

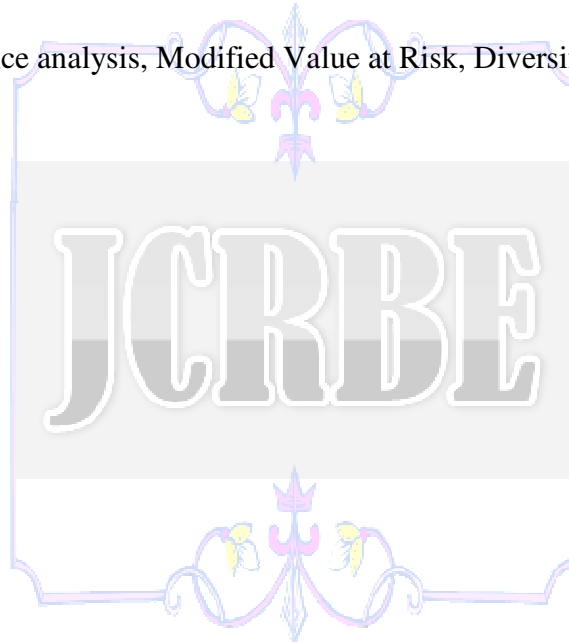
An empirical investigation of Markowitz Modern Portfolio Theory: A case of the Zimbabwe Stock Exchange

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ABSTRACT

This paper is an empirical study on Harry Markowitz's work on Modern Portfolio Theory (MPT). The model assumes the normality of assets' return. The paper examined the Zimbabwe Stock Exchange by mathematical and statistical methods for normality of assets' returns. We studied the effect of the parameters, Skewness and Kurtosis for different time series data. We tried to figure it out which data series is better to construct a portfolio and how these extra parameters can make us better informed in our investments.

Keywords: Mean-Variance analysis, Modified Value at Risk, Diversification, Skewness, Kurtosis



1.0 Introduction

The aim of this paper is to construct an empirical study on the Modern Portfolio Theory. The model was developed by Markowitz using mean-variance analysis. He states that the expected return (mean) and variance of returns of a portfolio are the whole criteria for portfolio selection. These two parameters can be used as a possible hypothesis about actual behaviour and a maxim for how investors ought to act.

Every model or theory is based on some assumption, basically some simplification tools. Markowitz's model relies on the following assumptions;

- Investors seek to maximize the expected return of total wealth.
- All investors have the same expected single period investment horizon.
- All investors are risk-averse, that is they will only accept a higher risk if they are compensated with a higher expected return.
- Investors base their investment decisions on the expected return and risk.
- All markets are perfectly efficient.

By having these assumptions in mind, we will go through some concepts and terminologies that will make us understand the model constructed in further part of this paper.

1.1 Risk and Reward (Mean and Variance Analysis)

Markowitz model relies on balancing risk and return, and it is important to understand the role of consumer's preferences in this balance. By assumption for the Markowitz model, investors are risk averse. Assuming equal returns, the investor prefers the one with less risk, which implies that an investor who seeks higher return must also accept the higher risk. There is no exact formula or definition for this and it is totally dependent on individual risk aversion characteristics of the investor.

1.1.0 Diversification

Diversification is a risk management technique that mixes a wide variety of investments within a portfolio. It is done to minimize the impact of any security on the overall portfolio performance. In order to have a diversified portfolio it is important that the assets chosen to be included in a portfolio do not have a perfect correlation, or a correlation coefficient of one. Diversification reduces the risk on a portfolio, but not necessarily the return, and that's why it is referred as "the only free lunch in finance". Diversification can be loosely measured by some statistical measurement, intra-portfolio correlation. It has a range from negative one to one and measures the degree to which the various asset in a portfolio can be expected to perform in a similar fashion or not.

In order to understand how to diversify a portfolio we should understand the risk. According to Ibbotson *et al* (1976), risk has two components, systematic and unsystematic. Where market forces affect all assets simultaneously in some systematic manner it generates Systematic risk or what so called, undiversifiable risk. Examples are bull markets, bear markets, wars, changes in the level of inflation. The other component of risk is unsystematic one, or so called diversifiable risk. These are idiosyncratic events that are statistically independent from the more widespread forces that generate undiversifiable risk. The examples of a diversifiable risk

are acts of God such as floods, inventions, management errors, lawsuits and good or bad news affecting one firm.

