CS109A Introduction to Data Science:

Homework 8: Ensembles: Bagging, Random Forests, and Boosting

Harvard University
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In [1]:
```
#RUN THIS CELL
import requests
from IPython.core.display import HTML
styles = requests.get("https://raw.githubusercontent.com/Harvard-IACS/2018-CS
HTML(styles)
```
Out[1]:

INSTRUCTIONS

- To submit your assignment follow the instructions given in Canvas (https://canvas.harvard.edu/courses/42693/pages/homework-policies-and-submission-instructions).
- If needed, clarifications will be posted on Piazza.
- This homework can be submitted in pairs.
- If you submit individually but you have worked with someone, please include the name of your **one** partner below.

Name of the person you have worked with goes here: Xi Han, Haoran Zhao

Learning Objectives

Completing this assignment will demonstrate success at the following objectives:

- Statistical
  - Predict when bagging will help model performance.
  - Identify how Random Forests improve over bagging.
  - Predict when boosting will help model performance.
  - Compare and contrast bagging and boosting.

- Coding
  - Identify and fix problems in poorly written code
- Communication
  - Visually explain a complex concept

In [2]:
```python
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt

from sklearn.model_selection import cross_val_score
from sklearn.utils import resample
from sklearn.tree import DecisionTreeClassifier
from sklearn.ensemble import RandomForestClassifier
from sklearn.ensemble import AdaBoostClassifier
from sklearn.metrics import accuracy_score

%matplotlib inline

import seaborn as sns
sns.set(style='whitegrid')
pd.set_option('display.width', 1500)
pd.set_option('display.max_columns', 100)

from IPython.display import display
```

Overview: Higgs Boson Discovery

The discovery of the Higgs boson in July 2012 marked a fundamental breakthrough in particle physics. The Higgs boson particle was discovered through experiments at the Large Hadron Collider at CERN, by colliding beams of protons at high energy. A key challenge in analyzing the results of these experiments is to differentiate between collisions that produce Higgs bosons and collisions that produce only background noise. We shall explore the use of ensemble methods for this classification task.

You are provided with data from Monte-Carlo simulations of collisions of particles in a particle collider experiment. The training set is available in `Higgs_train.csv` and the test set is in `Higgs_test.csv`. Each row in these files corresponds to a particle collision described by 28 features (columns 1-28), of which the first 21 features are kinematic properties measured by the particle detectors in the accelerator, and the remaining features are derived by physicists from the first 21 features. The class label is provided in the last column, with a label of 1 indicating that the collision produces Higgs bosons (signal), and a label of 0 indicating that the collision produces other particles (background).

The data set provided to you is a small subset of the HIGGS data set in the UCI machine learning repository. The following paper contains further details about the data set and the predictors used: Baldi et al., Nature Communications 5, 2014 (https://www.nature.com/articles/ncomms5308).

In [3]:
```python
# Load data
data_train = pd.read_csv('Higgs_train.csv')
data_test = pd.read_csv('Higgs_test.csv')

print(f"{len(data_train)} training samples, {len(data_test)} test samples")
print("\nColumns:")
print(', '.join(data_train.columns))
```

```
5000 training samples, 5000 test samples

Columns:
lepton pT, lepton eta, lepton phi, missing energy magnitude, missing energy ph
i, jet 1 pt, jet 1 eta, jet 1 phi, jet 1 b-tag, jet 2 pt, jet 2 eta, jet 2 phi,
jet 2 b-tag, jet 3 pt, jet 3 eta, jet 3 phi, jet 3 b-tag, jet 4 pt, jet 4 eta,
jet 4 phi, jet 4 b-tag, m_jj, m_jjj, m_lv, m_jlv, m_bb, m_wbb, m_wwbb, class
```

In [4]:
```python
display(data_train.head())
display(data_train.describe())
```

	lepton pT	lepton eta	lepton phi	missing energy magnitude	missing energy phi	jet 1 pt	jet 1 eta	jet 1 phi	jet 1 b-tag	jet 2 pt	jet 2 eta	jet 2 phi
0	0.377	-1.5800	-1.7100	0.991	0.114	1.250	0.620	-1.480	2.17	0.754	0.7750	-0.667
1	0.707	0.0876	-0.4000	0.919	-1.230	1.170	-0.553	0.886	2.17	1.300	0.7620	-1.060
2	0.617	0.2660	-1.3500	1.150	1.040	0.955	0.377	-0.148	0.00	1.060	-0.0194	1.110
3	0.851	-0.3810	-0.0713	1.470	-0.795	0.692	0.883	0.497	0.00	1.620	0.1240	1.180
4	0.768	-0.6920	-0.0402	0.615	0.144	0.749	0.397	-0.874	0.00	1.150	0.1270	1.320

	lepton pT	lepton eta	lepton phi	missing energy magnitude	missing energy phi	jet 1 pt	jet 1 eta
count	5000.000000	5000.000000	5000.000000	5000.000000	5000.000000	5000.000000	5000.000000
mean	0.978645	-0.014280	-0.018956	1.005793	0.002528	0.980390	0.025014
std	0.547025	1.011927	0.997945	0.591907	1.003337	0.463677	1.002018
min	0.275000	-2.410000	-1.740000	0.010000	-1.740000	0.170000	-2.920000
25%	0.587000	-0.764250	-0.877500	0.581000	-0.870000	0.676000	-0.659250
50%	0.846000	-0.009305	-0.016050	0.903500	0.001300	0.891000	0.049500
75%	1.220000	0.725500	0.837000	1.300000	0.866000	1.160000	0.716000
max	5.330000	2.430000	1.740000	6.260000	1.740000	4.190000	2.960000

In [5]:
```
# Split into NumPy arrays
X_train = data_train.iloc[:, data_train.columns != 'class'].values
y_train = data_train['class'].values
X_test = data_test.iloc[:, data_test.columns != 'class'].values
y_test = data_test['class'].values
```

Question 1: A Single Model [20 pts]

We start by fitting a basic model we can compare the other models to. We'll pick a decision tree as the base model, because we'll later include random forests and want a fair comparison. We'll tune the decision tree using cross-validation. As usual, we'll be tuning the maximum tree depth; we refer to this parameter as "depth" for simplicity.

Since we will only be using tree-based methods in this homework, we do not need to standardize or normalize the predictors.

1.1: Fit a decision tree model to the training set. Choose a range of tree depths to evaluate. Plot the estimated performance +/- 2 standard deviations for each depth using 5-fold cross validation. Also include the training set performance in your plot, but set the y-axis to focus on the cross-validation performance.

Hint: use `plt.fill_between` to shade the region.

1.2 Select an appropriate depth and justify your choice. Using your cross-validation estimates, report the mean +/- 2 stdev. Then report the classification accuracy on the test set. (Store the training and test accuracies in variables to refer to in a later question.)

1.3 What is the mechanism by which limiting the depth of the tree avoids over-fitting? What is one downside of limiting the tree depth? Your answer should refer to the bias-variance trade-off.

