# Supplementary Material for “Uncovering User Interest from Biased and Noised Watch Time in Video Recommendation”

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**ACM Reference Format:**
Haiyuan Zhao, Lei Zhang, Jun Xu, Guohao Cai, Zhenhua Dong, and Ji-Rong Wen. 2023. Supplementary Material for “Uncovering User Interest from Biased and Noised Watch Time in Video Recommendation”. In *Seventeenth ACM Conference on Recommender Systems (RecSys ’23), September 18–22, 2023, Singapore, Singapore.* ACM, New York, NY, USA, 3 pages. https://doi.org/10.1145/3604915.3608797

## 1 PROOF OF PROPOSITION 1

Proof. In section 4.2 of the paper, we propose D$^2$Co(S) for sensitivity control, that is:

$$r_{\mathbf{x}}^{\mathrm{D^2Co(S)}}(w, \widetilde{w}_d^+, \widetilde{w}_d^-) = \frac{\exp(\alpha w) - \exp(\alpha \widetilde{w}_d^-)}{\exp(\alpha \widetilde{w}_d^+) - \exp(\alpha \widetilde{w}_d^-)},$$

where $\alpha$ is the sensitivity control term. We can prove the sensitivity of D$^2$Co(S) which given the predicted value of duration bias and duration noise parameters $\widetilde{w}_d^+$ and $\widetilde{w}_d^-$ is:

$$\begin{aligned}
\mathbb{S}'_{\widetilde{w}_d^+} &= \left| \frac{\partial\, r_{\mathbf{x}}^{\mathrm{D^2Co(S)}}(w, \widetilde{w}_d^+, \widetilde{w}_d^-)}{\partial\, \widetilde{w}_d^+} \delta_{\widetilde{w}_d^+} \right| \\
&= \left| \frac{\alpha \exp(\alpha \widetilde{w}_d^+)(\exp(\alpha \widetilde{w}_d^-) - \exp(\alpha w))}{\left(\exp(\alpha \widetilde{w}_d^+) - \exp(\alpha \widetilde{w}_d^-)\right)^2} \delta_{\widetilde{w}_d^+} \right| \\
&= \frac{\alpha \exp(\alpha \widetilde{w}_d^+)(\exp(\alpha w) - \exp(\alpha \widetilde{w}_d^-))}{\left(\exp(\alpha \widetilde{w}_d^+) - \exp(\alpha \widetilde{w}_d^-)\right)^2} \left| \delta_{\widetilde{w}_d^+} \right|
\end{aligned}$$

$$\begin{aligned}
\mathbb{S}'_{\widetilde{w}_d^-} &= \left| \frac{\partial\, r_{\mathbf{x}}^{\mathrm{D^2Co(S)}}(w, \widetilde{w}_d^+, \widetilde{w}_d^-)}{\partial\, \widetilde{w}_d^-} \delta_{\widetilde{w}_d^-} \right| \\
&= \left| \frac{\alpha \exp(\alpha \widetilde{w}_d^-)(\exp(\alpha w) - \exp(\alpha \widetilde{w}_d^+))}{\left(\exp(\alpha \widetilde{w}_d^+) - \exp(\alpha \widetilde{w}_d^-)\right)^2} \delta_{\widetilde{w}_d^-} \right| \\
&= \frac{\alpha \exp(\alpha \widetilde{w}_d^-)(\exp(\alpha \widetilde{w}_d^+) - \exp(\alpha w))}{\left(\exp(\alpha \widetilde{w}_d^+) - \exp(\alpha \widetilde{w}_d^-)\right)^2} \left|\delta_{\widetilde{w}_d^-}\right|
\end{aligned}$$

where $\mathbb{S}'_{\widetilde{w}_d^+}$ and $\mathbb{S}'_{\widetilde{w}_d^-}$ denote the sensitivity of $r_{\mathbf{x}}^{\mathrm{D^2Co(S)}}$ towards $\widetilde{w}_d^+$ and $\widetilde{w}_d^-$ respectively. $\delta_{\widetilde{w}_d^+}$ and $\delta_{\widetilde{w}_d^-}$ is the disturbance of $\widetilde{w}_d^+$ and $\widetilde{w}_d^-$. The third equation holds due to $w \in [\widetilde{w}_d^-, \widetilde{w}_d^+]$. As we discussed in Theorem 3, the sensitivity of $\mathrm{D^2Co(A)}$ is:

