**University of Paris-Saclay** **Master 1 - AI**
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# Tutorial 3 with answers

**Exercise 1** Given the relation $R(A, B, C, D)$ below find the set of all functional dependencies of the form $X \to Y$ where $X$ and $Y$ are attributes of $R$.

| $R$ | $A$ | $B$ | $C$ | $D$ |
|---|---|---|---|---|
| | 1 | 1 | 2 | 3 |
| | 2 | 2 | 1 | 4 |
| | 1 | 3 | 2 | 2 |
| | 3 | 2 | 1 | 1 |

**Answer.** We remark that since all the values of $D$ are different, any functional dependency of the form $D \to Y$ is trivially true.

$A \to B$ **:** No. See 1st and 3rd tuple.

$A \to C$ **: Yes.**

$A \to D$ **:** No. See 1st and 3rd tuple.

$B \to A$ **:** No. See 2nd and 4th tuple.

$B \to C$ **: Yes.**

$B \to D$ **:** No. See 2nd and 4th tuple.

$C \to A$ **:** No. See 2nd and 4th tuple.

$C \to B$ **:** No. See 1st and 3rd tuple.

$C \to D$ **:** No. See 1st and 3rd tuple.

$D \to A$ **: Yes.**

$D \to B$ **: Yes.**

$D \to C$ **: Yes.**

So the set of all functional dependencies of $R$ is $\{A \to C, B \to C, D \to A, D \to B, D \to C\}$.

**Exercise 2** For the tables *Product*, *PC*, *Laptop* and *Printer* given below find the single attributes that can be keys.

**Answer.** *Product*: $\{model\}$
*PC*: $\{model\}$
*Laptop*: $\{model\}$, $\{price\}$
*Printer*: $\{model\}$

**Exercise 3** Consider a relation with schema *R(A, B, C, D)* and where the following functional dependencies hold:

$$AB \to C, \quad C \to D, \quad D \to A.$$

1. Find the closures of all subsets of $\{A, B, C, D\}$ and find all the superkeys.
2. Find all the keys.

**Answer.**

1. **Cardinality 1:** The closures of $\{A\}$ and $\{B\}$ are trivial. The closure of $\{D\}$ is $\{A, D\}$ because we have $D \to A$ and no other functional dependency determined by $\{A, D\}$.

   For the closure of $\{C\}$ we note that $C \to D$ implies that $D \in \{C\}^+$, thus $\{C\}^+ \supset \{C, D\}$. Also $D \to A$ implies that $A \in \{C\}^+$, thus $\{C\}^+ \supset \{A, C, D\}$. The attribute $B$ does not follow from any FD having on the left side some of the attributes of $\{A, C, D\}$, so $\{C\}^+ = \{A, C, D\}$.

   **Cardinality 2:** This implies that the closures of $\{A, C\}$ and $\{C, D\}$ are $\{A, C, D\}$.

   The closure of $\{A, B\}$ contains $C$ due to $AB \to C$ and $D$ due to $C \to D$, therefore $\{A, B\}$ is a superkey.

   The closure of $\{A, D\}$ is trivial.

   The closure of $\{B, C\}$ contains $D$ due to $C \to D$ and $A$ due to $D \to A$, therefore $\{B, C\}$ is a superkey.

   The closure of $\{B, D\}$ contains $A$ due to $D \to A$ and $C$ due to $AB \to C$, therefore $\{B, D\}$ is a superkey.

   **Cardinality 3:** Since $\{A, B\}$ is a superkey, then $\{A, B, C\}$ and $\{A, B, D\}$ are superkeys too.

   Since $\{B, C\}$ is a superkey, then $\{B, C, D\}$ is a superkey too.

   The closure of $\{A, C, D\}$ is trivial because $B$ is not determined by any other attributes.

2. The smallest superkeys have two elements and these are keys, so $\{A, B\}$, $\{B, C\}$ and $\{B, D\}$ are the keys.

**Exercise 4** Consider a relation with schema *S(A, B, C, D)* and where the following functional dependencies hold:

$$A \to B, \quad B \to C, \quad B \to D.$$

1. Find the closures of all subsets of $\{A, B, C, D\}$ and find all the superkeys.
2. Find all the keys.

**Answer.**

1. Let us consider the closure of $\{A\}$. From $A \to B$ we have that $B \in \{A\}^+$; from $B \to C$ and $B \to D$ we have that also $C$ and $D$ belong to $\{A\}^+$, thus $\{A\}$ is a superkey and indeed a key.

   This implies that any set of attributes containing $A$ is a superkey.

   From $B \to C$ and $B \to D$ we have that $C$ and $D$ belong to $\{B\}^+$. Since $A$ is not a consequence of any FD, then $\{B\}^+ = \{B, C, D\}$.

   This implies that the closure of any set of attributes containing $B$ and not $A$ is $\{B, C, D\}$.

   The closures of $\{C\}$ and $\{D\}$ and $\{C, D\}$ are trivial.

2. $\{A\}$ is a the only key.

