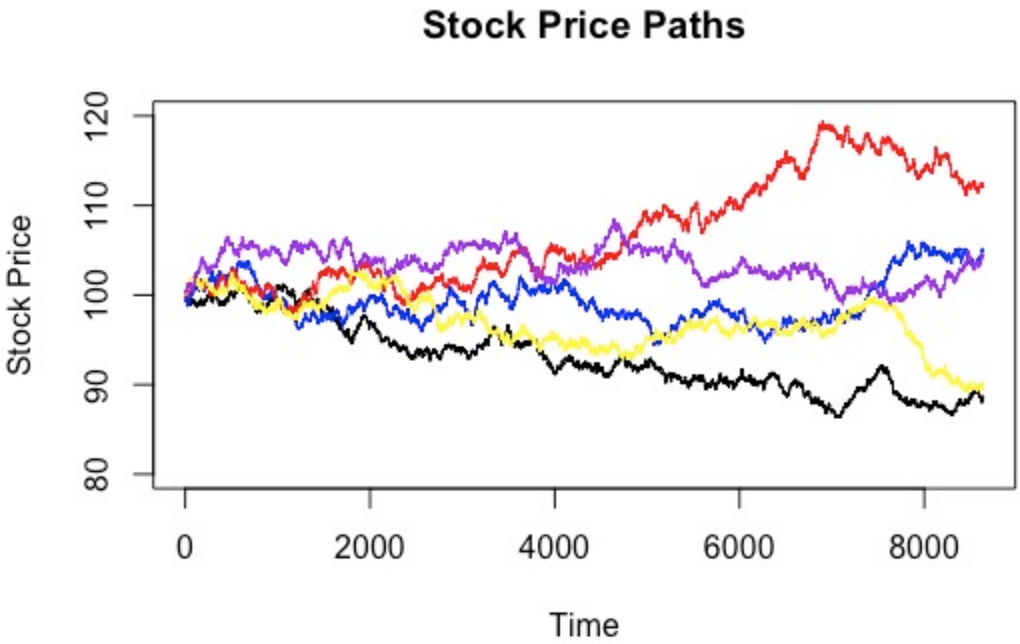
Problem 1.

a) Simulate and plot 5 paths for the stock price under the risk-neutral measure.



b) Black-Sholes call option price : 4.614997 $

c) Option price and the 95% confidence interval by Monte Carlo Simulation

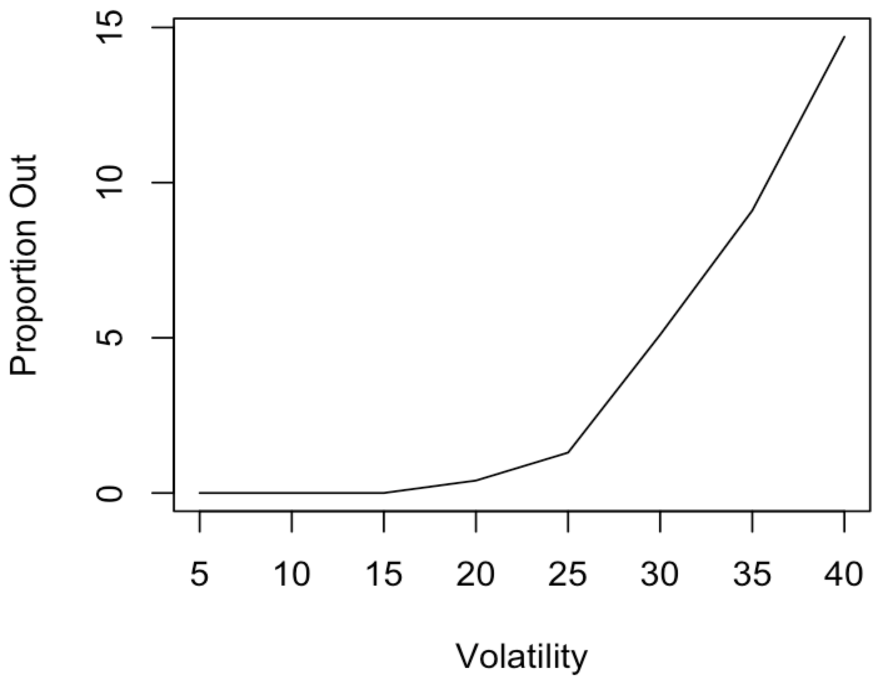
Number of Simulations	Option Price($)	95% Confidence Interval
100	4.748556	0 - 25.24191
1,000	4.757158	0 - 22.35109
1,000,000	4.613416	0 - 22.30513
10,000,000	4.614435	0 - 21.89742

Monte Carlo prices converges to the Black-Sholes price as the number of simulations increases and it is almost the same when the number of simulations are 100 million. The confidence interval shrinks as the number of simulations increase. Black-Sholes is an infinite version of a binomial tree, and as the number of Monte Carlo simulations increase it approaches infinity so it makes the confidence interval shrink around the option price.

Problem 2.

