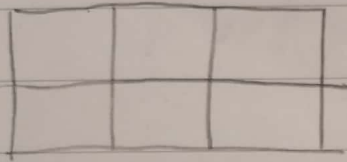


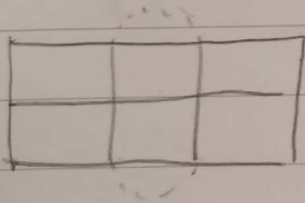
2. It is a non-Eulerian graph. We need to use minimum cost to turn it to a Eulerian graph.

A graph is Eulerian if two vertices have odd degree and all vertices in a connected graph.

there are 6 vertices with odd degree.



We can add 2 edges to eliminate 4 vertices with odd degree, thus the new graph is a Eulerian graph.



Thus, the shortest route is $17+2=19$ long.

1. Chi-square test is used to test the hypothesis that the difference between two categorical datasets is generated by randomness. It can test independence & goodness of fit. Yates' χ^2 test is used to test independence in a contingency table.

$$3. \quad P(X | X+Y > 0)$$

$$= \frac{P(X, X+Y > 0)}{P(X+Y > 0)}$$

$$= \frac{P(X+Y > 0 | X) P(X)}{P(X+Y > 0)}$$

$$= \frac{P(Y > -X | X) P(X)}{P(X+Y > 0)}$$

$$= \frac{(1 - P(X)) P(X)}{\frac{1}{2}}$$

$$= -2P^2(X) + 2P(X)$$

X, Y are i.i.d $N(0,1)$

so $X+Y$ is gaussian + 0

done

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4.1.

$$E(\text{loss}) = \frac{1}{6} \cdot 35 = \frac{35}{6}$$

$$E(\text{gain for next rolls}) = \frac{1}{6} (2+3+4+5+6)$$

$$+ \frac{1}{6} (1+2+3+4+5+6)$$

$$= \frac{41}{6}$$

$E(\text{gain}) > E(\text{loss})$, should continue

2, $\frac{41}{6}$ is most probable amount.

$43 - 35 = 8$ + next die rolls as 1, then stop.

Thus possible path are:

1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14

* 1, 1, 1, 1, 0, 0

2 rolls 0 1 2 3 4 5 4 3 2 1 0 0 0

3 rolls 1 3 6 10 15 19 21 21 19 15 10

4 rolls 1 4 10 20 35 54 74 92 105 110

5 rolls 1 5 15 35 70 124 199 285 380

6 rolls 1 6 21 56 126 250 446 726

7 1 7 28 84 210 460 905

8 1 8 36 122 332 792

9 1 44 166 498 1290

5

$$E \cdot L_{small} = \int_0^{\frac{1}{3}} \int_x^{1-2x} \frac{1}{1-x} dy \cdot dx$$

$$= \int_0^{\frac{1}{3}} \frac{1-3x}{1-x} \cdot x dx$$

$$= \int_0^{\frac{1}{3}} \left(3x - \frac{2x}{1-x} \right) dx$$

$$= \frac{3x^2}{2} \Big|_0^{\frac{1}{3}} + 2 \Big|_0^{\frac{1}{3}} - \int_0^{\frac{1}{3}} \frac{2}{1-x} dx$$

$$= \frac{1}{6} + \frac{2}{3} + 2 \log \frac{2}{3}$$

x
1-2x

1-x

$$E L_{big} = \int_{\frac{1}{3}}^{\frac{1}{2}} \left(\frac{1}{1-x} \int_{1-2x}^x dy \right) x dx$$

$$= \int_{\frac{1}{3}}^{\frac{1}{2}} \left(\frac{3x-1}{1-x} \right) x dx$$

$$= \int_{\frac{1}{3}}^{\frac{1}{2}} \left(-3x + \frac{2x}{1-x} \right) dx$$

$$= -\frac{3x^2}{2} \Big|_{\frac{1}{3}}^{\frac{1}{2}} - 2x \Big|_{\frac{1}{3}}^{\frac{1}{2}} - 2 \log(1-x) \Big|_{\frac{1}{3}}^{\frac{1}{2}}$$

$$= \frac{1}{6} - \frac{3}{8} - \frac{1}{3} - 2 \left(\log \frac{1}{2} - \log \frac{2}{3} \right)$$

$$E L_{middle} = 1 - E L_{small} - E L_{big}$$

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6 total # of combinations is $N!$

