MFE 409: Financial Risk Management
Week 1 Problem:
Trading Options with a VaR Constraint

March 30, 2020

You are a hedge fund manager with $100 million of capital, and you can take any long or short position as long as you keep the 99% 10-day VaR for your portfolio under your capital. You have limited liability.
You have access to long and short positions on a stock, 3-month European calls and puts on the underlying, and risk free bonds. You choose your positions today and must hold them for the next 10 days. Assume the risk-free asset has annual rate of return $r = 2\%$, and the stock price S_t has dynamics:

$$\frac{dS_t}{S_t} = \mu dt + \sigma dW_t$$

with $\mu = 7\%$ and $\sigma = 16\%$ (again, in annual units), and the current value $S_0 = 50$. Assume there are 252 trading days in a year.

1. Find a formula for the 10-day VaR for one share of the stock as a function of μ and σ. *Hint: if X is a random variable with quantile c equal to x_0, what is the quantile c of $g(X)$ if g is a monotone function?*

2. If you can only invest in stocks and bonds and want to maximize the average return on equity (subject to the VaR constraint), which portfolio do you choose?

3. If you can only invest in ATM calls and bonds, which portfolio do you choose?

4. If you can only invest in ATM puts and bonds, which portfolio do you choose?

5. Now you can choose one of the stock, call, or put (with arbitrary strike) to combine with bonds. Plot the optimal portfolio position as well as your expected return for each strike. Which strike and portfolio do you choose? Explain the intuition behind this result.

6. *Extra question.* What happens if you have to respect a constraint on expected shortfall instead of value-at-risk?