

## Project 6

MGMTMFE 405

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You will need to write codes for all the parts of the project. Make sure the codes work properly and understand the ideas behind each problem below. You may be asked to demonstrate how the codes work, by running them, and interpret the results. Code clarity and accuracy will determine the grades.

**Submit your codes and a PDF file of your answers to questions (including graphs, histograms, but no codes, in this PDF file) by 11PM PDT on Next Wednesday.**

1. Consider a 12-month **Fixed Strike Lookback Call and Put** options, when the interest rate is 3% per annum, the current stock price is \$98 and the strike price is \$100. Use the MC simulation method to estimate the prices of Call and Put options for the following range of volatilities: from 12% to 48%, in increments of 4%.

The payoff of the Call option is  $(S_{max} - X)^+$ , where  $S_{max} = \max\{S_t : t \in [0, T]\}$ , and the payoff of the Put option is:  $(X - S_{min})^+$ , where  $S_{min} = \min\{S_t : t \in [0, T]\}$ .

**Inputs:**  $S_0, X, T, r, \sigma, N$

**Outputs:** Graphs: Call and Put options prices as a function of the volatility. Place the Call graph in Proj6\_1a.jpg and the Put graph in Proj6\_1b.jpg.

2. Assume that the value of a collateral follows a jump-diffusion process:

$$\frac{dV_t}{V_t^-} = \mu dt + \sigma dW_t + \gamma dJ_t$$

where  $\mu, \sigma, \gamma < 0$ , and  $V_0$  are given,  $J$  is a Poisson process, with intensity  $\lambda_1$ , independent of the Brownian Motion process  $W$ .

$V_t^-$  is the value process before jump occurs at time  $t$  (if any).

Consider a collateralized loan, with a contract rate per period  $r$  and maturity  $T$  on the above-collateral, and assume the outstanding balance of that loan follows this process:

$$L_t = a - bc^{12t}$$

where  $a > 0, b > 0, c > 1$ , and  $L_0$  are given. We have that  $L_T = 0$ .

Define the following stopping time:

$$Q = \min\{t \geq 0: V_t \leq q_t L_t\}$$

This stopping time is the first time when the relative value of the collateral (with respect to the outstanding loan balance) crosses a threshold which will be viewed as the “optimal exercise boundary” of the option to default.

Define another stopping time, which is the first time an adverse event occurs:

$$S = \min\{t \geq 0: N_t > 0\}$$

Assume that  $N_t$  is a Poisson process with intensity of  $\lambda_2$ .

Define  $\tau = \min\{Q, S\}$ .

We assume the embedded default option will be exercised at time  $\tau$ , if and only if  $\tau < T$ .

If the option is exercised at time  $Q$  then the payoff to the borrower is:  $(L_Q - \epsilon V_Q)^+$ .

If the option is exercised at time  $S$  then the payoff to the borrower is:  $abs(L_S - \epsilon V_S)$ , where  $abs(.)$  is the absolute value function.

**Notes:**

1. If  $\min\{Q, S\} > T$  then there is no default option exercise.
2.  $\epsilon$  Should be viewed as the recovery rate of the collateral, so  $(1 - \epsilon)$  can be viewed as the legal and administrative expenses.

Assume  $J$  has intensity  $\lambda_1$  and  $N$  has intensity  $\lambda_2$ .  $N$  is independent of  $J$  and  $W$ .

Assume the APR of the loan is  $R = r_0 + \delta\lambda_2$  where  $r_0$  is the “risk-free” rate, and  $\delta$  is a positive parameter to measure the borrower’s creditworthiness in determining the contract rate per period:  $r$ .

We have monthly compounding here, so  $r = R/12$ .

Assume that  $q_t = \alpha + \beta t$ , where  $\beta > 0$ ,  $\alpha < V_0/L_0$  and  $\beta = \frac{\epsilon - \alpha}{T}$ .

Use  $r_0$  for discounting cash flows. Use the following base-case parameter values:

$V_0 = \$20,000$ ,  $L_0 = \$22,000$ ,  $\mu = -0.1$ ,  $\sigma = 0.2$ ,  $\gamma = -0.4$ ,  $\lambda_1 = 0.2$ ,  $T = 5$  years,  $r_0 = 0.02$ ,  $\delta = 0.25$ ,  $\lambda_2 = 0.4$ ,  $\alpha = 0.7$ ,  $\epsilon = 0.95$ . Notice that  $PMT = \frac{L_0 \cdot r}{\left[1 - \frac{1}{(1+r)^n}\right]}$ , where  $r = R/12$ ,  $n = T * 12$ , and

$a = \frac{PMT}{r}$ ,  $b = \frac{PMT}{r(1+r)^n}$ ,  $c = (1 + r)$ . Notice that  $q_T = \epsilon$ .

