Event studies

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Outline

References

- Campbell J., Lo A. and A.C. MacKinlay (1997), The Econometrics of Financial Markets, Princeton University Press
- Kothari S. and J. Warner, The Econometrics of Event Studies, Handbook of Corporate Finance, B. Eckbo (editor), Elsevier-North Holland

Event studies: What for?

Event studies aim at quantifying the effects of an (unexpected) economic event on the value of firms

- Financial economics: corporate events, market efficiency
- Macroeconomic policy: fed rates, trade deficits
- Accounting: earnings announcements
- Law and economics: changes in legal environment, and regulation (SOX), insider trading rules, infringements
- Marketing: brand strategy announcements
- ...

For a variety of reasons, the methodology of event-study was initiated and has been further developed in the field of financial economics

- Events are reflected in asset prices (assuming markets are informationnally efficient)
- Prices are easily observed
- Well-performing models are available to isolate the impact of a given event on asset prices

Pioneering works in event studies:

- J. Dolley (1933): sample of 95 splits from 1921 to 1931. Prices increase in 57 of the cases and prices decline in only 26 instances
- 1930's - 1960's: increased level of sophistication. Correction for general stock price movements and confounding events (Myers J. and A. Bakay, 1948; A. Barker, 1956,1957,1958; J. Ashley, 1962)
- Ball R. and P. Brown (1968): information content of earnings announcements
- Fama E, Fisher L, Jensen M. and R. Roll (1969): effect of stock splits (after removing the effect of simultaneous dividend increases)

Since the seminal work by Fama et al., the basic layout of event studies has not changed over time: the key focus is still on measuring the sample securities' mean, and potentially the cumulative mean abnormal return around the time of an event

Main changes include:

- Use of daily (and sometimes intradaily) rather than monthly security returns
- The methods used to estimate abnormal returns and calibrate their statistical significance have become more sophisticated, in particular with the advent of ML prediction methods
- Developments in the asset pricing literature

The making of an event studies involves 7 steps (Campbell J, Lo A. and A. MacKinlay)

1. Definition of the event
2. Selection criteria
3. Normal and abnormal returns
4. Estimation procedure
5. Test procedure
6. Results
7. Interpretation and conclusions

Step 1: Definition of the event

- Definition of the event of interest
- Identification of the period over which the security prices of the firms involved in the event will be examined: the event window
- The analysis will be conducted in relative time: date 0 corresponds to the event date
- It is customary to define an event window that is larger than the exact period of interest
  - Inclusion of the days prior to the event (-1, -2, ...) aims at accounting for possible anticipation of the event as well as information leakage
  - Inclusion of the days after the event ($+1$, $+2$, ...) aims at capturing posterior abnormal movements that occur after market close

Step 2: Selection criteria

- Which firms have to be included in the study?
- Restrictions imposed by data availability and reliability
- Restrictions imposed by representativeness issues
- Some summary statistics (market capitalization, average return, industry representation, distribution of events through time...) might prove useful to identify potential biases in the initial sample as well as outliers

Step 3: Normal and abnormal returns

- The event of interest is expected to induce specific price movements for the sample firms
- Issue: how to isolate those movements from contemporaneous movements in a given stock that are unrelated to the event?
- The isolation of the movements that are specific to the event is made in two steps
  1. Computation of the time series of abnormal returns for each firm
  2. Abnormal returns are then averaged in the cross-section: time series of average abnormal returns (AAR)
- Rationale for step 2: all selected firms experience the same event, therefore abnormal returns unrelated to the event should vanish through aggregation