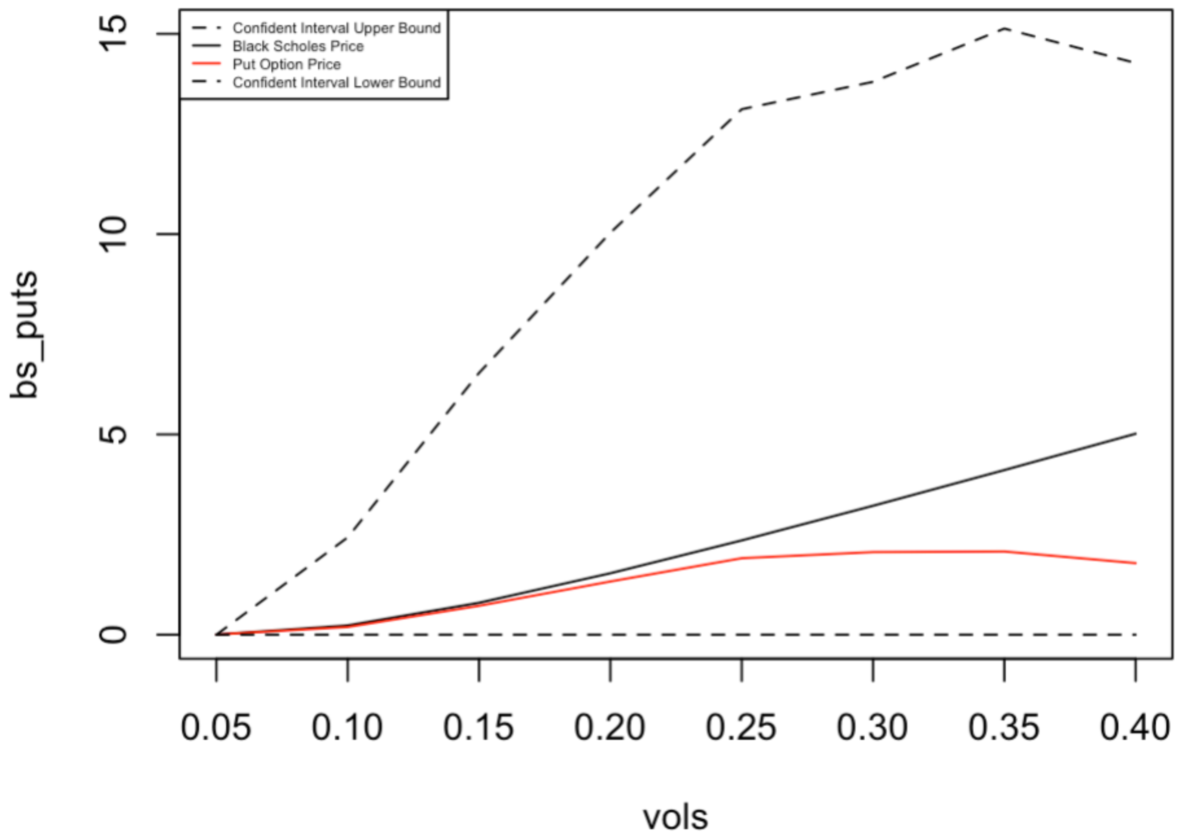
a). The 0.4% of the paths are out. The put option price is 1.328936. The 95% confidence interval is [0.00000, 10.03475]. Its Black-Schole price is 1.53426. The down-and-out put option price is lower than the BS price because BS did not capture the 0 payoff when the path hits the barrier and therefore has no value.

b).



It is shown that as volatility increases, the number of out paths increases at an increasing rate. As volatility increases, the stock price moves more dramatically. Thus, there is a higher chance that the stock price hits the barrier, thus more paths are out.

c).

