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Team Control Number

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T1 _____

T2 ____

T3 ____

Problem Chosen

T4 ____

B

2013 Mathematical Contest in Modeling (MCM) Summary Sheet

Our modeling aims at developing an overall national water strategy for China in 2025 to better distribute water resources and fill up the gap between freshwater demand and supply. Thus we build 5 models totally.

The first model predicts water shortage in 2025. We divide China into 9 main basin areas to discuss the problems more clearly and concisely. In southern China, as the water condition is relatively stable and regular, we use *regression models* to predict supply and demand, while in northern China, where the condition is more complex and fluctuating, *time series analysis* is applied to predict the supply and demand. *Supply minus demand equals the water shortage*.

The next three models compose an integrated water strategy:

The second model estimates possible droughts in the future. We apply *gray theory* and *GM(1,1) model* to obtain the years of potential droughts, their duration and occurrence cycle. Also, we suggest a *water storage plan* according predictions obtained from *gray model* to make up the supply gap during the drought spell.

In the third model we predict China's desalination capacity in 2025 using *Logistic model*, form the shortest route using *minimum spanning tree*, and give out *3 proposals on integrated planning* of desalination and water transfer for the government too choose from.

The fourth model advices on **several ways of water conservation** which can reduce the shortage by reducing the demand.

In the last model, we make an analysis of the water strategy in 3 aspects, namely economic, social and environmental influences. In particular, as for economic influences, we apply *Keynes's theory of investment multiplier* and figures out how much will the GDP increase due to the investment.

In conclusion, we have adopted 5 distinct models to cover every aspect of the problem, perusing a high accuracy and feasibility.

EASING THE THIRST. OF-CHINA! WATER STRATEGY FOR CHINA IN 2025

TEAM NUMBER: **22330** PROBLEM CHOSEN: B

2013 ANNUAL MATHEMATICAL CONTEST IN MODELLING

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1. Introduction

Human is born from water. When a baby feels the world for the first time it is surrounded by water, on which it will survive throughout his life. Water, the oxide with the smallest relative molecular mass, the substance that covers 70% of the earth's surface, makes the earth the unique lively planet as it is known.

Nevertheless, the number 70% does not mean that water supply is perfectly meeting people's critical demands for fresh water. On the contrary, fresh water is in shortage, limited by natural conditions and threatened by human activities.

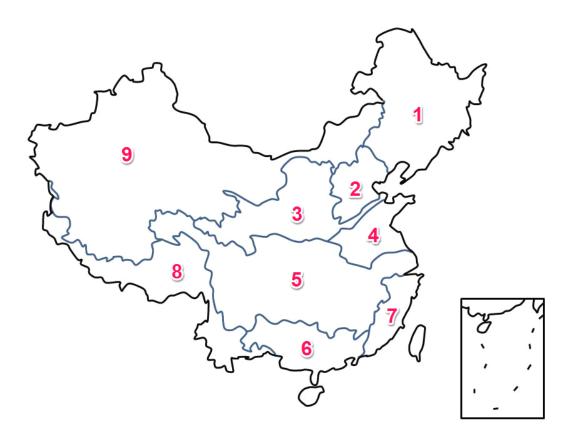
Entering the second decade of the 21st century, we must figure out a solution to ensure people's living standard and boom industrial development. That's the goal of this model we have been working on.

Here we have picked China as a study objective, which has undergone dramatic development, yet its development is hindered by the lack of fresh water. China boasts 6% of the world's water resources, ranking 3rd, but at the same time it is listed among the 13 most water-deficient countries in the world, due to its enormous population, huge demand and massive pollution. We are now confronted with the urgent need to minimize the gap between supply and demand of fresh water in China, currently measuring some 6 billion cubic meters, but it could have been lessened somehow. Desalination of sea water, better distribution of water resources, and water conservation, are the means we are going to discuss in our paper.

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2. Dividing Area

Since China boasts a vast territory, it is hardly feasible to build a model for China as a whole. Therefore, we divide China into 9 basin areas according to the major rivers in the areas, namely: (1)Northeast Rivers, (2)Haihe River, (3)Yellow River, (4)Huaihe River, (5)Yangtze River, (6)Pearl River, (7)Southeast Rivers, (8)Southwest Rivers, and (9)Northwest Rivers, as is shown below.



All the following models are discussed by the 9 basin areas, omitting individual situations of the cities within. Hainan, Taiwan and other remote islands are excluded.

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3. Water Consumption I-Southern China

3.1 Introduction

In this model, we mainly discuss water consumption in southern China's river basin areas, namely Yangtze River, Pearl River, Southeast Rivers and Southwest Rivers.

After observing historical data of these basin areas, we have found out that despite the fact that water use in an area varies with the passage of time, it demonstrates an overall trend. The reason is that water use is directly related to population, which generally grows or declines over a period of time. Regression model is an effective method to find potential relation between independent variables and dependent variables, the passage of time and water consumption in this case.

3.2 Restatement of the Problem

In order to establish a water strategy for China for 2025, as is stated in the problem, we must obtain the water consumption of that year in the first place by simulations that are based on available historical data. Thus, regression models are brought to the stage. Linearly fitted data helps get predicted values in the future with little effort but high precision.

3.3 Assumptions and Justifications

1. Chosen areas will not experience dramatic changes in water consumption habits at least before 2025.

Regression models do not work well when dramatic changes take place and influence current situation violently.

2. Chosen areas will not experience mass immigration.

As is clarified previously, water consumption is directly related to population. The models do not take abnormal population changes into consideration.

3. Chosen areas will not be affected by natural changes due to climate change, such as droughts and glacier melting.

It is obvious that natural changes impose huge impact on water consumption. Such abnormal conditions are excluded from the models.

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3.4 Variables

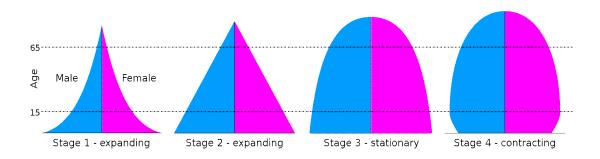
Mark	Meaning	Unit
x	Year 1996+x	/
У	Water consumption of an basin area	10 ⁸ m ³ per year

3.5 Establishment of the Model

3.5.1 Population Pyramid

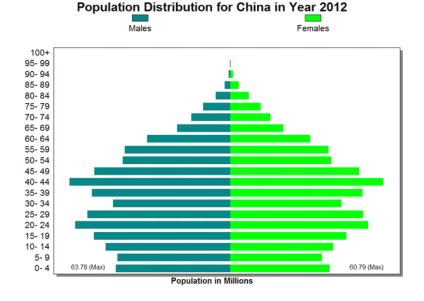
It is obvious that water consumption is closely related to population. The population of a typical developing country increases at a large rate, but it is not the case for China. We applied "Population Pyramid" to analyze China's population growth.

