Quiz 1 Scientific Computing

1. Solve the System of Linear Equation using Gauss-Seidel Method
   $-2x_1 - 4x_2 + x_3 = 1$
   $2x_1 + x_2 + 5x_3 = 8$
   $5x_1 + 2x_2 + x_3 = 9$
   Perform 3 iterations, calculate the relative approximate error at the end of each iteration, and use $x_1 = 1$, $x_2 = 0$, and $x_3 = 0$ as your initial guess.
2. Find how many minimum terms of the Maclaurin series for $f(x) = e^{2x}\cos(2x)$ are needed to approximate $f(x = 0.45)$ such that the relative approximation error is less than 10% !
3. A car's distance is recorded at different times during a journey, as shown in the table below:

Recorded Date	Drivers	Time in minutes	Distance
October 2nd	Nathanael Osborne Wahyudi	0	50
October 9th	Ahmad Husain	10	60
October 12th	Harry Santosa	20	72
October 16th	Andrew Widyanata	30	85

   Using Newton's interpolation polynomial of degree 3, estimate the car's distance at 15 minutes and 25 minutes !
4. Use the Newton-Raphson method to find the root of $f(x) = 2x^5 + 7x^4 + 9x^3 + 12x^2 + 7x + 5$ such that the relative approximation error is less than 5% when the initial guess $x = -4$!

- Kelvin Asclepius Minor -

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1. $-2x_1 - 4x_2 + x_3 = 1$ $\quad |-2| \ngtr |-4| + |1|$
$2x_1 + x_2 + 5x_3 = 8$ $\quad |1| \ngtr |2| + |5|$
$5x_1 + 2x_2 + x_3 = 9$ $\quad |1| \ngtr |5| + |2|$
} not diagonally dominant → need to rearrange →

$5x_1 + 2x_2 + x_3 = 9$ $\quad |5| > |2| + |1|$
$-2x_1 - 4x_2 + x_3 = 1$ $\quad |-4| > |-2| + |1|$
$2x_1 + x_2 + 5x_3 = 8$ $\quad |5| > |2| + |1|$
} diagonally dominant

iteration	$x_1 = \frac{9 - 2x_2 - x_3}{5}$	$x_2 = \frac{1 + 2x_1 - x_3}{-4}$	$x_3 = \frac{8 - 2x_1 - x_2}{5}$	Error $x_1 = \frac{x_{1\,now} - x_{1\,before}}{x_{1\,now}}$	Error $x_2 = \frac{x_{2\,now} - x_{2\,before}}{x_{2\,now}}$	Error $x_3 = \frac{x_{3\,now} - x_{3\,before}}{x_{3\,now}}$
0	1	0	0			
1	$\frac{9-2.0-0}{5} = 1.8$	$\frac{1+2(1.8)-0}{-4} = -1.15$	$\frac{8-2(1.8)-(-1.15)}{5} = 1.11$	$\left\lvert\frac{1.8-1}{1.8}\right\rvert \times 100\% = 44.44\%$	$\left\lvert\frac{-1.15-0}{-1.15}\right\rvert \times 100\% = 100\%$	$\left\lvert\frac{1.11-0}{1.11}\right\rvert \times 100\% = 100\%$
2	$\frac{9-2(-1.15)-1.11}{5} = 2.038$	-0.9915	0.9831	$\left\lvert\frac{2.038-1.8}{2.038}\right\rvert \times 100\% = 11.68\%$	$\left\lvert\frac{-0.9915-(-1.15)}{-0.9915}\right\rvert \times 100\% = 15.98\%$	$\left\lvert\frac{0.9831-1.11}{0.9831}\right\rvert \times 100\% = 12\%$
3	1.9998	-1.004215	1.000851	2%	1.266%	1.773%

$\therefore x_1 = 1.9998;\ x_2 = -1.004215;\ x_3 = 1.000851$

2. $f(x) = e^{2x}\cos(2x) \rightarrow f(x=0) = 1$

$f'(x) = 2e^{2x}\cos(2x) - 2e^{2x}\sin(2x) \rightarrow f'(x=0) = 2 - 0 = 2$

$f''(x) = [4e^{2x}\cos(2x) - 4e^{2x}\sin(2x)] - [4e^{2x}\sin(2x) + 4e^{2x}\cos(2x)] = -8e^{2x}\sin(2x) \rightarrow f''(x=0) = 0$

