\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}}\end{pmatrix} = \frac{1}{2} + \frac{1}{2} = 1$ (Normalized)

$\langle \vec{x'} | \vec{y'} \rangle = \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)\begin{pmatrix}\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}}\end{pmatrix} = \frac{1}{2} - \frac{1}{2} = 0$ (orthogonal)

$\langle \vec{y'} | \vec{x'} \rangle = \left(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)\begin{pmatrix}\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}}\end{pmatrix} = +\frac{1}{2} - \frac{1}{2} = 0$ (orthogonal)

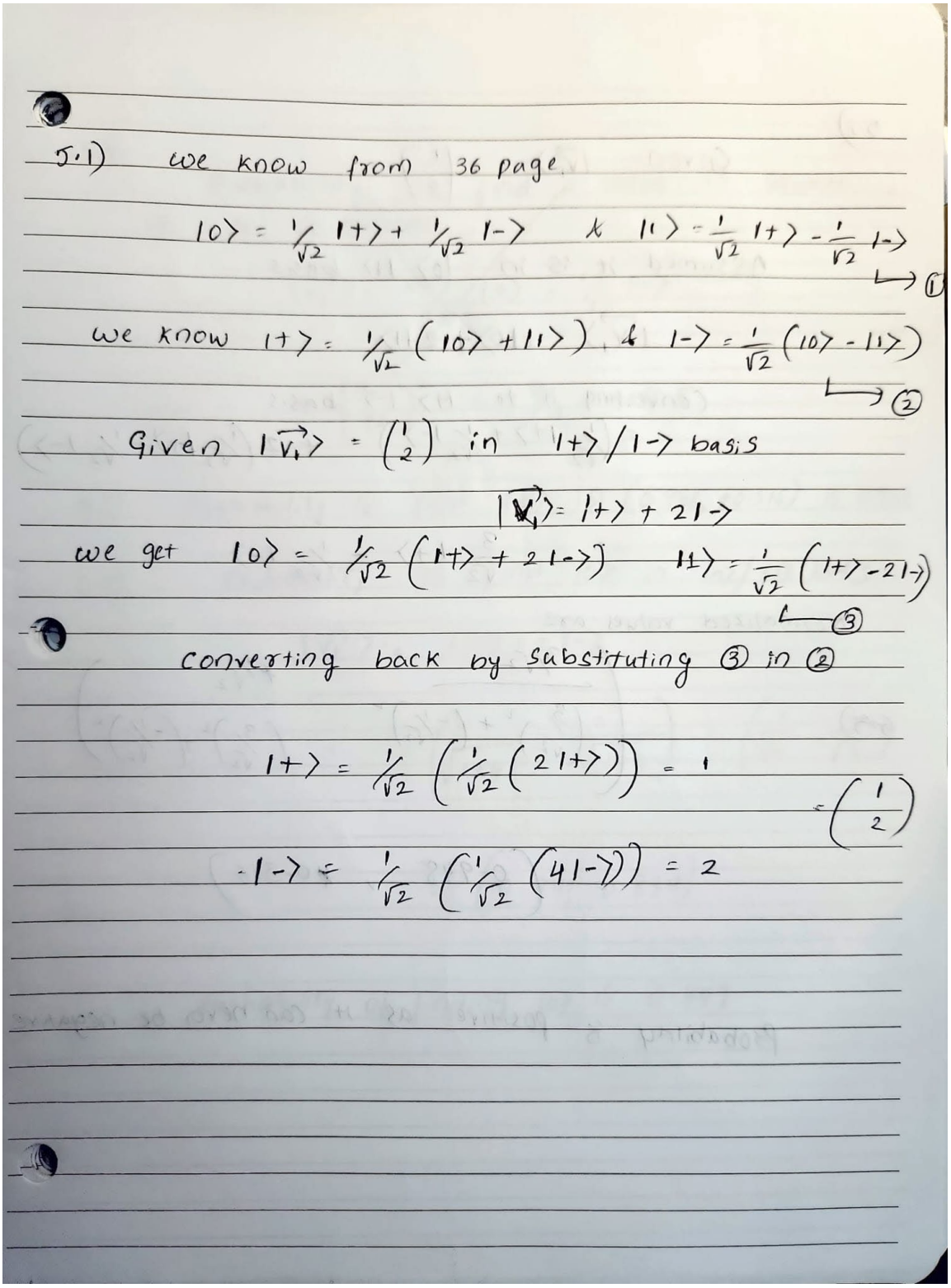
It is satisfying both orthoganality and normalization

It satisfies orthonormality

```
#Problem 4
v1 = np.array([[2],[1]])
print(v1)
v2 = v1/np.linalg.norm(v1)
print(v2)

v3 = np.array([[2],[1+1j]])
print(v3)
v4 = v3/np.linalg.norm(v3)
print(v4)
```

```
[[2]
 [1]]
[[0.89442719]
 [0.4472136 ]]
[[2.+0.j]
 [1.+1.j]]
[[0.81649658+0.j        ]
 [0.40824829+0.40824829j]]
```

Chapter 5



5.1) we know from 36 page.

$|0\rangle = \frac{1}{\sqrt{2}}|+\rangle + \frac{1}{\sqrt{2}}|-\rangle$ & $|1\rangle = \frac{1}{\sqrt{2}}|+\rangle - \frac{1}{\sqrt{2}}|-\rangle$ → (1)

we know $|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$ & $|-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$ → (2)

Given $|\vec{v_1}\rangle = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ in $|+\rangle / |-\rangle$ basis

$|\vec{v_1}\rangle = |+\rangle + 2|-\rangle$

we get $|0\rangle = \frac{1}{\sqrt{2}}(|+\rangle + 2|-\rangle)$ $|1\rangle = \frac{1}{\sqrt{2}}(|+\rangle - 2|-\rangle)$ → (3)

converting back by substituting (3) in (2)

$|+\rangle = \frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}(2|+\rangle)\right) = 1$

$= \begin{pmatrix} 1 \\ 2 \end{pmatrix}$

$|-\rangle = \frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}(4|-\rangle)\right) = 2$

5.2)

Given $|\vec{v_1}\rangle = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$

Assumed it is in $|0\rangle, |1\rangle$ basis

$$|\vec{v_1}\rangle = 1|0\rangle + 2|1\rangle$$

Converting it to $|+\rangle$ $|-\rangle$ basis

$$= \left(\frac{1}{\sqrt{2}}|+\rangle + \frac{1}{\sqrt{2}}|-\rangle\right) + 2\left(\frac{1}{\sqrt{2}}|+\rangle - \frac{1}{\sqrt{2}}|-\rangle\right)$$

$$= \frac{3}{\sqrt{2}}|+\rangle - \frac{1}{\sqrt{2}}|-\rangle$$

Normalized values are

5.3)

$$\left(\frac{3/\sqrt{2}}{\left(\frac{3}{\sqrt{2}}\right)^2 + \left(-\frac{1}{\sqrt{2}}\right)^2} \quad \frac{-1/\sqrt{2}}{\left(\frac{3}{\sqrt{2}}\right)^2 + \left(-\frac{1}{\sqrt{2}}\right)^2}\right)$$

$$= (0.948, -0.316)$$

Probability is positive as it can never be negative

5.3) Probability to find $|+\rangle$ in $(0.948, -0.316)$ is 0.948

5.4) Probability to find $|0\rangle$ in $|0\rangle / |1\rangle$ bases

$$|\vec{v_1}\rangle = 1|0\rangle + 2|-\rangle$$

$$= \left(\frac{1}{\sqrt{1+4}}, \frac{2}{\sqrt{1+4}}\right) = \left(\frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}}\right)$$

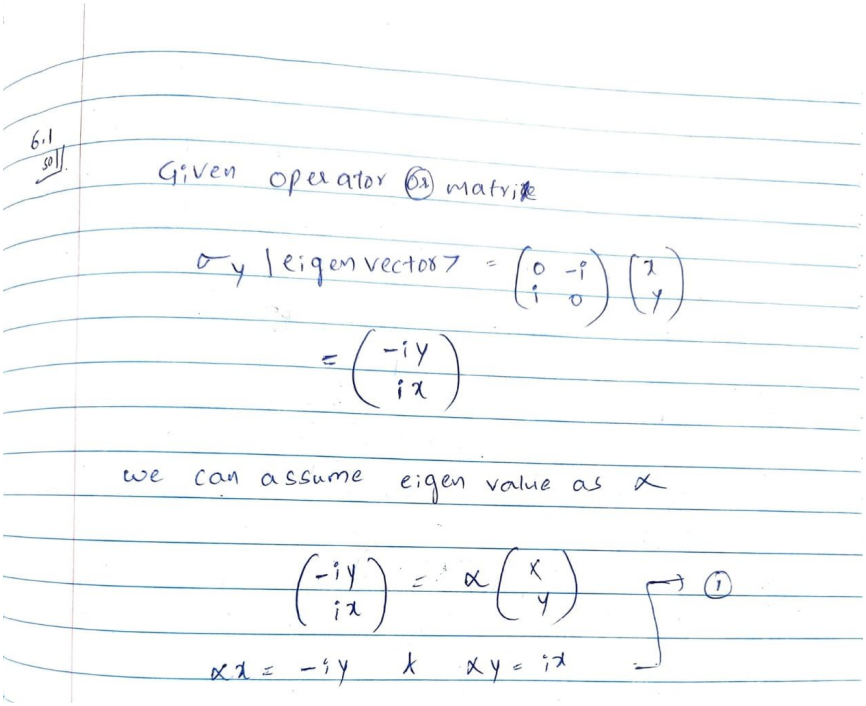
$$= (0.447, 0.894)$$

Probability of finding $|0\rangle$ is 0.447

```
#Problem 5.2
v1 = np.array([[3/np.sqrt(2)],[1/np.sqrt(2)]])
print(v1)
v2 = v1/np.linalg.norm(v1)
print(v2)

# Problem 5.4
v3 = np.array([[1] , [(2 )]])
print(v3)
v4 = v3/np.linalg.norm(v3)
print(v4)
```

```
[[2.12132034]
 [0.70710678]]
[[0.9486833 ]
 [0.31622777]]
[[1]
 [2]]
[[0.4472136 ]
 [0.89442719]]
```

Chapter 6



6.1
Sol:

Given operator (or) matrix

$$\sigma_y |\text{eigen vectors}\rangle = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

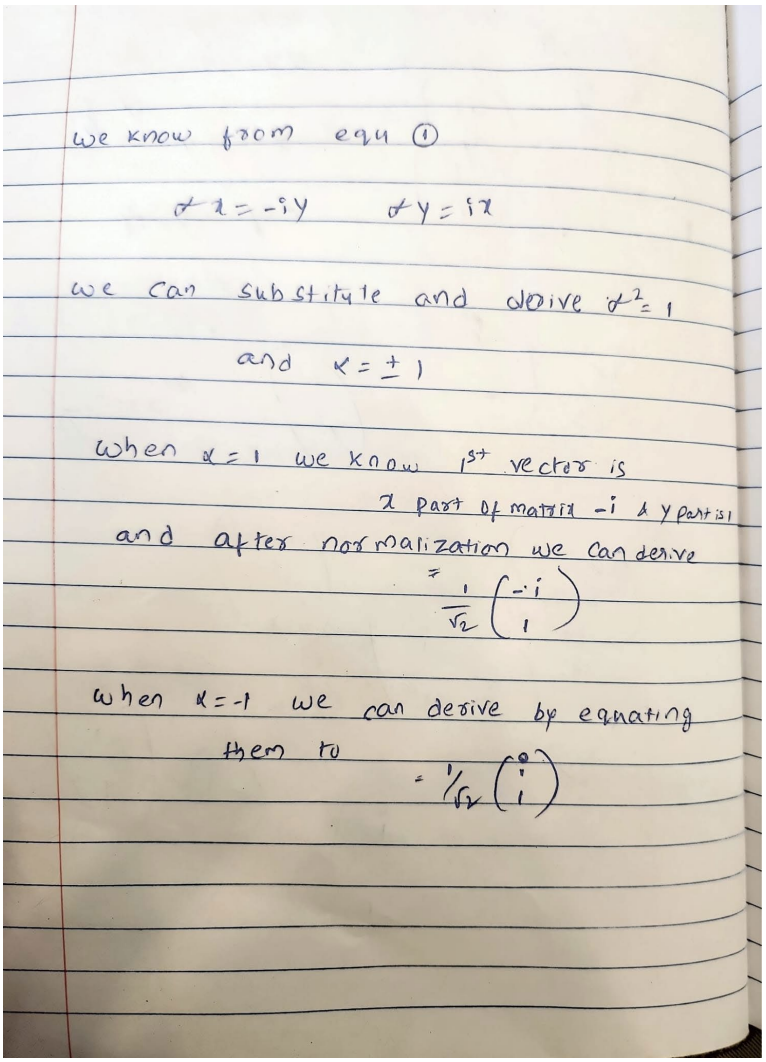
$$= \begin{pmatrix} -iy \\ ix \end{pmatrix}$$

we can assume eigen value as α

$$\begin{pmatrix} -iy \\ ix \end{pmatrix} = \alpha \begin{pmatrix} x \\ y \end{pmatrix} \rightarrow (1)$$

$\alpha x = -iy$ & $\alpha y = ix$

Introduction to Quantum computing (From a layperson to programmer in 30 steps)




```
#Lecture_6-Problems
import numpy as np
sigmay = np.array([[0, -1j],[1j,0]])
print(sigmay)
eigenvalues,eigenvectors = np.linalg.eig(sigmay)
print(eigenvalues)
print(eigenvectors)
```

```
[[ 0.+0.j -0.-1.j]
 [ 0.+1.j  0.+0.j]]
[ 1.+0.j -1.+0.j]
[[-0.         -0.70710678j  0.70710678+0.j        ]
 [ 0.70710678+0.j          0.         -0.70710678j]]
```

Chapter 7

Nithin Reddy Govindugari(nithinreddy1747@gmail.com)

Problems

7.1) $\sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$

To find $\sigma_y^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$

$= \begin{pmatrix} 0 - i^2 & 0 + 0 \\ 0 + 0 & -i^2 + 0 \end{pmatrix}$ $(\because i^2 = -1)$

$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I$

$\sigma_y^2 = I$

7.2) $\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

$\sigma_z^2 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

$= \begin{pmatrix} 1*1+0 & 0+0 \\ 0+0 & 0+1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I$

$\sigma_z^2 = I$

7.3) σ_y is Hermitian

We know $\sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$

Firstly conjugation

$$\sigma_y^* = \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix}$$

Transpose

$$\sigma_y^{*T} = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

$$\sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} = \sigma_y^{*T}$$

7.4) $\langle 0|\sigma_y$

$$\begin{pmatrix} 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} = \begin{bmatrix} 1\times 0 + 0\times i & 1\times -i + 0\times 0 \end{bmatrix}$$

$$\begin{pmatrix} 0 & -i \end{pmatrix}$$

It is in bra form and it is equal to $\sigma_y|0\rangle$ only if we do transpose and complex conjugate to $(\langle 0|\sigma_y)^*$ i.e, $(\langle 0|\sigma_y)^{T*} = \sigma_y|0\rangle$

```
#Lecture_7-Problems
#7.1
sigmay=np.array([[0,-1j],[1j,0]])
print(sigmay)
print(np.matmul(sigmay, sigmay))

#7.2
sigmaz=np.array([[1,0],[0,-1]])
print(sigmaz)
print(np.matmul(sigmaz, sigmaz))

#7.3

sigmayh = np.conjugate(np.transpose(sigmay))
print(sigmayh)

#7.4
zerospin=np.array([[1],[0]])
print(zerospin.shape)
print(np.dot(sigmay,zerospin))
```

```
[[ 0.+0.j -0.-1.j]
 [ 0.+1.j  0.+0.j]]
[[1.+0.j 0.+0.j]
 [0.+0.j 1.+0.j]]
[[ 1  0]
 [ 0 -1]]
[[1 0]
 [0 1]]
[[ 0.-0.j  0.-1.j]
 [-0.+1.j  0.-0.j]]
(2, 1)
[[0.+0.j]
 [0.+1.j]]
```


Nithin Reddy Govindugari(nithinreddy1747@gmail.com)

```
#Lecture_8-Problems

import numpy as np
sigmax=np.array([[0,1],[1,0]])
print(sigmax)
eigenvalues, eigenvectors=np.linalg.eig(sigmax)
print(eigenvalues)
print(eigenvectors)

sigmaxp=np.array([1/2,1/2])
sigmaxn=np.array([1/2,-1/2])
eigenvalues, eigenvectors=np.linalg.eig(sigmax)
print(eigenvalues)
print(eigenvectors)
print(np.dot(sigmaxp,sigmaxp))
print(np.dot(sigmaxn,sigmaxn))
```

```
[[ 0.+0.j -0.-1.j]
 [ 0.+1.j  0.+0.j]]
[[1.+0.j 0.+0.j]
 [0.+0.j 1.+0.j]]
[[ 1  0]
 [ 0 -1]]
[[1 0]
 [0 1]]
[[ 0.-0.j  0.-1.j]
 [-0.+1.j  0.-0.j]]
(2, 1)
[[0.+0.j]
 [0.+1.j]]
```

Nithin Reddy Govindugari(nithinreddy1747@gmail.com)

8.2) Given σ_z

we know $\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ it is ket notation

and even if find adjoint to get bra it is sa

so it is Hermitanian matrix

we know project operator is

$$P_{\sigma_z} = |\sigma_z\rangle\langle\sigma_z|$$

$$= \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$= \begin{pmatrix} 1+0 & 0+0 \\ 0+0 & 0+1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} =$$

$$I = P_{\sigma_z}$$

E.3)

Given

eigen vectors of σ_x

$|+\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix}$ $\quad |-\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\-1\end{pmatrix}$

we know for Probability

$$P_i = |i\rangle\langle i|$$

probability of $|+\rangle$

$$= \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix} \cdot \frac{1}{\sqrt{2}}(1 \; 1)$$

$$= \frac{1}{2}\begin{pmatrix}1 & 1\\1 & 1\end{pmatrix}$$

So it is 50%. & similarly for $|->$ is

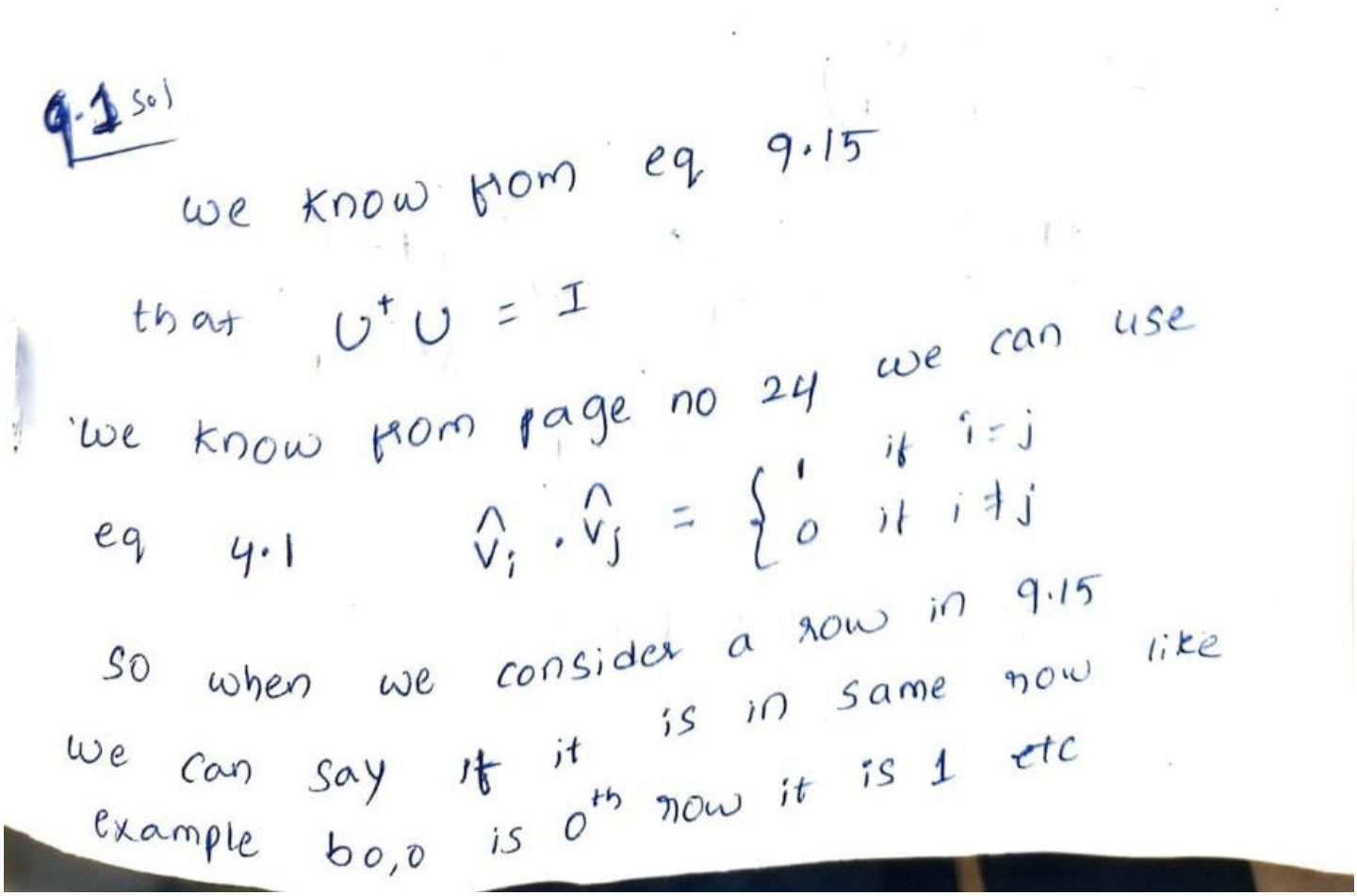
$$= \frac{1}{2}\begin{pmatrix}1 & -1\\-1 & 1\end{pmatrix}$$

```
#8.4
sigmaxp=np.array([1/np.sqrt(2),1/np.sqrt(2)])
sigmaxn=np.array([1/np.sqrt(2),-1/np.sqrt(2)])
eigenvalues, eigenvectors=np.linalg.eig(sigmax)
print(eigenvalues)
print(eigenvectors)
print(np.dot(sigmaxp,sigmaxp))
print(np.dot(sigmaxn,sigmaxn))
```

result

```
-
0.9999999999999998
0.9999999999999998
```


Nithin Reddy Govindugari(nithinreddy1747@gmail.com)



9.1 Sol

we know from eq 9.15

that $U^{\dagger}U = I$

we know from page no 24 we can use

eq 4.1 $\hat{v}_i \cdot \hat{v}_j = \begin{cases} 1 & \text{if } i = j \\ 0 & \text{if } i \neq j \end{cases}$

So when we consider a row in 9.15

we can say if it is in same row like

example $b_{0,0}$ is 0^{th} row it is 1 etc

9.2) Given 9.2 used it in (i) (ii) (iii)

$$\begin{vmatrix} a_{0,0}-\lambda_i & a_{0,1} \\ a_{1,0} & a_{1,1}-\lambda_i \end{vmatrix}$$

i) $\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

$$= \begin{vmatrix} 0-\lambda_i & 1 \\ 1 & 0-\lambda_i \end{vmatrix} = 0 \qquad +\lambda_i^2 - 1 = 0$$

$$\boxed{\lambda_i^2 = \pm 1}$$

ii) $\sigma_y = \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix}$

$$= \begin{vmatrix} 0-\lambda i & i \\ -i & 0-\lambda_i \end{vmatrix} = 0 \Rightarrow \lambda_i^2 - (i^2) = 0$$

$$\lambda^2 = -1$$

$$\boxed{\lambda = \pm 1}$$

iii) $\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

$$= \begin{vmatrix} 1-\lambda_i & 0 \\ 0 & -1-\lambda_i \end{vmatrix} = 0 \Rightarrow -(1-\lambda^2) = 0$$

$$\lambda^2 = 1$$

$$\boxed{\lambda = \pm 1}$$

eigen values are same for all

```
#Lecture_9-Problems
#9.3
# M=np.array([[1,0],[0,-1]])
sigmax=np.array([[0,1],[1,0]])
sigmay=np.array([[0,-1j],[1j,0]])
sigmaz=np.array([[1,0],[0,-1]])



print(np.linalg.det(sigmax))
print(np.linalg.det(sigmay))
print(np.linalg.det(sigmaz))
```

```
-1.0
(-1+0j)
-1.0
```

Nithin Reddy Govindugari(nithinreddy1747@gmail.com)

9.4)

For example we can take

A as $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ i.e, σ_x

we know pauli matrices are unitary

we can prove it now

$$|v_i\rangle = \frac{1}{\sqrt{2}}|\alpha\rangle + \frac{1}{\sqrt{2}}|\beta\rangle$$

$$|v_i\rangle = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{pmatrix}$$

$$A|v_i\rangle = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix}$$

we can prove orthonormality by

Adding squares of coefficients of transformed $A|v_i\rangle$ and other product with other matrices will be zero

$$\left(-\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2 = 1$$

10.i)

Transformation matrix from basis σ_y to σ_x

we know from problem 6.1 that eigen vectors of σ_y are $\frac{1}{\sqrt{2}}\begin{pmatrix}-i\\1\end{pmatrix}\begin{pmatrix}i\\1\end{pmatrix}$

$|+\rangle_{\sigma_x} = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix}$ & $|-\rangle_{\sigma_x} = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\-1\end{pmatrix}$

$$u = \langle new, y \rangle \langle old, x \rangle = \begin{bmatrix} \frac{1}{2}(-i\ i)^*\begin{pmatrix}1\\1\end{pmatrix} & \frac{1}{2}(-i,\ 1)^*\begin{pmatrix}1\\-1\end{pmatrix} \\ \frac{1}{2}(i\ 1)^+\begin{pmatrix}1\\1\end{pmatrix} & \frac{1}{2}(i\ 1)^+\begin{pmatrix}1\\-1\end{pmatrix} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{2}\begin{bmatrix} i+1 & i-1 \\ -i+1 & -i-1 \end{bmatrix} \end{bmatrix}$$

10.2) Given in Page 87

. complete basis is defined by outer product of each vectors sums to Identity matrix

Ide outer product is nothing but ket x bra

$$\sum_{i=0}^{n-1} |i\rangle\langle i| = |-\rangle\langle -| + |+\rangle\langle +|$$

By deriving

$$= \frac{1}{2}\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I$$

10.3) FOR σz we know

$$|-\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix} \qquad |+\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

we know from 10.19

$$\sigma x = \sum_{i=0}^{2-1} \lambda_i |i\rangle\langle i|$$

$$= 1\left(\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix} * \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \end{pmatrix}\right) + *$$

$$-1\left(\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix} \cdot \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & -1 \end{pmatrix}\right)$$

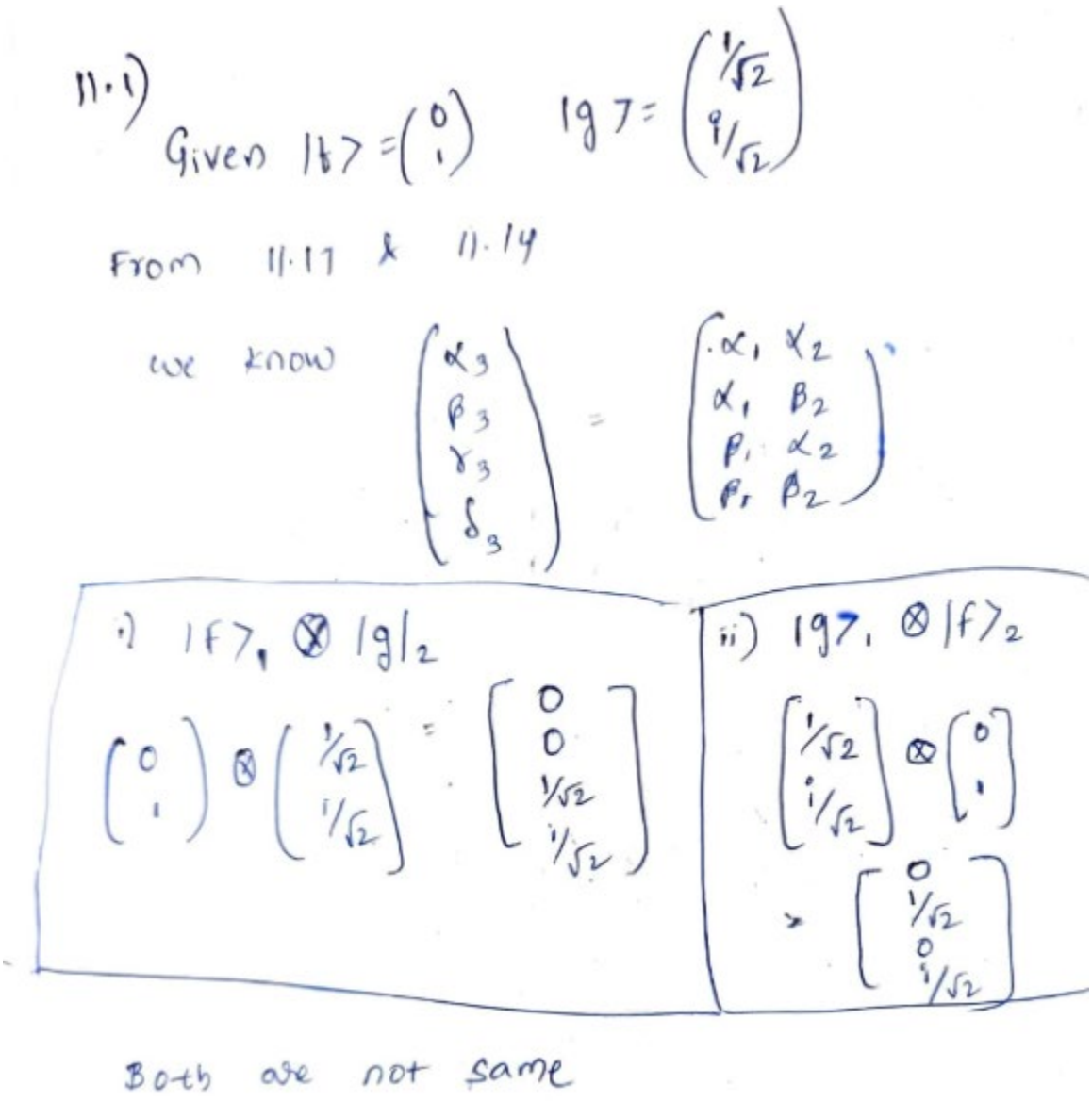
$$= \left(\frac{1}{2}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} - \left(\frac{1}{2}\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}\right.\right.$$

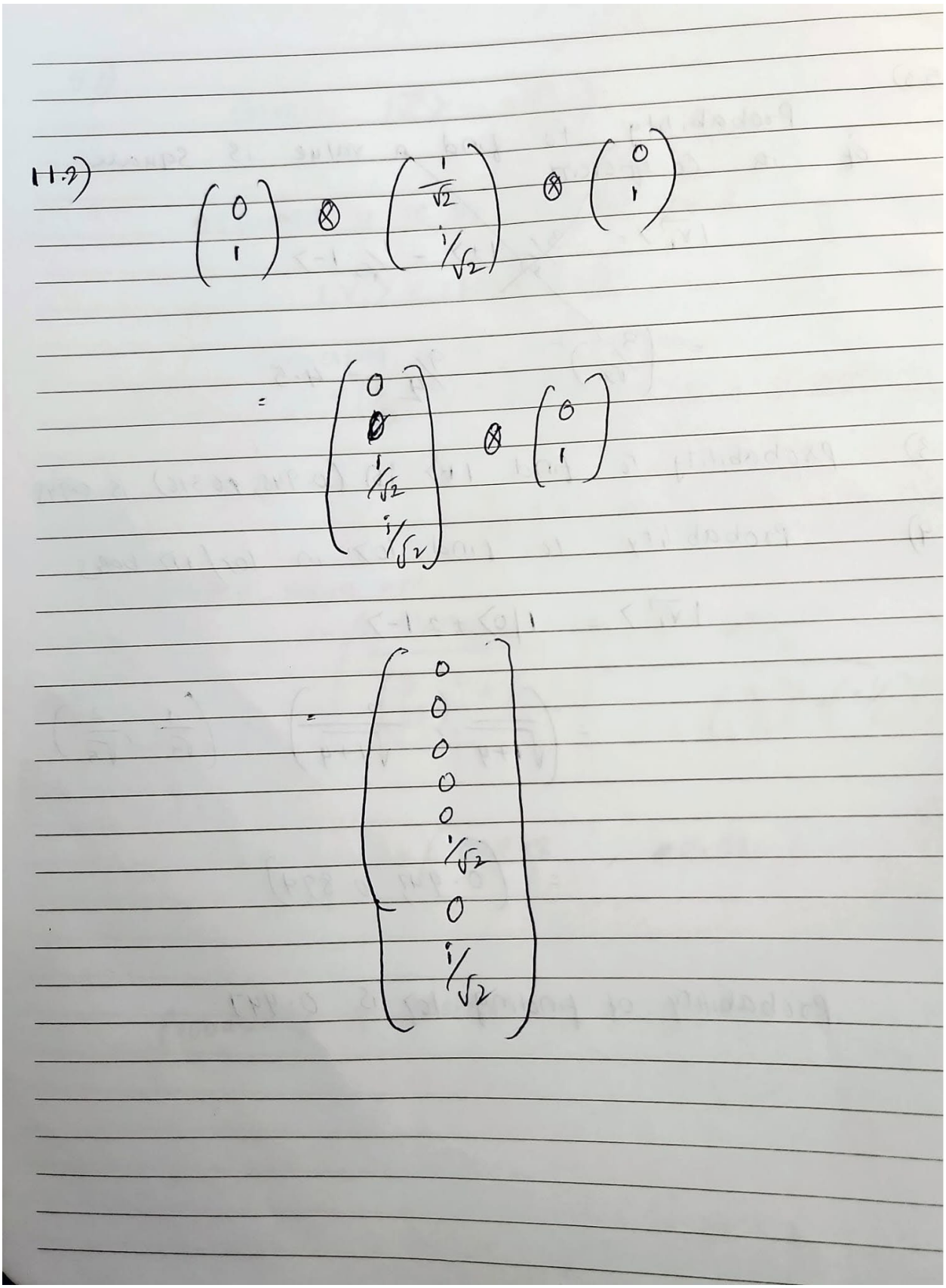
$$\sigma_x = \frac{1}{2}\begin{pmatrix} 0 & 2 \\ 2 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

```
#Lecture_10-Problems

#10.4

import numpy as np
sigmaxn=np.array([1/np.sqrt(2),-1])
sigmaxp=np.array([1/np.sqrt(2),1])
sigmayp=np.array([1/np.sqrt(2),+1j])
sigmayn=np.array([1/np.sqrt(2),-1j])

print(np.matmul(sigmaxn, sigmayn))
print(np.matmul(sigmaxn, sigmayp))
print(np.matmul(sigmaxp, sigmayn))
print(np.matmul(sigmaxp, sigmayp))
```

```
(0.4999999999999999+1j)
(0.4999999999999999-1j)
(0.4999999999999999-1j)
(0.4999999999999999+1j)
```

Chapter 11


11.1)
Given $|f\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ $|g\rangle = \begin{pmatrix} 1/\sqrt{2} \\ i/\sqrt{2} \end{pmatrix}$
From 11.17 & 11.14
we know $\begin{pmatrix} \alpha_3 \\ \beta_3 \\ \gamma_3 \\ \delta_3 \end{pmatrix} = \begin{pmatrix} \alpha_1 \gamma_2 \\ \alpha_1 \beta_2 \\ \beta_1 \alpha_2 \\ \beta_1 \beta_2 \end{pmatrix}$
i) $|f\rangle_1 \otimes |g\rangle_2$
$\begin{pmatrix} 0 \\ 1 \end{pmatrix} \otimes \begin{pmatrix} 1/\sqrt{2} \\ i/\sqrt{2} \end{pmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1/\sqrt{2} \\ i/\sqrt{2} \end{bmatrix}$
ii) $|g\rangle_1 \otimes |f\rangle_2$
$\begin{pmatrix} 1/\sqrt{2} \\ i/\sqrt{2} \end{pmatrix} \otimes \begin{pmatrix} 0 \\ 1 \end{pmatrix}$
$= \begin{bmatrix} 0 \\ 1/\sqrt{2} \\ 0 \\ i/\sqrt{2} \end{bmatrix}$
Both are not same



Nithin Reddy Govindugari(nithinreddy1747@gmail.com)

11.3) we need to prove distribution rule

11.20

$$(|\psi\rangle + |\varphi\rangle) \otimes |\eta\rangle = |\psi\rangle \otimes |\eta\rangle + |\varphi\rangle \otimes |\eta\rangle$$

$$\left(\begin{pmatrix}0\\1\end{pmatrix} + \begin{pmatrix}1\\0\end{pmatrix}\right) \otimes \begin{pmatrix}1/\sqrt{2}\\ i/\sqrt{2}\end{pmatrix}$$

$$= \begin{pmatrix}0\\1\end{pmatrix}\begin{pmatrix}1/\sqrt{2}\\ i/\sqrt{2}\end{pmatrix} + \begin{pmatrix}1\\0\end{pmatrix}\begin{pmatrix}1/\sqrt{2}\\ i/\sqrt{2}\end{pmatrix}$$

$$\begin{pmatrix}1/\sqrt{2}\\ i/\sqrt{2}\\ 1/\sqrt{2}\\ i/\sqrt{2}\end{pmatrix} = \begin{pmatrix}0\\ 0\\ 1/\sqrt{2}\\ i/\sqrt{2}\end{pmatrix} + \begin{pmatrix}1/\sqrt{2}\\ i/\sqrt{2}\\ 0\\ 0\end{pmatrix}$$

$$= \begin{pmatrix}1/\sqrt{2}\\ i/\sqrt{2}\\ 1/\sqrt{2}\\ i/\sqrt{2}\end{pmatrix} = \begin{pmatrix}1/\sqrt{2}\\ i/\sqrt{2}\\ 1/\sqrt{2}\\ i/\sqrt{2}\end{pmatrix}$$

L.H.S = R.H.S

11.4)

```
#Lecture_11-Problems

#11.4
#proving 11.1
import numpy as np
g=np.array([[0],[1]])
h=np.array([[1/np.sqrt(2)],[1j/np.sqrt(2)]])
print(np.kron(g,h))
print(np.kron(h,g))

# proving 11.2
x = np.kron(h,g)
print(np.kron(g,x))

#proving 11.3
y=np.array([[1],[0]])
z = (g + y)
print(np.kron(z,h))
a = np.kron(z,h)
```

```
l = np.kron(g,h)
m = np.kron(y,h)
b = l+m
print(l+m)

if  a.all() == b.all() :
  print(" L.H.S == R.H.S")
```

```
[[0.        +0.j        ]
 [0.        +0.j        ]
 [0.70710678+0.j        ]
 [0.        +0.70710678j]]
[[0.        +0.j        ]
 [0.70710678+0.j        ]
 [0.        +0.j        ]
 [0.        +0.70710678j]]
[[0.        +0.j        ]
 [0.        +0.j        ]
 [0.        +0.j        ]
 [0.        +0.j        ]
 [0.        +0.j        ]
 [0.70710678+0.j        ]
 [0.        +0.j        ]
 [0.        +0.70710678j]]
[[0.70710678+0.j        ]
 [0.        +0.70710678j]
 [0.70710678+0.j        ]
 [0.        +0.70710678j]]
[[0.70710678+0.j        ]
 [0.        +0.70710678j]
 [0.70710678+0.j        ]
 [0.        +0.70710678j]]
 L.H.S == R.H.S
```

Nithin Reddy Govindugari(nithinreddy1747@gmail.com)

11.5)

Given $n_1 = n_2 = 1$ & $|f\rangle = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$|e\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$, $|g\rangle = \begin{pmatrix} i \\ i \end{pmatrix}$ and $|k\rangle = \begin{pmatrix} i \\ 1 \end{pmatrix}$

eqy 11.21

$$\langle h_1 | h_2 \rangle = \left\langle \sum_{j=1}^{n_1} |f_j\rangle \otimes |g_j\rangle \middle| \sum_{i=1}^{n_2} |e_i\rangle \otimes |k_i\rangle \right\rangle$$

$$= \sum_{j=1}^{n_1} \sum_{i=1}^{n_2} \langle f_j | e_i \rangle \langle g_j | k_i \rangle$$

$\because n_1 = n_2 = 1$

L.H.S

$$= \left\langle \begin{pmatrix} 1 \\ 1 \end{pmatrix} \otimes \begin{pmatrix} i \\ i \end{pmatrix} \middle| \begin{pmatrix} 0 \\ 1 \end{pmatrix} \otimes \begin{pmatrix} i \\ 1 \end{pmatrix} \right\rangle$$

(∵ It is inner product)

$$= \begin{pmatrix} 1 & i & i & i \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ i \\ i \end{pmatrix} = i - 1$$

R.H.S

$$= \begin{pmatrix} 1 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & i \end{pmatrix} \begin{pmatrix} i \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} 0 + 1 \end{pmatrix} \begin{pmatrix} i - 1 \end{pmatrix}$$

$$= i - 1$$

12.1

Sol) Given $|f\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ and $|g\rangle = \begin{pmatrix} 1/\sqrt{2} \\ i/\sqrt{2} \end{pmatrix}$

Initially

$\sigma_x |f_0\rangle_1$

$= \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

$\sigma_x |g\rangle_2 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 1/\sqrt{2} \\ \frac{-i}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} i/\sqrt{2} \\ 1/\sqrt{2} \end{pmatrix}$

$\sigma_x |f\rangle_1 \otimes \sigma_x |g\rangle_2$

$= \begin{pmatrix} 1 \\ 0 \end{pmatrix} \begin{pmatrix} i/\sqrt{2} \\ 1/\sqrt{2} \end{pmatrix} = \begin{pmatrix} i/\sqrt{2} \\ 1/\sqrt{2} \\ 0 \\ 0 \end{pmatrix}$

Now to find $|f\rangle_1 \otimes |g\rangle_2$

$= \begin{pmatrix} 0 \\ 1 \end{pmatrix} \otimes \begin{pmatrix} 1/\sqrt{2} \\ i/\sqrt{2} \end{pmatrix} \Rightarrow \begin{pmatrix} 0 \\ 0 \\ 1/\sqrt{2} \\ i/\sqrt{2} \end{pmatrix}$

$$\sigma_x \otimes \sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}$$

$\sigma_x \otimes \sigma_x (|f_1\rangle \otimes |g|_2)$

$$= \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix}$$

$$= \begin{bmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \\ 0 \\ 0 \end{bmatrix}$$

So it is equal to $\sigma_x |f\rangle_1 \otimes \sigma_x |g\rangle_2$

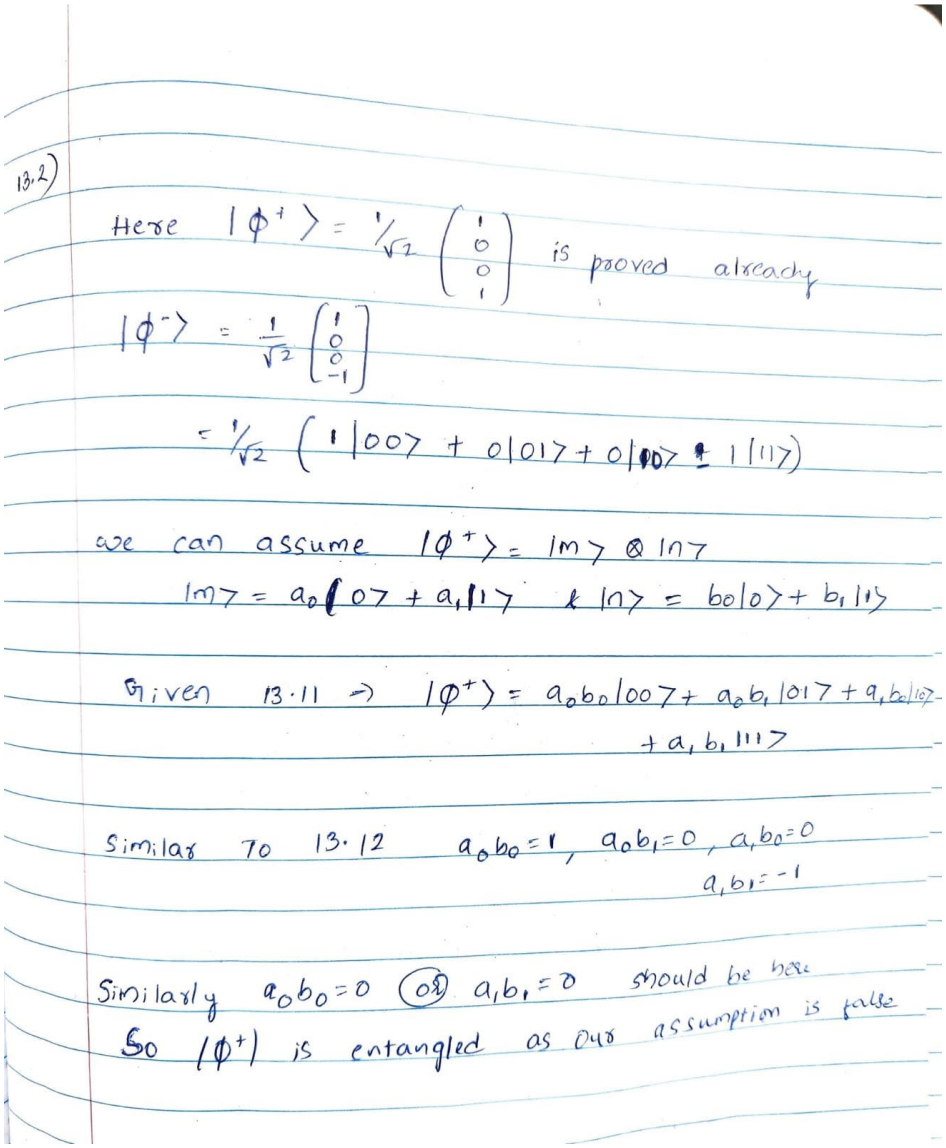
$\sigma_x |f\rangle_1 \otimes \sigma_x |g\rangle_2 = \sigma_x \otimes \sigma_x (|f_1\rangle \otimes |g|_2)$

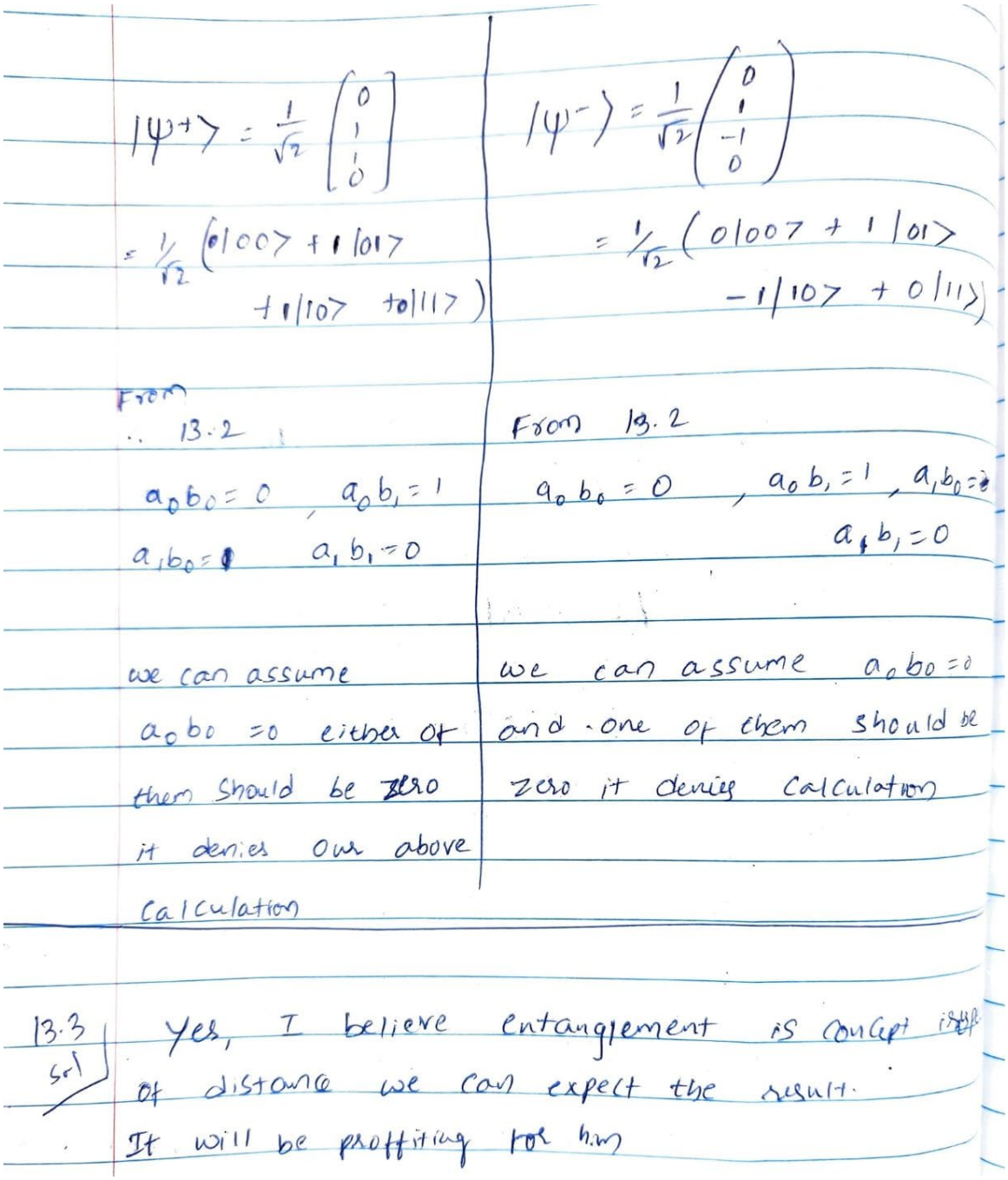
Chapter 13

13.1

As quantum computing is known for exponential speed and for quantum parallelism world can store up to $2^{10^{21}}$ quantum states due to superposition.

