CS109A Introduction to Data Science:

Homework 3 - Forecasting Bike Sharing Usage

Harvard University
Fall 2018
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In [1]:
```
#RUN THIS CELL
import requests
from IPython.core.display import HTML
styles = requests.get("https://raw.githubusercontent.com/Harvard-IACS/2018-CS
HTML(styles)
```

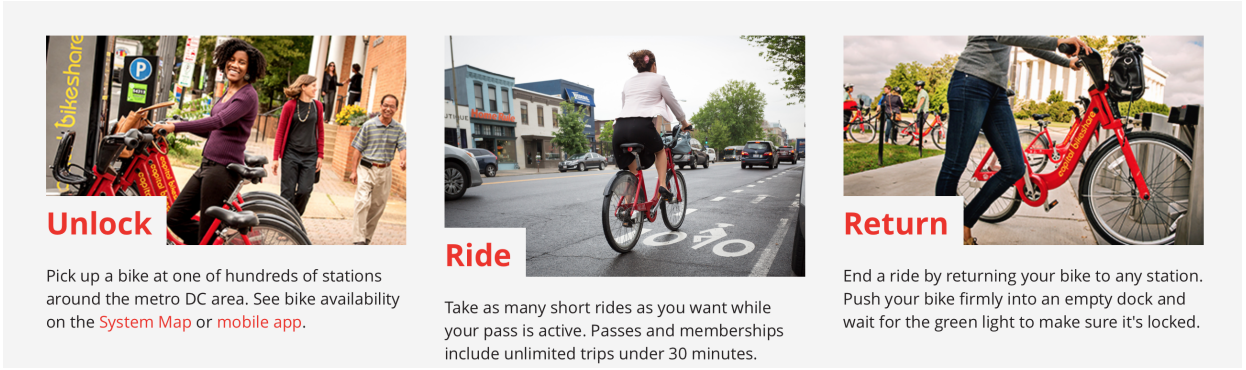
Out[1]:

INSTRUCTIONS

- To submit your assignment follow the instructions given in canvas.
- Restart the kernel and run the whole notebook again before you submit.
- If you submit individually and you have worked with someone, please include the name of your [one] partner below.
- As much as possible, try and stick to the hints and functions we import at the top of the homework, as those are the ideas and tools the class supports and is aiming to teach. And if a problem specifies a particular library you're required to use that library, and possibly others from the import list.

Names of people you have worked with goes here: Xi Han, Haoran Zhao

Type *Markdown* and LaTeX: α^2



Main Theme: Multiple Linear Regression, Subset Selection, Polynomial Regression

Overview

You are hired by the administrators of the Capital Bikeshare program (https://www.capitalbikeshare.com) program in Washington D.C., to **help them predict the hourly demand for rental bikes** and **give them suggestions on how to increase their revenue**. Your task is to prepare a short report summarizing your findings and make recommendations.

The predicted hourly demand could be used for planning the number of bikes that need to be available in the system at any given hour of the day. It costs the program money if bike stations are full and bikes cannot be returned, or empty and there are no bikes available. You will use multiple linear regression and polynomial regression and will explore techniques for subset selection to predict bike usage. The goal is to build a regression model that can predict the total number of bike rentals in a given hour of the day, based on all available information given to you.

An example of a suggestion to increase revenue might be to offer discounts during certain times of the day either during holidays or non-holidays. Your suggestions will depend on your observations of the seasonality of ridership.

The data for this problem were collected from the Capital Bikeshare program over the course of two years (2011 and 2012).

Use only the libraries below:

In [2]:
```
import numpy as np
import pandas as pd
import matplotlib
import matplotlib.pyplot as plt

import statsmodels.api as sm
from statsmodels.api import OLS

from sklearn import preprocessing
from sklearn.preprocessing import PolynomialFeatures
from sklearn.metrics import r2_score
from sklearn.model_selection import train_test_split

from pandas.plotting import scatter_matrix

import seaborn as sns

import itertools

%matplotlib inline
```

Data Exploration & Preprocessing, Multiple Linear Regression, Subset Selection

Overview

The initial data set is provided in the file `data/BSS_hour_raw.csv` . You will first add features that will help with the analysis and then separate the data into training and test sets. Each row in this file represents the number of rides by registered users and casual users in a given hour of a specific date. There are 12 attributes in total describing besides the number of users the weather if it is a holiday or not etc:

- `dteday` (date in the format YYYY-MM-DD, e.g. 2011-01-01)
- `season` (1 = winter, 2 = spring, 3 = summer, 4 = fall)
- `hour` (0 for 12 midnight, 1 for 1:00am, 23 for 11:00pm)
- `weekday` (0 through 6, with 0 denoting Sunday)
- `holiday` (1 = the day is a holiday, 0 = otherwise)
- `weather`
  - 1: Clear, Few clouds, Partly cloudy, Partly cloudy
  - 2: Mist + Cloudy, Mist + Broken clouds, Mist + Few clouds, Mist
  - 3: Light Snow, Light Rain + Thunderstorm
  - 4: Heavy Rain + Thunderstorm + Mist, Snow + Fog
- `temp` (temperature in Celsius)
- `atemp` (apparent temperature, or relative outdoor temperature, in Celsius)
- `hum` (relative humidity)
- `windspeed` (wind speed)
- `casual` (number of rides that day made by casual riders, not registered in the system)
- `registered` (number of rides that day made by registered riders)

General Hints

- Use pandas .describe() to see statistics for the dataset.
- When performing manipulations on column data it is useful and often more efficient to write a function and apply this function to the column as a whole without the need for iterating through the elements.
- A scatterplot matrix or correlation matrix are both good ways to see dependencies between multiple variables.
- For Question 2, a very useful pandas method is .groupby(). Make sure you aggregate the rest of the columns in a meaningful way. Print the dataframe to make sure all variables/columns are there!

Resources

http://pandas.pydata.org/pandas-docs/stable/generated/pandas.to_datetime.html (http://pandas.pydata.org/pandas-docs/stable/generated/pandas.to_datetime.html)

Question 1: Data Read-In and Cleaning

In this section, we read in the data and begin one of the most important analytic steps: verifying that the data is what it claims to be.

1.1 Load the dataset from the csv file `data/BSS_hour_raw.csv` into a pandas dataframe that you name `bikes_df`. Do any of the variables' ranges or averages seem suspect? Do the data types make sense?

1.2 Notice that the variable in column `dteday` is a pandas `object`, which is **not** useful when you want to extract the elements of the date such as the year, month, and day. Convert `dteday` into a `datetime` object to prepare it for later analysis.

1.3 Create three new columns in the dataframe:

- `year` with 0 for 2011, 1 for 2012, etc.
- `month` with 1 through 12, with 1 denoting January.
- `counts` with the total number of bike rentals for that **hour** (this is the response variable for later).

Answers

1.1 Load the dataset from the csv file `data/BSS_hour_raw.csv` into a pandas dataframe that you name `bikes_df`. Do any of the variables' ranges or averages seem suspect? Do the data types make sense?

In [3]:
```
# your code here
bikes_df = pd.read_csv("data/BSS_hour_raw.csv")
```

In [4]:
```
# your code here
bikes_df.head()
```

Out[4]:

	dteday	season	hour	holiday	weekday	workingday	weather	temp	atemp	hum	windspeed
0	2011-01-01	1	0	0	6	0	1	0.24	0.2879	0.81	0.0
1	2011-01-01	1	1	0	6	0	1	0.22	0.2727	0.80	0.0
2	2011-01-01	1	2	0	6	0	1	0.22	0.2727	0.80	0.0
3	2011-01-01	1	3	0	6	0	1	0.24	0.2879	0.75	0.0
4	2011-01-01	1	4	0	6	0	1	0.24	0.2879	0.75	0.0

In [5]:
```
# your code here
bikes_df.info()
```

```
<class 'pandas.core.frame.DataFrame'>
RangeIndex: 17379 entries, 0 to 17378
Data columns (total 13 columns):
dteday        17379 non-null object
season        17379 non-null int64
hour          17379 non-null int64
holiday       17379 non-null int64
weekday       17379 non-null int64
workingday    17379 non-null int64
weather       17379 non-null int64
temp          17379 non-null float64
atemp         17379 non-null float64
hum           17379 non-null float64
windspeed     17379 non-null float64
casual        17379 non-null int64
registered    17379 non-null int64
dtypes: float64(4), int64(8), object(1)
memory usage: 1.7+ MB
```

In [6]:
```
# your code here
bikes_df.describe()
```

Out[6]:

	season	hour	holiday	weekday	workingday	weather	
count	17379.000000	17379.000000	17379.000000	17379.000000	17379.000000	17379.000000	17379.
mean	2.501640	11.546752	0.028770	3.003683	0.682721	1.425283	0.
std	1.106918	6.914405	0.167165	2.005771	0.465431	0.639357	0.
min	1.000000	0.000000	0.000000	0.000000	0.000000	1.000000	0.
25%	2.000000	6.000000	0.000000	1.000000	0.000000	1.000000	0.
50%	3.000000	12.000000	0.000000	3.000000	1.000000	1.000000	0.
75%	3.000000	18.000000	0.000000	5.000000	1.000000	2.000000	0.
max	4.000000	23.000000	1.000000	6.000000	1.000000	4.000000	1.

Your answer here

The min of temp and atemp should be below 0 Celsius and the max of temp and atemp should be above 30 Celsius in DC. The average of temp and atemp should also be around 15 Celsius. Looks like these two column either have some data issues or are normalized already.

humidity and windspeed seem to have been normalized too.

dteday is a pandas object, which is not useful when we want to extract the elements of the date such as the year, month, and day.

1.2 Notice that the variable in column `dteday` is a pandas `object`, which is not useful when you want to extract the elements of the date such as the year, month, and day. Convert `dteday` into a `datetime` object to prepare it for later analysis.

In [7]:
```
# your code here
bikes_df['dteday'] = pd.to_datetime(bikes_df['dteday'], format='%Y-%m-%d')
```

1.3 Create three new columns in the dataframe:

- `year` with 0 for 2011, 1 for 2012, etc.
- `month` with 1 through 12, with 1 denoting January.
- `counts` with the total number of bike rentals for that hour (this is the response variable for later).

In [8]:
```
# your code here
bikes_df['year'] = bikes_df['dteday'].dt.year - 2011
bikes_df['month'] = bikes_df['dteday'].dt.month
bikes_df['counts'] = bikes_df['casual'] + bikes_df['registered']
```

In [9]:
```
# your code here
bikes_df.head()
```

Out[9]:

	dteday	season	hour	holiday	weekday	workingday	weather	temp	atemp	hum	windspeed
0	2011-01-01	1	0	0	6	0	1	0.24	0.2879	0.81	0.0
1	2011-01-01	1	1	0	6	0	1	0.22	0.2727	0.80	0.0
2	2011-01-01	1	2	0	6	0	1	0.22	0.2727	0.80	0.0
3	2011-01-01	1	3	0	6	0	1	0.24	0.2879	0.75	0.0
4	2011-01-01	1	4	0	6	0	1	0.24	0.2879	0.75	0.0

Question 2: Exploratory Data Analysis.

In this question, we continue validating the data, and begin hunting for patterns in ridership that shed light on who uses the service and why.

2.1 Use pandas' `scatter_matrix` command to visualize the inter-dependencies among all predictors in the dataset. Note and comment on any strongly related variables. [This will take several minutes to run. You may wish to comment it out until your final submission, or only plot a randomly-selected 10% of the rows]

2.2 Make a plot showing the *average* number of casual and registered riders during each hour of the day. `.groupby` and `.aggregate` should make this task easy. Comment on the trends you observe.

2.3 Use the variable `weather` to show how each weather category affects the relationships in question 2.2. What do you observe?

2.4 Make a new dataframe with the following subset of attributes from the previous dataset and with each entry being just **one** day:

- `dteday` , the timestamp for that day (fine to set to noon or any other time)
- `weekday` , the day of the week
- `weather` , the most severe weather that day
- `season` , the season that day falls in
- `temp` , the average temperature (normalized)
- `atemp` , the average atemp that day (normalized)
- `windspeed` , the average windspeed that day (normalized)
- `hum` , the average humidity that day (normalized)
- `casual` , the **total** number of rentals by casual users
- `registered` , the **total** number of rentals by registered users
- `counts` , the **total** number of rentals of that day

Name this dataframe `bikes_by_day` .

