

MFE405 Project 7

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Problem 1.

(a)

	Explicit FD dx1	Explicit FD dx2	Explicit FD dx3	Black Scholes	Error dx1	Error dx2	Error dx3
Stock Price (\$0)							
4	5.80198	5.80198	5.80198	5.80199	-0.00001	-0.00001	-0.00001
5	4.80198	4.80198	4.80198	4.80199	-0.00001	-0.00001	-0.00001
6	3.80210	3.80205	3.80211	3.80206	0.00004	-0.00000	0.00005
7	2.80550	2.80565	2.80558	2.80536	0.00014	0.00029	0.00022
8	1.84583	1.84737	1.84784	1.84427	0.00156	0.00311	0.00357
9	1.02760	1.02693	1.02773	1.02443	0.00317	0.00250	0.00330
10	0.46985	0.47229	0.47162	0.46469	0.00516	0.00759	0.00692
11	0.17249	0.17481	0.17234	0.17154	0.00095	0.00328	0.00080
12	0.05342	0.05495	0.05346	0.05246	0.00096	0.00249	0.00100
13	0.01408	0.01540	0.01460	0.01365	0.00043	0.00175	0.00094
14	0.00310	0.00332	0.00387	0.00311	-0.00001	0.00021	0.00076
15	0.00068	0.00071	0.00072	0.00063	0.00004	0.00008	0.00008
16	0.00021	0.00020	0.00020	0.00012	0.00009	0.00008	0.00008

(b)

	Implicit FD dx1	Implicit FD dx2	Implicit FD dx3	Black Scholes	Error dx1	Error dx2	Error dx3
Stock Price (\$0)							
4	5.80199	5.80199	5.80199	5.80199	0.00001	0.00001	0.00000
5	4.80199	4.80199	4.80199	4.80199	0.00001	0.00000	0.00000
6	3.80213	3.80209	3.80214	3.80206	0.00007	0.00003	0.00008
7	2.80568	2.80583	2.80576	2.80536	0.00032	0.00047	0.00041
8	1.84620	1.84772	1.84818	1.84427	0.00193	0.00345	0.00391
9	1.02744	1.02682	1.02762	1.02443	0.00301	0.00240	0.00319
10	0.46936	0.47175	0.47108	0.46469	0.00466	0.00705	0.00638
11	0.17214	0.17450	0.17202	0.17154	0.00060	0.00297	0.00048
12	0.05347	0.05500	0.05352	0.05246	0.00101	0.00254	0.00106
13	0.01424	0.01556	0.01475	0.01365	0.00059	0.00191	0.00110
14	0.00321	0.00343	0.00398	0.00311	0.00010	0.00032	0.00087
15	0.00073	0.00076	0.00077	0.00063	0.00009	0.00013	0.00014
16	0.00024	0.00023	0.00023	0.00012	0.00012	0.00011	0.00011

(c)

	C-N FD dx1	C-N FD dx2	C-N FD dx3	Black Scholes	Error dx1	Error dx2	Error dx3
Stock Price (\$0)							
4	5.80199	5.80198	5.80198	5.80199	-0.00000	-0.00000	-0.00000
5	4.80199	4.80198	4.80198	4.80199	-0.00000	-0.00000	-0.00000
6	3.80212	3.80207	3.80212	3.80206	0.00006	0.00001	0.00006
7	2.80559	2.80574	2.80567	2.80536	0.00023	0.00038	0.00031
8	1.84603	1.84754	1.84801	1.84427	0.00176	0.00328	0.00374
9	1.02749	1.02688	1.02767	1.02443	0.00307	0.00245	0.00324
10	0.46963	0.47202	0.47135	0.46469	0.00494	0.00732	0.00665
11	0.17230	0.17466	0.17218	0.17154	0.00076	0.00312	0.00064
12	0.05344	0.05497	0.05349	0.05246	0.00098	0.00251	0.00103
13	0.01416	0.01548	0.01468	0.01365	0.00051	0.00183	0.00102
14	0.00315	0.00338	0.00392	0.00311	0.00005	0.00027	0.00082
15	0.00070	0.00074	0.00074	0.00063	0.00007	0.00010	0.00011
16	0.00022	0.00021	0.00021	0.00012	0.00010	0.00010	0.00009

Three finite difference schemes yield very close results to the theoretical value (Black-Scholes Method). Some of the stock values does not exist in the grid, so the value of the corresponding option is interpolated. When the options are deep in-the-money or out-of-the-money, the finite difference method generated very accurate approximations of European Put Option. The largest errors of the finite difference approximations are incurred near at-the-money prices. The error generally increases with larger dx chosen

Problem 2

(a)

	call, ds = 0.25	call, ds = 1	call, ds = 1.25	put, ds = 0.25	put, ds = 1	put, ds = 1.25
S0=4	0.00112	0.02482	0.16571	6.00000	6.00000	6.00000
S0=5	0.00065	0.00772	0.03103	5.00000	5.00000	5.00000
S0=6	0.00085	0.01619	0.05318	4.00000	4.00000	4.00000
S0=7	0.00354	0.00674	0.01541	3.00000	3.00000	3.00000
S0=8	0.04207	0.04318	0.05954	2.00000	2.00000	2.00000
S0=9	0.22104	0.20105	0.22217	1.08128	1.05163	1.05437
S0=10	0.66080	0.62305	0.59631	0.48066	0.43998	0.41533
S0=11	1.36833	1.34813	1.36754	0.17506	0.15445	0.17371
S0=12	2.25003	2.24499	2.26158	0.05312	0.04815	0.06433
S0=13	3.21157	3.21159	3.21763	0.01378	0.01388	0.01973
S0=14	4.20112	4.20174	4.20311	0.00314	0.00380	0.00512
S0=15	5.19866	5.19901	5.19917	0.00065	0.00100	0.00118
S0=16	6.19814	6.19828	6.19860	0.00012	0.00026	0.00059