13.2)





13.4)

Here as mentioned basis states are orthonormal and if we have the same state in both two spaces then the inner product will be 1

Chapter 14

14.1

14.2

Given $\frac{1}{2}(|00\rangle + |01\rangle + |10\rangle + |11\rangle)$

From 14.12 $\Rightarrow$ $\frac{1}{2}|0\rangle\langle 0| + \frac{1}{2}|0\rangle\langle 1| + \frac{1}{2}|1\rangle\langle 0| + \frac{1}{2}|1\rangle\langle 1|$

$$\rho = \frac{1}{2}\begin{pmatrix}1\\0\end{pmatrix}(1\ 0) + \frac{1}{2}\begin{pmatrix}1\\0\end{pmatrix}(0\ 1) + \frac{1}{2}\begin{pmatrix}0\\1\end{pmatrix}(1\ 0) + \frac{1}{2}\begin{pmatrix}0\\1\end{pmatrix}(0\ 1)$$

$$= \frac{1}{2}\begin{pmatrix}1 & 0\\0 & 0\end{pmatrix} + \frac{1}{2}\begin{pmatrix}0 & 1\\0 & 0\end{pmatrix} + \frac{1}{2}\begin{pmatrix}0 & 0\\1 & 0\end{pmatrix} + \frac{1}{2}\begin{pmatrix}0 & 0\\0 & 1\end{pmatrix}$$

$$\rho = \frac{1}{2}\begin{pmatrix}1 & 1\\1 & 1\end{pmatrix}$$

As we know entaglement entropy is zero from previous chapter that we know our given state is not product of two subspaces and it should be zero

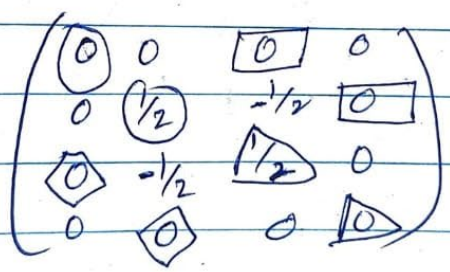
14.2)

Given $\rho = \frac{1}{2}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$ → 14.15

- We know $\rho_B = Tr_A(\rho)$

Figure 14.2 for tracing second qubit

$=$



Above same shape elements can be added

From above tracing B space

$=$

$$\begin{bmatrix} 0 & x & 0 & x \\ x & 1/2 & x & 0 \\ 0 & x & 1/2 & x \\ x & 0 & x & 0 \end{bmatrix}$$

15.1)

we know

From 15.6

$$U_{NOT} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

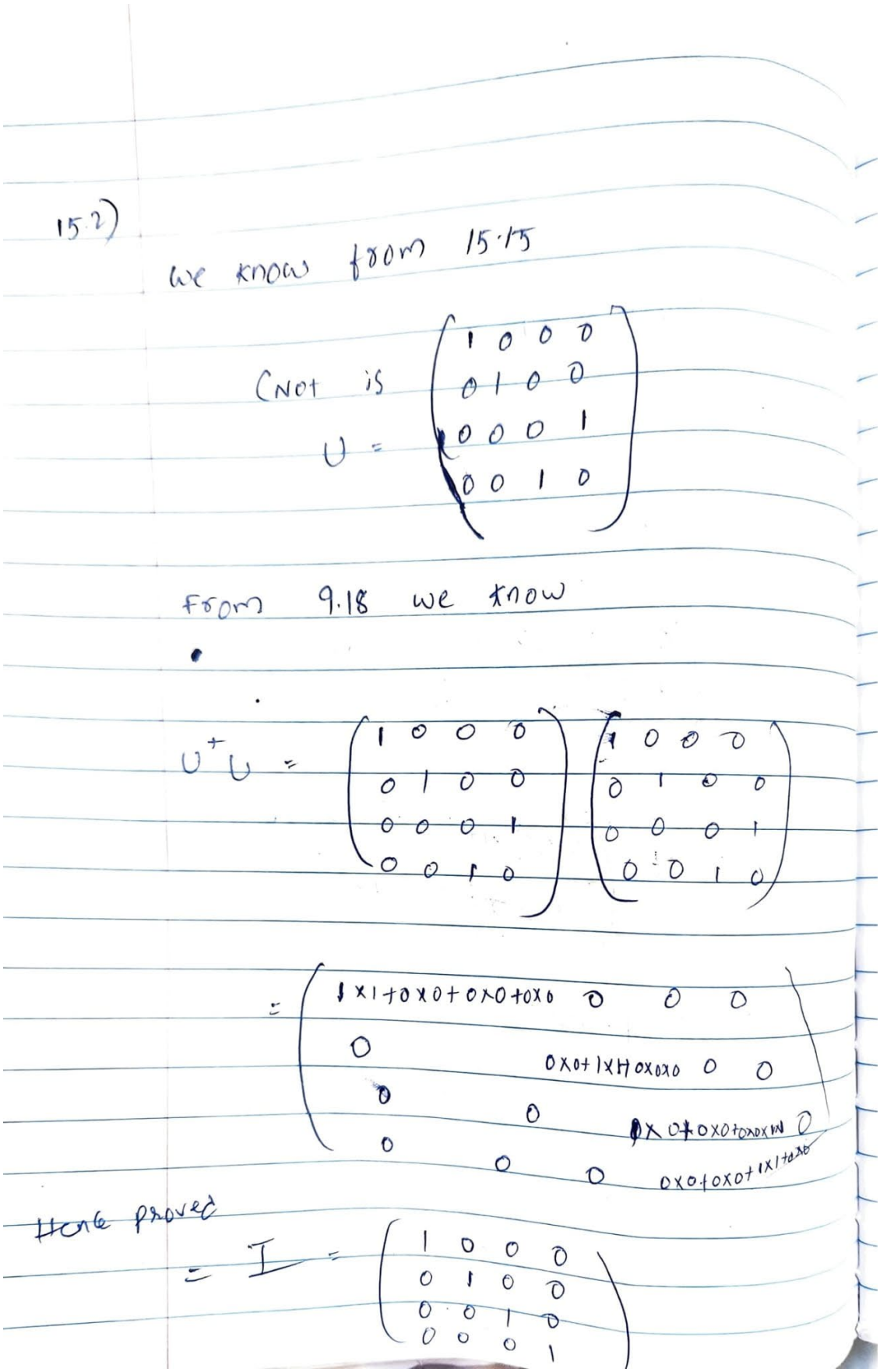
To prove it as unitary we know

$$U^{\dagger} \cdot U = I$$

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$ should be $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$$= \begin{pmatrix} 0\times0+1+1 & 0\times0+1\times0 \\ 1\times0+0\times1 & 1\times1+0\times0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

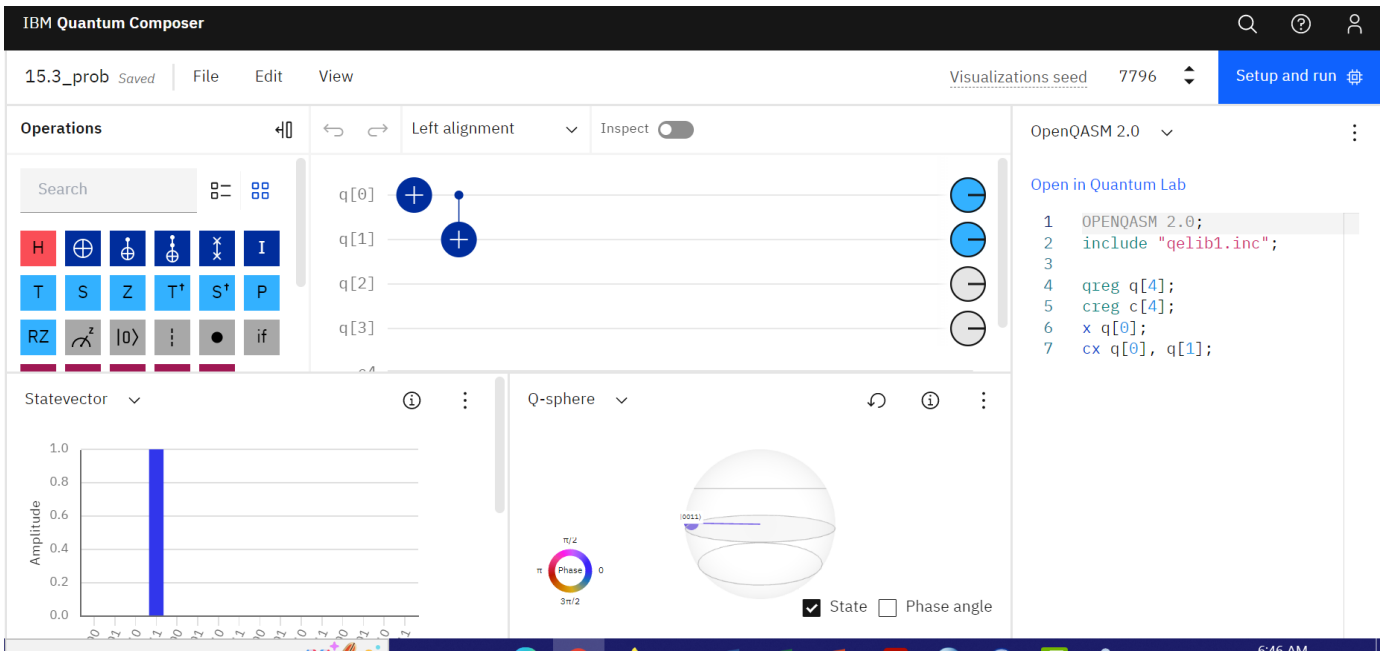
Hence proved


15.2)
We know from 15.15
CNot is
U =
From 9.18 we know
U⁺U =
Hence proved
= I =

```python
#Chapter 15

#15.1
import numpy as np
sigmay=np.array([[0,1],[1,0]])   #Not gate and transpose is same for this matrix
print(sigmay)
print(np.matmul(sigmay, sigmay))

#15.2
sigmac=np.array([[1,0,0,0],[0,1,0,0],[0,0,0,1],[0,0,1,0]])   #CNot gate and transpose is same for this matrix
print(sigmac)
print(np.matmul(sigmac, sigmac))
```

```
[[0 1]
 [1 0]]
[[1 0]
 [0 1]]
[[1 0 0 0]
 [0 1 0 0]
 [0 0 0 1]
 [0 0 1 0]]
[[1 0 0 0]
 [0 1 0 0]
 [0 0 1 0]
 [0 0 0 1]]
```

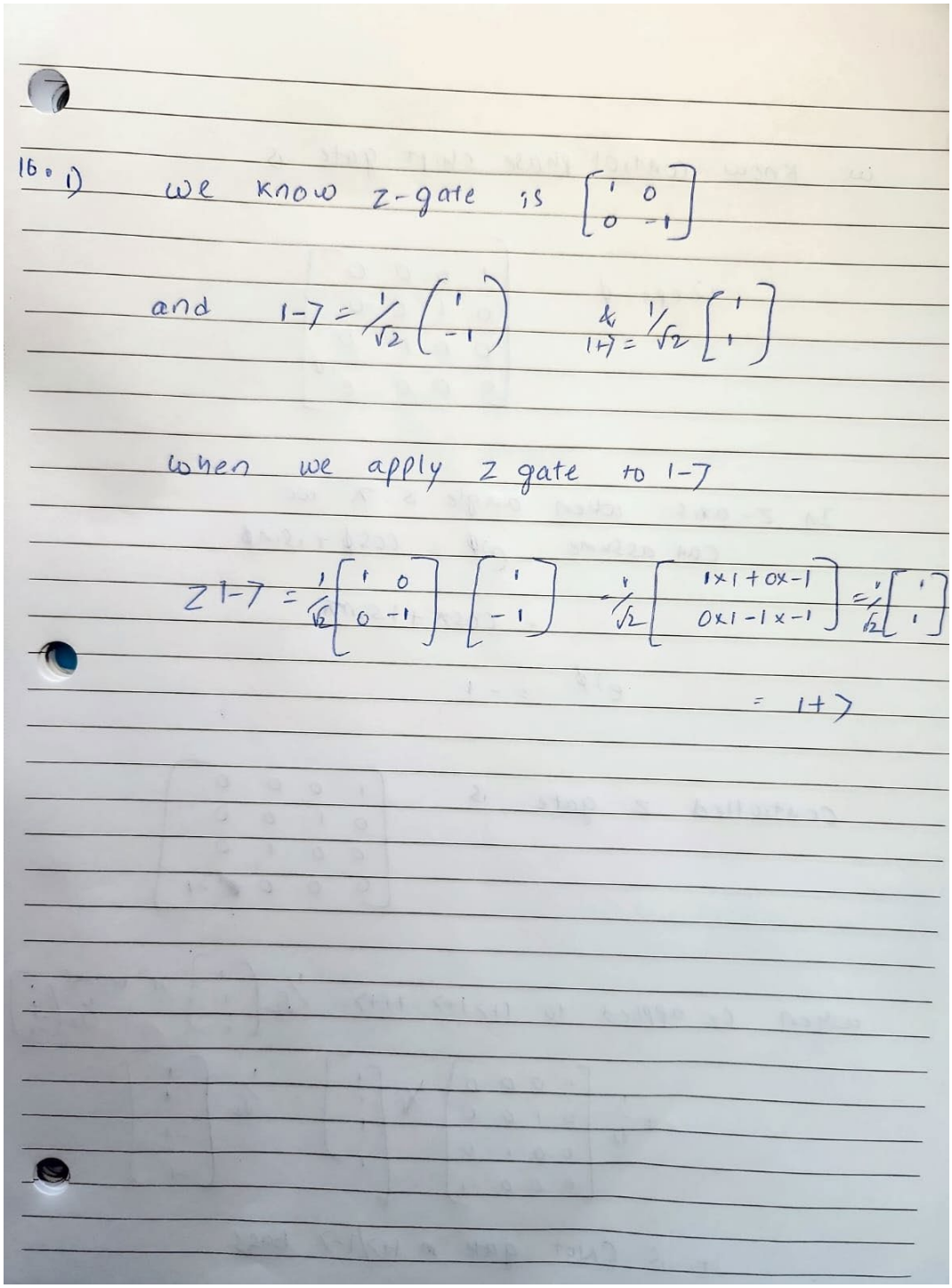
15.3)



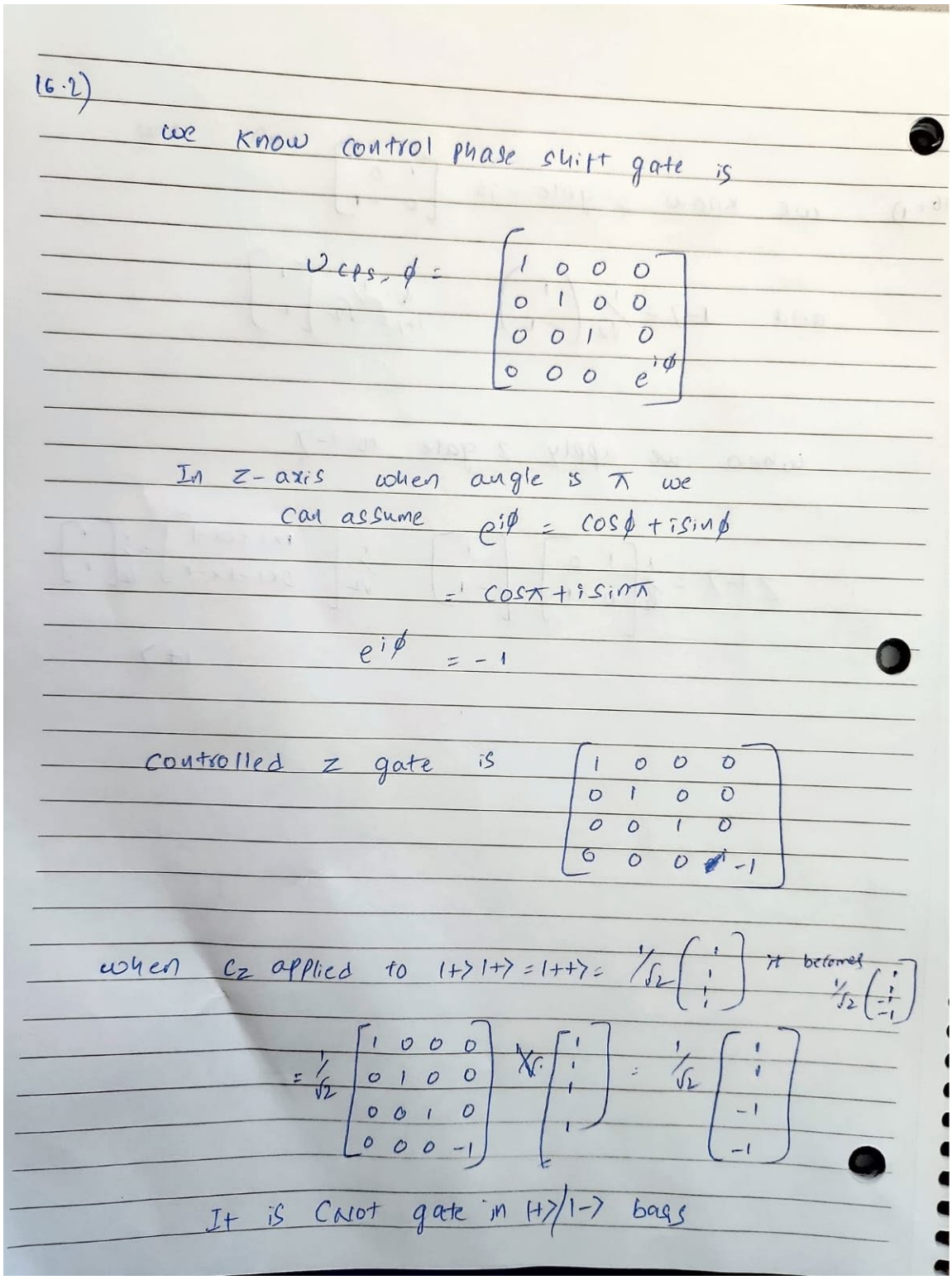
Chapter 16

16.1)

Nithin Reddy Govindugari(nithinreddy1747@gmail.com)



16.2)



16.2)

We know control phase shift gate is

$$U_{cps,\phi} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & e^{i\phi} \end{bmatrix}$$

In z-axis when angle is π we can assume $e^{i\phi} = \cos\phi + i\sin\phi$

$= \cos\pi + i\sin\pi$

$e^{i\phi} = -1$

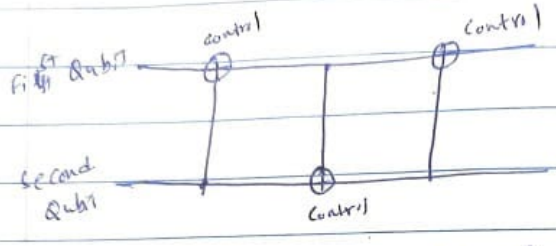
Controlled z gate is $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$

when C_Z applied to $|+\rangle|+\rangle = |++\rangle = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$ it becomes $\frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 1 \\ -1 \\ -1 \end{bmatrix}$

$$= \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \times \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 1 \\ -1 \\ -1 \end{bmatrix}$$

It is CNot gate in $|+\rangle|-\rangle$ bases

16.3) Here I have analyzed 3 CNOT gates usage in building swap gate



Here 1st Qubit converts (First Not gate)

1st CNot gate $CNot_1(|00\rangle = |00\rangle, |01\rangle = |01\rangle, |10\rangle = |11\rangle)$

control is with 2nd Qubit

2nd CNot gate $CNot_2(|00\rangle = |00\rangle, |01\rangle = |11\rangle, |10\rangle = |01\rangle)$

control is with 1st Qubit

3rd CNOT gate $CNot_3(|00\rangle = |00\rangle, |11\rangle = |10\rangle, |01\rangle = |01\rangle)$

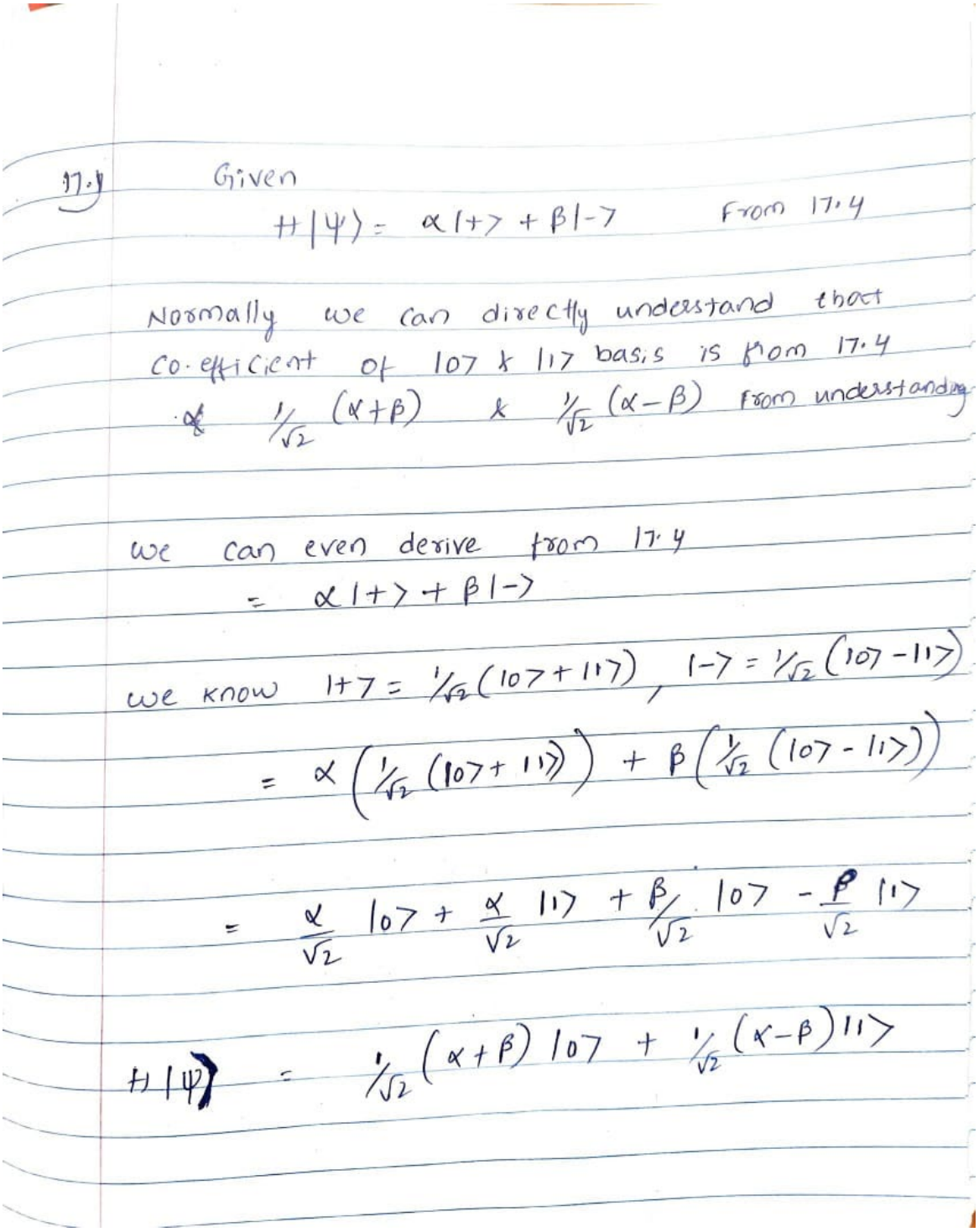
we can see swap is done and it is proved

For CNot 2 using state representation we can drive

$$CNot_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{matrix} |00\rangle \\ |11\rangle \\ |01\rangle \\ |10\rangle \end{matrix}$$

Chapter 17

17.1)



17.2)

17.2

Given $H^{\otimes n}|y\rangle = \frac{1}{2^{n/2}} \sum_{x=0}^{2^n-1} (-1)^{x \cdot y} |x\rangle$

Here n means n qubit Hadamard gate

$$H^{\otimes 2}|2\rangle = \frac{1}{2^{2/2}} \sum_{x=0}^{2^2-1} (-1)^{x \cdot 2} \cdot$$

$$= \frac{1}{2}\left((-1)^{0 \cdot 2}|0\rangle + (-1)^{1 \cdot 2}|1\rangle + (-1)^{2 \cdot 2}|2\rangle + (-1)^{3 \cdot 2}|3\rangle\right)$$

From 17.3 we have

$0 \cdot 2 = (0 \text{ AND } 1) \oplus (0 \text{ AND } 0) = 0 \oplus 0 = 0$

$1 \cdot 2 = (0 \text{ AND } 1) \oplus (1 \text{ AND } 0) = 0 \oplus 0 = 0$

$2 \cdot 2 = (1 \text{ AND } 1) \oplus (0 \text{ AND } 0) = 1 \oplus 0 = 1$

$3 \cdot 2 = (1 \text{ AND } 1) \oplus (1 \text{ AND } 0) = 1 \oplus 0 = 1$

$= H^{\otimes 2}|2\rangle = \frac{1}{2}(|0\rangle + |1\rangle - |2\rangle - |3\rangle)$

17.3)

17.3)

From 17.2 Example using 17.19

$$H^{\otimes 2}|2\rangle = H^{\otimes 2}|10\rangle$$

$$= (H|1\rangle) \otimes (H|0\rangle)$$

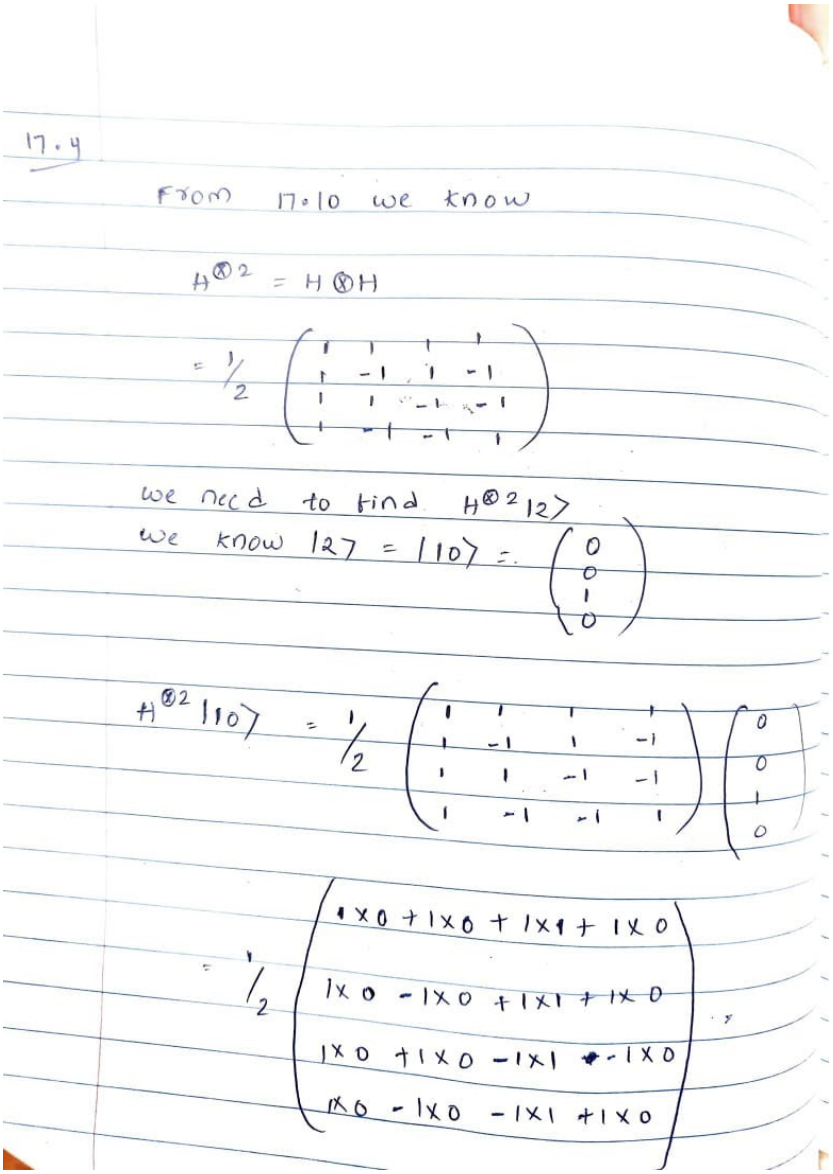
$$= \frac{1}{2^{2/2}} (|0\rangle + (-1)^1|1\rangle) \otimes (|0\rangle + (-1)^0|1\rangle)$$

$$= \frac{1}{2} (|0\rangle - |1\rangle) \otimes (|0\rangle + |1\rangle)$$

$$= \frac{1}{2} (|00\rangle + |01\rangle - |10\rangle - |11\rangle)$$

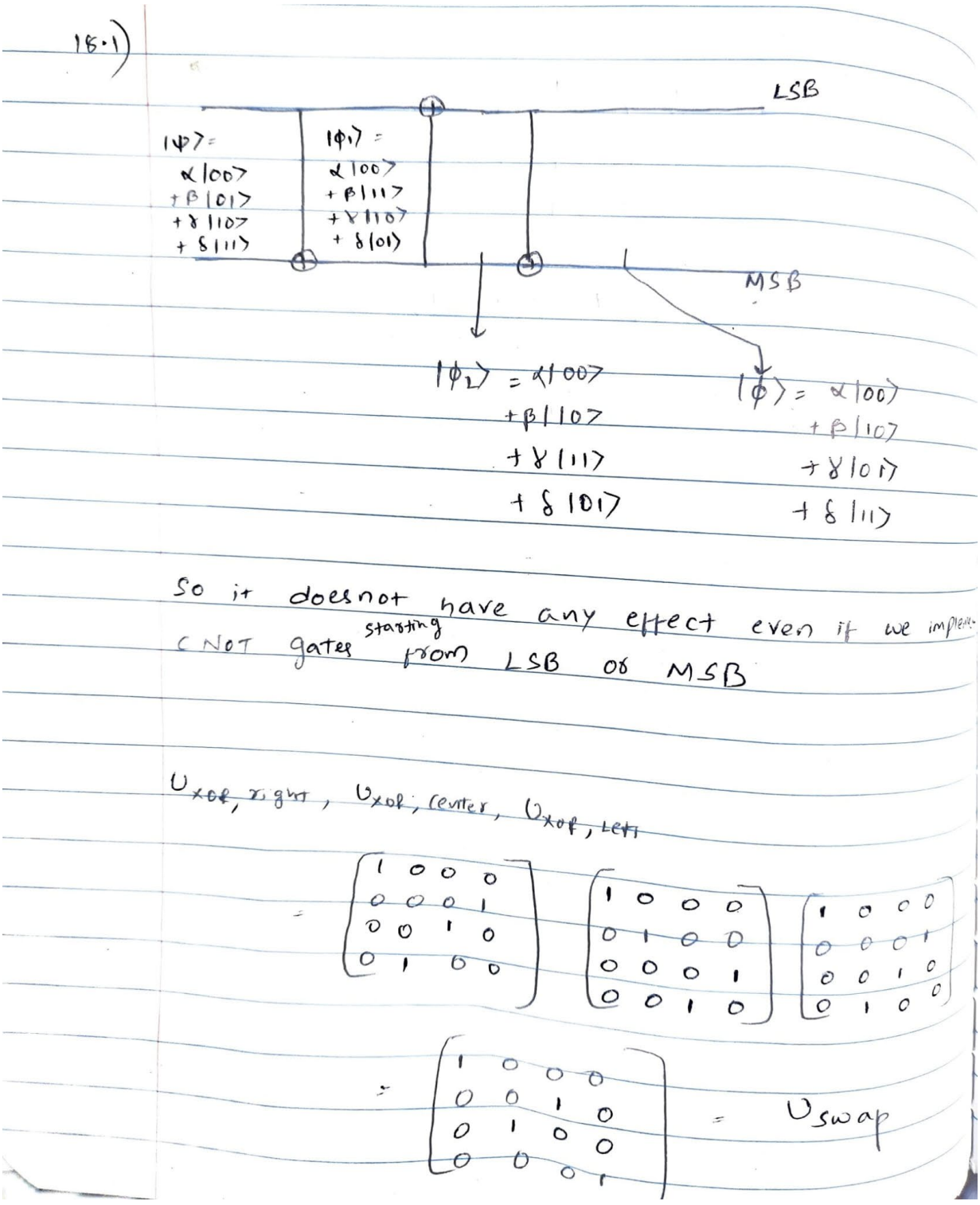
Here we proved same solution we achieved in previous problems

17.4)


17.4
From 17.10 we know
$H^{\otimes 2} = H \otimes H$
$= \frac{1}{2}\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{pmatrix}$
we need to find $H^{\otimes 2}|2\rangle$
we know $|2\rangle = |10\rangle = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}$
$H^{\otimes 2}|10\rangle = \frac{1}{2}\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{pmatrix}\begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}$
$= \frac{1}{2}\begin{pmatrix} 1\times0+1\times0+1\times1+1\times0 \\ 1\times0-1\times0+1\times1+1\times0 \\ 1\times0+1\times0-1\times1-1\times0 \\ 1\times0-1\times0-1\times1+1\times0 \end{pmatrix}$
$= \frac{1}{2}\begin{pmatrix} 1 \\ 1 \\ -1 \\ -1 \end{pmatrix}$
$= \frac{1}{2}(|00\rangle + |01\rangle - |10\rangle - |11\rangle)$
Hence proved

Chapter 18

18.1)



18.2)

18.2)

Given LSB as controlling bit From 15.13 we can derive

$U_{XOR}|00\rangle = |0\oplus 0, 0\rangle = |0,0\rangle = |00\rangle$

$U_{XOR}|01\rangle = |0\oplus 1, 01\rangle = |1,1\rangle = |11\rangle$

$U_{XOR}|10\rangle = |1\oplus 0, 10\rangle = |1,0\rangle = |10\rangle$

$U_{XOR}|11\rangle = |1\oplus 1, 11\rangle = |0,1\rangle = |01\rangle$

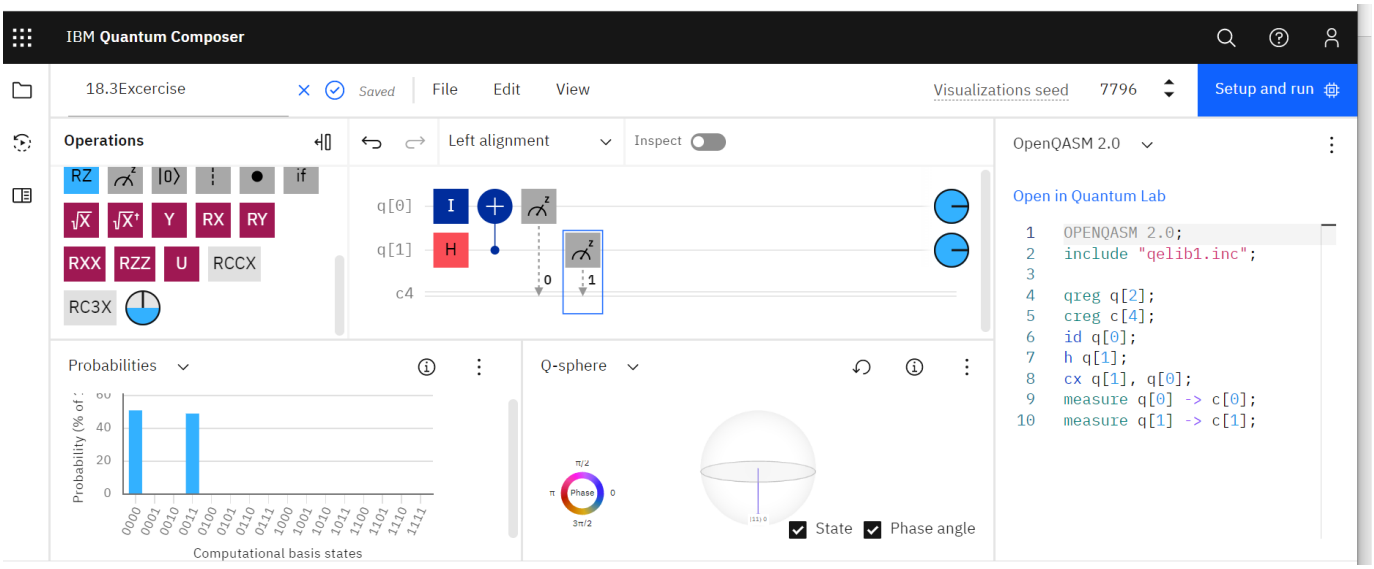
we can derive matrix as

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

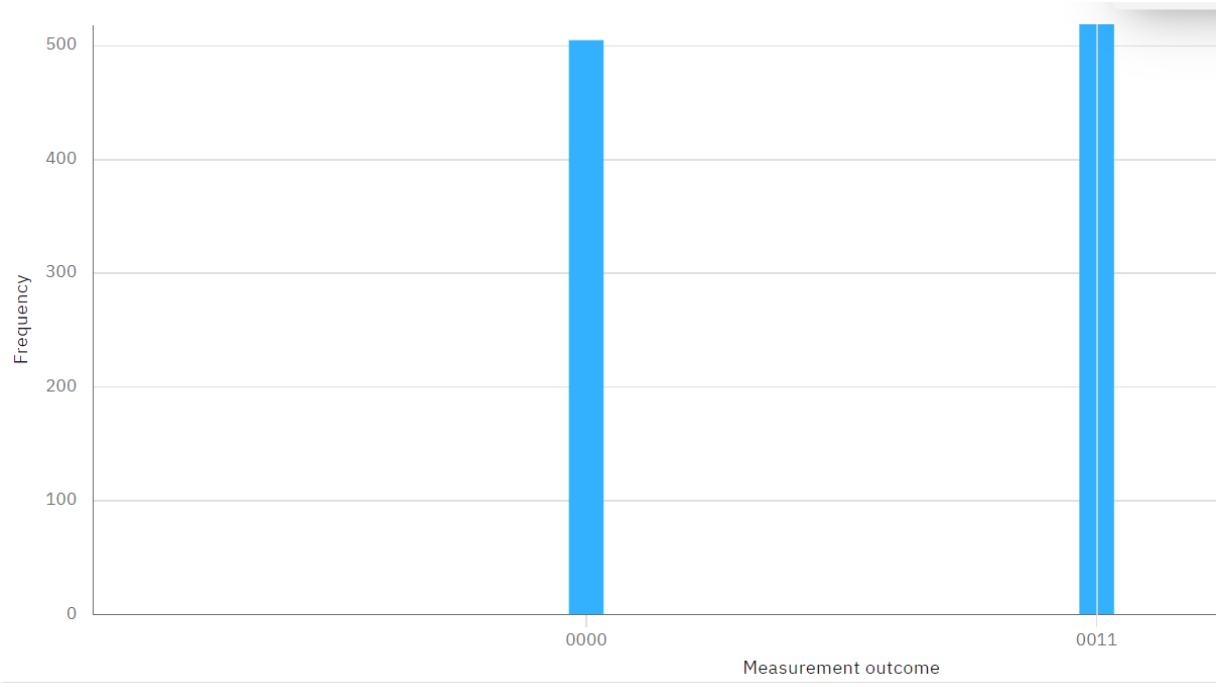
It matches our assumed value in previous problem

Introduction to Quantum computing (From a layperson to programmer in 30 steps)

18.3)



Simulation output



Nithin Reddy Govindugari(nithinreddy1747@gmail.com)

Hardware execution result

Details

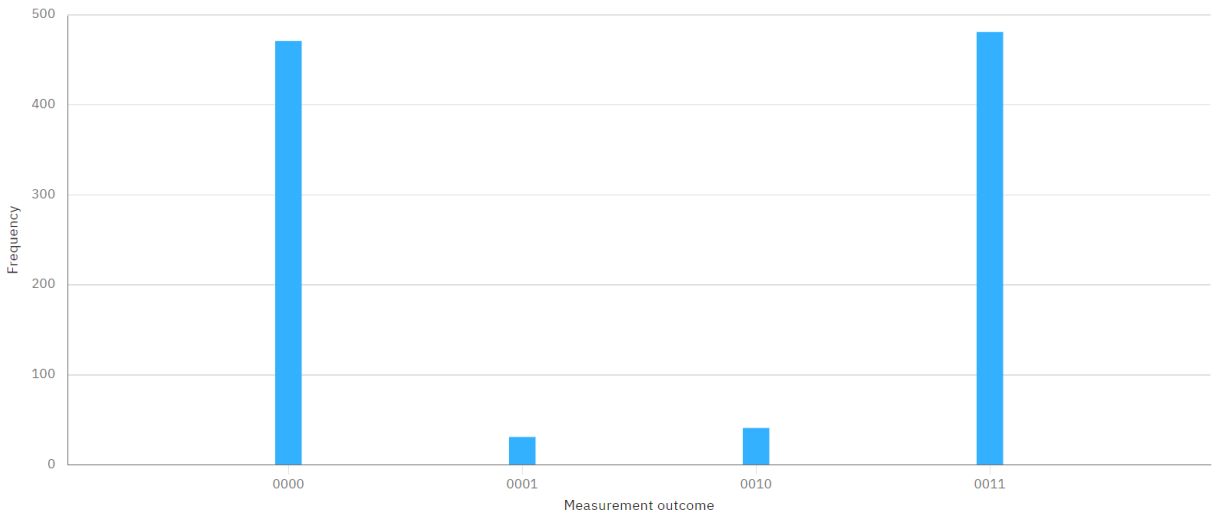
36m 30.5s
Total completion time

ibmq_manila
Backend

Sent from	Untitled circuit
Created on	Sep 07, 2022 11:50 AM
Sent to queue	Sep 07, 2022 11:50 AM
Provider	ibm-q/open/main
Run mode	fairshare
# of shots	1024
# of circuits	1

