MFE405 Project 2

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Problem 1.
Seed=713
When α = - 0.7, the value of ρ is -0.15426573594571363

Problem 2.
Seed=713
When ρ = 0.6, E [max(0, $(X^3 + \sin(Y) + X^2Y))] =$ 1.4963734679657592

Problem 3.
Seed=1000
(a)
A(1) is 1.0298987083057254 , B(1) is 0.9957905252502176 .
A(3) is 2.953365817162241 , B(3) is 1.0192296319785061 .
A(5) is 4.945213825708346 , B(5) is 1.0832608964550339 .

(b)
Let f(t) = $e^{t/2}cos(W_t)$. Apply Ito's lemma, we have

$$df(t) = \frac{\partial f}{\partial t}dt + \frac{\partial f}{\partial W}dW + \frac{1}{2}\frac{\partial^2 f}{\partial W^2}(dW)^2 = e^{\frac{t}{2}}(-sinWt)dWt$$

f(t) is a martingale. Then we have

$$E[f(Wt] = E[f(W0)] = 1$$

Hence the B(t)'s Expectation will always be 1 whatever t is.

(c)
After using Antithetic Variates Reduction Method,
A(5) is 5.033242827123166 , B(5) is 1.092483873978826 .
The variance before and after we adopt Antithetic Variates Reduction Method is in the table below. (seed=1000)

	t	A(t) Before	B(t) Before	A(t) After	B(t) After
1	1.0	2.596051	0.546878	1.939667	0.539896
3	3.0	18.773827	8.966538	18.360913	8.983966
5	5.0	52.202360	73.442551	51.266057	73.204344

From the table above, we find that, for t=1 and t=3, the variance of A(t) and B(t) are just reduced a little bit. However, when I change the seed to seed=9, the results in the table below suggests that the variance increases after using the Antithetic Variates Variance Reduction Method.(seed=9)

	t	A(t) Before	B(t) Before	A(t) After	B(t) After
1	1.0	2.438084	0.537973	1.930807	0.541703
3	3.0	17.125346	8.967294	17.563686	8.957311
5	5.0	50.971022	72.871350	51.721469	72.429686

I think we can't say that variances of estimator are significantly reduced by the Antithetic Variates Variance Reduction Method because the improvement is slight. Actually, as we change the seed, sometimes, the results in Part a are closer to true values than using Antithetic Variates Variance Reduction Method.

Problem 4.

(a)

The result of the call option price by the Monte Carlo Simulation is 18.017112538180456

(b)

The result of the call option price by the Black-Scholes formula is 18.28376570485581

(c)

The result of the call option price after using the Antithetic Variates reduction technique is 18.150630713788775.

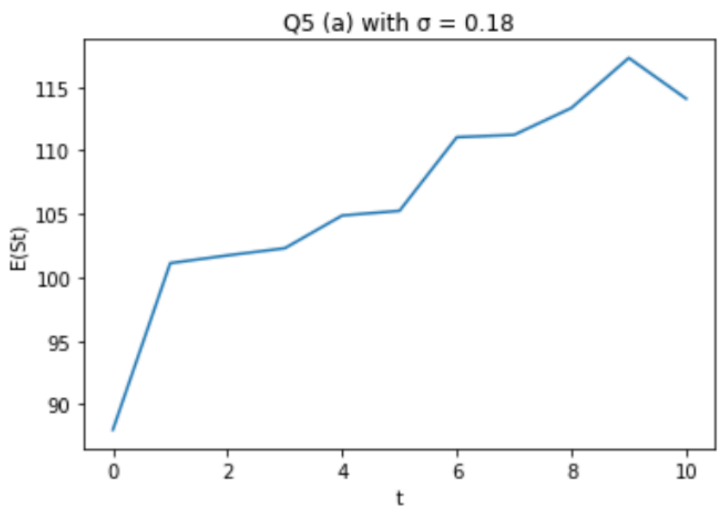
The variance before is 1207.7511366038145