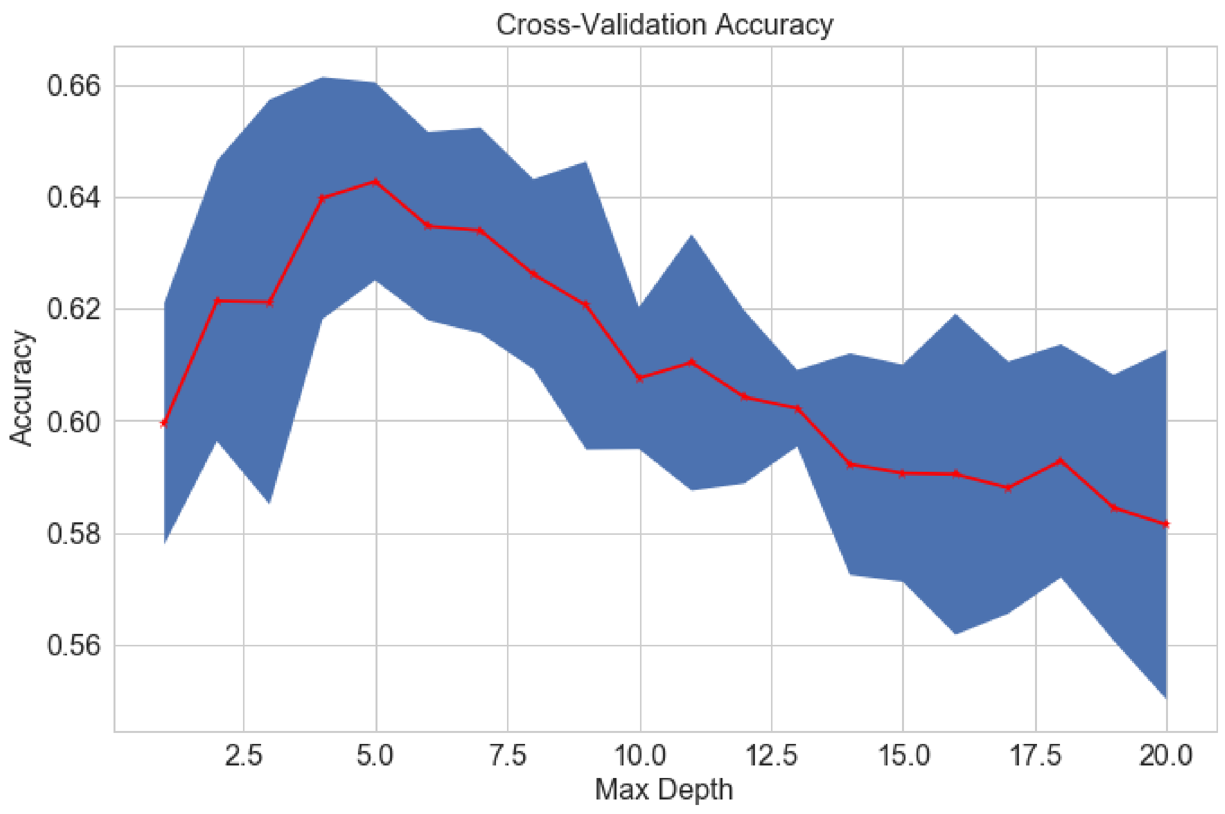
Answers

1.1: Fit a decision tree model to the training set. Choose a range of tree depths to evaluate. Plot the estimated performance +/- 2 standard deviations for each depth using 5-fold cross validation. Also include the training set performance in your plot, but set the y-axis to focus on the cross-validation performance.

Hint: use `plt.fill_between` to shade the region.

In [6]:
```
# your code here
result = []
for max_depth in range(1, 21):
    dt = DecisionTreeClassifier(max_depth=max_depth)
    cv = cross_val_score(estimator=dt, X=X_train, y=y_train, cv=5, n_jobs=-1)
    result.append([max_depth, cv.mean(), cv.std()])
```

In [7]:
```python
# your code here
result_array = np.array(result)
fig, ax = plt.subplots(1, 1, figsize=(11, 7))
ax.plot(result_array[:,0], result_array[:,1], 'r*-')
ax.fill_between(result_array[:,0], result_array[:,1] - 2*result_array[:,2], r
ax.tick_params(labelsize=16)
ax.set_title('Cross-Validation Accuracy', fontsize=16)
ax.set_xlabel("Max Depth", fontsize=16)
ax.set_ylabel("Accuracy", fontsize=16)
plt.show()
```



1.2 Select an appropriate depth and justify your choice. Using your cross-validation estimates, report the mean +/- 2 stdev. Then report the classification accuracy on the test set. (Store the training and test accuracies in variables to refer to in a later question.)

In [8]:
```python
# your code here
index = result_array.argmax(axis=0)[1]
depth_best = result_array[index, 0]
print("Best depth = {}".format(depth_best))
print("Mean Accuracy = {:.2%}".format(result_array[index, 1]))
print("Mean Accuracy - 2*Std = {:.2%}".format(result_array[index,1] - 2* resu
print("Mean Accuracy + 2*Std = {:.2%}".format(result_array[index,1] + 2* resu
```

```
Best depth = 5.0
Mean Accuracy = 64.28%
Mean Accuracy - 2*Std = 62.51%
Mean Accuracy + 2*Std = 66.05%
```

In [9]:
```python
# your code here
dt_single = DecisionTreeClassifier(max_depth=depth_best)
dt_single.fit(X_train, y_train)
dt_single_train_score = dt_single.score(X_train, y_train)
dt_single_test_score = dt_single.score(X_test, y_test)

print("Single Tree on training set = {:.2%}".format(dt_single_train_score))
print("Single Tree on test set = {:.2%}".format(dt_single_test_score))
```

```
Single Tree on training set = 68.12%
Single Tree on test set = 64.78%
```

Your answer here

We decided to use 5 as the max depth of the tree because it gives the highest cross validation accuracy with reasonable two standard deviation intervals.

1.3 What is the mechanism by which limiting the depth of the tree avoids over-fitting? What is one downside of limiting the tree depth? Your answer should refer to the bias-variance trade-off.

your answer here

- Q: What is the mechanism by which limiting the depth of the tree avoids over-fitting?
- A: If we don't limit the depth of the tree, the decision tree will become a full tree that each leaf node is corresponding to one observation. The single tree will have 100% classification accuracy in the training set, which means it has very low bias. However, the single tree is over-fitted to the training set, such that it's very sensitive to small changes in the dataset, in another word, with very high variance. Limitting the depth of the tree is helpful to avoid over-fitting, in terms of preventing the tree to become too complex and too specific to the training set. Once the depth of the tree is reasonablely deep enough, we will stop it. We can always use cross validation like in 1.1 and 1.2 to find the 'best' value to prevent overfitting and check with the testing accuracy as well.

- Q: What is one downside of limiting the tree depth?
- A: The one downside of limiting the tree depth is underfitting the model with high bias, although it is having lower variance compared to the tree with more depths. A decision tree withnot sufficient depth is not able to capture the complex relationships within the data.

Question 2: Bagging [25 pts]

Bagging is the technique of building the same model on multiple bootstraps from the data and combining each model's prediction to get an overall classification. In this question we build an example by hand and study how the number of bootstrapped datasets impacts the accuracy of the resulting classification.

2.1 Choose a tree depth that will overfit the training set. What evidence leads you to believe that this depth will overfit? Assign your choice to a variable here. (You may want to explore different settings for this value in the problems below.)

2.2 Create 45 bootstrapped replications of the original training data, and fit a decision tree to each. Use the tree depth you just chose in 2.1. Record each tree's prediction. In particular, produce a dataset like those below, where each row is a training (or test) example, each column is one of the trees, and each entry is that tree's prediction for that example. (Labeling the rows and columns is optional.)

Store these results as `bagging_train` and `bagging_test` . Don't worry about visualizing these results yet.

2.3 *Aggregate* all 45 *bootstrapped* models to get a combined prediction for each training and test point: predict a 1 if and only if a majority of the models predict that example to be from class 1. What accuracy does this *bagging* model achieve on the test set? Write an assertion that verifies that this test-set accuracy is at least as good as the accuracy for the model you fit in Question 1.

2.4 We want to know how the number of bootstraps affects our bagging ensemble's performance. Use the `running_predictions` function (given below) to get the model's accuracy score when using only 1,2,3,4,... of the bootstrapped models. Make a plot of training and test set accuracies as a function of number of bootstraps.

On your plot, also include horizontal lines for two baselines:

- the test accuracy of the best model from question 1
- the test accuracy of a single tree with the tree depth you chose in 2.1, trained on the full training set.

2.5 Referring to your graph from 2.4, compare the performance of bagging against the baseline of a single depth-10 tree. Explain the differences you see.

2.6 Bagging and limiting tree depth both affect how much the model overfits. Compare and contrast these two approaches. Your answer should refer to your graph in 2.4 and may duplicate something you said in your answer to 1.5.

2.7: In what ways might our bagging classifier be overfitting the data? In what ways might it be underfitting?

Hints

- Use `resample` from sklearn to easily bootstrap the x and y data.
- use `np.mean` to easily test for majority. If a majority of models vote 1, what does that imply about the mean?

Answers:

2.1 Choose a tree depth that will overfit the training set. What evidence leads you to believe that this depth will overfit? Assign your choice to a variable here. (You may want to explore different settings for this value in the problems below.)

In [10]:
```
# your code here
depth_ovf = 20
dt_single_ovf = DecisionTreeClassifier(max_depth=depth_ovf)
dt_single_ovf.fit(X_train, y_train)

dt_single_ovf_train_score = dt_single_ovf.score(X_train, y_train)
dt_single_ovf_test_score = dt_single_ovf.score(X_test, y_test)

print("Overfitted Depth = {}".format(depth_ovf))
print("Overfitted Single Tree on training set = {:.2%} ".format(dt_single_ovf
print("Overfitted Single Tree on test set = {:.2%} ".format(dt_single_ovf_tes
```

```
Overfitted Depth = 20
Overfitted Single Tree on training set = 100.00%
Overfitted Single Tree on test set = 59.78%
```

Your answer here

We believe depth = 20 will overfit the training set, because the training accuracy is 100%, while the test accuracy is only 60%. Another evidence is that cross-validation accuracy peaks at depth of 5, and then decreases after it, which is a sign of overfitting. Depth of 20 will definitely overfit the model.

2.2 Create 45 bootstrapped replications of the original training data, and fit a decision tree to each. Use the tree depth you just chose in 2.1. Record each tree's prediction. In particular, produce a dataset like those below, where each row is a training (or test) example, each column is one of the trees, and each entry is that tree's prediction for that example. (Labeling the rows and columns is optional.)

Store these results as `bagging_train` and `bagging_test` . Don't worry about visualizing these results yet.

Structure of `bagging_train` and `bagging_test` :

`bagging_train` :

	bootstrap model 1's prediction	bootstrap model 2's prediction	...	bootstrap model 45's prediction
training row 1	binary value	binary value	...	binary value
training row 2	binary value	binary value	...	binary value
...	...	...	...	...