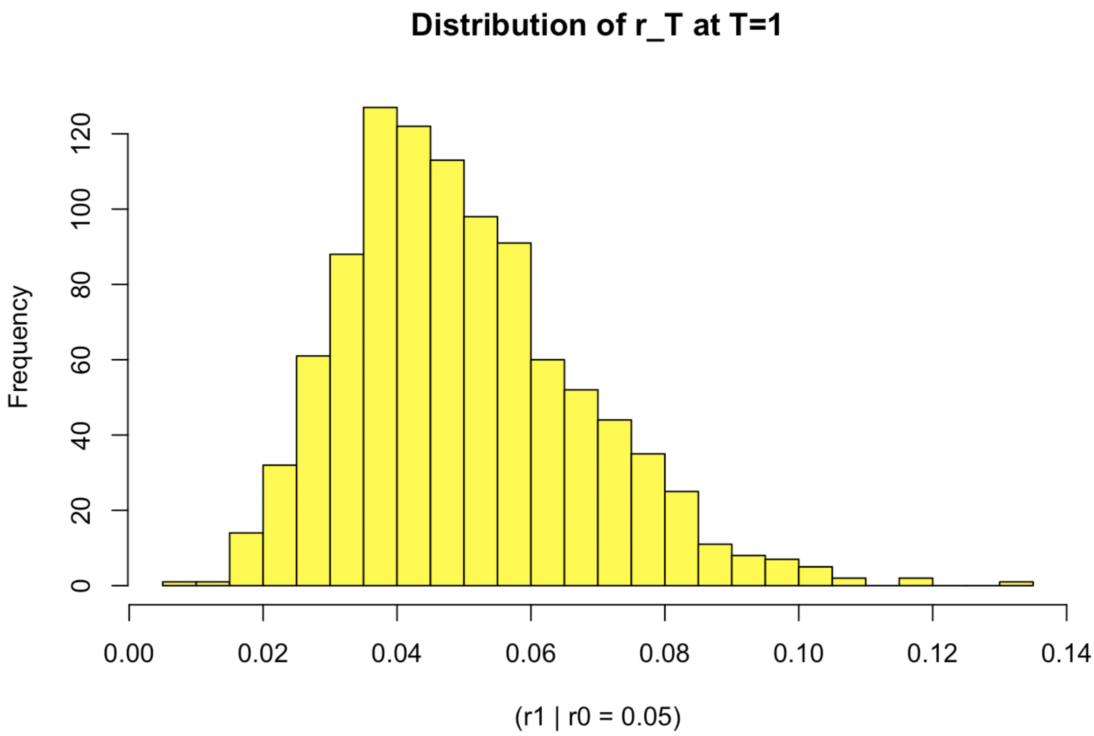


As volatility increases, Black Scholes put price increases, but, the down-and-out put option price increases at first and then drops.

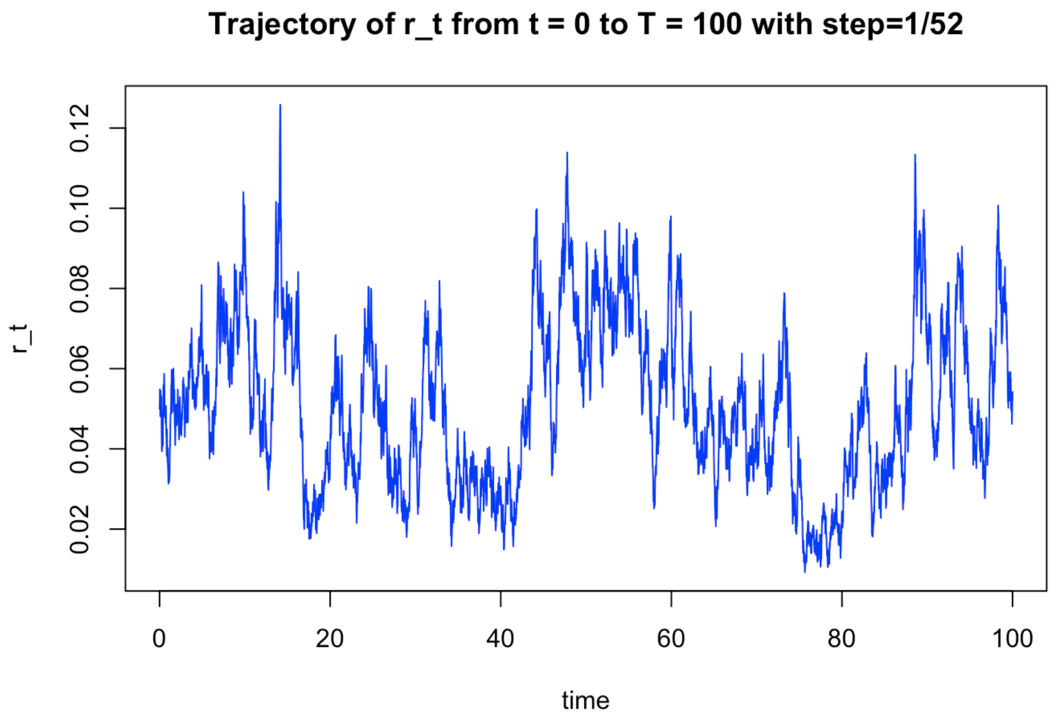
Black Schole put price increases because volatility parameter (sigma) is incorporated into the formula. When volatility is small, there are few out paths, so the price behaves similarly to Black Scholes model. That is increasing volatility increases the put option price. However, when volatility is big, the down-and-out put option price eventually drops because as the number of out paths increases, more payoffs are zero. Thus, the option has higher chance to have a terminal 0 value. Thus the option price drops.

Problem 3.

a) The distribution of rT at T=1 is shown below.



b) The trajectory of rt from t=0 to T=100 with step =1/52 is shown below



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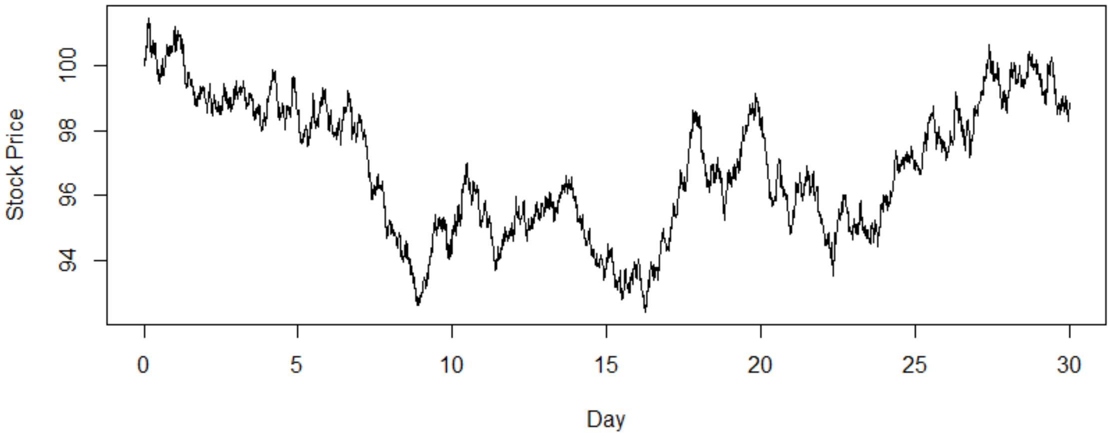
c) Based on the Monte-Carlo simulation, the call option price is $0.7006599.

