

Data acquisition, extraction, and storage

Relational Database Management

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Data management

Numerous applications (standalone software, Web sites, etc.) need to **manage data**:

- **Structure** data useful to the application
- Store them in a **persistent** manner (data retained even when the application is not running)
- **Efficiently query** information within large data volumes
- **Update** data without violating some structural **constraints**
- Enable data access and updates by **multiple users**, possibly **concurrently**

Often, desirable to access the same data from **several distinct applications**, from distinct computers.

Example: Information system of a hotel

Access from an in-house software (front desk), a Website (guests), an accounting software suite. Requirements:

- **Structured data** representing rooms, customers, guests, bookings, rates, etc.
- **No loss of data** when these applications are unused, or when a power cut arises
- **Find quasi-instantaneously** which rooms are booked, by whom, a given day, within a history containing several years of bookings
- Easily **add** a booking by ensuring the same room is not booked twice the same day
- The guest, the front desk employee, the accountant, must not have the same **view** of the data (confidentiality, ease of use, etc.)
- If a room is available, it cannot be booked by different guests **at the same time**

Naive implementation (1/2)

- Implementation in a classical programming language (C++, Java, Python, etc.) of data structures that can represent all useful data
- Definition of ad-hoc file formats to store data on disk, with regular synchronization and a mechanism for failure recovery
- In-memory storage of application data, with data structures (search trees, hash tables) and algorithms (search, sort, aggregation, graph navigation, etc.) for efficiently finding information
- Data update functions, with code ensuring no constraint is violated

Naive implementation (2/2)

- Definition within the application of user access rights and an authentication process; use of parallel programming to answer different requests at the same time, locks/semaphores on critical data manipulation steps
- Definition and implementation of a network communication protocol to access this software component from the Web, from a Windows application, from an accounting suite, etc.

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Lots of work!

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Lots of work! Requires a programmer that masters OOP, serialization, failure recovery, data structures, algorithms, integrity constraint verification, role management, parallel programming, concurrency control, network programming, etc.

Naive implementation (2/2)

- Definition within the application of user access rights and an authentication process; use of parallel programming to answer different requests at the same time, locks/semaphores on critical data manipulation steps
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Lots of work! Requires a programmer that masters OOP, serialization, failure recovery, data structures, algorithms, integrity constraint verification, role management, parallel programming, concurrency control, network programming, etc. Needs to be done again **for every new application** that manages data!

Role of a DBMS

Database Management System

Software that **simplifies the design** of applications that handle data, by providing a **unified access** to the functionalities required for **data management**, whatever the application.

Database

Collection of data (specific to a given application) managed by a DBMS

Features of DBMSs (1/2)

Physical independence. The user of a DBMS does not need to know how data are stored (in a file, on a raw partition, in a distributed filesystem, etc.); storage can be modified without impacting data access

Logical independence. It is possible to provide the user with a partial view of the data, corresponding to what he needs and is allowed to access

Ease of data access. Use of a declarative language describing queries and updates on the data, specifying the intent of a user rather than the way this will be implemented

Query optimization. Queries are automatically optimized to be implemented as efficiently as possible on the database

Features of DBMSs (2/2)

Logical integrity. The DBMS imposes constraints on data structure; every modification violating these constraints is denied

Physical integrity. The database remains in a coherent state, and data are durably preserved, even in case of software or hardware failure

Data sharing. Data are accessible by multiple users, concurrently, and these multiple and concurrent accesses cannot violate logical or physical data integrity

Standardization. The use of a DBMS is standardized, so that it may be possible to replace a DBMS with another without changing (in a major way) the code of the application

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Diversity of DBMSs

- **Dozens** of existing DBMSs that are broadly used
- All DBMSs do not provide **all these features**
- DBMSs can be **differentiated** based on:
 - data model used
 - trade-offs made between performance and features
 - ease of use
 - scalability
 - internal architecture

Major types of DBMSs

Relational (RDBMS). Tables, complex queries (SQL), rich features

XML. Trees, complex queries (XQuery), features similar to RDBMS

Graph/Triples. Graph data, complex queries expressing graph navigation

Objects. Complex data model, inspired by OOP

Documents. Complex data, organized in documents, relatively simple queries and features

Key-Value. Very basic data model, focus on performance

Column Stores. Data model in between key-value and RDBMS; focus on iteration and aggregation on columns

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NoSQL

Classical relational DBMSs

- Based on the **relational model**: decomposition of data into relations (i.e., tables)
- A standard query language: **SQL**
- Data **stored on disk**
- Relations (tables) stored **row by row**
- Centralized** system, with limited distribution possibilities

ORACLE
DATABASE


Microsoft®
SQL Server™

IBM
DB2

SAP **ASE**

 **SQLite**


MySQL®

PostgreSQL


Strengths of classical relational DBMSs

- **Independence** between:
 - data model and storage structures
 - declarative queries and the way queries are executed
- **Complex** queries
- Fine **optimization** of queries, **indexes** allowing quick access to data
- **Mature** technology, **stable**, **efficient**, rich in features and interfaces
- **Integrity constraints** ensuring invariants on data
- Efficient management of **very large volume of data** (up to terabytes)
- **Transactions** (sequences of elementary operations) with guaranties on concurrency control, isolation between users, failure recovery

ACID properties

Classical relational DBMS **transactions** satisfy **ACID** properties:

ACID properties

Classical relational DBMS **transactions** satisfy **ACID** properties:

Atomicity: The set of operations within a transaction is either executed as a whole or canceled as a whole

Consistency: Transactions ensure integrity constraints on the base are respected

Isolation: Two concurrent executions of transactions result in a state equivalent to serial execution of the transactions

Durability: Once transactions are committed, corresponding data stay durably in the base, even in case of system failure

Weaknesses of classical RDBMSs

- Incapable of managing **extremely large data volume** (of the order of a petabyte)
- Impossible to manage **extreme query rates** (beyond thousands of queries per second)
- The relational data model is sometimes poorly adapted to the storage and querying of **some data types** (hierarchical data, unstructured data, semi-structured data)
- ACID properties imply major **costs** in latency, disk accesses, processing time (locks, logging, etc.)
- Performances **limited by disk accesses**

NoSQL

- **No SQL** or **Not Only SQL**
- DBMSs with other trade-offs than those made by classical systems
- **Very diverse** ecosystem
- **Desiderata**: different data model, scalability, extreme performance
- **Abandoned features**: ACID, (sometimes) complex queries



