08_Linear_Model_for_Regression

Noting Paradigm

- x - Plain text: Scalar.
- $\mathbf{x}$ - Bold-Face lowercase: Vector of scalars.
    - e.g., $\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ \cdots \\ x_D \end{bmatrix}$, where $\mathbf{x} \in \mathbb{R}^D$
- $\mathbf{X}$ - Bold-Face uppercase: Set of vectors.
    - e.g., $\mathbf{X} = \{\mathbf{x}_1, \mathbf{x}_2, \cdots, \mathbf{x}_N\}$, where $\mathbf{X} \subset \mathbb{R}^D$ and $|\mathbf{X}| = N$.

8.0 Regression

8.0.0 Why Regression?

Problem Setup

Given

- A set of inputs:
    - $\mathbf{X} = \{\mathbf{x}_1, \mathbf{x}_2, \cdots, \mathbf{x}_N\}$, where, for each input:
        - $\mathbf{x}_i = \begin{bmatrix} x_{i1} \\ x_{i2} \\ \cdots \\ x_{iD} \end{bmatrix} \in \mathbb{R}^D$
- A set of corresponding outputs:
    - $\mathbf{y} = \{y_1, y_2, \cdots, y_N\}$, where, for each output:
        - $y_i \in \mathbb{R}$
- A labelling relation:
  - $D = \{(\mathbf{x}_1, y_1), (\mathbf{x}_2, y_2), \cdots, (\mathbf{x}_N, y_N)\}$
  Goal
- To learn a mapping
    - $f(x) : \mathbf{X} \to \mathbf{y}$, where
        - Input: An arbitrary vector $\mathbf{x} \notin \mathbf{X}$,
        - Output: A scalar $y \notin \mathbf{y}$.
- Such that we could make prediction

- about y_*
- on an unseen input $\mathbf{x}_*$ is encountered.

Different Kinds

- Parametric Regression 参数性回归
  - Assume a functional form for $f(x)$.
- Nonparametric Regression 非参数性回归
  - Does not assume a functional form for $f(x)$.

To sum up

- From given relations, learn a function that
  - Takes a vector as an input
  - Output a real number that
    - "Fits" the given pattern

8.0.1 Definition

❓ What is Regression?

- Regression aims at modelling the dependence of:
  - a response Y,
  - on a covariate X.
- That is, to predict the value of one or more continuous target variables y given the value of input vector x.

ℹ️ The regression model is described by

$$y = f(\mathbf{x}) + \epsilon$$

- where the dependence of a response y on a covariate $\mathbf{x}$ is captured via:
  - $p(y|\mathbf{x})$,
  - i.e., a conditional probability distribution.

8.1 Linear Regression