

Supplementary Material for “Uncovering User Interest from Biased and Noised Watch Time in Video Recommendation”

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1 PROOF OF PROPOSITION 1

PROOF. In section 4.2 of the paper, we propose $D^2Co(S)$ for sensitivity control, that is:

$$r_{\mathbf{x}}^{D^2Co(S)}(w, \tilde{w}_d^+, \tilde{w}_d^-) = \frac{\exp(\alpha w) - \exp(\alpha \tilde{w}_d^-)}{\exp(\alpha \tilde{w}_d^+) - \exp(\alpha \tilde{w}_d^-)},$$

where α is the sensitivity control term. We can prove the sensitivity of $D^2Co(S)$ which given the predicted value of duration bias and duration noise parameters \tilde{w}_d^+ and \tilde{w}_d^- is:

$$\begin{aligned} \mathbb{S}'_{\tilde{w}_d^+} &= \left| \frac{\partial r_{\mathbf{x}}^{D^2Co(S)}(w, \tilde{w}_d^+, \tilde{w}_d^-)}{\partial \tilde{w}_d^+} \delta_{\tilde{w}_d^+} \right| \\ &= \left| \frac{\alpha \exp(\alpha \tilde{w}_d^+) (\exp(\alpha \tilde{w}_d^-) - \exp(\alpha w))}{(\exp(\alpha \tilde{w}_d^+) - \exp(\alpha \tilde{w}_d^-))^2} \delta_{\tilde{w}_d^+} \right| \\ &= \frac{\alpha \exp(\alpha \tilde{w}_d^+) (\exp(\alpha w) - \exp(\alpha \tilde{w}_d^-))}{(\exp(\alpha \tilde{w}_d^+) - \exp(\alpha \tilde{w}_d^-))^2} \left| \delta_{\tilde{w}_d^+} \right| \end{aligned}$$

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$$\begin{aligned}
\mathbb{S}'_{\tilde{w}_d^-} &= \left| \frac{\partial r_{\mathbf{x}}^{\text{D}^2\text{Co(S)}}(w, \tilde{w}_d^+, \tilde{w}_d^-)}{\partial \tilde{w}_d^-} \delta_{\tilde{w}_d^-} \right| \\
&= \left| \frac{\alpha \exp(\alpha \tilde{w}_d^-) (\exp(\alpha w) - \exp(\alpha \tilde{w}_d^+))}{\left(\exp(\alpha \tilde{w}_d^+) - \exp(\alpha \tilde{w}_d^-) \right)^2} \delta_{\tilde{w}_d^-} \right| \\
&= \frac{\alpha \exp(\alpha \tilde{w}_d^-) (\exp(\alpha \tilde{w}_d^+) - \exp(\alpha w))}{\left(\exp(\alpha \tilde{w}_d^+) - \exp(\alpha \tilde{w}_d^-) \right)^2} \left| \delta_{\tilde{w}_d^-} \right|
\end{aligned}$$

where $\mathbb{S}'_{\tilde{w}_d^+}$ and $\mathbb{S}'_{\tilde{w}_d^-}$ denote the sensitivity of $r_{\mathbf{x}}^{\text{D}^2\text{Co(S)}}$ towards \tilde{w}_d^+ and \tilde{w}_d^- respectively. $\delta_{\tilde{w}_d^+}$ and $\delta_{\tilde{w}_d^-}$ is the disturbance of \tilde{w}_d^+ and \tilde{w}_d^- . The third equation holds due to $w \in [\tilde{w}_d^-, \tilde{w}_d^+]$. As we discussed in Theorem 3, the sensitivity of $\text{D}^2\text{Co(A)}$ is:

$$\begin{aligned}
\mathbb{S}_{\tilde{w}_d^+} &= \frac{w - \tilde{w}_d^-}{(\tilde{w}_d^+ - \tilde{w}_d^-)^2} \left| \delta_{\tilde{w}_d^+} \right| \\
\mathbb{S}_{\tilde{w}_d^-} &= \frac{\tilde{w}_d^+ - w}{(\tilde{w}_d^+ - \tilde{w}_d^-)^2} \left| \delta_{\tilde{w}_d^-} \right|
\end{aligned}$$

First of all, we will prove that when $\alpha < 0$, $\mathbb{S}'_{\tilde{w}_d^+} < \mathbb{S}_{\tilde{w}_d^+}$. The ratio of $\mathbb{S}'_{\tilde{w}_d^+}$ and $\mathbb{S}_{\tilde{w}_d^+}$ is:

$$\begin{aligned}
\frac{\mathbb{S}_{\tilde{w}_d^+}}{\mathbb{S}'_{\tilde{w}_d^+}} &= \frac{(w - \tilde{w}_d^-) \left(\exp(\alpha \tilde{w}_d^+) - \exp(\alpha \tilde{w}_d^-) \right)^2}{\alpha \exp(\alpha \tilde{w}_d^+) \left(\exp(\alpha w) - \exp(\alpha \tilde{w}_d^-) \right) (\tilde{w}_d^+ - \tilde{w}_d^-)^2} \\
&= \frac{\left(\exp(\alpha \tilde{w}_d^+) - \exp(\alpha \tilde{w}_d^-) \right)^2}{\exp(\alpha \tilde{w}_d^+) (\tilde{w}_d^+ - \tilde{w}_d^-)^2} \cdot \frac{(w - \tilde{w}_d^-)}{\alpha \left(\exp(\alpha w) - \exp(\alpha \tilde{w}_d^-) \right)}
\end{aligned}$$