Write the code as a function `Proj6_2func.*` that takes  $\lambda_1$ ,  $\lambda_2$  and  $T$  as parameters, setting defaults if these parameters are not supplied, and outputs the default option price, the default probability and the expected exercise time. Function specification:

`function [D, Prob, Et] = Proj6_2func(lambda1, lambda2, T)`

(a) Estimate the value of the default option for the following ranges of parameters:

- $\lambda_1$  from 0.05 to 0.4 in increments of 0.05;
- $\lambda_2$  from 0.0 to 0.8 in increments of 0.1;
- $T$  from 3 to 8 in increments of 1;

(b) Estimate the default probability for the following ranges of parameters:.

- $\lambda_1$  from 0.05 to 0.4 in increments of 0.05;
- $\lambda_2$  from 0.0 to 0.8 in increments of 0.1;
- $T$  from 3 to 8 in increments of 1;

(c) Find the Expected Exercise Time of the default option, conditional on  $\tau < T$ . That is, estimate  $E(\tau | \tau < T)$  for the following ranges of parameters:.

- $\lambda_1$  from 0.05 to 0.4 in increments of 0.05;
- $\lambda_2$  from 0.0 to 0.8 in increments of 0.1;
- $T$  from 3 to 8 in increments of 1;

**Inputs:** *seed*

**Outputs:**

- i. Values: the default option D, the default probability Prob and the expected exercise time  $E_t$  for parts (a), (b) and (c) with  $\lambda_1=0.2$ ,  $\lambda_2=0.4$  and  $T=5$ .
- ii. Graphs: For each of (a), (b) and (c) two graphs as a function of  $T$ , first with  $\lambda_1=0.2$  and  $\lambda_2$  from 0.0 to 0.8 in increments of 0.1, then with  $\lambda_2 = 0.4$  and  $\lambda_1$  from 0.05 to 0.4 in increments of 0.05. Put the two graphs in one .png file.

(d) **[Optional – NOT for grading]** Make additional assumptions as necessary to estimate the IRR of the investment.

**Note:** The drift of the  $V$  process should be a function of  $r_0, \lambda_1, \sigma$  under the risk-neutral measure, to be able to price the option, but not done so in this case.

### 3. **[Optional – NOT for grading]**

Compute, via MC simulation, the prices of the following options using 50,000 simulations of paths of the underlying stock price process and dividing the time-interval into 50 equal parts:

- (a) **Down-and-Out- Put:**  $S(0) = 50$ ,  $X = 50$ ,  $r = 0.04$ ,  $T = 2$  months,  $\sigma = 0.4$ , and for the following range of  $S_b$ : from \$36 to \$46, in increments of \$1.
- (b) **Down-and-In - Put:**  $S(0) = 50$ ,  $X = 50$ ,  $r = 0.04$ ,  $T = 2$  months,  $\sigma = 0.4$ , and for the following range of  $S_b$ : from \$36 to \$46, in increments of \$1.

### 4. **[Optional – NOT for grading]**

Compute the price of the **Asian Average Rate call** option by using:

- (a) Standard MC method, where  $S(0) = 50$ ,  $X = 50$ ,  $r = 0.04$ ,  $T = 2$  months,  $\sigma = 0.4$ .
- (b) Halton's Low-discrepancy sequences to generate the paths of the stock price, where  $S(0) = 50$ ,  $X = 50$ ,  $r = 0.04$ ,  $T = 2$  months,  $\sigma = 0.4$ .

### 5. **[Optional – NOT for grading]**

Assume the stock price follows an Arithmetic Brownian Motion.

- (a) Derive the formula for a price of a European call option, using all the other Black-Scholes assumptions.
- (b) Use MC Simulation techniques to compute the price and compare with the one implied by the formula derived.
- (c) Now compare the prices of the options using this model with the Black-Scholes prices (using the same parameters). Vary your parameters over a wide range to compare the prices. Any observations?

### 6. **[Optional – NOT for grading] Pricing of Exchange Options.**

Consider the following information on two stocks: stock of XX and stock of YY: The current stock price of XX is \$40, and the volatility of the stock price is  $\sigma_1 = 20\%$  per annum. The current stock price of YY is \$38.5, and the volatility of the stock price is  $\sigma_2 = 30\%$  per annum. Assume the correlation between the stock returns is  $\rho = 0.67$  and the prevailing risk-free rate is  $r = 6\%$  per annum. Use the following method to price the specified option:

Use the **LSMC** method with  $N=50,000$  paths simulations and time step of  $\Delta = \frac{1}{\sqrt{N}}$  to price an American Exchange Call option with strike price of  $K = \$0$  and maturity of 1-year. Use the first  $k$  of the **Simple**

**Monomials** for  $k = 3$  for both stocks, and their cross-products. (That is, you will use 9 basis functions). The price of the option is written as  $V_0 = \sup_{\tau \in [0, T]} \mathbb{E}_0^*(e^{-r\tau}(S_\tau^2 - S_\tau^1 - K))$ .