There are four general types of population structures in the world, namely: rapidly expanding (stage 1), expanding (stage 2), stationary (stage 3), and contracting (stage 4). The diagrams each form the shape of a pyramid, as is shown below:



According to China's population pyramid in 2012, the top of the pyramid resembles the shape of the "expanding" stage, while the bottom the shape of the "stationary" stage. However, considering the fact that future population growth is decided more by the younger generations and that the bottom of the pyramid represents the population structure of the younger generations, we have decided that China should fall in the category of the "stationary" stage, which means population growth is stable and gentle.

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3.5.2 Linear Regression Model

Based on the facts illustrated in the theory of "Population Pyramid", the population growth with the passage of time can be regarded as linear. Thus, we assume water consumption bears a linear relation to the passage of time.

We obtained historical water consumption data of each area from Chinese Ministry of Water Resources.

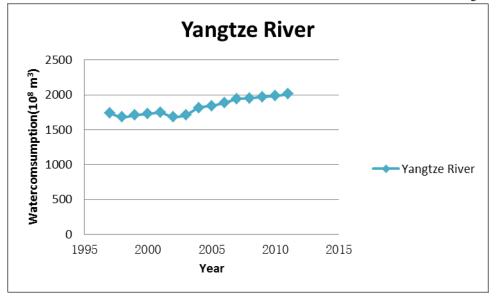
We take Yangtze River for an example.

Year	Consumption				
1997	1737.12	2002	1682.31	2007	1939.6
1998	1679	2003	1714	2008	1951.5
1999	1708.38	2004	1815.4	2009	1970.4
2000	1724.92	2005	1842.2	2010	1983.1
2001	1742.84	2006	1884.3	2011	2010

^{*}Unit:x108 m3

We drew a scatter plot of the data. It can be inferred from the graph below that with the passage of time, water consumption shows an obvious trend of increasing in spite of little fluctuation. This validates the assumption that water consumption bears linear relation to the passage of time. Regression model proves to be effective and efficient.





In order to investigate the exact relation between the passage of time and water consumption, we constructed an explanatory model:

$$y = \alpha x + \beta + \varepsilon$$
(1)

where

x = passage of time (year);

y =water consumption;

 α =the effect of the values of time on the values of water consumption;

 β =a constant;

 ε = stochastic variables.

Comparing equation (1) with the standard regression model:

$$\hat{y} = \hat{\alpha}x + \hat{\beta}$$

We can find that $\alpha = \hat{\alpha}$; $\hat{\beta} = \beta + \varepsilon$; $y = \hat{y}$.

Using least square method, we obtain:

$$\hat{\beta} = \frac{\sum_{i=1}^{n} (x - \overline{x})(y - \overline{y})}{\sum_{i=1}^{n} (x - \overline{x})^{2}}$$

$$\hat{\alpha} = \overline{y} - \hat{\beta}\overline{x}$$

The correlation coefficient is defined as:

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$$r^{2} = \frac{\sum_{i=1}^{n} (\hat{y} - \overline{y})^{2}}{\sum_{i=1}^{n} (y - \overline{y})^{2}}$$

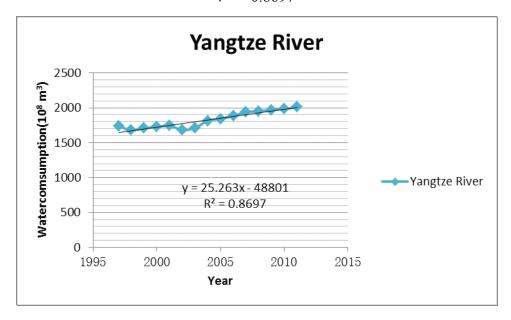
The closer the correlation coefficient is to 1, the better the fitting of the model is, which means the dependent variable y is closely related to the independent variable x.

We applied the methods above to the data of Yellow River and obtained the regression equation:

$$y = 25.263x - 48801$$

and the correlation coefficient:

$$r^2 = 0.8697$$



The correlation coefficient shows that the data is well fitted, and the expected volume of water consumption in the year 2025 is 2356.149x10⁸ cubic meters.

Similarly, we applied regression model to other southern basin areas and obtained predicted values of water consumption in 2025.

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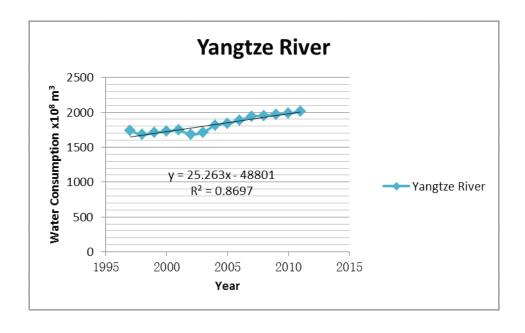
Basins	1997	1998	1999	2000	2001
Yangtze River	1737.12	1679	1708.38	1724.92	1742.84
Pearl River	834.09	842	839.74	824.69	838.54
Southeast	290.97	306	311.92	310.92	312.56
Southwest	85.86	82	98.03	99.27	99.95

Basins	2002	2003	2004	2005	2006
Yangtze River	1682.31	1714	1815.4	1842.2	1884.3
Pearl River	850.78	840	862.3	873.6	878.9
Southeast	319.31	317	316.3	325.1	327.6
Southwest	103.3	94	96.9	101.6	102.4

Basins	2007	2008	2009	2010	2011	2025
Yangtze River	1939.6	1951.5	1970.4	1983.1	2010	2246.76
Pearl River	879.9	881.2	876.8	883.5	876.8	873.4
Southeast	338	343.6	338.2	342.5	346.1	365.84
Southwest	108.7	111.8	104.3	108	107.8	99.16

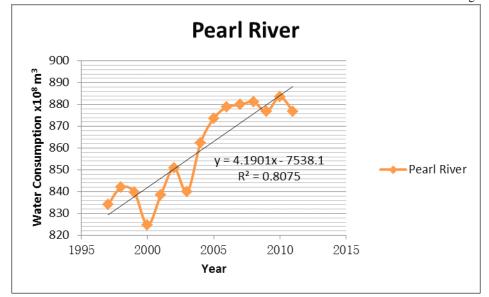
Abnormal Conditions Drought Severe pollution

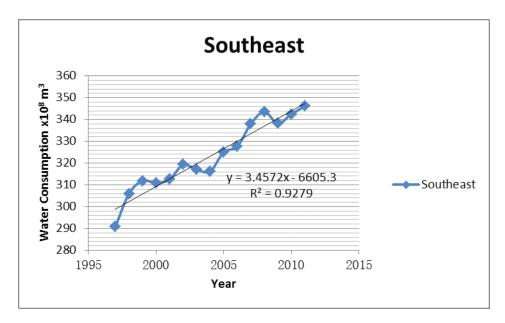
^{*}Unit:x108 m3

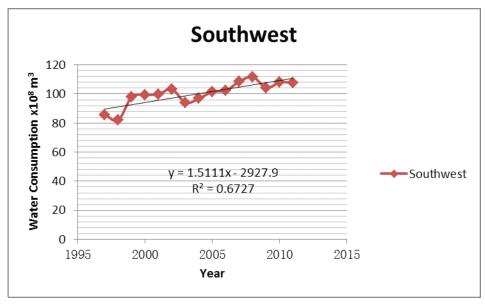


^{*} Droughts and Severe water pollutions are considered as events contributes to abnormal data.