Status Timeline

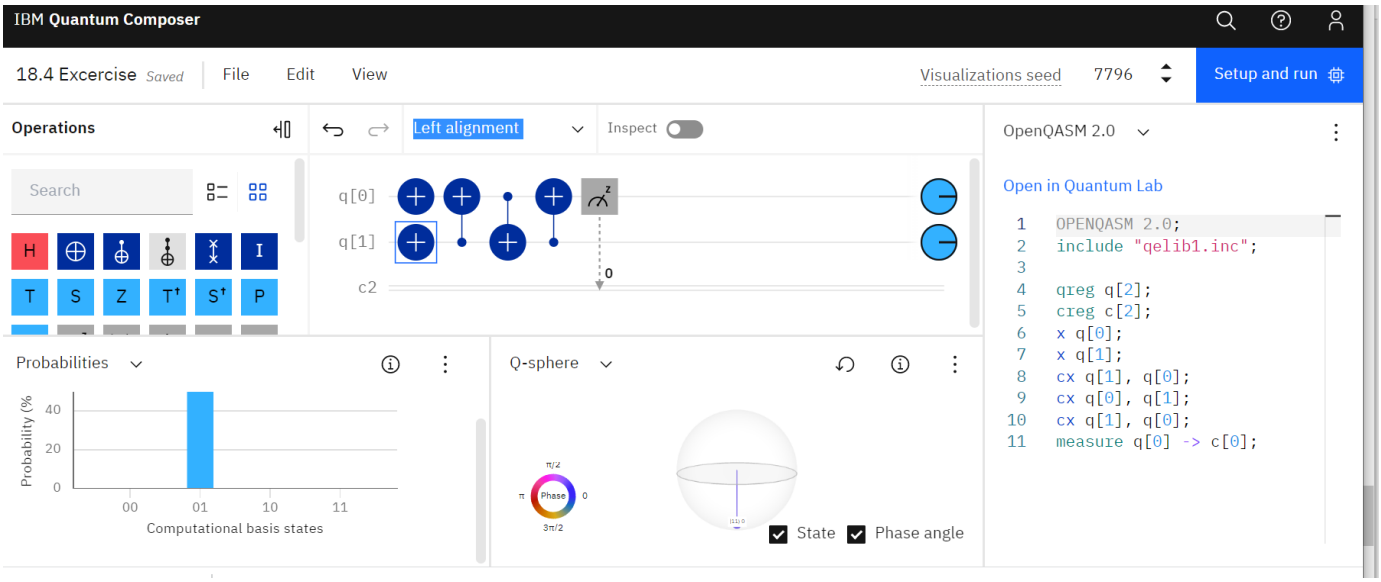
- Created: Sep 07, 2022 11:50 AM
- Transpiling: 1.3s
- Validating: 1.1s
- In queue: 35m 51.6s
- Running: 4.9s
  time in system 4.9s
- Completed: Sep 07, 2022 12:26 PM



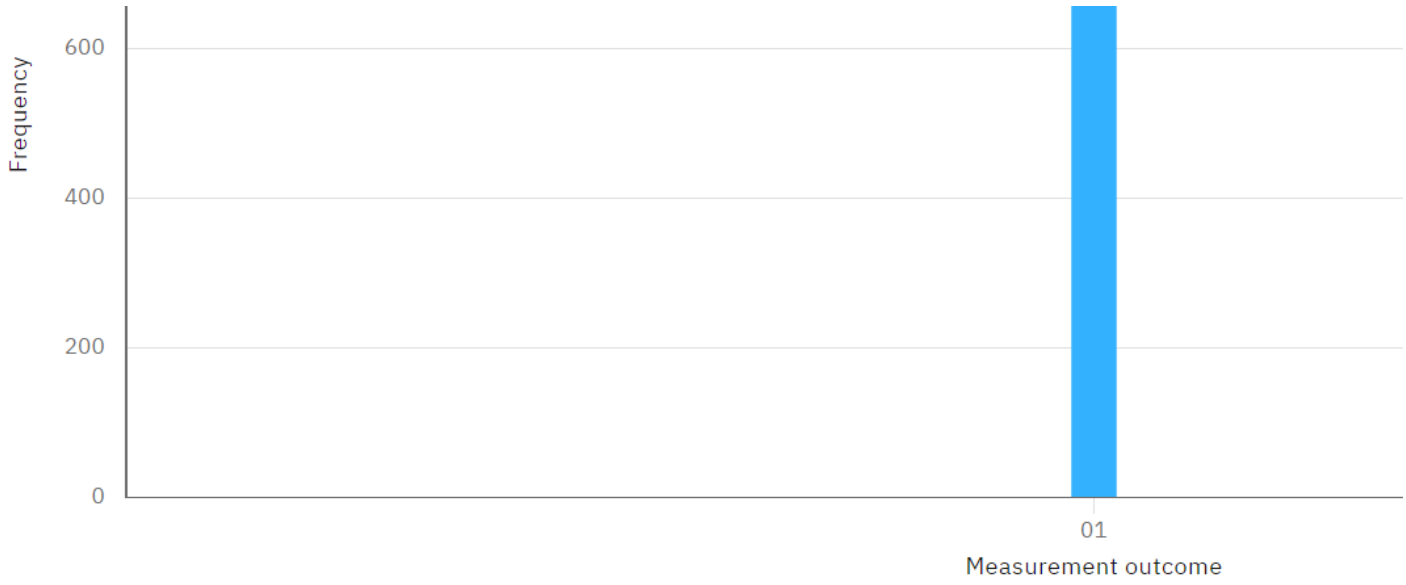
Nithin Reddy Govindugari(nithinreddy1747@gmail.com)

Introduction to Quantum computing (From a layperson to programmer in 30 steps)

18.4)



Simulation



Hardware execution result

Nithin Reddy Govindugari(nithinreddy1747@gmail.com)

Details

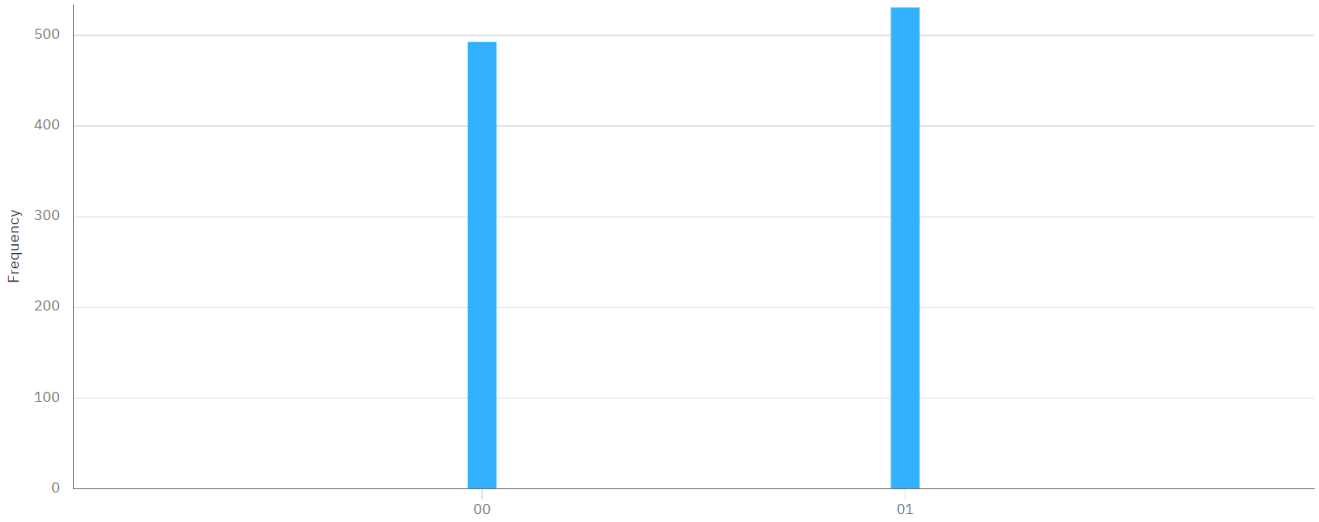
21h 2m 51.4s
Total completion time

ibm_oslo
Backend

Sent from	18.4 Excercise
Created on	Sep 07, 2022 12:14 PM
Sent to queue	Sep 07, 2022 12:14 PM
Provider	ibm-q/open/main
Run mode	fairshare
# of shots	1024
# of circuits	1

Status Timeline

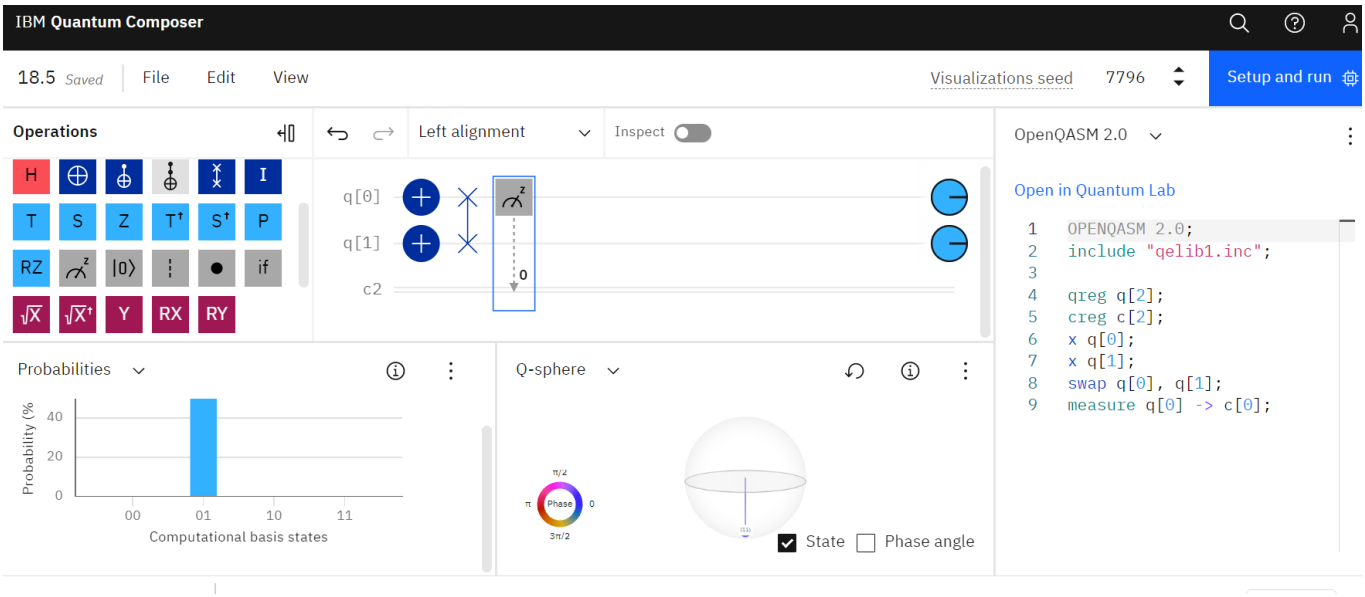
- Created: Sep 07, 2022 12:14 PM
- Transpiling: 1s
- Validating: 1.1s
- In queue: 21h 2m 41.1s
- Running: 4.4s
  time in system 4.4s
- Completed: Sep 08, 2022 9:16 AM



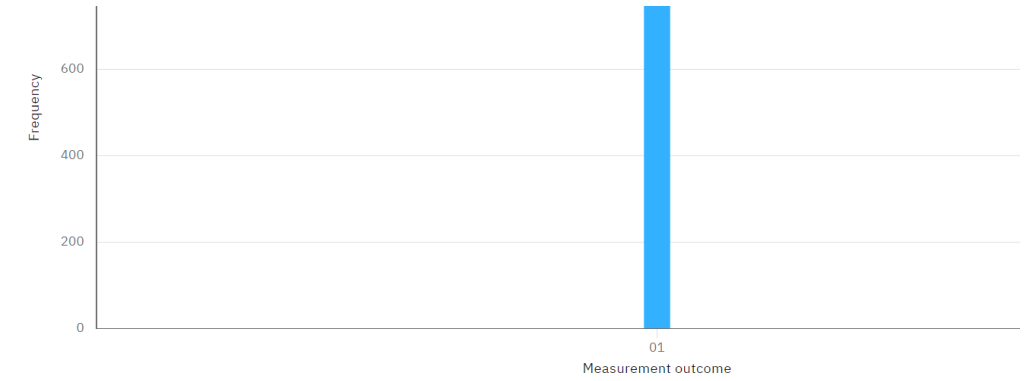
18.5)

Nithin Reddy Govindugari(nithinreddy1747@gmail.com)

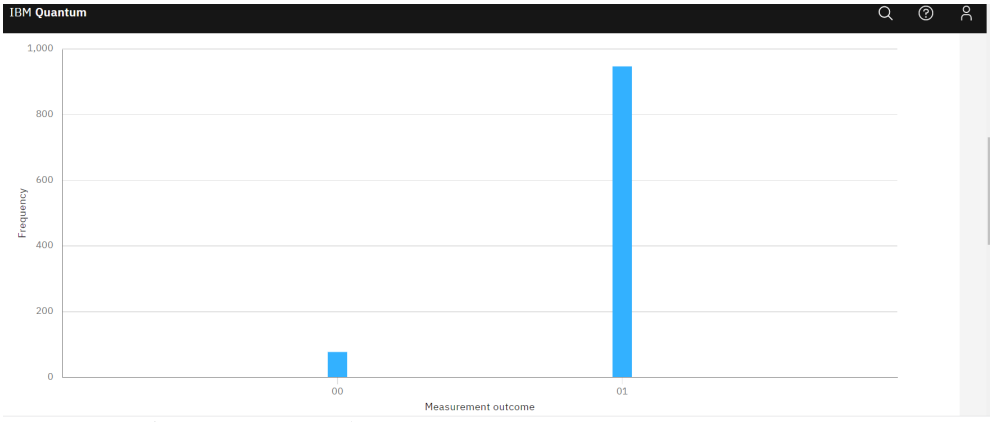
Introduction to Quantum computing (From a layperson to programmer in 30 steps)



Simulation results



Hardware execution result



Chapter 19

19.1)

Nithin Reddy Govindugari(nithinreddy1747@gmail.com)

19.1

Given

$$L\left(\frac{1}{\sqrt{2}}\left(|a\rangle_A + |b\rangle_A\right)|0\rangle_B\right) = \frac{1}{\sqrt{2}}\left(|a\rangle_A + |b\rangle_A\right) + \frac{1}{\sqrt{2}}\left(|a\rangle_B + |b\rangle_B\right)$$

$$L\left(|a\rangle_A|0\rangle_B + |b\rangle_A|0\rangle_B\right)$$

$$\Rightarrow |a\rangle_A|a\rangle_B + |b\rangle_A|b\rangle_B \rightarrow L.H.S \quad (1)$$

$\Rightarrow$ R.H.S (Tensor product)

$$\Rightarrow |a\rangle_A|a\rangle_B + |a\rangle_A|b\rangle_B + |b\rangle_A|a\rangle_B + |b\rangle_A|b\rangle_B \quad (2)$$

So Here L.H.S $\neq$ R.H.S because $|a\rangle_A|b\rangle_B$ & $|b\rangle_A|a\rangle_B$ probability is zero

Non cloning theorem survives on the concept of not able to copying linear combinations and linearity stands important

Ignored $1/\sqrt{2}$ for simplification

19.2)

(17.2) Here given $|+\rangle$ as input we know

$$|+\rangle = \frac{1}{\sqrt{2}}(\alpha|0\rangle_A + \beta|1\rangle_A) \longrightarrow (1)$$

we know from above 17.1 problem and as we are supposed to be normalizing the above it will work as general Quantum teleportation algorithm.

b) $H_2 C_{10} H_1 = \frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix}\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&0&1\\0&0&1&0\end{pmatrix}\frac{1}{\sqrt{2}}\begin{pmatrix}1&1\\1&-1\end{pmatrix}$

From 17.10 $= \frac{1}{2}\begin{pmatrix}1&1&1&1\\1&-1&1&-1\\1&1&-1&-1\\1&-1&-1&1\end{pmatrix}\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&0&1\\0&0&1&0\end{pmatrix}$

$$= \frac{1}{2}\begin{pmatrix}1&1&1&1\\1&-1&-1&1\\1&1&-1&-1\\1&-1&1&-1\end{pmatrix}$$

3) As it is $|+\rangle$ base we are converting to general convention and with linear combination of $|0\rangle$ & $|1\rangle$ is created with first Hadamard gate

4) MSB and Hadamard gates are not cancelled from 17.11 → 17.11 we know it has equal probability of all the four cases we can see same in output and CNOT is for entanglement

19.3)

19.3) Yes, It is possible to clone basis state as Non-cloning theorem only limits copying of linear combination of Alice to Bob but when we measure Alice state we can clone as many times as we want as when we ~~clone~~ measure the value remains same

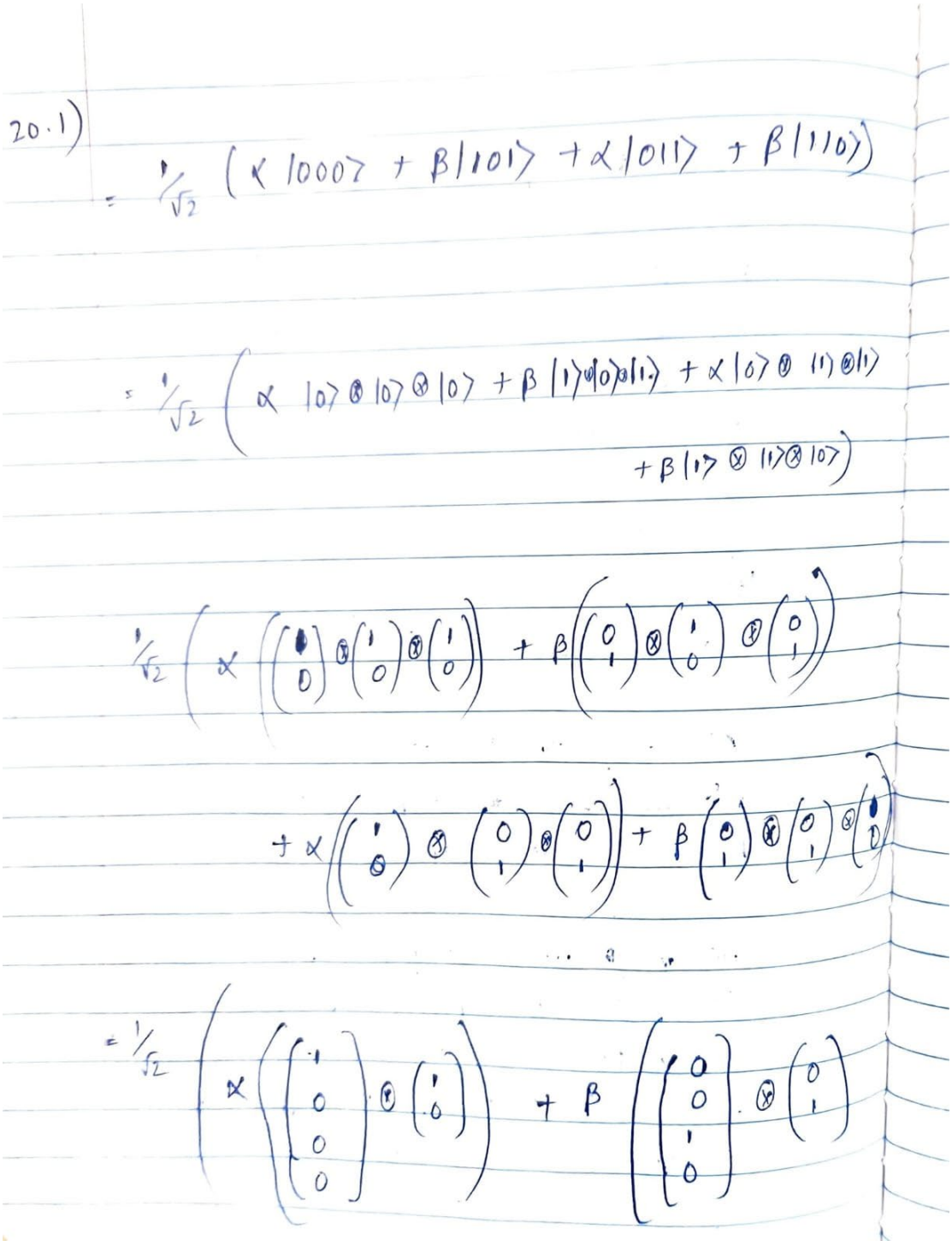
You can't clone arbitrary state due to linearity and we proved it 1st Question

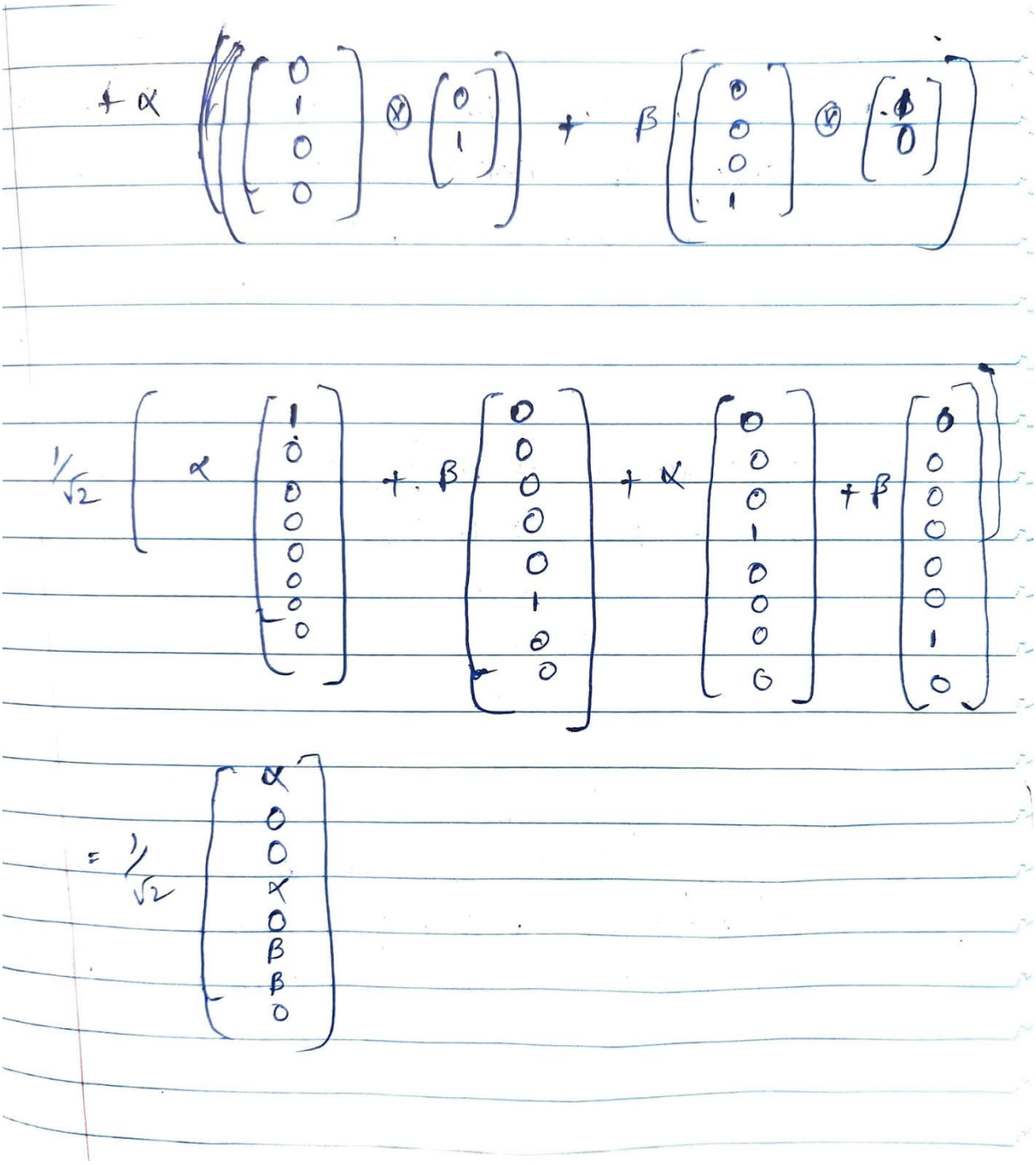
According to 19.7 we have to change basis and we can do teleportation but Alice won't be same after teleporting

Heinsberg uncertainity principle states that you can't calculate position of a things exactly at a time in two basis. we can't have exact basis states of two different basis at a time. we need to have linear combination of one state and base state of one state

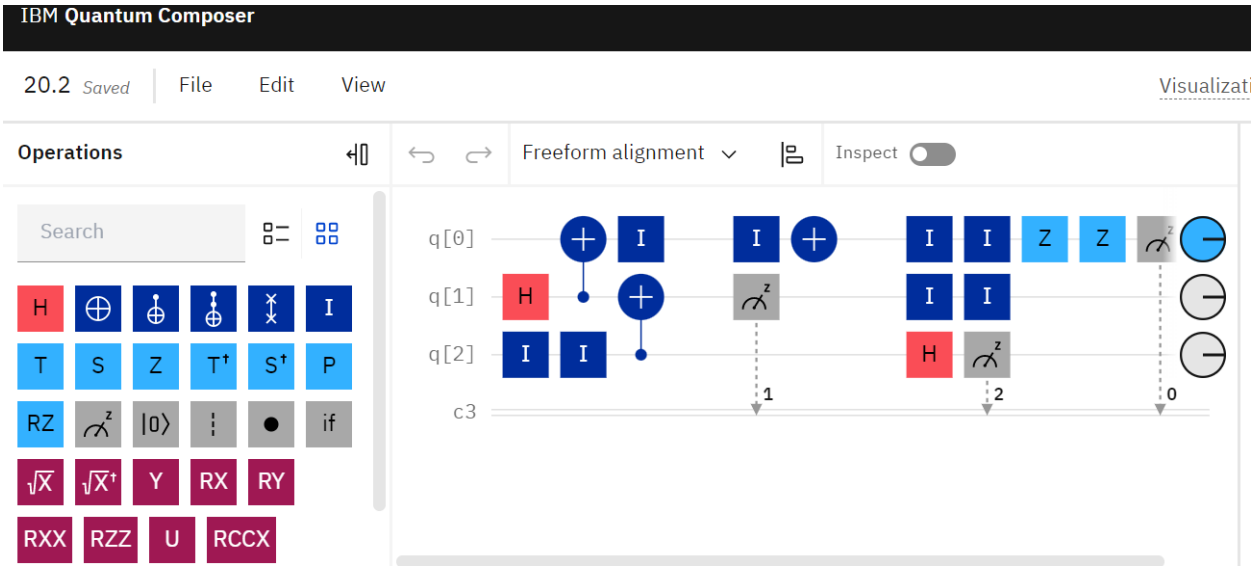
Chapter 20

20.1)

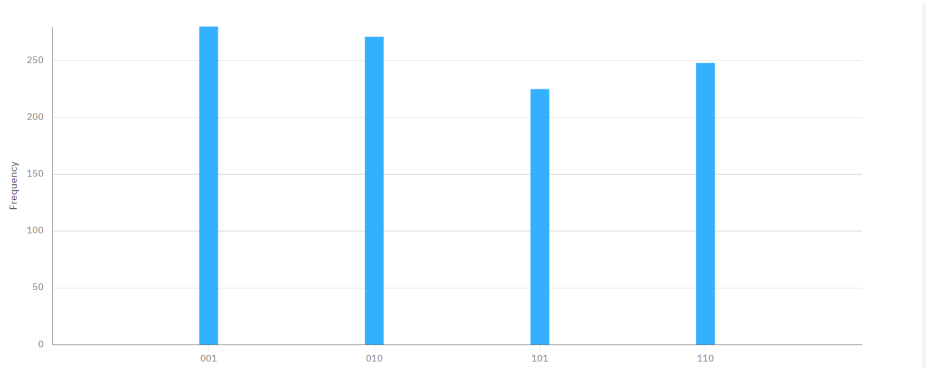




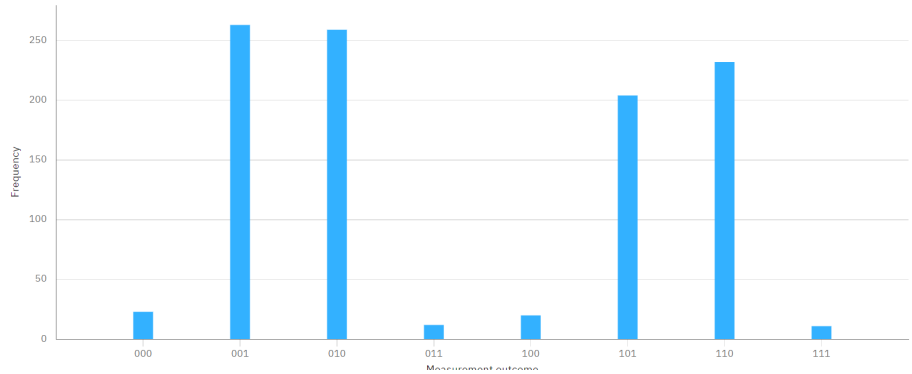
20.2)



Simulation result

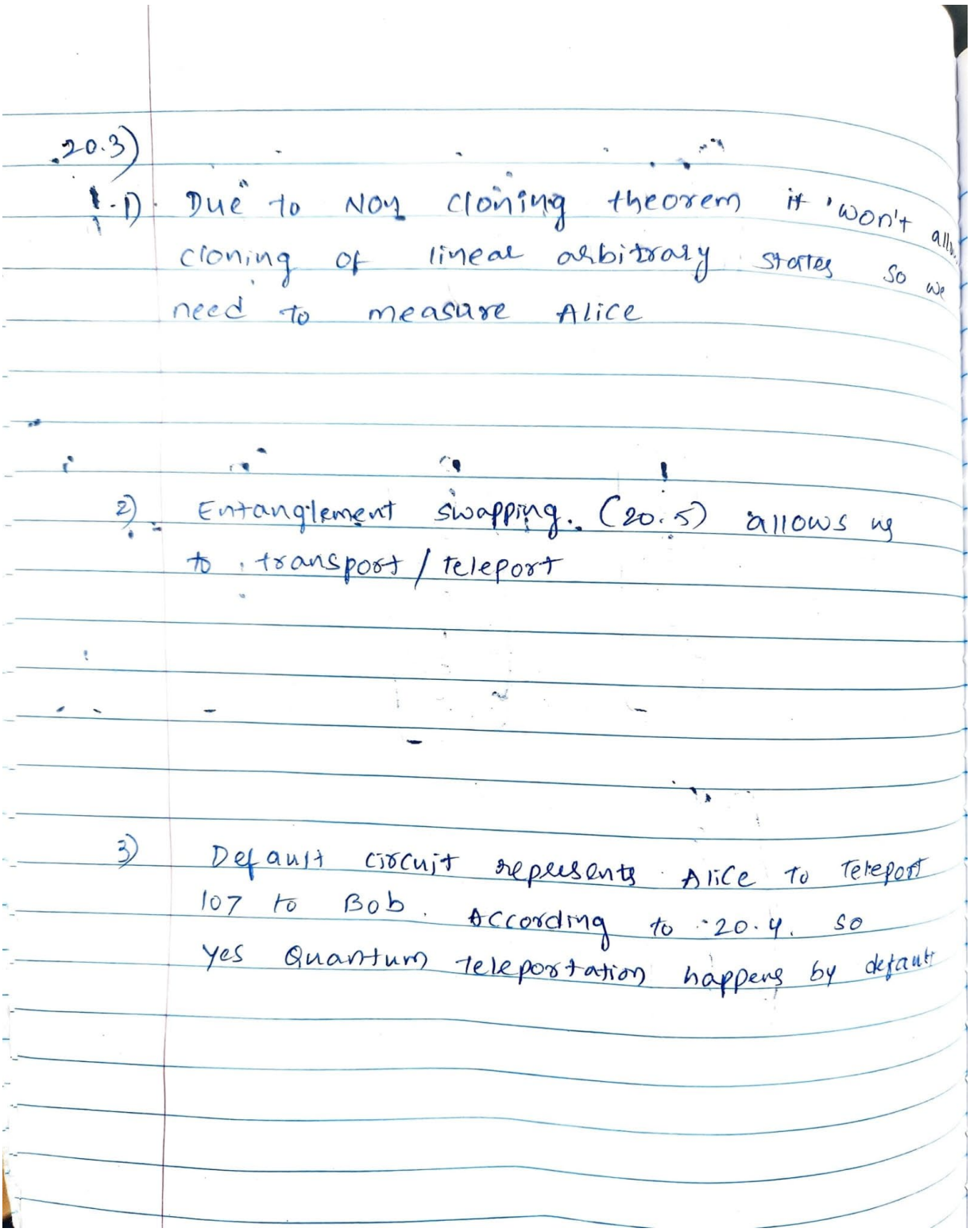


IBM



20.3)

Nithin Reddy Govindugari(nithinreddy1747@gmail.com)



20.3)

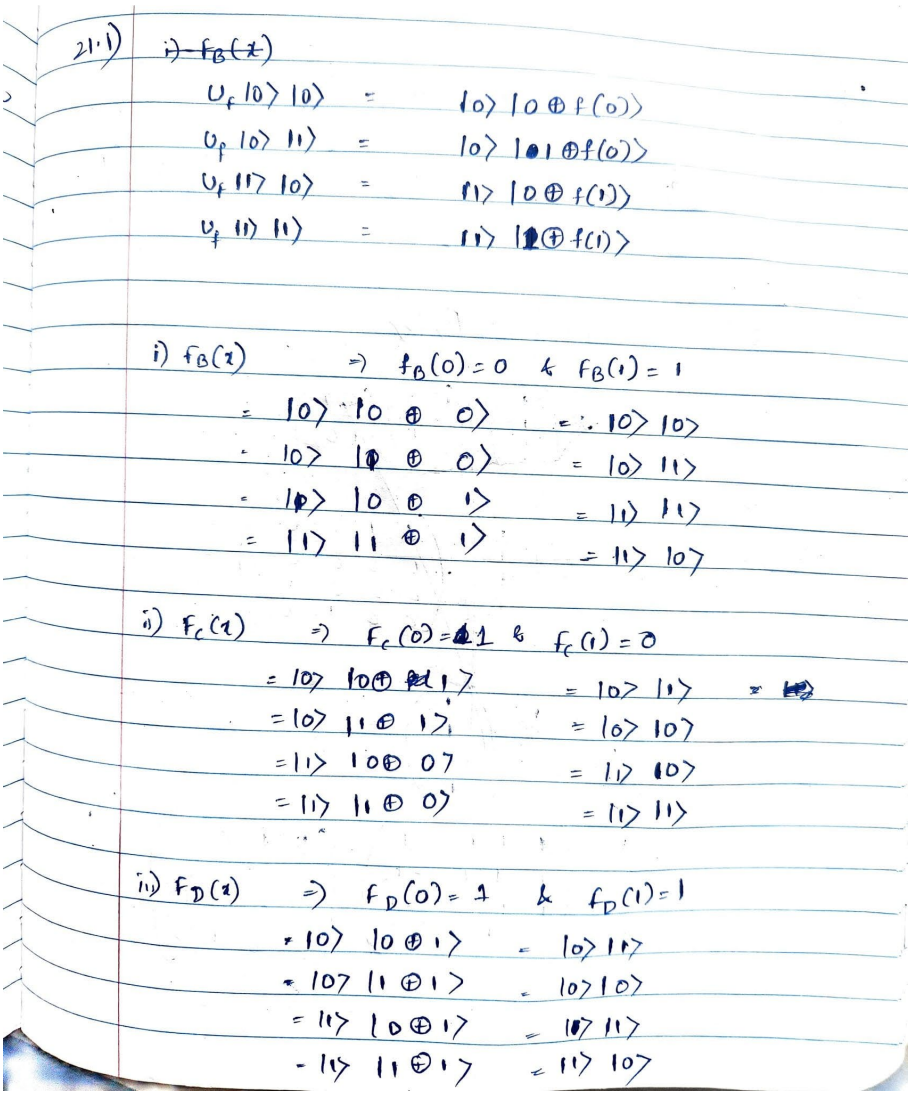
1) Due to NOy cloning theorem it won't allo. cloning of lineal arbitrary states so we need to measure Alice

2) Entanglement swapping. (20.5) allows us to transport / teleport

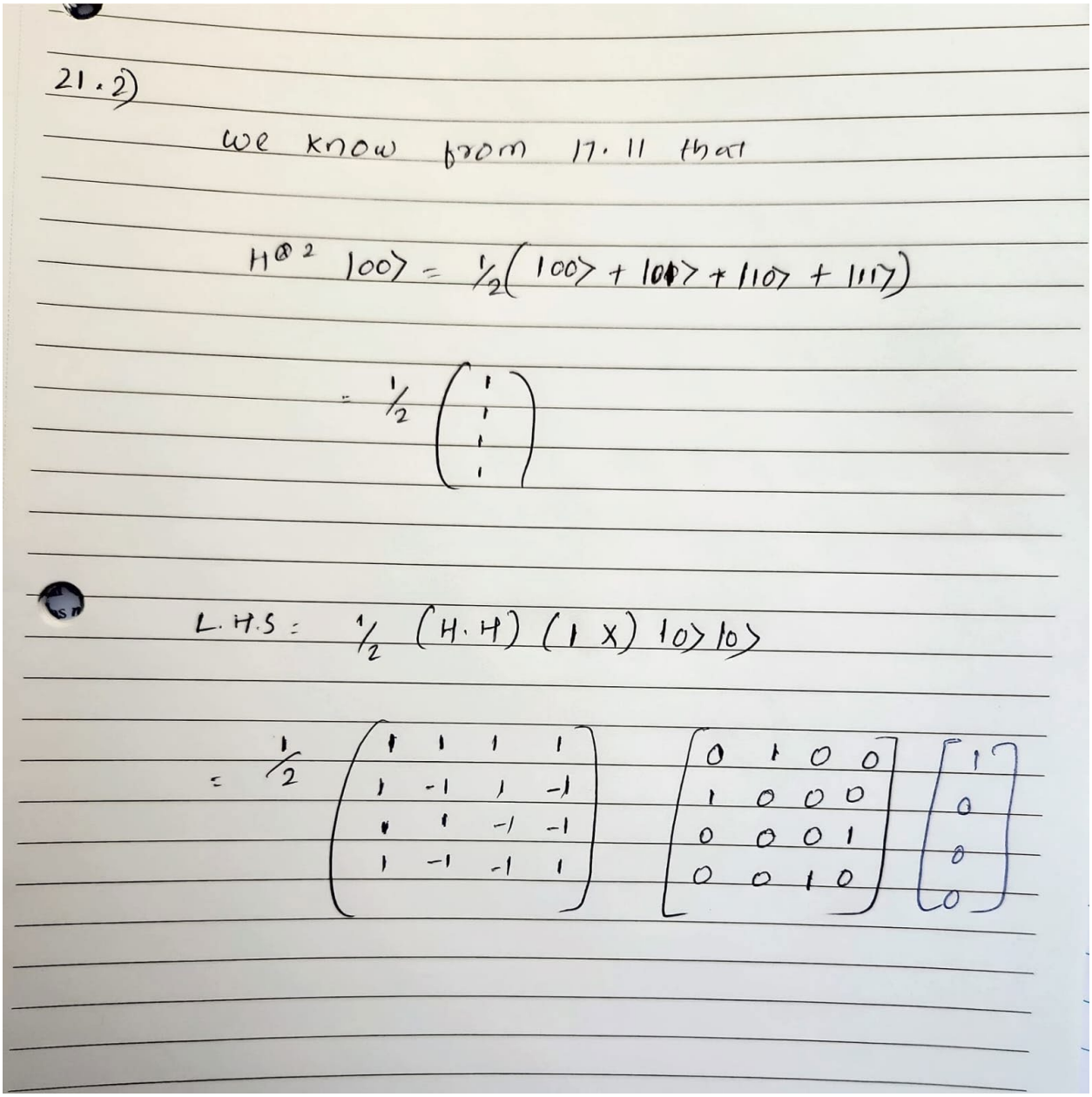
3) Default circuit represents Alice to Teleport 107 to Bob. According to 20.4. so yes Quantum teleportation happens by default

Nithin Reddy Govindugari(nithinreddy1747@gmail.com)

21.1)


21.1)
i) $f_B(x)$
$U_f|0\rangle|0\rangle = |0\rangle|0 \oplus f(0)\rangle$
$U_f|0\rangle|1\rangle = |0\rangle|1 \oplus f(0)\rangle$
$U_f|1\rangle|0\rangle = |1\rangle|0 \oplus f(1)\rangle$
$U_f|1\rangle|1\rangle = |1\rangle|1 \oplus f(1)\rangle$
i) $f_B(x)$ $\Rightarrow f_B(0)=0$ & $f_B(1)=1$
$= |0\rangle|0 \oplus 0\rangle \therefore |0\rangle|0\rangle$
$= |0\rangle|0 \oplus 0\rangle = |0\rangle|1\rangle$
$= |0\rangle|0 \oplus 1\rangle = |1\rangle|1\rangle$
$= |1\rangle|1 \oplus 1\rangle = |1\rangle|0\rangle$
ii) $f_C(x)$ $\Rightarrow f_C(0)=1$ & $f_C(1)=0$
$= |0\rangle|0 \oplus 1\rangle = |0\rangle|1\rangle$
$= |0\rangle|1 \oplus 1\rangle = |0\rangle|0\rangle$
$= |1\rangle|0 \oplus 0\rangle = |1\rangle|0\rangle$
$= |1\rangle|1 \oplus 0\rangle = |1\rangle|1\rangle$
iii) $f_D(x)$ $\Rightarrow f_D(0)=1$ & $f_D(1)=1$
$= |0\rangle|0 \oplus 1\rangle = |0\rangle|1\rangle$
$= |0\rangle|1 \oplus 1\rangle = |0\rangle|0\rangle$
$= |1\rangle|0 \oplus 1\rangle = |1\rangle|1\rangle$
$= |1\rangle|1 \oplus 1\rangle = |1\rangle|0\rangle$

21.2)



21.2)

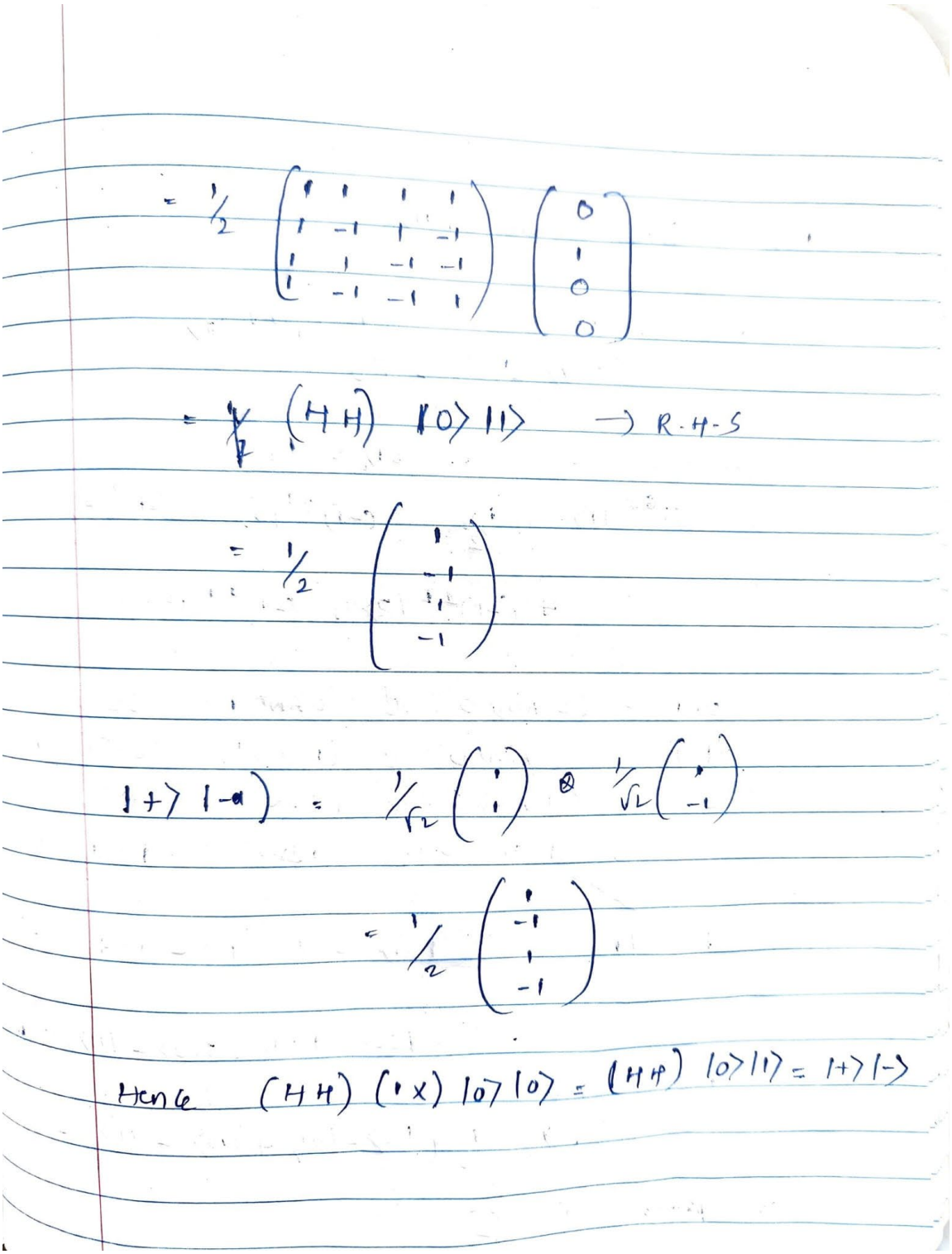
We know from 17.11 that

$$H^{\otimes 2}|00\rangle = \frac{1}{2}(|00\rangle + |01\rangle + |10\rangle + |11\rangle)$$

$$= \frac{1}{2}\begin{pmatrix}1\\1\\1\\1\end{pmatrix}$$

$$L.H.S: \frac{1}{2}(H.H)(I\ X)|0\rangle|0\rangle$$

$$= \frac{1}{2}\begin{pmatrix}1 & 1 & 1 & 1\\ 1 & -1 & 1 & -1\\ 1 & 1 & -1 & -1\\ 1 & -1 & -1 & 1\end{pmatrix}\begin{bmatrix}0 & 1 & 0 & 0\\ 1 & 0 & 0 & 0\\ 0 & 0 & 0 & 1\\ 0 & 0 & 1 & 0\end{bmatrix}\begin{bmatrix}1\\0\\0\\0\end{bmatrix}$$



$$= \frac{1}{2}\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}$$

$$= \frac{1}{2}(H \otimes H)\,|0\rangle|1\rangle \rightarrow \text{R.H.S}$$

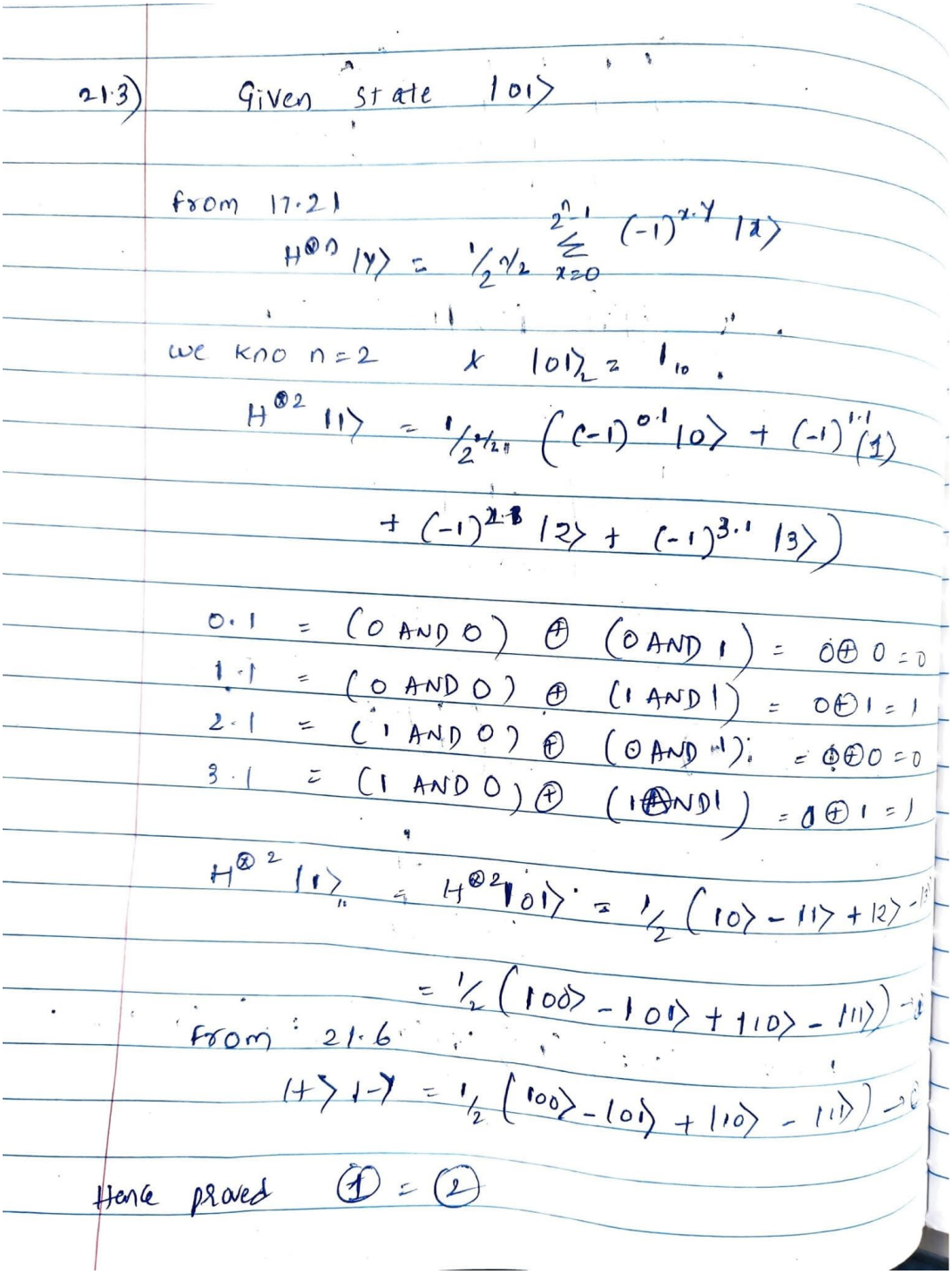
$$= \frac{1}{2}\begin{pmatrix} 1 \\ -1 \\ 1 \\ -1 \end{pmatrix}$$

$$|+\rangle|-\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix} \otimes \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

$$= \frac{1}{2}\begin{pmatrix} 1 \\ -1 \\ 1 \\ -1 \end{pmatrix}$$

Hence $(H \otimes H)(I \otimes X)|0\rangle|0\rangle = (H \otimes H)|0\rangle|1\rangle = |+\rangle|-\rangle$