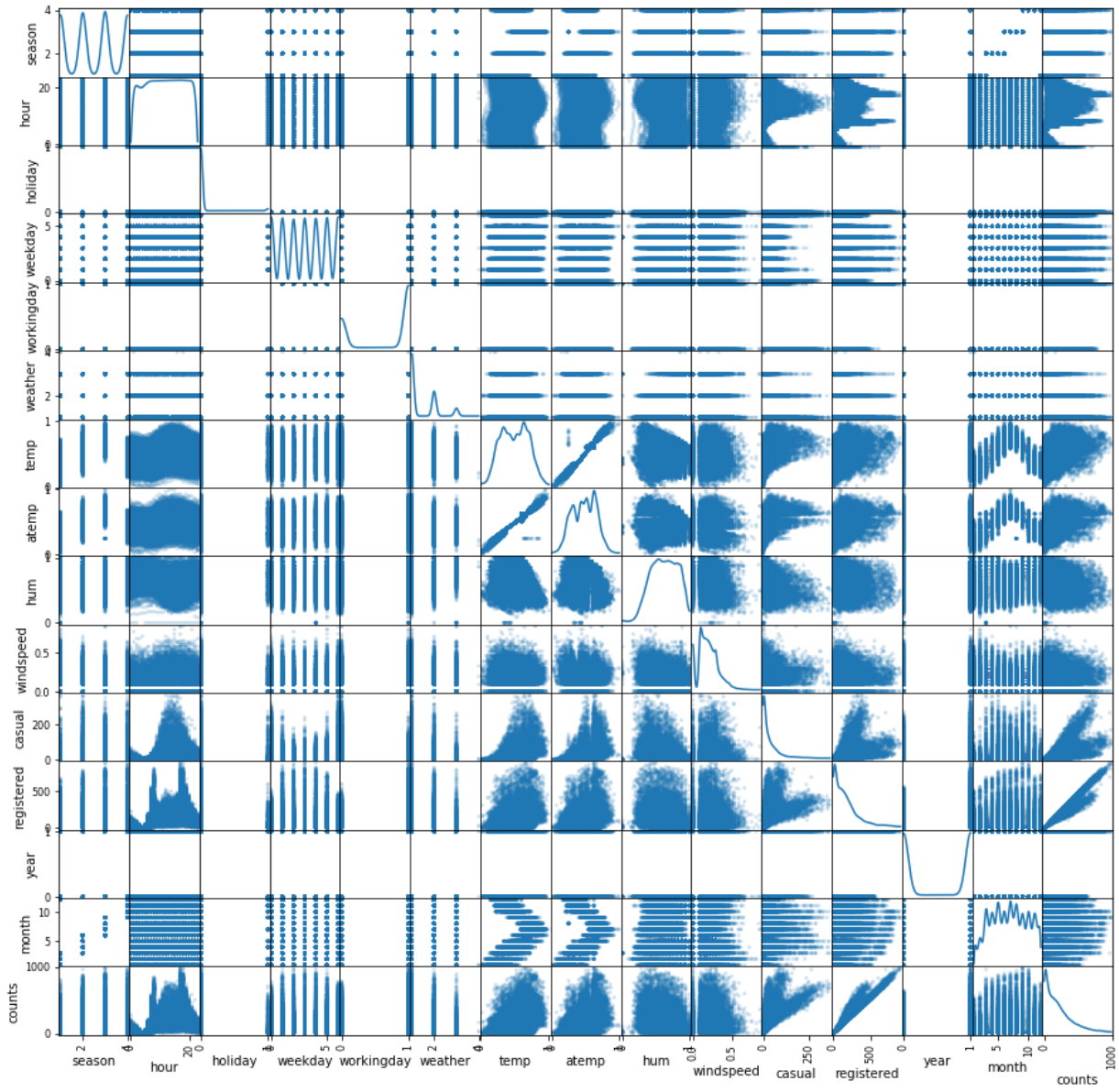
Make a plot showing the *distribution* of the number of casual and registered riders on each day of the week.

2.5 Use `bikes_by_day` to visualize how the distribution of **total number of rides** per day (casual and registered riders combined) varies with the **season**. Do you see any **outliers**? Here we use the pyplot's boxplot function definition of an outlier as any value 1.5 times the IQR above the 75th percentile or 1.5 times the IQR below the 25th percentiles. If you see any outliers, identify those dates and investigate if they are a chance occurence, an error in the data collection, or a significant event (an online search of those date(s) might help).

Answers

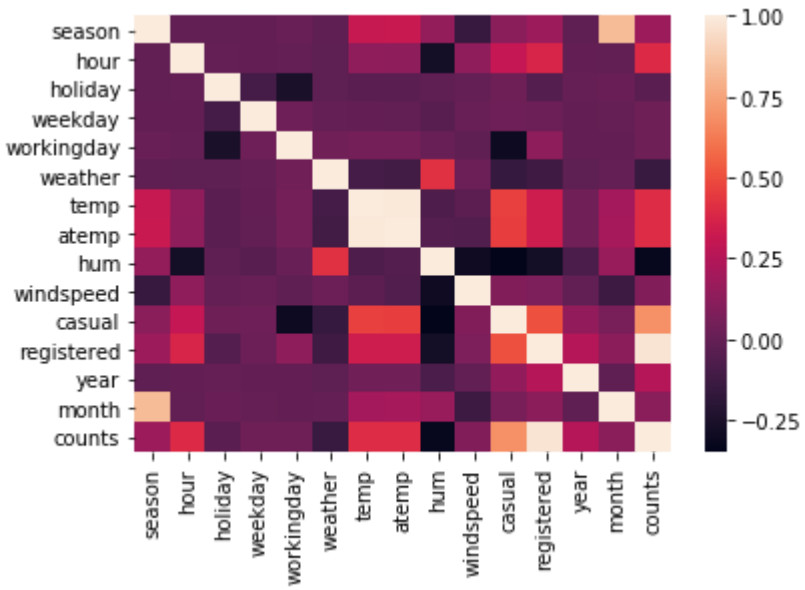
2.1 Use pandas' `scatter_matrix` command to visualize the inter-dependencies among all predictors in the dataset. Note and comment on any strongly related variables. [This will take several minutes to run. You may wish to comment it out until your final submission, or only plot a randomly-selected 10% of the rows]

In [268]:
```
# your code here
scatter_matrix(bikes_df, alpha=0.2, diagonal='kde', figsize=(15,15))
plt.show()
```



In [10]:
```
1 sns.heatmap(bikes_df.corr())
```

Out[10]: <matplotlib.axes._subplots.AxesSubplot at 0x23542ce2ef0>

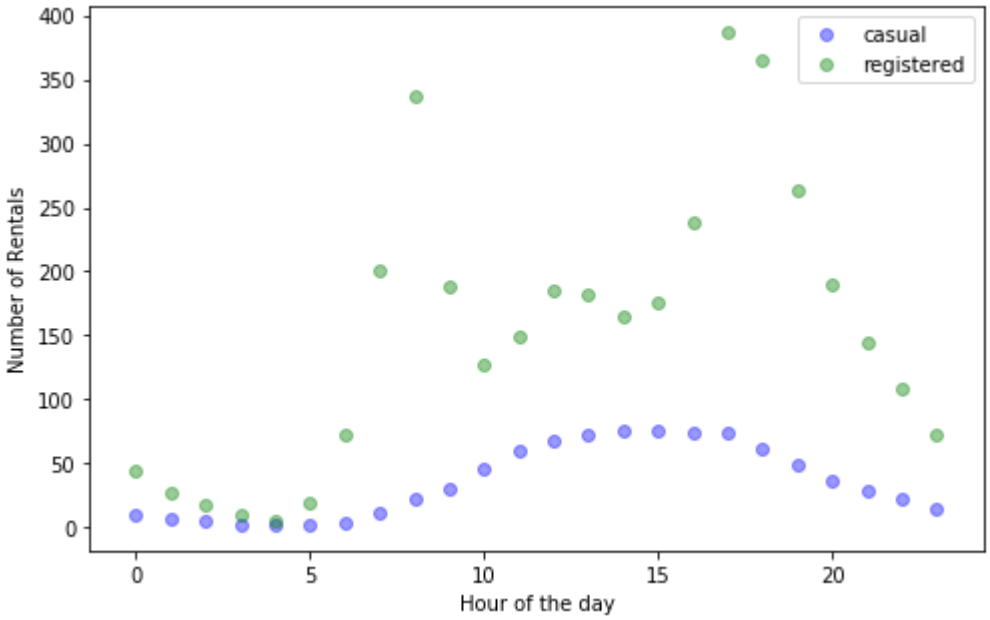


your answer here

- temp vs atemp: strong positive correlation
- month vs temp/atemp: temp/atemp is higher during the summer and cold during the winter.
- season vs month: 3 months are corresponding to 1 season.
- workingday vs weekday: weekday 1 to 5 has workingday 1, and weekday 0 and 6 has workingday 0.
- counts vs registered vs casuals: strong positive correlation
- counts vs windspeed: when windspeed is higher than 0.5, counts drops dramatically.
- counts vs hour: counts peak during commute hours and at minimum in the early morning and late night.
- counts vs month: counts are lower in January, Feburary, November and December compared to other months.

2.2 Make a plot showing the *average* number of casual and registered riders during each hour of the day. `.groupby` and `.aggregate` should make this task easy. Comment on the trends you observe.

In [11]:
```
# your code here
average = bikes_df.groupby(['hour']).agg({'casual':'mean', 'registered':'mean

fig, ax = plt.subplots(1, 1, figsize=(8, 5))
ax.set_ylabel('Number of Rentals')
ax.set_xlabel('Hour of the day')
ax.scatter(average.index.values, average['casual'].values, alpha=0.4 , c='b',
ax.scatter(average.index.values, average['registered'].values, alpha=0.4, c='
ax.legend()
fig.savefig("fig/riders.png")
```



your answer here

The registered rentals picked during commute hours from 8 am to 10am and from 4pm to 7pm. The causual rentals increased from 7 am, peaked from 12pm to 5pm and trending down afterwards.

2.3 Use the variable `weather` to show how each weather category affects the relationships in question 2.2. What do you observe?

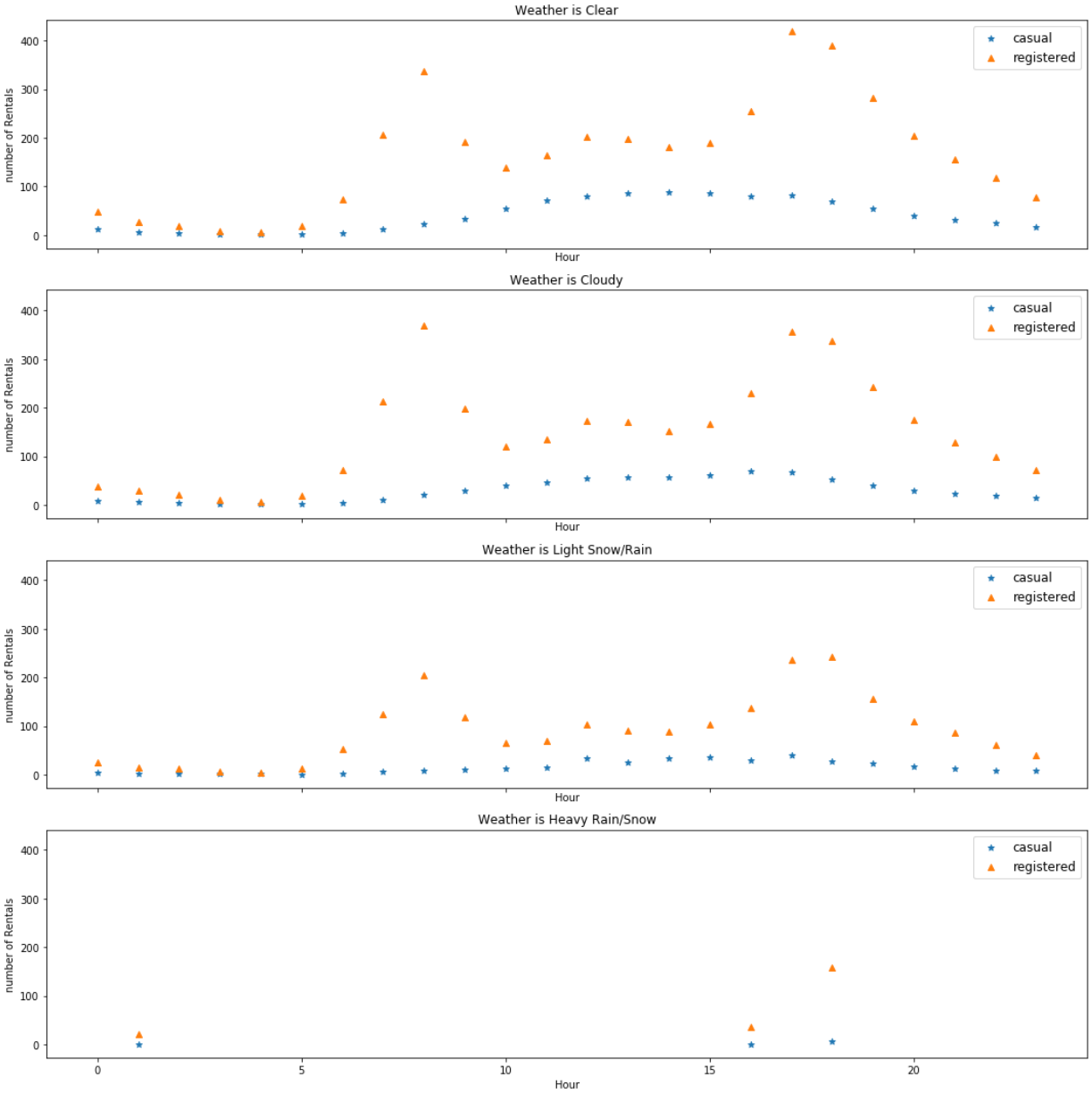
In [12]:
```
# your code here
average = bikes_df.groupby(['weather','hour']).agg({'casual':'mean','register
                                reset_index(level=['weather', 'hour'])
average.head()
```