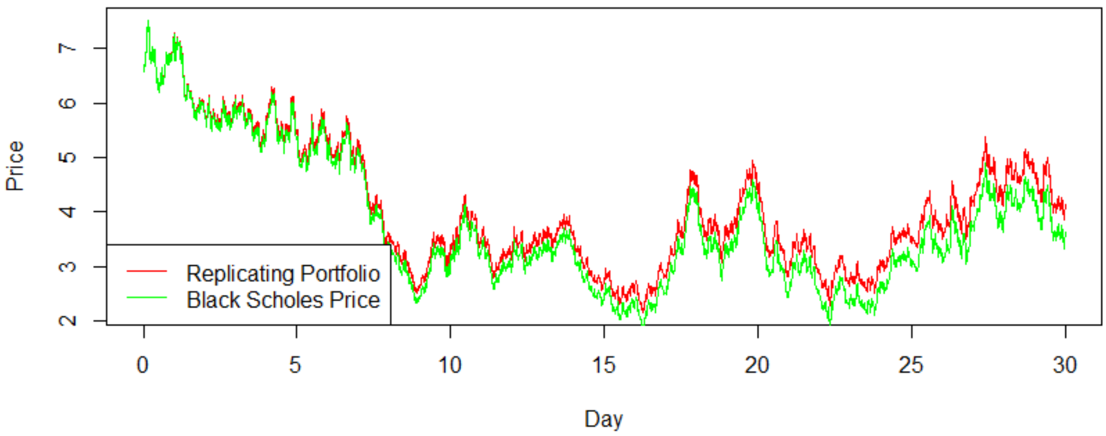
d) Based on the Monte-Carlo simulation, the call option price is $1.285735.

Problem 4

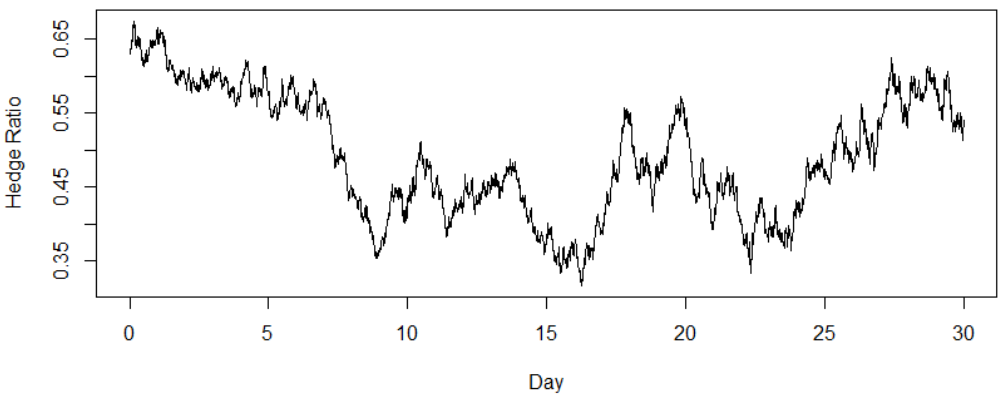
c)



Stock Price
94
96
98
100
0
5
10
15
20
25
30
Day

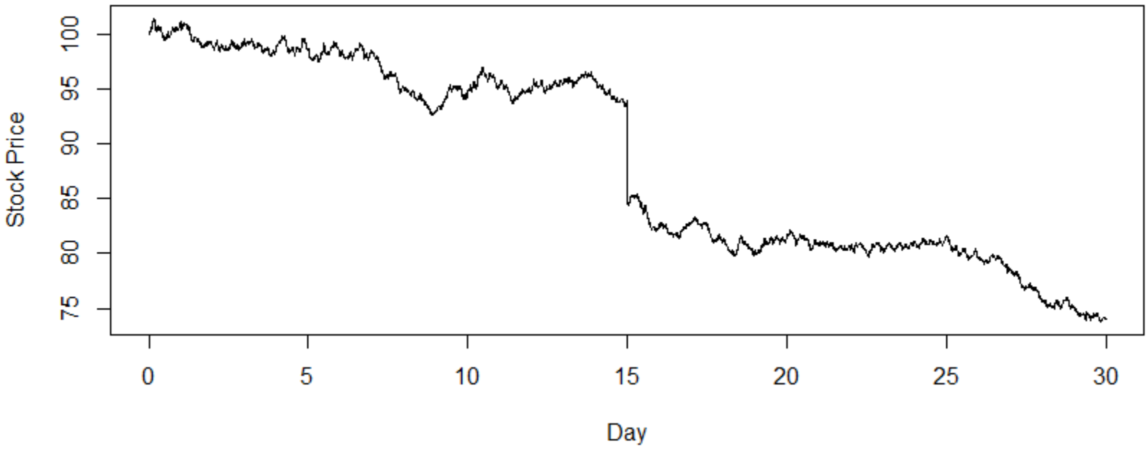


Price
2
3
4
5
6
7
Replicating Portfolio
Black Scholes Price
0
5
10
15
20
25
30
Day

