0.1 Gemometry, projections ‡

Let a P (or $\mathcal{P}$) plane of equation $\vec{p}\cdot\vec{x}+c=0$. We can note $|\vec{p}|$ the standard of p, i.e. $|\vec{p}|=\sqrt{\vec{p}\cdot\vec{p}}$.

1. Starting with the simple case $c=0$, and the sub-case $\vec{p}=(0,0,2)$ (in 3 dimensions, to fix the ideas), give a formula to calculate the projected $\vec{a}'$ of a point $\vec{a}$ on the P plane. Hint: first see what this P plane is, then see how to hack the coordinates of a to project them on this plane.
2. Generalize for any p (always with $c=0$, i.e. a plane passing through the origin of the system of coordinates). Hint: there is not so much reasoning to do, just convince yourself intuitively (drawings are allowed).
3. Generalize for any c. Remember that c is interpreted as the distance between the plane and the origin of the marker, i.e. the length of the OO' vector, if O is the origin of the marker and O' its projected on the plane.

0.2 Solution - Exo 0.1 - Gemometry, Projections ‡

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1. Starting with the simple case $c=0$, and the sub-case $\vec{p}=(0,0,2)$ (in 3 dimensions, to fix the ideas), give a formula to calculate the projected $\vec{a}'$ of a point $\vec{a}$ on the P plane. Hint: first see what this P plane is, then see how to hack the coordinates of a to project them on this plane.
   Solution: The plane is the plane $z=0$. Remember to remove the z component (3rd component) from the a vector. This component is obtained by$\vec{a}\cdot\vec{p}$, except that we must normalize $\vec{p}$, i.e. divide it by its norm: $\vec{a}\cdot\frac{\vec{p}}{|\vec{p}|}$. So you have to subtract that from $\vec{a}$. The formula is: $\vec{a}'=\vec{a}-\frac{\vec{a}\cdot\vec{p}}{|\vec{p}|}\frac{\vec{p}}{|\vec{p}|}$
2. Generalize for any p (always with $c=0$, i.e. a plane passing through the origin of the system of coordinates). Hint: there is not so much reasoning to do, just convince yourself intuitively (drawings are allowed).
   Solution: The formula is the same: $\vec{a}'=\vec{a}-\frac{\vec{a}\cdot\vec{p}}{|\vec{p}|}\frac{\vec{p}}{|\vec{p}|}$
3. Generalize for any c. Remember that c is interpreted as the distance between the plane and the origin of the marker, i.e. the length of the OO' vector, if O is the origin of the marker and O' its projected on the plane.
   Solution: For any a point, with the previous formula, we will project on the plane parallel to P which passes through O. So we need to shift the solution by an amount $\vec{OO'}$. With a picture, positionning $\vec{p}$ on O, we see that $OO'=|OO'|\frac{\vec{p}}{|\vec{p}|}=c.\frac{\vec{p}}{|\vec{p}|}$. The general formula is therefore $\vec{a}'=\vec{a}-\frac{\vec{a}\cdot\vec{p}}{|\vec{p}|}\frac{\vec{p}}{|\vec{p}|}+c.\frac{\vec{p}}{|\vec{p}|}$. If we assume that $\vec{p}$ is normalized (i.e. $|\vec{p}|=1$), it is prettier: $\vec{a}'=\vec{a}-(\vec{a}\cdot\vec{p})\vec{p}+c\vec{p}$ We see that a $\vec{a}$ point belonging to P would have projected itself, because it would check $\vec{p}\cdot\vec{a}+c=0$, and so the last 2 terms cancel each other out, and we have $\vec{a}'=\vec{a}$ (which is logical).

Note: this equation is valid in any size $d\in\mathbb{N}$! Nice, isn't it?