`bagging_test` :

	bootstrap model 1's prediction	bootstrap model 2's prediction	...	bootstrap model 45's prediction
test row 1	binary value	binary value	...	binary value
test row 2	binary value	binary value	...	binary value

	bootstrap model 1's prediction	bootstrap model 2's prediction	...	bootstrap model 45's prediction
...	...	...	...	...

In [11]:

```
# Creating model
np.random.seed(0)
dt_bagging = DecisionTreeClassifier(max_depth=depth_ovf)

# Initializing variables
n_trees = 45
bagging_train = np.zeros((data_train.shape[0], n_trees))
bagging_test = np.zeros((data_test.shape[0], n_trees))
bagging_fn = []

# Conduct bootstraping iterations
for i in range(n_trees):
    X_train_resample, y_train_resample = resample(X_train, y_train, replace=T

    dt_bagging.fit(X_train_resample, y_train_resample)
    bagging_train[:,i] = dt_bagging.predict(X_train)
    bagging_test[:,i] = dt_bagging.predict(X_test)

    # First node
    bagging_fn.append(dt_bagging.tree_.feature[0])

# Make Predictions Dataframe
columns = ["Bagging-Model_"+str(i+1) for i in range(n_trees)]
bagging_train = pd.DataFrame(bagging_train, columns=columns)
bagging_test = pd.DataFrame(bagging_test, columns=columns)
```

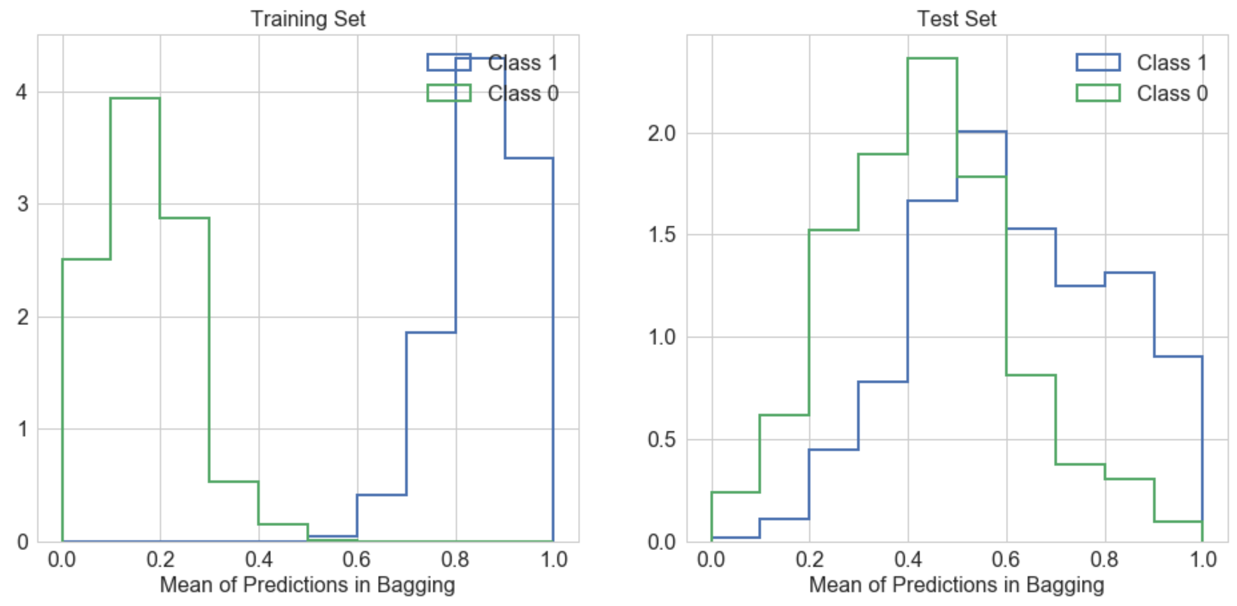
In [12]:
```
print(bagging_train.shape)
print(bagging_test.shape)
display(bagging_train.head())
display(bagging_test.head())
```

```
(5000, 45)
(5000, 45)
```

	Bagging-Model_1	Bagging-Model_2	Bagging-Model_3	Bagging-Model_4	Bagging-Model_5	Bagging-Model_6	Bagging-Model_7	Bagging-Model_8	Bagging-Model_9	Ba Mo
0	1.0	1.0	1.0	1.0	0.0	0.0	1.0	0.0	1.0	
1	0.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	0.0	
2	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	
3	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	0.0	
4	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	1.0	

	Bagging-Model_1	Bagging-Model_2	Bagging-Model_3	Bagging-Model_4	Bagging-Model_5	Bagging-Model_6	Bagging-Model_7	Bagging-Model_8	Bagging-Model_9	Ba Mo
0	1.0	0.0	1.0	1.0	1.0	0.0	1.0	0.0	0.0	
1	0.0	1.0	1.0	0.0	0.0	1.0	1.0	1.0	0.0	
2	1.0	0.0	1.0	1.0	1.0	0.0	1.0	0.0	1.0	
3	1.0	0.0	1.0	0.0	1.0	1.0	0.0	1.0	1.0	
4	0.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0	1.0	

In [13]:
```python
num_to_avg = n_trees
fig, axs = plt.subplots(1, 2, figsize=(16, 7))
for (ax, label, predictions, y) in [
    (axs[0], 'Training Set', bagging_train, y_train),
    (axs[1], 'Test Set', bagging_test, y_test)
]:
    mean_predictions = predictions.iloc[:,:num_to_avg].mean(axis=1)
    mean_predictions[y == 1].hist(density=True, histtype='step', range=[0,1],
    mean_predictions[y == 0].hist(density=True, histtype='step', range=[0,1],
    ax.legend(loc='best', fontsize=16)
    ax.set_xlabel("Mean of Predictions in Bagging", fontsize=16)
    ax.set_title(label, fontsize=16)
    ax.tick_params(labelsize=16)
```



2.3 *Aggregate* all 45 *bootstrapped* models to get a combined prediction for each training and test point: predict a 1 if and only if a majority of the models predict that example to be from class 1. What accuracy does this *bagging* model achieve on the test set? Write an assertion that verifies that this test-set accuracy is at least as good as the accuracy for the model you fit in Question 1.

In [14]:
```python
# Function to ensemble the prediction of each bagged decision tree model
def get_prediction(df, count=-1):
    count = df.shape[1] if count==-1 else count
    temp = df.iloc[:,0:count]
    return np.mean(temp, axis=1)>0.5

bagging_train_score = accuracy_score(y_train, get_prediction(bagging_train, c
bagging_test_score = accuracy_score(y_test, get_prediction(bagging_test, coun

print("Bagging on training set = {:.2%}".format(bagging_train_score))
print("Bagging on test set = {:.2%}".format(bagging_test_score))

print("Single Tree on training set = {:.2%}".format(dt_single_train_score))
print("Single Tree on test set = {:.2%}".format(dt_single_test_score))

assert bagging_test_score >= dt_single_train_score, "Test set accuracy for ba
```

```
Bagging on training set = 99.96%
Bagging on test set = 68.22%
Single Tree on training set = 68.12%
Single Tree on test set = 64.78%
```

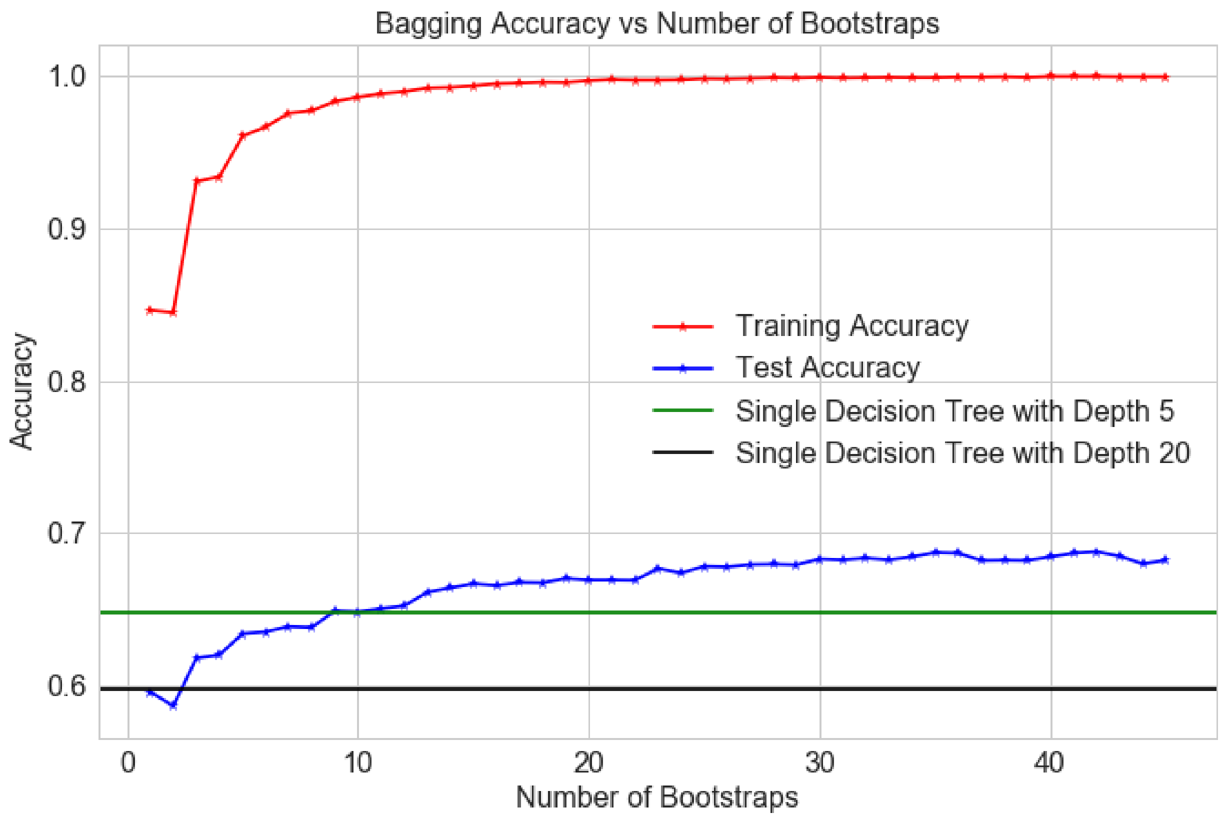
2.4 We want to know how the number of bootstraps affects our bagging ensemble's performance. Use the `running_predictions` function (given below) to get the model's accuracy score when using only 1,2,3,4,... of the bootstrapped models. Make a plot of training and test set accuracies as a function of number of bootstraps.