$f'''(x) = -16e^{2x}\sin(2x) - 16e^{2x}\cos(2x) \rightarrow f'''(x=0) = 0 - 16 = -16$

$f^{(4)}(x) = [-32e^{2x}\sin(2x) - 32e^{2x}\cos(2x)] + [-32e^{2x}\cos(2x) + 32e^{2x}\sin(2x)] = -64e^{2x}\cos(2x) \rightarrow f^{(4)}(x=0) = -64$

Maclaurin Series

$$f(x) = \sum_{n=0}^{n} \frac{f^{(n)}(x=0)}{n!}x^n = f(x=0) + f'(x=0)x + \frac{f''(x=0)}{2}x^2 + \frac{f'''(x=0)}{6}x^3 + \frac{f^{(4)}(x=0)}{24}x^4$$

$$f(x) = 1 + 2x + \frac{0}{2}x^2 + \left(\frac{-16}{6}\right)x^3 + \left(\frac{-64}{24}\right)x^4$$

$$f(x) = 1 + 2x - \frac{16}{6}x^3 - \frac{64}{24}x^4$$

using 1 term

$f(x) = 1 \rightarrow f(x=0.45) = 1$

using 2 terms

$f(x) = 1 + 2x \rightarrow f(x=0.45) = 1.9$

Error $= \left\lvert\frac{1.9-1}{1.9}\right\rvert \times 100\% = 47.368\%$

using 3 terms

$f(x) = 1 + 2x - \frac{16}{6}x^3 \rightarrow f(x=0.45) = 1.657$

Error $= \left\lvert\frac{1.657-1.9}{1.657}\right\rvert \times 100\% = 14.665\%$

using 4 terms

$f(x) = 1 + 2x - \frac{16}{6}x^3 - \frac{64}{24}x^4 \rightarrow f(x=0.45) = 1.54765$

Error $= \left\lvert\frac{1.54765-1.657}{1.54765}\right\rvert \times 100\% = 7.065\%$

$\therefore$ using the first 4 terms of Maclaurin Series results in an error of 7.065% which is less than 10%

3.

time (t)	distance (d)
5	50
10	60
20	72
30	85

$f(x) = a_0 + a_1(x-x_0) + a_2(x-x_0)(x-x_1) + a_3(x-x_0)(x-x_1)(x-x_2)$

$d(t) = a_0 + a_1(t-t_0) + a_2(t-t_0)(t-t_1) + a_3(t-t_0)(t-t_1)(t-t_2)$

$d(t) = a_0 + a_1(t-5) + a_2(t-5)(t-10) + a_3(t-5)(t-10)(t-20)$

$(5, 50) \rightarrow 50 = a_0$

$(10, 60) \rightarrow 60 = a_0 + 5a_1$

$(20, 72) \rightarrow 72 = a_0 + 15a_1 + a_2(15)(10)$

$(30, 85) \rightarrow 85 = a_0 + 25a_1 + a_2(25)(20) + a_3(25)(20)(10)$

$a_0 = 50$
$a_1 = 2$
$a_2 = -\frac{4}{75}$
$a_3 = \frac{7}{3000}$

so $d(t) = 50 + 2(t-5) - \frac{4}{75}(t-5)(t-10) + \frac{7}{3000}(t-5)(t-10)(t-20)$

$\therefore d(t=15) = 50 + 2(15-5) - \frac{4}{75}(15-5)(15-10) + \frac{7}{3000}(15-5)(15-10)(15-20) = 66.75$

$d(t=25) = 50 + 2(25-5) - \frac{4}{75}(25-5)(25-10) + \frac{7}{3000}(25-5)(25-10)(25-20) = 77.5$

4. $f(x) = 2x^5 + 7x^4 + 9x^3 + 12x^2 + 7x + 5$

$f'(x) = 10x^4 + 28x^3 + 27x^2 + 24x + 7$

iteration i	x_i	$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$	Error $= \left\lvert \frac{x_{i+1} - x_i}{x_{i+1}} \right\rvert \times 100\%$
0	-4	-3.40324	17.535 %
1	-3.40324	-2.96477	14.789%
2	-2.96477	-2.67738	10.734%
3	-2.67738	-2.53626	5.564%
4	-2.53626	-2.50188	$\boxed{1.37392}$

∴ the root of $f(x)$ is -2.50188 in an error of 1.37392 which is less than 5%