Quantum Information and Computation - Final Project
Quantum Risk Analysis in Finance

Group 36

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Abstract

Financial risk management is a cornerstone of the global economy, yet it presents significant computational challenges for classical computers. Techniques like Monte Carlo simulations, while standard, suffer from slow convergence rates. Quantum computing offers a promising alternative by leveraging algorithms that provide a quadratic speedup. This report first reviews the development of quantum algorithms for financial risk analysis, focusing on key metrics such as Value at Risk (VaR) and Conditional Value at Risk (CVaR). We detail the foundational framework using Quantum Amplitude Estimation (QAE), its application to credit risk, and recent advancements using Quantum Signal Processing (QSP) for the risk analysis of derivatives. After summarizing the state-of-the-art, I identify a key research gap: the generation of market scenarios. I then propose a novel research problem aimed at solving this gap: a hybrid quantum-classical generative model for dynamic scenario creation. This proposed method uses a variational quantum circuit, trained adversarially against a classical neural network, to produce realistic, non-Gaussian scenario distributions that can be fed directly into the QSP-based risk algorithm. This hybrid approach has the potential to improve the fidelity of risk models and further enhance the efficiency of quantum risk analysis.

Video Presentation: The video can be accessed via the following Google Drive link:

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1 Introduction

Risk management is a critical function within the financial system, enabling institutions to quantify and mitigate potential losses from various sources such as credit risk, market risk, and operational risk. The calculation of risk metrics for large, complex portfolios often relies on Monte Carlo (MC) simulations. However, these classical methods are computationally intensive, as their estimation error scales slowly, at a rate of $\mathcal{O}(M^{-1/2})$, where M is the number of samples. This slow convergence necessitates a vast number of simulation runs to achieve the high precision required for regulatory reporting (e.g., under Basel III) and internal decision-making, consuming significant time and computational resources.

Quantum computers, which operate based on the principles of quantum mechanics, offer new pathways to solve certain computational problems more efficiently. One of the most powerful tools in this domain is the Quantum Amplitude Estimation (QAE) algorithm, which can estimate a parameter with a convergence rate of $\mathcal{O}(M^{-1})$. This represents a quadratic speedup over classical MC methods and forms the basis for quantum advantage in a range of applications.

This report summarizes the development of quantum algorithms for financial risk analysis. I trace the evolution of this topic through three key papers. I review the foundational work of Woerner and Egger[1] , its extension to credit risk by Egger et al[2]. , and a recent paper by Stamatopoulos et al[3]. that introduces Quantum Signal Processing (QSP) for analyzing derivatives.

Subsequently, this report also proposes a new research problem. Based on the literature survey, I identify the creation of input scenarios as a critical challenge and propose a hybrid quantum-classical methodology to generate more robust and realistic market scenarios for risk analysis.

2 Financial Risk and Key Metrics

To analyze risk, financial institutions rely on several statistical metrics that quantify potential losses over a specific time horizon and at a given confidence level.

2.1 Value at Risk (VaR) and Conditional Value at Risk (CVaR)

Value at Risk (VaR) is one of the most widely used risk metrics. For a given portfolio, a random variable $\mathcal{L}$ representing the total loss, and a confidence level $\alpha \in [0, 1]$ (typically 99% or higher), the VaR is defined as the minimum loss value x that will not be exceeded with a probability of α. Formally, it is the quantile of the loss distribution:

$$VaR_\alpha[\mathcal{L}] = \inf_{x \geq 0}\{x|\mathbb{P}[\mathcal{L} \leq x] \geq \alpha\}$$

Conditional Value at Risk (CVaR), or Expected Shortfall, measures the expected loss in the tail of the distribution, given that the loss is greater than the VaR. It is considered more sensitive to extreme events in the tail of the loss distribution.

