Project 3

MGMTMFE 405
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You will need to write codes for all the parts of the project. Make sure the codes work properly and understand the ideas behind each problem below. You may be asked to demonstrate how the codes work, by running them, and interpret the results. Code clarity and accuracy will determine the grades.

Submit your codes and a PDF file of your answers to questions (including graphs, histograms, but no codes, in this PDF file) by 11PM PDT on Next Wednesday.

1. Evaluate the following expected values and probabilities:

$$p1 = P(Y_2 > 5),$$
$$e1 = E\left(X_2^{\frac{1}{3}}\right), \qquad e2 = E(Y_3), \qquad e3 = E\big(X_2 Y_2 1(X_2 > 1)\big),$$

where the Ito's processes X and Y evolve according to the following SDEs:

$$dX_t = \left(\frac{1}{5} - \frac{1}{2}X_t\right)dt + \frac{2}{3}dW_t,\ X_0 = 1; \quad dY_t = \left(\left(\frac{2}{1+t}\right)Y_t + \frac{1+t^3}{3}\right)dt + \frac{1+t^3}{3}dZ_t,\ \ Y_0 = \frac{3}{4},$$

and W, Z are independent Wiener Processes.

Inputs: $X_0,\ Y_0$
Outputs: Values: p1, e1. e2, e3.

2. Estimate the following expected values:

$$e1 = E(1 + X_3)^{1/3}, \quad e2 = E(X_1 Y_1)$$

where

$$dX_t = \frac{1}{4}X_t dt + \frac{1}{3}X_t dW_t - \frac{3}{4}X_t dZ_t,\ \ X_0 = 1; \ \text{ and } \ Y_t = e^{-0.08t + \frac{1}{3}W_t + \frac{3}{4}Z_t},$$

and W, Z are independent Wiener Processes.

Inputs: X_0
Outputs: Values: e*1*, *e2*

3.

(a) Write a code to compute prices of European Call options via Monte Carlo simulation. Use variance reduction techniques (e.g. Antithetic Variates) in your estimation. The function should be generic: for any input of the 5 parameters - $S_0, T, X, r, \sigma -$ the output is the corresponding price of the European call option.

(b) Write code to compute the prices of European Call options by using the Black-Scholes formula. Use the approximation of $N(\cdot)$ described in this chapter. The code should be generic: for any input values of the 5 parameters - S_0, T, X, r, σ - the output is the corresponding price of the European call option.

(c) Estimate the greeks –delta, gamma, theta, and vega of European Call options (all five Greeks) and graph them as functions of the initial stock price S_0. Use $X = 20$, $\sigma = 0.25$, $r = 0.04$ and $T = 0.5$ in your estimations. Use the range [15, 25] forS_0, with a step size of 1. You will have 4 different graphs for each of the 4 greeks.
In all cases, dt (time-step) should be user-defined. Use dt=0.004 (a day) as a default value.

Inputs: *dt, S_0, T, X, r, Sigma*
Outputs: Values: *C1* for part (a), *C2* for part (b), *D, G T, V* for part (c).

4. Consider the following 2-factor model for stock prices with stochastic volatility:

$$\begin{cases} dS_t = rS_t dt + \sqrt{V_t}\, S_t dW_t^1 \\ dV_t = \alpha(\beta - V_t)dt + \sigma\sqrt{V_t}\, dW_t^2 \end{cases}$$

where the Brownian Motion processes above are correlated: $dW_t^1 dW_t^2 = \rho dt$, where the correlation ρ is a constant in $[-1,1]$.
Estimate the price of a European Call option (via Monte Carlo simulation) that has a strike price of K and matures in T years.
Use the following default parameters of the model: $\rho = -0.6,\ r = 0.03,\ S_0 = \$48,\ V_0 = 0.05,\ \sigma = 0.42,\ \alpha = 5.8,\ \beta = 0.0625$.
Use the Full Truncation, Partial Truncation and Reflection methods, and provide 3 price estimates by using the three methods.

Inputs: $\rho,\ r,\ S_0,\ K, T,\ V_0,\ \sigma,\ \alpha,\ \beta$.
Outputs: Values: *C1*, C*2*, C*3*.

5. The objective of this exercise is to compare a sample of Pseudo-Random numbers with a sample of Quasi-MC numbers of $Uniform[0,1]x[0,1]$:

a) Generate 100 2-dimensional Uniform [0,1]x[0,1] vectors by using any one of the algorithms for random number generation.
b) Generate 100 points of the 2-dimensional Halton sequences, using bases 2 and 7.
c) Generate 100 points of the 2-dimensional Halton sequences, using bases 2 and 4. (Note: 4 is a non-prime number!).
d) Draw all 3 sequences of random numbers on separate graphs. Are there differences in the three sets (visual test only)? Comment on your observations.
e) Use 2-dimensional Halton sequences to compute the following integral:

$$I = \int_0^1 \int_0^1 e^{-xy}\left(sin6\pi x + cos^{\frac{1}{3}}2\pi y\right) dxdy$$

Use N=10,000 in your simulations. Try different couples for bases: (2,4), (2,7), (5,7).

Inputs: For e) b1 and b2 (bases)
Outputs: Values: *I* for part (e); Graphs: 3 plots for part (d); Writeup: comments for part (d).

6. ***OPTIONAL [NOT for grading*]**
(a) You hold two European Call options with similar characteristics, but one (the first) matures in 1 year and the other (the second) matures in 3 months. Which will have higher delta? Higher Gamma? Use explicit formulas to answer. Use simulations to answer.
(b) Which is more expensive: an ATM European call or an ATM European put on the same stock with the same maturity? Use explicit formulas to answer. Use simulations to answer.
(c) Which is more expensive: a 5% OTM European call or a5% OTM European put on the same stock with the same maturity? Use explicit formulas to answer. Use simulations to answer.
(d) Which is higher: the Gamma of an ATM European call or an ATM European put on the same stock with the same maturity? What if they both are 10% ITM or 10% OTM? Use explicit formulas to answer. Use simulations to answer.