Qishi Quiz 2

Instructions: Please post your solutions by **Monday, Oct. 12th**. You are encouraged to discuss the questions with other Qishi members. Please DO NOT share the problems with people outside Qishi.

1 Math/Stat.

1. What do you know about χ^2 test?
2. Given a 2 by 3 grid (which has 6 blocks and 17 edges), shortest route to visit all edges (assuming edge length is 1).
3. X, Y are iid $N(0,1)$, calculate $P(X|X+Y>0)$, try not use density function of joint distribution.
4. You have a six sided dice, you can keep rolling the dice and you get the dollars equal to the mount of the sum. However, if at some point, the sum is a square number, you must stop and will get zero dollars. (1) If at some point, your sum is 35, should you stop or keep rolling? (2) in (1), if you choose to continue and this is your strategy: you will keep rolling until you exceed 43, what is the most probable amount of dollar you win when you stop? (3) Is there a best strategy for this game, any number that you should stop ?
5. Given a stick, if randomly cut into 3 pieces, what's the average size of the smallest, of the middle-sized, and of the largest pieces?
6. At a party, N people throw their hats (all hats are different) into the center of room. The hats are mixed up and each people randomly selects one. Let Y be the number of people who select their own hats. Now ask (1) what is the expectation of Y? (2) what is the variance of Y? Now, the picking hats game rule is extended. For each hats pick round, the people choosing their own hats quit the game, while others (those picked wrong hats) put their selected hats back in the center of room, mix them up, and then reselect. Also, suppose that this game continues until each individual has his own hat. Suppose N individuals initially join the game, let $R(N)$ be the number of rounds that are run and $S(N)$ be the total number of selections made by the these N individuals, $(N>1)$. (3) Find the expectation of $R(N)$. (4) Find the expectation of $S(N)$. (5) Find the expected number of false selections made by one of the N people.
7. Consider linear regression of Y on features X_1, X_2: Model1-(Y, X_1), $R^2 = 0.1$; Model2-(Y, X_2), $R^2 = 0.2$; Model3-(Y, X_1, X_2), calculate the range of R^2 of Model3.
8. Given a function for a fair coin, write a function for a biased coin that returns heads with probability $\frac{1}{n}$ (n is a param).
9. 10 islands with 9 bridges. The bridges are either strong or weak (half half). Weak bridge falls if stepped on and the man is drifted to the 1st island, then all the bridges are miraculously fixed. To arrive the 10th island, how many bridges on average the man has to cross?

2 Programming.

10. Explain the following code: const int* const fun(const int* const& p) const;

11. How do you implement "delete" operation in a single-linked list?

12. Implement the interface for matrix class in C++.

13. Can the constructor of a class be virtual? How to realize a similar function as a virtual constructor?

14. Is it okay for a non-virtual function of the base class to call a virtual function?

15. Given a string, return the longest palindrome subsequence.

16. How to inverse a string of sentence (without reverse the word) ?

17. Say you have an array for which the i-th element is the price of a given stock on day i.

    Design an algorithm to find the maximum profit. You may complete at most two transactions.

    Note: You may not engage in multiple transactions at the same time (i.e., you must sell the stock before you buy again).

18. The book problem: There is a group of N ($2 \leq N \leq 1000$) people which are numbered 1 through N, and everyone of them has not less than $[\frac{N+1}{2}]$ friends. A man with number 1 has the book, which others want to read. Write the program which finds a way of transferring the book so that it will visit every man only once, passing from the friend to the friend, and, at last, has come back to the owner. Note: if A is a friend of B then B is a friend of A. INPUT: First line of input contains number N. Next N lines contain information about friendships. $(i+1)$-th line of input contains a list of friends of i-th man. OUTPUT: If there is no solution then your program must output 'No solution'. Else your program must output exactly $N+1$ number: this sequence should begin and should come to end by number 1, any two neighbors in it should be friends, and any two elements in it, except for the first and last, should not repeat.