Out[12]:

	weather	hour	casual	registered
0	1	0	11.429448	47.732106
1	1	1	6.951020	27.444898
2	1	2	5.008368	17.809623
3	1	3	2.868132	9.127473
4	1	4	1.273523	5.140044

In [13]:

```
# your code here
f, (ax1, ax2, ax3, ax4) = plt.subplots(4, figsize=(15, 15), sharex=True, shar

ax1.scatter(average.loc[average['weather']==1,['hour']], average.loc[average[
ax1.scatter(average.loc[average['weather']==1,['hour']], average.loc[average[
ax1.set_title('Weather is Clear')
ax1.set_xlabel('Hour')
ax1.set_ylabel('number of Rentals')
ax1.legend(prop={'size': 12})

ax2.scatter(average.loc[average['weather']==2,['hour']], average.loc[average[
ax2.scatter(average.loc[average['weather']==2,['hour']], average.loc[average[
ax2.set_title('Weather is Cloudy')
ax2.set_xlabel('Hour')
ax2.set_ylabel('number of Rentals')
ax2.legend(prop={'size': 12})

ax3.scatter(average.loc[average['weather']==3,['hour']], average.loc[average[
ax3.scatter(average.loc[average['weather']==3,['hour']], average.loc[average[
ax3.set_title('Weather is Light Snow/Rain')
ax3.set_xlabel('Hour')
ax3.set_ylabel('number of Rentals')
ax3.legend(prop={'size': 12})

ax4.scatter(average.loc[average['weather']==4,['hour']], average.loc[average[
ax4.scatter(average.loc[average['weather']==4,['hour']], average.loc[average[
ax4.set_title('Weather is Heavy Rain/Snow')
ax4.set_xlabel('Hour')
ax4.set_ylabel('number of Rentals')
ax4.legend(prop={'size': 12})
plt.tight_layout()

f.savefig('fig/weather.png')
```



your answer here

The relationship still maitains for weather = 1 ,2, 3 when the weather is nice or moderate, although the number of retental decreased as weather getting worse. However, when weather = 4 with heavy rain/snow, the relationship disappeared and almost no rentals for the entire day.

2.4 Make a new dataframe with the following subset of attributes from the previous dataset and with each entry being just one day:

- `dteday` , the timestamp for that day (fine to set to noon or any other time)
- `weekday` , the day of the week
- `weather` , the most severe weather that day
- `season` , the season that day falls in
- `temp` , the average temperature (normalized)
- `atemp` , the average atemp that day (normalized)
- `windspeed` , the average windspeed that day (normalized)

- `hum` , the average humidity that day (normalized)
- `casual` , the **total** number of rentals by casual users
- `registered` , the **total** number of rentals by registered users
- `counts` , the **total** number of rentals of that day

Name this dataframe `bikes_by_day` .

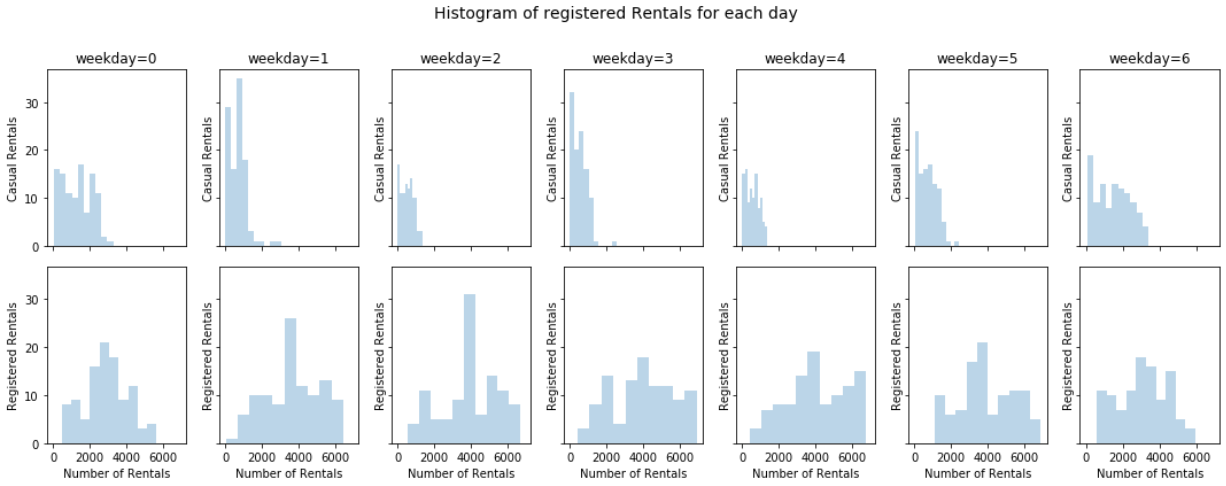
Make a plot showing the *distribution* of the number of casual and registered riders on each day of the week.

In [14]:
```
# your code here
bikes_df['dteday'] = bikes_df['dteday'].dt.normalize()

bikes_by_day = bikes_df.groupby(['dteday']).agg({'weekday':'mean',
                                                 'weather':'max',
                                                 'season':'mean',
                                                 'temp':'mean',
                                                 'atemp':'mean',
                                                 'windspeed':'mean',
                                                 'hum':'mean',
                                                 'casual':'sum',
                                                 'registered':'sum',
                                                 'counts':'sum'}).reset_index

bikes_by_day.head()
```

Out[14]:

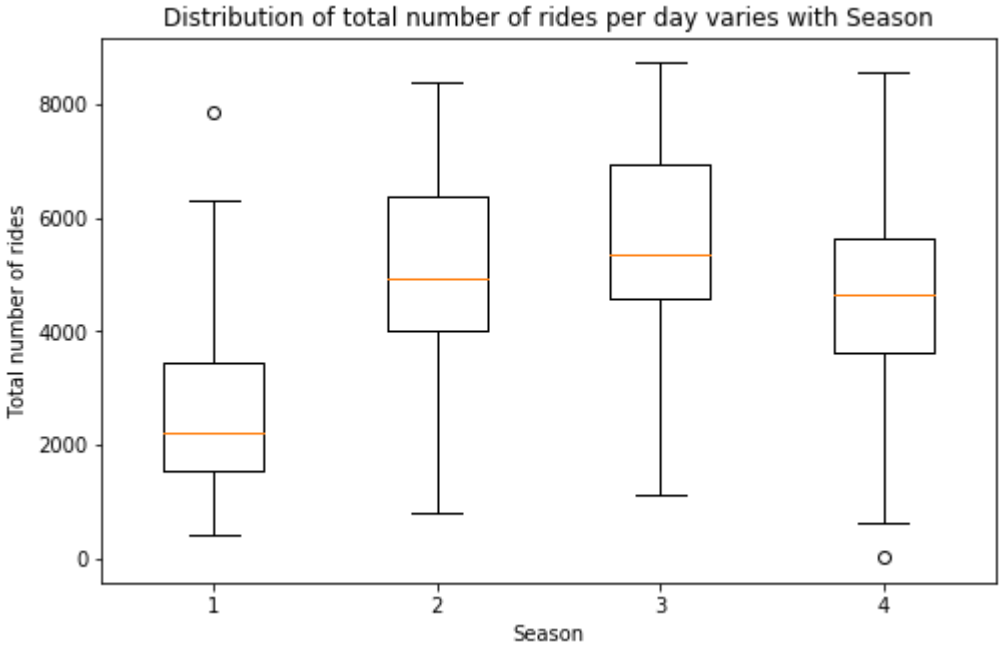
	dteday	weekday	weather	season	temp	atemp	windspeed	hum	casual	registered
0	2011-01-01	6	3	1	0.344167	0.363625	0.160446	0.805833	331	654
1	2011-01-02	0	3	1	0.363478	0.353739	0.248539	0.696087	131	670
2	2011-01-03	1	1	1	0.196364	0.189405	0.248309	0.437273	120	1229
3	2011-01-04	2	2	1	0.200000	0.212122	0.160296	0.590435	108	1454
4	2011-01-05	3	1	1	0.226957	0.229270	0.186900	0.436957	82	1518

In [15]:
```
# your code here
f, axarr = plt.subplots(2, 7, figsize=(15, 6), sharex=True, sharey=True)

for i in range(7):
    day = bikes_by_day[bikes_by_day['weekday']==i]
    axarr[0, i].hist(day['casual'], alpha=0.3)
    axarr[0, i].set_title('weekday=%i'%i)
    #axarr[0, i].set_xlabel('Number of Rentals')
    axarr[0, i].set_ylabel('Casual Rentals')

    day = bikes_by_day[bikes_by_day['weekday']==i]
    axarr[1, i].hist(day['registered'],alpha=0.3)
    #axarr[1, i].set_title('weekday=%i'%i)
    axarr[1, i].set_xlabel('Number of Rentals')
    axarr[1, i].set_ylabel('Registered Rentals')

f.suptitle('Histogram of registered Rentals for each day',fontsize=14)
f.tight_layout()
f.subplots_adjust(top=0.85)
```



2.5 Use `bikes_by_day` to visualize how the distribution of total number of rides per day (casual and registered riders combined) varies with the season. Do you see any outliers? Here we use the pyplot's boxplot function definition of an outlier as any value 1.5 times the IQR above the 75th percentile or 1.5 times the IQR below the 25th percentiles. If you see any outliers, identify those dates and investigate if they are a chance occurence, an error in the data collection, or a significant event (an online search of those date(s) might help).

In [16]:
```
# your code here
winter = bikes_by_day.loc[bikes_by_day['season']==1]
spring = bikes_by_day.loc[bikes_by_day['season']==2]
summer = bikes_by_day.loc[bikes_by_day['season']==3]
fall = bikes_by_day.loc[bikes_by_day['season']==4]
data = [winter['counts'],spring['counts'],summer['counts'],fall['counts']]

f, ax = plt.subplots(1, 1, figsize=(8, 5))
ax.set_xlabel('Season')
ax.set_ylabel('Total number of rides')
ax.set_title('Distribution of total number of rides per day varies with Seaso
ax.boxplot(data)
f.savefig('fig/season.png')
```



In [17]:
```
# your code here
print(winter.temp.mean())
print(spring.temp.mean())
winter.loc[winter['counts'].idxmax()]
```

```
0.29774755724431445
0.5444051275992435
```

Out[17]:
```
dteday        2012-03-17 00:00:00
weekday                         6
weather                         2
season                          1
temp                     0.514167
atemp                    0.505046
windspeed                0.110704
hum                      0.755833
casual                       3155
registered                   4681
counts                       7836
Name: 441, dtype: object
```

In [18]:
```
# your code here
fall.loc[fall['counts'].idxmin()]
```

Out[18]:
```
dteday        2012-10-29 00:00:00
weekday                         1
weather                         3
season                          4
temp                         0.44
atemp                      0.4394
windspeed                  0.3582
hum                          0.88
casual                          2
registered                     20
counts                         22
Name: 667, dtype: object
```

your answer here

3/17/2012 is an outlier date to have a much larger number of rentals compared to the rest of days during the summer. However, I do not think this is an error. The tempreture of the day is 0.51 which is much warmer than the rest of the winter and close to the average temperture in the spring time. In addition, weather and the windspeed are also very nice. Especially, it is Saturday. After a long cold winter, more people would like to go out at a warm and nice Saturday.

10/29/2018 is the day Hurricane Sandy, one of the worst hurricane, landed the east coast. It makes sense people stayed home for such day.

Question 3: Prepare the data for Regression

In order to build and evaluate our regression models, a little data cleaning is needed. In this problem, we will explicitly create binary variables to represent the categorical predictors, set up the train-test split in a careful way, remove ancillary variables, and do a little data exploration that will be useful to consider in the regression models later.

3.1 Using `bikes_df` , with hourly data about rentals, convert the categorical attributes ('season', 'month', 'weekday', 'weather') into multiple binary attributes using **one-hot encoding**.

3.2 Split the updated `bikes_df` dataset in a train and test part. Do this in a 'stratified' fashion, ensuring that all months are equally represented in each set. Explain your choice for a splitting algorithm.

3.3 Although we asked you to create your train and test set, but for consistency and easy checking, we ask that for the rest of this problem set you use the train and test set provided in the he files `data/BSS_train.csv` and `data/BSS_test.csv` . Read these two files into dataframes `BSS_train` and `BSS_test` , respectively. Remove the `dteday` column from both the train and the test dataset (its format cannot be used for analysis). Also, remove any predictors that would make predicting the `count` trivial. Note we gave more meaningful names to the one-hot encoded variables.

Answers

3.1 Using `bikes_df`, with hourly data about rentals, convert the categorical attributes ('season', 'month', 'weekday', 'weather') into multiple binary attributes using one-hot encoding.

In [19]:
```
# your code here
bikes_df_recoded = pd.get_dummies(bikes_df, columns=['season', 'month', 'week
```

In [20]:
```
#your code here
bikes_df_recoded.columns
```

Out[20]:
```
Index(['dteday', 'hour', 'holiday', 'workingday', 'temp', 'atemp', 'hum',
       'windspeed', 'casual', 'registered', 'year', 'counts', 'season_2',
       'season_3', 'season_4', 'month_2', 'month_3', 'month_4', 'month_5',
       'month_6', 'month_7', 'month_8', 'month_9', 'month_10', 'month_11',
       'month_12', 'weekday_1', 'weekday_2', 'weekday_3', 'weekday_4',
       'weekday_5', 'weekday_6', 'weather_2', 'weather_3', 'weather_4'],
      dtype='object')
```

3.2 Split the updated `bikes_df` dataset in a train and test part. Do this in a 'stratified' fashion, ensuring that all months are equally represented in each set. Explain your choice for a splitting algorithm.