21.3)



21.3) Given state $|01\rangle$

from 17.21

$$H^{\otimes n}|y\rangle = \frac{1}{2^{n/2}} \sum_{x=0}^{2^n-1} (-1)^{x.y} |x\rangle$$

we kno $n=2$ $\quad x\ |01\rangle_2 = 1_{10}$.

$$H^{\otimes 2}|1\rangle = \frac{1}{2^{2/2}} \left((-1)^{0.1}|0\rangle + (-1)^{1.1}|1\rangle + (-1)^{2.1}|2\rangle + (-1)^{3.1}|3\rangle \right)$$

$0.1 = (0 \text{ AND } 0) \oplus (0 \text{ AND } 1) = 0 \oplus 0 = 0$

$1.1 = (0 \text{ AND } 0) \oplus (1 \text{ AND } 1) = 0 \oplus 1 = 1$

$2.1 = (1 \text{ AND } 0) \oplus (0 \text{ AND } 1) = 0 \oplus 0 = 0$

$3.1 = (1 \text{ AND } 0) \oplus (1 \text{ AND } 1) = 0 \oplus 1 = 1$

$$H^{\otimes 2}|1\rangle_{10} = H^{\otimes 2}|01\rangle = \frac{1}{2}(|0\rangle - |1\rangle + |2\rangle - |3\rangle)$$

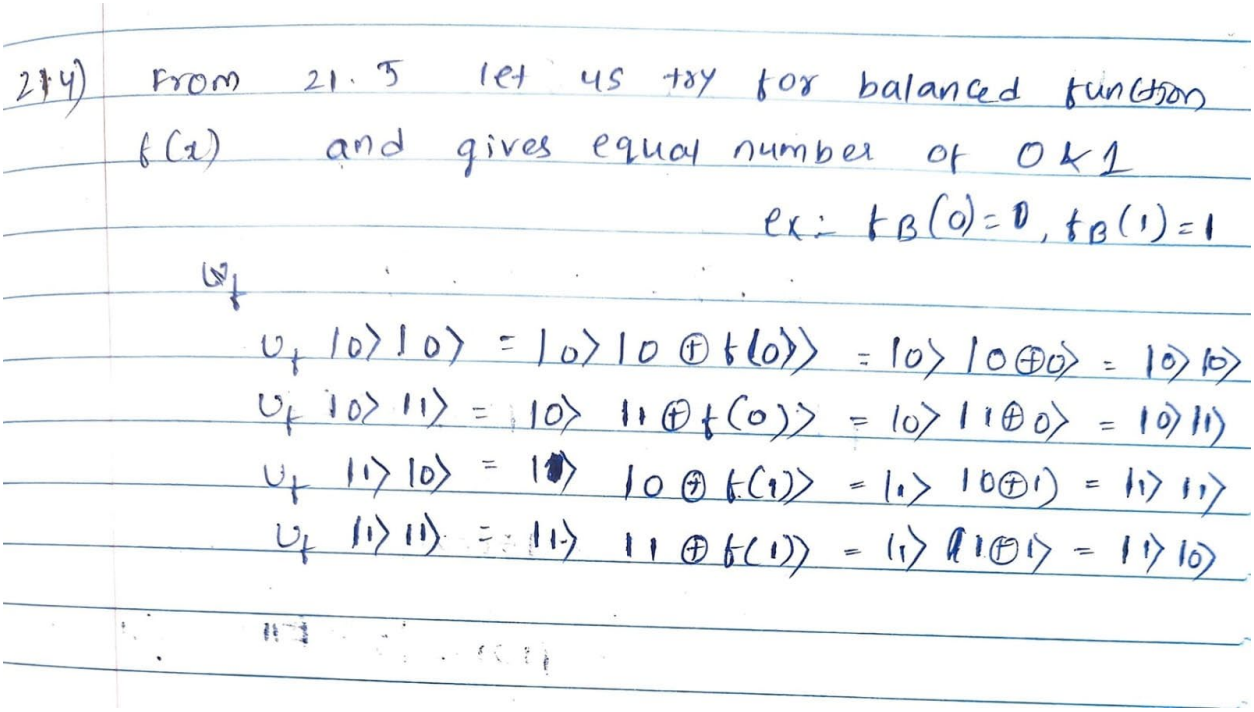
$$= \frac{1}{2}(|00\rangle - |01\rangle + |10\rangle - |11\rangle) \rightarrow (2)$$

From 21.6

$$|+\rangle|-\rangle = \frac{1}{2}(|00\rangle - |01\rangle + |10\rangle - |11\rangle) \rightarrow (1)$$

Hence proved (1) = (2)

21.4)



Chapter 22

22.1) & 22.2)

22.1 Given $x = 5$, $f(5) = 3$, $n = 3$, $m = 3$

y assumed as $|0\rangle_m$

we know from 22.1

$$U_f |x\rangle_n |y\rangle_m = |x\rangle_n |y \oplus f(x)\rangle_m$$

$$U_f |5\rangle_3 |0\rangle_3 = |5\rangle_3 |0 \oplus f(5)\rangle_3$$

~~$= |00101\rangle$~~

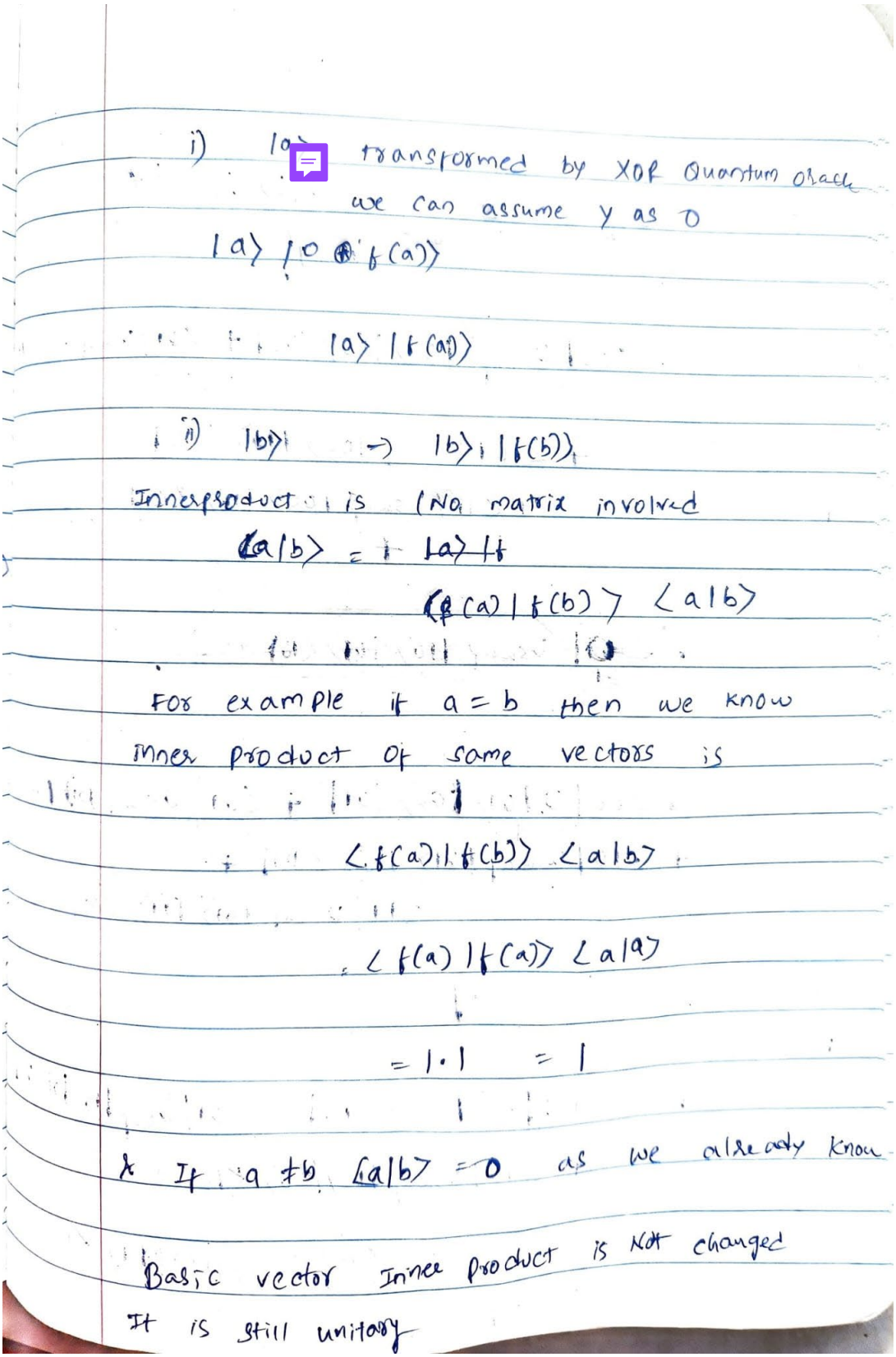
$$= |101\rangle_3 |0 \oplus (011)\rangle = |101\rangle_3 |000 \oplus$$

$$= |101\rangle |011\rangle$$

$$= |101011\rangle$$

22.2 If we are using same method as phase Quantum oracle

Two basis vectors $|a\rangle$ & $|b\rangle$ and inner product is $\langle a|b\rangle$



i) $|a\rangle$ transformed by XOR Quantum oracle
we can assume y as 0

$|a\rangle |0 \oplus f(a)\rangle$

$|a\rangle |f(a)\rangle$

ii) $|b\rangle \rightarrow |b\rangle |f(b)\rangle$

Innerproduct is (No matrix involved

~~$\langle a|b\rangle = |a\rangle |b\rangle$~~

$\langle f(a)|f(b)\rangle \langle a|b\rangle$

For example if $a = b$ then we know
Inner product of same vectors is

$\langle f(a)|f(b)\rangle \langle a|b\rangle$

$\langle f(a)|f(a)\rangle \langle a|a\rangle$

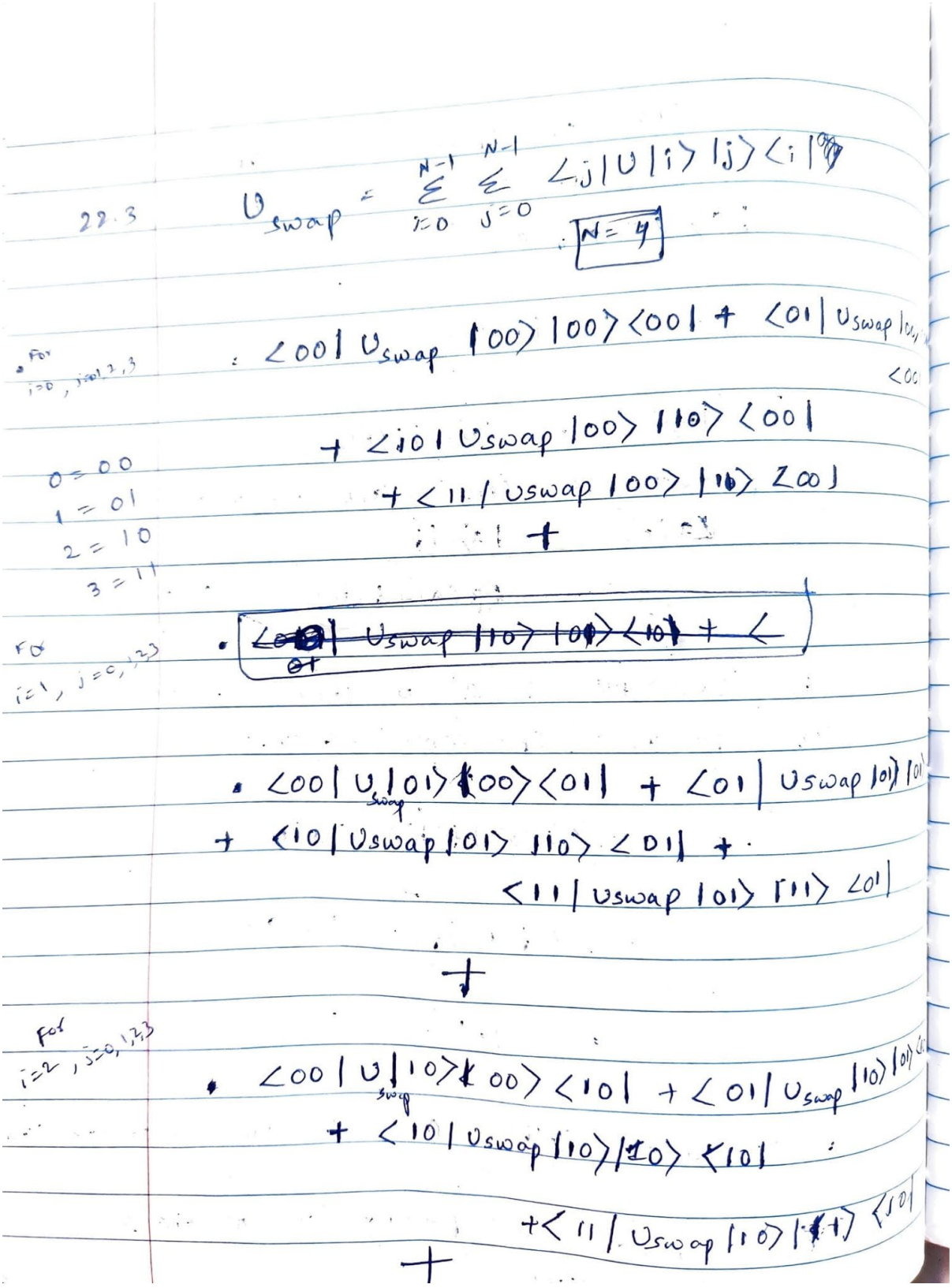
$= 1 \cdot 1 = 1$

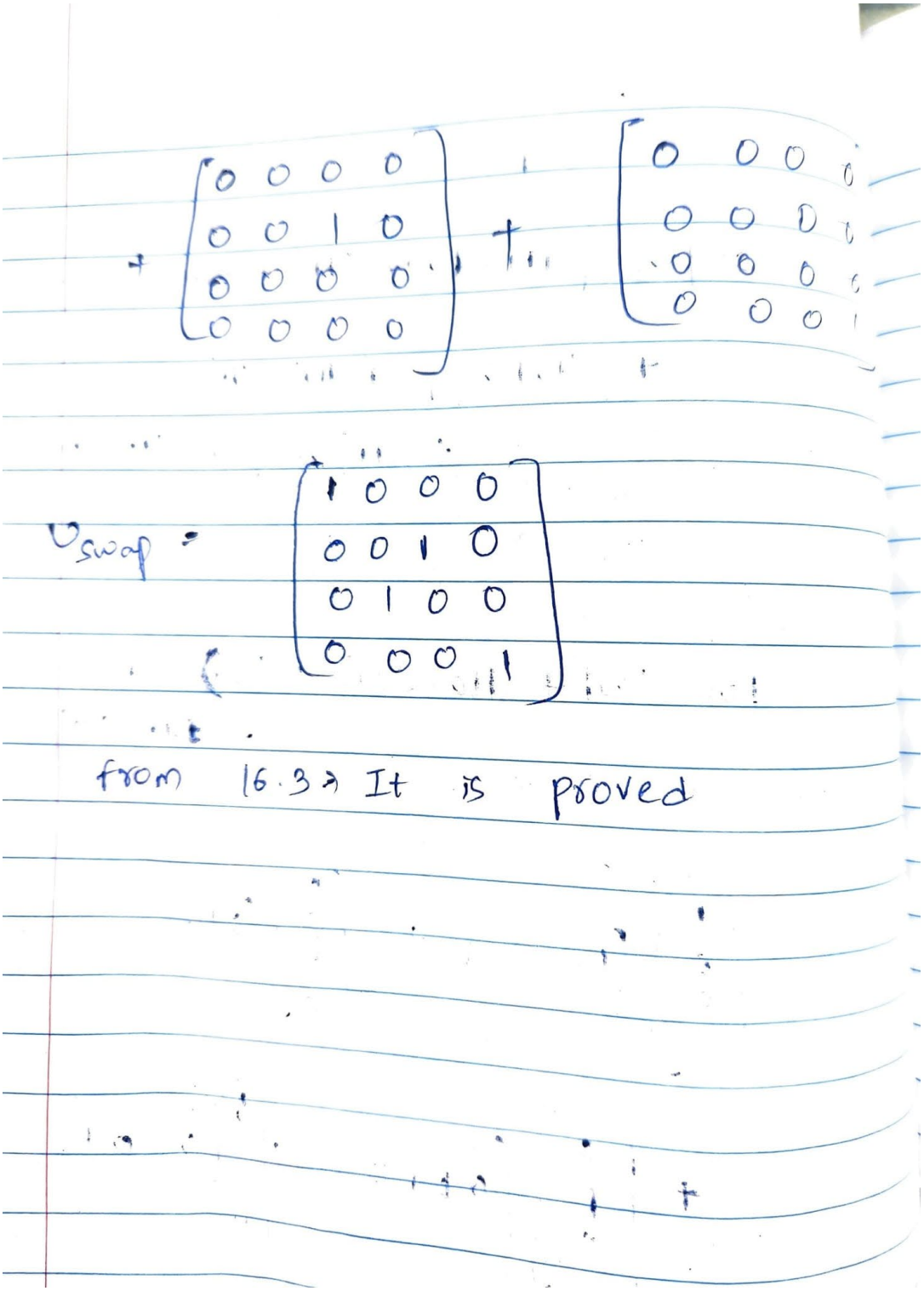
If $a \neq b$ $\langle a|b\rangle = 0$ as we already know

Basic vector Inner product is Not changed
It is still unitary

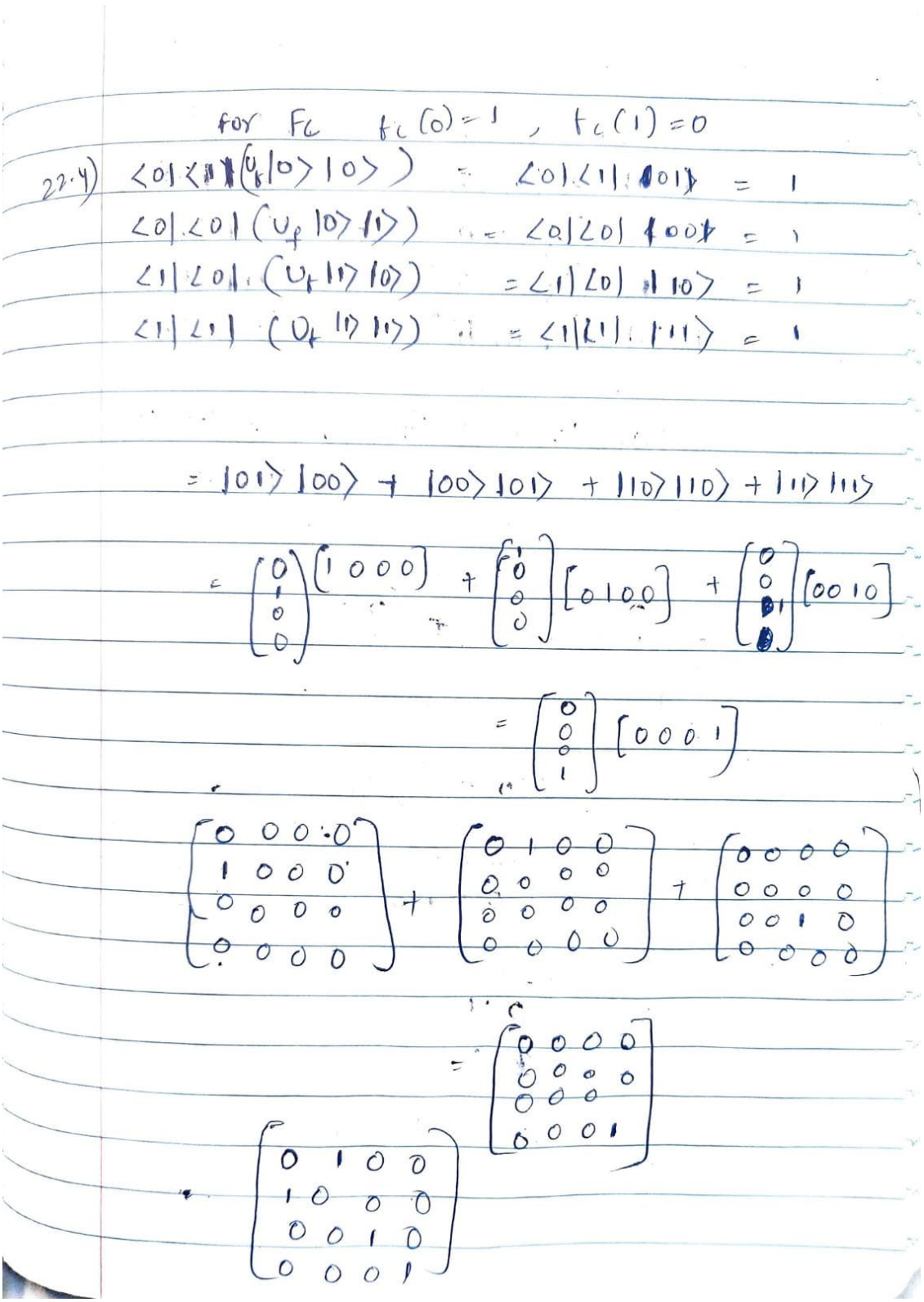
Nithin Reddy Govindugari(nithinreddy1747@gmail.com)

22.3)

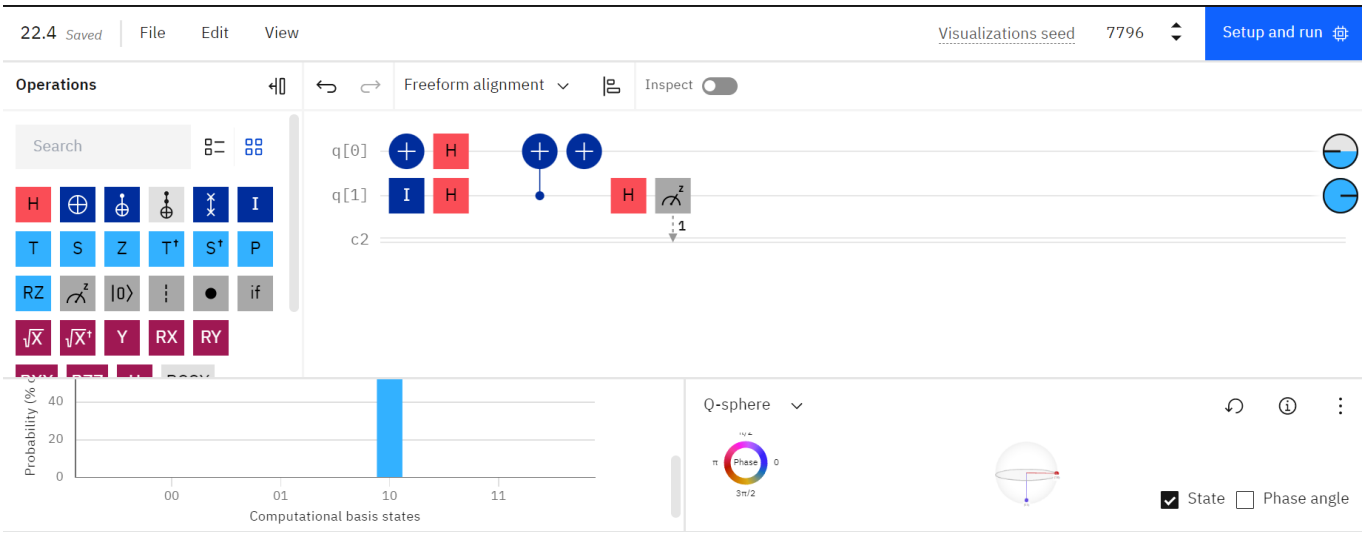

22.3
$U_{swap} = \sum_{i=0}^{N-1} \sum_{j=0}^{N-1} \langle j|U|i\rangle |j\rangle\langle i|$
N = 4
For i=0, j=0,1,2,3
$\langle 00|U_{swap}|00\rangle |00\rangle\langle 00| + \langle 01|U_{swap}|01\rangle$
$+ \langle 10|U_{swap}|00\rangle |10\rangle\langle 00|$
$+ \langle 11|U_{swap}|00\rangle |11\rangle\langle 00| +$
0 = 00
1 = 01
2 = 10
3 = 11
For i=1, j=0,1,2,3
$\langle 00|U_{swap}|10\rangle |00\rangle\langle 10| + \langle$
$\langle 00|U_{swap}|01\rangle |00\rangle\langle 01| + \langle 01|U_{swap}|01\rangle$
$+ \langle 10|U_{swap}|01\rangle |10\rangle\langle 01| +$
$\langle 11|U_{swap}|01\rangle |11\rangle\langle 01| +$
For i=2, j=0,1,2,3
$\langle 00|U_{swap}|10\rangle |00\rangle\langle 10| + \langle 01|U_{swap}|10\rangle$
$+ \langle 10|U_{swap}|10\rangle |10\rangle\langle 10|$
$+ \langle 11|U_{swap}|10\rangle |11\rangle\langle 10|$



22.4)

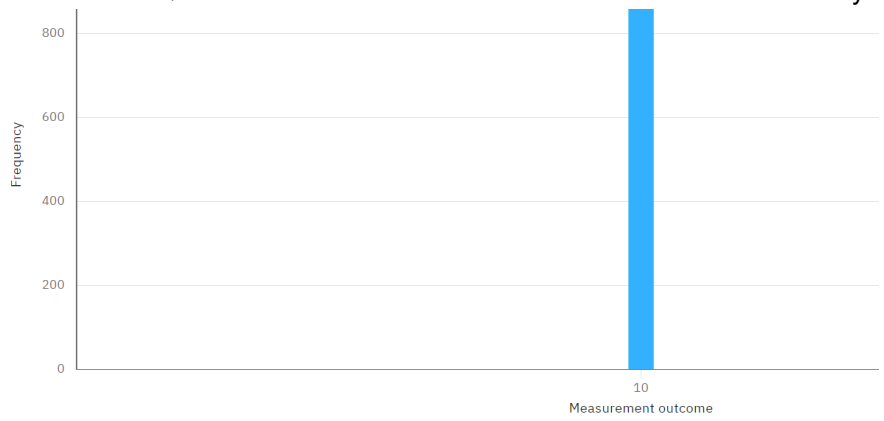

for Fc fc(0) = 1 , fc(1) = 0
22.4)
= |01⟩|00⟩ + |00⟩|01⟩ + |10⟩|10⟩ + |11⟩|11⟩

Introduction to Quantum computing (From a layperson to programmer in 30 steps)

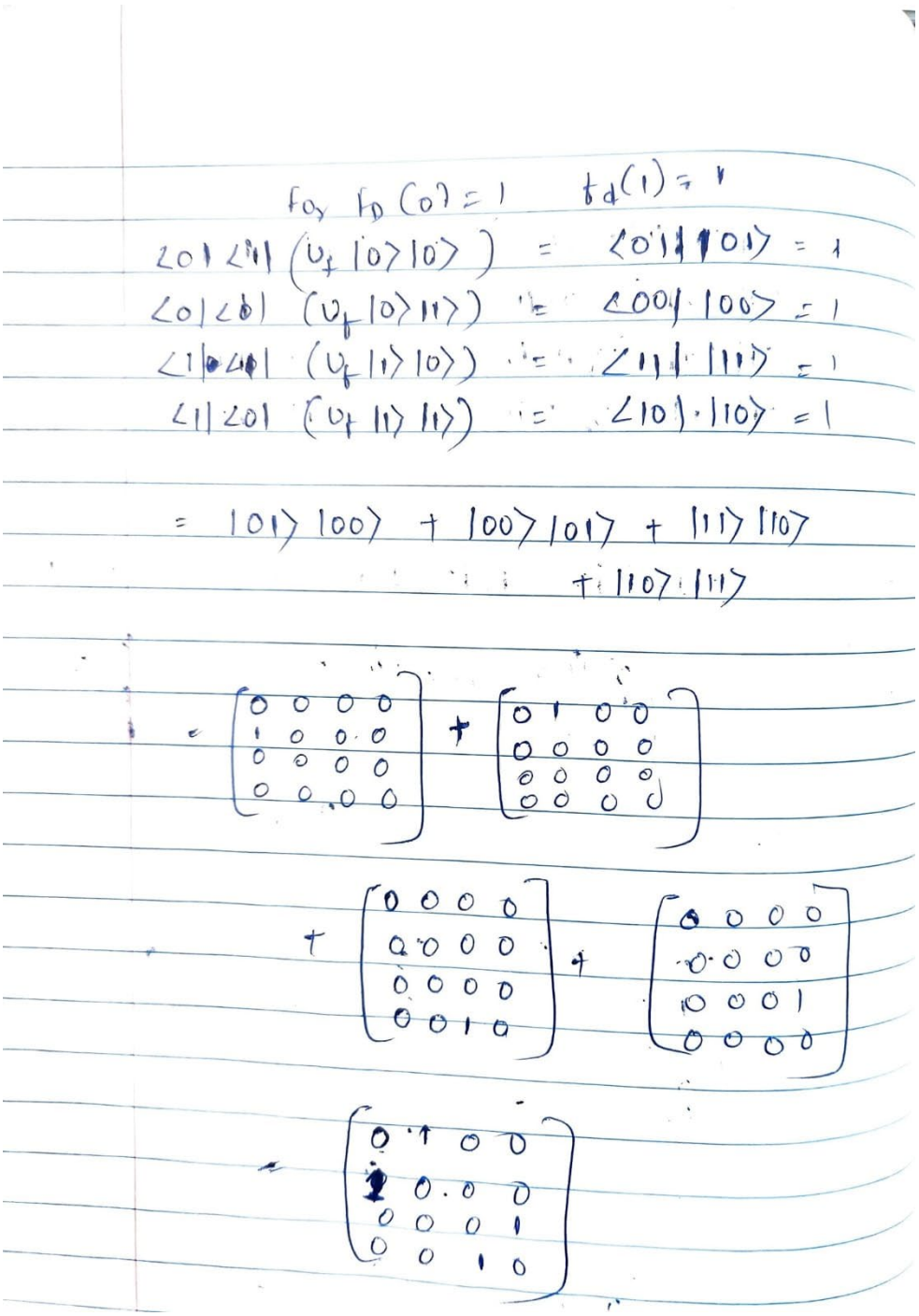


Simulator outcome

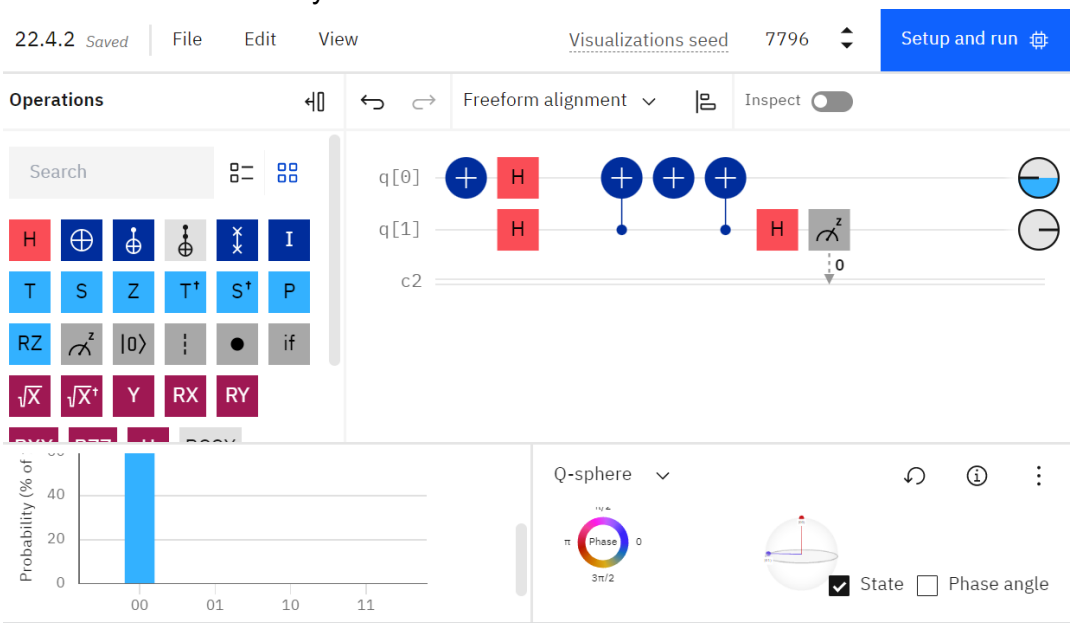
It is 10 in IBMQ instead of 01 because for balanced function MSB is always 1 (page no 212)



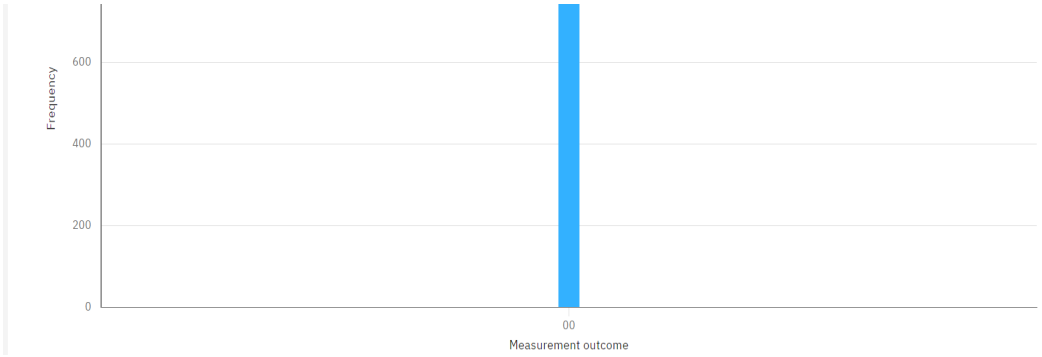
Nithin Reddy Govindugari(nithinreddy1747@gmail.com)


For $f_D(0) = 1$ $f_D(1) = 1$
$\langle 0|\langle 1|(U_f|0\rangle|0\rangle) = \langle 01|01\rangle = 1$
$\langle 0|\langle 0|(U_f|0\rangle|1\rangle) = \langle 00|00\rangle = 1$
$\langle 1|\langle 0|(U_f|1\rangle|0\rangle) = \langle 11|11\rangle = 1$
$\langle 1|\langle 0|(U_f|1\rangle|1\rangle) = \langle 10|10\rangle = 1$
$= |01\rangle|00\rangle + |00\rangle|01\rangle + |11\rangle|10\rangle + |10\rangle|11\rangle$

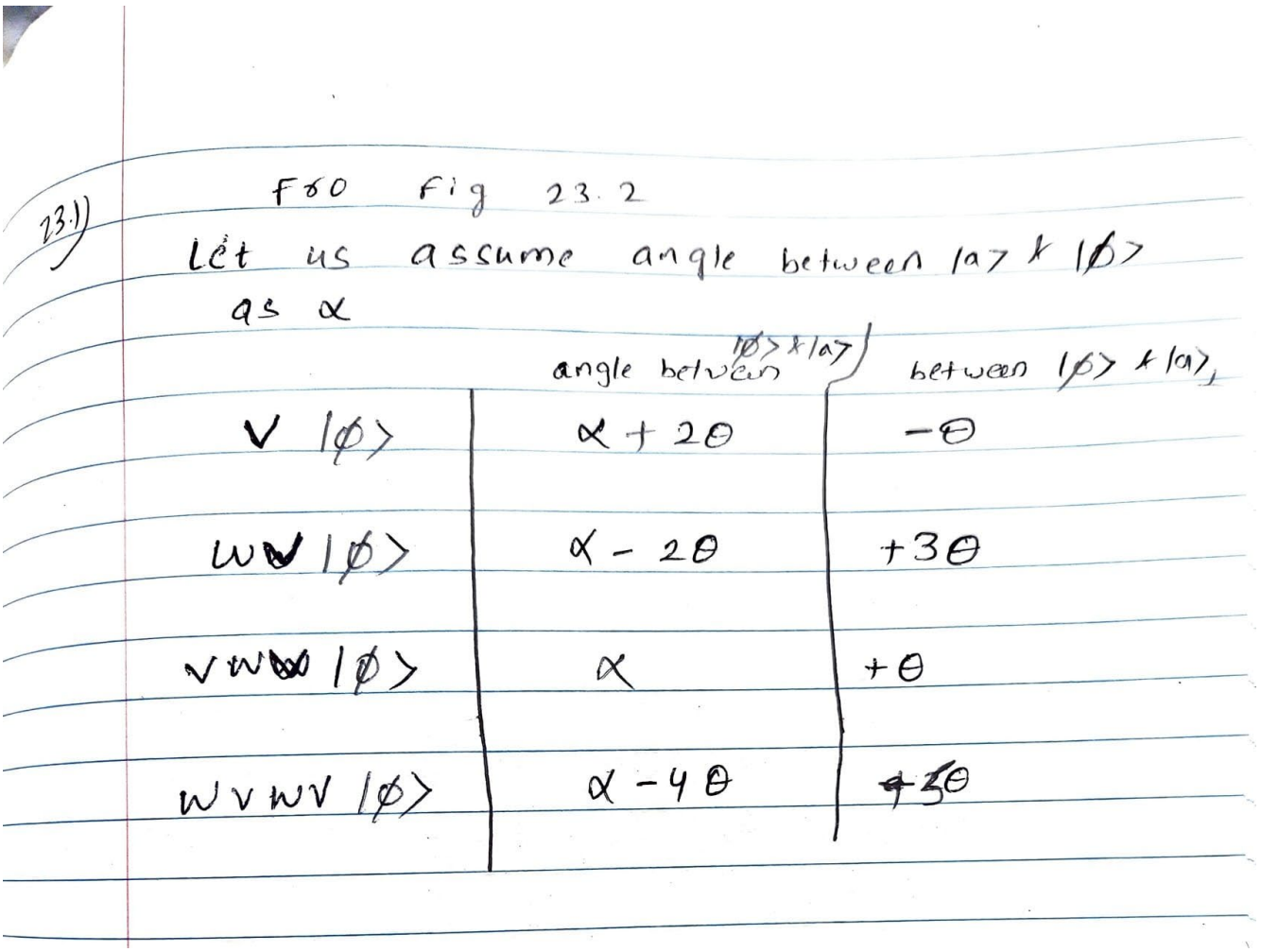
It is similar to above statement and as explained in page 200 it is always 0 for constant function here we can also remove both cnot gates as it wont make much difference. I have added it to describe the functionality



Simulator results



Chapter 23



23.1) For Fig 23.2

Let us assume angle between $|a\rangle$ & $|\phi\rangle$ as α

	angle between $|\phi\rangle$ & $|a\rangle$	between $|\phi\rangle$ & $|a\rangle$,
$V\,|\phi\rangle$	$\alpha + 2\theta$	$-\theta$
$WV\,|\phi\rangle$	$\alpha - 2\theta$	$+3\theta$
$VWV\,|\phi\rangle$	α	$+\theta$
$WVWV\,|\phi\rangle$	$\alpha - 4\theta$	-3θ

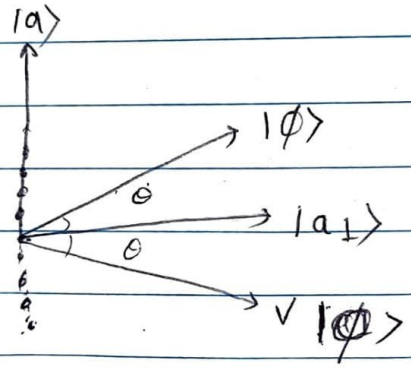
23.2)

23.2)

Given $V = I - 2|a\rangle\langle a|$

As we already know V is a quantum gate and it is linear combination of all basis vectors except our target entry $|a\rangle$

we know phase shift is added to target entry and it is multiplied by -1

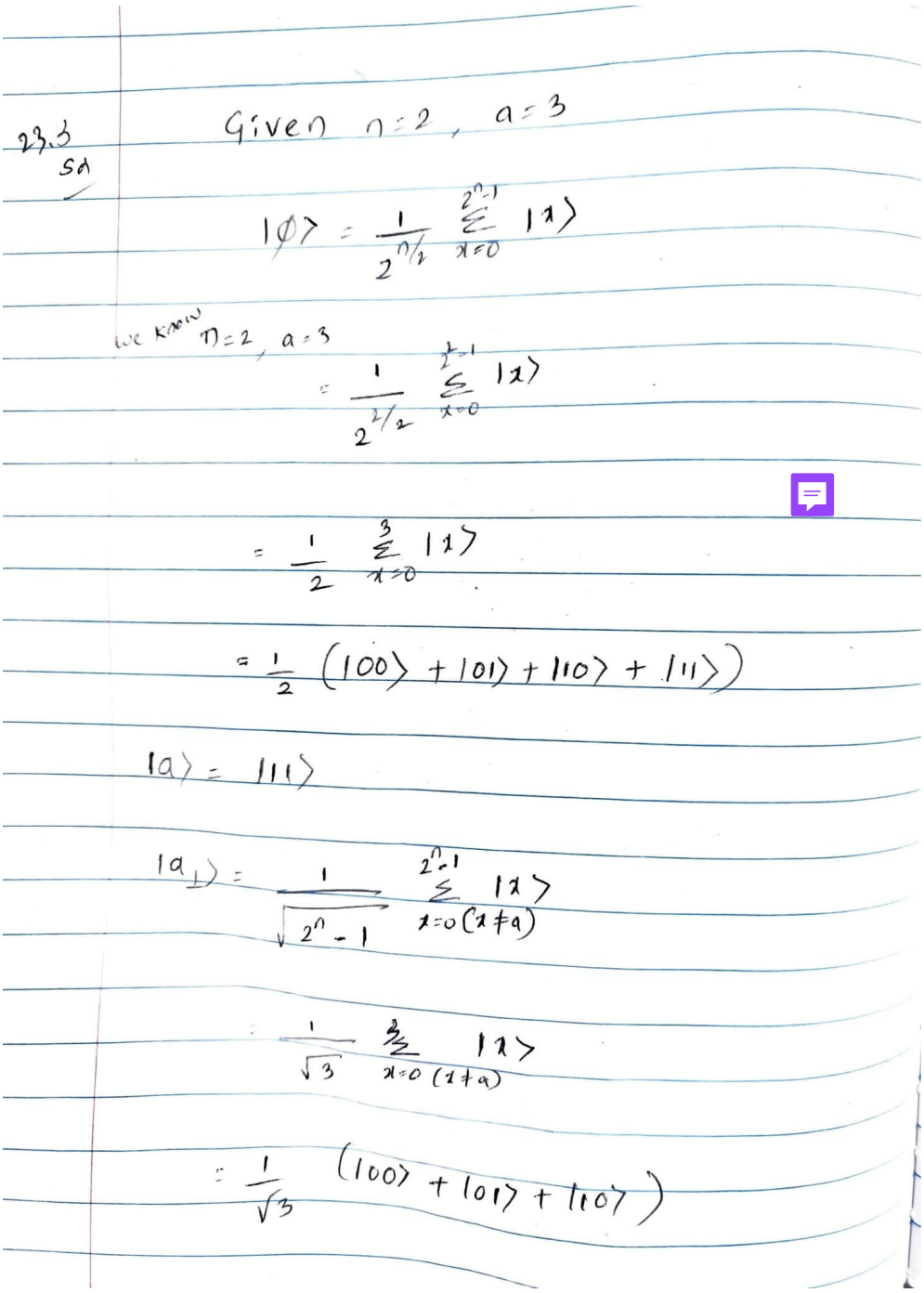


From ~~$V|\phi\rangle = (I - 2|a\rangle\langle a|)|\phi\rangle$~~

$= \sum_{i=0, i\neq a}^{i=1} |\phi\rangle$

Here when operator V is applied to $|\phi\rangle$ then it reflects $|\phi\rangle$ towards negation (phase shift)

23.3)

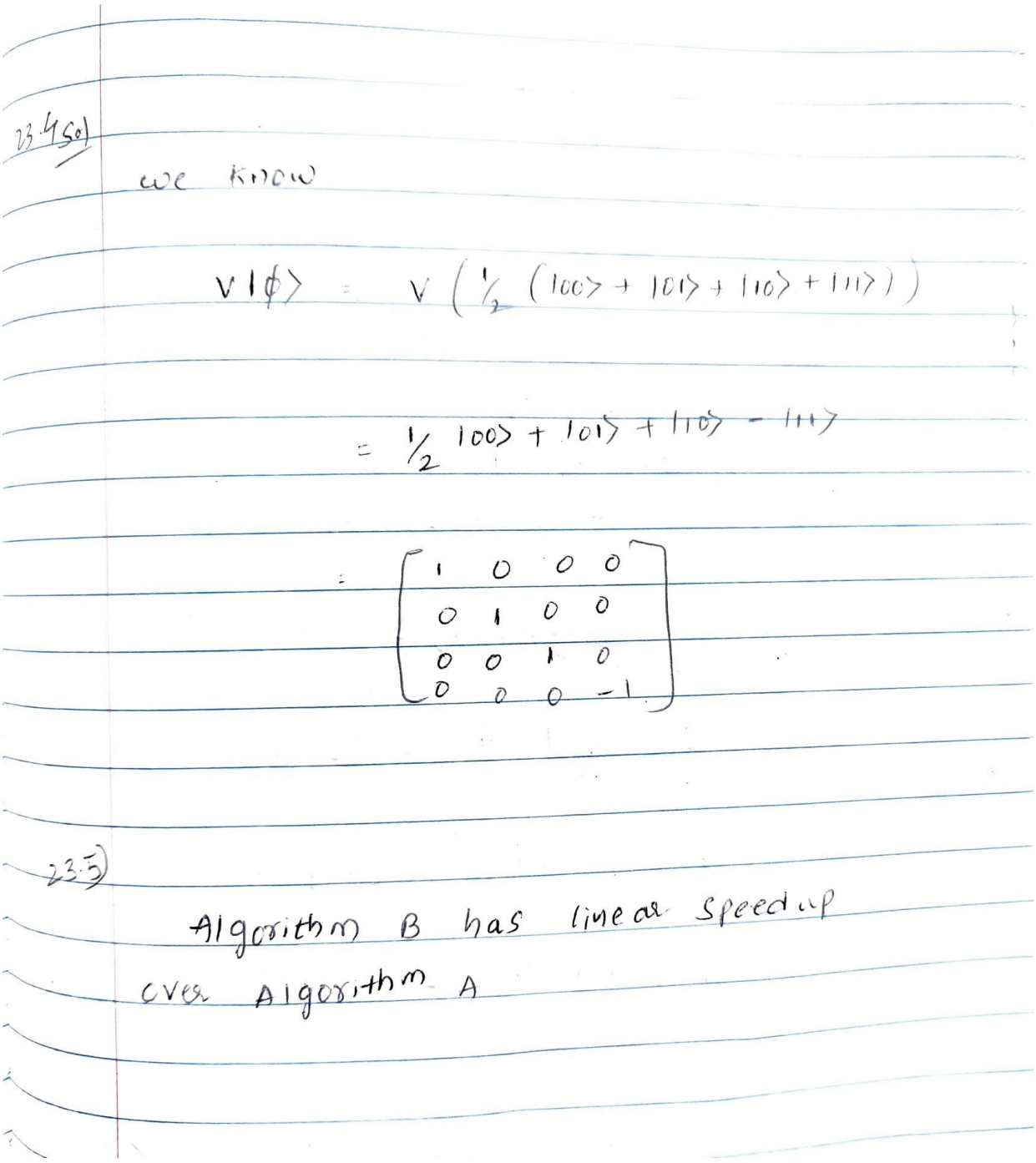


But it is possible to represent V|phi> as a combination of |phi> and |phi_perp>. Firstly, |phi>=cos(theta)|a_perp> +sin(theta)|a> using Eq. (23.5) to calculate cos and sin. Then, V|phi> = cos(theta)|a_perp> - sin(theta)|a> (23.11). Now, we can express |a_perp> (also |a>) as a linear combination of |phi> and |phi_perp>. Then you will get V|phi> as a function of |phi> and |phi_perp>.