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4. Water Consumption II-Northern China

4.1 Introduction

In this model, we mainly discuss water consumption in northern China's basin areas, namely Northeast Rivers, Haihe River, Yellow River, Huaihe River and Northwest Rivers.

Historical data of these areas appear to be fluctuating significantly, making linear regression inaccurate. However, the fluctuation is periodical according to the scatter plots. Therefore, we adopted "Time Series Analysis" to make prediction as precise as possible.

4.2 Assumptions and Justifications

See "Assumptions and Justifications" in 3.3.

4.3 Variables

Mark	Meaning	Unit
${\cal Y}_k$	Water consumption in the year 1996+k	10 ⁸ m ³ per year
p_1, p_2, p_3	Weights of y_k	/

4.4 Establishment of the Model

The principle of Time Series Analysis lies in the fact that every predicted value is calculated based on the three values preceding it, each of which is multiplied by a weight. The sum of the weights is 1.

Therefore, a recursion equation is established as follows:

$$y_{k+3} = p_1 y_k + p_2 y_{k+1} + p_3 y_{k+2}$$
,

where

$$p_1 + p_2 + p_3 = 1$$
, $p_1, p_2, p_3 \in [-1,1]$.

Using Matlab language, we obtained the optimal weights that fit the best with original data by

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enumerating all the numbers from -1 to 1 with the step of 0.01.

Optimal Weights

Basins	p1	p2	р3
Northeast	0.99	1	-0.99
Haihe River	0.22	0.84	-0.06
Yellow River	-0.63	0.78	0.84
Huaihe River	-0.37	0.84	0.53
Northwest	-0.99	1	0.99

And the predicted values of water consumption in 2025 are obtained:

2025 Water Consumption Prediction

Basins	2025 Consumption			
Northeast	619.8			
Haihe River	363.4			
Yellow River	406.1			
Huaihe River	562.6			
Northwest	627.9			

^{*}Unit:x108 m3

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5.Gap between Supply and Consumption

5.1 Introduction

The essence of China's water problem lies in the deficit of supply. Only when the gap between supply and demand is figured out can we plan for a strategy. The ultimate goal of our water strategy is to bridge the gap.

Supply minus demand equals gap. We have obtained the results of demand in the previous models. This model will illustrate the water supply scientifically. Then the gap can be easily obtained.

5.2 Variables

Mark	Meaning	Unit
x	Year	/
\mathcal{Y}	Precipitation	10 ⁸ m ³ per year
Con	Water Consumption	10 ⁸ m ³ per year
Con ₂₀₂₅	Estimated Water Consumption in 2025	10 ⁸ m ³ per year
S	Supply	10 ⁸ m ³ per year
S_{2025}	Estimated supply in 2025	10 ⁸ m ³ per year
C	Supply coefficient	/
C_{2025}	Estimated Supply coefficient in 2025	/
G	Gap between water supply and consumption	10 ⁸ m ³ per year
G_{2025}	Estimated gap between water supply and consumption in 2025	10 ⁸ m ³ per year

5.3 Establishment of the Model

5.3.1 Precipitation Prediction

It can be easily understood that the amount of water an area can supply to its habitants is mainly decided by precipitation. Thus, prediction of precipitation is necessary.

Data on precipitation are barely available on the Internet. Those we have found are of small samples and are discontinuous. So we applied Time Series Analysis to predict precipitation in

2025. The results are as follows:

Precipitation Prediction

Unit: 10 ⁸ m ³	1997	1999	2000	2001	2006	2025
Northeast	5612.3	5215.7	5415.68	4963.22	5817.2	4545.5
Haihe River	1164.6	1224.4	1559.36	1325.84	1402.5	1384.7
Yellow River	2630.8	3181.3	3043.46	3217.71	3237.1	3042.2
Huaihe River	2224.5	2212.3	3062.29	2100.73	2662.2	2150.8
Yangtze River	18338.1	20414	19561.45	17876.58	17366.9	19310.4
Pearl River	10829.2	8612.0	8548.94	10300.88	9369.0	9025.0
Southeast	4037	3668.6	3723.67	3521.97	3925.3	3344.0
Southwest	8661.7	9584.3	9517.54	9397.93	8474.0	7430.7
Northwest	4670.3	5589.9	5659.95	5417.35	5585.5	5402.5

5.3.2 Supply Coefficient

We define the supply coefficient:

$$C = \frac{S}{y}$$
,

where y represents precipitation while S represents supply.

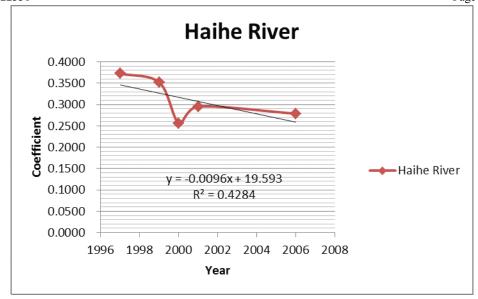
In Southern China

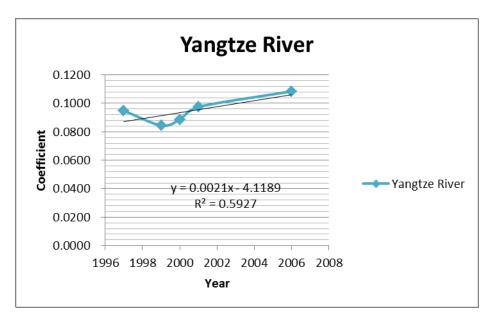
Southern China includes the basin areas of Haihe River, Yangtze River, Pearl River, Southeast Rivers and Southwest Rivers. By observing the data, we decide to apply linear regression to the calculation of supply coefficient in 2025 because data are of little fluctuation. The results are as follows:

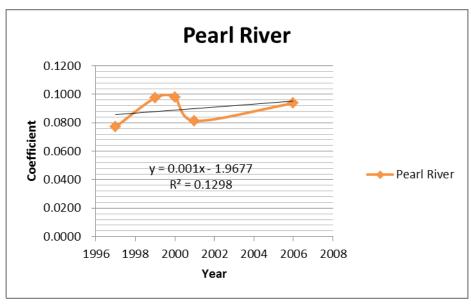
Supply Coefficient in Southern China

Coefficient	1997	1999	2000	2001	2006	2025
Haihe River	0.3728	0.3522	0.2562	0.2954	0.2788	0.0759
Yangtze River	0.0948	0.0845	0.0887	0.0975	0.1085	0.1462
Pearl River	0.0772	0.0975	0.0978	0.0814	0.0938	0.1146
Southeast	0.0714	0.0839	0.0848	0.0887	0.0835	0.1063
Southwest	0.0101	0.0102	0.0104	0.0106	0.0121	0.0164

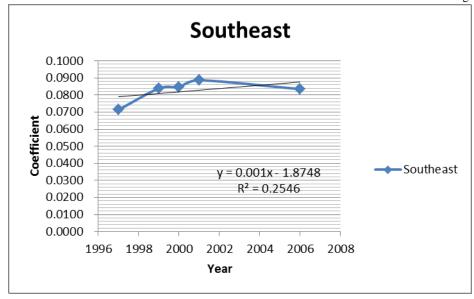
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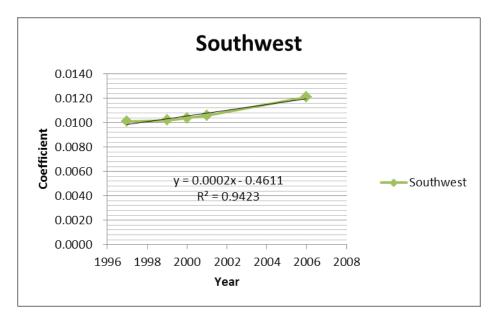






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In Northern China

Northern China includes the basin areas of Northeast Rivers, Yellow River, and Huaihe River. We apply time series analysis to the prediction of supply coefficient in 2025 because data fluctuates obviously. And the results are obtained as follows:

Supply Coefficient in Northern China

Coefficient	1997	1999	2000	2001	2006	2025
Northeast	0.1104	0.1188	0.1141	0.1201	0.0985	0.131
Yellow River	0.1538	0.1279	0.1293	0.1229	0.1224	0.127
Huaihe River	0.2999	0.2714	0.1811	0.2893	0.2225	0.234

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Then the estimated supply in 2025 can be obtained:

$$S_{2025} = y \cdot C_{2025}$$

Estimated 2025 Water supply

Basins	2025 Prec.(10 ⁸ m ³)	2025 Coef.	2025 Supply(10 ⁸ m ³)
Northeast	4545.5	0.1310	595.46
Haihe River	1384.7	0.0759	105.1
Yellow River	3042.2	0.1270	386.36
Huaihe River	2150.8	0.2340	503.29
Yangtze River	19310.4	0.1462	2823.18
Pearl River	9025.0	0.1146	1034.27
Southeast	3344.0	0.1063	355.47
Southwest	7430.7	0.0164	121.86
Northwest	5402.5	0.1055	570.0

We define the gap between water supply and consumption as:

$$G = S - Con$$

And the estimated gap in 2025:

$$G_{2025} = S_{2025} - Con_{2025}$$

Supply and Consumption Gap in 2025

Basins	2025 Supply	2025 Consu.	Gap	
Northeast	595.46	619.8	-24.34	
Haihe River	105.1	299.45	-194.35	
Yellow River	386.36	406.1	-19.74	
Huaihe River	503.29	562.6	-59.31	
Yangtze River	2823.18	2356.19	466.99	
Pearl River	1034.27	946.85	87.42	
Southeast	355.47	395.67	-40.2	
Southwest	121.86	131.99	-10.13	
Northwest	570	627.9	-57.9	
Total	6494.99	6346.55	148.44	
Surplus			554.41	
Insufficiency			-405.97	

Unit: $10^8 \, \text{m}^3$

The gap that we have predicted will be a basis of the water movement strategy elaborated further then.

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6.Drought Prediction and Water Storage Plan

6.1 Introduction

Droughts bring about significant impact on water consumption. In order to deal with droughts more efficiently, a model that predicts the time when droughts happen is needed. Gray theory and GM(1,1) model appears to be a good choice for prediction. Knowing the time when natural disaster is to strike, the government is able to carry out storage plans in advance correspondingly to make up the supply gap during the drought spell. We will make drought prediction in basin areas that are insufficient in water resources and make suggestions on water storage plan.

6.2 Restatement

Under most circumstances, droughts occur to a certain regularity and periodicity. Using the historical data, the prediction model is able to demonstrate the time when a drought is likely to emerge; that is to say, we will obtain the year of its occurrence, how long it will last and its cycle.

Then we multiply the duration of the drought with the annual water demand. We use the methods illustrated previously to figure out the total demand. Dividing it by its cycle, we can easily estimate how much water the reservoirs in that basin area should store every year in case there should be a drought.

6.3 Assumptions

- 1. When a drought occurs, the fresh water demand in the stricken area will be met only by the water storage of reservoirs.
- 2. Droughts have a certain regularity and periodicity to occur.

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6.4 Variables

Mark	Meaning		
$k_{(0)}$	The year that the data set begin with		
k	The year $k_{(0)} + k$		
п	The number of droughts that had happened		
$x^{(0)}$	The original sequence that donates the years of droughts that happened in history		
$\mathcal{X}^{(1)}$	The gray sequence generated from the $x^{(0)}$ through Accumulating Generation Operation (AGO)		
$z^{(1)}$	The generated mean sequence of $x^{(1)}$		
$x^{(0)}(k)/x^{(1)}(k)/z^{(1)}(k)$	The k^{\prime} th element in a sequence.		

6.5 Drought Prediction - Gray Model

We take Yellow River as an example.

6.5.1 Procedure I: Obtain original data and generate the original sequence

We have collected data since the year of 1950 and droughts happened in the years of:

1953, 1960, 1961, 1962, 1965, 1971, 1972, 1973, 1980, 1986, 1987, 1994, 1997, 1999, 2000, 2009, 2011.

The $x^{(0)}$ is obtained:

$$x^{(0)} = (3,10,11,12,15,21,22,23,30,36,37,44,47,49,50,59,61)$$

6.5.2 Procedure II: Obtain the gray sequence through AGO

The $x^{(1)}$ is obtained through accumulation:

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$$x^{(1)} = \sum_{i=1}^{17} x^{(0)}(i) = (3,13,24,36,51,72,94,117,147,183,220,264,311,360,410,469,530)$$

6.5.3 Procedure III: Obtain the mean sequence

 $z^{(1)}$ is decided by:

$$z^{(1)}(k) = \alpha x^{(1)}(k) + (1-\alpha)x^{(1)}(k-1), (k=2,3,...,17)$$

where α is valued 0.5.