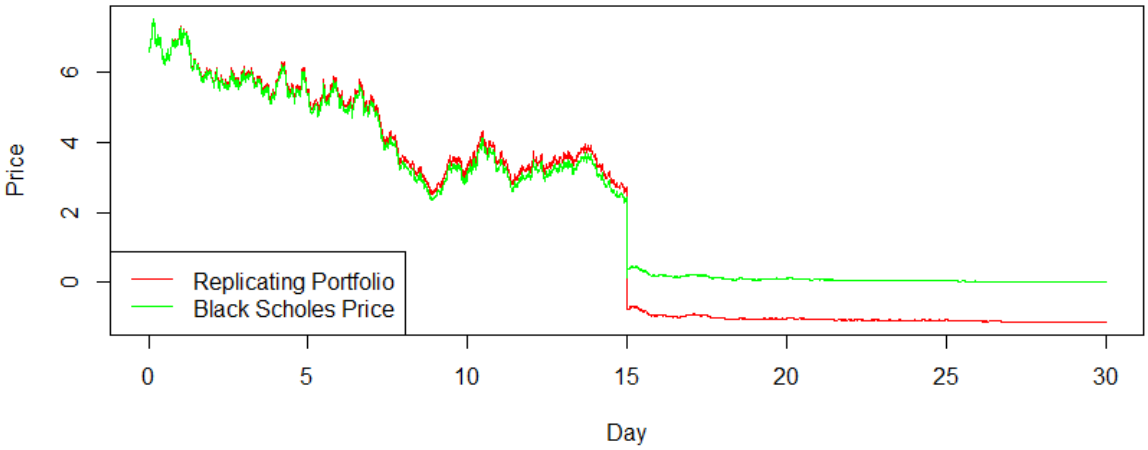


Hedge Ratio
0.35
0.45
0.55
0.65
0
5
10
15
20
25
30
Day

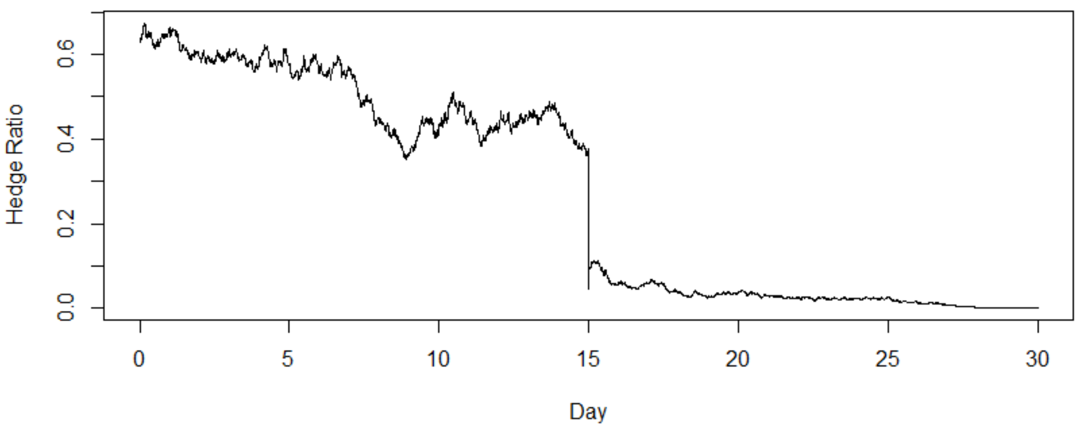
d)



Stock Price
100
95
90
85
80
75
0
5
10
15
20
25
30
Day



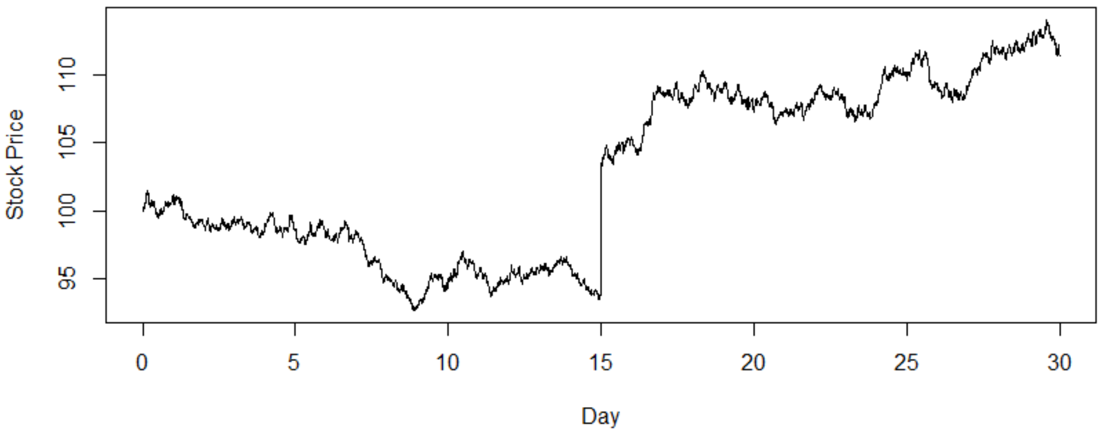
Price
6
4
2
0
Replicating Portfolio
Black Scholes Price
0
5
10
15
20
25
30
Day

