Quantum Risk Analysis in Finance

羅瑋宸 (B11902164)

Quantum Information and Computation, Final Project

National Taiwan University

June, 2025

MOTIVATION

The Classical Challenge: Risk Analysis is Slow

- Financial institutions rely on Monte Carlo (MC) simulations to calculate risk.
- This is computationally expensive due to slow convergence: $Error \propto \frac{1}{\sqrt{M}}$, where M is the number of samples.
- High precision requires huge computational resources and time.

The Quantum Promise: A Quadratic Speedup

Quantum algorithms like Quantum Amplitude Estimation (QAE) offer a path to faster solutions. QAE achieves a much faster convergence rate:

$$Error \propto \frac{1}{M}$$

This quadratic speedup could revolutionize computational finance.

Background: Key Financial Risk Metrics

- Value at Risk (VaR):
  - The most common risk metric. It is the maximum loss you expect not to exceed with a certain probability α.
  - $$VaR_\alpha[\mathcal{L}] = \inf_{x \geq 0}\{x | \mathbb{P}[\mathcal{L} \leq x] \geq \alpha\}$$
- Conditional Value at Risk (CVaR)
  - Also called "Expected Shortfall".
  - Measures the expected loss if the VaR threshold is breached. It is more sensitive to extreme, "black swan" events.
- Economic Capital Requirement (ECR)
  - An internal metric for how much capital a firm must hold to remain solvent.
  - $$ECR_\alpha[\mathcal{L}] = VaR_\alpha[\mathcal{L}] - \mathbb{E}[\mathcal{L}]$$

The Core Engine: Quantum Amplitude Estimation (QAE)

The Goal: To estimate the parameter 'a' in a quantum state:

$$|\psi\rangle = \sqrt{1-a}|\psi_0\rangle|0\rangle + \sqrt{a}|\psi_1\rangle|1\rangle$$

The 3-Step Process for Risk Analysis:

1. **Load Distribution (U)**:

   Create a superposition of all possible market outcomes or asset defaults.

$$|\psi\rangle_n = \sum_{i=0}^{N-1} \sqrt{p_i}|i\rangle_n$$

2. Compute Objective (F):

Apply a function $f(x)$ and encode its value into the amplitude of an ancilla qubit.

$$|i\rangle_n|0\rangle \mapsto |i\rangle_n(\sqrt{1-f(i)}|0\rangle + \sqrt{f(i)}|1\rangle)$$

3. Estimate with QAE:

Measure the amplitude to get the expected value $\mathbb{E}[f(X)]$.

Finding VaR: Use a comparator for $f(x)$ (is loss ≤ threshold?) and apply a classical bisection search to find the VaR efficiently.

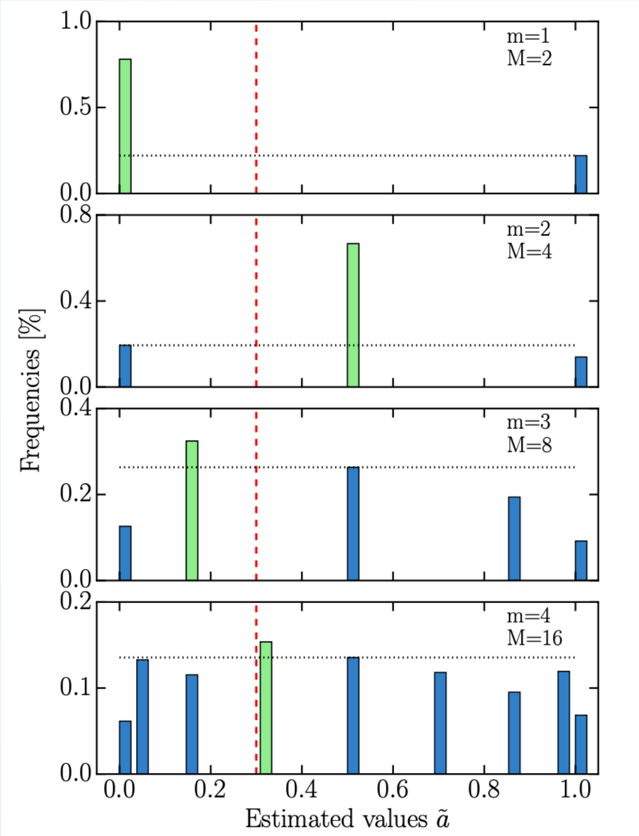
Foundational Work & Experimental Results

Ref: S. Woerner and D. J. Egger, "Quantum Risk Analysis," arXiv:1806.06893v1 [quant-ph], 2018.

Key Contribution: First to apply the QAE framework to calculate VaR and CVaR.