NewSQL

- Some applications require:
 - A **rich** (SQL) query language
 - conformity to **ACID** properties
 - but **greater performance** than classical RDBMSs

NewSQL

- Some applications require:
 - A **rich** (SQL) query language
 - conformity to **ACID** properties
 - but **greater performance** than classical RDBMSs
- Possible solutions:
 - Getting rid of classical **bottleneck** of RDBMSs: locks, logging, cache management
 - **In-memory** databases, with asynchronous copy on disk
 - **Lock-free** concurrency control (MVCC)
 - **Shared-nothing** distributed architecture with transparent **load balancing**



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Model

Relational Algebra

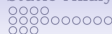
SQL

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Relational schema

We fix countably infinite sets:

- \mathcal{L} of **labels**
- \mathcal{V} of **values**
- \mathcal{T} of **types**, s.t., $\forall \tau \in \mathcal{T}, \tau \subseteq \mathcal{V}$

Definition

A **relation schema** (of **arity** n) is an n -tuple (A_1, \dots, A_n) where each A_i (called an **attribute**) is a pair (L_i, τ_i) with $L_i \in \mathcal{L}$, $\tau_i \in \mathcal{T}$ and such that all L_i are distinct

Definition

A **database schema** is defined by a finite set of labels $L \subseteq \mathcal{L}$ (**relation names**), each label of L being mapped to a relation schema.

Example database schema

- Universe:
 - \mathcal{L} the set of alphanumeric character strings starting with a letter
 - \mathcal{V} the set of finite sequences of bits
 - \mathcal{T} is formed of types such as INTEGER (representation as a sequence of bits of integers between -2^{31} and $2^{31} - 1$), REAL (representation of floating-point numbers following IEEE 754), TEXT (UTF-8 representation of character strings), DATE (ISO 8601 representation of dates), etc.
- Database schema formed of 2 relation names, Guest and Reservation
- Guest: ((id, INTEGER), (name, TEXT), (email, TEXT))
- Reservation:
 - ((id, INTEGER), (guest, INTEGER), (room, INTEGER), (arrival, DATE), (nights, INTEGER))

Database

Definition

An **instance** of a relation schema $((L_1, \tau_1), \dots, (L_n, \tau_n))$ (also called a **relation on this schema**) is a **finite set** $\{t_1, \dots, t_k\}$ of tuples of the form $t_j = (v_{j1}, \dots, v_{jn})$ with $\forall j \forall i v_{ji} \in \tau_i$.

Definition

An **instance** of a database schema (or, simply, a **database on this schema**) is a function that maps each relation name to an instance of the corresponding relation schema.

Note: **Relation** is used somewhat ambiguously to talk about a relation schema or an instance of a relation schema.

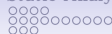
Example

Guest

id	name	email
1	John Smith	john.smith@gmail.com
2	Alice Black	alice@black.name
3	John Smith	john.smith@ens.fr

Reservation

id	guest	room	arrival	nights
1	1	504	2017-01-01	5
2	2	107	2017-01-10	3
3	3	302	2017-01-15	6
4	2	504	2017-01-15	2
5	2	107	2017-01-30	1



Some notation

- If $A = (L, \tau)$ is the i th attribute of a relation R , and t an n -tuple of an instance of R , we note $t[A]$ (or $t[L]$) the value of the i th component of t .
- Similarly, if \mathcal{A} is a k -tuple of attributes among the n attributes of R , $t[\mathcal{A}]$ is the k -tuple formed from t by concatenating the $t[A]$ for A in \mathcal{A} .
- A **tuple** is an n -tuple for some n .

Simple integrity constraints

One can add to the relational schema some **integrity constraints**, of different nature, to define a notion of **validity** of an instance

- **Key.** A tuple of attribute \mathcal{A} of a relation schema R is a **key** if there cannot exist two distinct tuples t_1 and t_2 in an instance of R such that $t_1[\mathcal{A}] = t_2[\mathcal{A}]$
- **Foreign key.** A k -tuple of attributes \mathcal{A} of a relation schema R is a **foreign key referencing** a k -tuple of attributes \mathcal{B} of a relation schema S if for all instances I^R and I^S of R and S , for every tuple t of I^R , there exists a **unique** tuple t' of I^S with $t[\mathcal{A}] = t'[\mathcal{B}]$
- **Check constraint.** Arbitrary condition on the values of the attributes of a relation (applying to each tuple of the instances of that relation)

Examples of constraints

- id is a **key** of Guest
- email is a **key** of Guest
- id is a **key** of Reservation
- (room, arrival) is a **key** of Reservation
- (guest, arrival) is a **key** of Reservation (?)
- guest is a **foreign key** of Reservation referencing id of Guest
- In Guest, email **must** contain a "@"
- In Reservation, room **must** be between 1 and 650
- In Reservation, nights **must** be positive

Impossible to express more complex constraints (e.g., a room cannot be occupied twice the same night, which depends on the date and the number of nights for multiple tuples of Reservation)

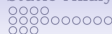
Variants: named and unnamed perspectives

The version presented considers the attributes of a relation are ordered and have a name. This is what best matches the way RDBMSs work, but not necessarily the most pleasant to reason on the relational model.

Named perspective. We forget the position of attributes, and consider they are uniquely identified by their names.

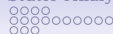
Unnamed perspective. We forget the name of attributes, and consider they are uniquely identified by their position. One uses notation such as $t[2]$ to access the value of the second attribute of a tuple.

No major impact, one will use one or the other depending on what is convenient.