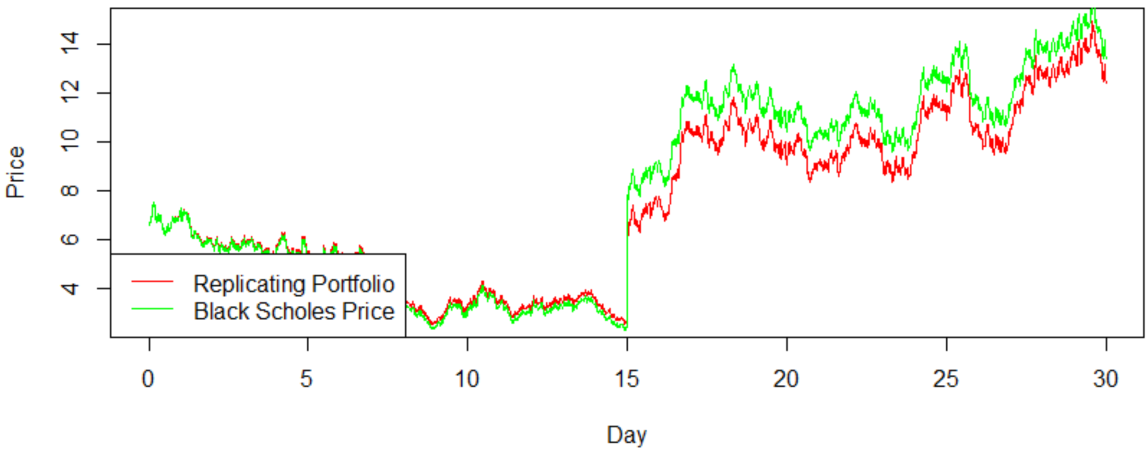


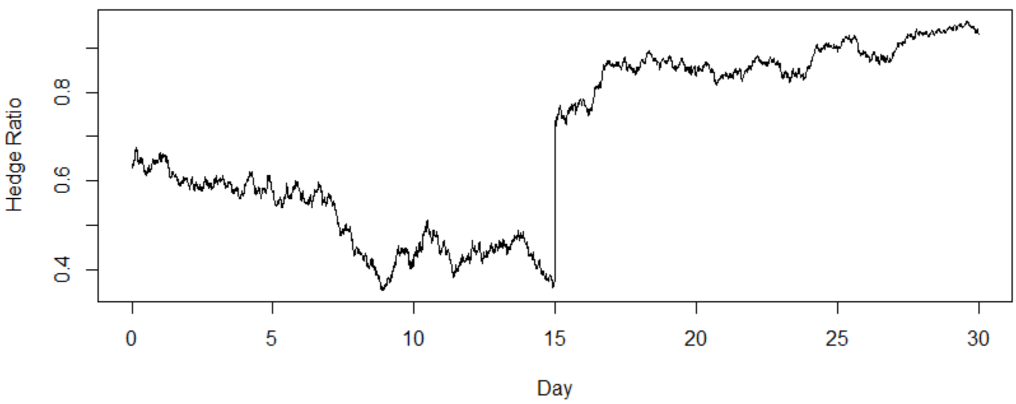
Hedge Ratio
0.6
0.4
0.2
0.0
0
5
10
15
20
25
30
Day

We observe that after the jump in price the replicating portfolio and black scholes price deviates from the black scholes price.

e)