As defined above, Total risk of a portfolio is the result of summation of systematic and unsystematic risks. On average, the total risk of a diversified portfolio tends to diminish as more randomly selected common stocks are added to the portfolio.

In the following section, “Data and Methodology” we introduce the type of the data under use for this study and some practical information about the data. The last section under title “Empirical investigation” is the main part of this research. In the first part we question the validity of one of the critical assumptions of the model and by some statistical test we support our claim, then we introduce a new ratio to handle this inefficiency regarding the model and finally we test these two ratios against each other by different combination of some extra parameters introduced during the process.

2.0 Data and Methodology

The data to investigate consists of 42 stocks listed on the Zimbabwean stock exchange. The data is chosen for a period 1997 - 2007, which is aimed to cover events on the stock market. Using this data set, we separate it into two parts, and we define the first period of the data set (1997 – 2002) as historical data and the latter (2003 – 2007) as future data. Throughout this paper they are referred to as historical and future data. The data is analyzed in 4 different time scales, weekly, monthly, quarterly and yearly. Practically in analysis of the data, there are always some missing cells due to discrepancies or simply the fact that no trade took place under those dates. To deal with this issue, we assumed no changes in the prices that occurred during those dates and consequently the assets' return was zero on those dates. The portfolio is constructed by Markowitz Model, where we emphasized it as the traditional model compared with adjustments to the parameters of this model.

2.1 The Sharpe Ratio

This ratio is a measurement for risk-adjusted returns and was developed by William F. Sharpe. The Sharpe ratio is defined by

$$\frac{E(R_p) - R_f}{\delta(R_p)}$$

$E(R_p)$ denotes, the expected return of the portfolio;

R_f denotes, the return on the risk-free asset; and

$\delta(R_p)$ denotes, the standard deviation of the portfolio returns.

This ratio measures the excess return, or the risk premium of a portfolio compared with the risk-free rate, and with the total risk of the portfolio, measured by the standard deviation. It is drawn from the capital market line, and it can be represented as follows:

$$\frac{E(R_p) - R_f}{\delta(R_p)} = \frac{E(R_m) - R_f}{\delta(R_m)}$$

This relation indicates that at equilibrium, the Sharpe ratio of the portfolio to be evaluated and the Sharpe ratio of the market portfolio are equal. The Sharpe ratio corresponds to the slope of the market line. If the portfolio is well diversified, then its Sharpe ratio is close to that of the market. The Sharpe ratio provides a good basis for comparing portfolios, and is widely used by investment firms for measuring portfolio performance.

2.2 Skewness

Skewness is a parameter that describes asymmetry in a random variable's probability distribution. In other words a distribution is skewed if one of its tails is longer than the other. Skewness can be positive, meaning that it has a long tail in the positive direction. It also can have a negative value, where it is called a negative skewness. Skewness is equal to zero where we have a perfect symmetry.

2.3 Kurtosis

In probability theory Kurtosis is the measure of peakedness of the probability distribution of a real valued random variable. A high kurtosis distribution has a sharper peak and fatter tails, while a low kurtosis distribution has a more rounded peak with wider shoulders. Mesokurtic curves take place when kurtosis is zero which means we have a normal distribution. Leptokurtic case happens when data are fat-tailed, we say so that we have a positive kurtosis. The last type is Platykurtic Curve, which the kurtosis is less than zero.

3.0 Empirical Investigation

This part tries to answer to some questions and use some statistical methods to motivate these answers. We are going to study some parameters on a group of constructed portfolios with up to forty two assets using Markowitz model. Before any further steps in analyzing the data we will examine the distributions' normality of our stream of data. We will examine the stream of data using the Jarque-Bera test. As it is clear here, for a risk manager that tries to guard against large losses, the deviation from the normality can not be neglected.

3.1 The Jarque Bera test of Normality

It is a goodness-of-fit measure of departure from normality, based on the sample kurtosis and skewness. When it comes to stock market, it is assumed that a return or change in the stock price is the result of many small influences and shocks and thus the return can be treated as a normal random variable. The result shows that under the hypothesis that X_i is independent observations from a normal distribution, for large n the distribution of the JB-test statistic is asymptotically Chi-square distributed. This will help us to do a test on normality. We calculate the JB-test statistic and compare it with the null hypothesis that the data represents a normal distribution. We know that in 95% of the cases the value of the JB test will be smaller than 5, 99 for the normally distributed samples. Consequently we reject the hypothesis of normality if the value of JB-test statistic exceeds this amount.