2.2 Economic Capital Requirement (ECR)

The Economic Capital Requirement (ECR) is an internal metric used by firms to determine the amount of capital they need to hold to remain solvent at a specific confidence level. It is defined as the difference between the VaR at that confidence level and the expected value of the total loss:

$$ECR_\alpha[\mathcal{L}] = VaR_\alpha[\mathcal{L}] - \mathbb{E}[\mathcal{L}]$$

2.3 Uncertainty Models

The calculation of these metrics depends on an underlying uncertainty model. Early work considered simple independent Bernoulli trials for asset losses. A more realistic credit risk model is the **Gaussian conditional independence model**, where individual asset defaults X_k are correlated through their dependence on a shared latent random variable Z (representing macroeconomic factors) that follows a standard normal distribution. The default probability p_k for asset k given a realization z of Z is:

$$p_k(z) = F\left(\frac{F^{-1}(p_k^0) - \sqrt{\rho_k}z}{\sqrt{1-\rho_k}}\right)$$

where p_k^0 is the baseline default probability, F is the CDF of the standard normal distribution, and ρ_k is the sensitivity of the asset to Z. For derivatives, risk is assessed by pricing the derivative under a set of future market scenarios, which can be generated from historical data or Monte Carlo simulations.

3 Quantum Amplitude Estimation for Risk Analysis

The foundational quantum approach to risk analysis is based on Quantum Amplitude Estimation (QAE).

3.1 The QAE Algorithm

Suppose we can construct a unitary operator $\mathcal{A}$ that acts on $n+1$ qubits to prepare the state:

$$\mathcal{A}|0\rangle_{n+1} = \sqrt{1-a}|\psi_0\rangle_n|0\rangle + \sqrt{a}|\psi_1\rangle_n|1\rangle$$

Here, the value a we wish to estimate is the probability of measuring the last qubit in the state $|1\rangle$. QAE uses Quantum Phase Estimation on a specially constructed operator $\mathcal{Q} = \mathcal{A}(\mathbb{I} - 2|0\rangle\langle 0|)\mathcal{A}^\dagger(\mathbb{I} - 2|\psi_0\rangle|0\rangle\langle\psi_0|\langle 0|)$ to determine a. The full error bound for an estimate $\tilde{a}$ using M evaluations is given by:

$$|a - \tilde{a}| \leq \frac{2\pi\sqrt{a(1-a)}}{M} + \frac{\pi^2}{M^2} = \mathcal{O}(M^{-1})$$

This provides a quadratic speedup over the $\mathcal{O}(M^{-1/2})$ convergence of classical Monte Carlo methods.

3.2 Mapping Risk to a Quantum Circuit

The application of QAE to risk analysis involves mapping the problem onto a quantum circuit. This begins with loading a probability distribution over a random variable X into a quantum state $|\psi\rangle_n = \sum_{i=0}^{N-1}\sqrt{p_i}|i\rangle_n$. An operator, let's call it $\mathcal{F}$, is then applied to encode a function $f(i)$ of the random variable into the amplitude of an ancilla qubit:

$$\mathcal{F} : |i\rangle_n|0\rangle \mapsto |i\rangle_n\left(\sqrt{1-f(i)}|0\rangle + \sqrt{f(i)}|1\rangle\right)$$

Applying $\mathcal{F}$ to the full superposition creates a state where the probability of measuring $|1\rangle$ is $\sum_i p_i f(i) = \mathbb{E}[f(X)]$. Different choices of $f(i)$ allow for the calculation of different risk metrics:

- **Expected Value:** Setting $f(i) = i/(N-1)$ allows for the estimation of $\mathbb{E}[X]$.
- **VaR:** A comparator function $f_l(i) = 1$ if $i \leq l$ and 0 otherwise is used. The resulting state is $\sum_{i=l+1}^{N-1}\sqrt{p_i}|i\rangle_n|0\rangle + \sum_{i=0}^{l}\sqrt{p_i}|i\rangle_n|1\rangle$. The probability of measuring $|1\rangle$ gives the CDF value $\mathbb{P}[X \leq l]$. A bisection search over l finds VaR_α.