We need to find |phi_perp>. We can assume it is m|a> + n|a_perp>. <phi|phi_perp> = 0 and <phi_perp|phi_perp>=1. I got m=sqrt(3)/2 and n=-1/2.

Nithin Reddy Govindugari(nithinreddy1747@gmail.com)

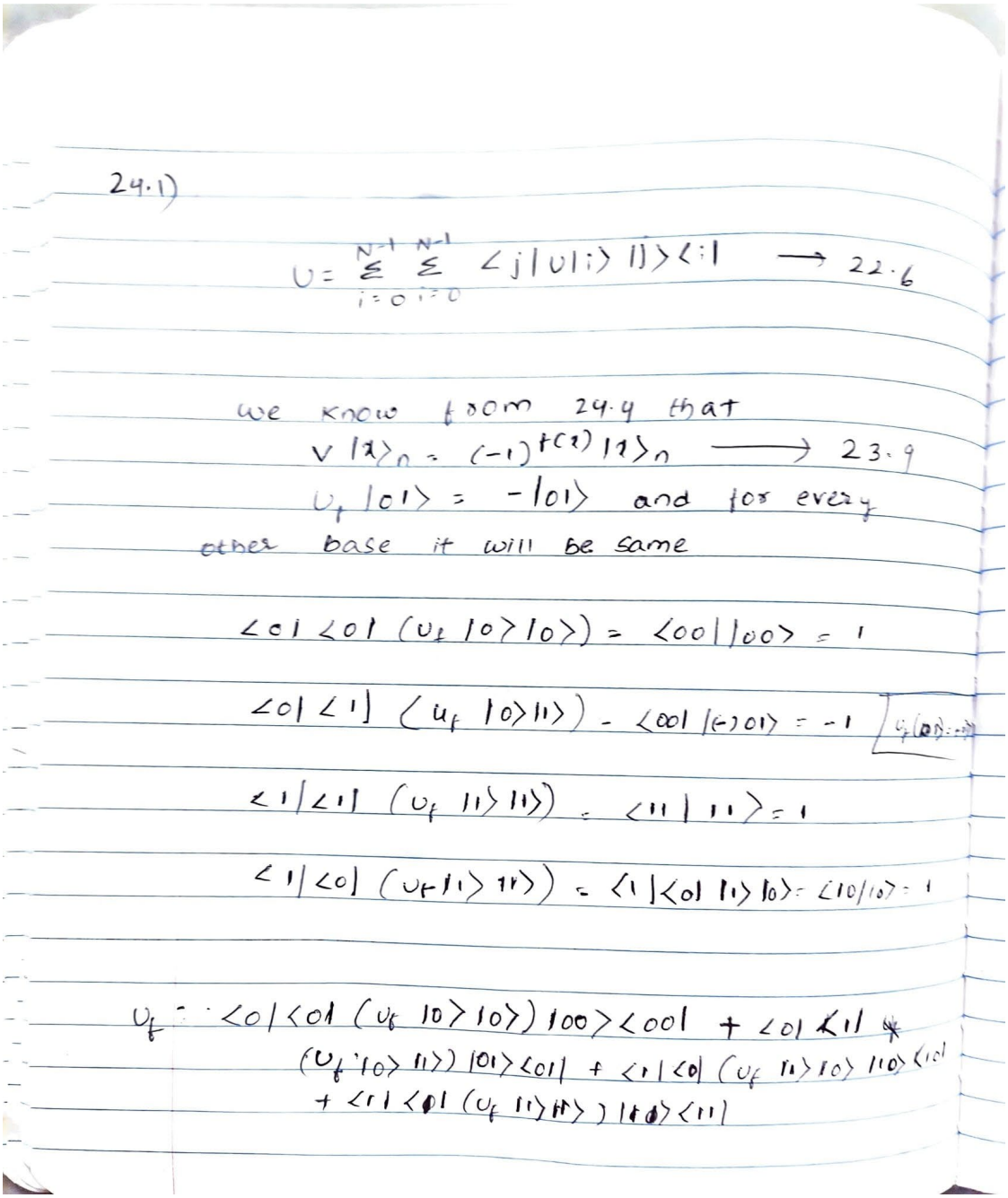
23.4 & 23.5)



Nithin Reddy Govindugari(nithinreddy1747@gmail.com)

Chapter 24

24.1)



24.1)

$$U = \sum_{i=0}^{N-1}\sum_{j=0}^{N-1} \langle j|U|i\rangle |j\rangle\langle i| \longrightarrow 22.6$$

we know from 24.4 that

$$V|x\rangle_n = (-1)^{f(x)}|x\rangle_n \longrightarrow 23.9$$

$U_f|01\rangle = -|01\rangle$ and for every other base it will be same

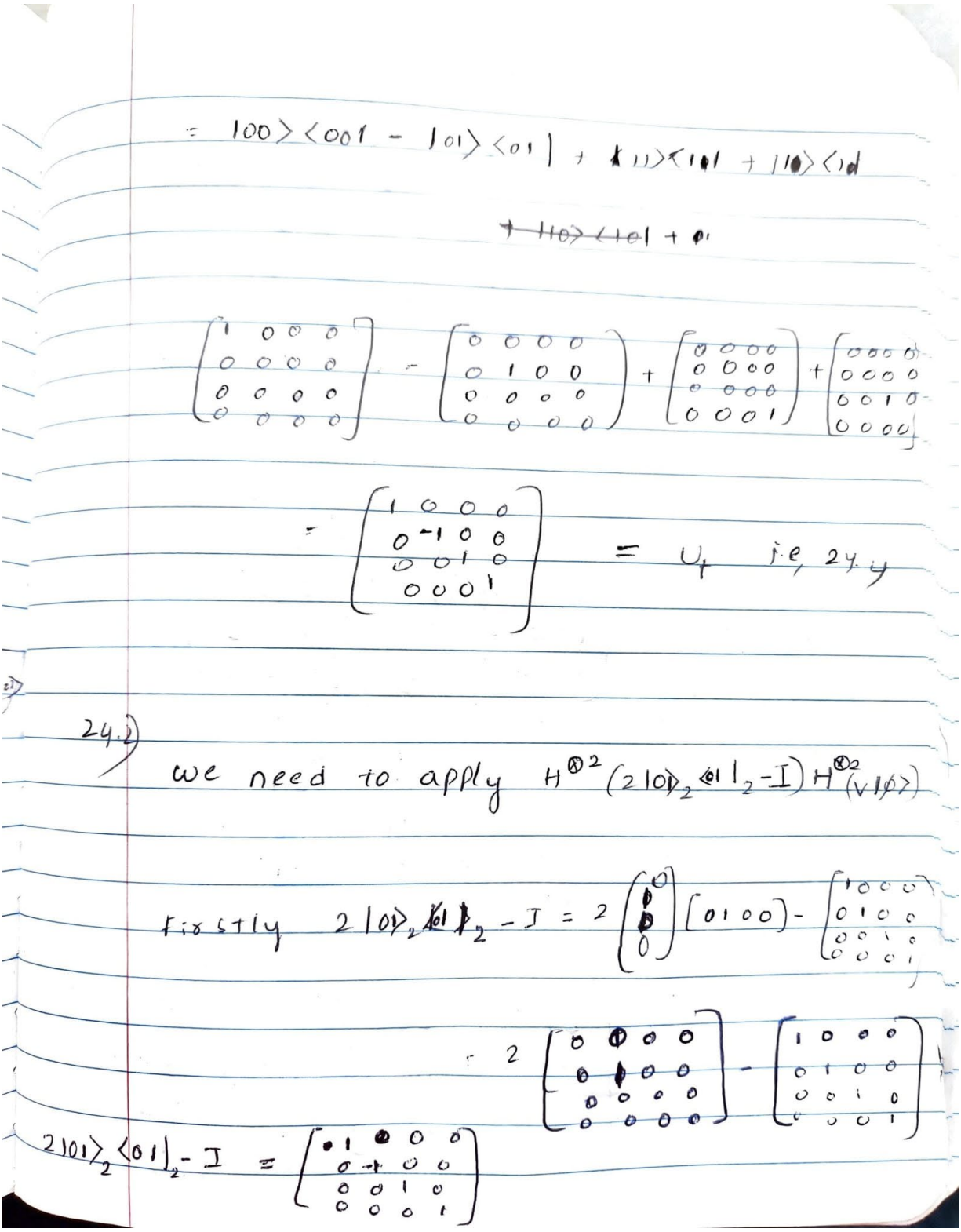
$$\langle 0|\langle 0|(U_f|0\rangle|0\rangle) = \langle 00||00\rangle = 1$$

$$\langle 0|\langle 1|(U_f|0\rangle|1\rangle) = \langle 01|(-|01\rangle) = -1$$

$$\langle 1|\langle 1|(U_f|1\rangle|1\rangle) = \langle 11|11\rangle = 1$$

$$\langle 1|\langle 0|(U_f|1\rangle|1\rangle) = \langle 1|\langle 0|1\rangle|0\rangle = \langle 10|10\rangle = 1$$

$$U_f = \langle 0|\langle 0|(U_f|0\rangle|0\rangle)|00\rangle\langle 00| + \langle 0|\langle 1|(U_f|0\rangle|1\rangle)|01\rangle\langle 01| + \langle 1|\langle 0|(U_f|1\rangle|0\rangle)|10\rangle\langle 10| + \langle 1|\langle 1|(U_f|1\rangle|1\rangle)|11\rangle\langle 11|$$



$= |00\rangle\langle 00| - |01\rangle\langle 01| + |11\rangle\langle 10| + |10\rangle\langle 11|$

~~$+ |10\rangle\langle 10| +$~~

$$\begin{bmatrix} 1&0&0&0\\0&0&0&0\\0&0&0&0\\0&0&0&0 \end{bmatrix} - \begin{bmatrix} 0&0&0&0\\0&1&0&0\\0&0&0&0\\0&0&0&0 \end{bmatrix} + \begin{bmatrix} 0&0&0&0\\0&0&0&0\\0&0&0&0\\0&0&0&1 \end{bmatrix} + \begin{bmatrix} 0&0&0&0\\0&0&0&0\\0&0&1&0\\0&0&0&0 \end{bmatrix}$$

$$= \begin{bmatrix} 1&0&0&0\\0&-1&0&0\\0&0&1&0\\0&0&0&1 \end{bmatrix} = U_f \quad \text{i.e, 24.4}$$

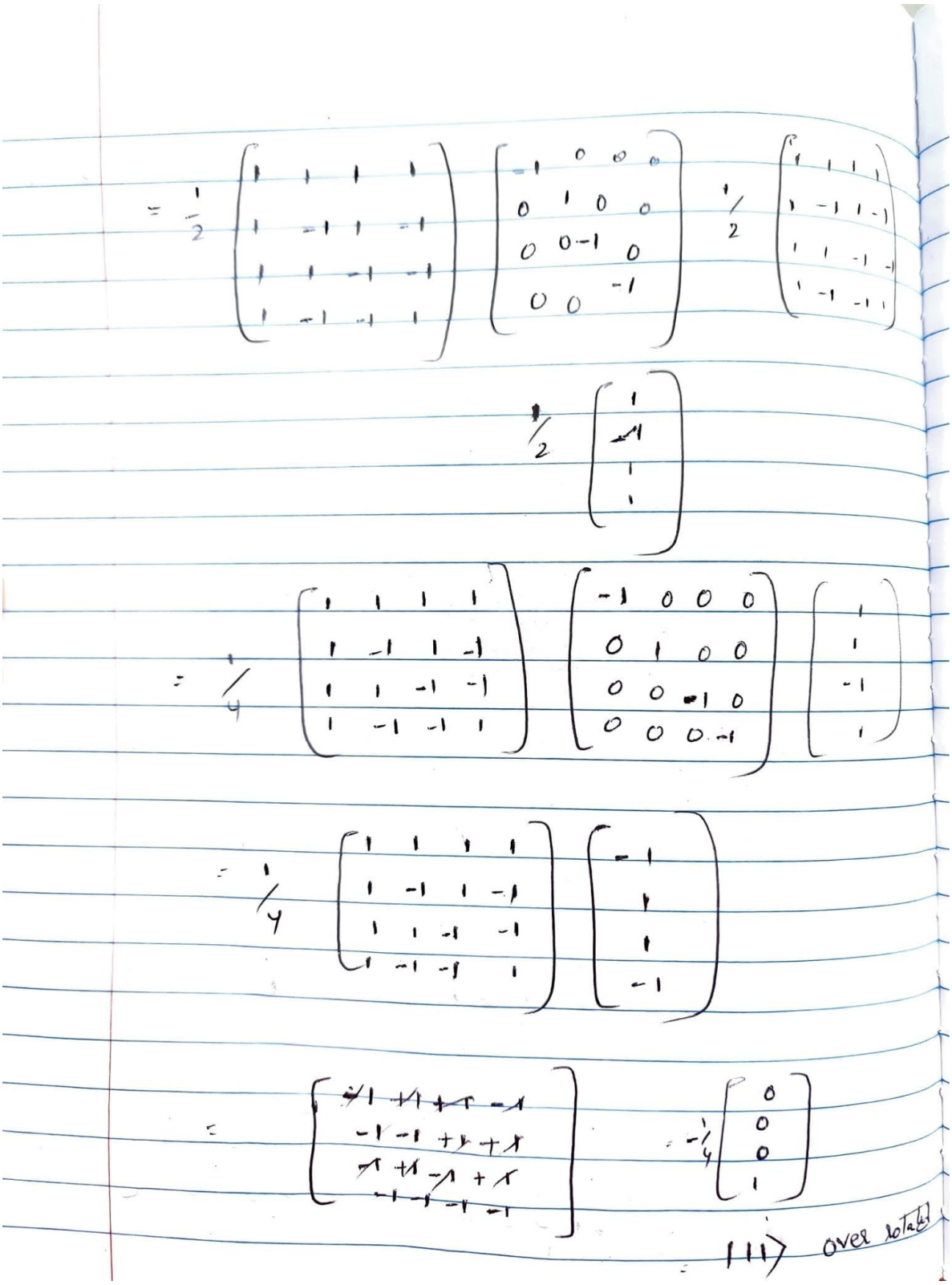
24.2)

we need to apply $H^{\otimes 2}(2|01\rangle_2\langle 01|_2 - I)H^{\otimes 2}(\sqrt{1}|\phi\rangle)$

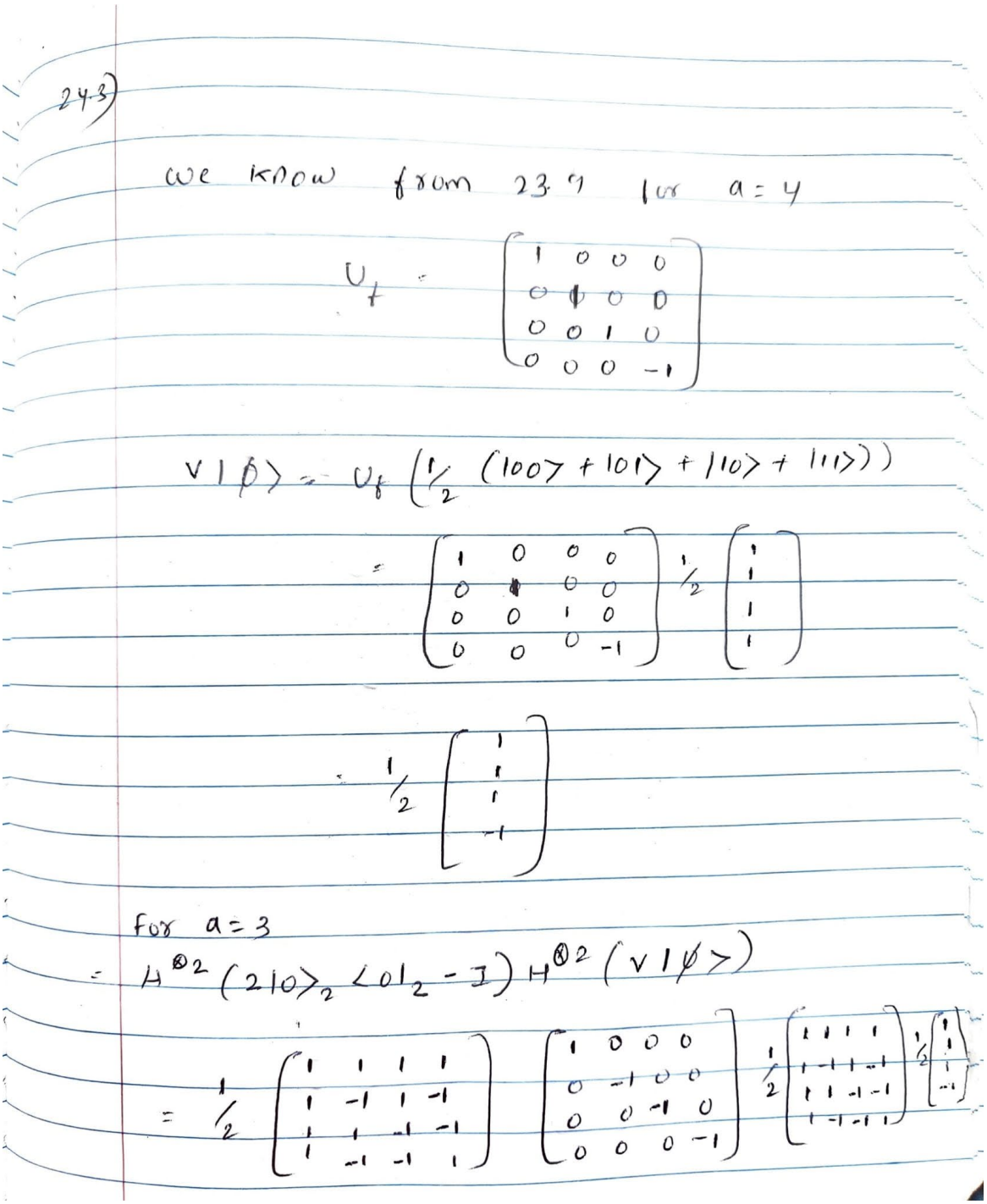
firstly $2|01\rangle_2\langle 01|_2 - I = 2\begin{pmatrix}0\\1\\0\\0\end{pmatrix}\begin{bmatrix}0&1&0&0\end{bmatrix} - \begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}$

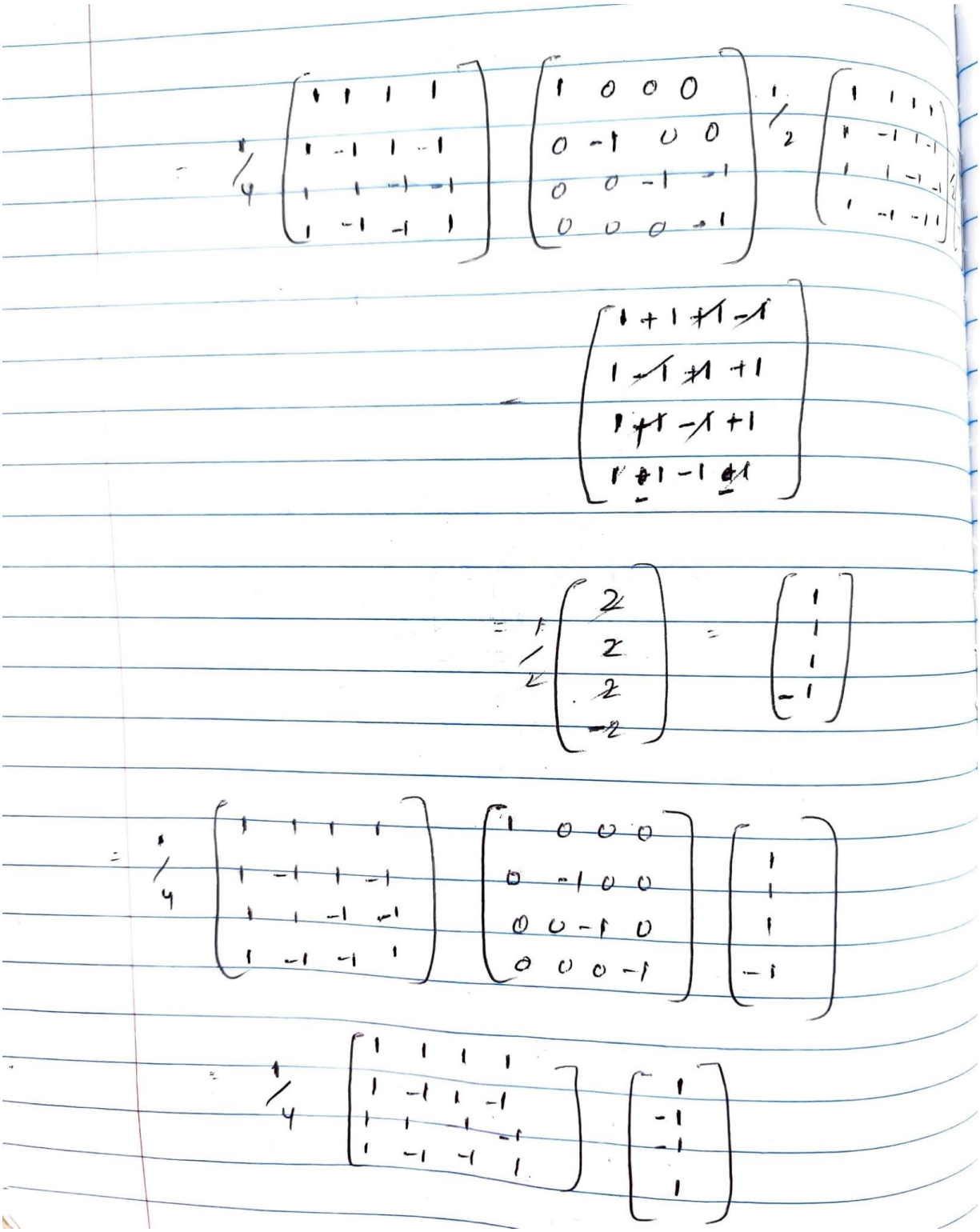
$$= 2\begin{bmatrix}0&0&0&0\\0&1&0&0\\0&0&0&0\\0&0&0&0\end{bmatrix} - \begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}$$

$$2|01\rangle_2\langle 01|_2 - I = \begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}$$



24.3)


24.3)
we know from 23.9 for a = 4
$U_f = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$
$V|\phi\rangle = U_f\left(\frac{1}{2}(|00\rangle + |01\rangle + |10\rangle + |11\rangle)\right)$
$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \frac{1}{2}\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$
$= \frac{1}{2}\begin{bmatrix} 1 \\ 1 \\ 1 \\ -1 \end{bmatrix}$
for a = 3
$= H^{\otimes 2}(2|0\rangle_2\langle 0|_2 - I)H^{\otimes 2}(V|\phi\rangle)$
$= \frac{1}{2}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}\frac{1}{2}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix}\frac{1}{2}\begin{bmatrix} 1 \\ 1 \\ 1 \\ -1 \end{bmatrix}$



$$= \frac{1}{4}\begin{bmatrix} 1 & -1 & -1 & +1 \\ 1 & +1 & -1 & -1 \\ 1 & +1 & +1 & -1 \\ 1 & +1 & +1 & +1 \end{bmatrix} = \frac{1}{4}\begin{pmatrix} 0 \\ 0 \\ 0 \\ 4 \end{pmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$= |11\rangle$$

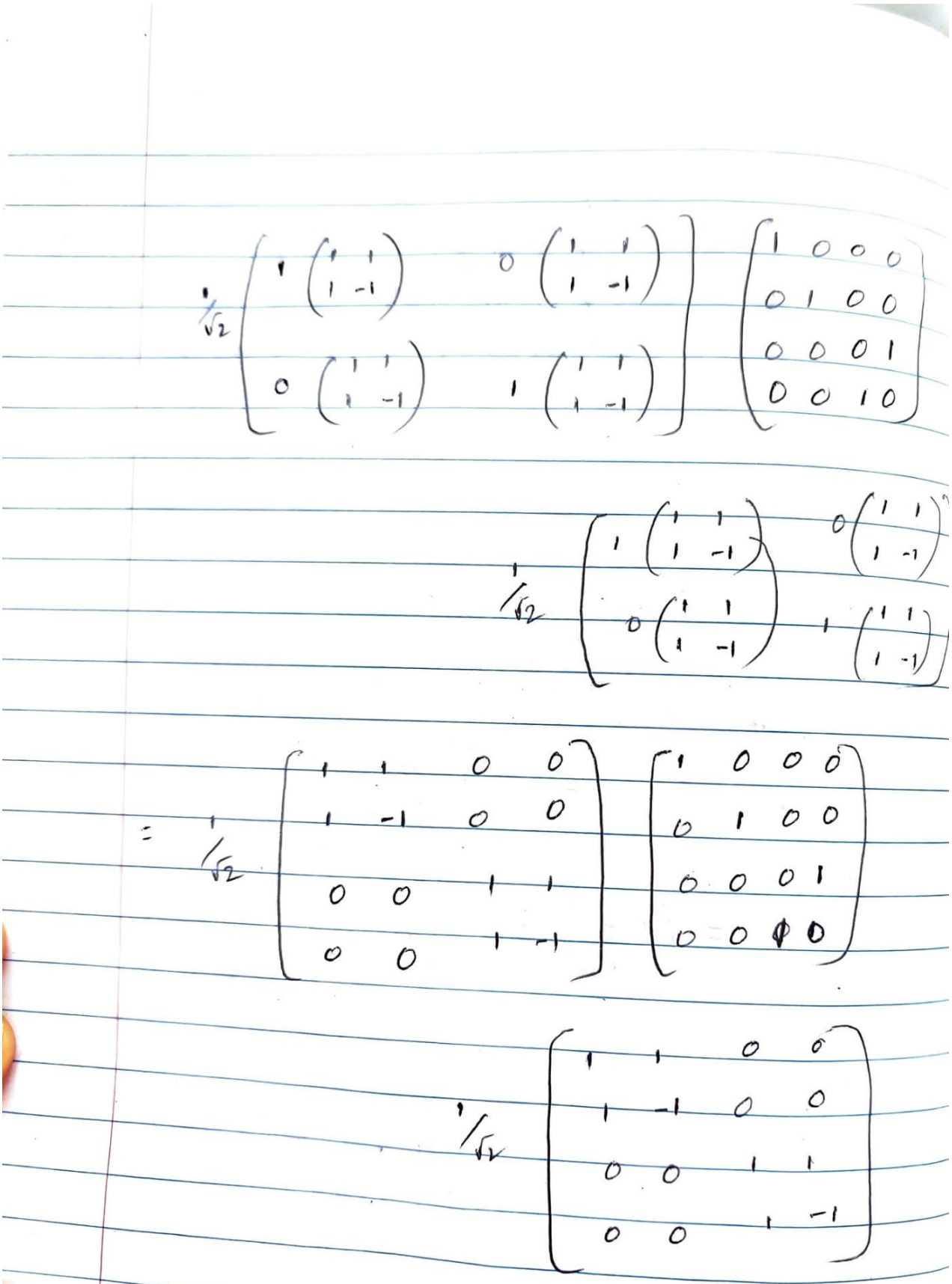
24.4)

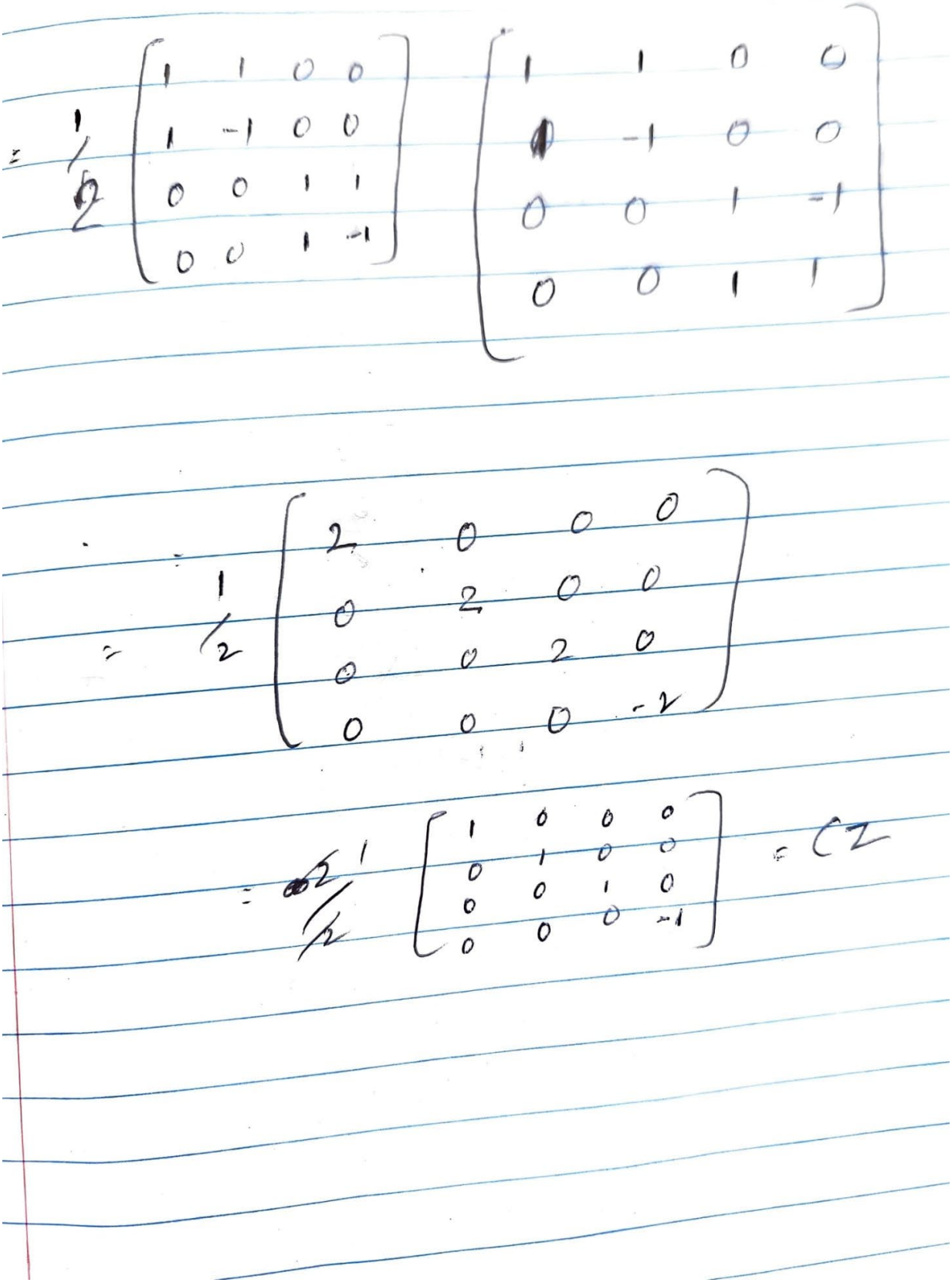
from 24.15

$$C_Z = (I \otimes H)\, U_{XOR}\, (I \otimes H)$$

$$= \left(\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \otimes \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\right)\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

$$\left(\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \otimes \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\right)$$





$$= \frac{1}{2}\begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & -2 \end{bmatrix}$$

$$= \frac{2}{2}\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} = CZ$$

24.5)

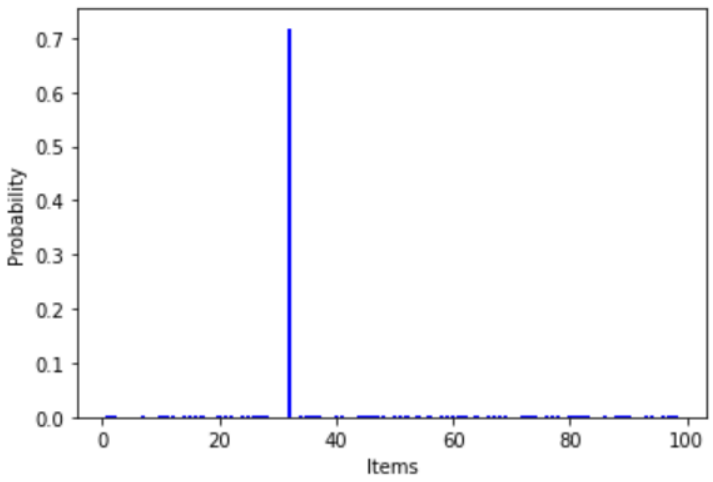
```
#24.5
#Keywords
#superposition is sup_pos
#amplitude  is amp

#Grovers Algorithm is implemented using standard procedure.
#Steps followed in books are implemented and tried to create classical grovers algorithm in different

plots = []

# Putting all qubits in superposition using hadamard gate
def  sup_pos(qubits):
    states = []
    total_states = int(math.pow(2,qubits))
    amp = 1/math.sqrt(total_states)
    for _ in range(0,total_states):
        states.append(amp)
    return states
```

```
# phase flip inverts the states amplitude
def grover_diffusion(states):
    average = sum(states)/len(states)
    for i in range(0,len(states)):
        states[i] = (average-states[i]) + average         #inversion about mean
    return states

#oracle to implement phase inversion
def oracle(states,datalist,key):
    for i in range(0,int(len(datalist))):
        if datalist[i] == key:
            states[i] *= -1                    #phase inversion
    return states
```

```
#classical main grover search function
def grover_search(qubits,datalist,key):
    states = sup_pos(qubits)
    num_iterations  = math.ceil(math.sqrt(math.pow(2,qubits)))        #iterations
    probability_states = []
    for _ in range(0,num_iterations):
        states = oracle(states,datalist,key)
        states = grover_diffusion(states)
        probability_states = [states[i]*states[i] for i in range(0,len(states))]
        plots.append(probability_states)
    return probability_states


def grover(datalist,key):
    size_datalist = len(datalist)
    qubits_needed = math.ceil(math.log(size_datalist,2))
    paddings_required = int(math.pow(2,qubits_needed) - size_datalist)
    #required if the number of data items is not a power of 2.
    for _ in range(0,paddings_required):
         datalist.append(0)
    grover_search(qubits_needed,datalist,key)
```

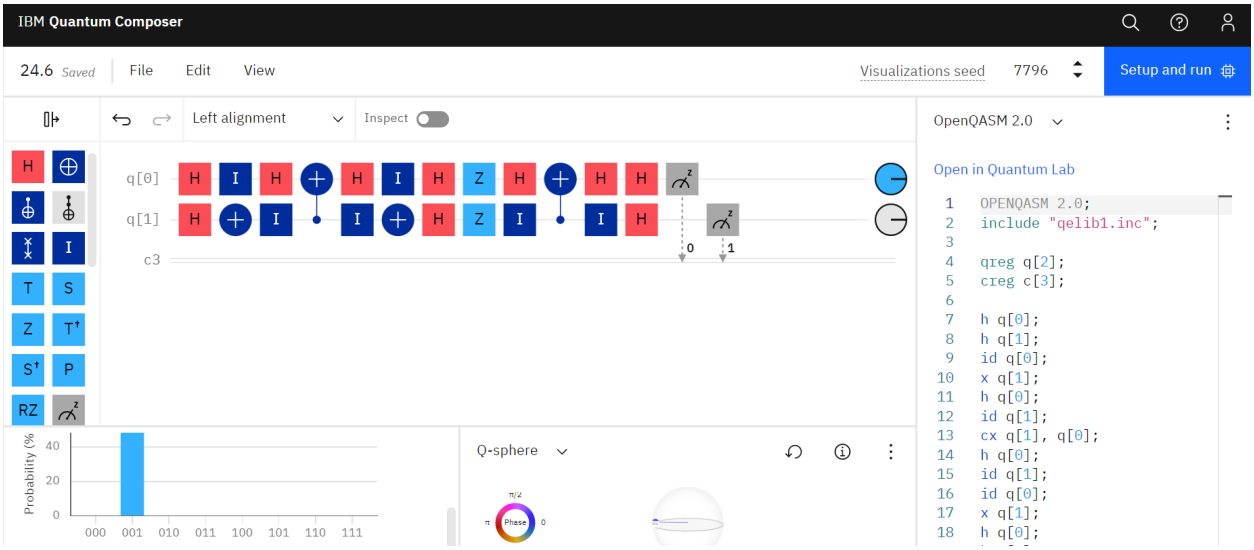
```
datalist = random.sample(range(1, 100), 64)
grover(datalist,32)
print(datalist)
print("plots showing the change in probabilities of items after every grover's iteration :")
result ={}
iteration = 1
for plot in plots:
    for i in range(len(plot)):
        result[datalist[i]] = plot[i]
    print("-------------------------------------------------------------------------------------------
    print("Iteration ",iteration," :")
    plt.bar(result.keys(), result.values(),color='b')
    plt.ylabel("Probability")
    plt.xlabel("Items")
    plt.show()
    iteration +=1
    print("-------------------------------------------------------------------------------------------
```

```
Iteration  8  :
```

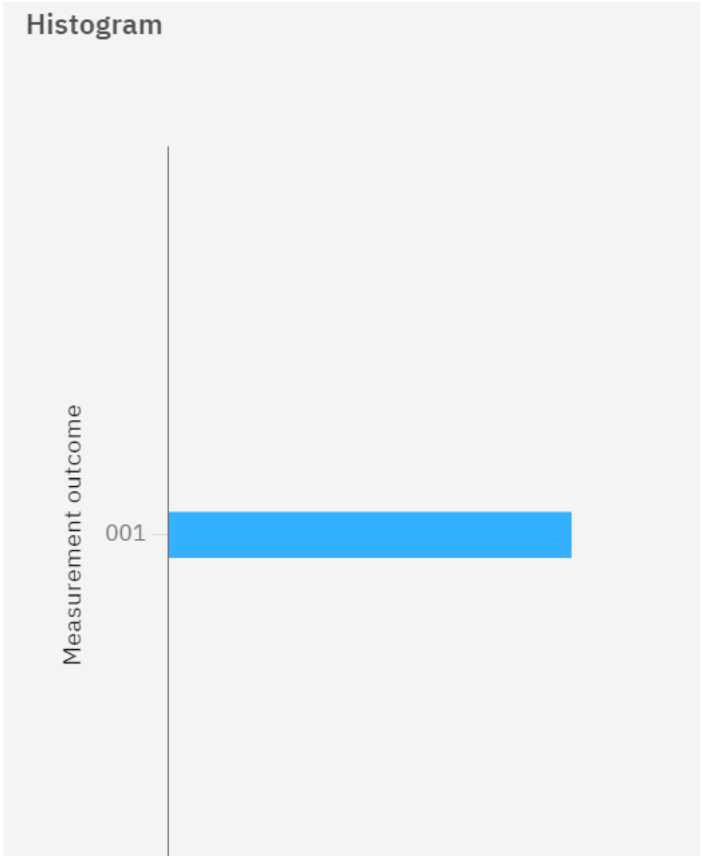


24.6)

Introduction to Quantum computing (From a layperson to programmer in 30 steps)

