# 积分表

维基百科,自由的百科全书

由于列表比较长,**积分表**被分为以下几个部分:

- 有理函数积分表
- 无理函数积分表
- 三角函数积分表
- 指数函数积分表
- 对数函数积分表
- 反三角函数积分表
- 双曲函数积分表
- 反双曲函数积分表

# 含有ax + b的积分

$$\int (ax+b)^n \mathrm{d}x = rac{(ax+b)^{n+1}}{a(n+1)} + C$$
 $\int rac{1}{ax+b} \mathrm{d}x = rac{1}{a} \ln |ax+b| + C$ 

## 目录

- 1 含有*ax* + *b*的积分
- 2 含有 $\sqrt{a+bx}$ 的积分
- 3 含有 $x^2 \pm \alpha^2$ 的积分
- 4 含有  $ax^2 + b$ 的积分
- 5 含有  $ax^2 + bx + c$ (a>0) 的积分
- 6 含有  $\sqrt{a^2 + x^2}$  (a > 0)的积分 7 含有  $\sqrt{x^2 a^2}$   $(x^2 > a^2)$ 的积分
- 8 含有 $\sqrt{a^2-x^2}$   $(a^2>x^2)$ 的积分
- 9 含有 $R = \sqrt{|a|x^2 + bx + c}$   $(a \neq 0)$ 的积分
- 10 含有三角函数的积分
- 11 含有反三角函数的积分
- 12 含有指数函数的积分
- 13 含有对数函数的积分
- 14 含有双曲函数的积分
- 15 定积分

$$\int rac{x}{ax+b} \mathrm{d}x = rac{1}{a^2} (ax+b-b \ln |ax+b|) + C \ \int rac{x^2}{ax+b} \mathrm{d}x = rac{1}{2a^3} \left[ (ax+b)^2 - 4b(ax+b) + 2b^2 \ln |ax+b| 
ight] + C \ \int rac{1}{x(ax+b)} \mathrm{d}x = -rac{1}{b} \ln \left| rac{ax+b}{x} 
ight| + C \ \int rac{1}{x^2(ax+b)} \mathrm{d}x = rac{a}{b^2} \ln \left| rac{ax+b}{x} 
ight| - rac{1}{bx} + C$$

# 含有 $\sqrt{a+bx}$ 的积分

$$\int x\sqrt{a+bx} dx = \frac{2}{15b^2} (3bx - 2a)(a+bx)^{\frac{3}{2}} + C$$

$$\int x^2 \sqrt{a+bx} dx = \frac{2}{105b^3} (15b^2x^2 - 12abx + 8a^2)(a+bx)^{\frac{3}{2}} + C$$

$$\int x^n \sqrt{a+bx} dx = \frac{2}{b(2n+3)} x^n (a+bx)^{\frac{3}{2}} - \frac{2na}{b(2n+3)} \int x^{n-1} \sqrt{a+bx} dx$$

$$\int \frac{\sqrt{a+bx}}{x} dx = 2\sqrt{a+bx} + a \int \frac{1}{x\sqrt{a+bx}} dx$$

$$\int \frac{\sqrt{a+bx}}{x^n} dx = \frac{-1}{a(n-1)} \frac{(a+bx)^{\frac{3}{2}}}{x^{n-1}} - \frac{(2n-5)b}{2a(n-1)} \int \frac{\sqrt{a+bx}}{x^{n-1}} dx, n \neq 1$$

$$\int \frac{1}{x\sqrt{a+bx}} dx = \frac{1}{\sqrt{a}} \ln \left( \frac{\sqrt{a+bx} - \sqrt{a}}{\sqrt{a+bx} + \sqrt{a}} \right) + C, a > 0$$

$$=rac{2}{\sqrt{-a}}rctan\sqrt{rac{a+bx}{-a}}+C, a<0$$
  $\int rac{1}{x^n\sqrt{a+bx}}\mathrm{d}x = rac{-1}{a(n-1)}rac{\sqrt{a+bx}}{x^{n-1}}-rac{(2n-3)b}{2a(n-1)}\int rac{1}{x^{n-1}}\sqrt{a+bx}\mathrm{d}x, n
eq 1$ 

# 含有 $x^2 \pm \alpha^2$ 的积分

$$\int rac{1}{x^2 + lpha^2} \mathrm{d}x = rac{rctan rac{x}{lpha}}{lpha} + C$$
  $\int rac{1}{\pm x^2 \mp lpha^2} \mathrm{d}x = rac{\ln \left(rac{x \mp lpha}{\pm x + lpha}
ight)}{2lpha} + C$ 

# 含有 $ax^2 + b$ 的积分

$$\int rac{1}{ax^2+b} \mathrm{d}x = rac{1}{\sqrt{ab}}rctanrac{\sqrt{a}x}{\sqrt{b}} + C$$