- **CVaR:** Once $l_\alpha = VaR_\alpha(X)$ is found, one can use the function $f(i) = \frac{i}{l_\alpha}$ for $i \leq l_\alpha$ and 0 otherwise. The probability of measuring $|1\rangle$ is then $\sum_{i=0}^{l_\alpha} \frac{i}{l_\alpha} p_i$. After accounting for the total probability of being in the tail, the CVaR is recovered via classical post-processing:

$$CVaR_\alpha(X) = \frac{l_\alpha}{\mathbb{P}[X \leq l_\alpha]} \sum_{i=0}^{l_\alpha} \frac{i}{l_\alpha} p_i$$

3.3 Experimental and Simulation Results

In Ref.[1], they demonstrated these principles with both real hardware experiments and simulations. For a simple T-bill model, they implemented the algorithm on the 5-qubit "IBM Q 5 Yorktown" processor. Their results, as shown in FIG.[1], show histograms of the estimated value for a growing number of evaluation qubits ($m = 1$ to $m = 4$). As m increases, the distribution of results sharpens, and the most frequent estimate converges towards the true value of 30%. Furthermore, FIG.[2] plots the estimation error versus the number of samples (M) on a log-log scale, demonstrating that for $M \geq 16$, the quantum algorithm's error falls below that of the classical simulation, confirming the superior $\mathcal{O}(M^{-1})$ convergence rate in practice.

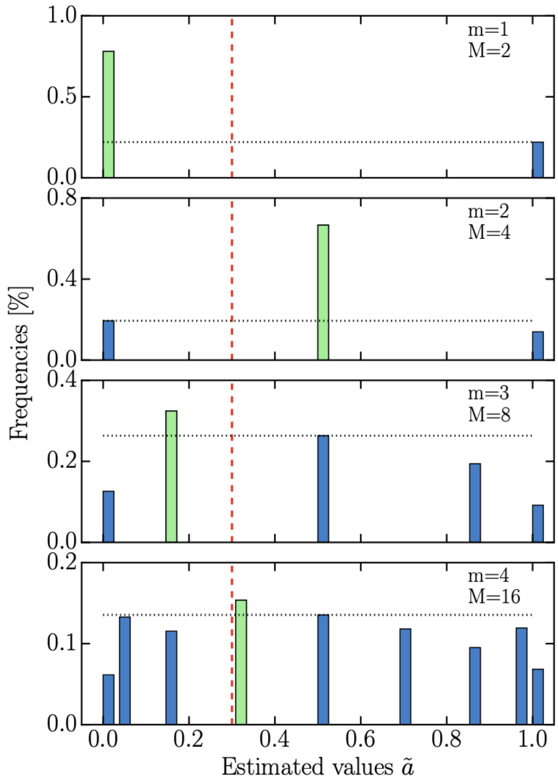

Figure 1: Figure taken from [1]. Results of running amplitude estimation on real hardware for m = 1, ..., 4 with 8192 shots each.

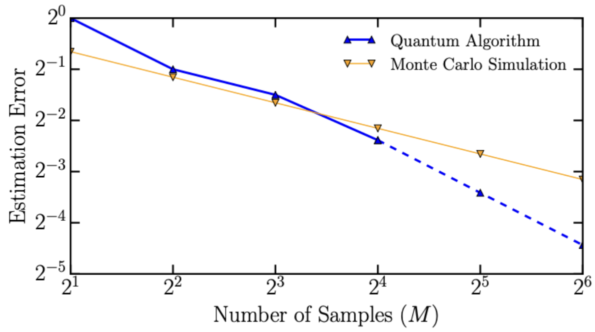

Figure 2: Figure taken from [1]. Comparison of the convergence of the error of Monte Carlo simulation and our algorithm with respect to the number of samples M.

4 Advanced Methods for Derivative Risk Analysis

The risk analysis of financial derivatives adds complexity, as the "loss" under a scenario is the result of another complex pricing calculation.