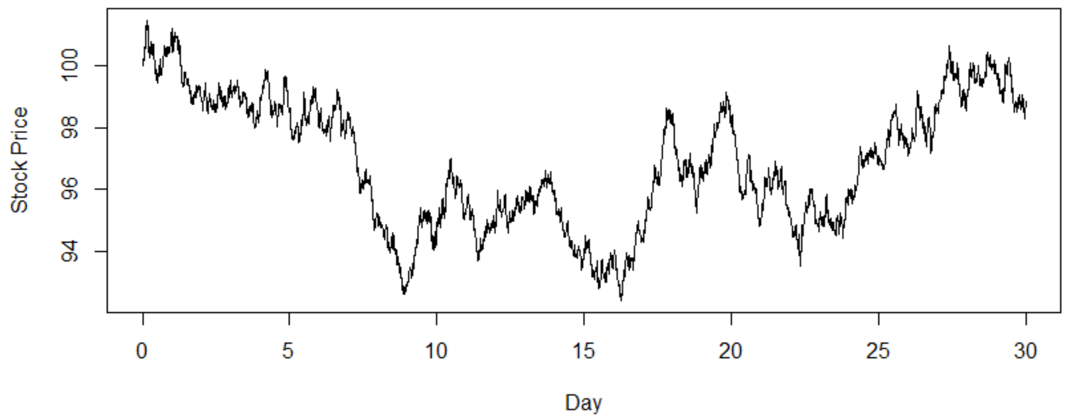


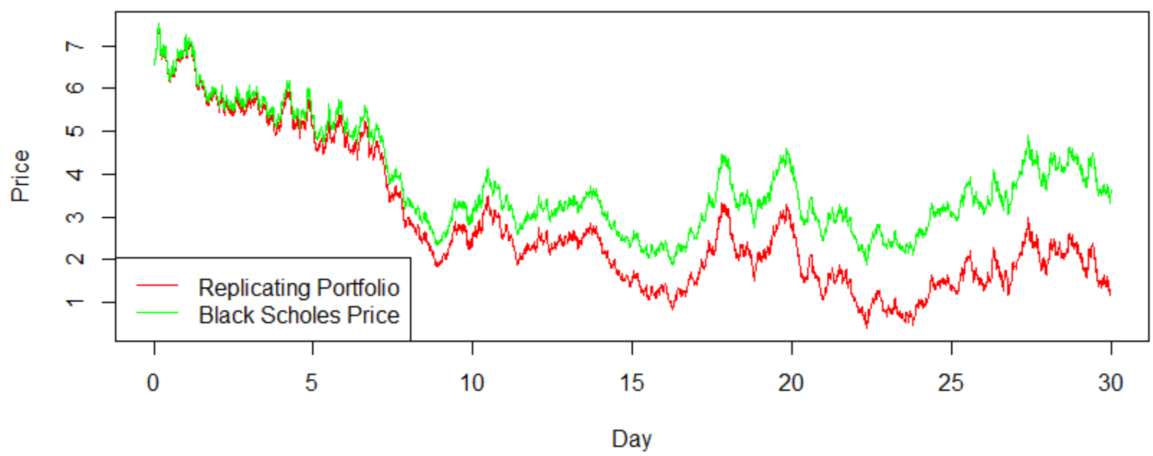


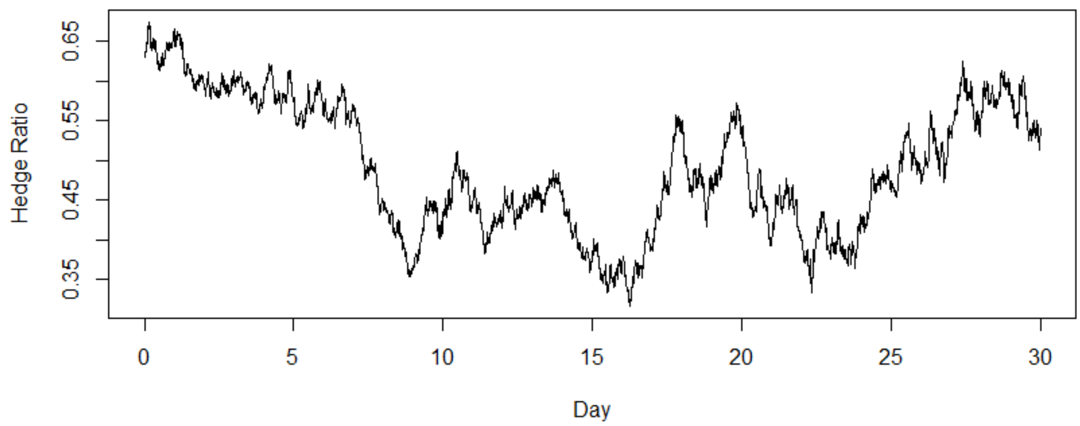
We observe that after the jump in price the replicating portfolio and black scholes price deviates from the black scholes price.

This implies that the replicating portfolio can’t handle the shocks.

f)







Due to the transaction costs, the replicating portfolio’s value decreases as time passes by because of accumulated transaction costs.

Problem 5

a. The Black-Scholes price is \$5.01 and the Heston price is \$4.68

b.