含有 
$$ax^2 + bx + c$$
  $(a > 0)$ 的积分

$$\int ax^2+bx+c\mathrm{d}x=rac{ax^3}{3}+rac{bx^2}{2}+cx+C$$

含有 
$$\sqrt{a^2+x^2}$$
  $(a>0)$ 的积分

$$\begin{split} &\int \sqrt{a^2 + x^2} \, \mathrm{d}x = \frac{1}{2} x \sqrt{a^2 + x^2} + \frac{1}{2} a^2 \ln \left( x + \sqrt{a^2 + x^2} \right) + C \\ &\int x^2 \sqrt{a^2 + x^2} \, \mathrm{d}x = \frac{1}{8} x (a^2 + 2x^2) \sqrt{a^2 + x^2} - \frac{1}{8} a^4 \ln \left( x + \sqrt{a^2 + x^2} \right) + C \\ &\int \frac{\sqrt{a^2 + x^2}}{x} \, \mathrm{d}x = \sqrt{a^2 + x^2} - a \ln \left( \frac{a + \sqrt{a^2 + x^2}}{x} \right) + C \\ &\int \frac{\sqrt{a^2 + x^2}}{x^2} \, \mathrm{d}x = \ln \left( x + \sqrt{a^2 + x^2} \right) - \frac{\sqrt{a^2 + x^2}}{x} + C \\ &\int \frac{1}{\sqrt{a^2 + x^2}} \, \mathrm{d}x = \ln \left( x + \sqrt{a^2 + x^2} \right) + C \\ &\int \frac{x^2}{\sqrt{a^2 + x^2}} \, \mathrm{d}x = \frac{1}{2} x \sqrt{a^2 + x^2} - \frac{1}{2} a^2 \ln \left( \sqrt{a^2 + x^2} + x \right) + C \\ &\int \frac{1}{x \sqrt{a^2 + x^2}} \, \mathrm{d}x = \frac{1}{a} \ln \left( \frac{x}{a + \sqrt{a^2 + x^2}} \right) + C \\ &\int \frac{1}{x \sqrt{a^2 + x^2}} \, \mathrm{d}x = -\frac{\sqrt{a^2 + x^2}}{a^2 x} + C \end{split}$$

含有
$$\sqrt{x^2-a^2}$$
  $(x^2>a^2)$ 的积分 $\int rac{1}{\sqrt{x^2-a^2}} \mathrm{d}x = \lnig(x+\sqrt{x^2-a^2}ig)+C$ 含有 $\sqrt{a^2-x^2}$   $(a^2>x^2)$ 的积分

$$\begin{split} \int \frac{1}{\sqrt{a^2 - x^2}} \mathrm{d}x &= \arcsin \frac{x}{a} + C = -\arccos \frac{x}{a} + C \\ \int \sqrt{a^2 - x^2} \mathrm{d}x &= \frac{1}{2} x \sqrt{a^2 - x^2} + \frac{a^2}{2} \arcsin \frac{x}{a} + C \\ \int x^2 \sqrt{a^2 - x^2} \mathrm{d}x &= \frac{1}{8} x (2x^2 - a^2) \sqrt{a^2 - x^2} + \frac{1}{8} a^4 \arcsin \frac{x}{a} + C \\ \int \frac{\sqrt{a^2 - x^2}}{x} \mathrm{d}x &= \sqrt{a^2 - x^2} - a \ln \left( \frac{a + \sqrt{a^2 - x^2}}{x} \right) + C \\ \int \frac{\sqrt{a^2 - x^2}}{x^2} \mathrm{d}x &= -\frac{\sqrt{a^2 - x^2}}{x} - \arcsin \frac{x}{a} + C \\ \int \frac{1}{x\sqrt{a^2 - x^2}} \mathrm{d}x &= -\frac{1}{a} \ln \left( \frac{a + \sqrt{a^2 - x^2}}{x} \right) + C \\ \int \frac{x^2}{\sqrt{a^2 - x^2}} \mathrm{d}x &= -\frac{1}{2} x \sqrt{a^2 - x^2} + \frac{1}{2} a^2 \arcsin \frac{x}{a} + C \end{split}$$