On your plot, also include horizontal lines for two baselines:

- the test accuracy of the best model from question 1
- the test accuracy of a single tree with the tree depth you chose in 2.1, trained on the full training set.

In [15]:
```
def running_predictions(prediction_dataset, targets):
    """A function to predict examples' class via the majority among trees (ti

    Inputs:
      prediction_dataset - a (n_examples by n_sub_models) dataset, where each
          for example i
      targets - the true class labels

    Returns:
      a vector where vec[i] is the model's accuracy when using just the first
    """

    n_trees = prediction_dataset.shape[1]

    # find the running percentage of models voting 1 as more models are consi
    running_percent_1s = np.cumsum(prediction_dataset, axis=1)/np.arange(1,n_

    # predict 1 when the running average is above 0.5
    running_conclusions = running_percent_1s > 0.5

    # check whether the running predictions match the targets
    running_correctnesss = running_conclusions == targets.reshape(-1,1)

    return np.mean(running_correctnesss, axis=0)
    # returns a 1-d series of the accuracy of using the first n trees to pred
```

In [16]:
```python
# your code here
train_prediction = running_predictions(bagging_train.iloc[:,:n_trees].values,
test_prediction = running_predictions(bagging_test.iloc[:,:n_trees].values, y

fig, ax = plt.subplots(1, 1, figsize=(11, 7))
ax.plot(range(1,n_trees+1), train_prediction, 'r*-', label="Training Accuracy
ax.plot(range(1,n_trees+1), test_prediction, 'b*-', label="Test Accuracy")
ax.axhline(y=dt_single_test_score, color='g', label="Single Decision Tree wit
ax.axhline(y=dt_single_ovf_test_score, color='black', label="Single Decision
ax.legend(loc='best', fontsize=16)
ax.tick_params(labelsize=16)
ax.set_title('Bagging Accuracy vs Number of Bootstraps', fontsize=16)
ax.set_xlabel("Number of Bootstraps", fontsize=16)
ax.set_ylabel("Accuracy", fontsize=16)
plt.show()
```



2.5 Referring to your graph from 2.4, compare the performance of bagging against the baseline of a single tree with the depth you picked in 2.1. Explain the differences you see.

your answer here

Almost all the bootrapping models test accuracy is higher than the 20 depth testing accuracy except for the first two bootstraps. This has illutrated that boostrapping does help reduce overfitting.

2.6 Bagging and limiting tree depth both affect how much the model overfits. Compare and contrast these two approaches. Your answer should refer to your graph in 2.4 and may duplicate something you said in your answer to 1.5.

your answer here

Depth 20 which is overfitting gives the worst accuracy with testing data. Both Bagging and depth 5 from cross validation are much better than depth 20 accuracy.

In addition, in most of the cases, bagging is also better than one sigle decision tree with depth 5 from cross validation because bagging uses bootstrapping to select the random samples to train the model which make the overall model more robust with higher performance and less variance.

2.7: In what ways might our bagging classifier be overfitting the data? In what ways might it be underfitting?

your answer here

If majority of our input samples are highly correlated, boostrapping may not help us reduce the overfitting if we choose a large depth.

If our input sample are highly uncorrelated or the classes are super unblanced (cancer or not), bagging could be underfitting because boostrapping could have missed out the important samples for the rare class (cancer).

Question 3: Random Forests [15 pts]

Random Forests are closely related to the bagging model we built by hand in question 2. In this question we compare our by-hand results with the results of using `RandomForestClassifier` directly.

3.1 Fit a `RandomForestClassifier` to the original `X_train` data using the same tree depth and number of trees that you used in Question 2.2. Evaluate its accuracy on the test set.

3.2 For each of the decision trees you fit in the bagging process, how many times is each feature used at the top node? How about for each tree in the random forest you just fit? What about the process of training the Random Forest causes this difference? What implication does this observation have on the accuracy of bagging vs Random Forest?

Hint: A decision tree's top feature is stored in `model.tree_.feature[0]`. A random forest object stores its decision trees in its `.estimators_` attribute.

3.3: Make a table of the training and test accuracy for the following models:

- Single tree with best depth chosen by cross-validation (from Question 1)
- A single overfit tree trained on all data (from Question 2, using the depth you chose there)
- Bagging 45 such trees (from Question 2)
- A Random Forest of 45 such trees (from Question 3.1)

(This problem should not require fitting any new models, though you may need to go back and store the accuracies from models you fit previously.)

What is the relative performance of each model on the training set? On the test set? Comment on how these relationships make sense (or don't make sense) in light of how each model treats the bias-variance trade-off.

Answers:

3.1 Fit a `RandomForestClassifier` to the original `X_train` data using the same tree depth and number of trees that you used in Question 2.2. Evaluate its accuracy on the test set.

In [17]:
```python
# your code here
print("Number of trees = {}".format(n_trees))
print("Depth of the trees = {}".format(depth_ovf))

# Training
dt_rf = RandomForestClassifier(n_estimators=n_trees, max_depth=depth_ovf, n_j
dt_rf.fit(X_train, y_train)

# Perfromance Evaluation
rf_train_score = accuracy_score(y_train, dt_rf.predict(X_train))
rf_test_score = accuracy_score(y_test, dt_rf.predict(X_test))

print("Random Forest on training set = {:.2%}".format(rf_train_score))
print("Random Forest on test set = {:.2%}".format(rf_test_score))
```

```
Number of trees = 45
Depth of the trees = 20
Random Forest on training set = 100.00%
Random Forest on test set = 69.14%
```

3.2 For each of the decision trees you fit in the bagging process, how many times is each feature used at the top node? How about for each tree in the random forest you just fit? What about the process of training the Random Forest causes this difference? What implication does this observation have on the accuracy of bagging vs Random Forest?

Hint: A decision tree's top feature is stored in `model.tree_.feature[0]`. A random forest object stores its decision trees in its `.estimators_` attribute.

In [18]:
```python
# your code here
from collections import Counter

rf_fn = [t.tree_.feature[0] for t in dt_rf.estimators_]

rf_fn_cnt = Counter(rf_fn)
bagging_fn_cnt = Counter(bagging_fn)

print("Random Forest First Node: {}".format(rf_fn_cnt))
print("Bagging First Node: {}".format(bagging_fn_cnt))
```

```
Random Forest First Node: Counter({25: 8, 0: 6, 24: 6, 27: 5, 5: 4, 3: 4, 14:
2, 10: 2, 7: 2, 1: 1, 26: 1, 21: 1, 9: 1, 22: 1, 4: 1})
Bagging First Node: Counter({25: 45})
```

your answer here

- Q: For each of the decision trees you fit in the bagging process, how many times is each feature used at the top node?
- A: The feature number with associated number of times at the top node is listed above. Feature 25 has always been the top node in all bootstrappings, and no other feature has ever been picked.

- Q: How about for each tree in the random forest you just fit?
- A: Feature 25 appreas the most frequently with total number of 8 times, followed by feature 0 and 24 with total number of 6 times.

- Q: What about the process of training the Random Forest causes this difference?
- A: Feature 25 seems to be a very strong predictor, such that bagging algorithm always picks it as the top node. To de-correlate the trees, random forest randomly select a set of predictors from the full set of predictors for each tree at each split. Thus, for some sample of predictors, feature 25 is not among it. Other features that might be correlated to feature 25 will take the leading role and appears as the top node.

