UCLA Anderson School of Management
Professor Eisfeldt, Derivative Markets MGMTMFE406

Assignment #1

1

1. Print a screenshot of option prices on Wellls Fargo equity from Bloomberg option monitor. Discuss how the call and put prices vary with strike and expiration, and provide an intuitive explanation.
2. Print a screenshot of a straddle on Apple equity from Bloomberg. Describe one interesting piece of data in these screenshots in less than two sentences. Discuss why you chose the maturity you did.
3. Print a screenshot of SPX index futures contracts (contract table), and the SPZ3 futures description for next month's contract (front month). Describe one interesting piece of data in these screenshots in less than two sentences.
4. Print a screenshot of the natural gas futures (futures contracts), and the description for next month's contract (front month). Describe one interesting piece of data in these screenshots in less than two sentences.

2 Pricing European Options

Use Matlab to write a computer code which takes as inputs:

- The initial stock price S_0
- The payoff function $F(S_T)$
- The interest rate r
- The length of the period h
- The up and down factors u and d
- The number of periods T

and which calculates the *European option* price as well as the composition of the replicating portfolio at every node of the tree.

1. Apply your code to compute the initial value of a straddle with $T = 4$, $r = 0.02$, $h = 0.25$, $u = e^{rh+0.2\sqrt{h}}$, $d = e^{rh-0.2\sqrt{h}}$, $S_0 = 100$, and strike $K = 90$.
2. Apply your code to compute the initial value of a straddle with $T = 40$, $r = 0.02$, $h = 0.025$, $u = e^{rh+0.2\sqrt{h}}$, $d = e^{rh-0.2\sqrt{h}}$, $S_0 = 100$, and strike $K = 90$.
3. Apply your code to compute the initial value of a binary call option with $T = 4$, $r = 0.02$, $h = 0.25$, $u = e^{rh+0.2\sqrt{h}}$, $d = e^{rh-0.2\sqrt{h}}$, $S_0 = 100$, and strike $K = 90$.

3 Pricing American Options

Use Matlab to write a computer code which takes as inputs:

- The initial stock price S_0
- The payoff function $g(S_T)$
- The interest rate r
- The length of the period h
- The up and down factors u and d
- The number of periods T

and which calculates the *American option* price as well as the composition of the replicating portfolio at every node of the tree and also determines the optimal exercise dates.

Apply your code to compute the initial value of an American put and an American call with strike $K = 10$ in a binomial model with $r = 0.01$, $u = e^{rh+0.15\sqrt{h}}$, $d = e^{rh-0.15\sqrt{h}}$, $h = 1/365$, $S_0 = 10$, and $T = 250$ periods.

4 Discrete Dividends

A very simple way to incorporate dividends is to assume a **constant dividend yield** at the payment date. A feature of this model is that the tree for the ex-dividend stock price S_t is **recombining**. Everything works out the same as before, except for u and d, which now are defined by $u = 1/d = e^{\sigma\sqrt{h}}$, and $p^* = \frac{e^{rh}-d}{u-d}$.

Consider a binomial model in which the ex-dividend price of the stock evolves according to

$$S_{t+1} = \left(1 - \mathbf{1}_{\{t+1\in\mathbb{D}\}}\delta\right) S_t \xi_{t+1} \tag{1}$$

where δ is a constant dividend yield, $\mathbb{D} \subseteq \{1, ..., T\}$ is the set of dividend dates, and $\xi_{t+1} \in \{u, d\}$. The variable $\mathbf{1}_{\{t+1\in\mathbb{D}\}}$ takes the value of 1 if $t+1$ is a dividend date and 0 otherwise. An important feature of this model is that the tree for the ex-dividend stock price is **recombining**.

Write a Matlab code which takes as inputs:

- The payoff function $g(S)$
- The nature of the derivative, i.e. *European* or *American*
- The number of periods T
- The interest rate r
- The length of the period h

- The up and down factors u and d
- The set of dividend dates $\mathbb{D} \subseteq \{1, ..., T\}$
- The dividend yield δ
- The initial ex-dividend stock price S_0

and which outputs the *price* as well as the *composition of the replicating portfolio* at every node of the tree and which also determines the *optimal exercise dates*.

1. Apply your code to compute the initial prices of an American put and an American call with strike $K = 10$ when $r = 0.02$, $u = 1/d = e^{0.2\sqrt{h}}$, $h = 1/365$, $S_0 = 10$, $T = 200$ periods, $\delta = 0.05$, and $\mathbb{D} = \{50, 100, 150\}$.

2. Apply your code to compute the initial price of an American straddle with the same parameters as above. If your calculations are correct, you should obtain $Straddle < Put + Call$. Why?

5 Pricing Asian Options by Monte Carlo

An Asian option is an option on the time average of the underlying asset. Consider the case of an Asian call option with discrete arithmetic averaging. An option with maturity of T years and strike K has the payoff

$$\max\left\{\frac{1}{N}\sum_{i=1}^{N} S(t_i) - K, 0\right\} \tag{2}$$

where $t_i = i \times h$ and $h = T/N$. Furthermore, assume $S(t_0) = 200$, $r = 0.02$, $\sigma = 0.20$, $K = 220$, $T = 1$, and $N = 365$. For this exercise, reset the random number generator before each simulation run by `randn('seed',0)`.

1. Price the option by Monte Carlo simulation using 100,000 paths.

2. State the associated 95 percent confidence interval of the option price.

6 Pricing American Options using the Longstaff and Schwartz Least-Square Method

Assume $S_0 = 200$, $r = 0.1$, and $\sigma = 0.3$. Build a Matlab code to price an American put option with $K = 220$ and $T = 1$. The code should take as inputs the number of steps N and the number of simulated paths. For this exercise, reset the random number generator before each simulation run by `randn('seed',0)`. ***Hint:*** *Paolo Brandimarte's website* ***http://staff.polito.it/paolo.brandimarte/*** *contains some useful Matlab codes. You can try to adapt those codes to this particular problem. If you do so, add comments documenting your understanding of the code.*

1. Price the option with $N = 250$ and 100,000 paths.

2. Keep $N = 250$ steps and price the option when the number of paths takes the values 10, 100, 1,000, 10,000, and 100,000. Make a graph.
3. Keep the number of paths at 100,000 and price the option when the number of steps N takes the values 3, 10, 100, 250, and 1,000. Make a graph.