含有
$$R=\sqrt{|a|x^2+bx+c}$$
  $(a
eq 0)$ 的积分

 $\int rac{1}{x^2 \sqrt{a^2 - x^2}} \mathrm{d}x = -rac{\sqrt{a^2 - x^2}}{a^2 x} + C$ 

$$\int \frac{\mathrm{d}x}{R} = \frac{1}{\sqrt{a}} \ln(2\sqrt{a}R + 2ax + b) \qquad (\text{for } a > 0)$$

$$\int \frac{\mathrm{d}x}{R} = \frac{1}{\sqrt{a}} \operatorname{arsinh} \frac{2ax + b}{\sqrt{4ac - b^2}} \qquad (\text{for } a > 0, 4ac - b^2 > 0)$$

$$\int \frac{\mathrm{d}x}{R} = \frac{1}{\sqrt{a}} \ln|2ax + b| \quad (\text{for } a > 0, 4ac - b^2 = 0)$$

$$\int \frac{\mathrm{d}x}{R} = -\frac{1}{\sqrt{-a}} \operatorname{arcsin} \frac{2ax + b}{\sqrt{b^2 - 4ac}} \qquad (\text{for } a < 0, 4ac - b^2 < 0, (2ax + b) < \sqrt{b^2 - 4ac})$$

$$\int rac{\mathrm{d}x}{R^3} = rac{4ax+2b}{(4ac-b^2)R}$$

$$\int rac{\mathrm{d}x}{R^5} = rac{4ax + 2b}{3(4ac - b^2)R} \left(rac{1}{R^2} + rac{8a}{4ac - b^2}
ight)$$

$$\int rac{\mathrm{d}x}{R^{2n+1}} = rac{2}{(2n-1)(4ac-b^2)} \left[rac{2ax+b}{R^{2n-1}} + 4a(n-1) \int rac{\mathrm{d}x}{R^{2n-1}}
ight]$$

$$\begin{split} &\int \frac{x}{R} \, \mathrm{d}x = \frac{R}{a} - \frac{b}{2a} \int \frac{\mathrm{d}x}{R} \\ &\int \frac{x}{R^3} \, \mathrm{d}x = -\frac{2bx + 4c}{(4ac - b^2)R} \\ &\int \frac{x}{R^{2n+1}} \, \mathrm{d}x = -\frac{1}{(2n-1)aR^{2n-1}} - \frac{b}{2a} \int \frac{\mathrm{d}x}{R^{2n+1}} \\ &\int \frac{\mathrm{d}x}{xR} = -\frac{1}{\sqrt{c}} \ln \left( \frac{2\sqrt{c}R + bx + 2c}{x} \right) \\ &\int \frac{\mathrm{d}x}{xR} = -\frac{1}{\sqrt{c}} \operatorname{arsinh} \left( \frac{bx + 2c}{|x|\sqrt{4ac - b^2}} \right) \end{split}$$

# 含有三角函数的积分

$$\int \cos x dx = \sin x + C$$

$$\int \sin x dx = -\cos x + C$$

$$\int \sec^2 x dx = \tan x + C$$

$$\int \csc^2 x dx = -\cot x + C$$

$$\int \sec x \tan x dx = \sec x + C$$

$$\int \csc x \cot x dx = -\csc x + C$$

$$\int \tan x dx = -\ln|\cos x| + C = \ln|\sec x| + C$$

$$\int \cot x dx = \ln|\sin x| + C$$

$$\int \sec x dx = \ln|\sec x + \tan x| + C$$

$$\int \csc x dx = -\ln|\csc x + \cot x| + C = \ln\left|\frac{\tan x - \sin x}{\sin x \tan x}\right| + C$$

$$\int \sin^n x \mathrm{d}x = -rac{1}{n} \sin^{n-1} x \cos x + rac{n-1}{n} \int \sin^{n-2} x \mathrm{d}x + C \quad orall n \geq 2 \ \int \sin^2 x \mathrm{d}x = rac{x}{2} - rac{\sin 2x}{4} + C$$

$$\int \cos^n x \mathrm{d}x = rac{1}{n} \cos^{n-1} x \sin x + rac{n-1}{n} \int \cos^{n-2} x \mathrm{d}x + C \quad orall n \geq 2 \ \int \cos^2 x \mathrm{d}x = rac{x}{2} + rac{\sin 2x}{4} + C$$

$$\int an^n x \mathrm{d}x = rac{1}{n-1} an^{n-1} x - \int an^{n-2} x \mathrm{d}x + C \quad orall n \geq 2 \ \int an^2 x \mathrm{d}x = an x - x + C$$

$$\int \cot^n x \mathrm{d}x = rac{1}{n-1}\cot^{n-1}x - \int \cot^{n-2}x \mathrm{d}x + C \quad orall n \geq 2 \ \int \cot^2 x \mathrm{d}x = -\cot x - x + C$$

$$\int \sec^n x \mathrm{d}x = rac{1}{n-1} \sec^{n-2} x an x + rac{n-2}{n-1} \int \sec^{n-2} x \mathrm{d}x + C \quad orall n \geq 2$$

$$\int \csc^n x \mathrm{d}x = -rac{1}{n-1}\csc^{n-2}x\cot x + rac{n-2}{n-1}\int \csc^{n-2}x \mathrm{d}x + C \quad orall n \geq 2$$