So:

$$z^{(1)} = (0.8, 18.5, 30, 43.5, 61.5, 83, 132, 165, 201.5, 242, 287.5, 335.5, 385.439.5, 499.5)$$

6.5.4 Procedure IV: Establishment and solution of gray differential equation

Assume the gray derivative of $x^{(0)}$ as d(k), and we get:

$$d(k) = x^{(0)}(k)$$
.

The gray differential equation of GM(1,1) model is defined as:

$$d(k) + az^{(1)}(k) = b$$

or

$$x^{(0)}(k) + az^{(1)}(k) = b$$
,

where a =developing coefficient, b =gray action.

Using least square method:

$$\hat{a} = (CD - (n-1)E)/((n-1)F - C^2), \hat{b} = (DF - CE)/((n-1)F - C^2),$$

$$C = \sum_{k=2}^{n} z^{(1)}(k), D = \sum_{k=2}^{n} x^{(0)}(k), E = \sum_{k=2}^{n} z^{(1)}(k) x^{(0)}(k), F = \sum_{k=2}^{n} (z^{(1)}(k))^{2},$$

where n=17.

Plug \hat{a} and \hat{b} into the gray differential equation:

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$$\begin{cases} \hat{x}^{(0)}(k+1) = (x^{(0)}(1) - a/b)e^{-ak} + b/a \\ \hat{x}^{(0)}(k+1) = \hat{x}^{(1)}(k+1) - \hat{x}^{(1)}(k) \end{cases}$$

And the final solution is obtained:

$$\hat{x}^{(1)}(k+1) = (1 - e^{a})(x^{(0)}(1) - b/a)e^{-ak}$$

So the estimated years of droughts are obtained (we only calculated ten following droughts, which is enough for our strategy):

76.2846, 84.8825, 94.4495, 105.0947, 116.9398, 130.1199, 144.7856, 161.1041, 179.2620, 199.4663,.....

That is to say, the nearest drought in Yellow River basin area will take place in the year of 2027.

Similarly, we applied the model to Huaihe Basin and Haihe Basin, where droughts are frequently witnessed. Here are the results.

Huaihe Basin:

73.1721, 81.2945, 90.3185, 100.3442, 111.4827, 123.8577, 137.6064, 152.8812, 169.8516, 188.7058.

Haihe Basin:

66.4624, 83.6914, 105.3866, 132.7059, 167.1070, 210.4260, 264.9745, 333.6635, 420.1588, 529.0760.

That is to say, in Huaihe Basin, drought is expected to take place in 2023, while Haihe Basin in 2017.

6.6 Water Storage Plan

According to the drought prediction, water storage plan may be carried out by the government in advance to address water supply issues during that hard time.

The following expression shows how much water the reservoirs in a region should store per year.

$$S_{y} = \frac{nD}{T}$$

where

 S_v : annual storage

n: duration of drought (n years)

D: total demand

6.7 Case Study - Yellow River

Using gray model, we have obtained the result that the next drought in Yellow River Basin will happen in 2027, and it will last for one year.

Using Time Series Analysis, we obtained predicted demand values in Yellow River Basin in 2027. That is 41.088 billion cubic meters.

There are altogether 15 years for the government to store water for the 2027 drought, so the average storage volume per year is around 2.7392 billion cubic meters.

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7. Desalination and Water Transfer

7.1 Desalination

7.1.1 Introduction

Desalination basically means the process of removing the salt and minerals in sea water so as to make it potable or usable.

Due to the great amount of energy required, this method is not economically practical. By implementing cogeneration, which is the process of using the excessive heat produced by other activities in desalination, costs of desalination can be reduced to a great extend. In addition, salt, desalination's byproduct, can be extracted from the procedure to increase its revenue.

Some of the technologies of desalination have already gone into practical use in certain countries, such as forward osmosis, geothermal desalination, nanotube membrane, biological membrane, etc.

7.1.2 Future Desalination Capacity Estimates

We obtained data on China's capacity of desalination per day:

Seawater Desalination Capacity

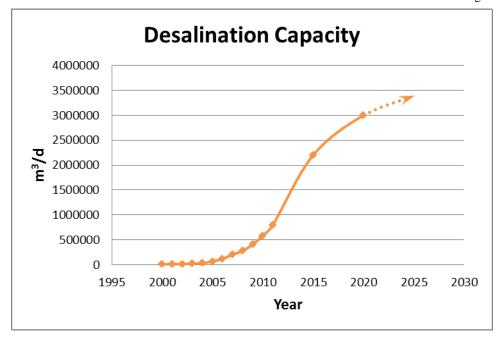
Year	2000	2001	2002	2003	2004	2005	2006
Capacity	9550	14550	16300	27960	33610	70090	120394
Year	2007	2008	2009	2010	2011	2015(Plan)	2020(Plan)
Capacity	205014	281024	408800	585400	800000	2200000	3000000

Unit: m³/d

Observing the scatter plot, we have found out that the curve of desalination capacity resembles a Logistic curve, so we have decided to predict desalination capacity of 2025 through simulating a Logistic equation.

The predicted desalination capacity of 2025 is around 3400000 cubic meters per day, or 12.41x10⁸ cubic meters per year.

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7.2 Project under Construction: South-to-North Water Diversion

South-to-North Water Diversion Project is a major strategic project that deploys part of the abundant water resources in southern China to northern and northwestern China, thus relieving flood in the south, drought and severe water shortage in the north. The purpose is to promote the coordinated development of economy, society, population, resources and environment in both southern and northern China.

South-to-North Water Diversion Project includes three routes for water transfer: the eastern, the western and the central.

	Location	Year to finish	Diversion Scale
			Ceiling (10^8 m^3)
Eastern	the lower reaches of Yangtze River	2013	148
Route	→Shandong Peninsula & North		
	China Plain		
Central	a tributary of Yangtze River & the	2014	130
Route	upper reaches of Han River→Beijing		
	& Tianjin		
Western	the upper reaches of Yangtze River	Before 2025	170
Route	→upper reaches of Yellow River		

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7.3 Integrated Planning

The desalination of sea water has its geographic limitations. It is suitable for large-scale construction and production in densely populated coastal areas. Then the desalinized water is transported into the central and western inland areas. Considering the undergoing construction of South-to-North Water Diversion, we want to make a combination of the two in order to reduce the costs.

Due to the difficulty of access to all the crucial information, we hereby provide several proposals with formulas of costs for the user to input data and obtain the results.

