1. Show that $m(a+bX) = a + b \times m(X)$.

$m(a+bX)$ ------------- $m(X) = E(X)$

$m(a) + m(bX)$ ------------- the mean of a constant "a" and "b" is itself

$\boxed{a + b\,m(X)}$

2. Show that $\text{cov}(X, a+bY) = b \times \text{cov}(X,Y)$

formula : $\text{cov}(X,Y) = \frac{1}{N}\sum_{i=1}^{N}(X_i - m(X))(Y_i - m(Y))$

plug in : $= \frac{1}{N}\sum_{i=1}^{N}(X_i - m(X))((a+bY_i - m(a+bY_i))$

we know from problem 1

$(a+bY_i) - (a+b\cdot m(Y))$

$\cancel{a}+bY_i - \cancel{a} - b\cdot m(Y)$

$bY_i - bm(Y)$

$\frac{1}{N}\sum_{i=1}^{N}(X_i - m(X)) \cdot b(Y_i - m(Y))$ factor out b

$\boxed{\text{cov}(X,Y)\cdot b}$

3. Show that $\text{cov}(a+bX, a+bX) = b^2\text{cov}(X,X)$, and in particular that $\text{cov}(X,X) = s^2$.

plug into formula from earlier

$\text{cov}(a+bX, a+bX) = \frac{1}{N}\sum_{i=1}^{N}(a+bX_i - m(a+bX_i))((a+bX_i) - m(a+bX_i))$

we did this same calculation in problem 2

$\underline{\underline{b}}(X_i - m(X)) \cdot \underline{\underline{b}}(X_i - m(X))$

$\frac{1}{N}\sum_{i=1}^{N} b^2(X_i - m(X))^2$

$\frac{1}{N}\sum_{i=1}^{N}(X_i - m(X))^2 \cdot b^2$

this is the formula for sample variance

$\boxed{s^2 \cdot b^2}$

4. Instead of the mean, consider the median. Consider transformations that are non-decreasing (if $x \geq x'$, then $g(x) \geq g(x')$), like $2 + 5 \times X$ or $\text{arcsinh}(X)$. Is a non-decreasing transformation of the median the median of the transformed variable? Explain. Does your answer apply to any quantile? The IQR? The range?

median of $X = \{1, 2, 3, 4, 5\}$, median is 3

transformation $X \longrightarrow g(X) = 1 + 2(X)$, now,

$g(X) = \{3, 5, 7, 9, 11\}$, median is 7

$g(3) = 7$

$$\boxed{\text{median}(g(X)) = g(\text{median}(X))}$$

this works because median $(g(X)) = 7$ and $g(\text{median}(X)) = g(3) = 7$

The transformations will also work for quantiles because the values are preserved under the non-decreasing transformation. For IQR and Range, it will not be in the same format of X to g(X) because they are scaled.

5. Consider a non-decreasing transformation $g()$. Is is always true that $m(g(X)) = g(m(X))$?

$X = \{1, 2, 3, 4, 5\}$ median is 3

non linear transformation $g(X) = X^2$

$g(X) = \{1, 4, 9, 16, 25\}$ median is 9

$g(m(3)) = 3^2 = 9$ ✓ this one works

if we are talking about m = median, then it is true because the order is preserved in the list of numbers, so even after transforming it, m(g(X)) = g(m(X))