Since since $\alpha < 0$, the second term can be regarded as a monotonically increasing function of w ($w \in [\tilde{w}_d^-, \tilde{w}_d^+]$). Also note that the first term is greater than zero, so above ratio is also a monotonically increasing function of w . When w tends to \tilde{w}_d^- , the minimum value of this ratio is:

$$\lim_{w \rightarrow \tilde{w}_d^-} \frac{\mathbb{S}_{\tilde{w}_d^+}}{\mathbb{S}'_{\tilde{w}_d^+}} = \frac{\left(\exp(\alpha \tilde{w}_d^+) - \exp(\alpha \tilde{w}_d^-) \right)^2}{\exp(\alpha \tilde{w}_d^+) (\tilde{w}_d^+ - \tilde{w}_d^-)^2} \cdot \frac{1}{\alpha^2 \exp(\alpha \tilde{w}_d^-)}$$

Then we will prove that this value is greater than one:

$$\begin{aligned}
&\frac{\left(\exp(\alpha \tilde{w}_d^+) - \exp(\alpha \tilde{w}_d^-) \right)^2}{\exp(\alpha \tilde{w}_d^+) (\tilde{w}_d^+ - \tilde{w}_d^-)^2 \alpha^2 \exp(\alpha \tilde{w}_d^-)} - 1 \\
&= \frac{\exp(2\alpha \tilde{w}_d^+) - \left(2 + \alpha^2 (\tilde{w}_d^+ - \tilde{w}_d^-)^2 \right) \exp(\alpha (\tilde{w}_d^+ + \tilde{w}_d^-)) + \exp(2\alpha \tilde{w}_d^-)}{\exp(\alpha \tilde{w}_d^+) (\tilde{w}_d^+ - \tilde{w}_d^-)^2 \alpha^2 \exp(\alpha \tilde{w}_d^-)}
\end{aligned}$$

Since the denominator is greater than zero, then we only need to prove that the numerator is greater than zero. We regard the numerator as the function of \tilde{w}_d^+ ($\tilde{w}_d^+ > w > \tilde{w}_d^-$):

$$f(\tilde{w}_d^+) = \exp(2\alpha\tilde{w}_d^+) - \left(2 + \alpha^2 (\tilde{w}_d^+ - \tilde{w}_d^-)^2\right) \exp(\alpha(\tilde{w}_d^+ + \tilde{w}_d^-)) + \exp(2\alpha\tilde{w}_d^-)$$

It can be prove that $f(\tilde{w}_d^+)$ is a monotonically increasing function, so its minimum value is when \tilde{w}_d^+ is tend to \tilde{w}_d^- :

$$\lim_{\tilde{w}_d^+ \rightarrow \tilde{w}_d^-} f(\tilde{w}_d^+) = 0$$

So we have $f(\tilde{w}_d^+) > 0$, thus proving that $\frac{\mathbb{S}_{\tilde{w}_d^+}}{\mathbb{S}'_{\tilde{w}_d^+}} > 1$.

Then we will prove that when $\alpha > 0$, $\mathbb{S}'_{\tilde{w}_d^-} < \mathbb{S}_{\tilde{w}_d^-}$. The ratio of $\mathbb{S}'_{\tilde{w}_d^-}$ and $\mathbb{S}_{\tilde{w}_d^-}$ is:

$$\begin{aligned} \frac{\mathbb{S}_{\tilde{w}_d^-}}{\mathbb{S}'_{\tilde{w}_d^-}} &= \frac{(\tilde{w}_d^+ - w) \left(\exp(\alpha\tilde{w}_d^+) - \exp(\alpha\tilde{w}_d^-) \right)^2}{\alpha \exp(\alpha\tilde{w}_d^-) \left(\exp(\alpha\tilde{w}_d^+) - \exp(\alpha w) \right) (\tilde{w}_d^+ - \tilde{w}_d^-)^2} \\ &= \frac{\left(\exp(\alpha\tilde{w}_d^+) - \exp(\alpha\tilde{w}_d^-) \right)^2}{\exp(\alpha\tilde{w}_d^-) (\tilde{w}_d^+ - \tilde{w}_d^-)^2} \cdot \frac{(\tilde{w}_d^+ - w)}{\alpha \left(\exp(\alpha\tilde{w}_d^+) - \exp(\alpha w) \right)} \end{aligned}$$

Similarly, since $\alpha > 0$, this ratio can be regarded as the monotonically decreasing of w ($w \in [\tilde{w}_d^-, \tilde{w}_d^+]$). When w tends to \tilde{w}_d^+ , the minimum value of this ratio is:

$$\lim_{w \rightarrow \tilde{w}_d^+} \frac{\mathbb{S}_{\tilde{w}_d^-}}{\mathbb{S}'_{\tilde{w}_d^-}} = \frac{\left(\exp(\alpha\tilde{w}_d^+) - \exp(\alpha\tilde{w}_d^-) \right)^2}{\exp(\alpha\tilde{w}_d^-) (\tilde{w}_d^+ - \tilde{w}_d^-)^2} \cdot \frac{1}{\alpha^2 \exp(\alpha\tilde{w}_d^+)}$$

It worth noting that this limit value is equal to $\lim_{w \rightarrow \tilde{w}_d^-} \frac{\mathbb{S}_{\tilde{w}_d^+}}{\mathbb{S}'_{\tilde{w}_d^+}}$. As we have proved before, $\lim_{w \rightarrow \tilde{w}_d^-} \frac{\mathbb{S}_{\tilde{w}_d^+}}{\mathbb{S}'_{\tilde{w}_d^+}} > 1$, we also

have $\lim_{w \rightarrow \tilde{w}_d^+} \frac{\mathbb{S}_{\tilde{w}_d^-}}{\mathbb{S}'_{\tilde{w}_d^-}} > 1$ equivalently. Then we have $\frac{\mathbb{S}_{\tilde{w}_d^-}}{\mathbb{S}'_{\tilde{w}_d^-}} > 1$. \square