4.1 The Nested QAE Approach

A direct extension of the QAE method to derivatives results in a "nested" algorithm where an inner QAE round digitizes the derivative's price in superposition, and an outer QAE round calculates the VaR probability. While fully quantum, this approach introduces approximation errors from the inner digitization step, where the value $V(s_i)$ for each scenario is represented by a distribution of binary strings weighted by probabilities $q_j(V(s_i))$ that peak around the true value.

4.2 The QSP-Based Approach

A more elegant and efficient algorithm uses Quantum Signal Processing (QSP). QSP applies polynomial transformations directly to singular values of a block-encoded matrix. The process is as follows:

1. The derivative pricing operator $\mathcal{A}$ is treated as a block-encoding unitary, where the value $\sqrt{V(s)}$ corresponds to a singular value.

2. QSP is used to apply a polynomial transformation $P(x)$ that approximates a sharp **threshold function** directly to the value amplitude:

$$\theta_\mu(x) = \begin{cases} 1, & x \leq \mu \\ 0, & x > \mu \end{cases}$$

   This is achieved by constructing a sequence of single-qubit rotations interleaved with applications of $\mathcal{A}$ and $\mathcal{A}^\dagger$.

3. This coherently maps all values below a threshold μ to an amplitude of ≈ 1 and all values above to ≈ 0, producing a state where the probability of measuring a target state $|G\rangle$ is $\mathbb{P}(|G\rangle) = \sum_{\{s|V(s)\leq\mu^2\}} p(s)$.

4. A single outer round of QAE then estimates this probability, and a bisection search over μ finds the VaR.

The polynomial approximation itself is found by solving a minimax optimization problem. For a polynomial $f(x) = \sum_{k=0}^{d/2} c_k T_{2k}(x)$, where $T_{2k}(x)$ are Chebyshev polynomials, the goal is to find coefficients $\{c_k\}$ that solve:

$$\min_{\{c_k\}} \max \left\{ \max_{x_j \in [0, \mu - \Delta/2]} |f(x_j) - c|, \max_{x_j \in [\mu + \Delta/2, 1]} |f(x_j)| \right\}$$

subject to $|f(x_j)| \leq c$ for all x_j in a discretized grid. Here Δ is the gap where the function transitions from $c \approx 1$ to 0.

The performance comparison between the QSP and nested QAE methods is starkly illustrated in FIG.[3]. This figure plots the VaR estimation error against the number of oracle calls for both methods. The results show that the QSP-based method consistently requires approximately **10x fewer oracle calls** than the nested QAE method to achieve the same target error.

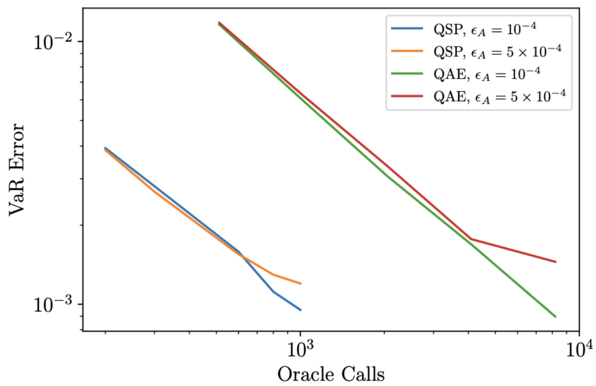

Figure 3: Figure taken from [3]. VaR estimation errors as a function of the number of oracle calls for the QSP and QAE quantum methods.

5 Resource Analysis, Quantum Advantage, and Noise Considerations

A key question is whether and when these quantum algorithms can provide a practical advantage. The papers provide detailed resource estimates and consider the impact of noise.

In Ref.[1], they provide a scaling analysis for a realistic credit risk problem size: a portfolio of $K = 2^{20}$ assets, $n_Z = 10$ qubits for the latent variable, and $m = 10$ evaluation qubits. For instance, FIG.[4] provides a concrete summary of the number of CNOT gates required. It shows that for $m = 5$ ($M = 32$ samples), the circuit requires over 55,000 CNOT gates for a realistic 20-qubit topology, highlighting the significant resources needed. With these parameters and assuming a fault-tolerant quantum computer with a T-gate clock rate of 10^{-4} seconds, they estimate the ECR could be calculated in approximately 30 to 60 minutes.