7.4 Proposals for Water Transfer and Desalination

7.4.1 Preconditions

Before demonstrating the concrete solution, we make a classification of water by their quality.

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Water	Category	Salinity	Desalination
Class		$(TDS=[Ca^{2+}+Mg^{2+}+Na^{+}+K^{+}]+$	Cost
		$[HCO_3^- + SO_4^{2-} + Cl^-]$	
I	Seawater	16—47‰	/
П	River Water	0.7-9.9‰	Relatively
			low
Ш	Freshwater	0.01-0.5‰	Relatively
			high

After that, by using the results of previous prediction models, we make a chart of shortage or surplus of the 9 basin areas:

Basins	2025 Supply	2025 Consume	Shortage/Surplus
Northeast Rivers	595.46	619.8	-24.34
Haihe River	105.1	299.45	-194.35
Yellow River	386.36	406.1	-19.74
Huaihe River	503.29	562.6	-59.31
Yangtze River	2823.18	2356.19	466.99
Pearl River	1034.27	946.85	87.42
Southeast Rivers	355.47	395.67	-40.2
Southwest Rivers	121.86	131.99	-10.13
Northwest Rivers	570	627.9	-57.9
Total	6494.99	6346.55	148.44
Surplus			554.41
Shortage in the North			-355.64

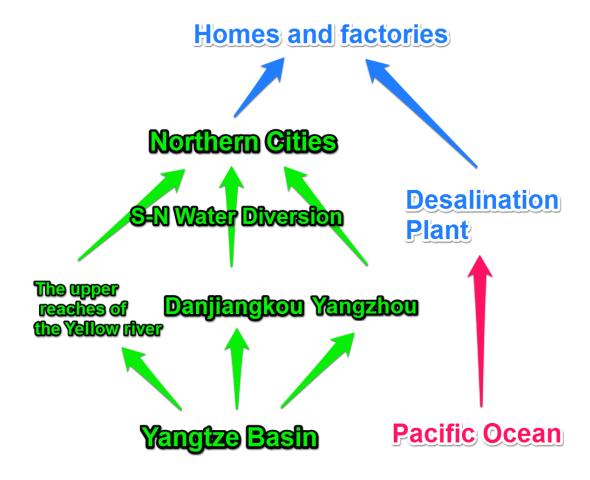
Unit: 108 m³

Since the population base in southern China is relatively large and the shortage per capita is small, therefore, we don't consider the shortage in southwest and southeast basin areas. Instead, we apply other solutions such as water conservation in those regions. In other words, in this model we only take account of the water shortage in northern China.

7.4.2 Proposal 1

In proposal 1, water desalination and South-to-North Water Diversion work separately. The desalination plants in northern coastal areas process the sea water into freshwater and transport it to destinations, then the diversion fills the rest of the insufficiency.

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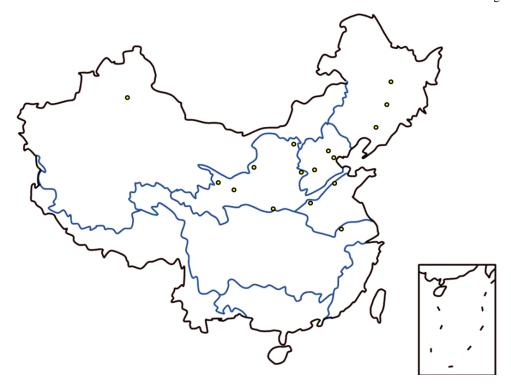
Blue=Freshwater; Green=River Water; Red=Seawater.

Minimum Spanning Tree (MST)

We choose the capital city of each province in the water-insufficient areas, i.e. northern China, as freshwater transport destinations. The destinations are listed as follows:

Harbin, Shijiazhuang, Lanzhou, Xining, Xi'an, Changchun, Shenyang, Zhengzhou, Jinan, Taiyuan, Hefei, Beijing, Tianjin, Hohhot, Yinchuan, Urumqi.

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We use minimum spanning tree model (MST) to create the route.

In graph theory, a "tree" refers to a connected non-cyclic graph. A minimum spanning tree is a spanning tree that connects all the dots in a weighted graph with the lowest edge weights.

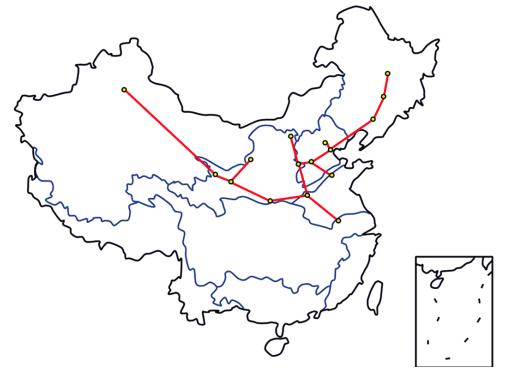
It can be described as:

where E(T) is the edge set of T, while W(e) is the weight of an edge.

Both Prim's and Kruskal's algorithms are common methods for MST. Since the dots are sparsely distributed on the graph, we have decided to use Kruskal's algorithm.

By programming in Matlab, we obtained the following graph of MST:

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The total distance of the routes is 6001 kilometers.

Total Cost

The total cost of proposal 1 is as follows:

$$C_1 = C_{d1}V_1 + C_tL_1 + C_{sn}(V_0 - V_1)$$

where

 C_1 : total cost

 C_{d1} : unit cost of desalination from seawater to freshwater

 C_i : unit cost of transport (including building, maintenance and management)

 $C_{{\scriptscriptstyle SN}}$: unit cost of South-to-North Water Diversion

 $V_{\scriptscriptstyle 0}$: volume of water resources gap

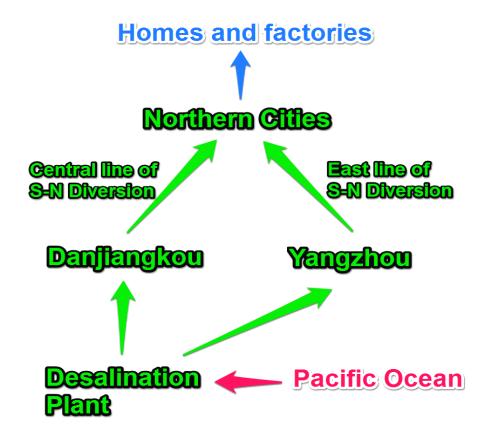
 V_1 : volume of desalinated water

 L_1 : distance of freshwater transport

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7.4.3 Proposal 2

In proposal 2 water diversion plays the principal role while the plants in the south also provide the whole diversion system with desalinated seawater (which is desalinated into river water standard). The natural river water and desalinated water flows into the three routes of the South-to-North Water Diversion Project.



Blue=Freshwater; Green=River Water; Red=Seawater.