Step 3: Normal and abnormal returns

Date	Firm 1	Firm 2	$\cdots$	Firm i	$\cdots$	Firm n	Sample
-2	$AR_{1,-2}$	$AR_{2,-2}$	$\cdots$	$AR_{i,-2}$	$\cdots$	$AR_{n,-2}$	AAR_{-2}
-1	$AR_{1,-1}$	$AR_{2,-1}$	$\cdots$	$AR_{i,-1}$	$\cdots$	$AR_{n,-1}$	AAR_{-1}
0	$AR_{1,0}$	$AR_{2,0}$	$\cdots$	$AR_{i,0}$	$\cdots$	$AR_{n,0}$	AAR_{0}
+1	$AR_{1,+1}$	$AR_{2,+1}$	$\cdots$	$AR_{i,+1}$	$\cdots$	$AR_{n,+1}$	AAR_{+1}
+2	$AR_{1,+2}$	$AR_{2,+2}$	$\cdots$	$AR_{i,+2}$	$\cdots$	$AR_{n,+2}$	AAR_{+2}
+3	$AR_{1,+3}$	$AR_{2,+3}$	$\cdots$	$AR_{i,+3}$	$\cdots$	$AR_{n,+3}$	AAR_{+3}
+4	$AR_{1,+4}$	$AR_{2,+4}$	$\cdots$	$AR_{i,+4}$	$\cdots$	$AR_{n,+4}$	AAR_{+4}
+5	$AR_{1,+5}$	$AR_{2,+5}$	$\cdots$	$AR_{i,+5}$	$\cdots$	$AR_{n,+5}$	AAR_{+5}

Step 3: Normal and abnormal returns

- Abnormal returns are computed as the difference between observed returns and normal returns
- Normal returns correspond to the expected would-be returns if the event had not taken place
- Computation of abnormal returns:

$$AR_{i,\tau} = R^{\star}_{i,\tau} - E(R_{i,\tau}|X_{\tau})$$

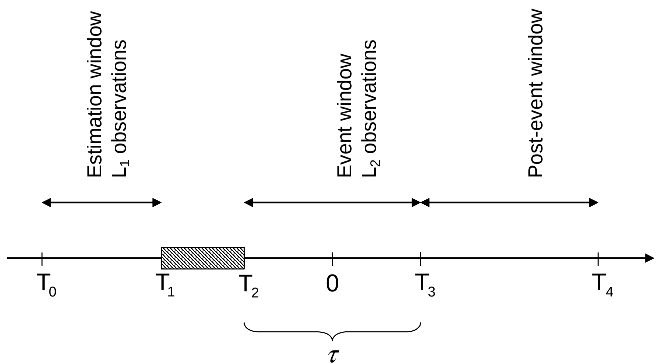
- A model is needed that permits computation (prediction) of normal returns
- There exist various solutions for the computation of normal returns
- In this session we will explore classical return-generating models as well as ML return-generating models

Step 4: Estimation procedure

- The parameters of the model that is used to generate normal returns over the event window have to be estimated
- The estimation is performed on the estimation window
- It is necessary to check that the estimation window is not contaminated by events that are likely to impact the parameters of the model that generates normal returns
- Intuitively, the event window (or part of it) should not be included in the estimation window (when feasible)
- It is quite common to introduce a buffer zone between the estimation window and the event window
- Common choices for the length of the estimation window are 120 days or 250 days

Step 4: Estimation procedure

Timeline for an event study (Source: Campbell, Lo and MacKinlay)



Step 5: Test procedure

Once abnormal returns are computed, the objective is to test their significance

The test procedure involves 2 steps

1. Step 1: aggregation of abnormal returns
   - Abnormal returns are first aggregated in the cross section of sample stocks to compute AARs on each event date
   - Average abnormal returns can also be aggregated in time-series to compute Cumulated Average Abnormal Returns (CAR) over the event window
   - CAR is useful to estimate the incidence of the event for claimholders' wealth

Step 5: Test procedure

2. Step 2: test of the null hypothesis
   - Comparison of the of actual distribution of returns with that of predicted returns
   - Typically, the specific null hypothesis to be tested is whether the mean abnormal return at time τ is equal to 0
   - Occasionally, other parameters of the cross-sectional variation in abnormal returns can be used
     - median
     - variance
   - To perform the test, one has to know the sampling distribution of AAR or CAR

Step 6: Results

The empiricist has to question the reliability of his results

- Robustness tests using various sub-samples
- Incidence of outliers?
- Sensitivity to the choice of the estimation window?
- Sensitivity to the normal-return generating model?

Step 7: Interpretation and conclusion

Either results are significant or not

- What do they mean?
- Are they different from those obtained in other comparable studies? Why?
- Differences in subsamples? Why?
- Sensitivity to outliers? Are outliers a proxy for some omitted factor?
- ...