Variant: bag semantics

- A relation instance is defined as a (finite) set of tuples. One can also consider a **bag semantics** of the relational model, where a relation instance is a multiset of tuples.
- This is what best matches how RDBMSs work...
- ... but most of relational database theory has been established for the set semantics, **more convenient** to work with
- We will **mostly discuss the set semantics** in this lecture, but explain where differences matter



Variant: untyped version

- In implementations, attributes are **always typed**
- In models and theoretical results, one often abstracts attribute types away and considers each attribute has a **universal type** \mathcal{V}
- We will most often omit **attribute types**

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The relational algebra

- **Algebraic language** to express queries
- A relational algebra expression produces a **new relation** from the database relations
- Each operator takes 0, 1, or 2 **subexpressions**
- Main operators:

Op.	Arity	Description	Condition
R	0	Relation name	$R \in \mathcal{L}$
$\rho_{A \rightarrow B}$	1	Renaming	$A, B \in \mathcal{L}$
$\Pi_{A_1 \dots A_n}$	1	Projection	$A_1 \dots A_n \in \mathcal{L}$
σ_φ	1	Selection	φ formula
\times	2	Cross product	
\cup	2	Union	
\setminus	2	Difference	
\bowtie_φ	2	Join	φ formula

Relation name

Guest			Reservation				
id	name	email	id	guest	room	arrival	nights
1	John Smith	john.smith@gmail.com	1	1	504	2017-01-01	5
2	Alice Black	alice@black.name	2	2	107	2017-01-10	3
3	John Smith	john.smith@ens.fr	3	3	302	2017-01-15	6
			4	2	504	2017-01-15	2
			5	2	107	2017-01-30	1

Expression: Guest

Result:

id	name	email
1	John Smith	john.smith@gmail.com
2	Alice Black	alice@black.name
3	John Smith	john.smith@ens.fr

Renaming

Guest			Reservation				
id	name	email	id	guest	room	arrival	nights
1	John Smith	john.smith@gmail.com	1	1	504	2017-01-01	5
2	Alice Black	alice@black.name	2	2	107	2017-01-10	3
3	John Smith	john.smith@ens.fr	3	3	302	2017-01-15	6
			4	2	504	2017-01-15	2
			5	2	107	2017-01-30	1

Expression: $\rho_{id \rightarrow \text{guest}}(\text{Guest})$

Result:

guest	name	email
1	John Smith	john.smith@gmail.com
2	Alice Black	alice@black.name
3	John Smith	john.smith@ens.fr

Projection

Guest

id	name	email
1	John Smith	john.smith@gmail.com
2	Alice Black	alice@black.name
3	John Smith	john.smith@ens.fr

Reservation

id	guest	room	arrival	nights
1	1	504	2017-01-01	5
2	2	107	2017-01-10	3
3	3	302	2017-01-15	6
4	2	504	2017-01-15	2
5	2	107	2017-01-30	1

Expression: $\Pi_{\text{email}, \text{id}}(\text{Guest})$

Result:

email	id
john.smith@gmail.com	1
alice@black.name	2
john.smith@ens.fr	3



Selection

Guest			Reservation				
id	name	email	id	guest	room	arrival	nights
1	John Smith	john.smith@gmail.com	1	1	504	2017-01-01	5
2	Alice Black	alice@black.name	2	2	107	2017-01-10	3
3	John Smith	john.smith@ens.fr	3	3	302	2017-01-15	6
			4	2	504	2017-01-15	2
			5	2	107	2017-01-30	1

Expression: $\sigma_{\text{arrival} > 2017-01-12 \wedge \text{guest} = 2}(\text{Reservation})$

Result:

id	guest	room	arrival	nights
4	2	504	2017-01-15	2
5	2	107	2017-01-30	1

The formula used in the selection can be any **Boolean combination** of **comparisons** of attributes to attributes or constants.

Cross product

Guest

id	name	email
1	John Smith	john.smith@gmail.com
2	Alice Black	alice@black.name
3	John Smith	john.smith@ens.fr

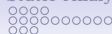
Reservation

id	guest	room	arrival	nights
1	1	504	2017-01-01	5
2	2	107	2017-01-10	3
3	3	302	2017-01-15	6
4	2	504	2017-01-15	2
5	2	107	2017-01-30	1

Expression: $\Pi_{id}(\text{Guest}) \times \Pi_{name}(\text{Guest})$

Result:

id	name
1	Alice Black
2	Alice Black
3	Alice Black
1	John Smith
2	John Smith
3	John Smith



Union

Guest			Reservation				
id	name	email	id	guest	room	arrival	nights
1	John Smith	john.smith@gmail.com	1	1	504	2017-01-01	5
2	Alice Black	alice@black.name	2	2	107	2017-01-10	3
3	John Smith	john.smith@ens.fr	3	3	302	2017-01-15	6
			4	2	504	2017-01-15	2
			5	2	107	2017-01-30	1

Expression: $\Pi_{\text{room}}(\sigma_{\text{guest}=2}(\text{Reservation})) \cup$
 $\Pi_{\text{room}}(\sigma_{\text{arrival}=2017-01-15}(\text{Reservation}))$

Result:

room
107
302
504

Union

Guest			Reservation				
id	name	email	id	guest	room	arrival	nights
1	John Smith	john.smith@gmail.com	1	1	504	2017-01-01	5
2	Alice Black	alice@black.name	2	2	107	2017-01-10	3
3	John Smith	john.smith@ens.fr	3	3	302	2017-01-15	6
			4	2	504	2017-01-15	2
			5	2	107	2017-01-30	1

Expression: $\Pi_{\text{room}}(\sigma_{\text{guest}=2}(\text{Reservation})) \cup$
 $\Pi_{\text{room}}(\sigma_{\text{arrival}=2017-01-15}(\text{Reservation}))$

Result:

room
107
302
504

This simple union could have been written

$\Pi_{\text{room}}(\sigma_{\text{guest}=2 \vee \text{arrival}=2017-01-15}(\text{Reservation}))$. Not always possible.



Difference

Guest			Reservation				
id	name	email	id	guest	room	arrival	nights
1	John Smith	john.smith@gmail.com	1	1	504	2017-01-01	5
2	Alice Black	alice@black.name	2	2	107	2017-01-10	3
3	John Smith	john.smith@ens.fr	3	3	302	2017-01-15	6
			4	2	504	2017-01-15	2
			5	2	107	2017-01-30	1

Expression: $\Pi_{\text{room}}(\sigma_{\text{guest}=2}(\text{Reservation})) \setminus$
 $\Pi_{\text{room}}(\sigma_{\text{arrival}=2017-01-15}(\text{Reservation}))$

Result:

room
107

Difference

Guest			Reservation				
id	name	email	id	guest	room	arrival	nights
1	John Smith	john.smith@gmail.com	1	1	504	2017-01-01	5
2	Alice Black	alice@black.name	2	2	107	2017-01-10	3
3	John Smith	john.smith@ens.fr	3	3	302	2017-01-15	6
			4	2	504	2017-01-15	2
			5	2	107	2017-01-30	1

Expression: $\Pi_{\text{room}}(\sigma_{\text{guest}=2}(\text{Reservation})) \setminus$
 $\Pi_{\text{room}}(\sigma_{\text{arrival}=2017-01-15}(\text{Reservation}))$

Result: $\frac{\text{room}}{107}$

This simple difference could have been written

$\Pi_{\text{room}}(\sigma_{\text{guest}=2 \wedge \text{arrival} \neq 2017-01-15}(\text{Reservation}))$. Not always possible.