In [21]:
```
# your code here
train_data, test_data = train_test_split(bikes_df_recoded, test_size = 0.2, s
```

In [22]:
```
# your code here
bikes_df['month'].value_counts().sort_index()
```

Out[22]:
```
1     1429
2     1341
3     1473
4     1437
5     1488
6     1440
7     1488
8     1475
9     1437
10    1451
11    1437
12    1483
Name: month, dtype: int64
```

In [23]:
```
# your code here
train_data[['month_2', 'month_3', 'month_4', 'month_5',
            'month_6', 'month_7', 'month_8', 'month_9',
            'month_10', 'month_11', 'month_12']].apply(pd.Series.value_counts
```

Out[23]:

	month_2	month_3	month_4	month_5	month_6	month_7	month_8	month_9	month_10	mor
0	12830	12725	12753	12713	12751	12713	12723	12753	12742	
1	1073	1178	1150	1190	1152	1190	1180	1150	1161	

In [24]:
```
# your code here
test_data[['month_2', 'month_3', 'month_4', 'month_5',
           'month_6', 'month_7', 'month_8', 'month_9',
           'month_10', 'month_11', 'month_12']].apply(pd.Series.value_counts)
```

Out[24]:

	month_2	month_3	month_4	month_5	month_6	month_7	month_8	month_9	month_10	mor
0	3208	3181	3189	3178	3188	3178	3181	3189	3186	
1	268	295	287	298	288	298	295	287	290	

your answer here

The argument of stratify=bikes_df['month'] along with the argument of test_size = 0.2 that I passed into train_test_split function makes the test set to have 20% of the total observations within each month group.

3.3 Although we asked you to create your train and test set, but for consistency and easy checking, we ask that for the rest of this problem set you use the train and test set provided in the he files `data/BSS_train.csv` and `data/BSS_test.csv` . Read these two files into dataframes `BSS_train` and `BSS_test` , respectively. Remove the `dteday` column from both the train and the test dataset (its format cannot be used for analysis). Also, remove any predictors that would make predicting the `count` trivial. Note we gave more meaningful names to the one-hot encoded variables.

In [25]:
```
# your code here
BSS_train = pd.read_csv("data/BSS_train.csv", index_col='Unnamed: 0')
BSS_test = pd.read_csv("data/BSS_test.csv", index_col='Unnamed: 0')
```

In [26]:
```
# your code here
BSS_train.drop(['dteday', 'casual', 'registered'], axis=1, inplace=True)
BSS_test.drop(['dteday', 'casual', 'registered'], axis=1, inplace=True)
```

In [27]:
```
1 BSS_test.columns
```

Out[27]:
```
Index(['hour', 'holiday', 'year', 'workingday', 'temp', 'atemp', 'hum',
       'windspeed', 'counts', 'spring', 'summer', 'fall', 'Feb', 'Mar', 'Apr',
       'May', 'Jun', 'Jul', 'Aug', 'Sept', 'Oct', 'Nov', 'Dec', 'Mon', 'Tue',
       'Wed', 'Thu', 'Fri', 'Sat', 'Cloudy', 'Snow', 'Storm'],
      dtype='object')
```

Question 4: Multiple Linear Regression

4.1 Use statsmodels to fit a multiple linear regression model to the training set using all the predictors (no interactions or polynomial terms) to predict `counts`, and report its R^2 score on the train and test sets.

4.2 Examine the estimated coefficients and report which ones are statistically significant at a significance level of 5% (p-value < 0.05). You should see some strange values, such as `July` producing 93 fewer rentals, all else equal, than January.

4.3 To diagnose the model, make two plots: first a histogram of the residuals, and second a plot of the residuals of the fitted model $e = y - \hat{y}$ as a function of the predicted value $\hat{y}$. Draw a horizontal line denoting the zero residual value on the Y-axis. What do the plots reveal about the OLS assumptions (linearity, constant variance, and normality)?

4.4 Perhaps we can do better via a model with polynomial terms. Build a dataset `X_train_poly` from `X_train` with added x^2 terms for `temp`, `hour`, and `humidity`. Are these polynomial terms important? How does predicted ridership change as each of `temp`, `hour`, and `humidity` increase?

4.5 The strange coefficients from 4.2 could also come from *multicolinearity*, where one or more predictors capture the same information as existing predictors. Why can multicolinearity lead to erroneous coefficient values? Create a temporary dataset `X_train_drop` that drops the following 'redundant' predictors from `X_train`: `workingday` `atemp` `spring` `summer` and `fall`. Fit a multiple linear regression model to `X_train_drop`. Are the estimates more sensible in this model?

Answers

4.1 Use statsmodels to fit a multiple linear regression model to the training set using all the predictors (no interactions or polynomial terms) to predict `counts`, and report its R^2 score on the train and test sets.

In [28]:
```
1 # your code here
2 x_train, y_train = BSS_train.drop(['counts'], axis=1), BSS_train['counts']
3 x_test, y_test = BSS_test.drop(['counts'], axis=1), BSS_test['counts']
```

In [29]:
```
1 # your code here
2 X_train = sm.add_constant(x_train)
3 OLSModel = sm.OLS(y_train, X_train)
4 ols = OLSModel.fit()
```

In [30]:
```
X_test = sm.add_constant(x_test)
print("R-squared of OLS for training set: %f" % ols.rsquared)
print("R-squared of OLS for testing set: %f" % r2_score(y_test, ols.predict(X
```

```
R-squared of OLS for training set: 0.406539
R-squared of OLS for testing set: 0.406386
```

4.2 Examine the estimated coefficients and report which ones are statistically significant at a significance level of 5% (p-value < 0.05). You should see some strange values, such as `July` producing 93 fewer rentals, all else equal, than January.

In [31]:
```
# your code here
ols.summary()
```

Out[31]:

OLS Regression Results

Dep. Variable:	counts	**R-squared:**	0.407
Model:	OLS	**Adj. R-squared:**	0.405
Method:	Least Squares	**F-statistic:**	316.8
Date:	Thu, 04 Oct 2018	**Prob (F-statistic):**	0.00
Time:	23:00:03	**Log-Likelihood:**	-88306.
No. Observations:	13903	**AIC:**	1.767e+05
Df Residuals:	13872	**BIC:**	1.769e+05
Df Model:	30		
Covariance Type:	nonrobust		

	coef	std err	t	P>|t|	[0.025	0.975]
const	-21.0830	8.641	-2.440	0.015	-38.020	-4.146
hour	7.2214	0.184	39.144	0.000	6.860	7.583
holiday	-18.0958	6.597	-2.743	0.006	-31.027	-5.165
year	76.3519	2.380	32.084	0.000	71.687	81.017
workingday	11.3178	2.751	4.114	0.000	5.926	16.710
temp	333.2482	44.162	7.546	0.000	246.684	419.812
atemp	74.6312	46.207	1.615	0.106	-15.940	165.202
hum	-205.4959	7.801	-26.343	0.000	-220.786	-190.205
windspeed	22.5168	10.753	2.094	0.036	1.439	43.595
spring	43.1541	7.417	5.818	0.000	28.615	57.693
summer	29.5426	8.773	3.367	0.001	12.346	46.739
fall	68.5953	7.492	9.156	0.000	53.911	83.280
Feb	-7.6430	5.966	-1.281	0.200	-19.336	4.050
Mar	-11.6737	6.665	-1.752	0.080	-24.737	1.390
Apr	-41.5244	9.878	-4.204	0.000	-60.886	-22.163
May	-33.2927	10.543	-3.158	0.002	-53.958	-12.628
Jun	-65.8039	10.716	-6.141	0.000	-86.809	-44.799
Jul	-93.4805	12.086	-7.734	0.000	-117.171	-69.789
Aug	-59.2081	11.832	-5.004	0.000	-82.401	-36.015
Sept	-16.0517	10.575	-1.518	0.129	-36.780	4.676
Oct	-16.1602	9.865	-1.638	0.101	-35.497	3.177
Nov	-25.8732	9.527	-2.716	0.007	-44.547	-7.199
Dec	-10.2043	7.614	-1.340	0.180	-25.128	4.719

Mon	-2.6601	2.978	-0.893	0.372	-8.498	3.177
Tue	-6.1425	3.208	-1.915	0.056	-12.430	0.145
Wed	2.2964	3.183	0.721	0.471	-3.943	8.536
Thu	-3.1611	3.185	-0.993	0.321	-9.404	3.082
Fri	2.8892	3.186	0.907	0.364	-3.355	9.133
Sat	14.9459	4.382	3.411	0.001	6.357	23.535
Cloudy	6.7868	2.900	2.341	0.019	1.103	12.470
Snow	-28.2859	4.819	-5.870	0.000	-37.731	-18.841
Storm	42.3569	98.377	0.431	0.667	-150.475	235.189

Omnibus:	2831.359	**Durbin-Watson:**	0.755
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	5657.789
Skew:	1.224	**Prob(JB):**	0.00
Kurtosis:	4.943	**Cond. No.**	1.17e+16

Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
[2] The smallest eigenvalue is 1.87e-26. This might indicate that there are
strong multicollinearity problems or that the design matrix is singular.

In [422]:
```
# your code here
ols.pvalues[ols.pvalues < 0.05]
```

Out[422]:
```
const          1.470264e-02
hour           0.000000e+00
holiday        6.095043e-03
year          6.205883e-218
workingday     3.905740e-05
temp           4.767468e-14
hum           2.797780e-149
windspeed      3.628163e-02
spring         6.082058e-09
summer         7.609902e-04
fall           6.106365e-20
Apr            2.640964e-05
May            1.592599e-03
Jun            8.447047e-10
Jul            1.110753e-14
Aug            5.685359e-07
Nov            6.619949e-03
Sat            6.490550e-04
Cloudy         1.926802e-02
Snow           4.454966e-09
dtype: float64
```

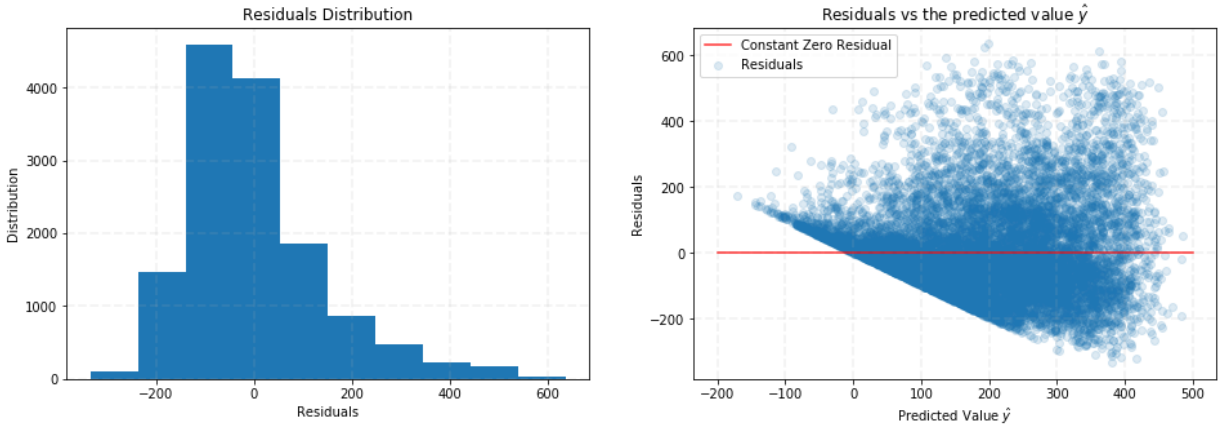
your answer here

July is indeed producing 93 fewer rentals, all else equal, than January.

4.3 To diagnose the model, make two plots: first a histogram of the residuals, and second a plot of the residuals of the fitted model $e = y - \hat{y}$ as a function of the predicted value $\hat{y}$. Draw a horizontal line denoting the zero residual value on the Y-axis. What do the plots reveal about the OLS assumptions (linearity, constant variance, and normality)?

In [32]:
```
# your code here
fig, ax = plt.subplots(1, 2, figsize=(16, 5))

ax[0].grid(True, lw=1.75, ls='--', alpha=0.15)
ax[0].hist(ols.resid)
ax[0].set_title(r'Residuals Distribution')
ax[0].set_xlabel(r'Residuals')
ax[0].set_ylabel(r'Distribution')

ax[1].grid(True, lw=1.75, ls='--', alpha=0.15)
ax[1].scatter(ols.fittedvalues, ols.resid, alpha=0.15, label='Residuals')
ax[1].plot(np.linspace(-200, 500, 2), [0] * len(np.linspace(-200, 500, 2)), c
ax[1].set_title(r'Residuals vs the predicted value $\hat{y}$')
ax[1].set_xlabel(r'Predicted Value $\hat{y}$')
ax[1].set_ylabel(r'Residuals')
ax[1].legend(loc='upper left')
```

Out[32]:
```
<matplotlib.legend.Legend at 0x23543f12d68>
```



your answer here

- linearity: Right plot shows clear non-linear relationship, which constradicts the linearity assumption.
- constant variance: Right plot shows that residuals are not evenly distributed around zero, which contradicts constant variance assumption.
- normality: Left plot shows that residual distribution is right skewed, which contradicts normality assumption.