- Q: What implication does this observation have on the accuracy of bagging vs Random Forest?
- A: The major drawback of bagging is that the trees are not independent. Bacause of the strong predictor, the greedy algorithm ensures that most of the models in the ensemble will choose to split on it in early iterations. That is, each tree in the ensemble is identically distributed, with the expected output of the averaged model the same as the expected output of any one of the trees. Random forest is a modified form of bagging that creates ensembles of independent decision trees, so overall the accuracy of random forest should be better or at least similar to the accuracy of bagging.

3.3: Make a table of the training and test accuracy for the following models:

- Single tree with best depth chosen by cross-validation (from Question 1)
- A single overfit tree trained on all data (from Question 2, using the depth you chose there)
- Bagging 45 such trees (from Question 2)
- A Random Forest of 45 such trees (from Question 3.1)

(This problem should not require fitting any new models, though you may need to go back and store the accuracies from models you fit previously.)

What is the relative performance of each model on the training set? On the test set? Comment on how these relationships make sense (or don't make sense) in light of how each model treats the bias-variance trade-off.

In [19]:
```
# your code here
print("Single Tree on training set = {:.2%}".format(dt_single_train_score))
print("Single Tree on test set = {:.2%}".format(dt_single_test_score))

print("Overfitted Single Tree on training set = {:.2%} ".format(dt_single_ovf
print("Overfitted Single Tree on test set = {:.2%} ".format(dt_single_ovf_tes

print("Bagging on training set = {:.2%}".format(bagging_train_score))
print("Bagging on test set = {:.2%}".format(bagging_test_score))

print("Random Forest on training set = {:.2%}".format(rf_train_score))
print("Random Forest on test set = {:.2%}".format(rf_test_score))
```

```
Single Tree on training set = 68.12%
Single Tree on test set = 64.78%
Overfitted Single Tree on training set = 100.00%
Overfitted Single Tree on test set = 59.78%
Bagging on training set = 99.96%
Bagging on test set = 68.22%
Random Forest on training set = 100.00%
Random Forest on test set = 69.14%
```

Fill in the following table (ideally in code, but ok to fill in this Markdown cell).

classifier	training accuracy	test accuracy
single tree with best depth chosen by CV	68.12%	64.80%
single depth-X tree	100.00%	59.94%
bagging 45 depth-X trees	99.96%	68.22%
Random Forest of 45 depth-X trees	100.00%	69.14%

your answer here

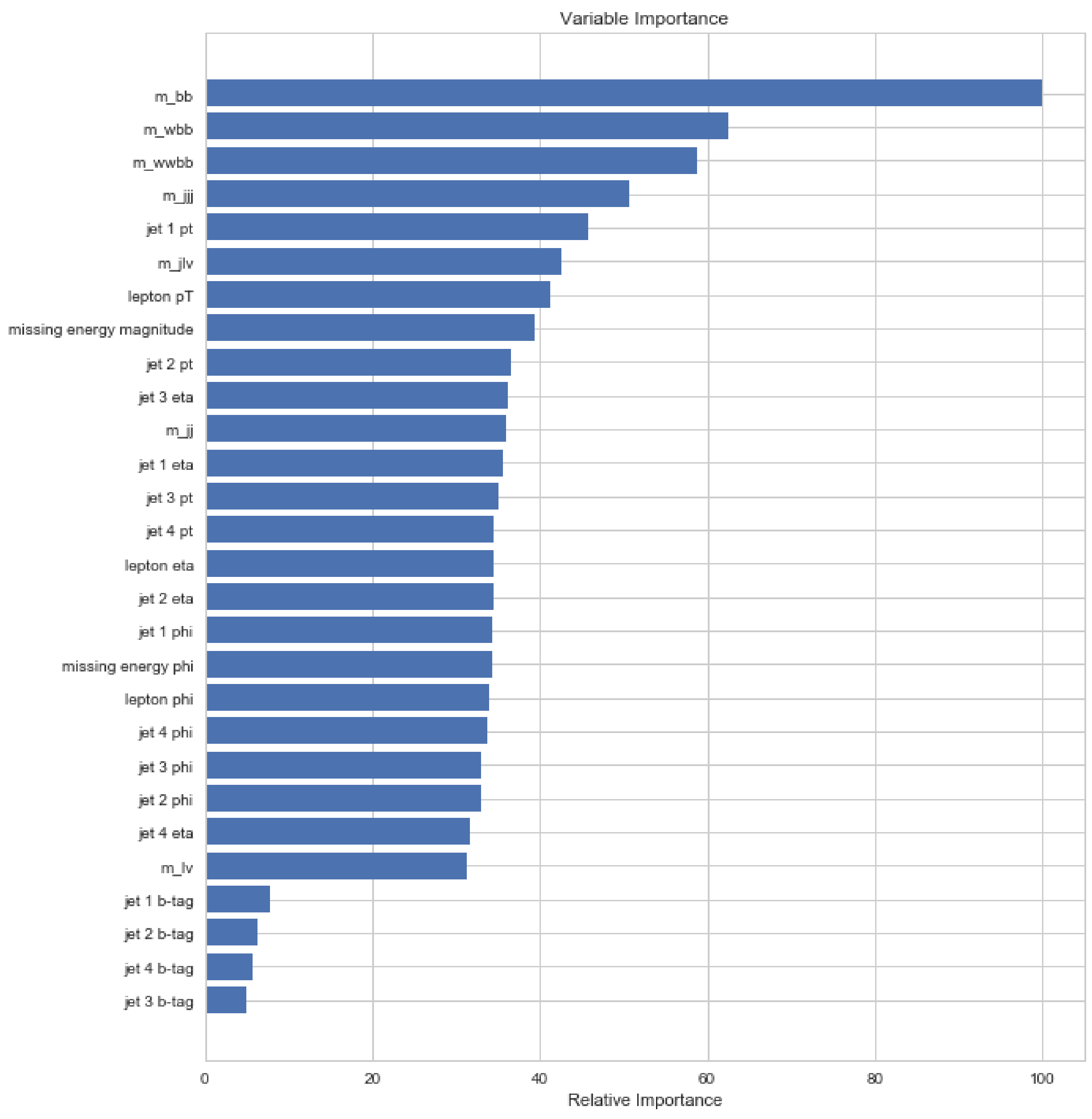
- Q: What is the relative performance of each model on the training set?
- A: Comparing four models:
    - Decision tree of depth 5: 68.12% accuracy, lowest.
    - Decision tree of depth 20: 100.00% accuracy, highest.
    - Bagging: 99.96% accuracy, third highest.
    - Random Forest: 100.00% accuracy, highest.

- Q: On the test set?
- A: Comparing four models:
    - Decision tree of depth 5: 64.80% accuracy, second lowest.
    - Decision tree of depth 20: 59.94% accuracy, lowest.
    - Bagging: 68.22% accuracy, second highest.
    - Random Forest: 69.14% accuracy, highest.

- Q: Comment on how these relationships make sense (or don't make sense) in light of how each model treats the bias-variance trade-off.
- A: Comparing four models:

- Decision tree of depth 5: Depth of 5 is chosen from cross validation, and the model has high bias (low accuracy on training set) with low variance (not overfitted, and robust). This model makes sense, with the advantage of interpretability.
- Decision tree of depth 20: This model has low bias (high accuracy on training set) with high variance (overfitted). This model does NOT make sense, since it's too overfitted, too sensitive and too specific.
- Bagging: This model has low bias (high accuracy on training set) with lower variance (not that overfitted) than the decision tree of depth 20. Bagging improves the variability of "high variance" models by leveraging the law of large numbers, but does not improve bias nearly as much. Essentially, it is an indirect way of "smoothing" these discreted step function by essentially jittering where those jumps occur. This model makes sense.
- Random Forest: This model has low bias (high accuracy on training set) with lower variance (still some overfitting) than bagging. Random forest is a modified form of bagging that creates ensembles of independent decision trees, so overall the accuracy of random forest should be better or at least similar to the accuracy of bagging. This model is the best model among the four choices.

In [20]:
```python
# Random Forest Feature Importance
feature_importance = dt_rf.feature_importances_
feature_importance = 100.0 * (feature_importance / feature_importance.max())
sorted_idx = np.argsort(feature_importance)
pos = np.arange(sorted_idx.shape[0]) + .5

# Plot
plt.figure(figsize=(10,12))
plt.barh(pos, feature_importance[sorted_idx], align='center')
plt.yticks(pos, data_train.columns[sorted_idx])
plt.xlabel('Relative Importance')
plt.title('Variable Importance')
plt.show()
```



Question 4: Boosting [15 pts]

In this question we explore a different kind of ensemble method, boosting, where each new model is trained on a dataset weighted towards observations that the current set of models predicts incorrectly.

We'll focus on the AdaBoost flavor of boosting and examine what happens to the ensemble model's accuracy as the algorithm adds more predictors to the ensemble.