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Total Cost

The total cost of proposal 2 is as follows:

$$C_2 = C_{d2}V_2 + C_{sn}V_0 + C_tL_2$$

where

C: total cost

 $C_{\it d2}$: unit cost of desalination from seawater to river water

 C_{sn} : unit cost of South-to-North Water Diversion

 V_0 : volume of water resources gap

 V_2 : volume of desalinated water

 L_2 : total distance

7.4.4 Proposal 3

With the absence of some of the crucial data, we cannot compare the cost of the first two proposals in number. However, once required values are input, results are worked out.

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Considering that the elements of the formula may not be linear, a third proposal combining the two is created where the first weight p and the second weight (1-p) are multiplied.

Since the costs on the construction of transfer pipelines are fixed, the total costs of proposal 3 is:

$$C_3 = pC_{d1}V_1 + (1-p)C_{d2}V_2 + C_{sn}V_0 - pC_{sn}V_1 + C_t(L_1 + L_2)$$

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8. Ways of saving water

Here are a few ways to save water that are practically available:

• Improved technology in water-consuming apparatus in industrial production and every-day life.

- Raising public awareness about water conservation, such as raising taxes or water prices for large per-capita users.
- Minimizing the pollution of water supplies through legislation.
- Using undrinkable water in industrial activities.
- Avoid wasting.

In conclusion, important measures should be to encourage the saving of water. Water is not saved in one day by one hero(ine) alone. It takes a world of people to do it.

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9. Analysis of the Strategy

9.1 Economic effect

9.1.1 Growth of GDP

The water strategy includes construction projects, thus booming industries such as building materials and construction. Meanwhile, it contributes to the GDP of China to a considerable extent by stimulating the domestic demand and multiply the total returns.

According to Keynes' theory of investment multiplier,

$$\Delta GDP = \Delta I \cdot MPC$$

$$MPC = \frac{1}{1 - \frac{\Delta C}{\Delta Y}}$$

where

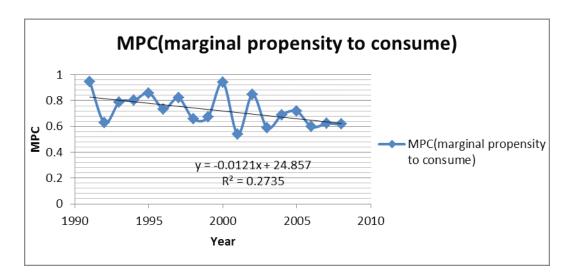
I: investment;*C*: consumption;*Y*: yield (income);

MPC: marginal propensity to consume.

	Data		
marginal disposable income(per capita)	marginal consumption(per capita)	MPC(marginal propensity to consume)	Year
127.3	120.7	0.948153967	1991
248	156	0.629032258	1992
353.9	278.8	0.787793162	1993
577	462.9	0.802253033	1994
870	747.7	0.859425287	1995
780.5	573.8	0.735169763	1996
557.6	457.3	0.820121951	1997
452.7	298	0.658272587	1998
473.3	318.9	0.673779844	1999
299.1	281.6	0.94149114	2000
581.7	313.8	0.539453326	200
968.7	818.8	0.845256529	200
616	362	0.587662338	2003

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724	499.7	0.69019337	2004
1207.2	865.3	0.716782638	2005
1257.9	750.6	0.5967088	2006
1606.5	998	0.621226268	2007
1751.7	1080.3	0.616715191	2008



Using the data of Chinese's income-consumption and a regression model, we've figured out the MPC of 2025 is 0.4. From that, we may moderately estimate that the investment of project will gain an aggregate economic increase of $\frac{5\Delta I}{3}$.

9.1.2 Water Price Fluctuation

9.1.2.1 Assumptions:

- 1. The water supply is monopolized by state-owned enterprise in China.
- 2. The national water supply mechanic is non-lucrative.
- 3. The government is a rational decision-maker.

9.1.2.2 Variables

Mark	Meaning	Unit
M_c	Marginal cost	RMB/m ³
P	Current price	RMB/m ³
P	Future price	RMB/m ³
S	Water supply	10 ⁸ m ³ per year

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9.1.2.3 Establishment of the Model

In the strategy,

$$M_c = \frac{\Delta I}{\Delta S}$$

Due to lack of data, we can not figure out the marginal cost, so hereby we'll discuss the following 3 situations:

- 1. When $P > M_c$, P < P;
- 2. When $P < M_c$, P < P;
- 3. When $P = M_c$, P = P.

The 3 situations are discussed in the short-term run; in the long run the influence of investment will gradually fade, and since the supply increases, the price will reduce on the base of $\cal P$.

Now we have drawn the conclusion that the water price will fall or slightly rise depending on the investment.

9.2 Social Influences

- 1. The water strategy redresses the imbalance between supply and demand of fresh water in northern China, benefits hundreds of millions of people and raises their living standard.
- 2. To accomplish this strategy, large amounts of labor force and personnel are in need, thus providing a decent many of employments.
- 3. This strategy facilitates other emerging industries like hydroelectric industry, chemical extracting industry, etc.
- 4. While recognizing the strength, we also face challenges: immigrant resettlement, historic relics preservation, etc.

9.3 Environmental Influences

1. By water diversion the distribution of water resources is managed more rationally in China. Combined with the desalination technology, the water resources condition in northern China will be greatly improved, preventing over-exploitation of local resources.

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2. Large scales of project construction will add to the burden of fuel consumption, greenhouse gas emission and rise of water temperature.

3. The solid or chemical wastes, if not wisely disposed, can result in environmental pollution and disturb the ecosystem and hydrological system.

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10. Evaluation

10.1 Prediction Model

Strength:

1. We use a combination of regression model and time series analysis to forecast the future freshwater demand and the water supply capacity in order to figure out the water shortage in 2025. Using data in different years, we strive to make the results accurate and close to real situations.

2. We have introduced a new concept: the water supply coefficient. Using this proportion, we can figure out the water supply capacity in 2025, thus determining the abstract data which are difficult to obtain.

Weakness:

1. Unable to find the data, we have to forecast the water shortage by calculating the gap between the expected water supply and the future freshwater demand. However, the large calculation process may distort the results.

10.2 Desalination & Water Transfer model:

Strength:

- 1. We have considered the existing projects in China and have combined them with our project to find an integrated proposal, which makes it more practical and economical.
- 2. We provide the users with a variety of proposals. The users only need to input the data and then choose one by the costs. We have made the process more flexible and manageable.
- 3. The scale of construction work is quiet small while it covers the situation in the whole country.