4.4 Perhaps we can do better via a model with polynomial terms. Build a dataset `X_train_poly` from `X_train` with added x^2 terms for `temp`, `hour`, and `humidity`. Are

these polynomial terms important? How does predicted ridership change as each of `temp`, `hour`, and `humidity` increase?

In [33]:
```
# your code here
x_train_poly = x_train.copy().reset_index(drop=True)
x_train_poly = x_train_poly.drop(['temp', 'hour', 'hum'], axis=1)
print(x_train_poly.shape)
print(x_train_poly.index)
x_train_poly.head()
```

```
(13903, 28)
RangeIndex(start=0, stop=13903, step=1)
```

Out[33]:

	holiday	year	workingday	atemp	windspeed	spring	summer	fall	Feb	Mar	...	Dec	Mon
0	0	0	0	0.2879	0.0	0	0	0	0	0	...	0	0
1	0	0	0	0.2727	0.0	0	0	0	0	0	...	0	0
2	0	0	0	0.2727	0.0	0	0	0	0	0	...	0	0
3	0	0	0	0.2879	0.0	0	0	0	0	0	...	0	0
4	0	0	0	0.2879	0.0	0	0	0	0	0	...	0	0

5 rows × 28 columns

In [34]:
```
tra = PolynomialFeatures(2, include_bias=False)
```

In [35]:
```
array_temp = tra.fit_transform(x_train['temp'].values.reshape(-1,1))
df_temp = pd.DataFrame(array_temp)
df_temp = df_temp.rename(columns={0: 'temp', 1: 'temp_2'})
print(df_temp.shape)
df_temp.head()
```

```
(13903, 2)
```

Out[35]:

	temp	temp_2
0	0.24	0.0576
1	0.22	0.0484
2	0.22	0.0484
3	0.24	0.0576
4	0.24	0.0576

In [36]:
```
array_hour = tra.fit_transform(x_train['hour'].values.reshape(-1,1))
df_hour = pd.DataFrame(array_hour)
df_hour = df_hour.rename(columns={0: 'hour', 1: 'hour_2'})
print(df_hour.shape)
df_hour.head()
```

```
(13903, 2)
```

Out[36]:

	hour	hour_2
0	0.0	0.0
1	1.0	1.0
2	2.0	4.0
3	3.0	9.0
4	4.0	16.0

In [37]:
```
array_hum = tra.fit_transform(x_train['hum'].values.reshape(-1,1))
df_hum = pd.DataFrame(array_hum)
df_hum = df_hum.rename(columns={0: 'hum', 1: 'hum_2'})
print(df_hum.shape)
df_hum.head()
```

```
(13903, 2)
```

Out[37]:

	hum	hum_2
0	0.81	0.6561
1	0.80	0.6400
2	0.80	0.6400
3	0.75	0.5625
4	0.75	0.5625

In [38]:
```
x_train_poly = x_train_poly.merge(df_temp, left_index=True, right_index=True)
x_train_poly = x_train_poly.merge(df_hour, left_index=True, right_index=True)
x_train_poly = x_train_poly.merge(df_hum, left_index=True, right_index=True)
```

In [39]:
```
y_train_poly = y_train.copy().reset_index(drop=True)
X_train_poly = sm.add_constant(x_train_poly)
OLSModel = sm.OLS(y_train_poly, X_train_poly)
ols_poly = OLSModel.fit()
ols_poly.summary()
```

Out[39]:

OLS Regression Results

Dep. Variable:	counts	**R-squared:**	0.501
Model:	OLS	**Adj. R-squared:**	0.500
Method:	Least Squares	**F-statistic:**	421.8
Date:	Thu, 04 Oct 2018	**Prob (F-statistic):**	0.00
Time:	23:00:17	**Log-Likelihood:**	-87102.
No. Observations:	13903	**AIC:**	1.743e+05
Df Residuals:	13869	**BIC:**	1.745e+05
Df Model:	33		
Covariance Type:	nonrobust		

	coef	std err	t	P>|t|	[0.025	0.975]
const	-185.2131	14.016	-13.214	0.000	-212.687	-157.739
holiday	-13.0061	6.056	-2.148	0.032	-24.877	-1.135
year	81.0305	2.199	36.854	0.000	76.721	85.340
workingday	13.2894	2.524	5.265	0.000	8.342	18.237
atemp	67.4957	43.532	1.550	0.121	-17.833	152.824
windspeed	-6.9100	9.920	-0.697	0.486	-26.354	12.534
spring	43.7116	6.805	6.424	0.000	30.374	57.049
summer	33.9087	8.066	4.204	0.000	18.098	49.720
fall	72.1937	6.878	10.497	0.000	58.712	85.675
Feb	1.6487	5.538	0.298	0.766	-9.207	12.504
Mar	9.5583	6.304	1.516	0.129	-2.798	21.914
Apr	-10.7152	9.238	-1.160	0.246	-28.824	7.393
May	-2.7388	9.789	-0.280	0.780	-21.926	16.449
Jun	-23.0368	9.922	-2.322	0.020	-42.485	-3.588
Jul	-53.5230	11.163	-4.795	0.000	-75.405	-31.642
Aug	-23.6944	10.965	-2.161	0.031	-45.188	-2.201
Sept	10.9055	9.887	1.103	0.270	-8.475	30.286
Oct	2.8452	9.262	0.307	0.759	-15.309	20.999
Nov	-16.5926	8.897	-1.865	0.062	-34.032	0.846
Dec	-6.9106	7.078	-0.976	0.329	-20.784	6.963
Mon	-2.4620	2.731	-0.901	0.367	-7.816	2.892

Tue	-3.8629	2.943	-1.313	0.189	-9.631	1.905
Wed	2.1275	2.920	0.729	0.466	-3.596	7.851
Thu	-0.2540	2.922	-0.087	0.931	-5.981	5.473
Fri	4.7347	2.923	1.620	0.105	-0.995	10.465
Sat	16.7983	4.021	4.178	0.000	8.917	24.679
Cloudy	-8.4327	2.680	-3.146	0.002	-13.687	-3.179
Snow	-47.3269	4.583	-10.327	0.000	-56.310	-38.344
Storm	35.5800	90.246	0.394	0.693	-141.315	212.475
temp	132.7247	58.298	2.277	0.023	18.452	246.997
temp_2	109.4437	36.470	3.001	0.003	37.958	180.930
hour	39.5786	0.662	59.777	0.000	38.281	40.876
hour_2	-1.3570	0.027	-50.567	0.000	-1.410	-1.304
hum	11.8636	36.114	0.329	0.743	-58.925	82.652
hum_2	-108.7057	28.944	-3.756	0.000	-165.440	-51.971

Omnibus:	2946.543	**Durbin-Watson:**	0.890
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	6094.033
Skew:	1.253	**Prob(JB):**	0.00
Kurtosis:	5.059	**Cond. No.**	1.35e+16

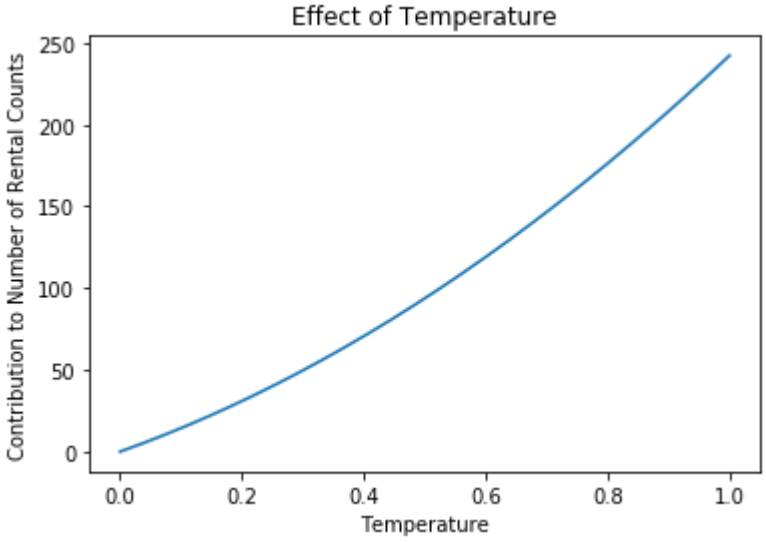
Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
[2] The smallest eigenvalue is 4.56e-24. This might indicate that there are
strong multicollinearity problems or that the design matrix is singular.

In [40]:
```
x_test_poly = x_test.copy().reset_index(drop=True)
x_test_poly = x_test_poly.drop(['temp', 'hour', 'hum'], axis=1)

array_temp = tra.fit_transform(x_test['temp'].values.reshape(-1,1))
df_temp = pd.DataFrame(array_temp)
df_temp = df_temp.rename(columns={0: 'temp', 1: 'temp_2'})

array_hour = tra.fit_transform(x_test['hour'].values.reshape(-1,1))
df_hour = pd.DataFrame(array_hour)
df_hour = df_hour.rename(columns={0: 'hour', 1: 'hour_2'})

array_hum = tra.fit_transform(x_test['hum'].values.reshape(-1,1))
df_hum = pd.DataFrame(array_hum)
df_hum = df_hum.rename(columns={0: 'hum', 1: 'hum_2'})

x_test_poly = x_test_poly.merge(df_temp, left_index=True, right_index=True)
x_test_poly = x_test_poly.merge(df_hour, left_index=True, right_index=True)
x_test_poly = x_test_poly.merge(df_hum, left_index=True, right_index=True)

X_test_poly = sm.add_constant(x_test_poly)
y_test_poly = y_test.copy().reset_index(drop=True)
```

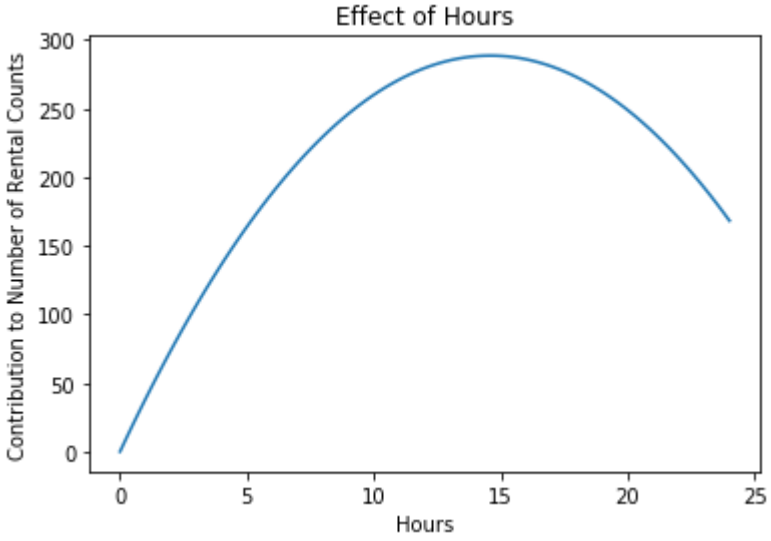
In [41]:
```
print("R-squared of OLS with polynomial terms for training set: %f" % ols_pol
print("R-squared of OLS with polynomial terms for testing set: %f" % r2_score
```

```
R-squared of OLS with polynomial terms for training set: 0.500904
R-squared of OLS with polynomial terms for testing set: 0.496531
```

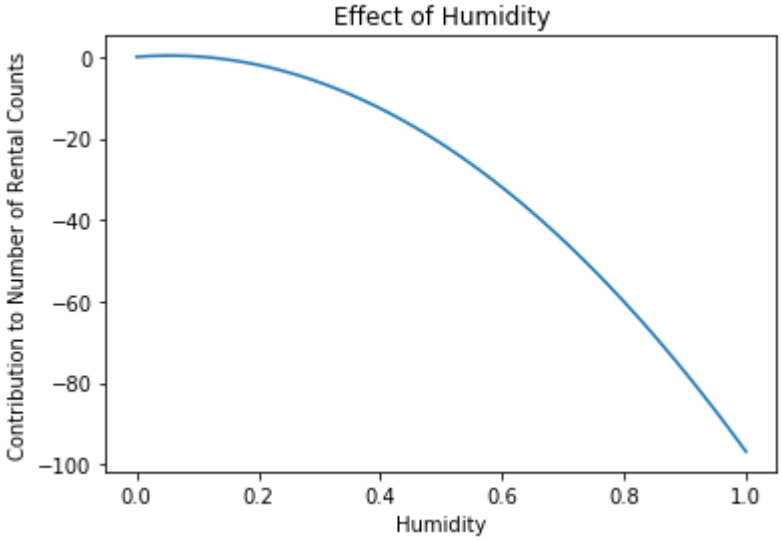
In [42]:
```
x_vals = np.linspace(0, 1, 1000)
y_vals = ols_poly.params['temp'] * x_vals + ols_poly.params['temp_2'] * x_val
plt.plot(x_vals, y_vals)
plt.title('Effect of Temperature')
plt.xlabel('Temperature')
plt.ylabel('Contribution to Number of Rental Counts')
plt.show()
```



In [43]:
```
x_vals = np.linspace(0, 24, 100)
y_vals = ols_poly.params['hour'] * x_vals + ols_poly.params['hour_2'] * x_val
plt.plot(x_vals, y_vals)
plt.title('Effect of Hours')
plt.xlabel('Hours')
plt.ylabel('Contribution to Number of Rental Counts')
plt.show()
```



In [44]:
```
x_vals = np.linspace(0, 1, 100)
y_vals = ols_poly.params['hum'] * x_vals + ols_poly.params['hum_2'] * x_vals*
plt.plot(x_vals, y_vals)
plt.title('Effect of Humidity')
plt.xlabel('Humidity')
plt.ylabel('Contribution to Number of Rental Counts')
plt.show()
```



your answer here

The OLS model with polynomial terms vs origial OLS model:

- R-squared in training set: 0.500904 vs 0.406539

- R-squared in testing set: 0.496531 vs 0.406386
- AIC: 1.743e+05 vs 1.767e+05
- BIC: 1.745e+05 vs 1.769e+05

So OLS polynomil model is better.