Join

Guest			Reservation				
id	name	email	id	guest	room	arrival	nights
1	John Smith	john.smith@gmail.com	1	1	504	2017-01-01	5
2	Alice Black	alice@black.name	2	2	107	2017-01-10	3
3	John Smith	john.smith@ens.fr	3	3	302	2017-01-15	6
			4	2	504	2017-01-15	2
			5	2	107	2017-01-30	1

Expression: $\text{Reservation} \bowtie_{\text{guest}=\text{id}} \text{Guest}$

Result:

id	guest	room	arrival	nights	name	email
1	1	504	2017-01-01	5	John Smith	john.smith@gmail.com
2	2	107	2017-01-10	3	Alice Black	alice@black.name
3	3	302	2017-01-15	6	John Smith	john.smith@ens.fr
4	2	504	2017-01-15	2	Alice Black	alice@black.name
5	2	107	2017-01-30	1	Alice Black	alice@black.name

The formula used in the join can be any **Boolean combination** of **comparisons** of attributes of the table on the left to attributes of the table on the right.

Note on the join

- The join is not an **elementary** operator of the relational algebra (but it is very useful)
- It can be seen as a **combination** of renaming, cross product, selection, projection
- Thus:

$$\begin{aligned}
 & \text{Reservation} \bowtie_{\text{guest=id}} \text{Guest} \\
 \equiv & \Pi_{\text{id,guest,room,arrival,nights,name,email}}(\\
 & \sigma_{\text{guest=temp}}(\text{Reservation} \times \rho_{\text{id} \rightarrow \text{temp}}(\text{Guest})))
 \end{aligned}$$

- If R and S have for attributes \mathcal{A} and \mathcal{B} , we note $R \bowtie S$ the **natural join** of R and S , where the join formula is $\bigwedge_{A \in \mathcal{A} \cap \mathcal{B}} A = A$.

Illegal operations

- All expressions of the relational algebra are not **valid**
- The validity of an expression generally depends on the database schema
- For example:
 - No reference to the name of a relation that doesn't exist in the database schema
 - One cannot reference (within a renaming, projection, selection, join) an attribute that does not exist in the result of a sub-expression
 - One cannot union two relations with different attributes
 - One cannot produce (cross product, join, renaming) a table with two attributes with the same name
- Systems implementing the relational algebra may do a static or dynamic verification of these rules, or sometimes ignore them



Bag semantics

In bag semantics (what is actually used by RDBMS):

- All operations return **multisets**
- In particular, projection and union can **introduce** multisets even when initial relations are sets

Extension: Aggregation

- Various extensions have been proposed to the relational algebra to add **additional features**
- In particular, **aggregation and grouping** [Klug, 1982, Libkin, 2003] of results
- With a syntax inspired from [Libkin, 2003]:

$$\sigma_{\text{avg} > 3}(\gamma_{\text{room}}^{\text{avg}}[\lambda x. \text{avg}(x)](\Pi_{\text{room}, \text{night}}(\text{Reservation})))$$

computes the average number of nights per reservation for each room having an average greater than 3

room	avg
302	6
504	3.5

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SQL

- **Structured Query Language**, standard language (ISO/IEC 9075, several versions [ISO, 1987, 1999]) to **interact with an RDBMS**
- Unfortunately, implementation of the standard very **variable** from one RDBMS to the next
- Many little things (e.g., available types) vary between RDBMSs instead of following the standard
- Differences more **syntactical** than major
- Where it makes a difference, we use the **PostgreSQL** version
- Two main parts: DDL (**Data Definition Language**) to define the schema and DML (**Data Manipulation Language**) to query and update data
- **Declarative** language: express what you mean, the system will take care of translating this into an **efficient execution plan**

Syntax of SQL

- Quite **verbose**, designed to be almost readable as English words [Chamberlin and Boyce, 1974]
- Keywords are **case-insensitive**, traditionally written in all uppercase
- Identifiers often **case-insensitive** (depends of the RDBMS), often written with an initial uppercase for table names, in all lower case for attribute names
- **Comments** introduced with --
- SQL statements end with a “;” in some contexts (e.g., command line client) but the “;” is not properly part of the statement



NULL

- In SQL, NULL is a special value that an attribute can take within a tuple
- Denotes **absence of value**
- Different from 0, from an empty string, etc.
- Weird **tri-valued** logic: True, False, NULL
- A normal comparison (equality, inequality, etc.) with NULL always returns NULL
- **IS NULL, IS NOT NULL** can be used to test whether a value is NULL
- NULL est ultimately converted to False
- Weird consequences, poor integration with the formal relational model

Data Definition Language

```
CREATE TABLE Guest(id INTEGER, name TEXT, email TEXT);
CREATE TABLE Reservation(id INTEGER, guest INTEGER,
    room INTEGER, arrival DATE, nights INTEGER);
```

But also:

- **DROP TABLE** Guest; to destroy a table
- **ALTER TABLE** Guest **RENAME TO** Guest2; to rename a table
- **ALTER TABLE** Guest **ALTER COLUMN** id **TYPE** TEXT; to change the type of a column

Constraints

Specified at the creation of a table, or added later on (with **ALTER TABLE**)

PRIMARY KEY for the primary key; only one per table, it is the key that will be used for physical organization of data; implies **NOT NULL**

UNIQUE for other keys

REFERENCES for foreign keys

CHECK for Check constraints

NOT NULL to indicate that an attribute cannot take the NULL value

Constraints

```
CREATE TABLE Guest(
    id INTEGER PRIMARY KEY,
    name TEXT NOT NULL,
    email TEXT UNIQUE CHECK (email LIKE '%@%')
);
```

```
CREATE TABLE Reservation(
    id INTEGER PRIMARY KEY,
    guest INTEGER NOT NULL REFERENCES Guest(id),
    room INTEGER NOT NULL CHECK (room>0
        AND room<651),
    arrival DATE NOT NULL,
    nights INTEGER NOT NULL CHECK (nights>0),
    UNIQUE(room, arrival),
    UNIQUE(guest, arrival)
);
```