In Ref.[3], they estimate that for risk analysis on derivatives, the required logical clock rate for quantum advantage could be reduced by up to **30x** compared to pricing a single derivative, provided the scenario superposition can be prepared efficiently.

To assess hardware readiness, Woerner and Egger also simulated the effect of noise, with results presented in FIG.[5]. They studied the effects of energy relaxation and cross-talk and found that for their algorithm to succeed (identify the correct value with >50% probability), relaxation rates must be below $\gamma < 10^{-4}s^{-1}$ and cross-talk strength $|\alpha| < 1\%$. This provides tangible hardware targets for achieving quantum advantage in the near future.

m	M	#qubits	#CX all-to-all	#CX IBM Q 20	overhead
1	2	13	795	1’817	2.29
2	4	14	2’225	5’542	2.49
3	8	15	5’085	12’691	2.50
4	16	16	10’803	26’457	2.45
5	32	17	22’235	55’520	2.50

Figure 4: Table taken from [1]. Summary of the number of CNOT gates to estimate VaR as a function of m for a processor architecture featuring an all-to-all qubit connectivity and an architecture with a qubit connectivity corresponding to the IBM Q 20 Austin chip with 20 qubits.

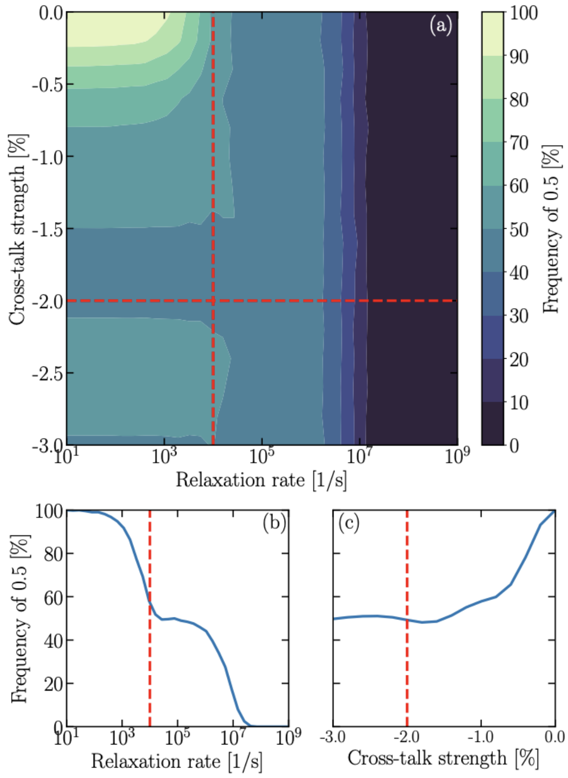

Figure 5: Figure taken from [1]. Results from noisy simulation for estimating the expected value of the two-asset portfolio using two evaluation qubits.

6 Research Gaps and Future Directions

The reviewed literature demonstrates a clear path to quantum advantage in risk analysis, culminating in the highly efficient QSP-based algorithm. However, a critical component is largely treated as a given: the creation of the initial state representing the market scenarios. The papers assume that the superposition over scenarios is either created from classical samples or loaded from a known, often simple, probability distribution (e.g., a log-concave distribution).

This presents two significant research gaps:

1. **Fidelity of Scenario Generation:** Financial markets exhibit complex, non-Gaussian features, including heavy tails and clustered volatility. Simple distributions may not capture these critical features, leading to an underestimation of risk.

2. **Efficiency of State Preparation:** The resource estimates for the quantum algorithms depend heavily on the T-depth of the scenario-loading unitary, T_S. The overall quantum advantage can be lost if this initial state preparation step is too slow.

A promising future direction lies in developing a quantum-native method for generating high-fidelity scenario distributions that can be used directly within the risk analysis algorithm, addressing both gaps simultaneously.