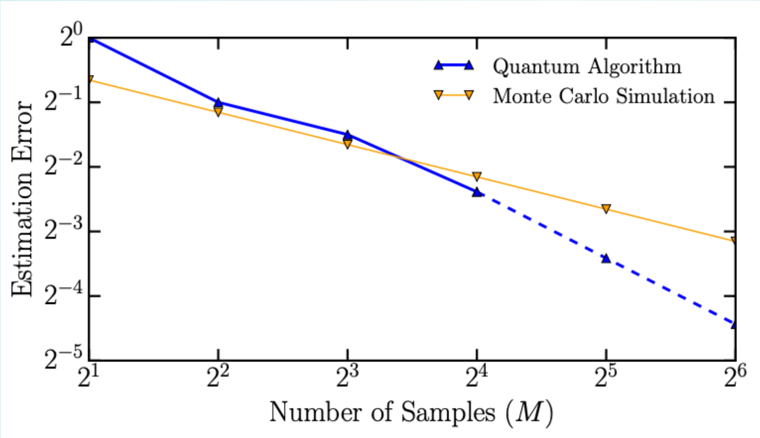
1. Real Hardware Demonstration

- A simple T-bill model was run on a **5-qubit IBM Q processor**.
- The results show convergence to the true value of 30% as more evaluation qubits(m) are used.



2. **Simulation of Quantum Speedup**

- A simulation directly compared the convergence error of QAE vs. classical Monte Carlo.
- The plot shows the quantum algorithm's error dropping below the classical error at just M=16 samples, confirming the quadratic advantage in practice.



Advanced Method for Derivative Risk Analysis

Ref: N. Stamatopoulos, B. D. Clader, S. Woerner, and W. J. Zeng, ”Quantum Risk Analysis of Financial Derivatives,” arXiv:2404.10088v1 [quant-ph], 2024.

The Challenge:

For derivatives, the "loss function" is itself a complex pricing calculation. A nested QAE approach is inefficient.

Solution — Quantum Signal Processing (QSP):

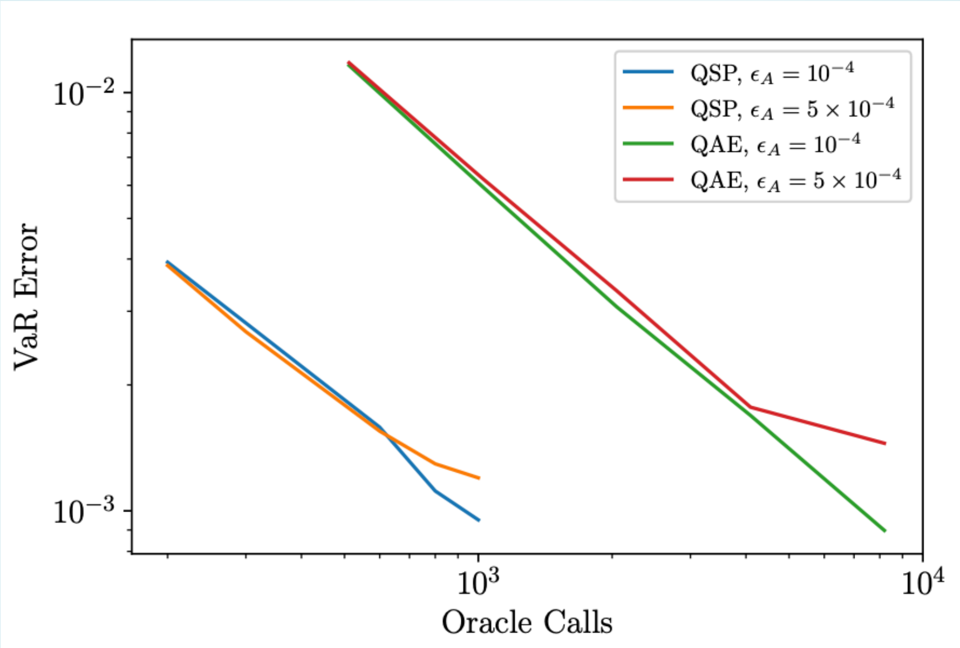
- Instead of digitizing the price, QSP applies a **polynomial transformation** directly to the quantum amplitude representing the price.
- The polynomial is engineered to approximate a sharp **threshold function**:

$$\theta_\mu(x) = \begin{cases} 1 & x \leq \mu \\ 0 & x > \mu \end{cases}$$

- This coherently "flags" all scenarios with values below the threshold μ.

Key Result:

QSP is far more efficient. Simulations show it requires ~10x fewer oracle calls than the nested QAE method to achieve the same accuracy.



Resource Analysis & Hardware Readiness

Ref: *S. Woerner and D. J. Egger, "Quantum Risk Analysis," arXiv:1806.06893v1 [quant-ph], 2018.*

D. J. Egger, R. G. Guti ´errez, J. C. Mestre, and S. Woerner, "Credit Risk Analysis using Quantum Computers," arXiv:1907.03044v1 [quant-ph], 2019.