Updates

- Insertions:

INSERT INTO Guest(id,name) **VALUES** (5, 'John');

- Deletions:

DELETE FROM Reservation **WHERE** id>4;

- Modifications:

UPDATE Reservation **SET** room=205 **WHERE** room=204;

Updates

INSERT INTO Guest VALUES

```
(1, 'Jean Dupont', 'jean.dupont@gmail.com'),
(2, 'Alice Dupuis', 'alice@dupuis.name'),
(3, 'Jean Dupont', 'jean.dupont@ens.fr');
```

INSERT INTO Reservation VALUES

```
(1, 1, 504, '2017-01-01', 5),
(2, 2, 107, '2017-01-10', 3),
(3, 3, 302, '2017-01-15', 6),
(4, 2, 504, '2017-01-15', 2),
(5, 2, 107, '2017-01-30', 1);
```

Queries

General following form:

SELECT ... **FROM** ... **WHERE** ...

GROUP BY ... **HAVING** ...

UNION SELECT ... **FROM** ...

SELECT projection, renaming, aggregation

FROM cross product

WHERE selection (optional)

GROUP BY grouping (optional)

HAVING selection on the grouping (optional)

UNION union (optional)

Other keywords: **ORDER BY** to reorder, **LIMIT** to limit to first k results, **DISTINCT** to impose set semantics, **EXCEPT** for set difference, etc.

Renaming

Guest		
id	name	email
1	John Smith	john.smith@gmail.com
2	Alice Black	alice@black.name
3	John Smith	john.smith@ens.fr

Reservation				
id	guest	room	arrival	nights
1	1	504	2017-01-01	5
2	2	107	2017-01-10	3
3	3	302	2017-01-15	6
4	2	504	2017-01-15	2
5	2	107	2017-01-30	1

$$\rho_{id \rightarrow guest}(\text{Guest})$$

```
SELECT id AS guest, name, email
FROM Guest;
```

Projection

Guest		
id	name	email
1	John Smith	john.smith@gmail.com
2	Alice Black	alice@black.name
3	John Smith	john.smith@ens.fr

Reservation				
id	guest	room	arrival	nights
1	1	504	2017-01-01	5
2	2	107	2017-01-10	3
3	3	302	2017-01-15	6
4	2	504	2017-01-15	2
5	2	107	2017-01-30	1

$$\Pi_{\text{email}, \text{id}}(\text{Guest})$$

```
SELECT DISTINCT email, id
FROM Guest;
```

Selection

Guest			Reservation				
id	name	email	id	guest	room	arrival	nights
1	John Smith	john.smith@gmail.com	1	1	504	2017-01-01	5
2	Alice Black	alice@black.name	2	2	107	2017-01-10	3
3	John Smith	john.smith@ens.fr	3	3	302	2017-01-15	6
			4	2	504	2017-01-15	2
			5	2	107	2017-01-30	1

$$\sigma_{\text{arrival} > 2017-01-12 \wedge \text{guest}=2}(\text{Reservation})$$

SELECT *

FROM Reservation

WHERE arrival > '2017-01-12' **AND** guest=2;

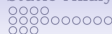
Cross product

Guest			Reservation				
id	name	email	id	guest	room	arrival	nights
1	John Smith	john.smith@gmail.com	1	1	504	2017-01-01	5
2	Alice Black	alice@black.name	2	2	107	2017-01-10	3
3	John Smith	john.smith@ens.fr	3	3	302	2017-01-15	6
			4	2	504	2017-01-15	2
			5	2	107	2017-01-30	1

$$\Pi_{id}(Guest) \times \Pi_{name}(Guest)$$

SELECT *
FROM

(SELECT DISTINCT id FROM Guest) AS temp1,
 (SELECT DISTINCT name FROM Guest) AS temp2
ORDER BY name, id;



Union

Guest		
id	name	email
1	John Smith	john.smith@gmail.com
2	Alice Black	alice@black.name
3	John Smith	john.smith@ens.fr

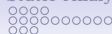
Reservation				
id	guest	room	arrival	nights
1	1	504	2017-01-01	5
2	2	107	2017-01-10	3
3	3	302	2017-01-15	6
4	2	504	2017-01-15	2
5	2	107	2017-01-30	1

$$\Pi_{\text{room}}(\sigma_{\text{guest}=2}(\text{Reservation})) \\ \cup \Pi_{\text{room}}(\sigma_{\text{arrival}=2017-01-15}(\text{Reservation}))$$

```

SELECT room
FROM Reservation
WHERE guest=2
UNION
SELECT room
FROM Reservation
WHERE arrival='2017-01-15';

```



Difference

Guest		
id	name	email
1	John Smith	john.smith@gmail.com
2	Alice Black	alice@black.name
3	John Smith	john.smith@ens.fr

Reservation				
id	guest	room	arrival	nights
1	1	504	2017-01-01	5
2	2	107	2017-01-10	3
3	3	302	2017-01-15	6
4	2	504	2017-01-15	2
5	2	107	2017-01-30	1

$$\Pi_{\text{room}}(\sigma_{\text{guest}=2}(\text{Reservation}))$$

$$\setminus \Pi_{\text{room}}(\sigma_{\text{arrival}=2017-01-15}(\text{Reservation}))$$

```

SELECT room
FROM Reservation
WHERE guest=2
EXCEPT
SELECT room
FROM Reservation
WHERE arrival='2017-01-15';

```

Join

Guest			Reservation				
id	name	email	id	guest	room	arrival	nights
1	John Smith	john.smith@gmail.com	1	1	504	2017-01-01	5
2	Alice Black	alice@black.name	2	2	107	2017-01-10	3
3	John Smith	john.smith@ens.fr	3	3	302	2017-01-15	6
			4	2	504	2017-01-15	2
			5	2	107	2017-01-30	1

Reservation $\bowtie_{\text{guest=id}}$ Guest

```
SELECT Reservation.*, name, email
FROM Reservation JOIN Guest ON guest=Guest.id;
```

```
SELECT Reservation.*, name, email
FROM Reservation, Guest
WHERE guest=Guest.id;
```