(1)(2) # of k people have wrong hats:

$$\binom{k}{N} !k \text{ where } !k = k! \sum_{i=0}^k \frac{(-1)^i}{i!}$$

$$E(k) = \sum_{k=0}^N \frac{1}{N!} \binom{k}{N} !k \cdot k$$

$$= \sum_{k=0}^N \frac{1}{(N-k)!} \left(\sum_{i=0}^k \frac{(-1)^i}{i!} \right) \cdot k$$

$$E(k^2) = \sum_{k=0}^N \frac{1}{N!} \binom{k}{N} !k \cdot k^2$$

$$= \sum_{k=0}^N \frac{1}{(N-k)!} \left(\sum_{i=0}^k \frac{(-1)^i}{i!} \right) k^2$$

$$\text{Var}(k) = E(k^2) - (E(k))^2$$

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$$\sum (\hat{y}_i - \bar{y}_i)^2 = 0.1 \text{ SSTO} \quad , \quad \sum (\hat{y}_i^2 - \bar{y}_i)^2 = 0.2 \text{ SSTO}$$

$$7. \quad \sum (y_i - \hat{y}_i)^2 = 0.9 \text{ SSTO} = \sum (y_i - \beta_1 x_{1i})^2 = 0.9 \text{ SSTO}$$

$$\sum (y_i - \hat{y}_i)^2 = 0.8 \text{ SSTO} = \sum (y_i - \beta_2 x_{2i})^2 = 0.8 \text{ SSTO}$$

$$\sum (\bar{y}_i - \alpha_1 x_{1i} - \alpha_2 x_{2i})^2$$

$$= \sum (\bar{y}_i - \alpha_1 x_{1i})^2 - \sum 2(\bar{y}_i - \alpha_1 x_{1i}) \alpha_2 x_{2i} + \sum (\alpha_2 x_{2i})^2$$

$$= 0.1 \text{ SSTO} + \sum (-2\bar{y}_i \alpha_2 x_{2i} + (\alpha_2 x_{2i})^2) + \sum 2\alpha_1 \alpha_2 x_{1i} x_{2i}$$

$$= 0.1 \text{ SSTO} + \sum (\alpha_2 x_{2i} - \bar{y}_i)^2 - \sum (\bar{y}_i)^2 + \sum 2\alpha_1 \alpha_2 x_{1i} x_{2i}$$

$$\leq 0.1 \text{ SSTO} + 0.2 \text{ SSTO} + \sum 2\alpha_1 \alpha_2 x_{1i} x_{2i} - \sum (\bar{y}_i)^2$$

$$\leq 0.3 \text{ SSTO}$$

\therefore r -squared in $[0.2, 0.3]$

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8. assume $2^k < n < 2^{k+1}$

Then any path with beginning of T is considered as failure

a path consisting of all $(k+1)$ H is assumed as accept.

Choose $(n - 1 - 2^k)$ randomly from all other paths starting with H as failures.

The rest are regarded as retry.

$$9. E(L) = \frac{1}{2}(10) + \frac{1}{2} \cdot \frac{1}{2}(1+10) + \frac{1}{2^3}(2+10) + \dots + \frac{1}{2^{10}}(9+10) + \frac{1}{2^{10}} \cdot 10.$$

$$= 10 + \underbrace{\frac{1}{4} + \frac{2}{8} + \dots + \frac{9}{2^{10}}}_A$$

$$= 10 + 1 - \frac{11}{2^{10}} = 11 - \frac{11}{2^{10}}$$

$$\begin{aligned} \text{Prepared by } A &= \frac{1}{4} + \frac{1}{8} + \frac{1}{2^{10}} - \frac{1}{2^{11}} \\ \text{Date } A &= \frac{1}{4} \left(1 - \left(\frac{1}{2} \right)^9 \right) - \frac{9}{2^{11}} \end{aligned}$$

```

11. class MyNode:
    def __init__(self, val):
        self.value = val
        self.next = None

```

To delete the node next to A, defined as temp
temp = A.next ; A.next = temp.next.

13 :

1. No, to create an object, complete information is needed.

2. similar function as virtual constructor

Base * p = new B()

Base * p1 = new p->A()

where B() is a derived class of A()

14 Yes, it is.

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15. will implement it using recursive method:

```
def myfun (mystr, i, j):
```

```
    if i == j:
```

```
        return 1
```

```
    if j == i+1 & mystr[i] == mystr[j]:
```

```
        return 2
```

```
    if mystr[i] == mystr[j]:
```

```
        return myfun [mystr, i+1, j-1] + 2
```

```
    if mystr[i] != mystr[j]:
```

```
        return max (myfun (seq, i, j-1), myfun (seq, i+1, j))
```

16. First, split the string by ' '. it generates a list.

Second, reverse the list.

Third: ' '.join (List) to form a new string.

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17. pseudo-code:

1. find the local minima & day 0 is local minimum if $P_0 < P_1$, $[min_1, min_2, \dots]$
2. find the local maxima & day N 's local maxima if $P_N > P_{N-1}$, $[max_1, max_2, \dots]$
3. choose two largest number from $[max_1 - min_1, \dots, max_k - min_k]$

18. design a recursive solution:

```
def myfun (N, List, index)
    if 1 not in index & len(index) > 0: return -1
    for i in List[0]:
```

or condition
1 is not
connected in the
residual

$N = N - 1$

$List = List.remove(List[List[0][i]])$

$index = index.remove(i)$

$result.append(i)$

$val = myfun(N, List, index)$

if $val == -1$:

$result.remove(i)$; return -1

else:

return i

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