Classical return-generating models

Two main standard models:

- Constant mean-return model
- Market model

Though simple, these models have proven reliable for the detection of abnormal returns (see below)

The constant mean-return model

Definition and assumptions

- The constant mean-return model posits that firm i's return on date t is given by

$$R_{i,t} = \mu_i + \varepsilon_{i,t}$$

- μ_i is firm i's average return
- $\varepsilon_{i,t}$ is the innovation on date t
- The following (simplifying) assumptions are made:
  - $E(\varepsilon_i) = 0$
  - $\sigma^2(\varepsilon_{i,t}) = \sigma^2(\varepsilon_i), \ \forall t$ (homoscedasticity)
  - $E(\varepsilon_{i,t} \times \varepsilon_{i,t'}) = 0, \ \forall t \neq t'$ (random walk - market efficiency)

The market model (Sharpe, 1963)

Definition and assumptions

- The market model posits a linear relationship between $R_{i,t}$ and the market return on the same date, denoted $R_{m,t}$ (proxied by the return of an appropriate stock index - eg S&P 500)

$$R_{i,t} = \alpha_i + \beta_i R_{m,t} + \xi_{i,t}$$

- α_i and β_i are the two parameters of the market model for firm i
- $\xi_{i,t}$ is a disturbance term
    - $E(\xi_i) = 0$
    - $\sigma^2(\xi_{i,t}) = \sigma^2(\xi_i), \ \forall t$ (homoscedasticity)
    - $E(\xi_{i,t} \times \xi_{i,t'}) = 0, \ \forall t \neq t'$

Other return-generating models

Statistical models

- Index model – Equivalent to the market model where α_i is constrained to 0 and β_i is constrained to 1: $R_{i,t} = R_{m,t}$
- Factor models – The goal is to reduce the variance of the abnormal returns by explaining a larger fraction of the variation in normal returns. Common choices include:
  - The Fama-French 3-factor model:

$$R_{i,t} = \alpha_i + \beta_{1,i} R_{m,t} + \beta_{2,i} SMB_t + \beta_{3,i} HML_t + \eta_{i,t}$$

  - The Fama-French 5-factor model:

$$R_{i,t} = \alpha_i + \beta_{1,i} R_{m,t} + \beta_{2,i} SMB_t + \beta_{3,i} HML_t + \beta_{4,i} RMW_t + \beta_{5,i} CMA_t + \zeta_{i,t}$$

- New trend: normal return predictors relying on ML models

- The parameters of the various statistical models – constant-mean, market, Fama-French, Carhart – must be estimated on the estimation window
- It is possible to estimate the parameter(s) of any of these models using a matrix representation and applying the appropriate specification so as to estimate the set of parameters of the selected return-generating model

Estimation: matrix representation of data

- Firm i returns on the estimation window can be written as:

$$\mathbf{R_i} = \mathbf{X}_i\theta_i + \epsilon_i \tag{1}$$

  where $\mathbf{R}_i = [R_{i,T_0}, \ldots, R_{i,T_1}]'$ is the vector of firm i L_1 returns on the estimation window $[T_0; T_1]$
  - $\mathbf{X_i}$ reduces to a L_1-row column vector of ones for the constant-mean model
  - $\mathbf{X_i}$ is a $L_1 \times 2$ matrix with ones in the firm column and the L_1 market returns on dates T_0 to T_1 for the market model
- θ_i corresponds to the vector of parameters associated with the model of interest
  - θ_i reduces to the scalar μ_i for the constant-mean model
  - θ_i is equal to vector $[\alpha_i, \beta_i]'$ for the market model