$$\begin{aligned}
\mathbb{S}_{\widetilde{w}_d^+} &= \frac{w - \widetilde{w}_d^-}{(\widetilde{w}_d^+ - \widetilde{w}_d^-)^2} \left|\delta_{\widetilde{w}_d^+}\right| \\
\mathbb{S}_{\widetilde{w}_d^-} &= \frac{\widetilde{w}_d^+ - w}{(\widetilde{w}_d^+ - \widetilde{w}_d^-)^2} \left|\delta_{\widetilde{w}_d^-}\right|
\end{aligned}$$

First of all, we will prove that when $\alpha < 0, \quad \mathbb{S}'_{\widetilde{w}_d^+} < \mathbb{S}_{\widetilde{w}_d^+}$. The ratio of $\mathbb{S}'_{\widetilde{w}_d^+}$ and $\mathbb{S}'_{\widetilde{w}_d^+}$ is:

$$\begin{aligned}
\frac{\mathbb{S}_{\widetilde{w}_d^+}}{\mathbb{S}'_{\widetilde{w}_d^+}} &= \frac{(w - \widetilde{w}_d^-)\left(\exp(\alpha \widetilde{w}_d^+) - \exp(\alpha \widetilde{w}_d^-)\right)^2}{\alpha \exp(\alpha \widetilde{w}_d^+)\left(\exp(\alpha w) - \exp(\alpha \widetilde{w}_d^-)\right)(\widetilde{w}_d^+ - \widetilde{w}_d^-)^2} \\
&= \frac{\left(\exp(\alpha \widetilde{w}_d^+) - \exp(\alpha \widetilde{w}_d^-)\right)^2}{\exp(\alpha \widetilde{w}_d^+)(\widetilde{w}_d^+ - \widetilde{w}_d^-)^2} \cdot \frac{(w - \widetilde{w}_d^-)}{\alpha\left(\exp(\alpha w) - \exp(\alpha \widetilde{w}_d^-)\right)}
\end{aligned}$$

Since since $\alpha < 0$, the second term can be regarded as a monotonically increasing function of $w$ ($w \in [\widetilde{w}_d^-, \widetilde{w}_d^+]$). Also note that the first term is greater than zero, so above ratio is also a monotonically increasing function of $w$. When $w$ tends to $\widetilde{w}_d^-$, the minimum value of this ratio is:

$$\lim_{w \to \widetilde{w}_d^-} \frac{\mathbb{S}_{\widetilde{w}_d^+}}{\mathbb{S}'_{\widetilde{w}_d^+}} = \frac{\left(\exp(\alpha \widetilde{w}_d^+) - \exp(\alpha \widetilde{w}_d^-)\right)^2}{\exp(\alpha \widetilde{w}_d^+)(\widetilde{w}_d^+ - \widetilde{w}_d^-)^2} \cdot \frac{1}{\alpha^2 \exp(\alpha \widetilde{w}_d^-)}$$

Then we will prove that this value is greater than one:

$$\begin{aligned}
&\frac{\left(\exp(\alpha \widetilde{w}_d^+) - \exp(\alpha \widetilde{w}_d^-)\right)^2}{\exp(\alpha \widetilde{w}_d^+)(\widetilde{w}_d^+ - \widetilde{w}_d^-)^2 \alpha^2 \exp(\alpha \widetilde{w}_d^-)} - 1 \\
&= \frac{\exp(2\alpha \widetilde{w}_d^+) - \left(2 + \alpha^2\left(\widetilde{w}_d^+ - \widetilde{w}_d^-\right)^2\right)\exp\left(\alpha(\widetilde{w}_d^+ + \widetilde{w}_d^-)\right) + \exp(2\alpha \widetilde{w}_d^-)}{\exp(\alpha \widetilde{w}_d^+)(\widetilde{w}_d^+ - \widetilde{w}_d^-)^2 \alpha^2 \exp(\alpha \widetilde{w}_d^-)}
\end{aligned}$$