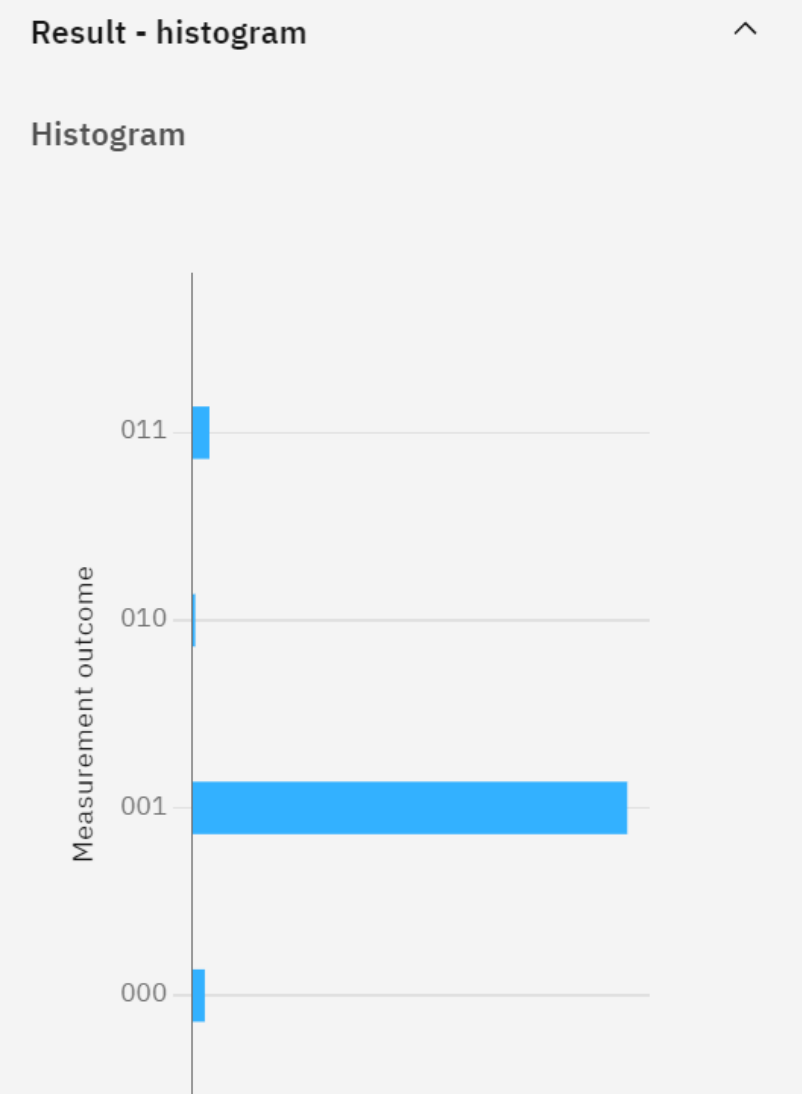


Simulation



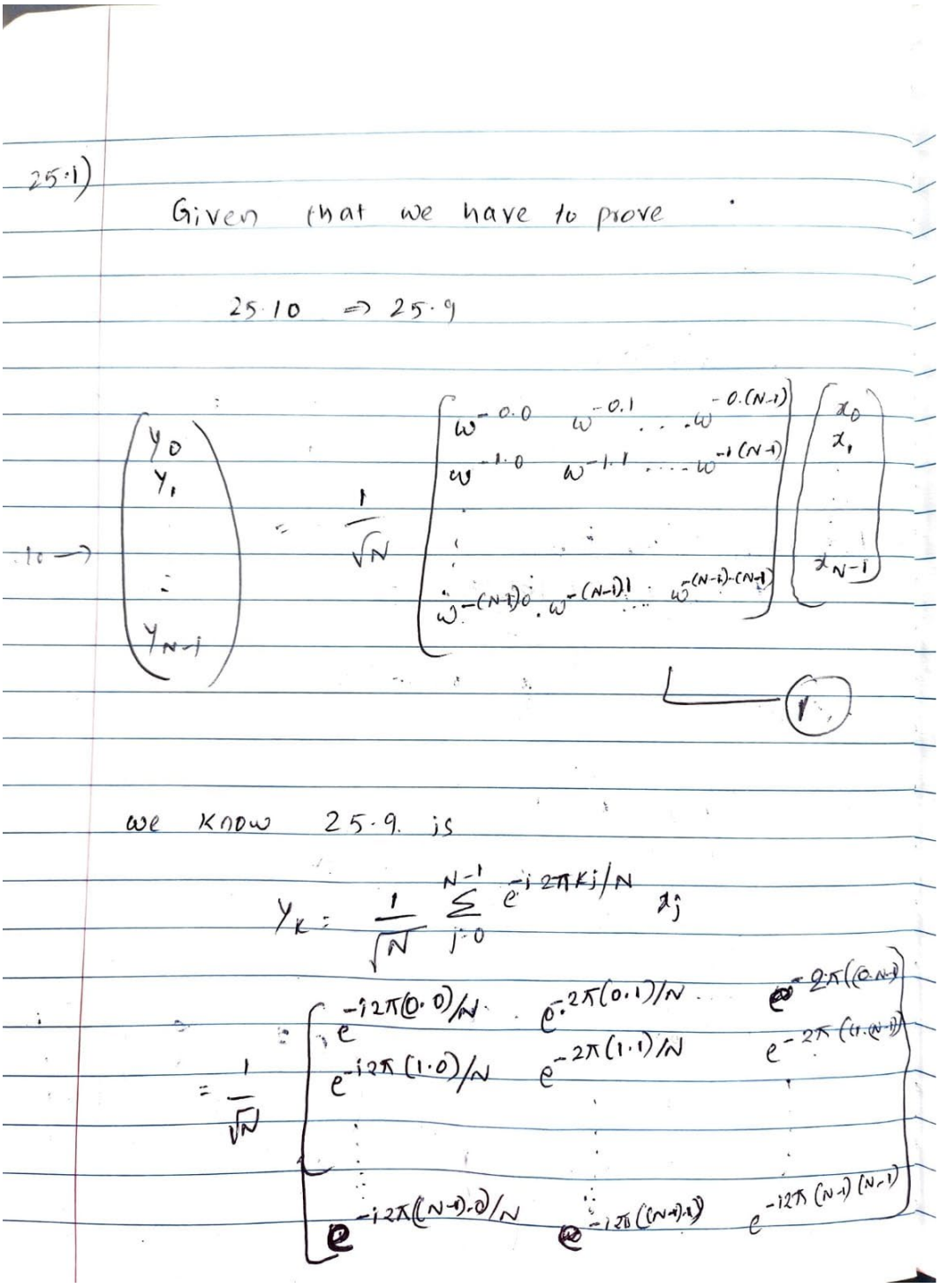
Hardware circuit result

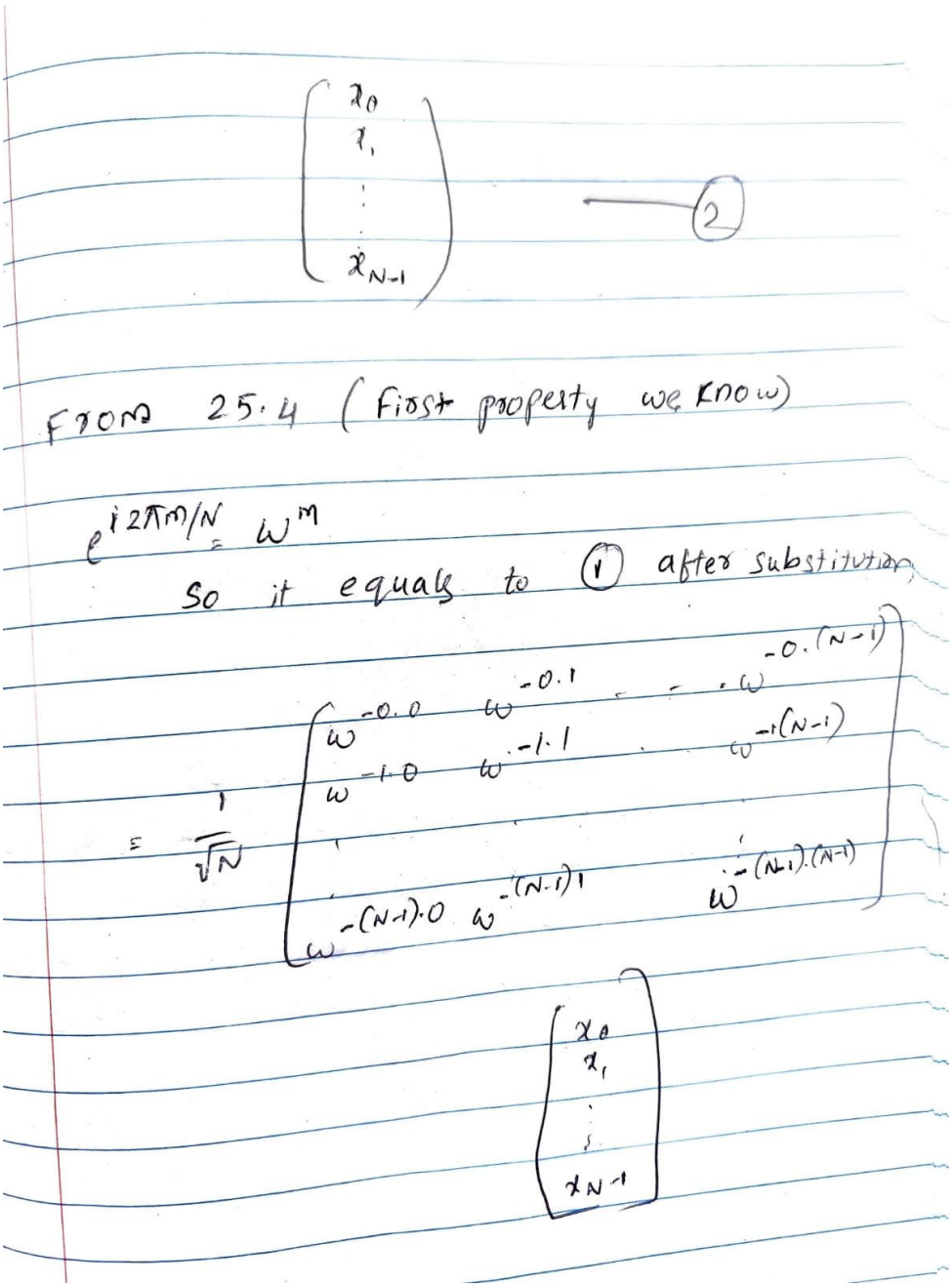
Nithin Reddy Govindugari(nithinreddy1747@gmail.com)



Nithin Reddy Govindugari(nithinreddy1747@gmail.com)

Chapter 25




$\begin{pmatrix} x_0 \\ x_1 \\ \vdots \\ x_{N-1} \end{pmatrix}$ — (2)
From 25.4 (First property we know)
$e^{i2\pi m/N} = \omega^m$
So it equals to (1) after substitution
$= \frac{1}{\sqrt{N}} \begin{pmatrix} \omega^{-0.0} & \omega^{-0.1} & \cdots & \omega^{-0.(N-1)} \\ \omega^{-1.0} & \omega^{-1.1} & & \omega^{-1(N-1)} \\ \vdots & & & \vdots \\ \omega^{-(N-1).0} & \omega^{-(N-1)1} & & \omega^{-(N-1).(N-1)} \end{pmatrix}$
$\begin{pmatrix} x_0 \\ x_1 \\ \vdots \\ x_{N-1} \end{pmatrix}$

25.2)

25.2 yes. U_{QFT} is Hermitian

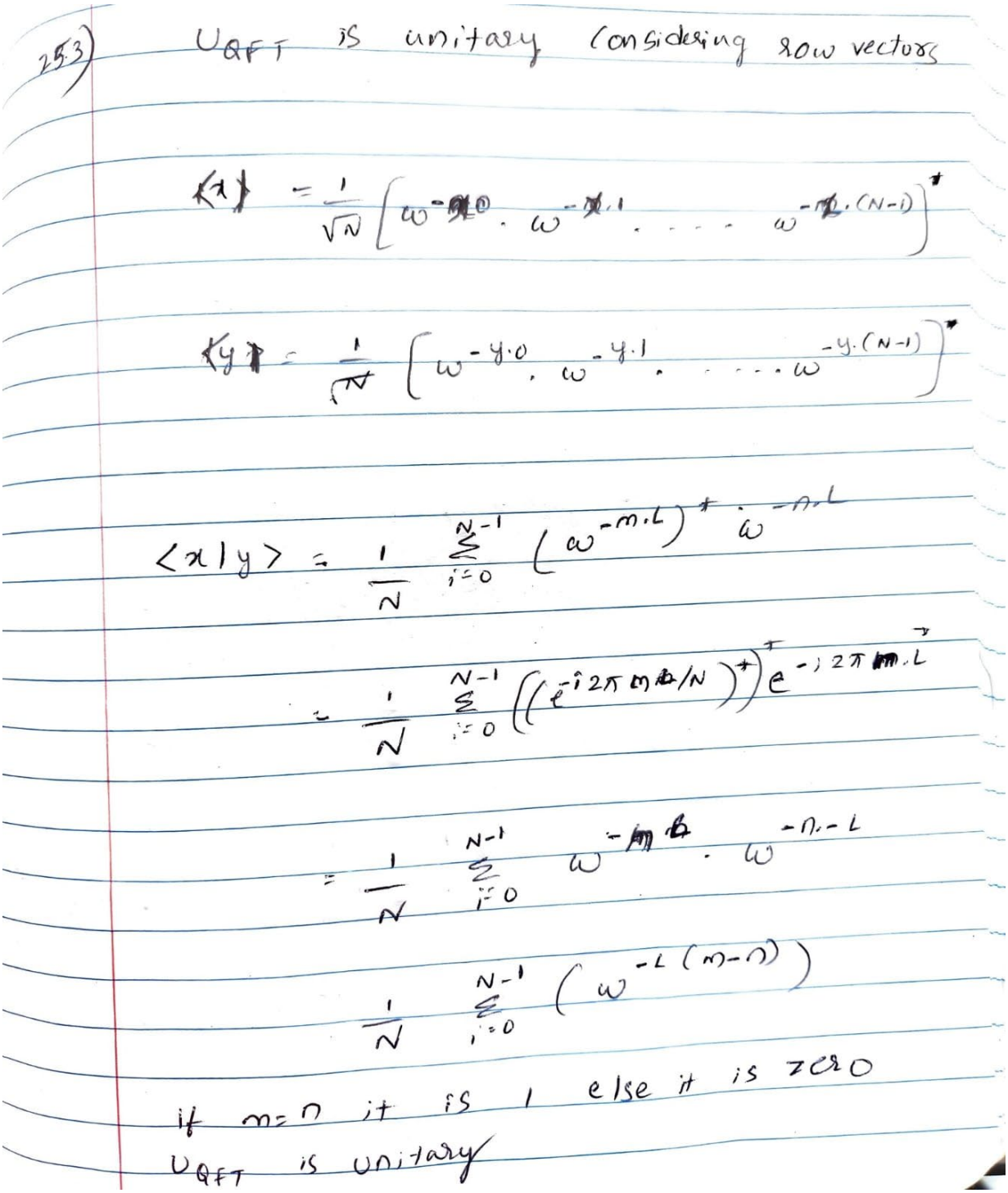
From 25.20 we know

$$U_{QFT} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$ i.e, equals to Hadamard gate

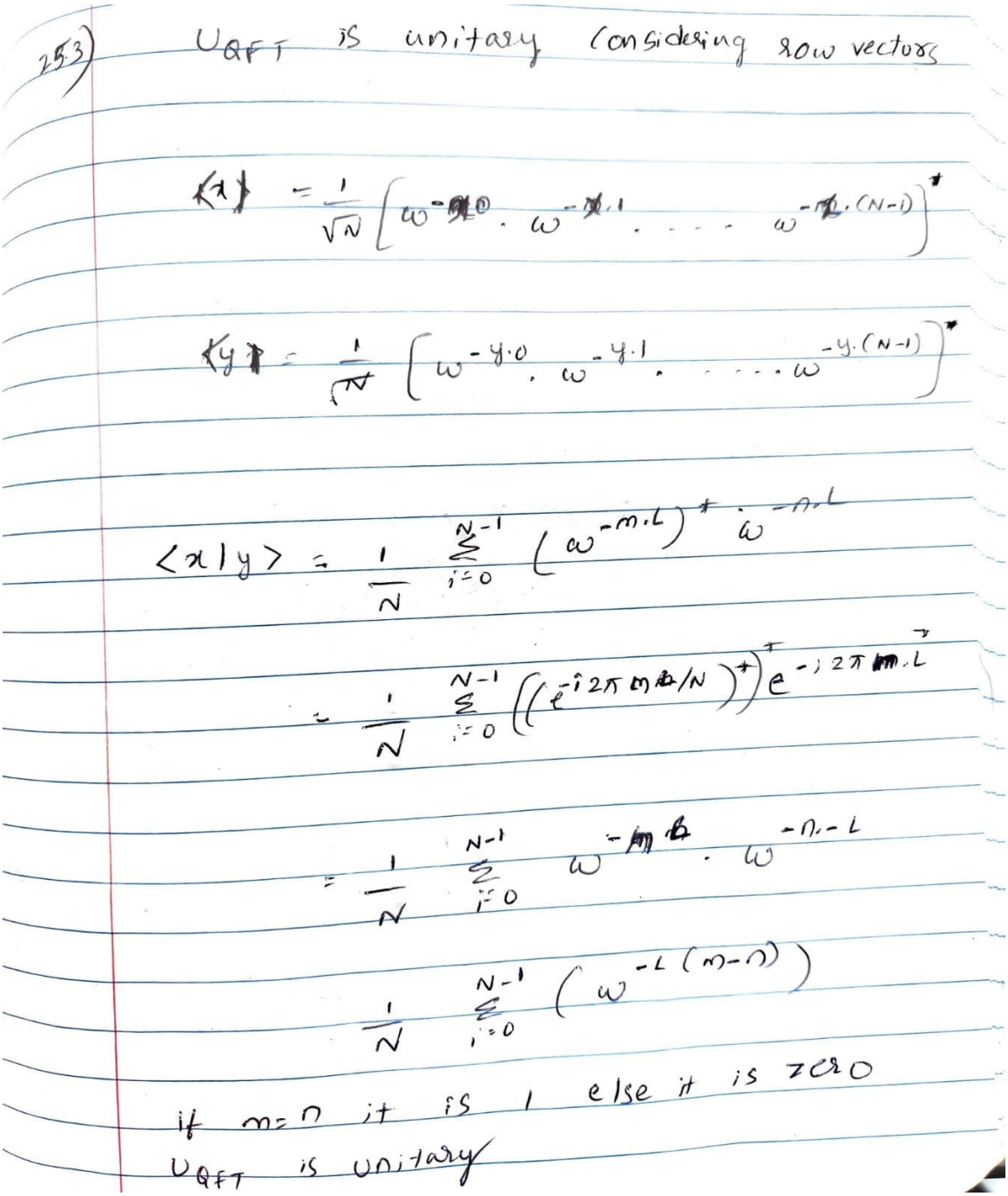
$$U_{QFT}^{\dagger} = U_{QFT}$$ (Hermitian Rule is satisfied)

Here, U_{QFT} is Symmetric (25.12) & satisfies Hermitian requirements. I believe it should be Hermitian because according to Fourier transforms domains changes i.e, bases in QFT it should remain same

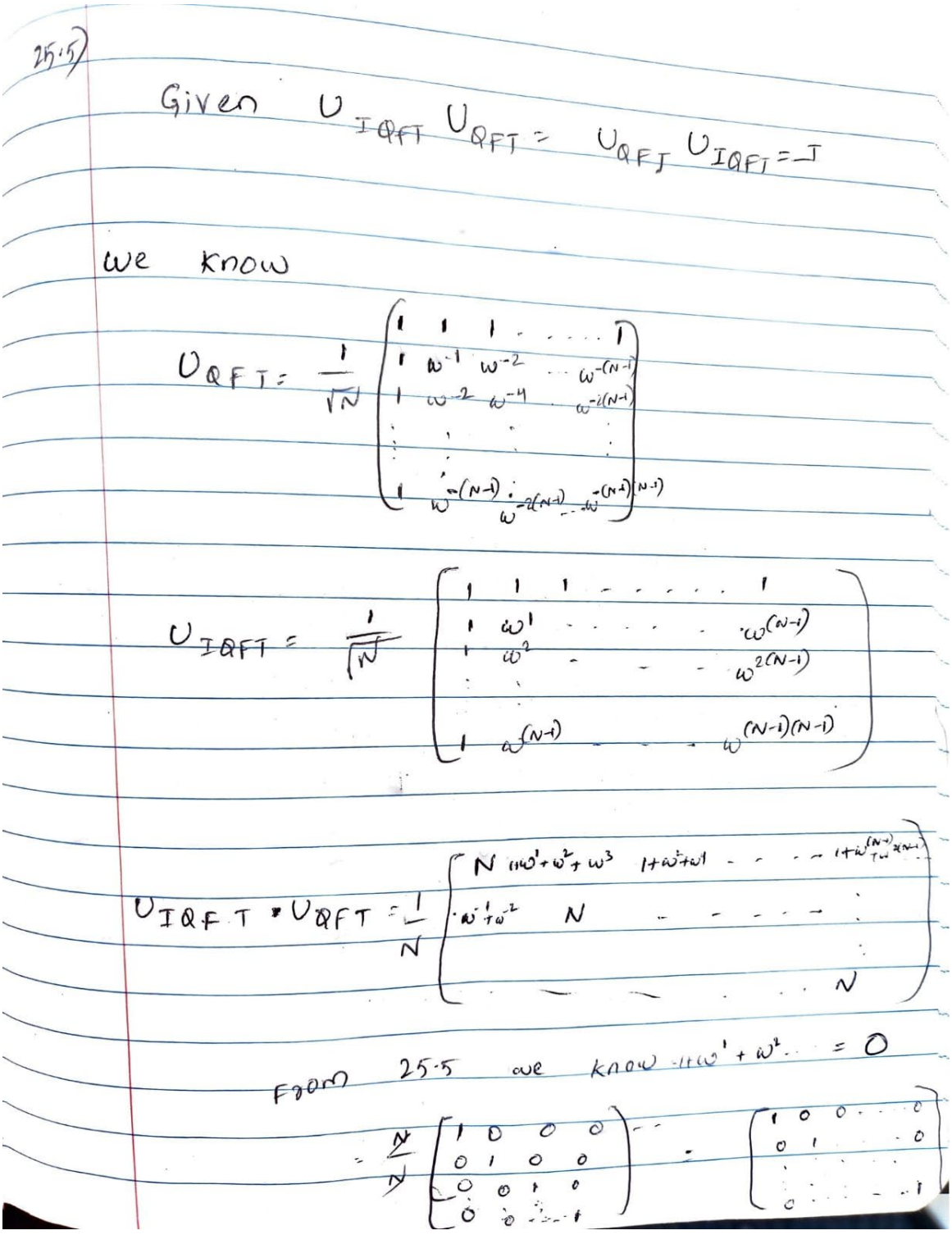
25.3)

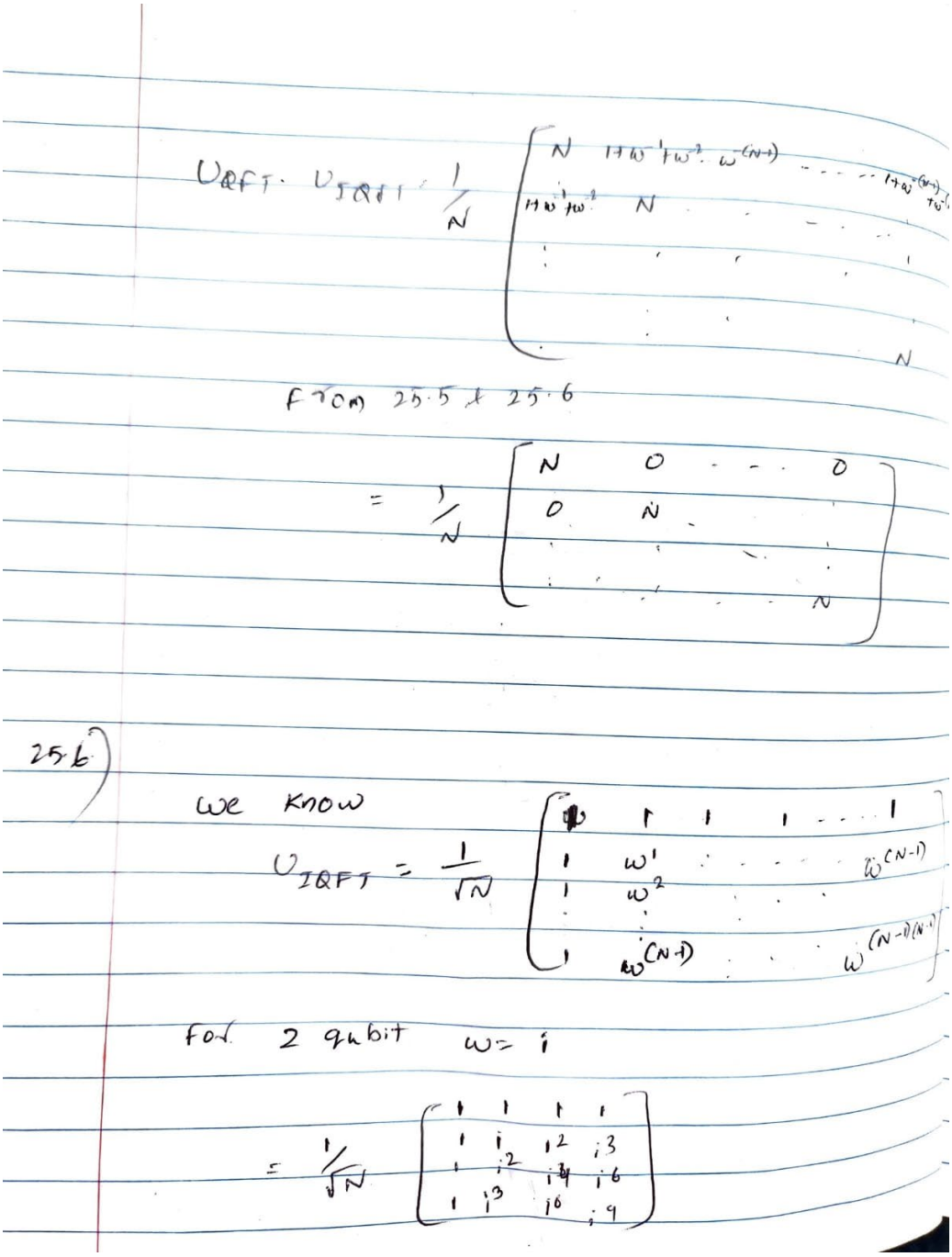

25.3) U_{QFT} is unitary considering row vectors
$\langle x| = \frac{1}{\sqrt{N}} \left[\omega^{-x.0} \cdot \omega^{-x.1} \cdot \ldots \cdot \omega^{-x.(N-1)} \right]^*$
$\langle y| = \frac{1}{\sqrt{N}} \left[\omega^{-y.0} \cdot \omega^{-y.1} \cdot \ldots \cdot \omega^{-y.(N-1)} \right]^*$
$\langle x|y\rangle = \frac{1}{N} \sum_{i=0}^{N-1} \left(\omega^{-m.L}\right)^* \cdot \omega^{-n.L}$
$= \frac{1}{N} \sum_{i=0}^{N-1} \left(\left(e^{-i 2\pi m L/N} \right)^* \right) e^{-i 2\pi m.L}$
$= \frac{1}{N} \sum_{i=0}^{N-1} \omega^{-mL} \cdot \omega^{-n.L}$
$\frac{1}{N} \sum_{i=0}^{N-1} \left(\omega^{-L(m-n)} \right)$
if $m=n$ it is 1 else it is zero
U_{QFT} is unitary

25.4)


25.3) U_{QFT} is unitary considering row vectors
$\langle x| = \frac{1}{\sqrt{N}} \left[\omega^{-x.0} \cdot \omega^{-x.1} \cdots \omega^{-x.(N-1)} \right]^*$
$\langle y| = \frac{1}{\sqrt{N}} \left[\omega^{-y.0} \cdot \omega^{-y.1} \cdots \omega^{-y.(N-1)} \right]^*$
$\langle x|y\rangle = \frac{1}{N} \sum_{i=0}^{N-1} (\omega^{-m.L})^* \cdot \omega^{-n.L}$
$= \frac{1}{N} \sum_{i=0}^{N-1} \left((e^{-i2\pi m/N})^* e^{-i2\pi m.L} \right)^*$
$= \frac{1}{N} \sum_{i=0}^{N-1} \omega^{-m} \cdot \omega^{-n.L}$
$\frac{1}{N} \sum_{i=0}^{N-1} \left(\omega^{-L(m-n)} \right)$
if $m=n$ it is 1 else it is zero
U_{QFT} is unitary

25.5)


25.5)
Given $U_{IQFT} U_{QFT} = U_{QFT} U_{IQFT} = I$
we know
$U_{QFT} = \frac{1}{\sqrt{N}} \begin{pmatrix} 1 & 1 & 1 & \cdots & 1 \\ 1 & \omega^{-1} & \omega^{-2} & \cdots & \omega^{-(N-1)} \\ 1 & \omega^{-2} & \omega^{-4} & \cdots & \omega^{-2(N-1)} \\ \vdots & & & & \\ 1 & \omega^{-(N-1)} & \omega^{-2(N-1)} & \cdots & \omega^{-(N-1)(N-1)} \end{pmatrix}$
$U_{IQFT} = \frac{1}{\sqrt{N}} \begin{pmatrix} 1 & 1 & 1 & \cdots & 1 \\ 1 & \omega^{1} & \cdots & \cdots & \omega^{(N-1)} \\ 1 & \omega^{2} & \cdots & \cdots & \omega^{2(N-1)} \\ \vdots & & & & \\ 1 & \omega^{(N-1)} & \cdots & \cdots & \omega^{(N-1)(N-1)} \end{pmatrix}$
$U_{IQFT} * U_{QFT} = \frac{1}{N} \begin{pmatrix} N & 1+\omega^{1}+\omega^{2}+\omega^{3} & 1+\omega^{2}+\omega^{4} & \cdots & 1+\omega^{(N-1)}+\omega^{2(N-1)} \\ 1+\omega^{1}+\omega^{2} & N & \cdots & \cdots & \vdots \\ \vdots & & & & \\ & & & & N \end{pmatrix}$
From 25.5 we know $1+\omega^{1}+\omega^{2}\ldots = 0$
$= \frac{N}{N} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & \cdots & 1 \end{pmatrix} \cdots \begin{pmatrix} 1 & 0 & 0 & \cdots & 0 \\ 0 & 1 & & & 0 \\ \vdots & & & & \\ 0 & \cdots & & & 1 \end{pmatrix}$



$$U_{QFT} \cdot U_{IQFT} = \frac{1}{N} \begin{bmatrix} N & 1+\omega^{1}+\omega^{2}+\omega^{(N-1)} & \cdots & 1+\omega^{(N-1)}+\omega^{(N-1)} \\ 1+\omega^{1}+\omega^{2} & N & & \\ \vdots & & \ddots & \\ & & & N \end{bmatrix}$$

From 25.5 & 25.6

$$= \frac{1}{N} \begin{bmatrix} N & 0 & \cdots & 0 \\ 0 & N & & \\ \vdots & & \ddots & \\ & & & N \end{bmatrix}$$

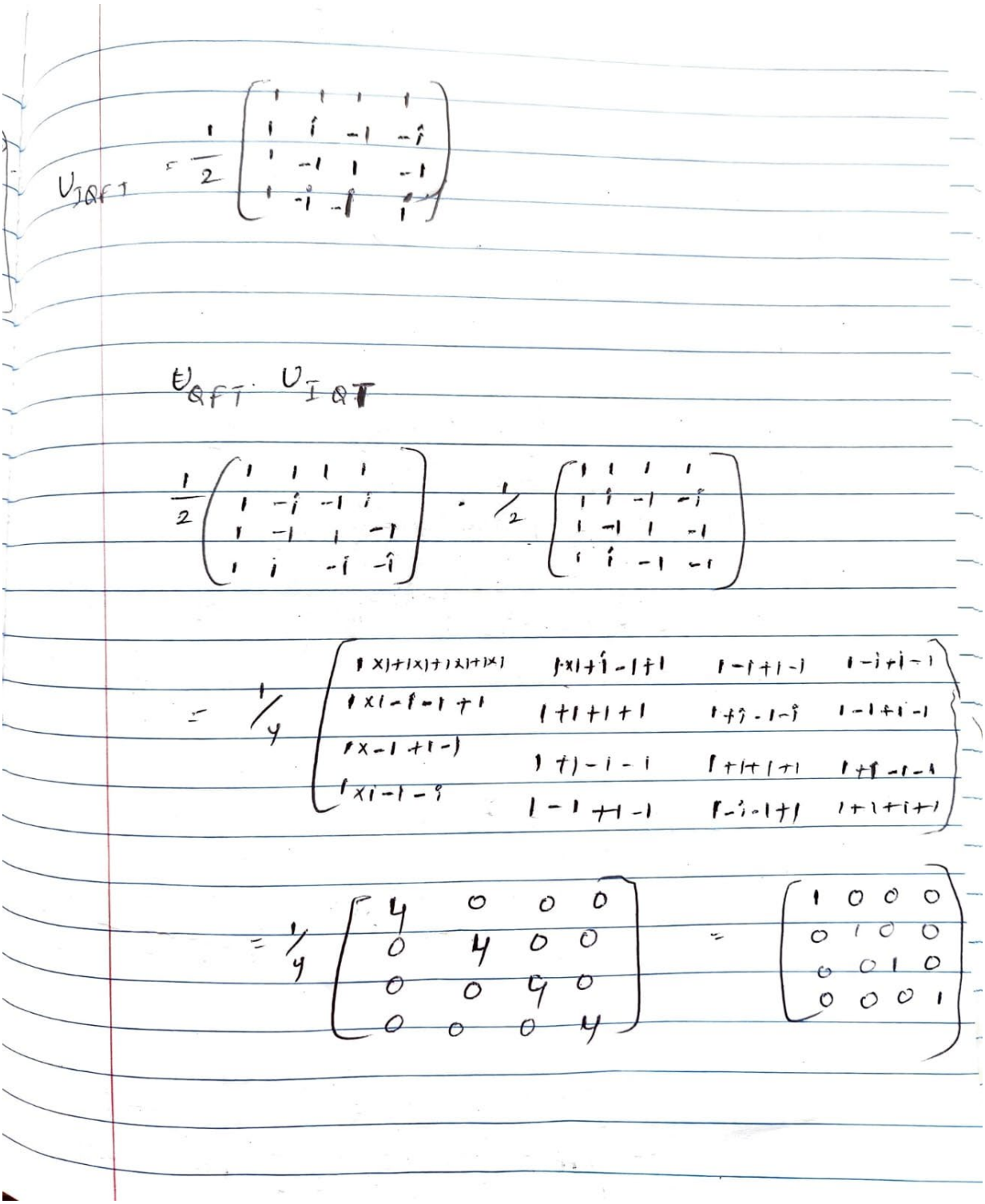
25.6)

We know

$$U_{IQFT} = \frac{1}{\sqrt{N}} \begin{bmatrix} 1 & 1 & 1 & \cdots & 1 \\ 1 & \omega^{1} & & & \omega^{(N-1)} \\ 1 & \omega^{2} & & & \\ \vdots & & & & \\ 1 & \omega^{(N-1)} & & & \omega^{(N-1)(N-1)} \end{bmatrix}$$

For 2 qubit $\omega = i$

$$= \frac{1}{\sqrt{N}} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & i & i^{2} & i^{3} \\ 1 & i^{2} & i^{4} & i^{6} \\ 1 & i^{3} & i^{6} & i^{9} \end{bmatrix}$$



$$U_{IQFT} = \frac{1}{2}\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & i & -1 & -i \\ 1 & -1 & 1 & -1 \\ 1 & -i & -1 & i \end{pmatrix}$$

$$U_{QFT} \cdot U_{IQFT}$$

$$\frac{1}{2}\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & -i & -1 & i \\ 1 & -1 & 1 & -1 \\ 1 & i & -i & -i \end{pmatrix} \cdot \frac{1}{2}\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & i & -1 & -i \\ 1 & -1 & 1 & -1 \\ 1 & i & -1 & -i \end{pmatrix}$$

$$= \frac{1}{4}\begin{pmatrix} 1\times1+1\times1+1\times1+1\times1 & 1\times1+1\times i-1\times1+1\times-i & 1\times1+1\times-1+1\times1+1\times-1 & 1\times1+1\times-i+1\times-1+1\times i \\ 1\times1-1\times1-1\times1+1\times1 & 1\times1+1\times1+1\times1+1\times1 & 1\times1+1\times i-1\times1-1\times i & 1\times1+1\times-1+1\times1-1\times1 \\ 1\times1-1\times1+1\times1-1\times1 & 1\times1-1\times i-1\times1+1\times i & 1\times1-1\times-1+1\times1-1\times-1 & 1\times1-1\times-i+1\times-1+1\times i \\ 1\times1-1\times1-1\times1+1\times1 & 1\times1-1\times-i-1\times-1+1\times-i & 1\times1-1\times1+1\times1-1\times1 & 1\times1-1\times i+1\times-1+1\times-i \end{pmatrix}$$

$$= \frac{1}{4}\begin{pmatrix} 4 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 \\ 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 4 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Nithin Reddy Govindugari(nithinreddy1747@gmail.com)

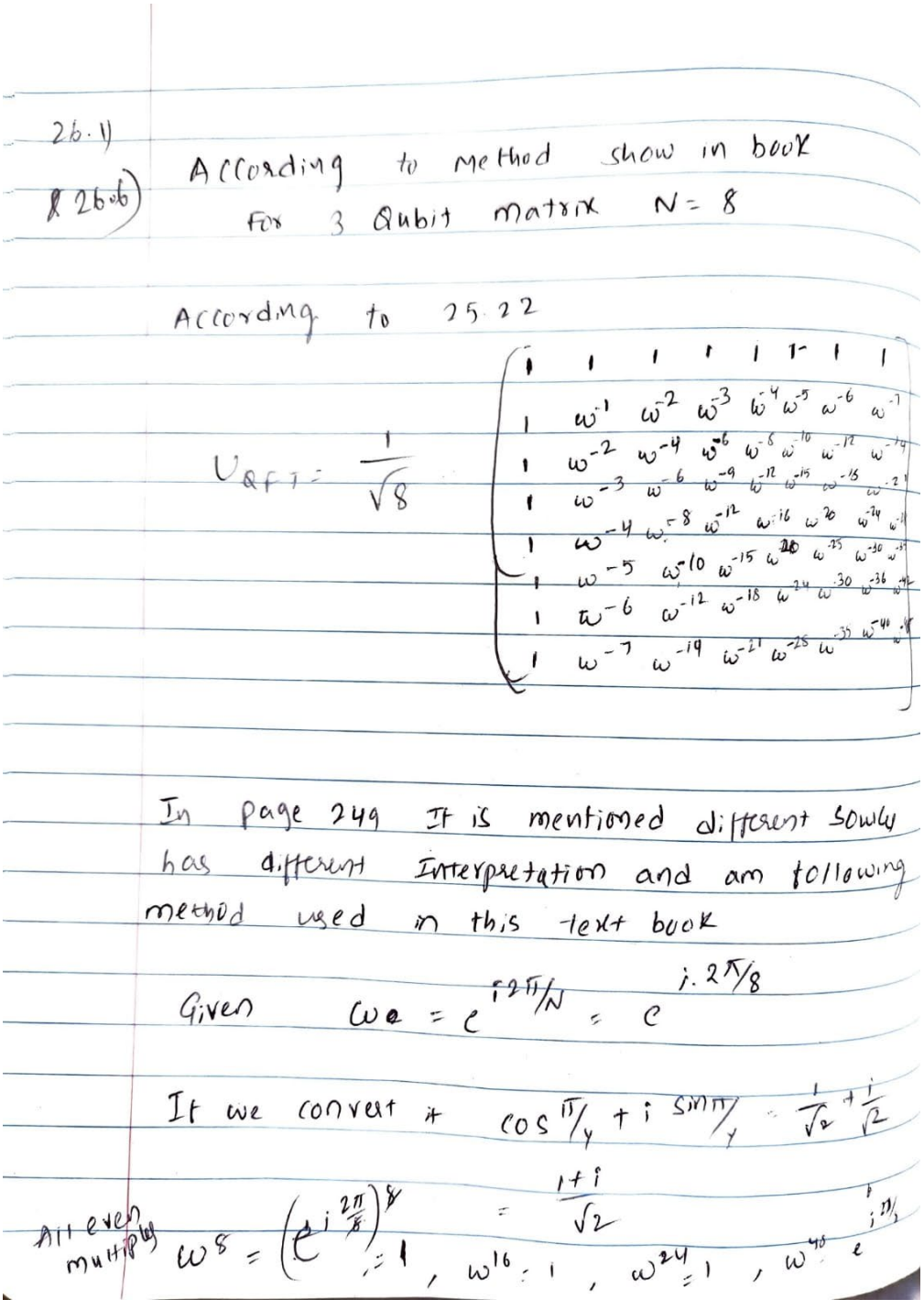
```
#25.7

from sympy.abc import w
N = int(input("Enter a value", ))
# w = 'w'

matrix=[]
row=[]
for i in range(N): #total row is 3
    row=[]
    for j in range(N): #total column is 3
        # row.append( a \N{SUPERSCRIPT (i*j)})
         row.append( (w)** (i*j) )

    matrix.append(row) #add fully defined column into the row
print( matrix )
```

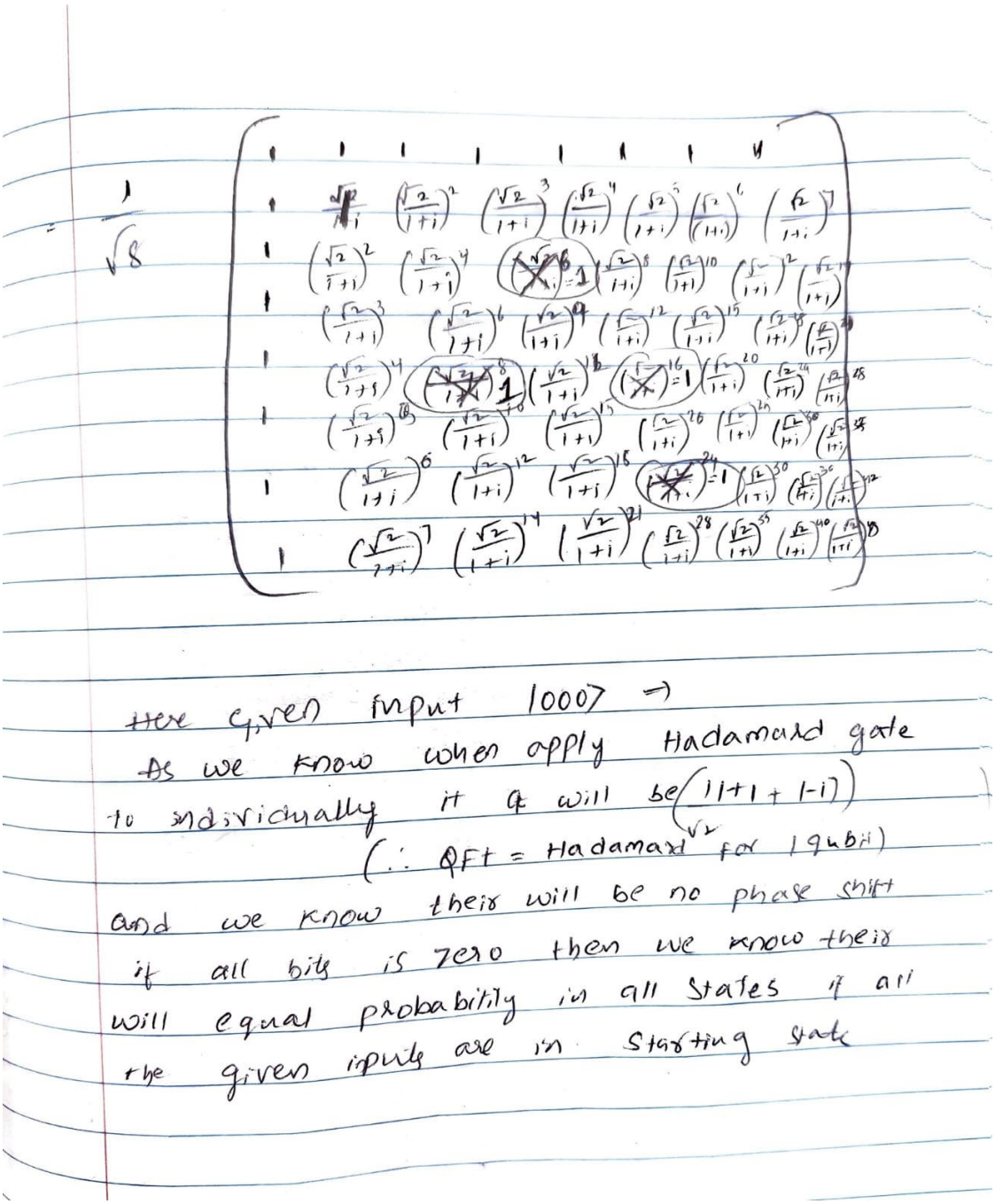
```
Enter a value4
[[1, 1, 1, 1], [1, w, w**2, w**3], [1, w**2, w**4, w**6], [1, w**3, w**6, w**9]]
```

Nithin Reddy Govindugari(nithinreddy1747@gmail.com)

Chapter 26

26.1)





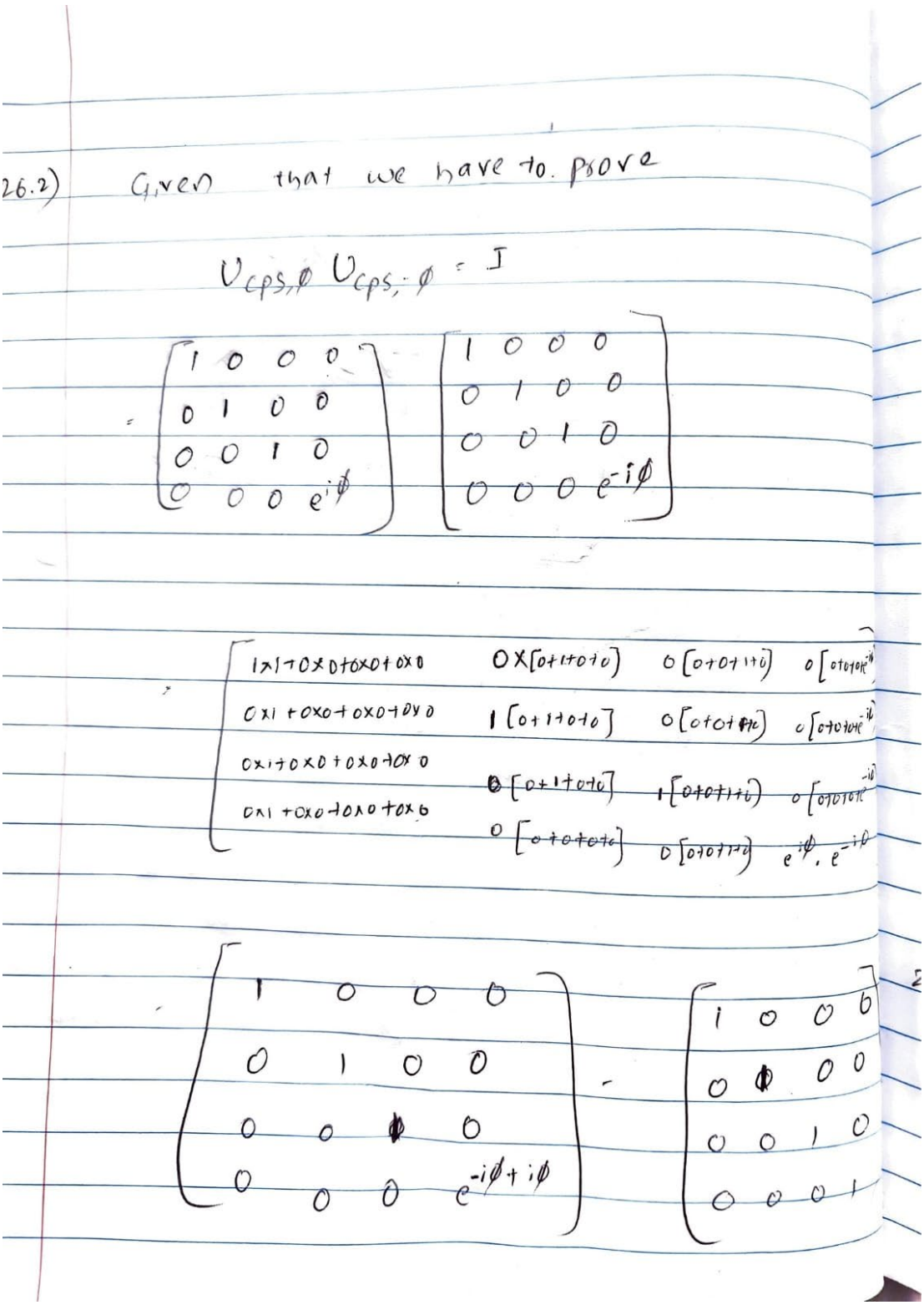
Here given input 1000? →

As we know when apply Hadamard gate to individually it o/p will be $\left(\frac{1}{\sqrt{2}}(1+1 \;\; 1-1)\right)$