Weakness:

- 1. Some important data (probably confidential) being inaccessible, we have kept some symbols in the solution, which makes the solution hard to access and the proposal lacks integrity.
- 2. We can only figure out the straight-line distance between two cities with the Minimum Spanning Tree. The figure we get is certainly different from the actual distance that is needed to cover.

10.3 Drought Prediction & Storage Plan

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Strength:

1. We employed a gray model to predict the cycle and years of the occurrence of droughts. Based on the results we got, we calculated the amount of water needed to be stored each year. By starting to prepare a decade before the disaster comes and making a small amount of storage each year, much work at the last minute is no longer necessary.

2. We did a case study, and tested our model using the Yellow River basin, a typical water-deficient area.

Weakness:

1. Our assumption that that all the demand for water resources on years of drought can be met from storage in water reservoirs may have oversized the needed storage capacities of reservoirs.

10.4 Economic Effect

Strength:

- 1. Employing a classic model in economics, the results are convincing and authoritative.
- 2. The calculation is easily accessible, enabling direct perception of the model.

Weakness

1. The model is theoretical and idealistic, which indicates possible discrepancy with the reality.

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11. Appendix

11.1 References:

```
http://www.mwr.gov.cn/zwzc/hygb/szygb/
http://www.yellowriver.gov.cn/
http://www.nsbd.gov.cn/zx/english/mrp.htm
http://www.zjwater.gov.cn/
http://wenku.baidu.com/view/b43de84033687e21af45a97c.html
http://www.baidu.com/
http://en.wikipedia.org/wiki/Main_Page

Principles of Economics, N.Gregory Mankiw,
```

11.2 Matlab Program Code

How to use Matlab to realize Time Series Analysis

```
f=999999999999
e=0
aa=0
bb=0
cc=0
data = [402.56398 403.28 391.38 395.39 388.61 354 372.1
                                                                 381.5
                                                                          396.1
                   385.7
            384.2
                              392.3
                                       404.4]
result=[402.56398 403.28 391.38 395.39 388.61 354 372.1
                                                                 381.5
                                                                          396.1
    381.1
             384.2 385.7
                              392.3
                                       404.400000000000000
n=length(data);
m=length(result);
for i=1:201
   a=(i-101)*0.01;
   for ii=1:201
      b=(ii-101)*0.01;
      c=1-b-a;
      for j=4:n
           result(j)=result(j-3)*a+result(j-2)*b+result(j-1)*c;
          temp=data(j)-result(j);
          if temp<0
              temp=-temp;
           end
      end
      e=e+temp;
```

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How to use Matlab to realize minimum spanning tree model

```
x=[7 9 10 11 15 22 31 38 49 51]
n=length(x)
temp=0
for i=1:n
    for ii=1:i
    temp=temp+x(ii)
    end
    y(i)=temp
    temp=0
end
for i=2:n
    z(i)=0.5*y(i)+0.5*y(i-1)
end
c=0
d=0
e=0
f=0
for i=2:n
    c=c+z(i)
end
for i=2:n
     d=d+x(i)
end
for i=2:n
    e=e+z(i)*x(i)
```

```
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                                                                              Page 45 of 47
end
for i=2:n
    f=f+z(i).^2
a=(c*d-(n-1)*e)/((n-1)*f-c.^2)
b=(d*f-c*e)/((n-1)*f-c.^2)
year=10
for i=1:year
    m(i)=(1-exp(a))*(4-b/a)*exp(-(n+i-1)*a)
End
How to use Matlab to realize gray model prediction
n=input('Please input the number of vertex:n= ')
W=input('Please
                           input
                                                          weighted
                                                                              adjacency
matrix:[W(1,1),...,W(1,n);...;W(n,1),...,W(n,n)]=')
T=zeros(n);
WW=W;
   for i=1:n
       for j=1:n
           if W(i,j)==\inf WW(i,j)=0;
           end
       end
    end
m=((nnz(WW))/2);
j=0;
for i=1:m
    if j<(n-1)
       min=inf; a=0; b=0;
       for k=1:n
           for I=(k+1):n
               if W(k,l) \le \min \min W(k,l); a=k; b=l; end
```

end

T(a,b)=W(a,b); T(b,a)=W(a,b);

if $T(i,v)^{\sim}=0$ y=y+1; P(1,y)=i; P(2,y)=v;

if P(1,I)==P(2,y) P(1,I)=P(1,y);

P=zeros(2,m); y=0;

for v=(i+1):n

end

if P(1,y) < P(2,y)

for I=(y+1):m

end

f=0;

for i=1:n

end

for y=1:m

end

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```
elseif P(2,I) == P(2,y) P(2,I) = P(1,y);
                  end
               end
               P(2,y)=P(1,y);
           elseif P(2,y) < P(1,y)
               for I=(y+1):m
                   if P(1,I)==P(1,y) P(1,I)=P(2,y);
                  elseif P(2,I) == P(1,y) P(2,I) = P(2,y);
                  end
               end
               P(1,y)=P(2,y);
           elseif (P(1,y)+P(2,y))^{-}=0 f=1;
               break
           end
       end
       if f==1
                 T(a,b)=0; T(b,a)=0;
       else j=j+1;
       end
       W(a,b)=inf;
   else
       MST=T;
       input('The weighted adjacency matrix of the minimum spanning tree is:')
       MST
       break
      end
   end
if j<(n-1)
   input('No minimum spanning tree available in the graph.')
```

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11.3 Position Paper for the State Council of PRC

China has abundant water resources in absolute amount, but hardly can we say that it is a water-sufficient country when considering a large population base, an imbalanced spatial distribution and the lack of fresh water. Studies show that China will face a severe water crisis in 2025. Considering all these facts, we provide the State Council with the following strategies to wisely distribute the water resources.

Our strategy includes:

A seawater desalination project and a water diversion project:

We provide two integrated plans for the government to choose from. Plan I and Plan II can also be weighted and then combined in order to find the best solution.

Reservoir storage:

We figure out the amount of water we need to store for a drought to meet an urgent need and to stabilize the price of water.

Water-saving measures:

We give advice of taking macro or personal measures to reduce the demand of water so as to relieve water shortage and ensure everyone in the country has enough water to use.

WHY CHOOSE US?

Firstly, this water strategy includes construction projects, thus booming construction industries and emerging industries like hydroelectric industry. Meanwhile, it contributes to stimulating domestic demand and multiplies the total revenues.

Secondly, it benefits the citizens. The water strategy redresses the imbalance between supply and demand of fresh water in northern China so as to raise people's living standard and help maintain social stability. This strategy also facilitates other emerging industries like hydroelectric industry, chemical extracting industry, etc.

In addition, by water diversion the distribution of resources is managed more rationally in China. Combined with the desalination technology, the water resource condition in northern China will be greatly improved.

We hope that this strategy could be adopted and could contribute to the current condition of China.

Team 22330