Aggregation

Guest			Reservation				
id	name	email	id	guest	room	arrival	nights
1	John Smith	john.smith@gmail.com	1	1	504	2017-01-01	5
2	Alice Black	alice@black.name	2	2	107	2017-01-10	3
3	John Smith	john.smith@ens.fr	3	3	302	2017-01-15	6
			4	2	504	2017-01-15	2
			5	2	107	2017-01-30	1

$$\sigma_{\text{avg} > 3}(\gamma_{\text{room}}^{\text{avg}}[\lambda x. \text{avg}(x)](\Pi_{\text{room}, \text{nights}}(\text{Reservation})))$$

```

SELECT room, AVG(nights) AS avg
FROM Reservation
GROUP BY room
HAVING AVG(nights) > 3
ORDER BY room;

```

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Model

Relational Algebra

SQL

Relational Calculus

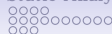
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Relational calculus

- **Logical language** to express queries
- **First-order logic** formula, without function symbols, and with **relation symbols** the labels of the database schema (plus comparison predicates)
- **Unnamed, untyped** perspective
- Fix:
 - A set \mathcal{X} of variables
 - A set \mathcal{V} of values
 - A database schema S



Relational calculus: Syntax

- For every relation $R \in S$ of arity n , for every $(\alpha_1, \dots, \alpha_n) \in (\mathcal{X} \cup \mathcal{V})^n$: $R(\alpha_1, \dots, \alpha_n) \in \text{FO}$
- Also allow equality predicate, possibly inequality
- For every $(\varphi_1, \varphi_2) \in \text{FO}^2$, for every $x \in \mathcal{X}$:
 - $\varphi_1 \wedge \varphi_2 \in \text{FO}$
 - $\varphi_1 \vee \varphi_2 \in \text{FO}$
 - $\neg \varphi_1 \in \text{FO}$
 - $\forall x \varphi_1 \in \text{FO}$
 - $\exists x \varphi_1 \in \text{FO}$
- **Free variables** of $\varphi \in \text{FO}$: variables x appearing in φ and not qualified by a $\forall x$ or a $\exists x$
- One writes a relational calculus **query** in the form $Q(x_1, \dots, x_m) = \varphi$ where x_1, \dots, x_m are free variables of φ

Relational calculus: Semantics

- A relational calculus query on schema S can be seen as a **function** with input a database D over S and producing a relation as output
- $\text{adom}(D)$: **active domain** of D , set of values in D
- If $Q(x_1, \dots, x_n) = \varphi$ is a calculus query over S and D a database over S , then:

$$Q(D) = \{ (v_1, \dots, v_n) \in (\text{adom}(D))^n \mid D \models \varphi[x_1/v_1, \dots, x_n/v_n] \}$$

where $D \models \varphi$ is defined inductively:

- $D \models R(u_1, \dots, u_m) \iff R(u_1, \dots, u_m) \in D$
- $D \models \varphi_1 \wedge \varphi_2 \iff D \models \varphi_1 \wedge D \models \varphi_2$
- $D \models \varphi_1 \vee \varphi_2 \iff D \models \varphi_1 \vee D \models \varphi_2$
- $D \models \neg \varphi_1 \iff D \not\models \varphi_1$
- $D \models \forall x \varphi_1 \iff \forall v \in \text{adom}(D) D \models \varphi_1[x/v]$
- $D \models \exists x \varphi_1 \iff \exists v \in \text{adom}(D) D \models \varphi_1[x/v]$

Codd's theorem

Theorem ([Codd, 1972])

The relational algebra and the relational calculus are equivalent:

- *for every relational algebra query q over a schema S , there exists a relational calculus query Q over S such that for every database D over S , $q(D) = Q(D)$*
- *for every relational calculus query Q over a schema S , there exists a relational algebra query q over S such that for every database D over S , $q(D) = Q(D)$*

Furthermore, translating from one formalism to the other can be done in polynomial time.

Why is this important?

- Allows using a **declarative formalism** to specify queries: logics... or SQL
- These queries are then compiled via Codd's transformation into an **algebraic formalism**
- Algebraic queries are then **optimized**, by using the properties of the relational algebra (transformation rules, e.g., pushing selection within joins, exploiting associativity of joins, etc.)
- Optimized queries can then be **evaluated**, by exploiting the fact that each operator of the relational algebra can easily be implemented (in several different ways, to be chosen based on a cost function)
- This is RDBMS Implementation 101, a main reason of the success of RDBMSs!



Subclasses of queries

- **Conjunctive query (CQ)**: relational calculus query without \vee, \neg, \forall
- **Positive query (PQ)**: relational calculus query without \neg, \forall
- **Union of conjunctive queries (UCQ)**: special case of positive query where the \vee and \wedge form a DNF formula

Subclasses of queries

- **Conjunctive query (CQ)**: relational calculus query without \vee, \neg, \forall
- **Positive query (PQ)**: relational calculus query without \neg, \forall
- **Union of conjunctive queries (UCQ)**: special case of positive query where the \vee and \wedge form a DNF formula

Expressiveness

- CQs are **equivalent** to the relational algebra without \cup and \setminus , and where σ does not feature disjunction
- UCQs are **equivalent** to PQs (but exponential blow-up), and equivalent to the relational algebra without \setminus

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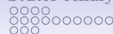
References

Motivation: recursive queries

- Query languages considered so far (relational algebra, calculus) have a **limited horizon**
- Some data structures (trees, graphs) require **arbitrarily deep** navigation, **recursion**
- How can we build a theory of **recursive query languages**?
- RDBMSs are **not always** adapted to this type of data/queries, cf. XML or graph DBMSs
- **Example application**: transitive closure of a graph $G(\textit{from}, \textit{to})$

Datalog

- Simplest recursive query language: adding recursion to **conjunctive queries**
- Inspired from **logic programming**
- Datalog query (or **program**): set of rules that produce **intensional facts**
- **Schema** of a Datalog program: classical database schema (**extensional schema**) + (disjoint) schema of intensional facts (**intensional schema**)
- Fix a distinguished relation *Goal* of the intensional schema, whose arity is the arity of the query



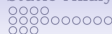
Syntax

Finite set of rules r of the form:

$$\underbrace{S(\mathbf{y})}_{\text{head}} \leftarrow \underbrace{R_1(\mathbf{x}_1), \dots, R_n(\mathbf{x}_n)}_{\text{body}}$$

with:

- S relation of the **intensional** schema
- R_1, \dots, R_n **relations** of the intensional or extensional schemas
- $\mathbf{x}_1, \dots, \mathbf{x}_n, \mathbf{y}$: tuples of **variables** (or possibly constants), of arity compatible with the relations
- Each variable in the head is **present** in the body