Since the denominator is greater than zero, then we only need to prove that the numerator is greater than zero. We regard the numerator as the function of $\widetilde{w}_d^+$ ($\widetilde{w}_d^+ > w > \widetilde{w}_d^-$):

$$f(\widetilde{w}_d^+) = \exp(2\alpha\widetilde{w}_d^+) - \left(2 + \alpha^2 \left(\widetilde{w}_d^+ - \widetilde{w}_d^-\right)^2\right) \exp\left(\alpha(\widetilde{w}_d^+ + \widetilde{w}_d^-)\right) + \exp(2\alpha\widetilde{w}_d^-)$$

It can be prove that $f(\widetilde{w}_d^+)$ is a monotonically increasing function, so its minimum value is when $\widetilde{w}_d^+$ is tend to $\widetilde{w}_d^-$:

$$\lim_{\widetilde{w}_d^+ \to \widetilde{w}_d^-} f(\widetilde{w}_d^+) = 0$$

So we have $f(\widetilde{w}_d^+) > 0$, thus proving that $\frac{\mathbb{S}_{\widetilde{w}_d^+}}{\mathbb{S}'_{\widetilde{w}_d^+}} > 1$.

Then we will prove that when $\alpha > 0$, $\quad \mathbb{S}'_{\widetilde{w}_d^-} < \mathbb{S}_{\widetilde{w}_d^-}$. The ratio of $\mathbb{S}'_{\widetilde{w}_d^-}$ and $\mathbb{S}_{\widetilde{w}_d^-}$ is:

$$\begin{aligned}\frac{\mathbb{S}_{\widetilde{w}_d^-}}{\mathbb{S}'_{\widetilde{w}_d^-}} &= \frac{(\widetilde{w}_d^+ - w)\left(\exp(\alpha\widetilde{w}_d^+) - \exp(\alpha\widetilde{w}_d^-)\right)^2}{\alpha\exp(\alpha\widetilde{w}_d^-)\left(\exp(\alpha\widetilde{w}_d^+) - \exp(\alpha w)\right)(\widetilde{w}_d^+ - \widetilde{w}_d^-)^2} \\ &= \frac{\left(\exp(\alpha\widetilde{w}_d^+) - \exp(\alpha\widetilde{w}_d^-)\right)^2}{\exp(\alpha\widetilde{w}_d^-)(\widetilde{w}_d^+ - \widetilde{w}_d^-)^2} \cdot \frac{(\widetilde{w}_d^+ - w)}{\alpha\left(\exp(\alpha\widetilde{w}_d^+) - \exp(\alpha w)\right)}\end{aligned}$$

Similarly, since $\alpha > 0$, this ratio can be regarded as the monotonically decreasing of $w$ ($w \in [\widetilde{w}_d^-, \widetilde{w}_d^+]$). When $w$ tends to $\widetilde{w}_d^+$, the minimum value of this ratio is:

$$\lim_{w \to \widetilde{w}_d^+} \frac{\mathbb{S}_{\widetilde{w}_d^-}}{\mathbb{S}'_{\widetilde{w}_d^-}} = \frac{\left(\exp(\alpha\widetilde{w}_d^+) - \exp(\alpha\widetilde{w}_d^-)\right)^2}{\exp(\alpha\widetilde{w}_d^-)(\widetilde{w}_d^+ - \widetilde{w}_d^-)^2} \cdot \frac{1}{\alpha^2\exp(\alpha\widetilde{w}_d^+)}$$

It worth noting that this limit value is equal to $\lim_{w \to \widetilde{w}_d^-} \frac{\mathbb{S}_{\widetilde{w}_d^+}}{\mathbb{S}'_{\widetilde{w}_d^+}}$. As we have proved before, $\lim_{w \to \widetilde{w}_d^-} \frac{\mathbb{S}_{\widetilde{w}_d^+}}{\mathbb{S}'_{\widetilde{w}_d^+}} > 1$, we also have $\lim_{w \to \widetilde{w}_d^+} \frac{\mathbb{S}_{\widetilde{w}_d^-}}{\mathbb{S}'_{\widetilde{w}_d^-}} > 1$ equivalently. Then we have $\frac{\mathbb{S}_{\widetilde{w}_d^-}}{\mathbb{S}'_{\widetilde{w}_d^-}} > 1$. □