## 含有反三角函数的积分

$$\int \arcsin x dx = x \arcsin x + \sqrt{1 - x^2} + C$$

$$\int \arccos x dx = x \arccos x - \sqrt{1 - x^2} + C$$

$$\int \arctan x dx = x \arctan x - \ln \sqrt{1 + x^2} + C$$

$$\int \operatorname{arccot} x dx = x \operatorname{arccot} x + \ln \sqrt{1 + x^2} + C$$

$$\int \operatorname{arccot} x dx = x \operatorname{arccot} x + \ln \sqrt{1 + x^2} + C$$

$$\int \operatorname{arcsec} x dx = x \operatorname{arcsec} x - \operatorname{sgn}(x) \ln |x + \sqrt{x^2 - 1}| + C = x \operatorname{arcsec} x + \operatorname{sgn}(x) \ln |x - \sqrt{x^2 - 1}| + C$$

$$\int \operatorname{arccsc} x dx = x \operatorname{arccsc} x + \operatorname{sgn}(x) \ln |x + \sqrt{x^2 - 1}| + C = x \operatorname{arccsc} x - \operatorname{sgn}(x) \ln |x - \sqrt{x^2 - 1}| + C$$

## 含有指数函数的积分

$$\int e^x \mathrm{d}x = e^x + C$$

$$\int \alpha^x \mathrm{d}x = \frac{\alpha^x}{\ln \alpha} + C$$

$$\int xe^{ax} \mathrm{d}x = \frac{1}{a^2}(ax - 1)e^{ax} + C$$

$$\int x^n e^{ax} \mathrm{d}x = \frac{1}{a}x^n e^{ax} - \frac{n}{a}\int x^{n-1}e^{ax} \mathrm{d}x$$

$$\int e^{ax} \sin bx \mathrm{d}x = \frac{e^{ax}}{a^2 + b^2}(a\sin bx - b\cos bx) + C$$

$$\int e^{ax} \cos bx \mathrm{d}x = \frac{e^{ax}}{a^2 + b^2}(a\cos bx + b\sin bx) + C$$

## 含有对数函数的积分

$$egin{split} \int \ln x \mathrm{d}x &= x \ln x - x + C \ \int \log_lpha x \mathrm{d}x &= rac{1}{\lnlpha} \left(x \ln x - x 
ight) + C \ \int x^n \ln x \mathrm{d}x &= rac{x^{n+1}}{(n+1)^2} [(n+1) \ln x - 1] + C \ \int rac{1}{x \ln x} \mathrm{d}x &= \ln \left(\ln x 
ight) + C \end{split}$$

## 含有双曲函数的积分

$$\int \sinh x \mathrm{d}x = \cosh x + C$$

$$\int \cosh x \mathrm{d}x = \sinh x + C$$

$$\int \tanh x \mathrm{d}x = \ln(\cosh x) + C$$

$$\int \coth x \mathrm{d}x = \ln(\sinh x) + C$$

$$\int \operatorname{sech} x \mathrm{d}x = \arcsin(\tanh x) + C = \arctan(\sinh x) + C$$

$$\int \operatorname{csch} x \mathrm{d}x = \ln\left(\tanh \frac{x}{2}\right) + C$$

## 定积分

$$\int_{-\infty}^{\infty} e^{-\alpha x^2} dx = \sqrt{\frac{\pi}{\alpha}}$$

$$\int_{0}^{\frac{\pi}{2}} \sin^n x dx = \int_{0}^{\frac{\pi}{2}} \cos^n x dx = \begin{cases} \frac{n-1}{n} \cdot \frac{n-3}{n-2} \cdot \dots \cdot \frac{4}{5} \cdot \frac{2}{3}, & \text{if } n > 1 \text{ and } n \text{ is odd} \\ \frac{n-1}{n} \cdot \frac{n-3}{n-2} \cdot \dots \cdot \frac{3}{4} \cdot \frac{1}{2} \cdot \frac{\pi}{2}, & \text{if } n > 0 \text{ and } n \text{ is even} \end{cases}$$

取自"https://zh.wikipedia.org/w/index.php?title=积分表&oldid=44076140"

- 本页面最后修订于2017年4月21日 (星期五) 06:48。
- 本站的全部文字在知识共享署名-相同方式共享3.0协议之条款下提供,附加条款亦可能应用。(请参阅使用条款) Wikipedia®和维基百科标志是维基媒体基金会的注册商标;维基™是维基媒体基金会的商标。 维基媒体基金会是在美国佛罗里达州登记的501(c)(3)免税、非营利、慈善机构。