**Exercise 5** For any of the relations below with the given functional dependencies prove that the relation is not in BCNF and decompose it into relations in BCNF using the algorithm seen during the course. Prove any claim that you make.
**Use this result:** A relation with exactly two attributes is always in BCNF.

1. *R(A, B, C, D)* with
$$AB \to C, \quad C \to D, \quad D \to A;$$
2. *R(A, B, C, D)* with
$$B \to C, \quad B \to D;$$

**Answer.**

1. We have that $C \to D$ is a BCNF violation. Indeed $\{C\}^+ = \{C, D, A\}$ because of $C \to D$ and $D \to A$ and because $B$ is not determined by any other attributes since it is not on the right side of any of the FD shown above.

   Thus we split the set of attributes $\{A, B, C, D\}$ into $\{A, C, D\} \cup \{B, C\}$.

   The relation $R_1(B, C)$ is in BCNF because it has two attributes, while $R_2(A, C, D)$ is not because $D \to A$ holds and $D$ is not a key since $D$ does not determine $C$.

   Since the closure of $D$ in $R_2$ is $\{A, D\}$, then we split $R_2(A, C, D)$ into $R_3(A, D)$ and $R_4(C, D)$ which are in BCNF.

2. Trivially we have that $\{B\}^+ = \{B, C, D\}$ and that $\{A, B\}$ is the only key, thus $B \to C$ is a BCNF violation.

   We decompose $\{A, B, C, D\}$ into $\{B, C, D\} \cup \{A, B\}$. The relation $R_1(A, B)$ is in BCNF because it has two attributes. Also $R_2(B, C, D)$ is in BCNF because $\{B\}$ is a key and the only FD holding in $R_2$ are $B \to C$ and $B \to D$.

| *maker* | *model* | *type* |
|---|---|---|
| A | 1001 | pc |
| A | 1002 | pc |
| A | 1003 | pc |
| A | 2004 | laptop |
| A | 2005 | laptop |
| A | 2006 | laptop |
| B | 1004 | pc |
| B | 1005 | pc |
| B | 1006 | pc |
| B | 2007 | laptop |
| C | 1007 | pc |
| D | 1008 | pc |
| D | 1009 | pc |
| D | 1010 | pc |
| D | 3004 | printer |
| D | 3005 | printer |
| E | 1011 | pc |
| E | 1012 | pc |
| E | 1013 | pc |
| E | 2001 | laptop |
| E | 2002 | laptop |
| E | 2003 | laptop |
| E | 3001 | printer |
| E | 3002 | printer |
| E | 3003 | printer |
| F | 2008 | laptop |
| F | 2009 | laptop |
| G | 2010 | laptop |
| H | 3006 | printer |
| H | 3007 | printer |

Figure 20: Sample data for `Product`

| *model* | *speed* | *ram* | *hd* | *price* |
|---|---|---|---|---|
| 1001 | 2.66 | 1024 | 250 | 2114 |
| 1002 | 2.10 | 512 | 250 | 995 |
| 1003 | 1.42 | 512 | 80 | 478 |
| 1004 | 2.80 | 1024 | 250 | 649 |
| 1005 | 3.20 | 512 | 250 | 630 |
| 1006 | 3.20 | 1024 | 320 | 1049 |
| 1007 | 2.20 | 1024 | 200 | 510 |
| 1008 | 2.20 | 2048 | 250 | 770 |
| 1009 | 2.00 | 1024 | 250 | 650 |
| 1010 | 2.80 | 2048 | 300 | 770 |
| 1011 | 1.86 | 2048 | 160 | 959 |
| 1012 | 2.80 | 1024 | 160 | 649 |
| 1013 | 3.06 | 512 | 80 | 529 |

(a) Sample data for relation `PC`

| *model* | *speed* | *ram* | *hd* | *screen* | *price* |
|---|---|---|---|---|---|
| 2001 | 2.00 | 2048 | 240 | 20.1 | 3673 |
| 2002 | 1.73 | 1024 | 80 | 17.0 | 949 |
| 2003 | 1.80 | 512 | 60 | 15.4 | 549 |
| 2004 | 2.00 | 512 | 60 | 13.3 | 1150 |
| 2005 | 2.16 | 1024 | 120 | 17.0 | 2500 |
| 2006 | 2.00 | 2048 | 80 | 15.4 | 1700 |
| 2007 | 1.83 | 1024 | 120 | 13.3 | 1429 |
| 2008 | 1.60 | 1024 | 100 | 15.4 | 900 |
| 2009 | 1.60 | 512 | 80 | 14.1 | 680 |
| 2010 | 2.00 | 2048 | 160 | 15.4 | 2300 |

(b) Sample data for relation `Laptop`

| *model* | *color* | *type* | *price* |
|---|---|---|---|
| 3001 | true | ink-jet | 99 |
| 3002 | false | laser | 239 |
| 3003 | true | laser | 899 |
| 3004 | true | ink-jet | 120 |
| 3005 | false | laser | 120 |
| 3006 | true | ink-jet | 100 |
| 3007 | true | laser | 200 |

(c) Sample data for relation `Printer`