UNIVERSITY -1911-1-1911-

§ 2. n元函数与n元向量值函数

1.n元函数f

$$f: \Omega \subset \mathbb{R}^n \to \mathbb{R}$$
$$x \mapsto y$$

 $x \in \mathbb{R}^n$:自变量

 $y \in \mathbb{R}$:因变量

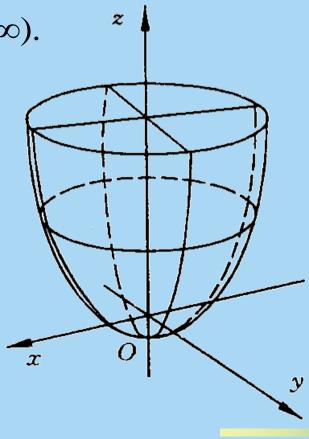
 Ω : f的定义域

 $f(\Omega) \triangleq \{y \in \mathbb{R} | \exists x \in \Omega, s.t.y = f(x) \}$ 称为f的值域

例. 旋转抛物面 $z = x^2 + y^2, (x, y) \in \mathbb{R}^2$.

这是一个2元函数.

定义域: ℝ². 值域:[0,+∞).



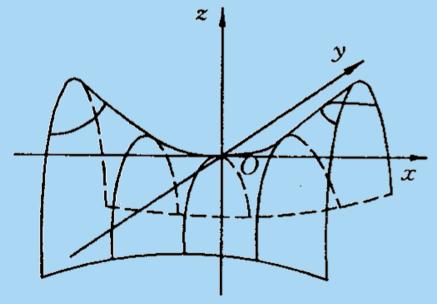
例. 马鞍面 $z = \frac{x^2}{a^2} - \frac{y^2}{b^2}, (x, y) \in \mathbb{R}^2, (a, b > 0$ 为给定参数).

●用
$$x = x_0$$
截曲面,得

$$z = -\frac{y^2}{b^2} + \frac{x_0^2}{a^2},$$

●用 $y = y_0$ 截曲面

$$z = \frac{x^2}{a^2} - \frac{y_0^2}{b^2},$$



•用
$$z = z_0$$
截曲面,得 $\frac{x^2}{a^2} - \frac{y^2}{b^2} = z_0, z_0 > 0$? $z_0 < 0$?

Question. z = xy, $(x, y) \in \mathbb{R}^2$ 也是马鞍面, 为什么?

2. n元函数的隐函数表示法与参数表示法

例. 半球面
$$z = \sqrt{R^2 - x^2 - y^2}$$
, $(R > 0)$

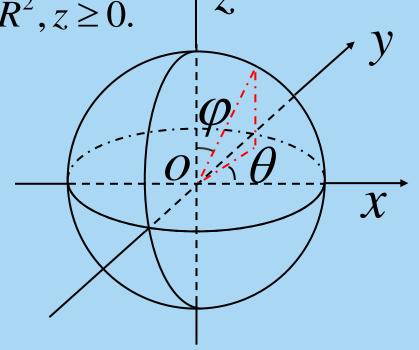
隐函数表示法: $x^2 + y^2 + z^2 = R^2, z \ge 0$.

参数表示法:

$$\begin{cases} x = R \sin \varphi \cos \theta, \\ y = R \sin \varphi \sin \theta, \\ z = R \cos \varphi, \end{cases}$$

$$0 \le \varphi \le \frac{\pi}{2}, 0 \le \theta < 2\pi$$

Question. 椭球面 $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ 的参数方程?



$3.\mathbb{R}^n \to \mathbb{R}^m$ 的向量值函数f

$$f: \Omega \subset \mathbb{R}^n \to \mathbb{R}^m \qquad x = (x_1, x_2, \dots, x_n),$$
$$x \mapsto y \qquad y = (y_1, y_2, \dots, y_m)$$

 Ω : f的定义域

$$f(\Omega) \triangleq \{ y \in \mathbb{R}^m | \exists x \in \Omega, s.t.y = f(x) \}$$
 称为f的值域

f可以看成m个n元函数构成的向量 $f = (f_1, f_2, \dots, f_m)^T$,即

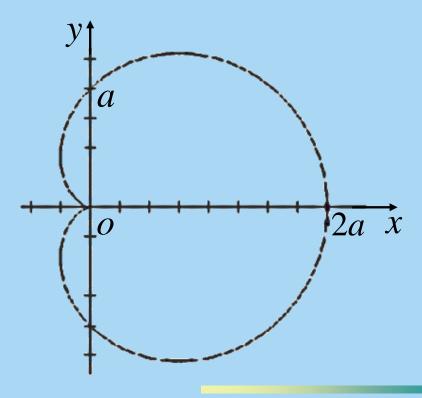
$$\begin{cases} y_{1} = f_{1}(x_{1}, x_{2}, \dots, x_{n}), \\ y_{2} = f_{2}(x_{1}, x_{2}, \dots, x_{n}), \\ \vdots \\ y_{m} = f_{m}(x_{1}, x_{2}, \dots, x_{n}). \end{cases}$$

例. 给定常数a > 0,

$$\begin{cases} x = a(1 + \cos \theta) \cos \theta \\ y = a(1 + \cos \theta) \sin \theta \end{cases}, \quad \theta \in [-\pi, \pi)$$

表示一条平面曲线,

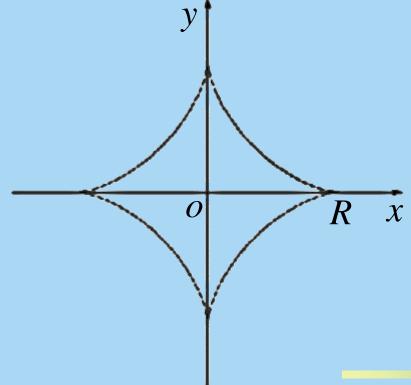
称为心脏线.



例. 给定常数R > 0,

$$\begin{cases} x = R\cos^3 t \\ y = R\sin^3 t \end{cases}, \quad t \in [0, 2\pi)$$

为星型线.



例. 给定常数r, w, v > 0,

$$\begin{cases} x = r \cos wt \\ y = r \sin wt , & t \in \mathbb{R} \\ z = vt \end{cases}$$

为空间螺线.