Fix-point semantics

- Each rule r of a program P can be seen as a **conjunctive query** on the database D :

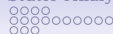
$$r(D) := \{S(\mathbf{y}) \mid \exists z_1 \dots z_k R_1(\mathbf{x}_1) \in D \wedge \dots \wedge R_n(\mathbf{x}_n) \in D\}$$

where the z_i 's are the variables of the rule body

- Consequence operator** Γ_P defined by:

$$\Gamma_P(D) := D \cup \bigcup_{r \in P} \{r(D)\}$$

- We consider the **sequence** (D_n) defined by:
 $D_0 = D, D_{n+1} = \Gamma_P(D_n)$
- The semantics of P over D is the set of facts of the relation *Goal* in D_∞ , the **fixpoint** of the sequence (D_n)



Example: transitive closure

$$Goal(x, y) \leftarrow G(x, y)$$

$$Goal(x, y) \leftarrow Goal(x, z), G(z, y)$$

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Common Table Expressions (CTE)

- Only way in SQL to introduce recursion (also called **hierarchical queries**)
- **Idea:** We declare a table, whose definition refers to itself
- Inspired from **inflationary fixpoint logics** but also from recursive functions in programming languages

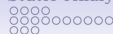
Syntax

```

WITH RECURSIVE T(x1, ..., xn) AS (
    SELECT -- base case
UNION
    SELECT -- recursive case, using table T
)
SELECT * FROM T;

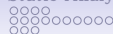
```

This precise form, using **UNION** (or **UNION ALL**) between base and recursive cases is **compulsory**



Semantics

- The query is evaluated **iteratively**, from $T = \emptyset$ and by replacing at every step T with the result of evaluating the definition of T
- This terminates when a **fixpoint is reached**
- The query defining T must be **monotone** (T can only grow at each step)
- Optimizations possible (and used) in order to only take into account **new facts** at every step (thanks to monotonicity)
- Infinite loops **possible** if new values are created



Example: transitive closure

```

WITH RECURSIVE C(f,t) AS (
    SELECT * FROM G
    UNION
    SELECT C.f, G.t FROM C JOIN G ON C.t=G.f
)
SELECT * FROM C;

```


Notes on historical support

- CTE normalized **late** [ISO, 1999] in SQL
- **Non-compatible** historical extensions: (**CONNECT BY** introduced by Oracle, reused by some other DBMSs)
- Support is now **correct**
- Recursive queries are not a priority of DBMSs, poor optimization (in PostgreSQL, recursive CTEs are an **optimization fence**)

Query evaluation

- Query Q in some query language (CQ, FO, FO+ μ , FO+ μ^+ ...) – we will use a logical formalism here
- Database D (always **finite**!)
- **Query evaluation**: Computing $Q(D)$
- **Complexity** of this problem?
- To simplify the study of complexity, we often assume that Q is a **Boolean** query, i.e., it returns \top or \perp

Data complexity

For some fixed Q , what is the complexity of computing $Q(D)$ in terms of the **size of the database D** ?

Combined complexity

For some query language \mathcal{Q} , what is the complexity of computing $Q(D)$ in terms of the **size of the query $Q \in \mathcal{Q}$ and of the database D** ?

Complexity classes

- We restrict to **Boolean** problems (returning \top or \perp)
- Set of all problems solvable by a **resource-constrained computing method**:
- For example:

PTIME: deterministic Turing machine in polynomial time

NP: non-deterministic Turing machine in polynomial time

PSPACE: deterministic Turing machine in polynomial space

AC⁰: Boolean circuit of polynomial size and constant depth

- We know that: $AC^0 \subsetneq PTIME \subseteq NP \subseteq PSPACE$
- Open whether $PSPACE \subseteq PTIME$ (!)

Membership and hardness for a class

- A problem P **belongs** to a complexity class \mathcal{C} (or **in** \mathcal{C}) if it is **solvable** by the corresponding resource-constrained computing method
- A problem P is **hard** for a complexity class \mathcal{C} (or **\mathcal{C} -hard**) if there exists a **reduction** that transforms whatever problem $P' \in \mathcal{C}$ into an instance of the problem P
- **complete**: in $\mathcal{C} + \mathcal{C}$ -hard
- Several ways to define reductions
- Here, we assume that there exists a function computable **in polynomial time** that **transforms** one instance I' of problem P' into an instance I of P such that $P(I) = P'(I')$

Descriptive complexity

- A query language \mathcal{Q} **captures** a complexity class \mathcal{C} if:
 - For all $Q \in \mathcal{Q}$, query evaluation of query Q in \mathcal{C} (data complexity)
 - For all problem P in \mathcal{C} , there exists a query $Q \in \mathcal{Q}$ such that evaluating Q **exactly solves** P (without a reduction)!
- If \mathcal{Q} captures \mathcal{C} and if \mathcal{C} has problems that are complete for \mathcal{C} , then there exists $Q \in \mathcal{Q}$ such that Q is \mathcal{C} -complete, but **the converse is not true**

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Data complexity

Theorem

*CQ evaluation is **PTIME** in data complexity.*

Proof.

By enumerating all valuations of variables of the query in the database. We will see later a much stronger result. □

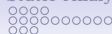
Combined complexity

Theorem

*CQ evaluation is **NP-complete** in combined complexity.*

Proof.

Membership in NP is easy. Hardness for NP can be proved by reduction from graph 3-colorability. □



α -acyclic query

- A CQ can be seen as a **hypergraph** (vertices are variables, hyperedges the atoms of the CQ, labeled by the relation name)
- A hypergraph \mathcal{H} has a **join tree** where one can find a tree whose nodes are labeled by the hyperedges of \mathcal{H} and such that:
 - every hyperedge of \mathcal{H} appears as the label of one node of the tree;
 - for every vertex x of \mathcal{H} , the set of tree nodes labeled by a hyperedge referring to x is a connected subtree
- A query is **α -acyclic** if its hypergraph has a **join tree**
- Can be obtained in **linear** time if it exists [Tarjan and Yannakakis, 1984]

Yannakakis's algorithm [Yannakakis, 1981]

- Algorithm to evaluate acyclic queries (non-necessarily Boolean):
 - Construct the **join tree**
 - Eliminate all useless tuples of a relation with the **semijoin** operator \bowtie : $R \bowtie S = \Pi_{R.*}(R \bowtie S)$ by navigating twice in the join tree: from bottom up, then from top down
 - Evaluate the query **bottom up**, by computing joins following the tree and by projecting useless variables out as you go
- Polynomial** complexity in the size of the query, the input, and the output (combined complexity)

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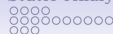
Theorem

*FO evaluation is **PTIME** in data complexity.*

Proof.