The Result of Jarque-Bera Test on Our Portfolio Assets

In order to see if we can reject the normality of the data set, we performed a JB-test on the data sets. As mentioned before, our study compares 4 different sorts of data on Zimbabwe Stock Exchange, weekly, monthly, quarterly and yearly. Surprisingly the quarterly data set has a larger number of normally distributed assets, which can be due to the lack of data (the length of the data set is shorter than the latter categories). In the yearly data set, most of the assets successfully pass the JB-test, but it can not be a reliable result considering the number of data in each data set. We considered 10 years data, for two periods which will result in an analysis of a data set of five.

4.0 Analysis of the Empirical Investigation

In this part of the empirical investigation we will try to answer the following questions:

1. What will be the difference between two optimized portfolios when;
 - The first one is optimized by traditional mean-variance with the Sharpe model, and then by sorting out the stocks' skewness and kurtosis and study the importance of these parameters.
 - Second case, when we optimize the portfolio considering a new risk measure, Modified Value at Risk (MVaR). Then sorting out data by skewness and kurtosis and perform the same study done already on the last group. Then compare these results with future data.
2. Compare the portfolios calculated in part one with other portfolios that have different time series (monthly, weekly, etc).

Weekly Portfolio

Let's consider the first portfolio constructed by the traditional Sharpe ratio where skewness and kurtosis effect were not considered by the original model. This is illustrated on table 1. The traditional Sharpe ratio is almost double the Modified one in the first case. This case introduces the second highest return for the historical portfolio and the lowest risk. But it will be interesting to compare and return kept their positions. The next portfolio is the one with both positive skewness and kurtosis greater than 3. In this case we have the lowest Sharpe ratio for both historical and future portfolios. Returns are second best, but considering the high risks they are not worth to consider. But it is interesting to consider the velocity of losing value of the returns from the historical portfolio to the future one, from 23, 21% to 9, 12%.

The third portfolio is the one with just positive skewness. It has still a traditional Sharpe ratio greater than 1, highest return and simultaneously lowest risk. For the future portfolio, Sharpe is still relatively high. The return is not the highest but the risk managed to be the lowest for the future data. The fourth portfolio that we analyzed is the portfolio with stocks which have kurtosis greater than 3. As it is predictable by looking back again in the second case it is not a good method to construct a portfolio. Low Sharpe ratio for both periods, and the risk which is high for both periods and the return which is not so high compared with other cases for the first period, but interestingly not diminished as much as other portfolios for the second period. The following table illustrates results for weekly portfolio.

Table 1: Weekly portfolio: Sharpe

	Historical data		Future data		Increase/decrease %	
	Sharpe ratio	Modified Sharpe ratio	Sharpe ratio	Modified Sharpe ratio	Sharpe ratio	Modified Sharpe ratio
Traditional Markowitz Model	1,12922	0,429416	0,68652	0,706234	-37,46%	-37,45%
Optimization with Positive Skewness and Kurtosis Greater than 3	0,52150	0,31705	0,251157	0,152692	-51,84%	-51,84%
Optimization with Positive Skewness	1,08602	0,66025	0,611901	0,372110	-43,66%	-43,64%
Optimization with Kurtosis Greater than 3	0,55674	0,33848	0,587054	0,3571378	5,44 %	5,51 %
			Weekly Portfolio: Returns			
Traditional Markowitz Model	24,90% -	24,91%	13,67%	13,68%	-45,09%	-45,10%
Optimization with Positive Skewness and Kurtosis Greater than 3	23,21%	23,21%	9,12%	9,12%	-60,71%	-60,71%
Optimization with Positive Skewness	25,17%	25,17%	12,59%	12,60%	-49,97%	-49,96%
Optimization (Kurtosis Greater than 3)	20,24%	20,24%	15,34%	15,35%	-24,22%	-24,16%
			Weekly Portfolio: Risk			
Traditional Markowitz Model	18,63%	18,64%	15,76%	15,77%	-15,37%	-15,39%
Optimization with Positive Skewness and Kurtosis Greater than 3	37,10%	37,10%	26,21%	26,21%	-29,37%	-29,37%
Optimization with Positive Skewness	19,62%	19,62%	16,43%	16,43%	-16,24%	-16,24%
Optimization with Kurtosis Greater than 3	29,42%	29,42%	21,81%	21,81%	-25,88%	-25,86%