(b)

	call, ds = 0.25	call, ds = 1	call, ds = 1.25	put, ds = 0.25	put, ds = 1	put, ds = 1.25
S0=4	0.00121	0.02513	0.16632	6.00000	6.00000	6.00000
S0=5	0.00072	0.00792	0.03141	5.00000	5.00000	5.00000
S0=6	0.00095	0.01657	0.05378	4.00000	4.00000	4.00000
S0=7	0.00371	0.00691	0.01569	3.00000	3.00000	3.00000
S0=8	0.04241	0.04351	0.05982	2.00000	2.00000	2.00000
S0=9	0.22092	0.20106	0.22219	1.08057	1.05125	1.05419
S0=10	0.66022	0.62229	0.59553	0.47973	0.43908	0.41443
S0=11	1.36800	1.34788	1.36733	0.17459	0.15417	0.17349
S0=12	2.25008	2.24508	2.26159	0.05313	0.04825	0.06437
S0=13	3.21172	3.21172	3.21774	0.01394	0.01402	0.01986
S0=14	4.20121	4.20181	4.20318	0.00325	0.00389	0.00521
S0=15	5.19869	5.19903	5.19920	0.00069	0.00105	0.00122
S0=16	6.19814	6.19828	6.19861	0.00014	0.00028	0.00061

(c)

	call, ds = 0.25	call, ds = 1	call, ds = 1.25	put, ds = 0.25	put, ds = 1	put, ds = 1.25
S0=4	0.00116	0.02497	0.16600	6.00000	6.00000	6.00000
S0=5	0.00069	0.00782	0.03121	5.00000	5.00000	5.00000
S0=6	0.00090	0.01638	0.05347	4.00000	4.00000	4.00000
S0=7	0.00362	0.00682	0.01554	3.00000	3.00000	3.00000
S0=8	0.04224	0.04334	0.05967	2.00000	2.00000	2.00000
S0=9	0.22098	0.20105	0.22218	1.08092	1.05144	1.05428
S0=10	0.66051	0.62267	0.59592	0.48019	0.43953	0.41488
S0=11	1.36816	1.34800	1.36744	0.17482	0.15431	0.17360
S0=12	2.25005	2.24503	2.26159	0.05312	0.04820	0.06435
S0=13	3.21165	3.21165	3.21768	0.01386	0.01395	0.01979
S0=14	4.20117	4.20178	4.20315	0.00320	0.00384	0.00517
S0=15	5.19868	5.19902	5.19918	0.00067	0.00103	0.00120
S0=16	6.19814	6.19828	6.19861	0.00013	0.00027	0.00060

Figure 1 is a line graph showing Call Option Prices (Y-axis, ranging from 0 to 6) versus Initial Stock Price (X-axis, ranging from 4 to 16). The graph compares the performance of three methods (EFD, IFD, and CNFD) for different values of ds (0.25, 1, and 1.25). The legend indicates the following series:

- EFD $ds=0.25$ (Blue line with 'x' markers)
- IFD $ds=0.25$ (Orange line with diamond markers)
- CNFD $ds=0.25$ (Green line with triangle markers)
- EFD $ds=1$ (Red line with 'x' markers)
- IFD $ds=1$ (Purple line with triangle markers)
- CNFD $ds=1$ (Brown line with diamond markers)
- EFD $ds=1.25$ (Pink line with 'x' markers)
- IFD $ds=1.25$ (Grey line with diamond markers)
- CNFD $ds=1.25$ (Yellow line with star markers)

The graph shows that the Call Option Price increases significantly as the Initial Stock Price increases, particularly for $ds=1.25$. The CNFD method generally shows slightly higher prices than EFD and IFD for $ds=0.25$ and $ds=1$, but they converge for $ds=1.25$.

Figure 1 is a line graph showing Put Option Prices (Y-axis, ranging from 0 to 6) versus Initial Stock Price (X-axis, ranging from 4 to 16). The graph compares the results of three methods (EFD, IFD, CNFD) for different values of ds (0.25, 1, 1.25). The legend indicates the following series:

- EFD $ds=0.25$ (Blue line with 'x' markers)
- IFD $ds=0.25$ (Orange line with diamond markers)
- CNFD $ds=0.25$ (Green line with star markers)
- EFD $ds=1$ (Red line with 'x' markers)
- IFD $ds=1$ (Purple line with star markers)
- CNFD $ds=1$ (Brown line with diamond markers)
- EFD $ds=1.25$ (Pink line with 'x' markers)
- IFD $ds=1.25$ (Grey line with diamond markers)
- CNFD $ds=1.25$ (Yellow line with star markers)

The graph shows that the Put Option Price decreases as the Initial Stock Price increases. The three methods (EFD, IFD, CNFD) produce very similar results for all values of ds , with the prices converging to zero as the Initial Stock Price increases beyond 12.