4.1 We'll motivate AdaBoost by noticing patterns in the errors that a single classifier makes. Fit `tree1` , a decision tree with depth 3, to the training data. For each predictor, make a plot that compares two distributions: the values of that predictor for examples that `tree1` classifies correctly, and the values of that predictor for examples that `tree1` classifies incorrectly. Do you notice any predictors for which the distributions are clearly different?

4.2 The following code attempts to implement a simplified version of boosting using just two classifiers (described below). However, it has both stylistic and functionality flaws. First, imagine that you are a grader for a Data Science class; write a comment for the student who submitted this code. Then, imagine that you're the TF writing the solutions; make an excellent example implementation. Finally, use your corrected code to compare the performance of `tree1` and the boosted algorithm on both the training and test set.

4.3 Now let's use the sklearn implementation of AdaBoost: Use `AdaBoostClassifier` to fit another ensemble to `X_train` . Use a decision tree of depth 3 as the base learner and a learning rate 0.05, and run the boosting for 800 iterations. Make a plot of the effect of the number of estimators/iterations on the model's train and test accuracy.

Hint: The `staged_score` method provides the accuracy numbers you'll need. You'll need to use `list()` to convert the "generator" it returns into an ordinary list.

4.4 Repeat the plot above for a base learner with depth of (1, 2, 3, 4). What trends do you see in the training and test accuracy?

(It's okay if your code re-fits the depth-3 classifier instead of reusing the results from the previous problem.)

4.5 Based on the plot you just made, what combination of base learner depth and number of iterations seems optimal? Why? How does the performance of this model compare with the performance of the ensembles you considered above?

Answers

4.1 We'll motivate AdaBoost by noticing patterns in the errors that a single classifier makes. Fit `tree1` , a decision tree with depth 3, to the training data. For each predictor, make a plot that compares two distributions: the values of that predictor for examples that `tree1` classifies correctly, and the values of that predictor for examples that `tree1` classifies incorrectly. Do you notice any predictors for which the distributions are clearly different?

Hints:

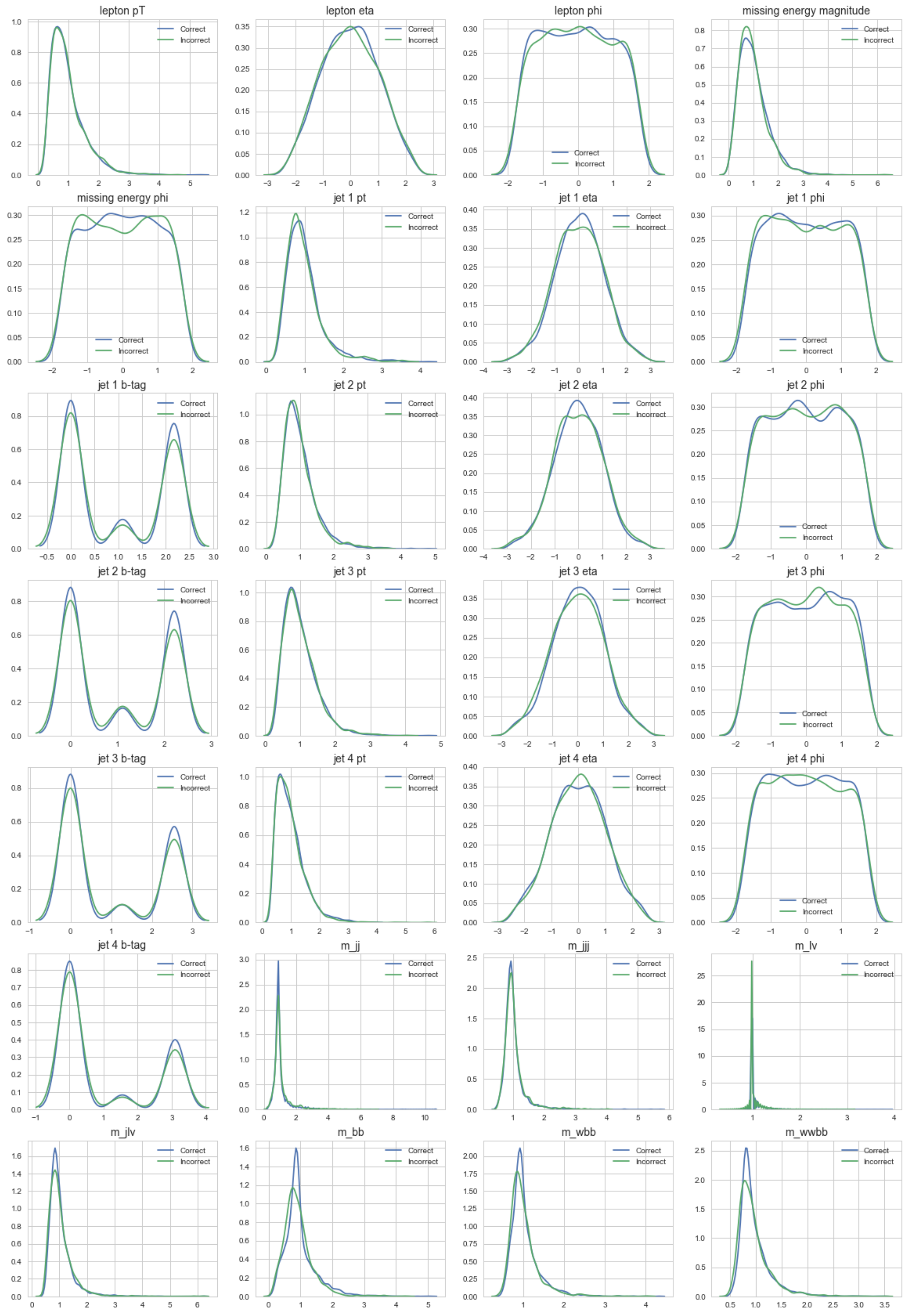
- If you have `fig, axs = plt.subplots(...)` , then `axs.ravel()` gives a list of each plot in reading order.

- `sns.kdeplot` (https://seaborn.pydata.org/generated/seaborn.kdeplot.html) takes `ax` and `label` parameters.

In [21]:
```
# your code here
tree1 = DecisionTreeClassifier(max_depth=3)
tree1.fit(X_train, y_train)
tree1_pred = tree1.predict(X_train)

print("Tree1 on training set = {:.2%}".format(tree1.score(X_train, y_train)))
print("Tree1 on test set = {:.2%}".format(tree1.score(X_test, y_test)))
```

```
Tree1 on training set = 64.18%
Tree1 on test set = 64.42%
```

In [22]:
```
features = data_train.columns[:-1]

f, axs = plt.subplots(7, 4, figsize = (20, 30))

for i in range(len(axs)):
    for j in range(len(axs[0])):
        feature = features[i * len(axs[0]) + j]
        sns.kdeplot(data_train[feature][y_train == tree1_pred], ax=axs[i][j],
        sns.kdeplot(data_train[feature][y_train != tree1_pred], ax=axs[i][j],
        axs[i][j].set_title(feature, fontsize=14)
```



your answer here

- Q: Do you notice any predictors for which the distributions are clearly different?

- A: Yes, some of the predictors show very different distributions, like m_bb, m_wbb, m_wwbb.

4.2 The following code attempts to implement a simplified version of boosting using just two classifiers (described below). However, it has both stylistic and functionality flaws. First, imagine that you are a grader for a Data Science class; write a comment for the student who submitted this code. Then, imagine that you're the TF writing the solutions; make an excellent example implementation. Finally, use your corrected code to compare the performance of `tree1` and the boosted algorithm on both the training and test set.

The intended functionality is the following:

1. Fit `tree1` , a decision tree with max depth 3.
2. Construct an array of sample weights. Give a weight of 1 to samples that `tree1` classified correctly, and 2 to samples that `tree1` misclassified.
3. Fit `tree2` , another depth-3 decision tree, using those sample weights.
4. To predict, compute the probabilities that `tree1` and `tree2` each assign to the positive class. Take the average of those two probabilities as the prediction probability.

In [23]:

```python
# Original Code
def boostmeup():
    tree = DecisionTreeClassifier(max_depth=3)
    tree1 = tree.fit(X_train, y_train)
    sample_weight = np.ones(len(X_train))
    q = 0
    for idx in range(len(X_train)):
      if tree1.predict([X_train[idx]]) != y_train[idx]:
         sample_weight[idx] = sample_weight[idx] * 2
         q = q + 1
    print("tree1 accuracy:", q / len(X_train))
    tree2 = tree.fit(X_train, y_train, sample_weight=sample_weight)

# Train
    q = 0
    for idx in range(len(X_train)):
        t1p = tree1.predict_proba([X_train[idx]])[0][1]
        t2p = tree2.predict_proba([X_train[idx]])[0][1]
        m = (t1p + t2p) / 2
        if m > .5:
            if y_train[idx] == True:
                q = q + 0
            else:
                q = q + 1
        else:
            if y_train[idx] == True:
                q = q + 1
            else:
                q = 0
    print("Boosted accuracy:", q / len(X_train))

# Test
    q = 0
    for idx in range(len(X_test)):
        t1p = tree1.predict_proba([X_test[idx]])[0][1]
        t2p = tree2.predict_proba([X_test[idx]])[0][1]
        m = (t1p + t2p) / 2
        if m > .5:
            if y_train[idx] == True:
                q = q + 0
            else:
                q = q + 1
        else:
            if y_train[idx] == True:
                q = q + 1
            else:
                q = 0
    print("Boosted accuracy:", q / len(X_test))

boostmeup()
```

```
tree1 accuracy: 0.3582
Boosted accuracy: 0.0008
Boosted accuracy: 0.002
```

Your answer here

If I were a grader for a Data Science class...