Monthly Portfolio

In the following data set, we can see that values for the two portfolios with “Skewness and Kurtosis” and “Kurtosis greater than 3” is not included on table 2. The reason for this is that

the numbers of stocks after sorting out for these portfolios were not reaching the desired level for an efficient diversification. This is one of the most important concepts of portfolio construction. Considering the first available category which is the portfolio constructed with the traditional Markowitz model where only mean and variance are considered, the Sharpe ratio is the highest for both methods of calculation, modified and traditional Sharpe.

When we move to future portfolios for the same category the Sharpe ratios almost became half. When we are analyzing the Sharpe ratios, it would make more sense to look at risk and return closely. Return is still highest for this category while representing the least risk. But surprisingly while the return became almost half of the historical portfolios the risk is decreased only by 6%. The next category is where we have stocks included in the portfolio with only positive skewness. In this category we have almost the same figures as the last case, but in general 1-2 percent less.

In this category we have a minimization of only 43 percent for modified Sharpe ratio against 52 percent of the traditional case. In order to analyze this result, we can compare these figures with the case of considering stocks with positive skewness. In the case of constructing a portfolio with only positive skewness, the figures for both methods of calculation of the Sharpe ratios are identical. It clarified that the modified Sharpe ratio considers the positive skewness even in the case of traditional Markowitz model. The following table illustrates monthly portfolio results.

Table 2: Monthly portfolio: Sharpe

	Historical data		Future data		Increase/decrease %	
	Sharpe ratio	Modified Sharpe ratio	Sharpe ratio	Modified Sharpe ratio	Sharpe ratio	Modified Sharpe ratio
Traditional Markowitz Model	1,41441	0,85990	0,67654	0,48224	-52,17%	-43,92%
Optimization with Positive Skewness and Kurtosis Greater than 3	N/A	N/A	N/A	N/A	N/A	N/A
Optimization with Positive Skewness	1,32204	0,80374	0,62795	0,38177	-52,50%	-52,50%
Optimization with Kurtosis Greater than 3	N/A	N/A	N/A	N/A	N/A	N/A
			Monthly Portfolio: Returns			
Traditional Markowitz	25,62%	25,62%	12,33%	12,33%	-51,89%	-51,88%

Model						
Optimization with Positive Skewness and Kurtosis Greater than 3	N/A	N/A	N/A	N/A	N/A	N/A
Optimization with Positive Skewness	24,67%	24,67%	11,99%	11,99%	51,41 %	-51,42 %
Optimization with Kurtosis Greater than 3	N/A	N/A	N/A	N/A	N/A	N/A
			Monthly Portfolio: Risk			
Traditional Markowitz Model	15,38%	15,38%	14,45%	14,45%	-6,03 %	-6,02 %
Optimization with Positive Skewness and Kurtosis Greater than 3	N/A	N/A	N/A	N/A	N/A	N/A
Optimization with Positive Skewness	15,74%	15,74%	15,03%	15,03%	-4,50 %	-4,50 %
Optimization (Kurtosis Greater than 3)	N/A	N/A	N/A	N/A	N/A	N/A

Quarterly Portfolio

Based on the same reason we mentioned on the last type of the portfolio, we have only two categories to analyze for quarterly portfolio as shown on table 3. The first category is where we have the general model applied. Looking at Sharpe ratios and their developments we will see that this category has the highest Sharpe ratios both traditional and modified while the development for the traditional case is worse compared with other categories, a figure equal to almost 80%. Compared to the case of the portfolio with positive skewness, the return of 14% for the same category is not at all satisfying considering 15% of risk.

Considering the portfolio with stocks which has only positive skewness, we have a good Sharpe ratio for both methods compared with the first category analyzed where the return is almost 26% and risk is relatively low, only 16%. Development of the figures from the historical portfolios to future is interesting. While return diminished, the risk has risen for both categories. The development of the Modified Sharpe ratio remained almost constant for the case of the portfolio with positive skewness while it has fallen for the first category.