Predicted ridership with respect to

- $temperature$: Statistically significant with p-value < 0.05. All else equal, 1 unit of temperature increase produces 333 more rentals.
- $temperature^2$: Statistically significant with p-value < 0.05. All else equal, 1 unit of 2nd degree polynomial in temperature increase produces 109 more rentals. We expect rentals to increase at higher pace with higher temperature.
- $hour$: Statistically significant with p-value < 0.05. All else equal, 1 unit of hour increase produces 39 more rentals.
- $hour^2$: Statistically significant with p-value < 0.05. All else equal, 1 unit of 2nd degree polynomial in hour increase produces 1 fewer rentals. We expect rentals to increase during the day which peaks at around 3pm, but then decrease after that.
- $humidity$: Statistically not significant with p-value > 0.05. No strong relationship can be inferred.
- $humidity^2$: Statistically significant with p-value < 0.05. All else equal, 1 unit of 2nd degree polynomial in humidity increase produces 109 fewer rentals.

4.5 The strange coefficients from 4.2 could also come from *multicolinearity*, where one or more predictors capture the same information as existing predictors. Why can multicolinearity lead to erroneous coefficient values? Create a temporary dataset `X_train_drop` that drops the following 'redundant' predictors from `X_train` : `workingday` `atemp` `spring` `summer` and `fall` . Fit a multiple linear regression model to `X_train_drop` . Are the estimates more sensible in this model?

In [45]:
```
# your code here
BSS_train_drop = BSS_train.drop(['workingday', 'atemp', 'spring', 'summer', '
BSS_test_drop = BSS_test.drop(['workingday', 'atemp', 'spring', 'summer', 'fa
x_train_drop, y_train_drop = BSS_train_drop.drop(['counts'], axis=1), BSS_tra
x_test_drop, y_test_drop = BSS_test_drop.drop(['counts'], axis=1), BSS_test_d
```

In [46]:
```
X_train_drop = sm.add_constant(x_train_drop)
OLSModel = sm.OLS(y_train_drop, X_train_drop)
ols_drop = OLSModel.fit()
```

In [47]:
```
X_test_drop = sm.add_constant(x_test_drop)
print("R-squared of OLS after dropping redundant predictors for training set:
print("R-squared of OLS after dropping redundant predictors for testing set:
```

```
R-squared of OLS after dropping redundant predictors for training set: 0.401686
R-squared of OLS after dropping redundant predictors for testing set: 0.400475
```

In [439]:
```
1 ols_drop.summary()
```

Out[439]:

OLS Regression Results

Dep. Variable:	counts	**R-squared:**	0.402
Model:	OLS	**Adj. R-squared:**	0.401
Method:	Least Squares	**F-statistic:**	358.3
Date:	Thu, 04 Oct 2018	**Prob (F-statistic):**	0.00
Time:	20:38:31	**Log-Likelihood:**	-88363.
No. Observations:	13903	**AIC:**	1.768e+05
Df Residuals:	13876	**BIC:**	1.770e+05
Df Model:	26		
Covariance Type:	nonrobust		

	coef	std err	t	P>|t|	[0.025	0.975]
const	-20.0627	8.541	-2.349	0.019	-36.805	-3.321
hour	7.2378	0.185	39.095	0.000	6.875	7.601
holiday	-35.8906	7.395	-4.854	0.000	-50.385	-21.396
year	76.3039	2.389	31.945	0.000	71.622	80.986
temp	406.2359	13.279	30.593	0.000	380.208	432.264
hum	-201.5103	7.800	-25.835	0.000	-216.799	-186.221
windspeed	11.9668	10.448	1.145	0.252	-8.512	32.446
Feb	-7.6897	5.986	-1.285	0.199	-19.422	4.043
Mar	2.8889	6.158	0.469	0.639	-9.182	14.960
Apr	1.0237	6.594	0.155	0.877	-11.902	13.950
May	7.2426	7.613	0.951	0.341	-7.680	22.165
Jun	-30.6611	8.346	-3.674	0.000	-47.020	-14.302
Jul	-67.7620	9.062	-7.477	0.000	-85.525	-49.999
Aug	-34.2712	8.628	-3.972	0.000	-51.183	-17.359
Sept	20.6406	7.882	2.619	0.009	5.191	36.090
Oct	50.7025	6.823	7.431	0.000	37.329	64.076
Nov	42.3211	6.111	6.926	0.000	30.344	54.299
Dec	34.2134	5.952	5.748	0.000	22.546	45.881
Mon	9.2907	4.570	2.033	0.042	0.333	18.248
Tue	4.7929	4.442	1.079	0.281	-3.914	13.500
Wed	13.2143	4.417	2.992	0.003	4.557	21.871
Thu	8.0051	4.445	1.801	0.072	-0.708	16.718
Fri	13.0474	4.429	2.946	0.003	4.367	21.728
Sat	14.1461	4.397	3.217	0.001	5.528	22.764

Cloudy	6.7192	2.909	2.310	0.021	1.018	12.421
Snow	-29.1668	4.828	-6.041	0.000	-38.631	-19.703
Storm	40.3125	98.759	0.408	0.683	-153.267	233.893

Omnibus:	2850.389	**Durbin-Watson:**	0.749
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	5702.134
Skew:	1.231	**Prob(JB):**	0.00
Kurtosis:	4.944	**Cond. No.**	1.13e+03

Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
[2] The condition number is large, 1.13e+03. This might indicate that there are
strong multicollinearity or other numerical problems.

your answer here

Multicolinearity can lead correlated factors to compete with each other to explain the variation in response variable, which will overestimate the standard error, and thus underestimate the t-stats of the correlated factors.

The estimates are more sensible in the model:

- temp vs atemp: The origianl coefficient on atemp is now added to the factor temp, which makes temp more significnt.
- workingday vs weekday: Droping workingday from the regression model makes some of the weekdays more significant.
- season vs month: Droping season from the regression model makes some of the months more signficant.

R-squared, AIC and BIC are all comparable before and after droping the redundant variables.

Question 5: Subset Selection

Perhaps we can automate finding a good set of predictors. This question focuses on forward stepwise selection, where predictors are added to the model one by one.

5.1 Implement forward step-wise selection to select a minimal subset of predictors that are related to the response variable. Run your code on the richest dataset, `X_train_poly` , and determine which predictors are selected.

We require that you implement the method **from scratch**. You may use the Bayesian Information Criterion (BIC) to choose the best subset size.

Note: Implementing from scratch means you are not allowed to use a solution provided by a Python library, such as sklearn or use a solution you found on the internet. You have to write all of the code on your own. However you MAY use the `model.bic` attribute implemented in statsmodels.

5.2 Does forward selection eliminate one or more of the colinear predictors we dropped in Question 4.5 (`workingday` `atemp` `spring` `summer` and `fall`)? If any of the five predictors are not dropped, explain why.

5.3 Fit the linear regression model using the identified subset of predictors to the training set. How do the train and test R^2 scores for this fitted step-wise model compare with the train and test R^2 scores from the polynomial model fitted in Question 4.4?

Answers

5.1 Implement forward step-wise selection to select a minimal subset of predictors that are related to the response variable. Run your code on the richest dataset, `X_train_poly` , and determine which predictors are selected.

We require that you implement the method from scratch. You may use the Bayesian Information Criterion (BIC) to choose the best subset size.

Note: Implementing from scratch means you are not allowed to use a solution provided by a Python library, such as sklearn or use a solution you found on the internet. You have to write all of the code on your own. However you MAY use the `model.bic` attribute implemented in statsmodels.

In [48]:
```
# your code here
print(x_train_poly.shape, y_train_poly.shape)
x_train_poly.head()
```

```
(13903, 34) (13903,)
```

Out[48]:

	holiday	year	workingday	atemp	windspeed	spring	summer	fall	Feb	Mar	...	Sat	Cloudy
0	0	0	0	0.2879	0.0	0	0	0	0	0	...	1	0
1	0	0	0	0.2727	0.0	0	0	0	0	0	...	1	0
2	0	0	0	0.2727	0.0	0	0	0	0	0	...	1	0
3	0	0	0	0.2879	0.0	0	0	0	0	0	...	1	0
4	0	0	0	0.2879	0.0	0	0	0	0	0	...	1	0

5 rows × 34 columns

In [49]:

```
# your code here
def find_best_subset(x, y, best_subset):
    best_bic = np.inf
    best_model_data = None

    predictors = x.columns
    candidates = set(best_subset) ^ set(predictors)
    subsets = [best_subset + [c] for c in candidates]

    # Inner loop: iterate through subsets
    for subset in subsets:

        # Fit regression model using 'subset'
        # Keep track of subset with lowest BIC
        features = list(subset)
        x_subset = sm.add_constant(x[features])

        model = sm.OLS(y, x_subset).fit()
        bic = model.bic

        # Check if we get a lower BIC value than than current min BIC.
        # If so, update our best subset
        if bic < best_bic:
            best_bic = bic
            best_model_data = {
                'subset': features,
                'model': model
            }
    return best_model_data
```

In [50]:
```
def forward_step_wise_selection(x, y):
    """Forward step wise predictor combinations

    Parameters:
    -----------
    x : DataFrame of predictors/features
    y : response varible


    Returns:
    -----------

    Dataframe of model comparisons and OLS Model with
    lowest BIC for subset with the lowest BIC

    """

    predictors = x.columns

    stats = []
    models = dict()
    best_subset = []

    # Outer loop: iterate over sizes 1, 2 .... d
    for k in range(1, len(predictors)):
        best_size_k_model = find_best_subset(x, y, best_subset)
        best_subset = best_size_k_model['subset']
        best_model = best_size_k_model['model']

        stats.append({
            'k': k,
            'formula': "y ~ {}".format(' + '.join(best_subset)),
            'bic': best_model.bic,
            'features': best_subset,
            'model': best_model
        })
        models[k] = best_model

    return pd.DataFrame(stats), models
```

In [51]:
```
stats, models = forward_step_wise_selection(x_train_poly, y_train_poly)
```

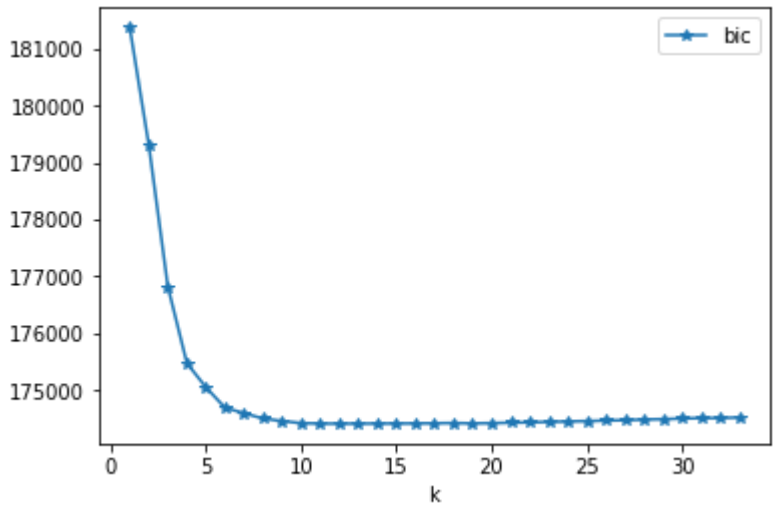
In [52]:
```
1 stats[['k', 'bic', 'features']]
```