Estimation: matrix representation of data

- Irrespective of the underlying model, the estimation of the parameters can be made through OLS and the following results obtain

$$\hat{\theta}_i = (\mathbf{X}_i'\mathbf{X}_i)^{-1}\mathbf{X}_i'\mathbf{R}_i \quad (2)$$

$$\hat{\sigma}^2(\epsilon_i) = \frac{1}{L_1 - k}\hat{\epsilon}_i'\hat{\epsilon}_i \quad (3)$$

$$\hat{\epsilon}_i = \mathbf{R}_i - \mathbf{X}_i\hat{\theta}_i \quad (4)$$

$$\sigma^2(\hat{\theta}_i) = (\mathbf{X}_i'\mathbf{X}_i)^{-1}\sigma^2(\epsilon_i) \quad (5)$$

where $k = 1$ for the constant-mean model and $k = 2$ for the market model

- Abnormal returns are computed for each firm on each date $[T_2; T_3]$ of the event window as the difference between observed returns and predicted returns
- Using the previously defined notations:

$$\hat{\epsilon}_i^\star = \mathbf{R}_i^\star - \mathbf{X}_i^\star \hat{\theta}_i$$

where:

  - $\mathbf{R}_i^\star$ is the L_2 -row vector of observed returns on the event date
  - $\mathbf{X}_i^\star$ is either a L_2-row vector of ones or a $L_2 \times 2$ matrix with ones on the first column and market returns on dates T_2 to T_3 on the second column
  - $\hat{\theta}_i$ is the vector of parameter estimates

The case of a single asset

- The abnormal returns for firm i on dates $[T_2; T_3]$ will be distributed with zero mean and a $L_2 \times L_2$ covariance matrix whose value is given by:

$$V_i = \sigma^2(\epsilon_i)\mathbf{I} + \sigma^2(\epsilon_i)[\mathbf{X}_i^\star(\mathbf{X}_i'\mathbf{X}_i)^{-1}\mathbf{X}_i^{\star\prime}] \quad (6)$$

  where $\mathbf{I}$ is the $L_2 \times L_2$ identity matrix
- The first term in (6) is the variance due to future disturbances whereas the second term is the additional variance due to sampling error in $\hat{\theta}_i$
- Notice that the second term goes to 0 when the length L_1 of the estimation window becomes large
- Most tests assume that V_i reduces to $\sigma^2(\epsilon_i)\mathbf{I}$: the variance of the abnormal returns is thus $\sigma^2(\epsilon_i)$

The case of a single asset

- For the market model, (6) reduces to

$$Q_{i,\tau} = \sigma^2(\epsilon_i) \underbrace{\left\{ 1 + \frac{1}{L_1} + \frac{(R_{m,\tau} - \bar{R}_m)^2}{\sum_{t=T_0}^{T_1} (R_{m,t} - \bar{R}_m)^2} \right\}}_{C_{i,\tau}}$$

- C_i reflects the increase in variance due to prediction outside the estimation window (estimation is made out-of-sample)
- Notice that C_i goes to 1 when L_1 goes to $+\infty$

The case of a single asset

- To test whether $\hat{\epsilon}^{\star}_{i,\tau}$ is different from 0 on a given date, we need the additional restrictive assumption that the (abnormal) return distributions are normal
- Under this assumption, the following results obtain:

$$\frac{\hat{\epsilon}^{\star}_{i,\tau}}{\sigma(\epsilon_i)} \sim N(0,1) \text{ or } \frac{\hat{\epsilon}^{\star}_{i,\tau}}{\sigma(\epsilon_i)\sqrt{C_{i,\tau}}} \sim N(0,1) \tag{7}$$

$$\frac{(L_1 - k)\hat{\sigma}^2(\epsilon_i)}{\sigma^2(\epsilon_i)} \sim \chi^2(L_1 - k) \tag{8}$$

$$\frac{\hat{\epsilon}^{\star}_{i,\tau}}{\hat{\sigma}(\epsilon_i)} \sim t(L_1 - k) \text{ or } \frac{\hat{\epsilon}^{\star}_{i,\tau}}{\hat{\sigma}(\epsilon_i)\sqrt{C_{i,\tau}}} \sim t(L_1 - k) \tag{9}$$

Cross-sectional aggregation

Brown and Warner (1980) – Constant-mean return

- As cross-sectional independence may not hold in the constant-mean return model, one way to deal with this issue is to form equally-weighted portfolios over both the estimation window and the event window
- On a given event date, the abnormal return for the portfolio, assuming N event firms, is computed just as

$$AAR_\tau = \frac{1}{N} \sum_{i=1}^{N} \hat{\epsilon}^\star_{i,\tau} \qquad (10)$$