Table 3: Quarterly portfolio: Sharpe

	Historical data		Future data		Increase/decrease %	
	Sharpe ratio	Modified Sharpe ratio	Sharpe ratio	Modified Sharpe ratio	Sharpe ratio	Modified Sharpe ratio
Traditional Markowitz Model	1,49323	0,90782	0,30437	0,42635	-79,62%	-53,04%
Optimization with Positive Skewness and Kurtosis Greater than 3	N/A	N/A	N/A	N/A	N/A	N/A
Optimization with Positive Skewness	1,39841	0,85017	0,61989	0,37686	-55,67%	-55,67%
Optimization with Kurtosis Greater than 3	N/A	N/A	N/A	N/A	N/A	N/A
			Quarterly Portfolio: Returns			
Traditional Markowitz Model	25,78%	25,78%	8,52%	8,53%	-66,93%	-66,90%
Optimization with Positive Skewness and Kurtosis Greater than 3	N/A	N/A	N/A	N/A	N/A	N/A
Optimization with Positive Skewness	26,66%	26,66%	14,82%	14,82%	-44,41%	-44,41%
Optimization (Kurtosis Greater than 3)	N/A	N/A	N/A	N/A	N/A	N/A
			Quarterly Portfolio: Risk			
Traditional Markowitz Model	15,38%	15,38%	19,63%	19,65%	27,64%	27,79%
Optimization with Positive	N/A	N/A	N/A	N/A	N/A	N/A

Skewness and Kurtosis Greater than 3						
Optimization with Positive Skewness	16,30%	16,30%	19,80%	19,80%	21,44%	21,45%
Optimization with Kurtosis Greater than 3	N/A	N/A	N/A	N/A	N/A	N/A

Yearly Portfolio

For this category, since the time series is not long, we can not construct portfolios with reasonable structures. So we won't consider this category in our investigation. The reason for this is the unrealistic results of Sharpe ratio calculations.

Analysis for the different type of time series for constructing a portfolio

Weekly time series:

This time series is divided into two periods as mentioned above and four types of portfolios constructed considering combinations of 2 additional parameters, skewness and kurtosis. As mentioned earlier on, we are not going to consider the case of a portfolio with positive skewness and kurtosis greater than 3, since the number of the stocks available is limited and diversification can not take place.

The traditional Markowitz model shows the lowest decrease in Sharpe ratio which is due to the low decrease in return and the lowest decrease in risk of the portfolio for the two periods. This case, compared with other portfolios has the highest Sharpe ratio in this time horizon and also the least risk and a high return. The next category is where we have a portfolio of positive skewness. 34 stocks out of our 42 in the sample have this characteristic. This obviously gives a good level of diversification. The decrease of both traditional and modified Sharpe ratios seems to be moderately low compared with other categories. Despite the last case, with only kurtosis greater than 3 we have a positive development.

A high traditional Sharpe ratio of 1.08, an annual portfolio return of 25% followed by a risk of 19% makes this investment attractive for its time horizon. The difference in this category is not much from the traditional Markowitz model. The last case is the only one with positive development of the Sharpe ratio. The reason is that the decrease in risk is greater than the decrease for the return of the portfolio.

Monthly Time series:

Before considering this category it should be noted that two types of portfolios were not possible to establish, that is, the portfolios with positive skewness and kurtosis greater than 3, and the one with only kurtosis greater than 3. The reason for this was the lack of data, a limited number of stocks to perform a portfolio and consequently low level of diversification. Considering the portfolio based on traditional Markowitz model, the Modified Sharpe ratio gave

us a more stable result in these two periods as compared to the traditional Sharpe ratio. One explanation to this can be the low level of decrease in the risk associated with this type of portfolio. Monthly portfolio in comparison with weekly portfolio has a lower risk in association with almost the same level of return. In other two cases we have a radical decrease in the risk measure in these two periods while the level of decrease for return remained almost constant for these two types of portfolios. One might conclude that it is a good sign. A larger decrease in the level of risk for two periods, associated with almost the same level of return might be attractive. More detailed data is shown on the following table.