Can this scale to real-world problems?

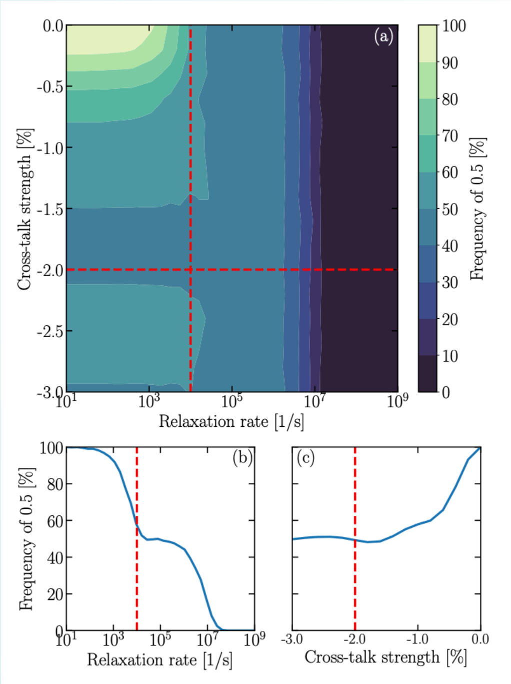
1. Scaling for Credit Risk

- Analysis of a portfolio with 1 million assets on a future Fault-Tolerant Quantum Computer (FTQC) estimates a runtime of only 30-60 minutes.
- However, the resource requirements are high: a small problem requires over 55,000 CNOT gates.

			#CX		
m	M	#qubits	all-to-all	IBM Q 20	overhead
1	2	13	795	1'817	2.29
2	4	14	2'225	5'542	2.49
3	8	15	5'085	12'691	2.50
4	16	16	10'803	26'457	2.45
5	32	17	22'235	55'520	2.50

2. Noise Considerations

- Simulations show the algorithm can tolerate some noise.
- It is feasible if hardware meets certain targets:
  - Relaxation rates $< 10^{-4}\, s^{-1}$
  - Cross-talk strength < 1%



Identifying the Research Gap

Where We Are: We have highly efficient quantum algorithms (QAE and QSP) to analyze risk... given a probability distribution of scenarios.

The Bottleneck: The final result is only as good as the input scenarios.

The Two Key Research Gaps:

1. Fidelity Gap: Financial markets are complex and non-Gaussian. How can we generate scenarios that capture "real-world" features like heavy tails and volatility clustering?

2. Efficiency Gap: How can we load this complex scenario distribution onto a quantum computer efficiently? The cost of this state preparation (the T_s depth) can erase the quantum advantage.

My Research Proposal: Hybrid qGAN for Scenario Generation

The Problem: To design a framework for dynamic, high-fidelity scenario generation that integrates seamlessly with our best quantum risk algorithms.

Proposed Method: Quantum Generative Adversarial Network (qGAN)

- A hybrid quantum-classical machine learning model.
- **Quantum Generator** G_θ - a trainable quantum circuit—learns to produce quantum states that mimic real market data.
- **Classical Discriminator** D_ϕ - a neural network—tries to distinguish between the real data and the "fake" data from the generator.

They are trained against each other in a minimax game:

$$\min_{\theta} \max_{\phi} \left(\mathbb{E}_{x \sim p_{\text{data}}}[\log D(x|\phi)] + \mathbb{E}_{x \sim p_{G(\theta)}}[\log(1 - D(x|\phi))]\right)$$

Proposed Framework & Justification

Integration: The trained quantum generator circuit becomes the state preparation unitary S for the QSP-based VaR algorithm.

Why This Approach is Powerful:

- Higher Fidelity: Quantum models may be better at capturing the complex, high-dimensional correlations in financial data than classical models.
- Quantum-Native Integration: It directly produces the required quantum state, bypassing the need for classical sampling and loading data with QRAM. This directly addresses the efficiency gap and the T_s bottleneck.
- Novelty: This creates a fully integrated, end-to-end quantum risk analysis pipeline. It moves beyond using quantum for just the calculation part and uses it for the modeling part as well.

Conclusion

What I have Learned

- Quantum computing offers a proven quadratic speedup for financial risk calculations.
- The state-of-the-art has evolved from foundational QAE to highly efficient QSP-based algorithms.
- The primary bottleneck for practical advantage is now shifting towards the efficient preparation of high-fidelity states that represent market scenarios.

The Path Forward

- My proposed hybrid qGAN framework offers a promising solution to this bottleneck.
- It combines the generative power of quantum models with the best-in-class QSP algorithm for risk analysis.

Future Work

- Numerically simulate the proposed qGAN-QSP pipeline.
- Analyze the resource costs of training the qGAN.
- Test the framework on real, high-dimensional financial time-series data.

THE END