7 Proposed Research Problem: Hybrid Quantum-Classical Methods for Dynamic Scenario Generation

Building on the identified research gaps, I propose a new research problem that integrates the state-of-the-art QSP-based risk algorithm with a quantum generative model.

7.1 Formal Problem Statement

The research problem is to design and analyze a **hybrid quantum-classical generative adversarial framework for dynamic scenario generation in financial risk analysis**.

The goal is to create a quantum state $|\psi_{scenarios}\rangle = \sum_i \sqrt{p_i}|s_i\rangle$ that accurately represents a complex, empirically-derived distribution of market scenarios, and to use this state directly as the input to the QSP-based VaR algorithm. The framework should be able to learn the complex correlations and non-Gaussian features present in historical financial data.

7.2 Proposed Methodology

I propose a methodology based on Quantum Generative Adversarial Networks (qGANs). The architecture consists of a hybrid quantum-classical feedback loop:

1. **Classical Discriminator $D(\phi)$:** A classical deep neural network with parameters ϕ is trained on historical market data. Its job is to distinguish between real historical data and synthetic data.

2. **Quantum Generator $G(\theta)$:** A Variational Quantum Circuit (VQC) with parameters θ acts as the generator. It prepares a quantum state $|\psi_G(\theta)\rangle$. Measurements of this state yield a probability distribution $p_G(\theta)$ over classical data samples.

3. **Adversarial Training Loop:** The generator and discriminator are trained in a minimax game. The objective can be formulated as finding the optimal parameters θ^* and ϕ^* for the cost function:

$$\min_\theta \max_\phi \left(\mathbb{E}_{x\sim p_{data}}[\log D(x|\phi)] + \mathbb{E}_{x\sim p_G(\theta)}[\log(1 - D(x|\phi))]\right)$$

This training process tunes the VQC parameters θ such that the generator learns to produce a state $|\psi_G(\theta^*)\rangle$ whose measurement statistics mimic the true data distribution p_{data}.

4. **Integration with QSP-VaR Algorithm:** Once the training is complete, the VQC has learned the parameters required to generate a quantum state $|\psi_{scenarios}\rangle$ that models the true data distribution. This state preparation circuit now serves as the **scenario-loading unitary $\mathcal{S}$** in the QSP-based VaR algorithm. Instead of generating classical samples via measurement, the prepared state is coherently fed into the rest of the quantum risk algorithm.

7.3 Justification and Novelty

This proposed method is promising for several reasons:

1. **Higher Fidelity:** Quantum models like VQCs may capture complex, high-dimensional correlations inherent in financial data more effectively than classical generative models.

2. **Quantum-Native Integration:** The generator directly produces the quantum state needed for the risk algorithm. This bypasses the potentially costly process of loading classical data into a quantum state (QRAM), which could reduce the critical T_S cost.

3. **Novelty:** The existing literature on quantum risk analysis treats scenario generation as a separate, classical pre-processing step. This proposal introduces a novel, integrated framework where the scenario distribution is learned and prepared natively on the quantum computer. This directly addresses the open question of efficient preparation of relevant probability distributions highlighted as a key challenge for the field.

8 Conclusion

The application of quantum computing to financial risk analysis has evolved from a foundational concept to a sophisticated field with a credible path toward practical quantum advantage. The journey began with the core insight that QAE can provide a quadratic speedup for calculating fundamental risk metrics. This was extended to realistic credit risk models and further refined with the introduction of the highly efficient QSP-based algorithm for derivatives.

However, the field's full potential is limited by the challenge of preparing quantum states that faithfully represent complex market dynamics. The research problem proposed in this report—a hybrid qGAN framework for dynamic scenario generation—aims to address this critical gap. By training a quantum circuit to learn and prepare realistic market scenario distributions, this approach could enhance the accuracy of risk models while simultaneously improving the efficiency of the overall quantum algorithm. If successful, this integrated framework would represent a significant step toward a truly end-to-end quantum solution for financial risk management, bringing the promise of quantum advantage closer to reality.

References

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