Cross-sectional aggregation

Brown and Warner (1980) – Constant-mean return (cont'd)

- The variance of the AARs on the estimation window can be estimated as

$$\frac{1}{L_1 - 1} \sum_{t=T_0}^{T_1} (AAR_t - \overline{AAR})^2 \tag{11}$$

where

$$\overline{AAR} = \frac{1}{NL_1} \sum_{t=T_0}^{T_1} \sum_{i=1}^{N} AR_{i,t}$$

Cross-sectional aggregation

Brown and Warner (1980) – Constant-mean return (cont'd)

The statistic

$$\frac{\frac{1}{N}\sum_{i=1}^{N} AR_{i,\tau}}{\sqrt{\frac{1}{L_1 - 1}\sum_{t=T_0}^{T_1}(AAR_t - \overline{AAR})^2}}$$

has a Student-t distribution with $(L_1 - 1)$ degrees of freedom

Cross-sectional aggregation

Brown and Warner (1980) – Market model

- Under the market model, residual cross-correlation is likely to be small
- AAR_τ on the event window are calculated in the same way as in (10)
- Since we assume cross-sectional independence, the standard deviation of AAR_τ is given by

$$\frac{1}{N}\sqrt{\sum_{i=1}^{N} \sigma_i^2(\epsilon_i)} \tag{12}$$

Cross-sectional aggregation

Brown and Warner (1980) – Market model (cont'd)

- Since, (12) is unknown, it must be estimated. An unbiased estimator is:

$$\frac{1}{N}\sqrt{\sum_{i=1}^{N}\hat{\sigma}_i^2(\epsilon_i)}$$

- Thus, the statistic

$$\frac{\sum_{i=1}^{N}\hat{\epsilon}^{\star}_{i,\tau}}{\sqrt{\sum_{i=1}^{N}\hat{\sigma}_i^2(\epsilon_i)}}$$

has a Student-t distribution with $(L_1 - 2)$ degrees of freedom

Times-series aggregation – CAR

Brown and Warner (1980) – Constant-mean return model

- Let denote $CAAR = \sum_\tau AAR_\tau$
- Clearly, $E(CAAR) = 0$, and assuming time-series independence, $\sigma(ACAR) = \sqrt{L_2}\sigma(AAR_\tau)$
- The unbiased estimator $\hat{\sigma}^2(AAR_\tau)$ is the same as in (11), so that

$$\frac{\sum_\tau AAR_\tau}{\sqrt{\frac{L_2}{L_1 - 1}\sum_{t=T_0}^{T_1}(AAR_t - \overline{AAR})^2}}$$

has a Student-t distribution with $(L_1 - 1)$ degrees of freedom

Times-series aggregation – CAR

Brown and Warner (1980) – Market model

- Let $CAR_i = \sum_\tau \hat{\epsilon}^\star_{i,\tau}$
- $E(CAR_i) = 0$, and assuming time-series independence, $\sigma(CAR_i) = \sqrt{L_2}\sigma(\epsilon_i)$
- Let $ACAR = 1/N \sum_{i=1}^N CAR_i$
- $E(ACAR) = 0$, and assuming cross-sectional independence, $\sigma(ACAR) = 1/N\sqrt{\sum_{i=1}^N \sigma^2(CAR_i)}$, which is equal to $\sqrt{L_2}/N\sqrt{\sum_{i=1}^N \sigma^2(\epsilon_i)}$

Times-series aggregation – CAR

Brown and Warner (1980) – Market model (cont'd)

- Since $\sigma^2(\epsilon_i)$ is unknown, we must use the unbiased estimator $\hat{\sigma}_i^2(\epsilon_i)$
- Thus the statistic

$$\frac{\sum_{i=1}^{N} \sum_{\tau} \hat{\epsilon}^{\star}_{i,\tau}}{\sqrt{L_2}\sqrt{\sum_{i=1}^{N} \hat{\sigma}_i^2(\epsilon_i)}}$$

has a Student-t distribution with $(L_1 - 2)$ degrees of freedom

Non parametric tests

- So far, we have assumed that abnormal returns (residuals) are normally distributed, which allowed us to derive the sampling distribution of AR, AAR as well as CAR
- We thus performed parametric tests
- In non parametric tests, no assumption is made as to the distribution of abnormal returns