Out[52]:

	k	bic	features
0	1	181379.567621	[temp]
1	2	179312.318838	[temp, hour]
2	3	176822.942085	[temp, hour, hour_2]
3	4	175475.152084	[temp, hour, hour_2, year]
4	5	175050.969747	[temp, hour, hour_2, year, hum_2]
5	6	174691.049974	[temp, hour, hour_2, year, hum_2, fall]
6	7	174595.468053	[temp, hour, hour_2, year, hum_2, fall, Jul]
7	8	174500.568450	[temp, hour, hour_2, year, hum_2, fall, Jul, S...
8	9	174457.138957	[temp, hour, hour_2, year, hum_2, fall, Jul, S...
9	10	174420.945354	[temp, hour, hour_2, year, hum_2, fall, Jul, S...
10	11	174412.175681	[temp, hour, hour_2, year, hum_2, fall, Jul, S...
11	12	174411.588858	[temp, hour, hour_2, year, hum_2, fall, Jul, S...
12	13	174411.851241	[temp, hour, hour_2, year, hum_2, fall, Jul, S...
13	14	174413.244261	[temp, hour, hour_2, year, hum_2, fall, Jul, S...
14	15	174414.014055	[temp, hour, hour_2, year, hum_2, fall, Jul, S...
15	16	174415.376751	[temp, hour, hour_2, year, hum_2, fall, Jul, S...
16	17	174417.944690	[temp, hour, hour_2, year, hum_2, fall, Jul, S...
17	18	174422.737349	[temp, hour, hour_2, year, hum_2, fall, Jul, S...
18	19	174415.093484	[temp, hour, hour_2, year, hum_2, fall, Jul, S...
19	20	174420.804369	[temp, hour, hour_2, year, hum_2, fall, Jul, S...
20	21	174427.379862	[temp, hour, hour_2, year, hum_2, fall, Jul, S...
21	22	174434.833497	[temp, hour, hour_2, year, hum_2, fall, Jul, S...
22	23	174442.340363	[temp, hour, hour_2, year, hum_2, fall, Jul, S...
23	24	174449.298473	[temp, hour, hour_2, year, hum_2, fall, Jul, S...
24	25	174457.016553	[temp, hour, hour_2, year, hum_2, fall, Jul, S...
25	26	174465.504295	[temp, hour, hour_2, year, hum_2, fall, Jul, S...
26	27	174473.659632	[temp, hour, hour_2, year, hum_2, fall, Jul, S...
27	28	174482.492449	[temp, hour, hour_2, year, hum_2, fall, Jul, S...
28	29	174491.549111	[temp, hour, hour_2, year, hum_2, fall, Jul, S...
29	30	174500.941093	[temp, hour, hour_2, year, hum_2, fall, Jul, S...
30	31	174510.368271	[temp, hour, hour_2, year, hum_2, fall, Jul, S...
31	32	174510.368271	[temp, hour, hour_2, year, hum_2, fall, Jul, S...
32	33	174519.802374	[temp, hour, hour_2, year, hum_2, fall, Jul, S...

In [53]:
```
1 stats.plot(x='k', y='bic', marker='*')
```

Out[53]: <matplotlib.axes._subplots.AxesSubplot at 0x235467515f8>



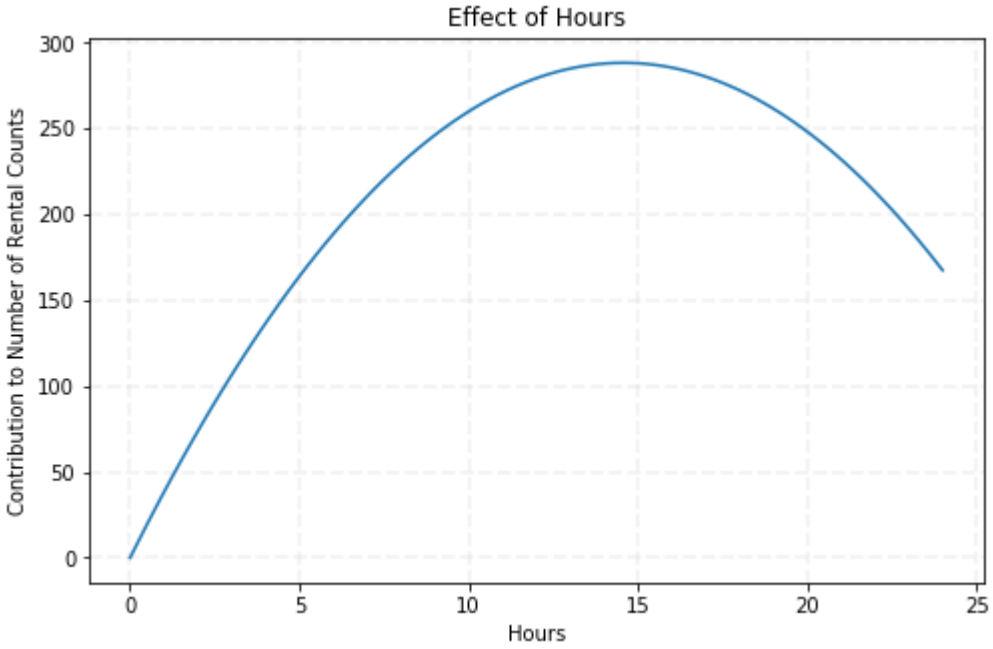
In [54]:
```
1 stats.info()
```

```
<class 'pandas.core.frame.DataFrame'>
RangeIndex: 33 entries, 0 to 32
Data columns (total 5 columns):
bic         33 non-null float64
features    33 non-null object
formula     33 non-null object
k           33 non-null int64
model       33 non-null object
dtypes: float64(1), int64(1), object(3)
memory usage: 1.4+ KB
```

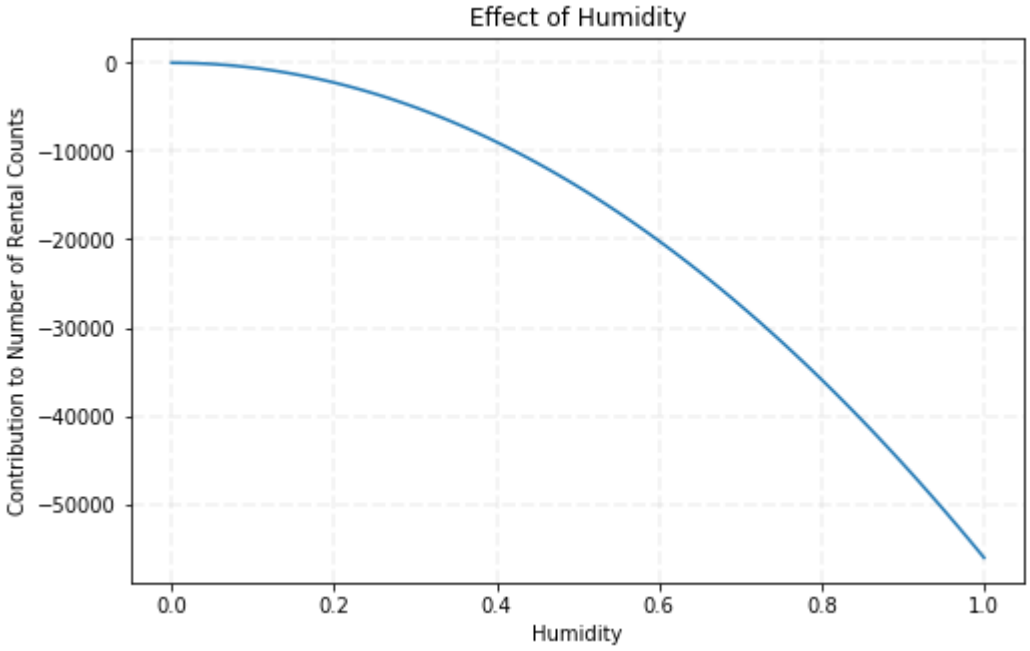
In [55]:
```
best_stat = stats.iloc[stats.bic.idxmin()]
print(best_stat.model.summary())
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                 counts   R-squared:                       0.498
Model:                            OLS   Adj. R-squared:                  0.497
Method:                 Least Squares   F-statistic:                     1148.
Date:                Thu, 04 Oct 2018   Prob (F-statistic):               0.00
Time:                        23:01:03   Log-Likelihood:                -87144.
No. Observations:               13903   AIC:                         1.743e+05
Df Residuals:                   13890   BIC:                         1.744e+05
Df Model:                          12
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const       -189.1223      5.283    -35.795      0.000    -199.479    -178.766
temp         310.9097      6.890     45.125      0.000     297.404     324.415
hour          39.5749      0.647     61.177      0.000      38.307      40.843
hour_2        -1.3582      0.026    -51.420      0.000      -1.410      -1.306
year          80.3153      2.182     36.804      0.000      76.038      84.593
hum_2        -97.3713      5.534    -17.595      0.000    -108.219     -86.524
fall          56.0162      2.797     20.026      0.000      50.533      61.499
Jul          -25.6107      4.679     -5.473      0.000     -34.783     -16.438
Snow         -48.1081      4.447    -10.819      0.000     -56.824     -39.392
spring        25.8171      2.910      8.873      0.000      20.114      31.520
Sept          29.2489      4.302      6.798      0.000      20.816      37.682
holiday      -27.4291      6.432     -4.265      0.000     -40.037     -14.822
Cloudy        -8.4814      2.666     -3.181      0.001     -13.707      -3.256
==============================================================================
Omnibus:                     2990.921   Durbin-Watson:                   0.884
Prob(Omnibus):                  0.000   Jarque-Bera (JB):             6264.394
Skew:                           1.265   Prob(JB):                         0.00
Kurtosis:                       5.102   Cond. No.                     1.85e+03
==============================================================================

Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is correctl
y specified.
[2] The condition number is large, 1.85e+03. This might indicate that there are
strong multicollinearity or other numerical problems.
```

In [56]:
```
fig, ax = plt.subplots(1, 1, figsize=(8,5))

x_vals_hour = np.linspace(0, 24, 100)
y_vals_hour = best_stat.model.params['hour'] * x_vals_hour + best_stat.model.
ax.grid(True, lw=1.75, ls='--', alpha=0.15)
ax.plot(x_vals_hour, y_vals_hour)
ax.set_title('Effect of Hours')
ax.set_xlabel('Hours')
ax.set_ylabel('Contribution to Number of Rental Counts')
fig.savefig("fig/hours.png")
```



In [57]:
```
fig, ax = plt.subplots(1, 1, figsize=(8, 5))

x_vals_hum = np.linspace(0, 1, 100)
y_vals_hum = best_stat.model.params['hum_2'] * x_vals_hour**2
ax.grid(True, lw=1.75, ls='--', alpha=0.15)
ax.plot(x_vals_hum, y_vals_hum)
ax.set_title('Effect of Humidity')
ax.set_xlabel('Humidity')
ax.set_ylabel('Contribution to Number of Rental Counts')
fig.savefig("fig/humidity.png")
```



5.2 Does forward selection eliminate one or more of the colinear predictors we dropped in Question 4.5 (`workingday` `atemp` `spring` `summer` and `fall`)? If any of the five predictors are not dropped, explain why.

your answer here

Yes, forward selection method eliminated workingday, atemp, and summer, but kept spring and fall.

Spring includes March, April, and May. It's not surprising that Spring is kept since none of the March, April, or May is included in the model. There is no perfect collinearity in the model that can replace Spring.

Fall includes September, October, and November. Even though September is included, Fall still includes information about October and November. There is no perfect collinearity in the model that can replace Fall.

5.3 Fit the linear regression model using the identified subset of predictors to the training set. How do the train and test R^2 scores for this fitted step-wise model compare with the train and test R^2 scores from the polynomial model fitted in Question 4.4?

In [58]:
```
# your code here
x_test_step = x_test_poly[best_stat.features]
X_test_step = sm.add_constant(x_test_step)
print("R-squared of OLS step-wise model for training set: %f" % best_stat.mod
print("Adjusted R-squared of OLS step-wise model for training set: %f" % best
print("R-squared of OLS step-wise model for testing set: %f" % r2_score(y_tes

print("R-squared of OLS polynomial model for training set: %f" % ols_poly.rsq
print("Adjusted R-squared of OLS polynomial model for training set: %f" % ols
print("R-squared of OLS polynomial model for testing set: %f" % r2_score(y_te
```

```
R-squared of OLS step-wise model for training set: 0.497927
Adjusted R-squared of OLS step-wise model for training set: 0.497494
R-squared of OLS step-wise model for testing set: 0.494310
R-squared of OLS polynomial model for training set: 0.500904
Adjusted R-squared of OLS polynomial model for training set: 0.499717
R-squared of OLS polynomial model for testing set: 0.496531
```

In [59]:
```
print("BIC of OLS step-wise model: %f" % best_stat.model.bic)
print("BIC of OLS polynomial model: %f" % ols_poly.bic)
```

```
BIC of OLS step-wise model: 174411.588858
BIC of OLS polynomial model: 174529.253392
```

your answer here

The polynomial model is slightly better than the step-wise model in terms of R-squared in training and testing set, but step-wise model is better in terms of BIC.

Written Report to the Administrators [20 pts]

Question 6

Write a short repost stating some of your findings on how the administrators can increase the bike share system's revenue. You might want to include suggestions such as what model to use to predict ridership, what additional services to provide, or when to give discounts, etc. Include your report as a pdf file in canvas. The report should not be longer than one page (300 words) and should include a maximum of 5 figures.