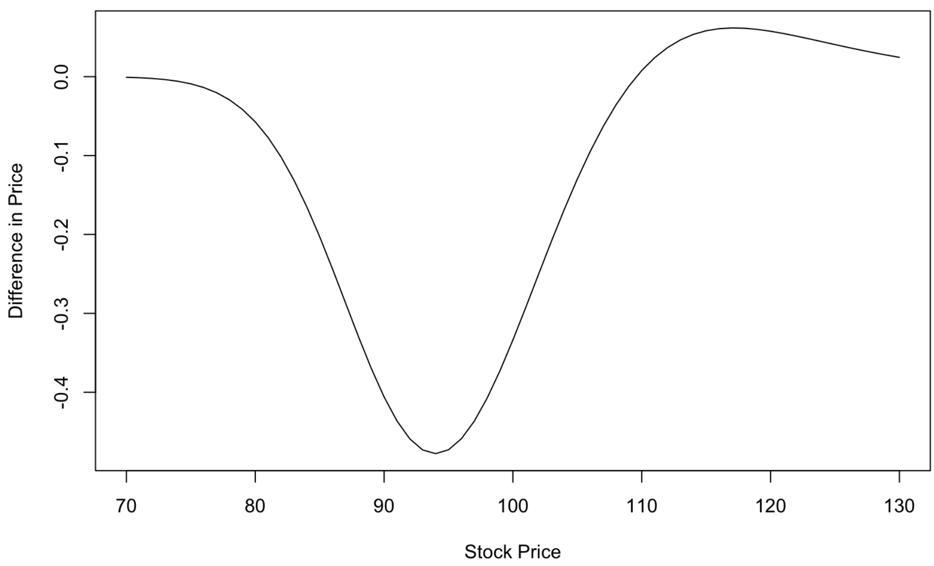


Figure 1: Difference in Heston and Black-Scholes Prices

c.

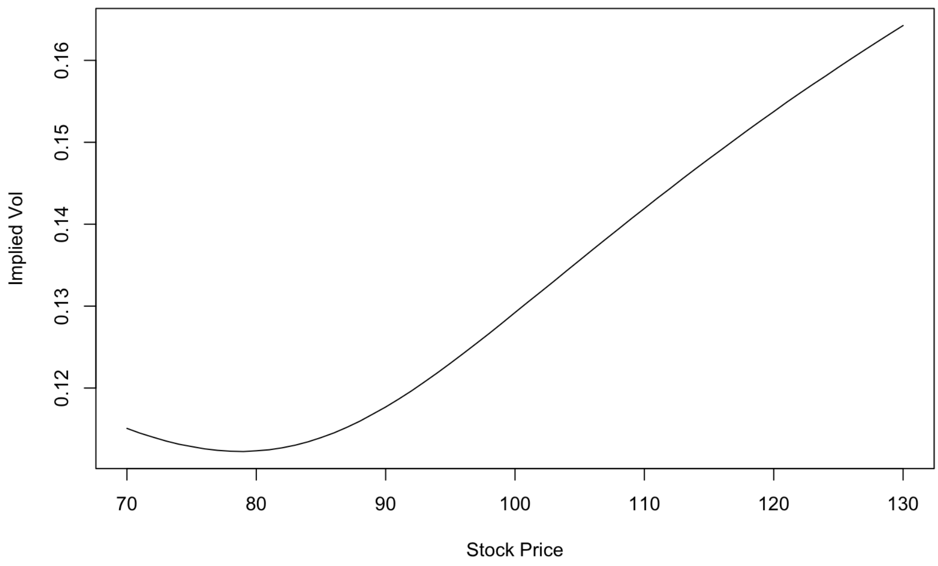


Figure 2: Implied Volatility with ρ= -0.5

d.

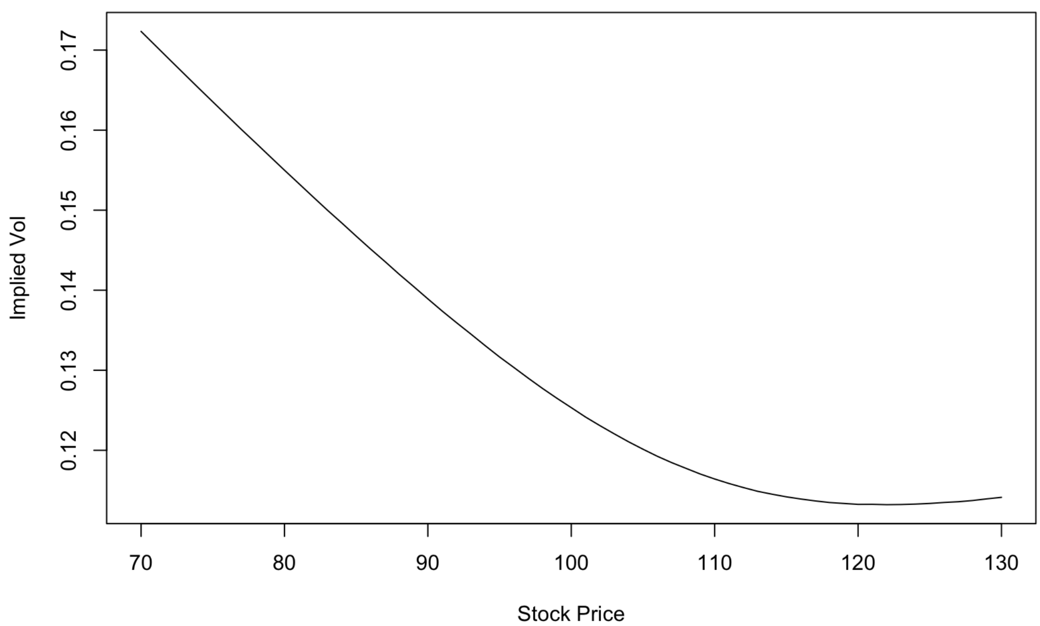


Figure 2: Implied Volatility with ρ= 0.5

When the brownian motions are negatively correlated the implied volatility increases with stock price, but when the correlation is positive the implie d volatility decreases with stock price.