Non parametric tests

The sign test

- The sign test is a cross-sectional test that is based on the size of the abnormal returns (both the sample of $AR_i s$ or $CAR_i s$ can be used)
- It requires that the $AR_i s$ or $CAR_i s$ are independent across securities and that the expected proportion of positive (cumulated) abnormal returns under the null hypothesis is .5
- If, for example, the alternative hypothesis is that that there is a positive abnormal return associated with the event, the null hypothesis is $H_0: \ p \leq 0.5$ and the alternative is $H_a: \ p > 0.5$, where $p = Pr(AR \ or \ CAR \geq 0)$

Non parametric tests

The sign test (cont'd)

- Under the previously defined null hypothesis, the statistic requires the computation of N^+, where N^+ is the number of cases when the (cumulated) abnormal return is positive
- Asymptotically when N increases, we have:

$$\left[\frac{N^+}{N} - 0.5\right]\frac{\sqrt{N}}{0.5} \sim N(0,1)$$

Non parametric tests

Corrado (1989) rank test

- A weakness of the sign test is that it may not be well specified if the distribution of abnormal return is skewed
- This is likely to happen with daily or intradaily data
- With skewed abnormal returns, the expected proportion of positive abnormal returns can differ from 0.5 even under the null hypothesis
- The Corrado (1989) rank test adresses this shortcoming

Non parametric tests

Corrado (1989) rank test (cont'd)

- Drawing on previous notations, consider the sample of $(L_1 + L_2)$ returns for the N event firms
- The $(L_1 + L_2)$ abnormal returns on each firm i are ranked from 1 to $(L_1 + L_2)$
- Let denote $K_{i,\tau}$ the rank of the abnormal return of firm i for the event date τ
- The rank test uses the fact that the expected rank under the null hypothesis is $(L_1 + L_2)/2$

Non parametric tests

Corrado (1989) rank test (cont'd)

- The test statistics (whose asymptotic distribution is standard normal) for the null hypothesis of no abnormal return on event day τ is computed as

$$\frac{1}{N}\sum_{i=1}^{N}\left(K_{i,\tau}-\frac{L_1+L_2}{2}\right)\Big/ S(K)$$

where $S(K)$ is equal to

$$S(K)=\sqrt{\frac{1}{L_1+L_2}\sum_{t=1}^{L_1+L_2}\left(\frac{1}{N}\sum_{i=1}^{N}\left(K_{i,t}-\frac{L_1+L_2}{2}\right)\right)^2}$$

- Using test statistics, errors are of two types:
  - Type I error occurs when the null hypothesis is falsely rejected (false positive)
  - Type II error occurs when the null hypothesis is falsely accepted (false negative)
- Accordingly, two key properties of event study tests have been investigated
  - The first is whether the test statistic is correctly specified. A correctly specified test yields a Type I error probability equal to the assumed size of the test (eg 1%, 5%)
  - The second concern is power, i.e. a test's ability to detect abnormal performance when it is present.
  - When comparing tests that are well specified, those with higher power are preferred

The joint test issue

- While the specification and power of a test can be statistically determined, economic interpretation is not straightforward because all tests are joint tests
- Indeed, an event study is a test of whether abnormal returns are zero <u>and</u> whether the assumed return-generating model is correct

Brown-Warner simulations

- To address the issue of event study properties, the standard tool is to employ simulation procedures that use actual security returns
- Different event study methods are simulated repeatedly by applying each method to samples that have been constructed through a random selection of securities and random assignment of an event date to each
- If abnormal returns are measured correctly, these samples should show no abnormal performance, on average

This makes possible to study test statistic specification, i.e. rejecting the null hypothesis when it is known to be true

Brown-Warner simulations (cont'd)

- Further, various levels of a deterministic abnormal performance (eg 0.1%, 0.5%, 1%) are artificially introduced in the sample
- This permits direct study of the power of the event study tests, i.e. the ability to detect a given level of abnormal performance