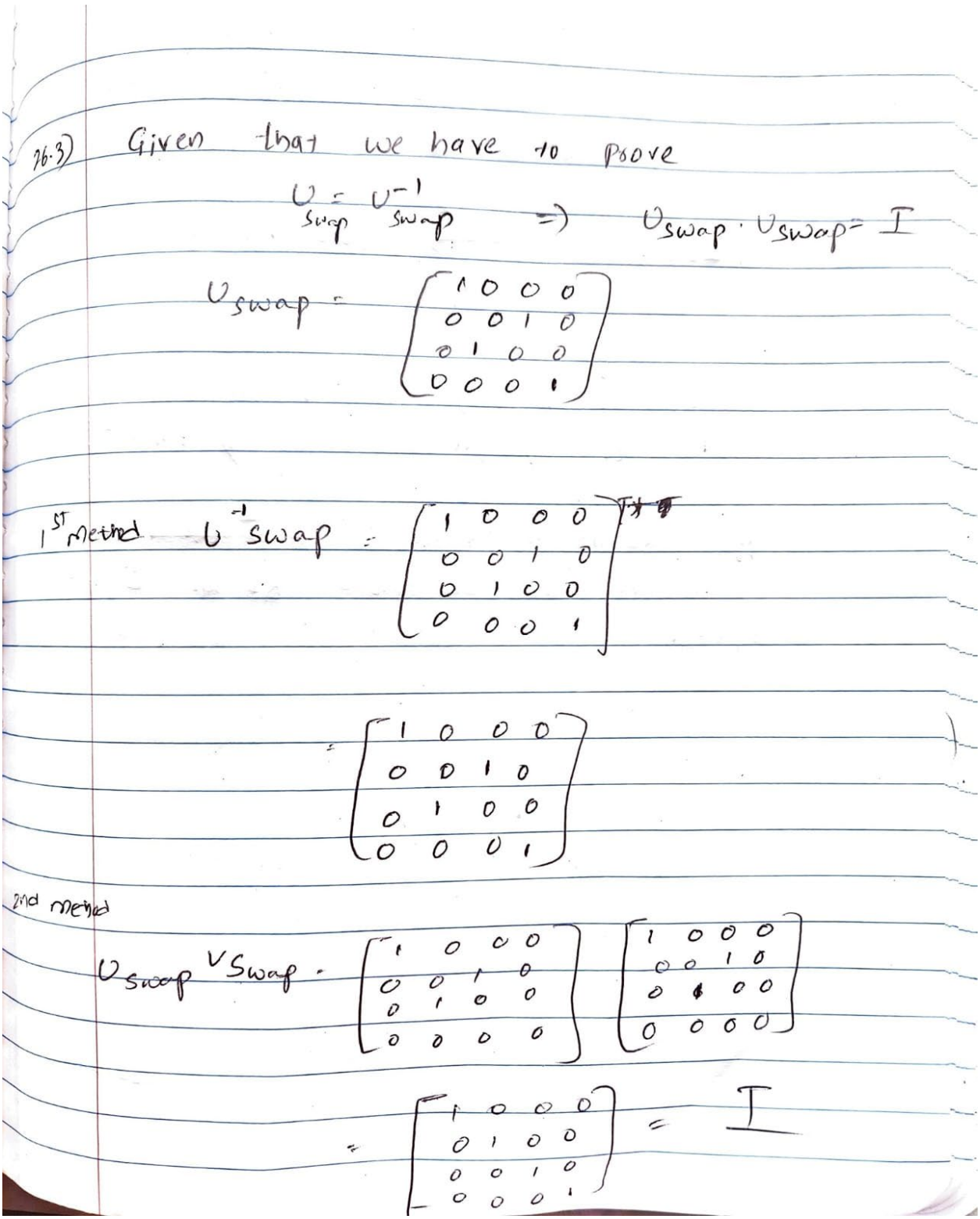
(∵ QFt = Hadamard for 1 qubit)

and we know their will be no phase shift if all bits is zero then we know their will equal probability in all states if all the given inputs are in starting state

26.2)

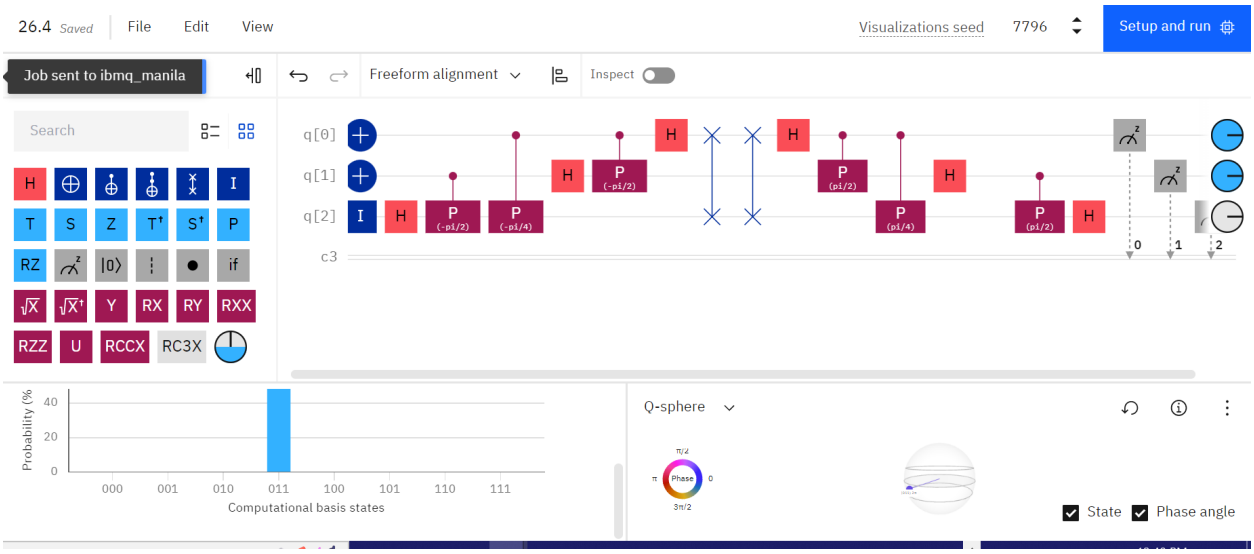


26.3)

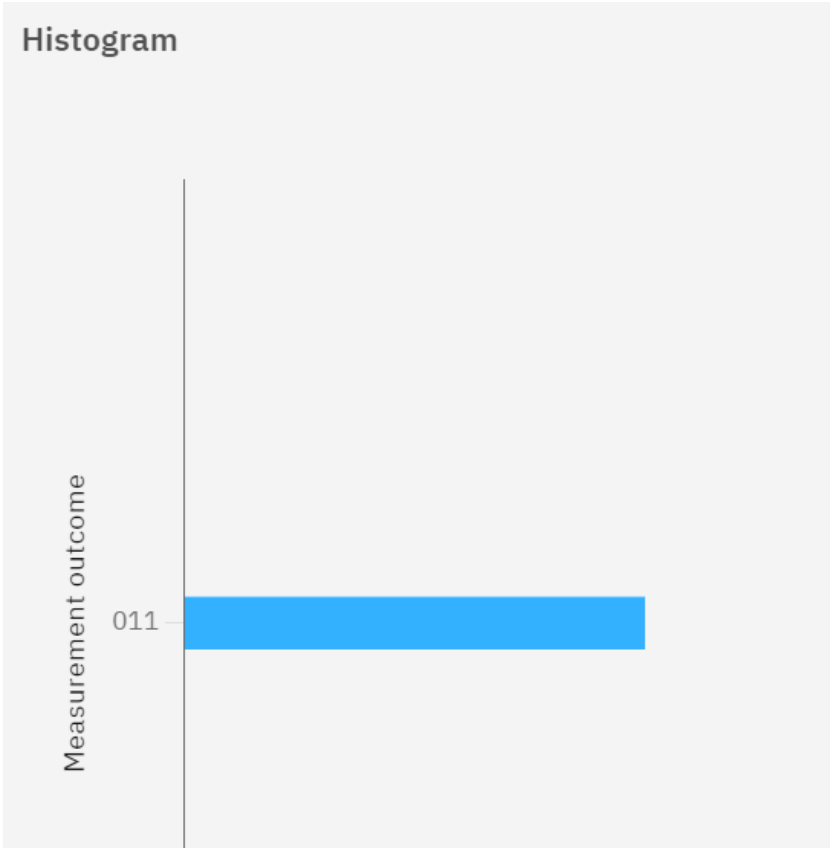

26.3) Given that we have to prove
$U_{swap} = U_{swap}^{-1} \Rightarrow U_{swap} \cdot U_{swap} = I$
$U_{swap} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$
1st Method $U^{-1}_{swap} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}^{T}$
$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$
2nd method
$U_{swap} U_{swap} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$
$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = I$

26.4)

Introduction to Quantum computing (From a layperson to programmer in 30 steps)

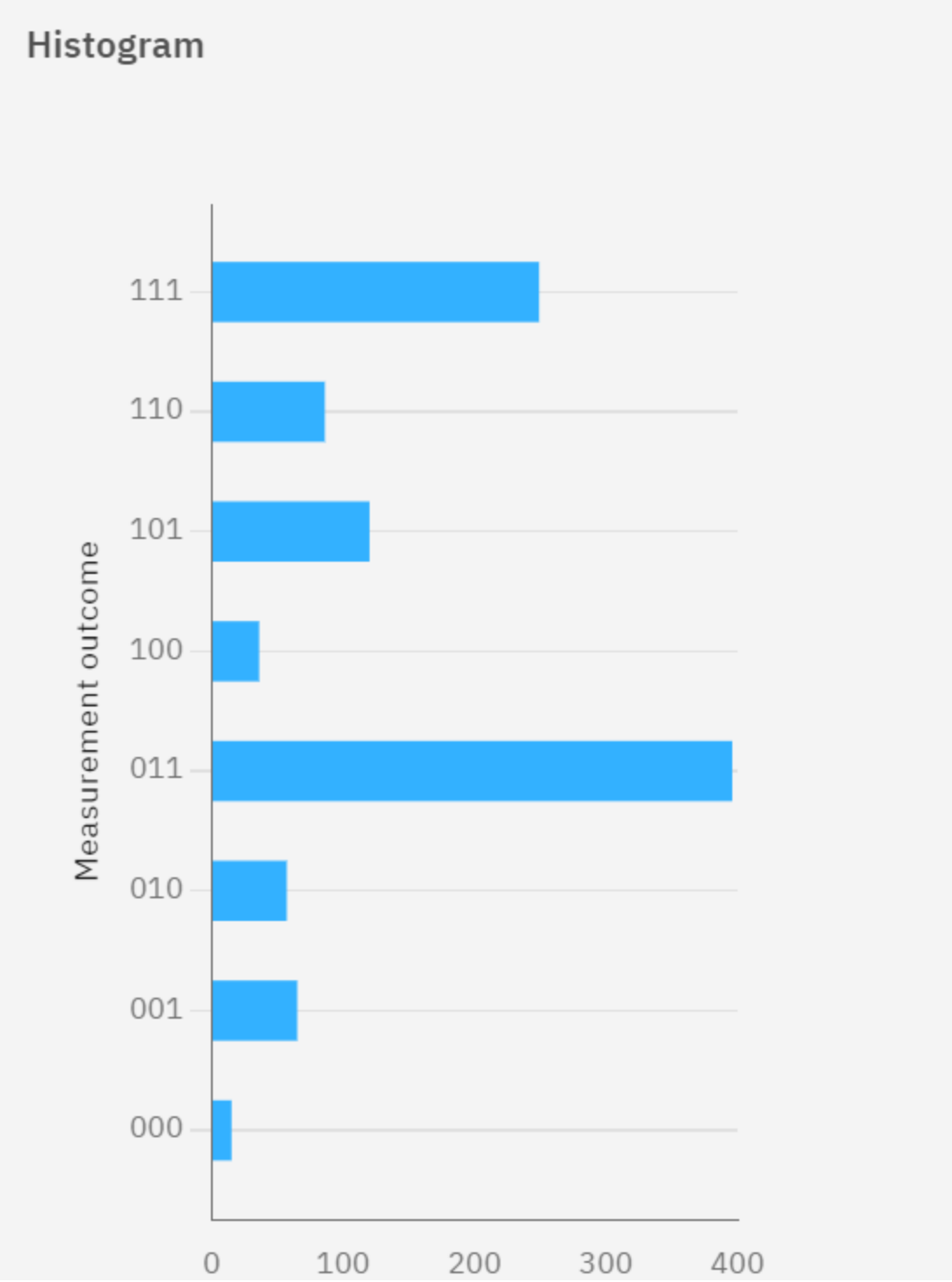


Simulation result



Hardware circuit result

Nithin Reddy Govindugari(nithinreddy1747@gmail.com)



Nithin Reddy Govindugari(nithinreddy1747@gmail.com)

26.5) according to figure 26.7 we need $n-1$ $V_{c.p.s}$, ϕ gates with ϕ from $\frac{-2\pi}{2^2}$ to $\frac{-2\pi}{2^n}$

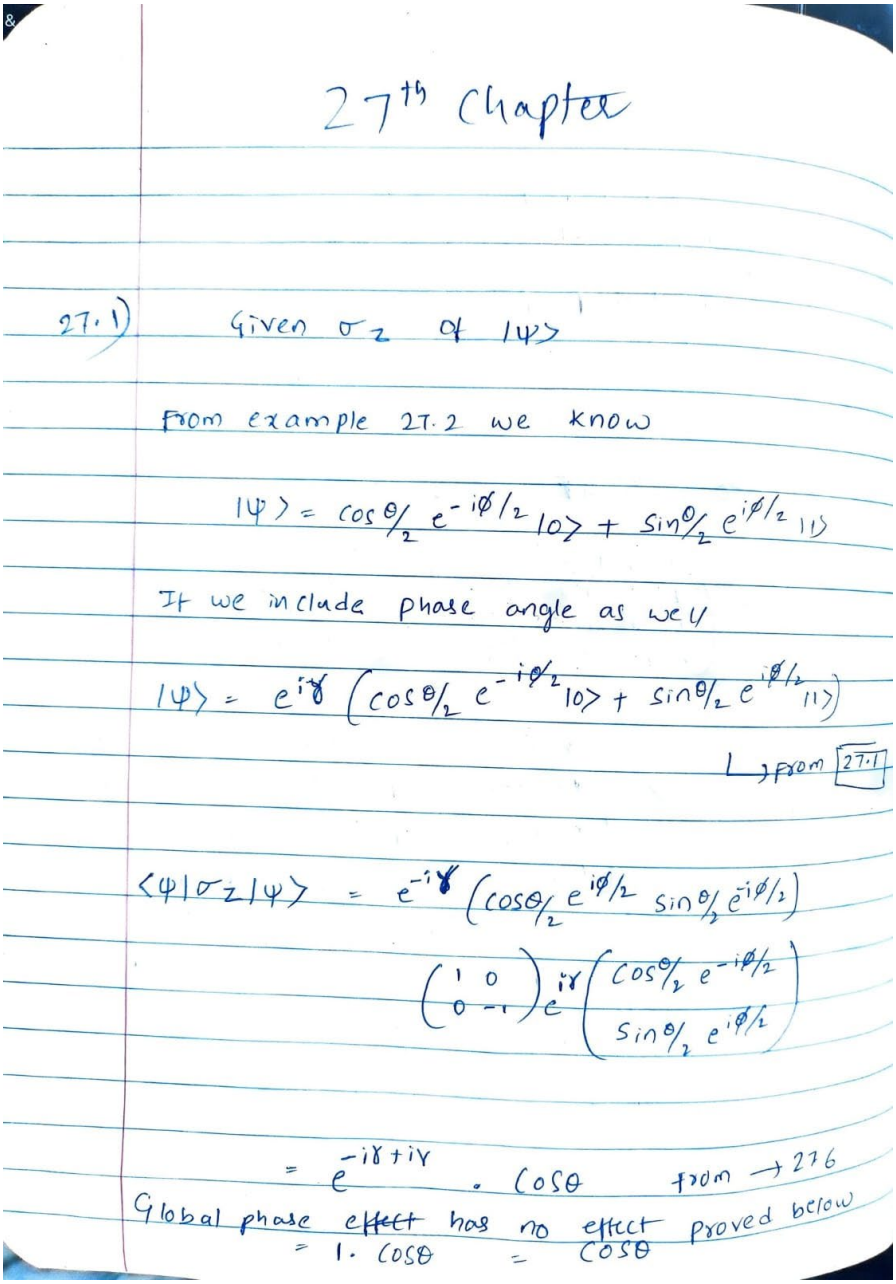
if execution time is $y = 2^n$ seconds then i believe as we know QFT has $O(n^2)$ speed

26.6) we have derived and elaborated matrix in 26.1

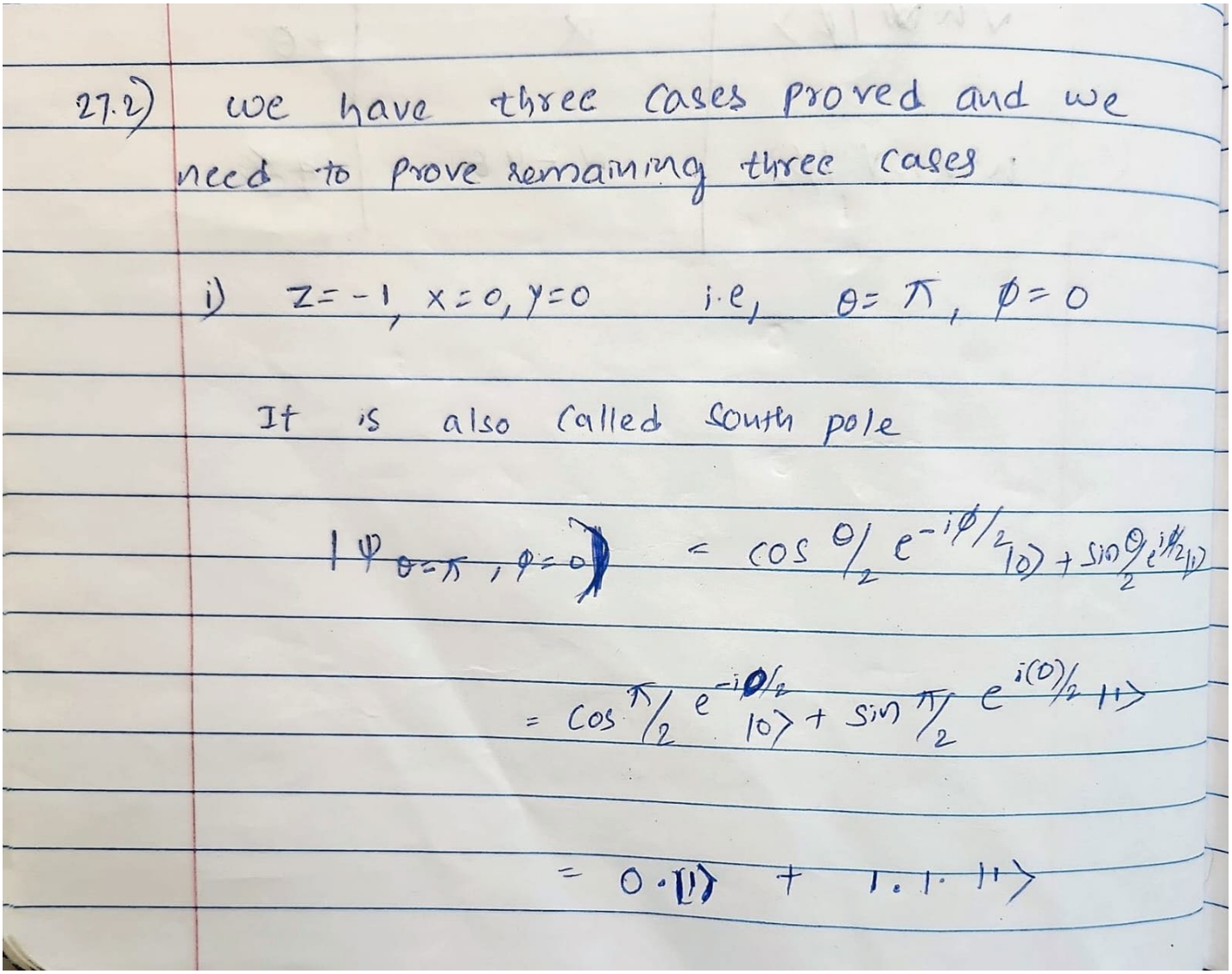
26.7) $|1001100\rangle \xrightarrow{\text{swap}} |0011001\rangle$

Chapter 27

27.1)



27.2)



27.2) we have three cases proved and we need to prove remaining three cases.

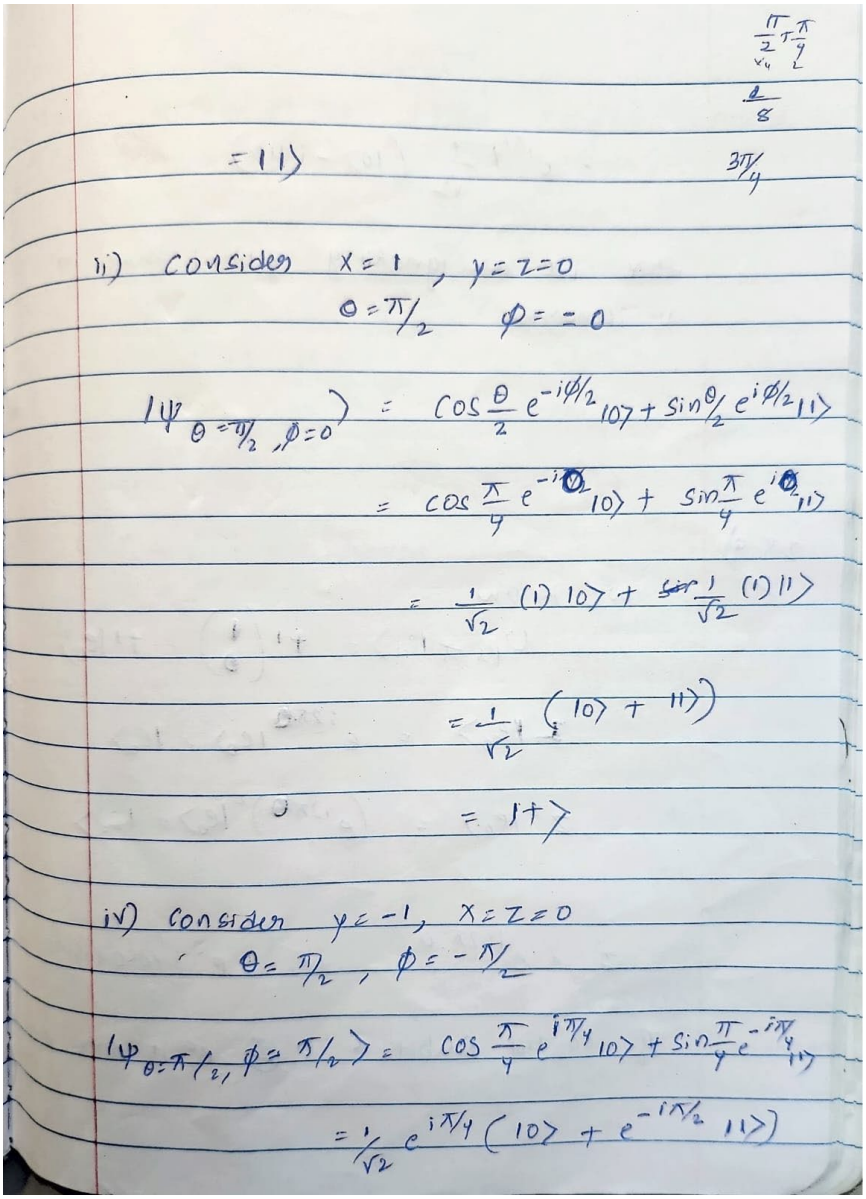
i) $z=-1, x=0, y=0$ i.e, $\theta=\pi, \phi=0$

It is also called south pole

$$|\psi_{\theta=\pi,\phi=0}\rangle = \cos\frac{\theta}{2} e^{-i\phi/2}|0\rangle + \sin\frac{\theta}{2} e^{i\phi/2}|1\rangle$$

$$= \cos\frac{\pi}{2} e^{-i0/2}|0\rangle + \sin\frac{\pi}{2} e^{i(0)/2}|1\rangle$$

$$= 0\cdot|1\rangle + 1\cdot 1\cdot|1\rangle$$



$= |1\rangle$

ii) Consider $x = 1$, $y = z = 0$

$\theta = \pi/2 \quad \phi = 0$

$$|\psi_{\theta=\pi/2, \phi=0}\rangle = \cos\frac{\theta}{2} e^{-i\phi/2} |0\rangle + \sin\frac{\theta}{2} e^{i\phi/2} |1\rangle$$

$$= \cos\frac{\pi}{4} e^{-i0} |0\rangle + \sin\frac{\pi}{4} e^{i0} |1\rangle$$

$$= \frac{1}{\sqrt{2}} (1) |0\rangle + \frac{1}{\sqrt{2}} (1) |1\rangle$$

$$= \frac{1}{\sqrt{2}} (|0\rangle + |1\rangle)$$

$$= |+\rangle$$

iv) Consider $y = -1$, $x = z = 0$

$\theta = \pi/2, \quad \phi = -\pi/2$

$$|\psi_{\theta=\pi/2, \phi=\pi/2}\rangle = \cos\frac{\pi}{4} e^{i\pi/4} |0\rangle + \sin\frac{\pi}{4} e^{-i\pi/4} |1\rangle$$

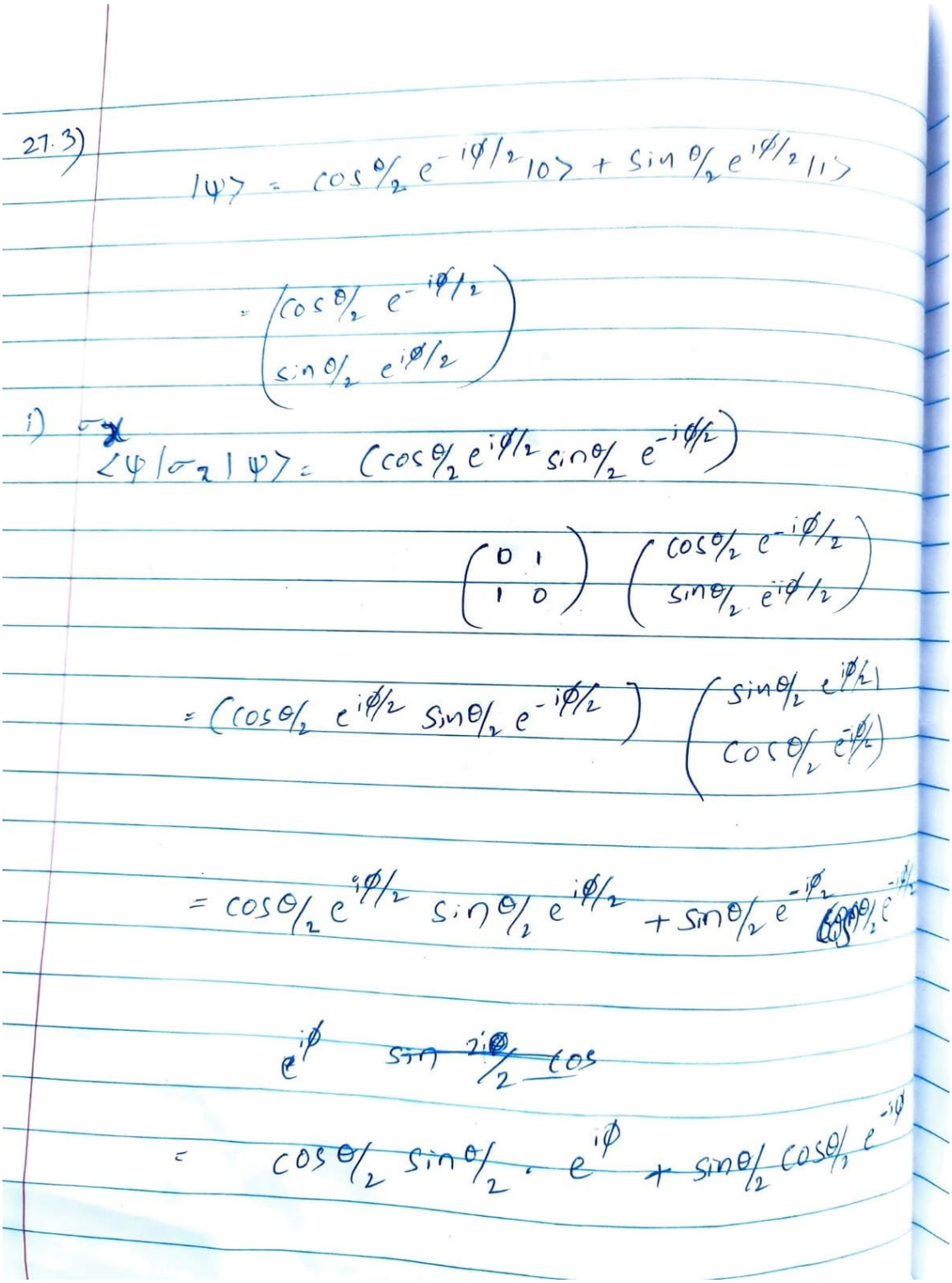
$$= \frac{1}{\sqrt{2}} e^{i\pi/4} (|0\rangle + e^{-i\pi/2} |1\rangle)$$

$$= e^{i\pi/4} \frac{1}{\sqrt{2}} (|0\rangle - i|1\rangle)$$

Here we are ignoring global phase in all scenarios

Nithin Reddy Govindugari(nithinreddy1747@gmail.com)

27.3)


27.3)
$|\psi\rangle = \cos\theta/2\, e^{-i\phi/2}|0\rangle + \sin\theta/2\, e^{i\phi/2}|1\rangle$
$= \begin{pmatrix} \cos\theta/2\, e^{-i\phi/2} \\ \sin\theta/2\, e^{i\phi/2} \end{pmatrix}$
i) σ_x
$\langle\psi|\sigma_x|\psi\rangle = (\cos\theta/2\, e^{i\phi/2} \quad \sin\theta/2\, e^{-i\phi/2})$
$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} \cos\theta/2\, e^{-i\phi/2} \\ \sin\theta/2\, e^{i\phi/2} \end{pmatrix}$
$= (\cos\theta/2\, e^{i\phi/2} \quad \sin\theta/2\, e^{-i\phi/2}) \begin{pmatrix} \sin\theta/2\, e^{i\phi/2} \\ \cos\theta/2\, e^{-i\phi/2} \end{pmatrix}$
$= \cos\theta/2\, e^{i\phi/2} \sin\theta/2\, e^{i\phi/2} + \sin\theta/2\, e^{-i\phi/2} \cos\theta/2\, e^{-i\phi/2}$
$e^{i\phi}$
$= \cos\theta/2 \sin\theta/2 \cdot e^{i\phi} + \sin\theta/2 \cos\theta/2\, e^{-i\phi}$

$= \cos\theta/2 \sin\theta/2 \left(e^{i\phi} + e^{-i\phi} \right)$

$= \cos\theta/2 \sin\theta/2 \left(\cos\phi/2 + i\sin\phi/2 + \cos\phi/2 - i\sin\phi/2 \right)$

$= \cos\theta/2 \sin\theta/2 \left(\cos\phi \right)$

$= \sin\theta \cos\phi$

ii) $\langle\psi|\sigma_y|\psi\rangle = \cos\theta/2\, e^{i\phi/2} \sin\theta/2\, e^{-i\phi/2} \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$

$\begin{bmatrix} \cos\theta/2\, e^{-i\phi/2} \\ \sin\theta/2\, e^{i\phi/2} \end{bmatrix}$

$= \left(\cos\theta/2\, e^{i\phi/2} \sin\theta/2\, e^{-i\phi/2} \right)$

$\begin{bmatrix} -i\sin\theta/2\, e^{i\phi/2} \\ i\cos\theta/2\, e^{-i\phi/2} \end{bmatrix}$

$= \cos\theta/2\, e^{i\phi/2} \cdot -i\sin\theta/2\, e^{i\phi/2} + \sin\theta/2\, e^{-i\phi/2} \cdot i\cos\theta/2\, e^{-i\phi/2}$

$$i \sin\theta/2 \cos\theta/2 e^{-i\phi} - i \cos\theta/2 \sin\theta/2 e^{i\phi}$$

$$= i \sin\theta/2 \cos\theta/2 \left(e^{-i\phi} - e^{i\phi} \right)$$

$$= i \sin\theta/2 \cos\theta/2 \left(\cos\phi - i\sin\phi - \cos\theta - i\sin\phi \right)$$

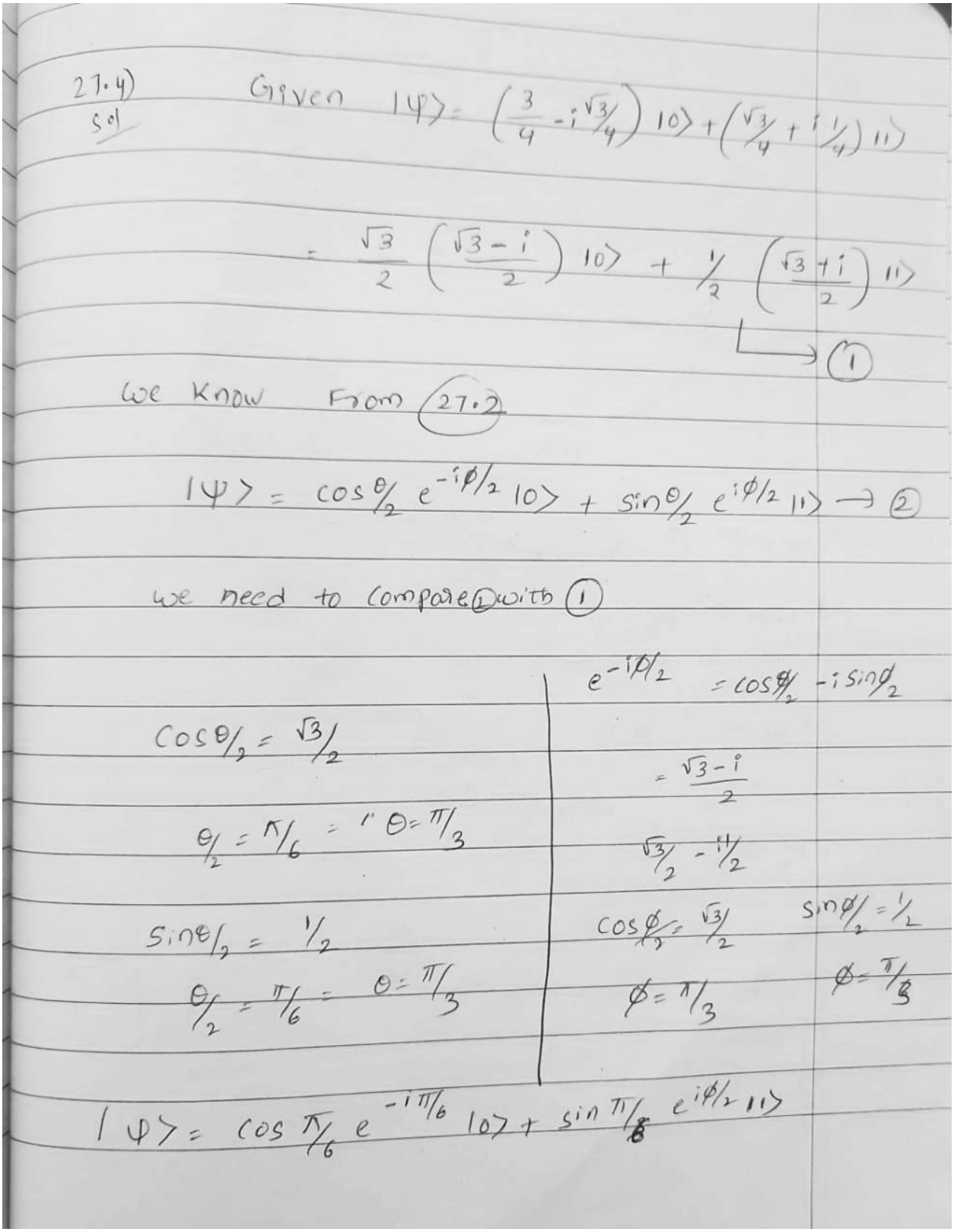
$$= i \sin\theta/2 \cos\theta/2 \left(-i \sin\phi \right)$$

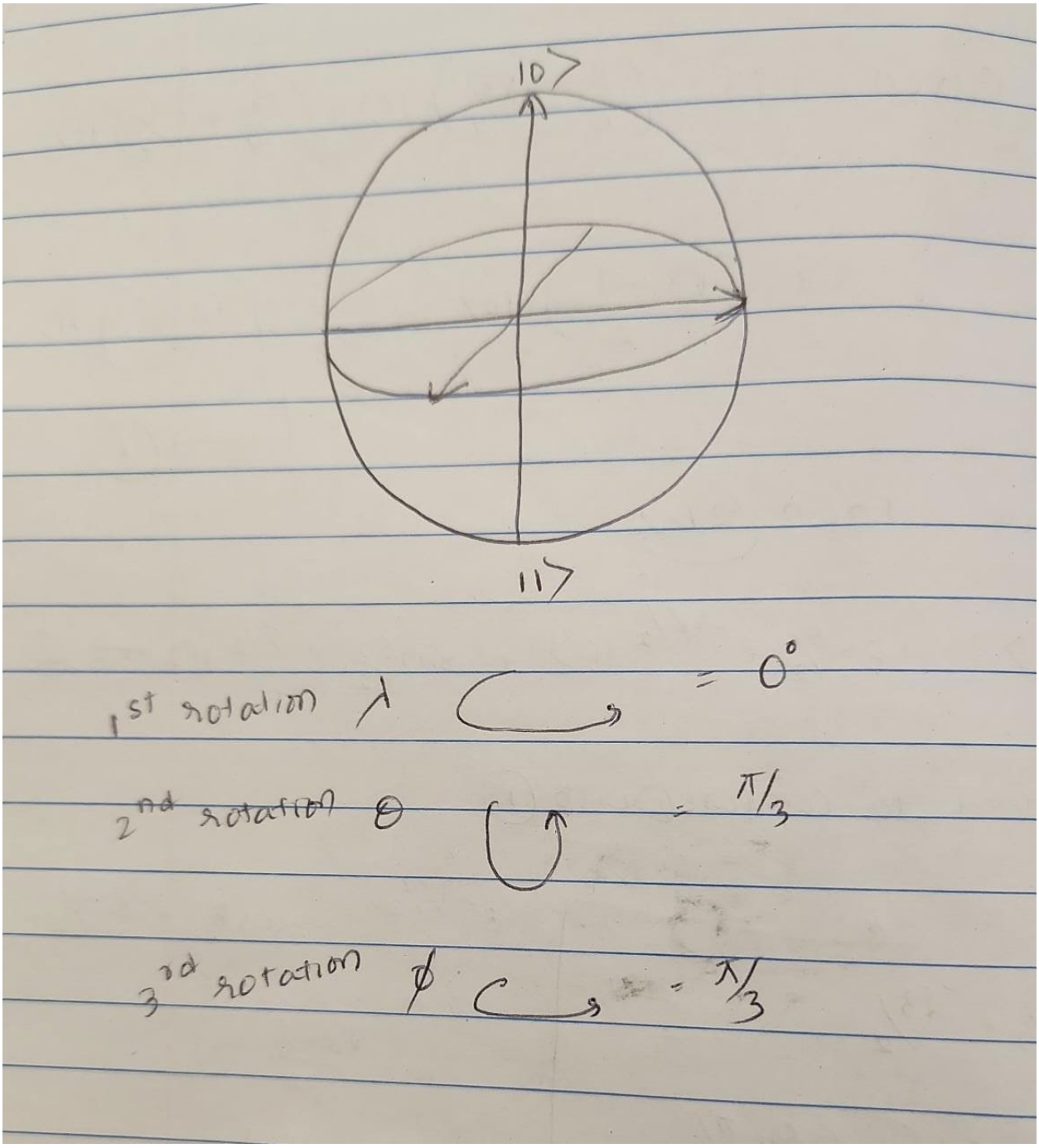
$$= -i^2 \sin\theta \sin\phi \left(\right.$$

$$= \sin\theta \sin\phi$$

Nithin Reddy Govindugari(nithinreddy1747@gmail.com)

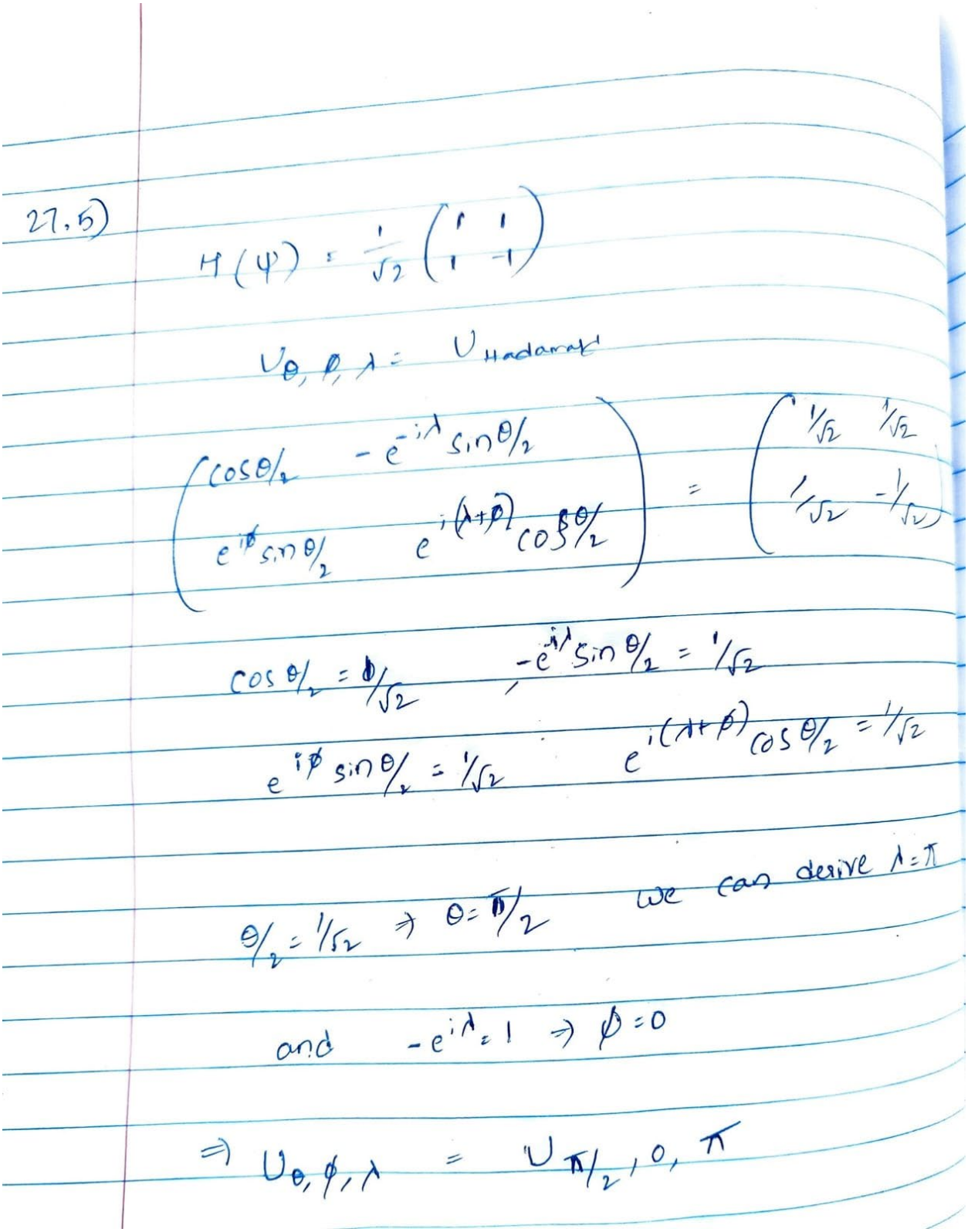
27.4


27.4)
Sol
Given $|\psi\rangle = \left(\frac{3}{4} - i\frac{\sqrt{3}}{4}\right)|0\rangle + \left(\frac{\sqrt{3}}{4} + i\frac{1}{4}\right)|1\rangle$
$= \frac{\sqrt{3}}{2}\left(\frac{\sqrt{3}-i}{2}\right)|0\rangle + \frac{1}{2}\left(\frac{\sqrt{3}+i}{2}\right)|1\rangle$ → (1)
We know From (27.2)
$|\psi\rangle = \cos\frac{\theta}{2} e^{-i\phi/2}|0\rangle + \sin\frac{\theta}{2} e^{i\phi/2}|1\rangle$ → (2)
we need to compare (2) with (1)
$\cos\frac{\theta}{2} = \frac{\sqrt{3}}{2}$
$\frac{\theta}{2} = \frac{\pi}{6} \Rightarrow \theta = \frac{\pi}{3}$
$\sin\frac{\theta}{2} = \frac{1}{2}$
$\frac{\theta}{2} = \frac{\pi}{6} \Rightarrow \theta = \frac{\pi}{3}$
$e^{-i\phi/2} = \cos\frac{\phi}{2} - i\sin\frac{\phi}{2}$
$= \frac{\sqrt{3}-i}{2}$
$\frac{\sqrt{3}}{2} - \frac{i1}{2}$
$\cos\frac{\phi}{2} = \frac{\sqrt{3}}{2}$
$\sin\frac{\phi}{2} = \frac{1}{2}$
$\phi = \frac{\pi}{3}$
$\phi = \frac{\pi}{3}$
$|\psi\rangle = \cos\frac{\pi}{6} e^{-i\pi/6}|0\rangle + \sin\frac{\pi}{6} e^{i\phi/2}|1\rangle$