- Your code has several stylistic and functionality flaws that I would listed below:
  - Stylistic flaws:
    - Watch out for consistent indentation in Python. Python is a language that's style restrict, and indentation is one of the most beautiful part of it, which has saved us countless curly brackets.
    - Train and test parts have the exact code structure, with only input data difference. Considering about code productivity and usability, you should make the input file as argument passed into the function.
    - Python is a concise and efficient language, so you'd better avoid for loop if not necessary. Build-in list and array operations are computationally faster.
  - Functional flaws:
    - Make sure that you understand prediction accuracy vs prediction error. Your calculation of prediction accuracy is actually prediction error.
    - In the test part of your code, I guess you meant to write "q = q + 0" on line 47 above, while you wrote it as "q = 0". This explained why your "prediction error" is so low.
    - In order to calculate prection accuracy on the test set, you need to compare the prediction to the actual response in the test set, while you were comparing to the response in the training set.

In [25]:
```python
# TF version
def boostmeup(X, y, depth, label, test=False, tree1=None, tree2=None):
    tree = DecisionTreeClassifier(max_depth=depth)

    # Tree 1
    if test is False:
        tree1 = tree.fit(X, y)
    tree1_pred = tree1.predict(X) == y
    tree1_sw = [1 if x == True else 2 for x in tree1_pred]
    print("tree1 accuracy on {} set = {:.2%}".format(label, tree1_sw.count(1)

    # Tree 2
    if test is False:
        tree2 = tree.fit(X, y, sample_weight=tree1_sw)

    t1p = np.array(tree1.predict_proba(X))[:,1]
    t2p = np.array(tree2.predict_proba(X))[:,1]
    tp = (t1p + t2p) / 2
    tree2_pred = [1 if x > 0.5 else 0 for x in tp] == y
    tree2_sw = [1 if x == True else 2 for x in tree2_pred]
    print("tree2 accuracy on {} set = {:.2%}".format(label, tree2_sw.count(1)

    return tree1, tree2

tree1, tree2 = boostmeup(X=X_train, y=y_train, depth=3, label="training")
boostmeup(X=X_test, y=y_test, depth=3, label="test", test=True, tree1=tree1,
```

```
tree1 accuracy on training set = 64.18%
tree2 accuracy on training set = 61.34%
tree1 accuracy on test set = 60.88%
tree2 accuracy on test set = 60.88%
```

Out[25]:
```
(DecisionTreeClassifier(class_weight=None, criterion='gini', max_depth=3,
             max_features=None, max_leaf_nodes=None,
             min_impurity_decrease=0.0, min_impurity_split=None,
             min_samples_leaf=1, min_samples_split=2,
             min_weight_fraction_leaf=0.0, presort=False, random_state=None,
             splitter='best'),
 DecisionTreeClassifier(class_weight=None, criterion='gini', max_depth=3,
             max_features=None, max_leaf_nodes=None,
             min_impurity_decrease=0.0, min_impurity_split=None,
             min_samples_leaf=1, min_samples_split=2,
             min_weight_fraction_leaf=0.0, presort=False, random_state=None,
             splitter='best'))
```

My simplified boosting accuracy on training set is actually lower than the single tree accuray, and I think that's because this simplified boosting doesn't implement several aspects of the full Adaboost that ensure that at least training accuracy doesn't decrease. Testing accuracies are the same.

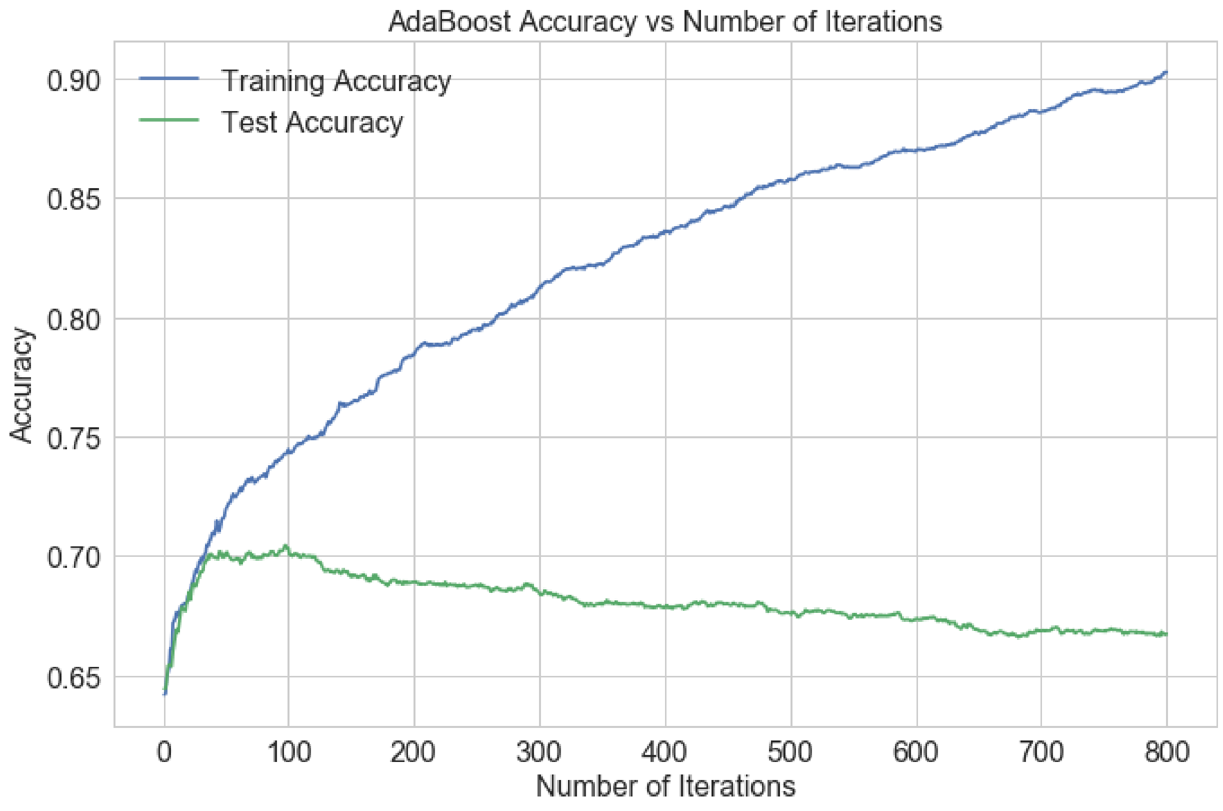
4.3 Now let's use the sklearn implementation of AdaBoost: Use `AdaBoostClassifier` to fit another ensemble to `X_train`. Use a decision tree of depth 3 as the base learner and a learning rate 0.05, and run the boosting for 800 iterations. Make a plot of the effect of the number of estimators/iterations on the model's train and test accuracy.

Hint: The `staged_score` method provides the accuracy numbers you'll need. You'll need to use `list()` to convert the "generator" it returns into an ordinary list.

In [26]:
```
# your code here
# Training
dt_ab = AdaBoostClassifier(base_estimator=DecisionTreeClassifier(max_depth=3)
dt_ab.fit(X_train, y_train)

# Perfromance Evaluation
dt_ab_train_score = accuracy_score(y_train, dt_ab.predict(X_train))
dt_ab_test_score = accuracy_score(y_test, dt_ab.predict(X_test))

print("AdaBoost on training set = {:.2%}".format(dt_ab_train_score))
print("AdaBoost on test set = {:.2%}".format(dt_ab_test_score))
```

```
AdaBoost on training set = 90.26%
AdaBoost on test set = 66.72%
```

In [27]:
```
dt_ab_train_scores = list(dt_ab.staged_score(X_train, y_train))
dt_ab_test_scores = list(dt_ab.staged_score(X_test, y_test))

fig, ax = plt.subplots(1, 1, figsize=(11, 7))
ax.plot(range(1, len(dt_ab_train_scores)+1), dt_ab_train_scores, label="Train
ax.plot(range(1, len(dt_ab_test_scores)+1), dt_ab_test_scores, label="Test Ac
ax.legend(loc='best', fontsize=16)
ax.tick_params(labelsize=16)
ax.set_title('AdaBoost Accuracy vs Number of Iterations', fontsize=16)
ax.set_xlabel("Number of Iterations", fontsize=16)
ax.set_ylabel("Accuracy", fontsize=16)
plt.show()
```