Table 4 - Sharpe Ratio

	Sharpe ratio/ modified Sharpe ratio	Weekly portfolio	Monthly Portfolio	Quarterly Portfolio
	Traditional Markowitz Model			
	Sharpe Ratio	1,12922393	1,414414883	1,493233205
Historical	Modified Sharpe Ratio	0,686519981	0,859903226	0,907821329
	Sharpe Ratio	0,706233886	0,676540412	0,304372119
Future	Modified Sharpe Ratio	0,429416123	0,48223651	0,426347861
	Positive Skewness and Kurtosis Greater than 3			
	Sharpe Ratio	0,521497735	N/A	N/A
Historical	Modified Sharpe Ratio	0,317048111	N/A	N/A
	Sharpe Ratio	0,2511566	N/A	N/A
Future	Modified Sharpe Ratio	0,15269236	N/A	N/A
	Positive Skewness			
	Sharpe Ratio	1,08602251	1,322041672	1,39841282
Historical	Modified Sharpe Ratio	0,6602549	0,803744036	0,850174664
	Sharpe Ratio	0,611900981	0,627952737	0,619887812
Future	Modified Sharpe Ratio	0,372109973	0,381771894	0,376864504
	Kurtosis Greater than 3			
	Sharpe Ratio	0,556742372	N/A	N/A
Historical	Modified Sharpe Ratio	0,338475322	N/A	N/A
	Sharpe Ratio	0,587054964	N/A	N/A
Future	Modified Sharpe Ratio	0,357137817	N/A	N/A

The second portfolio in this time category is constructed with stocks which have only positive skewness. As it was not far from imagination, we have again the same level of decrease

in the both portfolios, optimized by traditional Sharpe ratio or by the modified Sharpe ratio. This can be the result of our choice of stocks, the stocks with only positive skewness. It can be seen that the Modified Value at Risk used to measure the risk for constructing the modified Sharpe ratio can consider the right skewed effect, but still giving the same level of the risk and return for both of the portfolios optimized by traditional or modified Sharpe ratio.

Quarterly Time Series:

The last time series we are going to analyze more in detail is the quarterly time series. The data to construct two types of portfolios was not available to us as indicated in the table by N/A. This type of time horizon is quite different from other time horizons. The reason is the release of quarterly reports by companies. Almost all companies try to clean up their financial losses and show a good performance, although it might come quite late into the analysis of investors, but it has its impact on stock markets, both on liquidity and volatility of the market. For this time horizon the difference in return of the portfolios is almost in the same range of the other time horizons, that is why we exempt this parameter and go directly to the risk for finding out the reason for this dramatic decrease in traditional Sharpe ratio.

The pattern of changes in the difference of ratios for risk which started from weekly time horizon just turned the sign and became an increase for quarterly time horizon. This increase in the risk can be due to release of the quarterly reports by corporations and of course followed by an increase in trade for stocks. This results in more liquidity in the market. The other reason can be the cumulative return of the stocks during the quarter, while we ignore the volatility of the market in this period. We should also consider positive or mostly overestimated effect of these reports; the annual returns based on figures deviated long from the mean, and the annual risk based on the not so frequent return statistics, but cumulated and long away from the mean.

The portfolio with positive skewness has a less difference in risk development in comparison with the Markowitz model and also a much small difference in return's developments. This consequently is followed by a small difference in traditional Sharpe ratio. In contrast with the latter portfolio, the one with skewness shows the same development for traditional compared to the modified Sharpe ratio.

Yearly Time Series

This data series can not be used to construct a portfolio, since the value obtained by solver for optimized Sharpe ratio is irrelevant.

Table 5 - Portfolio Risk in Different Time Horizons.

	Risk	Weekly portfolio	Monthly Portfolio	Quarterly Portfolio
	Traditional Markowitz Model			
	Sharpe Ratio	18,63%	15,38%	15,38%
Historical	Modified Sharpe Ratio	18,64%	15,38%	15,38%
	Sharpe Ratio	15,76%	14,45%	19,63%

Future	Modified Sharpe Ratio	15,77%	14,45%	19,65%
	Positive Skewness and Kurtosis Greater than 3			
	Sharpe Ratio	37,10%	N/A	N/A
Historical	Modified Sharpe Ratio	37,10%	N/A	N/A
	Sharpe Ratio	26,21%	N/A	N/A
Future	Modified Sharpe Ratio	26,21%	N/A	N/A
	Positive Skewness			
	Sharpe Ratio	19,62%	15,74%	16,30%
Historical	Modified Sharpe Ratio	19,62%	15,74%	16,30%
	Sharpe Ratio	16,43%	15,03%	19,80%
Future	Modified Sharpe Ratio	16,43%	15,03%	19,80%
	Kurtosis Greater than 3			
	Sharpe Ratio	29,42%	N/A	N/A
Historical	Modified Sharpe Ratio	29,42%	N/A	N/A
	Sharpe Ratio	21,81%	N/A	N/A
Future	Modified Sharpe Ratio	21,81%	N/A	N/A

Table 6 - Portfolio Return in Different Time Horizon.