Nithin Reddy Govindugari(nithinreddy1747@gmail.com)

27.5)

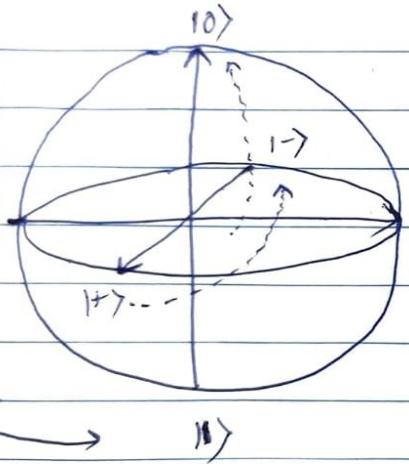

27.5)
$H(\psi) = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$
$U_{\theta,\phi,\lambda} = U_{Hadamard}$
$\begin{pmatrix} \cos\theta/2 & -e^{i\lambda}\sin\theta/2 \\ e^{i\phi}\sin\theta/2 & e^{i(\lambda+\phi)}\cos\theta/2 \end{pmatrix} = \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & -1/\sqrt{2} \end{pmatrix}$
$\cos\theta/2 = 1/\sqrt{2}$
$-e^{i\lambda}\sin\theta/2 = 1/\sqrt{2}$
$e^{i\phi}\sin\theta/2 = 1/\sqrt{2}$
$e^{i(\lambda+\phi)}\cos\theta/2 = 1/\sqrt{2}$
$\theta/2 = 1/\sqrt{2} \Rightarrow \theta = \pi/2$
we can derive $\lambda = \pi$
and $-e^{i\lambda} = 1 \Rightarrow \phi = 0$
$\Rightarrow U_{\theta,\phi,\lambda} = U_{\pi/2,0,\pi}$

27.6)

27.6) Hadamard applied to $|+\rangle$ To $|1\rangle$

We know Hadamard's $U\theta, \phi, \lambda$ is $U\pi/2, 0, \pi$

$H \rightarrow |+\rangle \rightarrow |0\rangle$

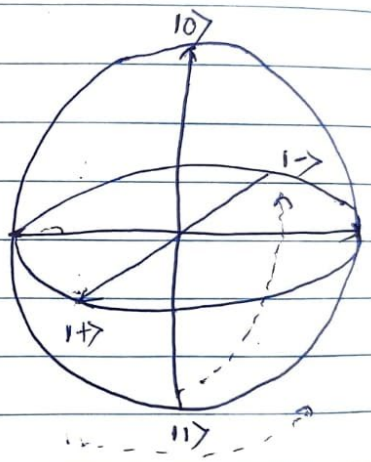


1st rotation $\lambda = \pi$

2nd rotation $\theta = \pi/2$

3rd rotation $\phi = 0$

$H \rightarrow |1\rangle \rightarrow |-\rangle$



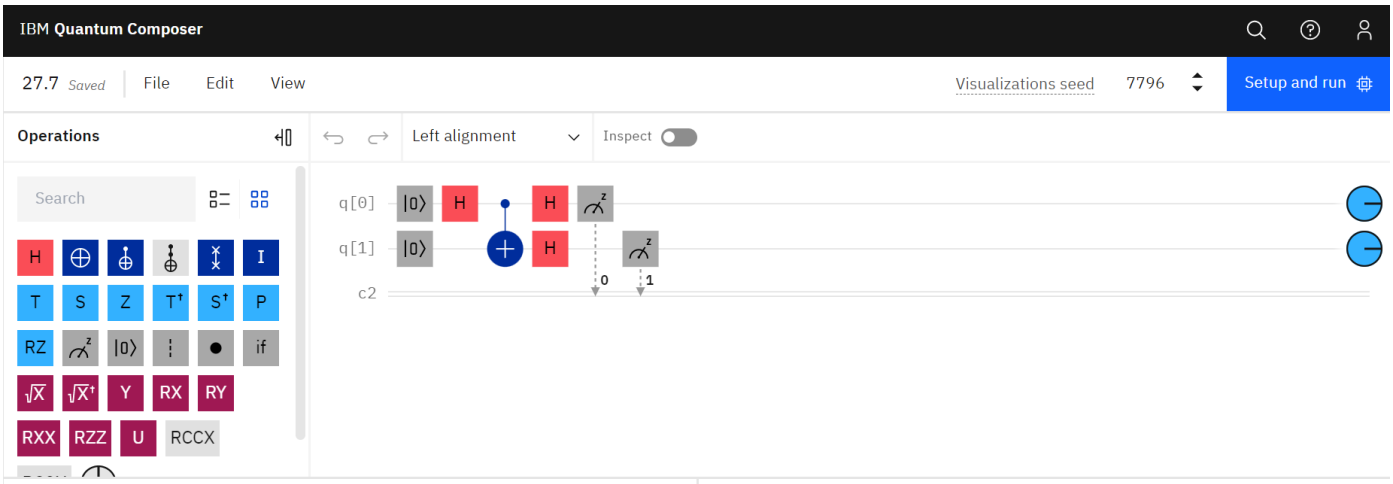
1st rotation $\lambda = \pi$ (No use)

2nd rotation $\theta = \pi/2$

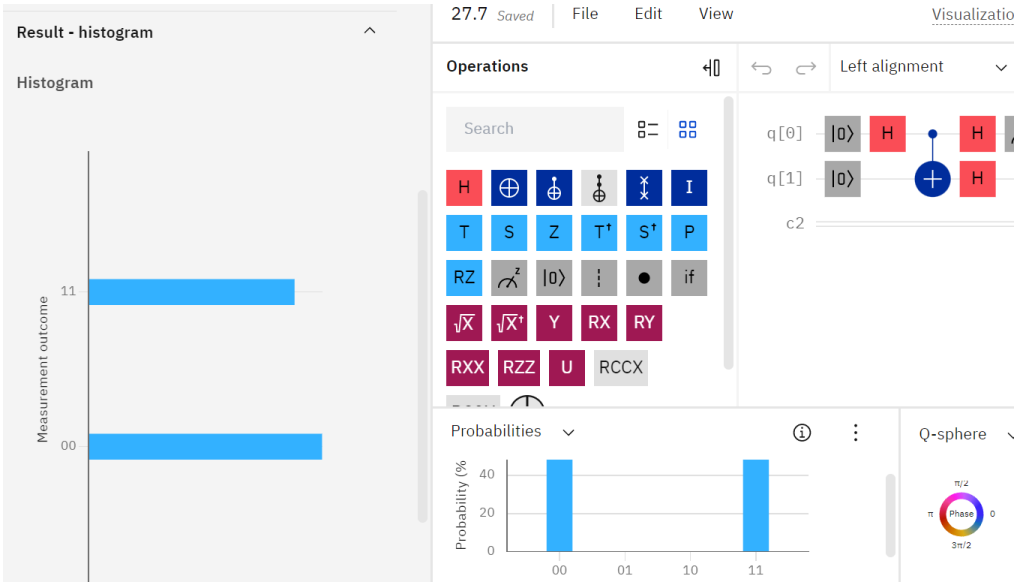
3rd rotation $\phi = 0$

Introduction to Quantum computing (From a layperson to programmer in 30 steps)

27.7

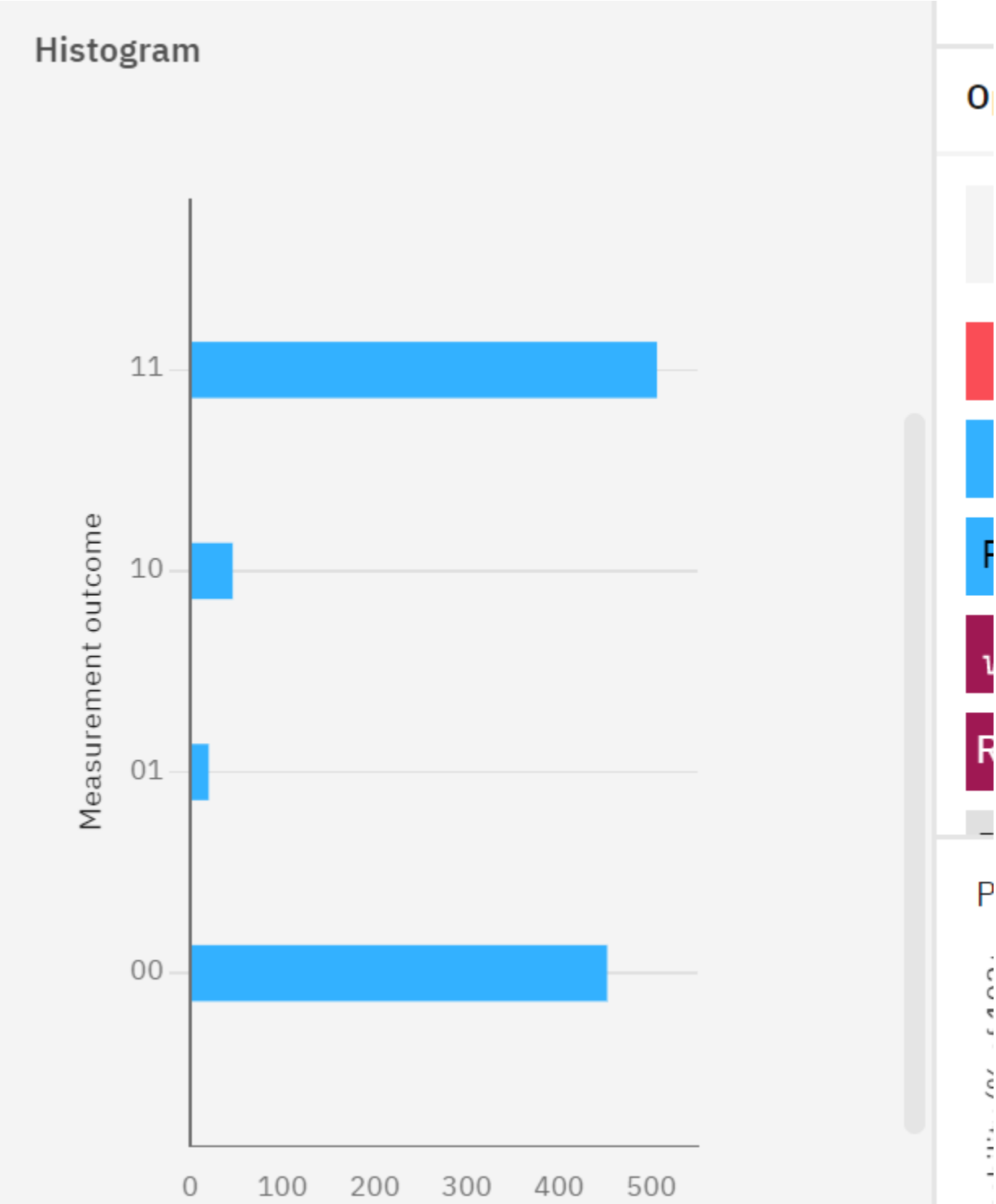


Result



Hardware execution result

Nithin Reddy Govindugari(nithinreddy1747@gmail.com)


Histogram
Measurement outcome
11
10
01
00
0
100
200
300
400
500

Chapter 28

28.1)

Given $\frac{1}{\sqrt{2}}\begin{pmatrix} -i & -i \\ -i & i \end{pmatrix}$

we can extract $-i$ from the above gate it becomes hadamard gate

$$-i \cdot \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

$$-i\begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix} = e^{i\gamma}\begin{pmatrix} \cos\theta/2 & -e^{i\lambda}\sin\theta/2 \\ e^{i\phi}\sin\theta/2 & e^{i(\lambda+\phi)}\cos\theta/2 \end{pmatrix}$$

$\cos\theta/2 = \frac{1}{\sqrt{2}} \Rightarrow \theta/2 = \frac{1}{\sqrt{2}}$ $\boxed{\theta = \pi/2}$

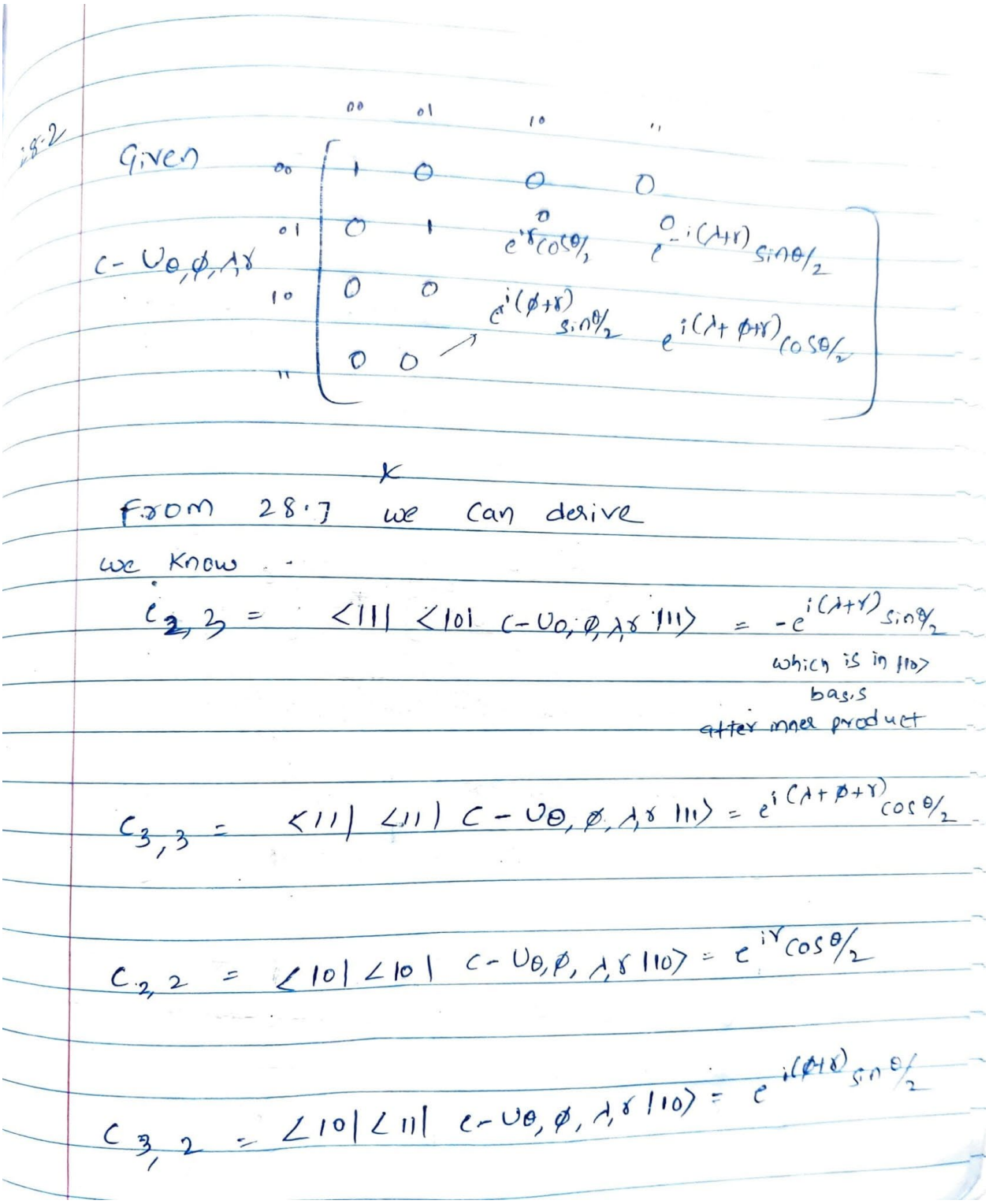
$-e^{i\lambda}\sin\theta/2 \Rightarrow \boxed{\sin\theta/2 = \frac{1}{\sqrt{2}}} \Rightarrow -e^{i\lambda} \Rightarrow \lambda = \pi$

$e^{i\phi}\sin\theta/2 = e^{i\phi} \Rightarrow \phi = 0$

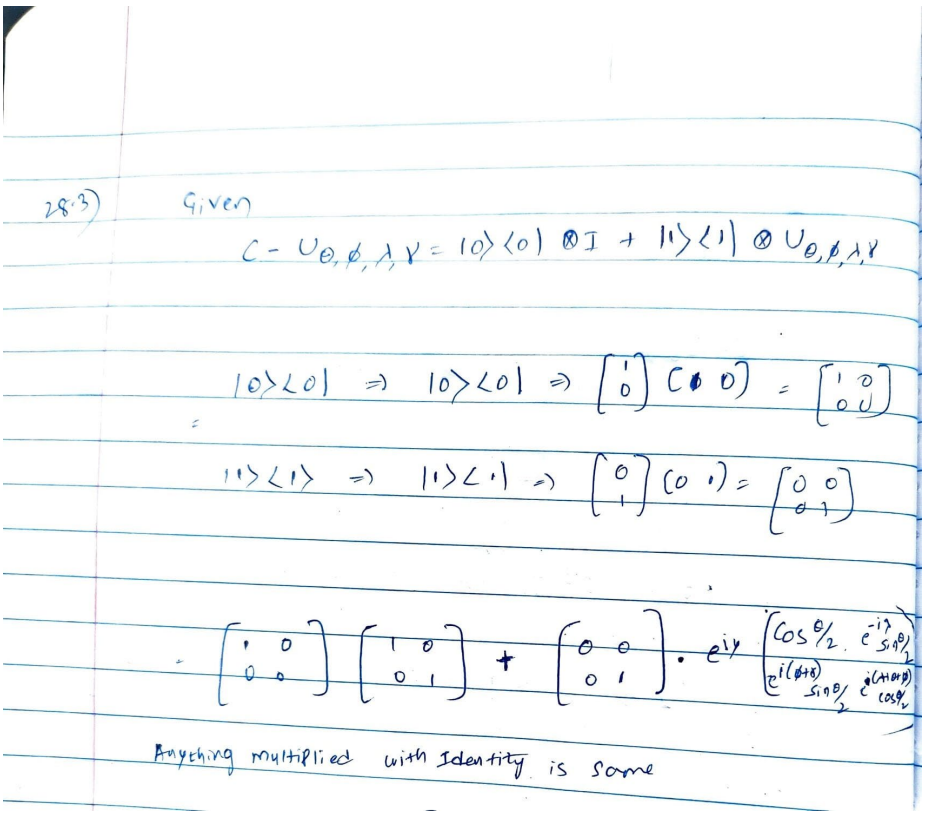
$e^{i\gamma} = -i \Rightarrow -(\cos\gamma + i\sin\gamma) = \gamma = \pi/2$

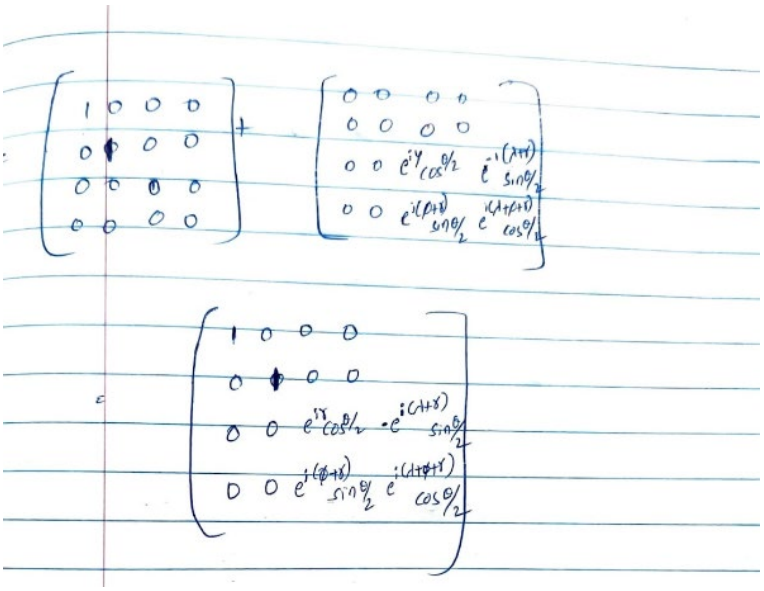
$\boxed{U\theta, \phi, \lambda, \gamma = U_{\pi/2, 0, \pi, \pi/2}}$

28.2)

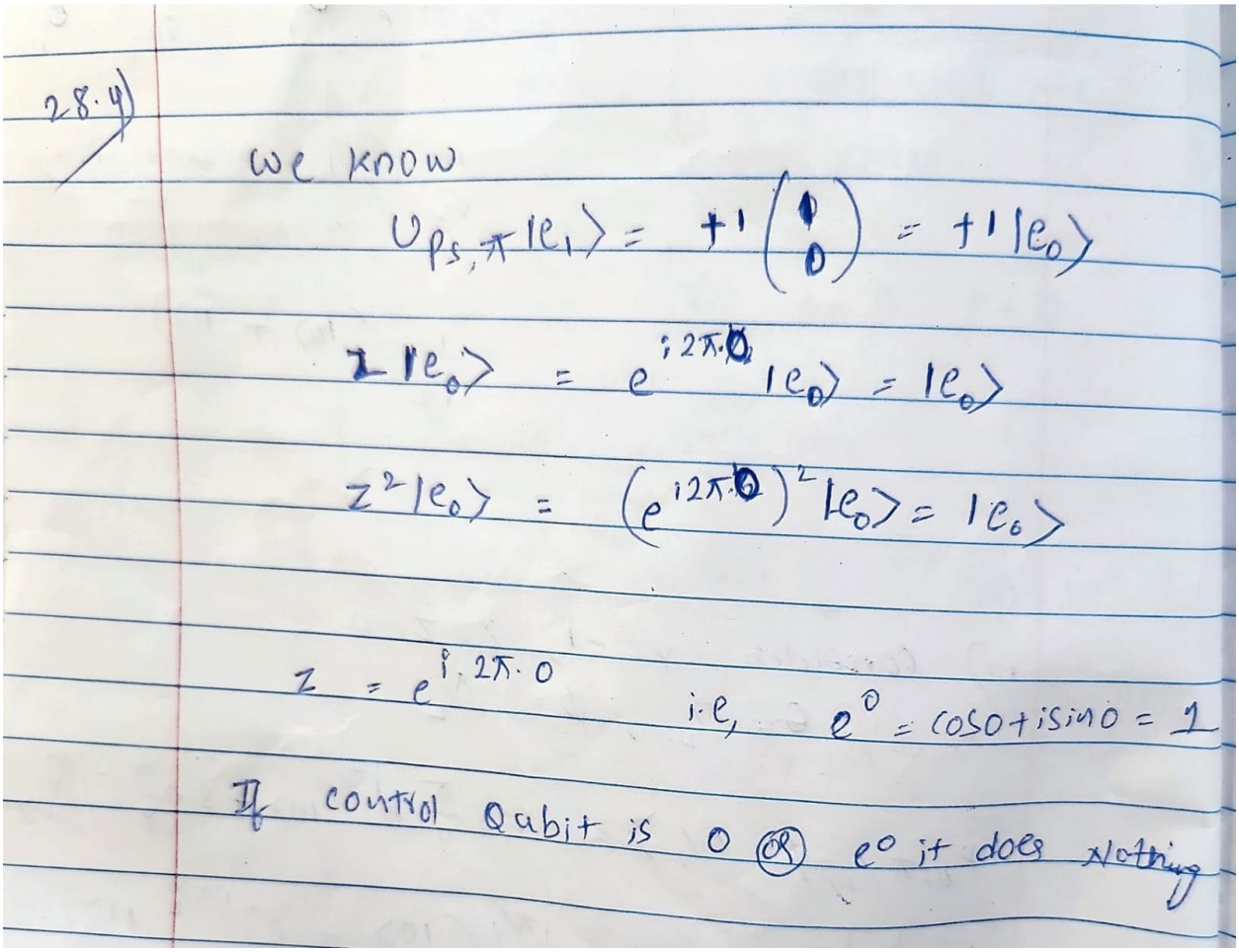

28:2
Given
c - U θ, φ, λ, γ
00 01 10 11
00 1 0 0 0
01 0 1 $e^{i\gamma}\cos\theta/2$ $e^{-i(\lambda+\gamma)}\sin\theta/2$
10 0 0 $e^{i(\phi+\gamma)}\sin\theta/2$ $e^{i(\lambda+\phi+\gamma)}\cos\theta/2$
11 0 0
From 28:7 we can derive
we know :-
$c_{3,3} = \langle 11| \langle 10| c-U_{\theta,\phi,\lambda,\gamma} |11\rangle = -e^{i(\lambda+\gamma)}\sin\theta/2$
which is in |10> basis after inner product
$c_{3,3} = \langle 11| \langle 11| c-U_{\theta,\phi,\lambda,\gamma} |11\rangle = e^{i(\lambda+\phi+\gamma)}\cos\theta/2$
$c_{2,2} = \langle 10| \langle 10| c-U_{\theta,\phi,\lambda,\gamma} |10\rangle = e^{i\gamma}\cos\theta/2$
$c_{3,2} = \langle 10| \langle 11| c-U_{\theta,\phi,\lambda,\gamma} |10\rangle = e^{i(\phi+\gamma)}\sin\theta/2$

28.3)


28.3) Given
$C-U_{\theta,\phi,\lambda,\gamma} = |0\rangle\langle 0| \otimes I + |1\rangle\langle 1| \otimes U_{\theta,\phi,\lambda,\gamma}$
$|0\rangle\langle 0| \Rightarrow |0\rangle\langle 0| \Rightarrow \begin{bmatrix}1\\0\end{bmatrix}\begin{bmatrix}1 & 0\end{bmatrix} = \begin{bmatrix}1 & 0\\0 & 0\end{bmatrix}$
$|1\rangle\langle 1| \Rightarrow |1\rangle\langle 1| \Rightarrow \begin{bmatrix}0\\1\end{bmatrix}\begin{bmatrix}0 & 1\end{bmatrix} = \begin{bmatrix}0 & 0\\0 & 1\end{bmatrix}$
$\begin{bmatrix}1 & 0\\0 & 0\end{bmatrix}\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix} + \begin{bmatrix}0 & 0\\0 & 1\end{bmatrix} \cdot e^{i\gamma}\begin{pmatrix}\cos\theta/2 & e^{-i\lambda}\sin\theta/2\\ e^{i(\phi+\lambda)}\sin\theta/2 & e^{i(\lambda+\phi+\theta)}\cos\theta/2\end{pmatrix}$
Anything multiplied with Identity is same

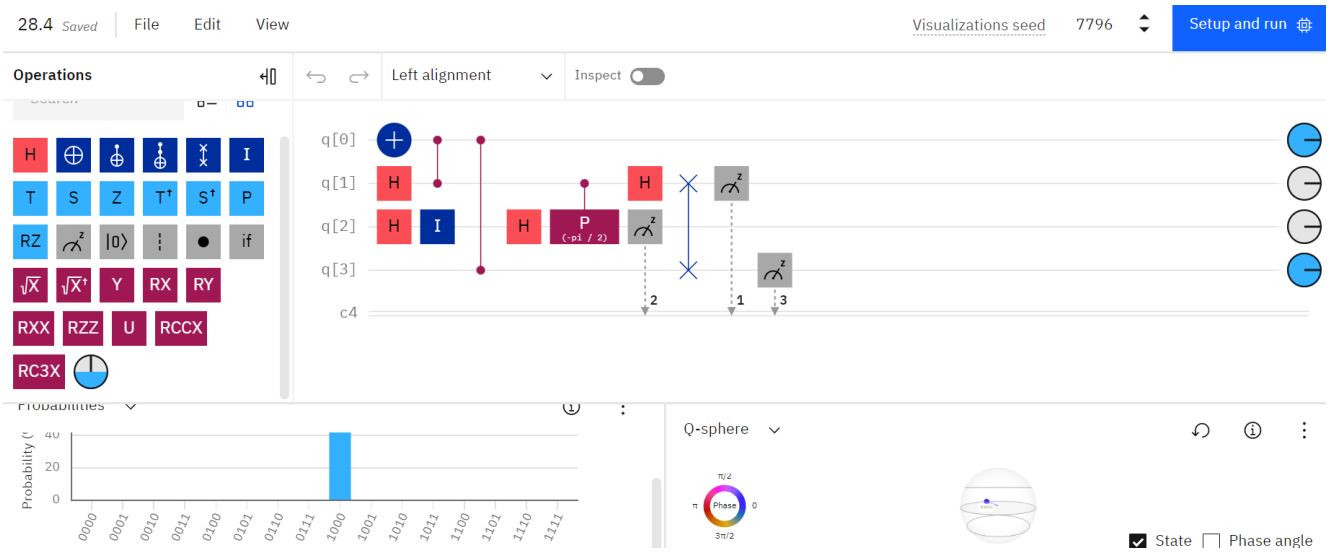

$\begin{bmatrix}1&0&0&0\\0&0&0&0\\0&0&0&0\\0&0&0&0\end{bmatrix} + \begin{bmatrix}0&0&0&0\\0&0&0&0\\0&0&e^{i\gamma}\cos\theta/2 & -e^{i(\lambda+\gamma)}\sin\theta/2\\0&0&e^{i(\phi+\gamma)}\sin\theta/2 & e^{i(\lambda+\phi+\gamma)}\cos\theta/2\end{bmatrix}$
$= \begin{bmatrix}1&0&0&0\\0&0&0&0\\0&0&e^{i\gamma}\cos\theta/2 & -e^{i(\lambda+\gamma)}\sin\theta/2\\0&0&e^{i(\phi+\gamma)}\sin\theta/2 & e^{i(\lambda+\phi+\gamma)}\cos\theta/2\end{bmatrix}$

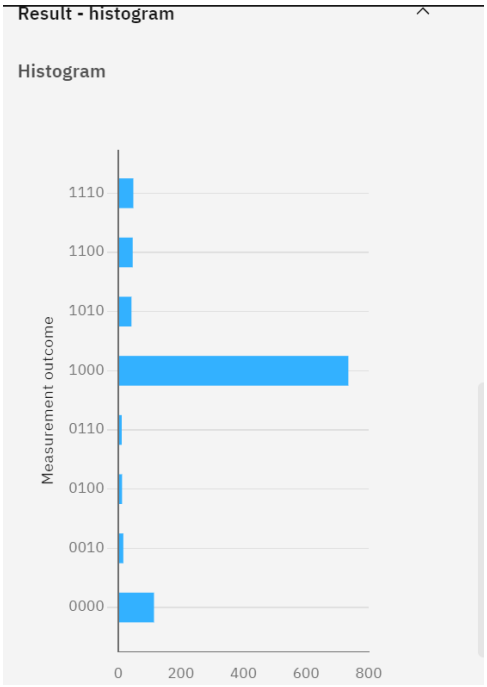
28.4)


28.4)
we know
$U_{PS,\pi}|e_1\rangle = +1\begin{pmatrix}1\\0\end{pmatrix} = +1|e_0\rangle$
$Z|e_0\rangle = e^{i2\pi\cdot 0}|e_0\rangle = |e_0\rangle$
$Z^2|e_0\rangle = (e^{i2\pi\cdot 0})^2|e_0\rangle = |e_0\rangle$
$Z = e^{i\cdot 2\pi\cdot 0}$ i.e, $e^0 = \cos 0 + i\sin 0 = 1$
If control Qubit is 0 (or) e^0 it does nothing

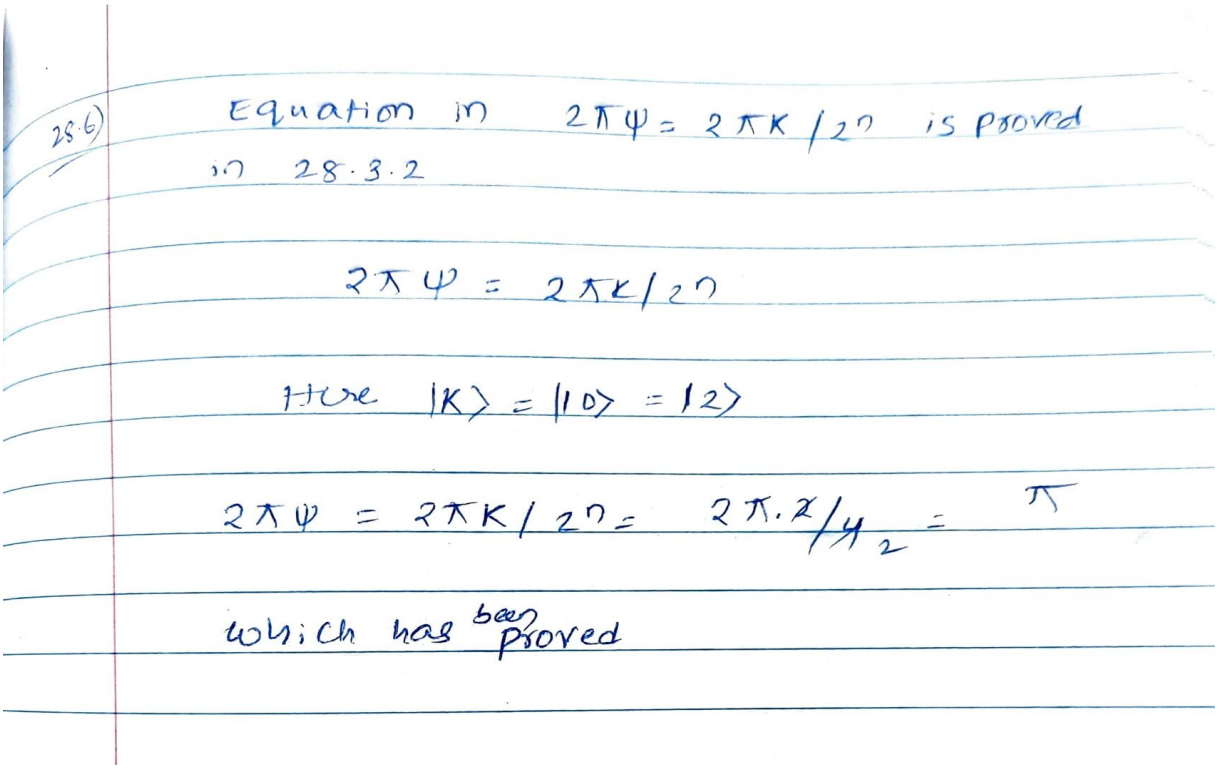
28.5)

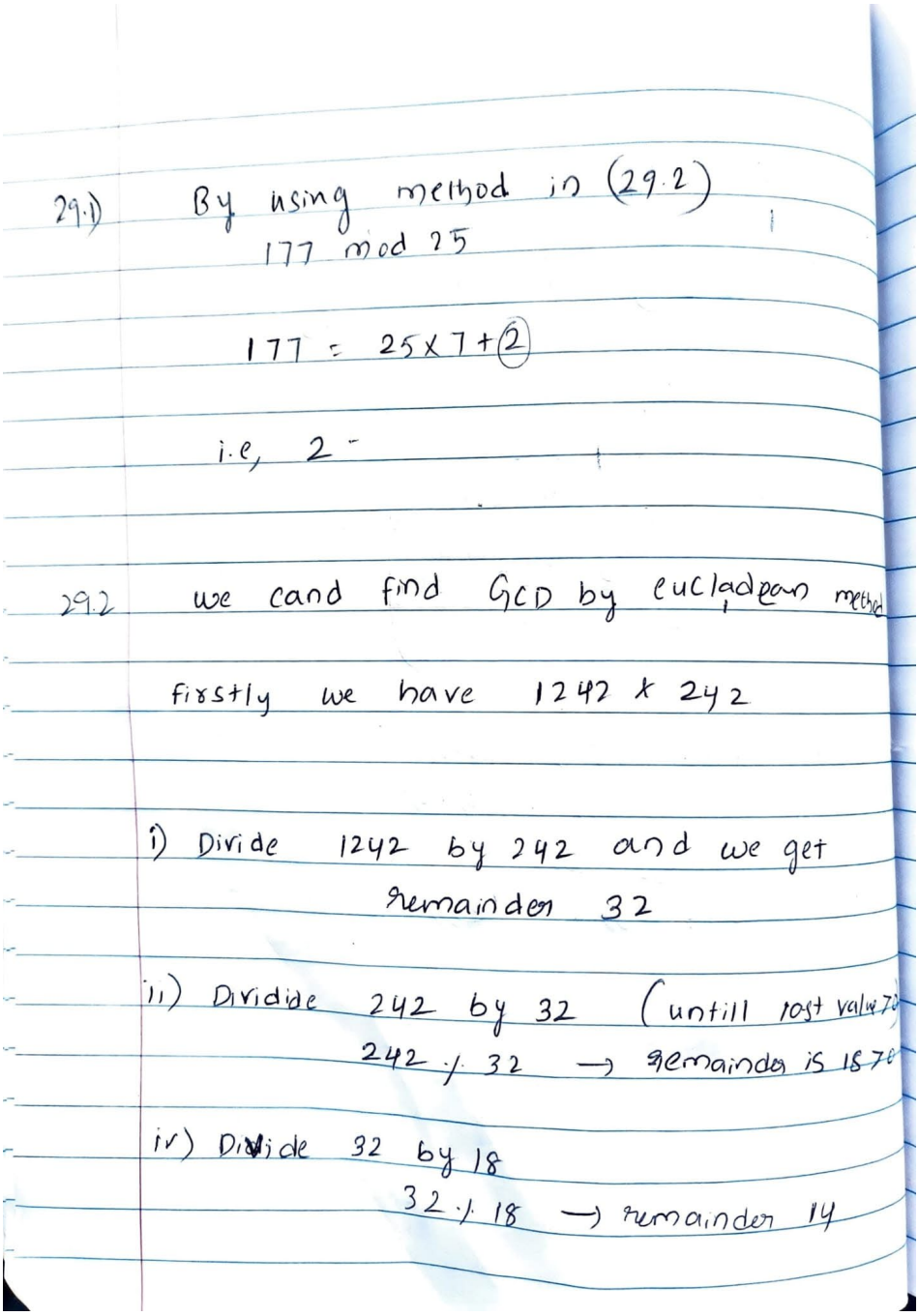
Introduction to Quantum computing (From a layperson to programmer in 30 steps)





Nithin Reddy Govindugari(nithinreddy1747@gmail.com)


28.6)
Equation in $2\pi\psi = 2\pi k/2^n$ is proved in 28.3.2
$2\pi\psi = 2\pi k/2^n$
Here $|k\rangle = |10\rangle = |2\rangle$
$2\pi\psi = 2\pi k/2^n = 2\pi.2/4_2 = \pi$
which has been proved



29.1) By using method in (29.2)
177 mod 25

$177 = 25 \times 7 + 2$

i.e, 2

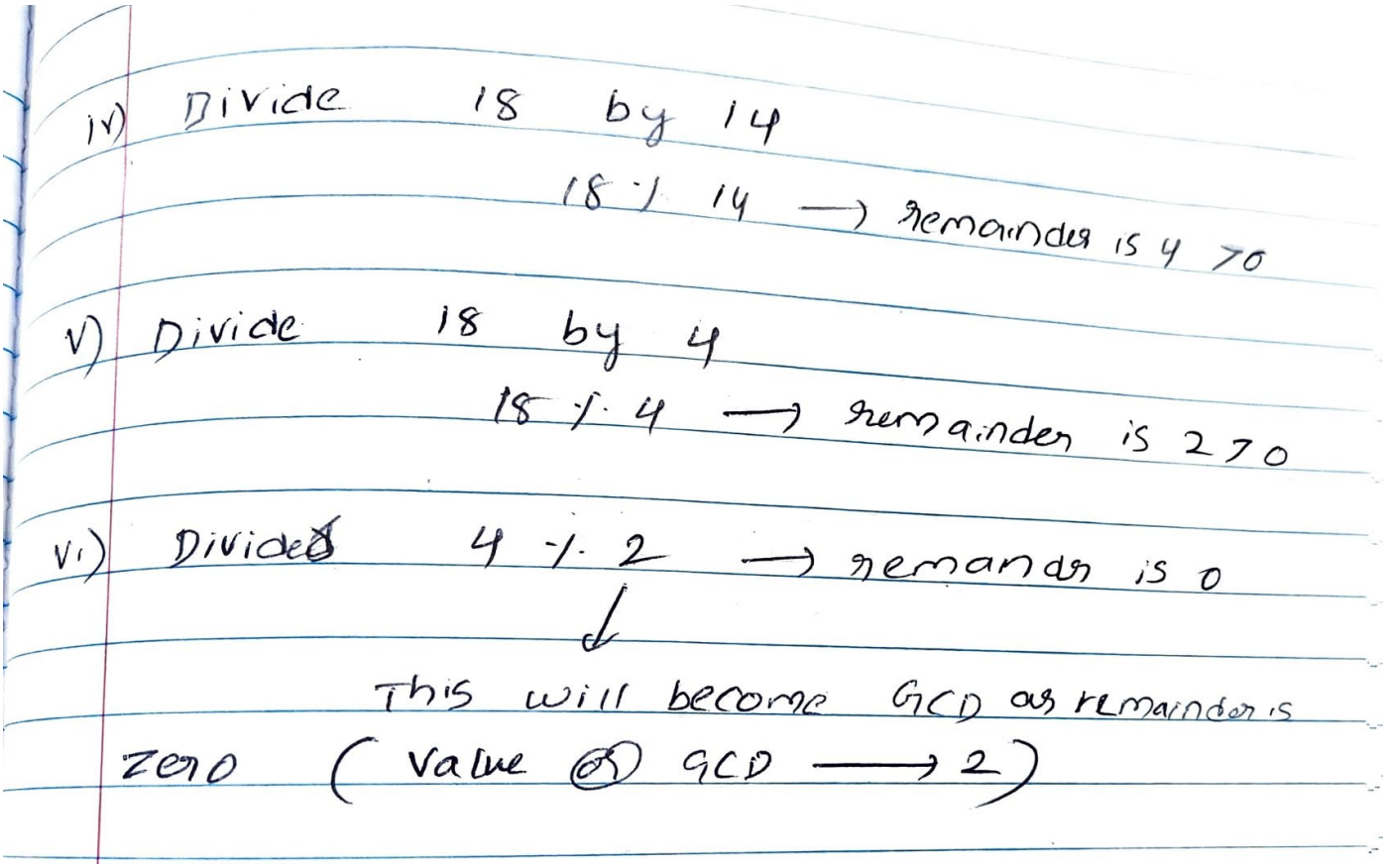
29.2 we cand find GCD by euclad ean method

firstly we have 1242 & 242

i) Divide 1242 by 242 and we get remainder 32

ii) Dividide 242 by 32 (untill last value)
242 % 32 → remainder is 18

iv) Divide 32 by 18
32 % 18 → remainder 14


iv) Divide 18 by 14
18 % 14 → remainder is 4 ?0
v) Divide 18 by 4
18 % 4 → remainder is 2 ?0
vi) Divide 4 % 2 → remainder is 0
This will become GCD as remainder is zero (value of GCD → 2)

29.3)

```
# Shors simplified algorithm in python but cant handle big prime numbers
#29.3
# gcf is calculated
def get_gcf(a, b):
    while a != b:
        if a > b:
            a -= b
        elif b > a:
            b -= a
    return a

# I have used below integers as a in our text book we can add any number of numbers

# 12 is not suitable a so i have used incrementation to get better a

Randomint = [8, 10, 12]

for g in Randomint:
  N = 63
  done = False
  p = 0
  p_values = []
  while not done:
    p += 1
    r = g**p % N
    if g**p % N == 1:
        print(g, p)
        p_values.append(p)
    if len(p_values) >= 1:
        p = p_values[0]
        if p_values[0] % 2 != 0:
            g += 1
            p = 0
            p_values = []
        else:
            x = g**(p/2)+1
            y = g**(p/2)-1
            a = int(get_gcf(N, x))
            b = int(get_gcf(N, y))
            if a == 1 or b == 1:
                g += 1
                p = 0
                p_values = []
            else:
                print(g, p, a, b)
                done = True
```

```
    if p > 500:
        g += 1
        p = 0
        p_values = []

  print(a, "x", b, "=", N)
```

```
8 2
8 2 9 7
9 x 7 = 63
10 6
10 6 7 9
7 x 9 = 63
13 6
13 6 7 9
7 x 9 = 63
```

29.4)

29.4) Here we have four steps to do before measurement and as informed it is practice for substitution

For the last question where it is asked that if $n=1$ will be possible and according to my understanding it is not possible because as explained in textbook 29.3 [page No 282] we need have n where $2^n \geq n$ if $n=1$ $2^1 \geq n$ so we need to have N which is less than (equal) 2 which is not appropriate and i estimate we need to have atleast $3n+2$ qubits → means N should be atleast 2

Step 1

$$|\varepsilon_1\rangle = (H^{\otimes 2n} I^{\otimes n})(|0\rangle^{\otimes 2n} |0\rangle^{\otimes n})$$

$$= \frac{1}{2^n} \sum_{x=0}^{2^n-1} |x\rangle_{2n} |0\rangle_n$$

If $n=1$

$$= \frac{1}{2} \sum_{x=0}^{3} |3\rangle_2 |0\rangle_1 \longrightarrow (1)$$

we can expand it to

$$= \frac{1}{2} [|00\rangle|0\rangle + |01\rangle|0\rangle + |10\rangle|0\rangle + |11\rangle|0\rangle]$$

But we are keeping it as (1) for simplification

$$= 1$$

Step 2 Applying Quantum oracle & evaluating function

$|\varepsilon_2\rangle = U_f |\varepsilon_1\rangle$

From 29.11

$$|\varepsilon_2\rangle = \frac{1}{2} \sum_{x=0}^{3} |3\rangle_2 |f(x)\rangle_3$$

Step 3

$$U_{QFT}|x\rangle = \frac{1}{\sqrt{2^{2n}}} \sum_{K=0}^{2^{2n}-1} \omega^{-Kx} |K\rangle \quad \begin{pmatrix} K \text{ is row} \\ x \text{ is column} \end{pmatrix}$$

$$|\varepsilon_3\rangle = (U_{QFT} \otimes I^{\otimes n}) |\varepsilon_2\rangle$$

From 29.13

$$= \frac{1}{4^n} \sum_{x=0}^{3} \sum_{x=0}^{3} \omega^{-Kx} |K\rangle_2 |f(x)\rangle.$$

Step 4

It is measurement on the least significant bit

which we can ignore according to the question asked

It is not possible to have 1 in my opinion as we need to have atleast 5 qubits to execute shors Algorithm

Nithin Reddy Govindugari(nithinreddy1747@gmail.com)

Introduction to Quantum computing (From a layperson to programmer in 30 steps)

Nithin Reddy Govindugari(nithinreddy1747@gmail.com)