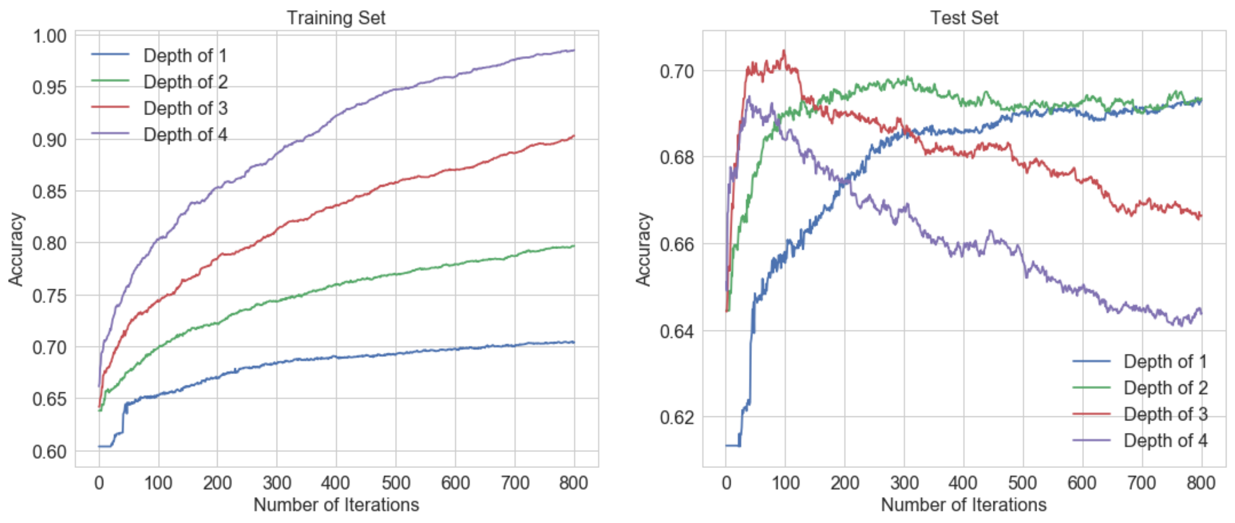


4.4 Repeat the plot above for a base learner with depth of (1, 2, 3, 4). What trends do you see in the training and test accuracy?

(It's okay if your code re-fits the depth-3 classifier instead of reusing the results from the previous problem.)

In [28]:
```
# your code here
dt_ab_depths = [1, 2, 3, 4]
dt_ab_train_score_depths = []
dt_ab_test_score_depths = []

for depth in dt_ab_depths:
    dt_ab = AdaBoostClassifier(base_estimator=DecisionTreeClassifier(max_dept
    dt_ab.fit(X_train, y_train)

    dt_ab_train_score_depths.append(list(dt_ab.staged_score(X_train, y_train)
    dt_ab_test_score_depths.append(list(dt_ab.staged_score(X_test, y_test)))
```

In [29]:
```python
# your code here
fig, ax = plt.subplots(1, 2, figsize=(18, 7))

for depth in dt_ab_depths:
    ax[0].plot(range(1, len(dt_ab_train_score_depths[depth-1])+1),
              dt_ab_train_score_depths[depth-1],
              label="Depth of {}".format(depth))

    ax[0].legend(loc='best', fontsize=16)
    ax[0].tick_params(labelsize=16)
    ax[0].set_title('Training Set', fontsize=16)
    ax[0].set_xlabel("Number of Iterations", fontsize=16)
    ax[0].set_ylabel("Accuracy", fontsize=16)

    ax[1].plot(range(1, len(dt_ab_test_score_depths[depth-1])+1),
              dt_ab_test_score_depths[depth-1],
              label="Depth of {}".format(depth))

    ax[1].legend(loc='best', fontsize=16)
    ax[1].tick_params(labelsize=16)
    ax[1].set_title('Test Set', fontsize=16)
    ax[1].set_xlabel("Number of Iterations", fontsize=16)
    ax[1].set_ylabel("Accuracy", fontsize=16)

plt.show()
```



Your answer here

- Q: What trends do you see in the training and test accuracy?
- A:
    - Training set: Training accuray monotonically becomes higher with higher number of iterations regardless of depth of the trees. Given same number of iterations, more complex models (higher depth) fit better than the simple models (lower depth) in the training set.
    - Test set: Model with depth of 1 has monotonical increasing accuracy in test set with higher number of iterations. Model with depths of 2, 3, and 4 first have higher test accuracy, then peaks around at iterations of 300, 100, and 50, after that decreases with with higher number of iterations.

4.5 Based on the plot you just made, what combination of base learner depth and number of iterations seems optimal? Why? How does the performance of this model compare with the performance of the ensembles you considered above?

In [30]:
```python
# your code here
dt_ab_itr = np.array(dt_ab_test_score_depths[2]).argmax(axis=0) + 1
print("Best Number of iterations for depth of 3 base learner = {}".format(dt_

# Training
dt_ab = AdaBoostClassifier(base_estimator=DecisionTreeClassifier(max_depth=3)
dt_ab.fit(X_train, y_train)

# Perfromance Evaluation
dt_ab_train_score = accuracy_score(y_train, dt_ab.predict(X_train))
dt_ab_test_score = accuracy_score(y_test, dt_ab.predict(X_test))

print("AdaBoost on training set = {:.2%}".format(dt_ab_train_score))
print("AdaBoost on test set = {:.2%}".format(dt_ab_test_score))
```

```
Best Number of iterations for depth of 3 base learner = 98
AdaBoost on training set = 74.28%
AdaBoost on test set = 70.38%
```

Your answer here

classifier	training accuracy	test accuracy
single tree with best depth chosen by CV	68.12%	64.80%
single depth-X tree	100.00%	59.94%
bagging 45 depth-X trees	99.96%	68.22%
Random Forest of 45 depth-X trees	100.00%	69.14%
AdaBoost	74.28%	70.38%

Best base learner is the decision tree with depth of 3 and 98 number of iterations, because it gives the best performance in the test set. The AddBoost model with 70.38% accuracy in testing set beats all the previous models we considered.

Question 5: Understanding [15 pts]

This question is an overall test of your knowledge of this homework's material. You may need to refer to lecture notes and other material outside this homework to answer these questions.

5.1 How do boosting and bagging relate: what is common to both, and what is unique to each?

5.2 Reflect on the overall performance of all of the different classifiers you have seen throughout this assignment. Which performed best? Why do you think that may have happened?

5.3 What is the impact of having too many trees in boosting and in bagging? In which instance is it worse to have too many trees?

5.4 Which technique, boosting or bagging, is better suited to parallelization, where you could have multiple computers working on a problem at the same time?

5.5 Which of these techniques can be extended to regression tasks? How?

Answers:

5.1 How do boosting and bagging relate: what is common to both, and what is unique to each?

Your answer here

Common: both boosting and bagging are ensember methods. Difference: For bagging, each run is independent to predict the final target predictor. But, for boosting, each run is consecutive to predict the residule from the previous run.

5.2 Reflect on the overall performance of all of the different classifiers you have seen throughout this assignment. Which performed best? Why do you think that may have happened?

your answer here

Boosting gave the best accuracy after carefully tuning the parameters of the model such as depth, number of iterations etc. For each iteration, boosting is to compute on the residule so the performance improved each time if the model is underfitting. However, this required more carefully selection of the parameters compared to random forest.

5.3 What is the impact of having too many trees in boosting and in bagging? In which instance is it worse to have too many trees?

Your answer here

Too many tree in both ramdom forest and boosting need more time to compute, especially for boosting which can not be run parallelly.

In addition, too many trees in boosting could cause serious overfitting. Too many trees in random forest can also lead to have more correlated trees to increase variance but this problem is not going to be as bad as boosting.

5.4 Which technique, boosting or bagging, is better suited to parallelization, where you could have multiple computers working on a problem at the same time?

Your answer here

Bagging is good for parallelization because each run is independent to predict the target variable but boosting always depends on the previous run so that you can not run it in parallel.

5.5 Which of these techniques can be extended to regression tasks? How?

your answer here

Both bagging and boosting can be used for regression. Average of all the run can be used for bagging. Boosting may be better to be used for regression since each iteration is using the previous run's residule as target varaible.

Question 6: Explaining Complex Concepts Clearly [10 pts]

One of the core skills of a data scientist is to be able to explain complex concepts clearly. To practice this skill, you'll make a short presentation of one of the approaches we have recently studied.

Choose one of the following topics:

- Decision Trees
- Random Forests
- Bagging
- Boosting
- Simple Neural Nets (like the MLP we saw in Homework 6)
- (other topics are possible, but get staff approval first)

Make 3 slides explaining the concept.

- Focus on **clear explanations**, NOT aesthetic beauty. Photos of pen-and-paper sketches are fine if they're legible.
- For your audience, choose **future CS109A students**.
- You may take inspiration from anywhere, but explain in **your own words** and **make your own illustrations**.

Submit your slides as a PDF and the source format (`.pptx` , Google Slides, etc.)

NOTE: If you would be okay with us using your slides for future classes (with attribution, of course), please include a note to that effect. This will not affect your grade either way.