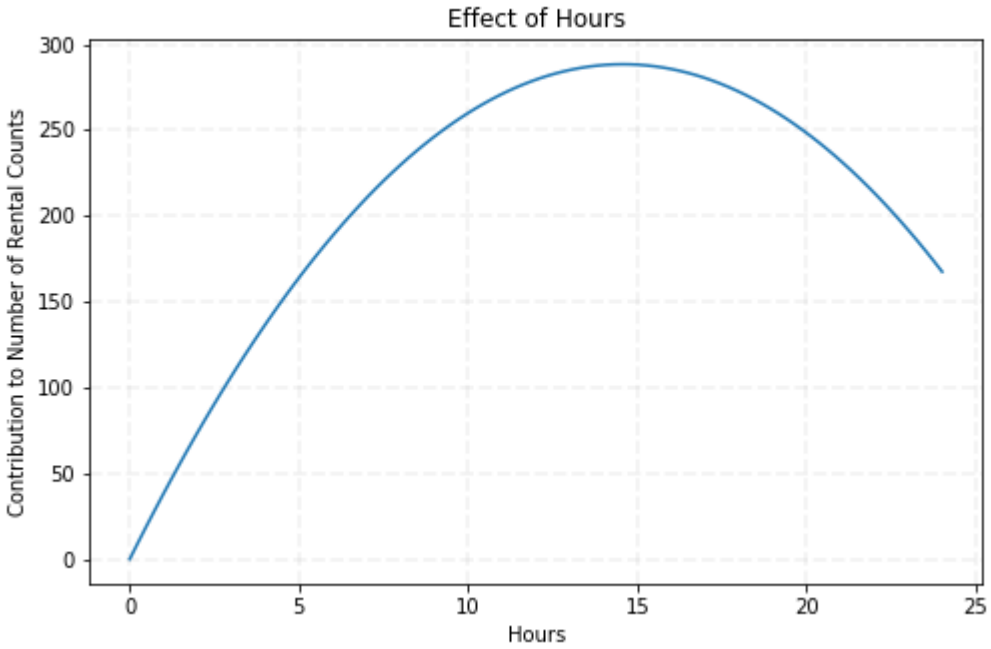
Answers 6

After our exploratory data analysis and modeling, we would like to recommend the following model to predict the hourly demand for rental bikes:

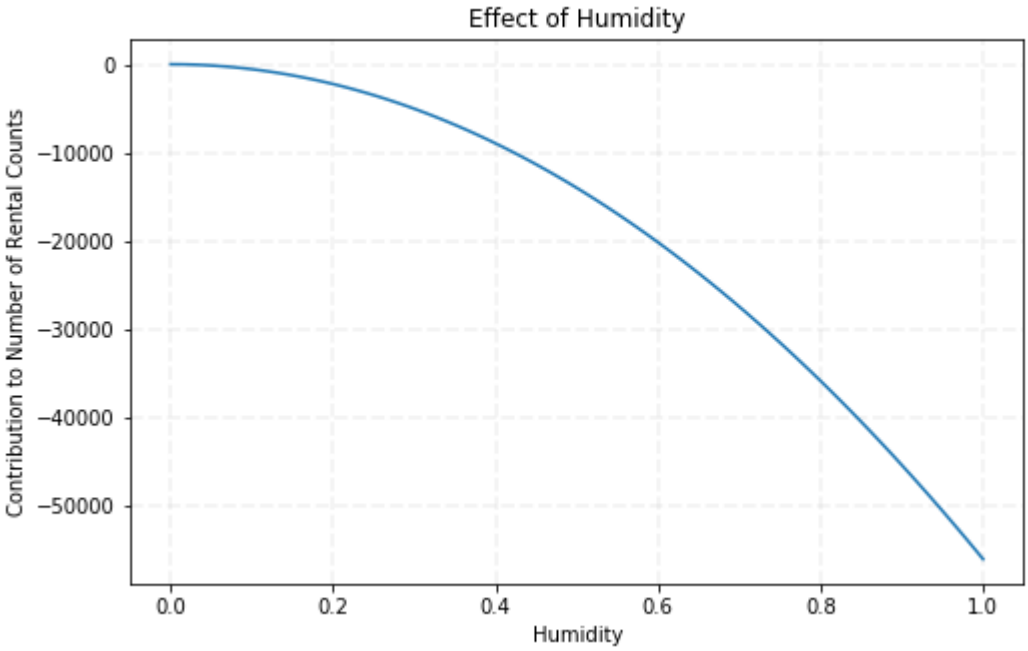
$$Rentals = -189 + 310Temp + 39Hour - Hour^2 + 80Year - 97Hum^2 + 56Fall - 26July - $$

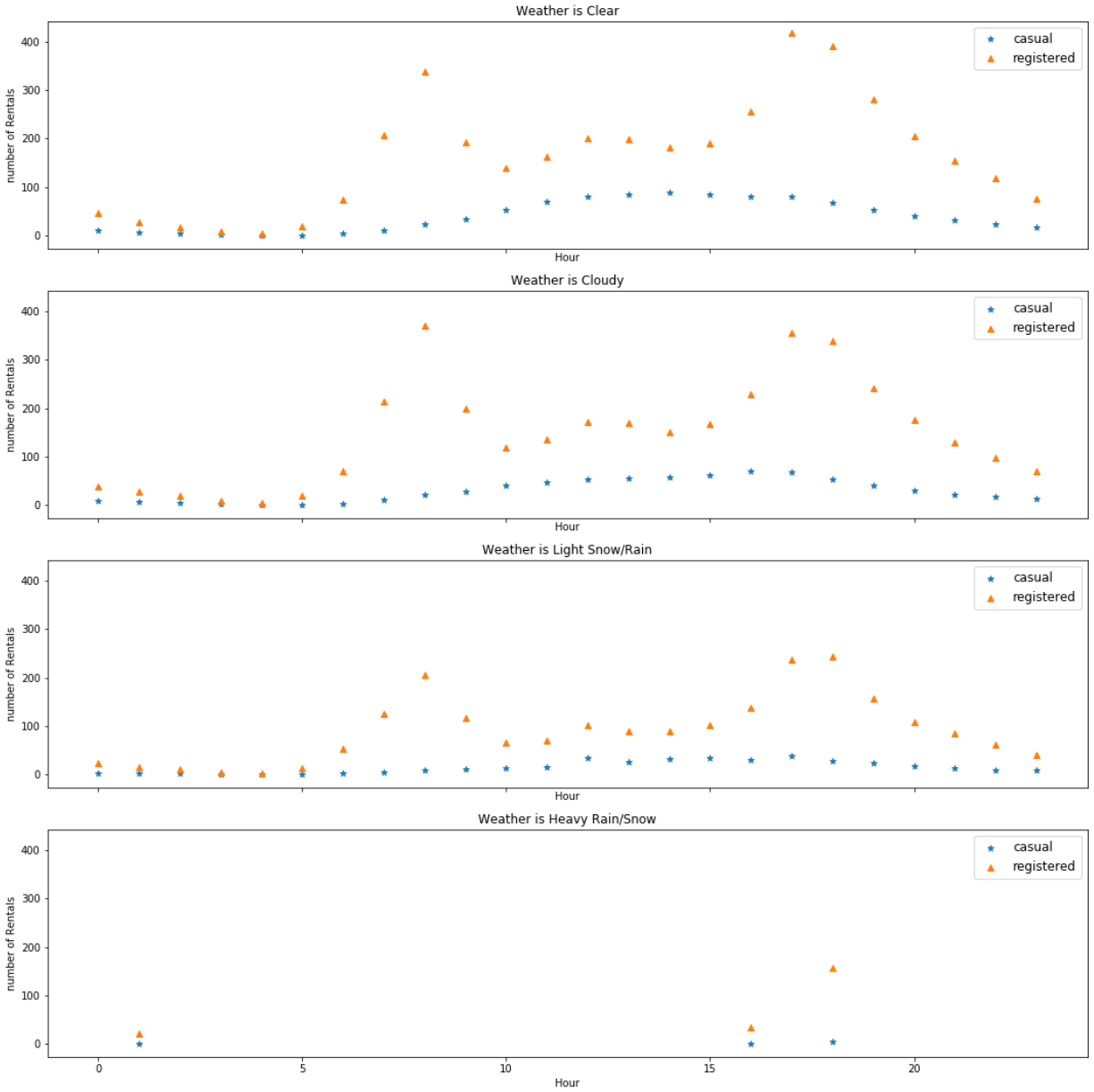
The model performs reasonably well, which can explain around 50% of the variation in the hourly demand all significant predictors. The prediction power is very close to the polynomial model, but with less predictors.

Regarding the operating hours, we found that ridership quickly becomes higher from midnight till peaks around 3pm in the afternoon, and dramatically decreases after 8pm in the night. Based on this pattern, we would like to recommend differentiating rental fare by hours, for example, 30% discount rate before 8am and after 8pm.

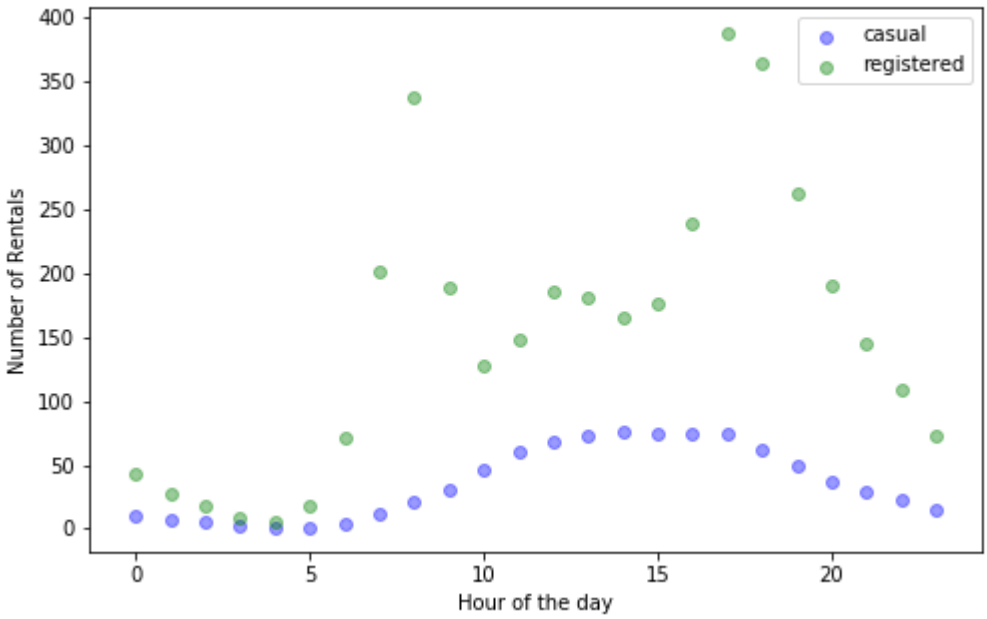


Regarding the humidity and weather, we found high humidity has a quadratic negative impact on the ridership, since fewer people would ride in the rain. Of couse, people also don't like to ride in the snow or cloudy days. Luckily, weather prediction might be helpful for us to give a discount rates beforehand. In the meanwhile, we should warn the danger of riding during snowing or raining days to potential riders.

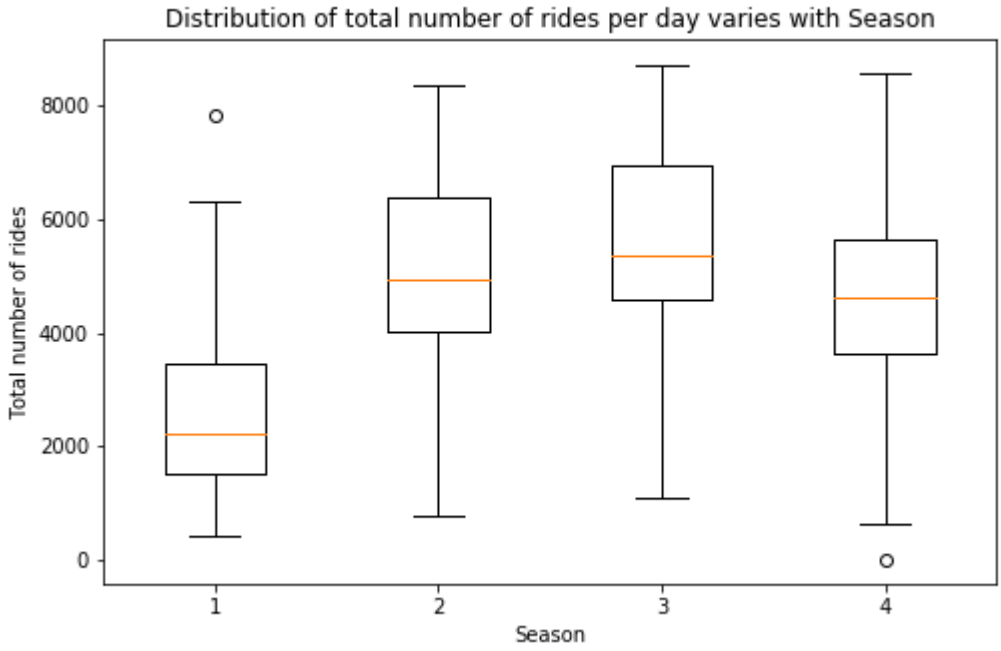

Effect of Humidity
Contribution to Number of Rental Counts
0
−10000
−20000
−30000
−40000
−50000
0.0
0.2
0.4
0.6
0.8
1.0
Humidity



Regarding riders, we found that registered riders are more consistent than casual riders. We should have more promotion events in good weather and good season to attract more people to register for the program for revenue growth.



Regarding seasonality, we found that riders prefer spring and fall seasons, like September but don't like July and holidays. Accordingly, we would suggest dicsounts event during holidays to attract price-sensitive customers, hold community events like riding competitions to let more people know the healthy aspect of riding during spring and fall season.



Overall, we love this program, and firmly believe our recommendations would help for revenue growth.