The variance after is 406.53462669639896

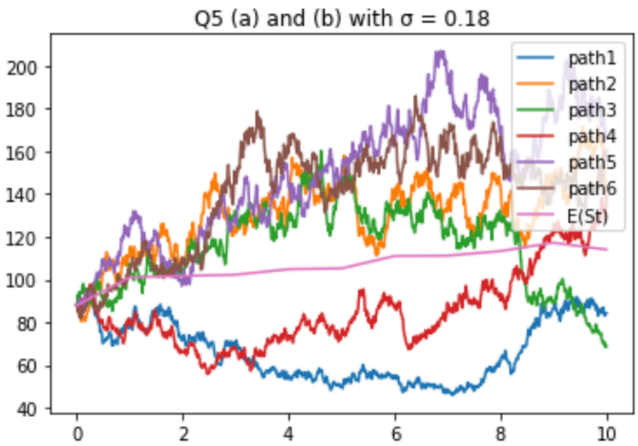
we can see that we reduce the variance of observation significantly. This method improves the accuracy of simulation. We can also find that call price in (c) is closer to the BS call price which is accurate.

Problem 5.

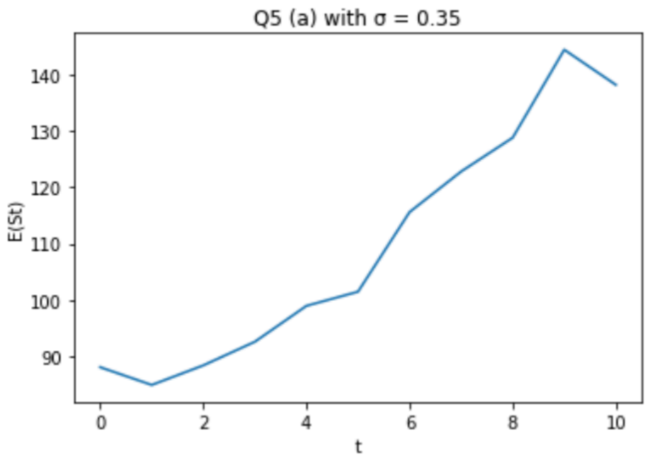
(a).

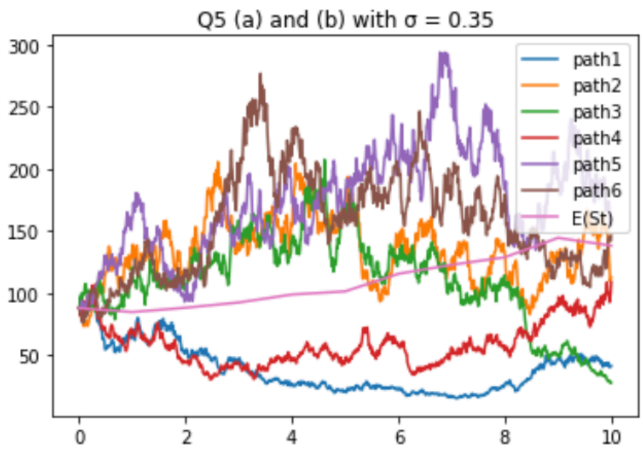


(c)



(d)





After we increase σ, the expected value of St decreases a little bit instead of increasing when t=1, and the it then dramatically increases after t=2. The paths of price perform more volatile compared to the expected value of St after σ is increased. The paths are also more dispersive than before.

Question 6

(a)

The integral by using the Euler's discretization scheme is 3.141391477611317

(b)

The integral by using the Monte Carlo Simulation is 3.1364573458377736

(c)

The integral by using the Importance Sampling method is 3.1403760135839502.

The algorithm is that we have $g(x) = 4\sqrt{1-x^2}$, x~U(0,1), then f(x)=1 for x within (0,1), t(x) is

$$t(x) = \begin{cases} \dfrac{1-ax^2}{1-\frac{a}{3}} & \text{for } 0 \le x \le 1 \\ 0 & \text{otherwise} \end{cases}$$

The first thing is that we need to generate a random variable y having density function as t(y) from uniform distributed variables.

First, we need to find the cumulative probability function of y by integral.

$$T(y) = \int_0^y \frac{1-ax^2}{1-\frac{a}{3}} dx = \frac{3y - ay^3}{3-a} = u \sim U(0,1)$$

$$ay^3 - 3y + (3-a)u = 0$$

Random variable y is the root of the equation above, which is within (0,1). We use np.root function to find the solution of y.

Second, we need to find the a that minimize the variance of g(x)f(x)/t(x). By generating a list of random variables x following U(0,1), we find the a=0.755. Then we use the formula above to calculate the expectation value of g(x)f(x)/t(x) which is the estimated value of π.

From the results in (a), (b) and (c), we find that by Importance Sampling method, the result is much closer to the true value of π than other methods.