	Return	Weekly portfolio	Monthly Portfolio	Quarterly Portfolio
	Traditional Markowitz Model			
	Sharpe Ratio	24,90%	25,62%	25,78%
Historical	Modified Sharpe Ratio	24,91%	25,62%	25,78%
	Sharpe Ratio	13,67%	12,33%	8,52%
Future	Modified Sharpe Ratio	13,68%	12,33%	8,53%
	Positive Skewness and Kurtosis Greater than 3			
	Sharpe Ratio	23,21%	N/A	N/A
Historical	Modified Sharpe Ratio	23,21%	N/A	N/A
	Sharpe Ratio	9,12%	N/A	N/A
Future	Modified Sharpe Ratio	9,12%	N/A	N/A
	Positive Skewness			

	Sharpe Ratio	25,17%	24,67%	26,66%
Historical	Modified Sharpe Ratio	25,17%	24,67%	26,66%
	Sharpe Ratio	12,59%	11,99%	14,82%
Future	Modified Sharpe Ratio	12,60%	11,99%	14,82%
	Kurtosis Greater than 3			
	Sharpe Ratio	20,24%	N/A	N/A
Historical	Modified Sharpe Ratio	20,24%	N/A	N/A
	Sharpe Ratio	15,34%	N/A	N/A
Future	Modified Sharpe Ratio	15,35%	N/A	N/A

5.0 Summary of the Results

Weekly time series

In this time series we found that the Sharpe ratio for the Markowitz model had the smallest change compared to all other time series. The Markowitz portfolio had the lowest return compared to the other time series for this portfolio, but had the second highest risk. The skewed portfolios had the second highest risk but the third highest return. In the future data for these portfolios the Markowitz model had the third highest risk and the highest return, compared to other time series. For the skewed one we had the third highest risk and the second highest return. In this case, weekly data lean a little more in favour for the Markowitz model.

Monthly time series

In this data series we omitted the portfolios with positive skewness and kurtosis greater than 3, and the one with only kurtosis greater than 3. The reason for this was that it did not satisfy the concept of diversification. We found that, for the traditional Markowitz model the level of decrease in the traditional and modified Sharpe is not equal. However, the modified Sharpe showed a more stable result. A reason for this could be the low level of risk associated with that type of portfolio. The monthly portfolio has a lower level of risk compared to the weekly, whereas the return was approximately the same. For the portfolio with positive skewness, we obtained again the same level of decrease in both portfolios, optimizing using traditional and modified Sharpe ratio. It was shown that the Modified Value at Risk used to measure the risk for constructing the modified Sharpe ratio can consider the right skewed effect, since it still gives the same level of the risk and return for both of the portfolios optimized by traditional or modified Sharpe ratio.

Quarterly time series

We realized that one of the underlying reasons for the large shifts in the stock returns is, from how the stock markets are affected by the speculators and analysts in the market during quarterly reports. We found that in this time series the decrease in the traditional Sharpe is the highest and that it had the highest Sharpe ratio in the historical time series than the other ones. It is also interesting to see in the investigation that the risk when looking into the future has a

positive change (increase), than all the other time sets. It can be concluded that the market speculations on the quarterly reports had an impact of large effects on the stock prices.

5.1 Conclusion

From our empirical research we are able to draw the following conclusion of the study we made.

- ❖ The concept of diversification on portfolio selection showed its importance in the mean-variance optimization approach, due to the balancing of risk and reward.
- ❖ Incorporating higher statistical moments in decision-making has shown both weaknesses and strengths. The incorporation of Skewness has shown slightly better effect on the mean-variance optimization compared to future portfolios.
- ❖ The data set which replicated best for the future portfolios was the monthly time series. It showed moderate accurate estimates of the future, when risk and return were taken into account.
- ❖ In general, the traditional Markowitz model showed an inconsistent estimation compared with modified version when two time periods collated. This was mainly due to extreme events.

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