By rewriting in prenex form and naive evaluation.





FO does not capture the whole of PTIME

Theorem

One cannot compute in FO that a relation containing a total order has an even number of elements, or that a graph is connected.

Fairly complex to prove, relies on Ehrenfeucht–Fraïssé games (see [Libkin, 2004]).

Data complexity, more precise

Theorem

FO evaluation is AC^0 in data complexity.

Proof.

By rewriting to the relational algebra.



Combined complexity

Theorem

*FO evaluation is **PSPACE-complete** in combined complexity.*

Proof.

Membership in PSPACE easy. Hardness for PSPACE from the QSAT problem. □

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Query optimization

- **Goal:** Given a query q in some query language \mathcal{Q} and a database D , find a query **equivalent to q on D** and faster to evaluate on D
- **Here:** \mathcal{Q} in the relational calculus (or a fragment thereof), and one looks for a query faster **on whatever database** (we do not look D , we perform **static analysis**)
- **In actual RDBMSs:** \mathcal{Q} is the set of **query execution plans** (a specialization of the relational algebra where implementations are chosen for each operator) and statistics on D are used

Global optimization

- We consider **global** optimization techniques, considering a query in its entirety (techniques on execution plans are more local, e.g., local rewritings)
- We formally define:

Equivalence: $q \equiv q'$ if for all database D , $q(D) = q'(D)$

Minimality: q' is the “best” query equivalent to q in \mathcal{Q}

Containment and equivalence

Definition

A query q is **contained** in a query q' (denoted $q \sqsubseteq q'$) if for all database D , $q(D) \subseteq q'(D)$

Containment and equivalence

Definition

A query q is **contained** in a query q' (denoted $q \sqsubseteq q'$) if for all database D , $q(D) \subseteq q'(D)$

Proposition

$q \equiv q'$ iff $q \sqsubseteq q'$ and $q' \sqsubseteq q$.

Proof.

Immediate. □

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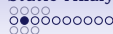
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Relational Calculus

References



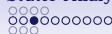
Case of conjunctive queries

- We consider **conjunctive queries** (CQ) of the form:

$$q(\mathbf{x}) \leftarrow \exists \mathbf{y} : R_1(\mathbf{z}_1) \wedge \cdots \wedge R_n(\mathbf{z}_n)$$

where each \mathbf{z}_i is a tuple of variables among \mathbf{x} and \mathbf{y} , and
 where each x_j appears at least in one \mathbf{z}_j

- Set** semantics: for all database D , $q(D)$ is a finite set of tuples



Homomorphism

Definition

A **homomorphism** from a CQ q to a CQ q' is a function φ from the variables x, y of q to the variables x', y' of q' such that:

- $\varphi(x) = x'$
- for every atom $R(z_i)$ of q , there exists an atom $R(z'_{i'})$ of q' such that $\varphi(z_i) = z'_{i'}$

Definition

A homomorphism is an **isomorphism** if it is one-to-one and its converse is a homomorphism.



Instance associated to a query

Definition

For all conjunctive query

$$q(\mathbf{x}) \leftarrow \exists \mathbf{y} : R_1(\mathbf{z}_1) \wedge \cdots \wedge R_n(\mathbf{z}_n)$$

one can construct the **instance associated to q** , denoted I_q , where the active domain is $\{a_z \mid z \in \mathbf{x} \cup \mathbf{y}\}$ and which is formed of the n tuples $R(a_{z_{i1}}, \dots, a_{z_{ik}})$ for $R(z_{i1}, \dots, z_{ik})$ atom of q



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Proposition

For all CQs $q(\mathbf{x})$, $q'(\mathbf{x}')$, there exists a homomorphism from q to q' iff $(a_{x'_1}, \dots, a_{x'_j}) \in q(I_{q'})$.



Homomorphism theorem

Theorem ([Chandra and Merlin, 1977])

For all CQs q, q' , $q \sqsubseteq q'$ iff there exists a homomorphism from q' to q .

Minimal query

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A conjunctive query is **minimal** if it has a minimal number of atoms among all equivalent conjunctive queries.

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- Translation of a CQ to an algebra query: if there are n atoms, we obtain $n - 1$ joins
- Joins are the **most costly** operations of the relational algebra (bar cross products)
- Finding a minimal query amounts to **global optimization**

Unicity of minimal query

Proposition ([Chandra and Merlin, 1977])

Let q be a CQ. Then there exists a CQ q' obtained by **removing atoms** from q which is minimal.

Proof.

Consider a minimal query equivalent to q and apply the homomorphism theorem. □

Proposition ([Chandra and Merlin, 1977])

Let q, q' be two equivalent minimal CQs. Then there exists an **isomorphism** from q to q' .

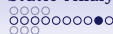
Proof.

Apply the homomorphism theorem. The image by the homomorphism is an equivalent minimal query. □

Minimization algorithm

Apply the following procedure to **minimize a query**:

For every atom of the query, test if there exists an equivalent query not containing this atom, and thus if there exists a homomorphism sending this atom to another atom of the query. If so, delete it, and continue until obtaining an equivalent minimal query.



Complexity issues

Proposition

The following problems are **NP-complete**:

- given two CQs q, q' , determine whether $q \sqsubseteq q'$
- given two CQs q, q' , determine whether $q \equiv q'$
- given a CQ q , determine if q is non-minimal

Proof.

NP-hardness is by reduction from 3-colorability, as for combined complexity of query evaluation. Membership in NP is direct. □

Complexity issues

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NP-hard... in the queries. Queries may be small enough so that an exponential algorithm may not be an issue.

Bag semantics

[Chaudhuri and Vardi, 1993]

- In practice, RDBMSs implement a bag semantics
- Two queries in bag semantics are **equivalent** if and only if they are **isomorphic** (intuitively, because two similar but non isomorphic queries can introduce a different number of results)
- Query **containment**: Π_2^P -hard originally claimed (but not proved, and more or less withdrawn since). Decidability (and precise complexity if decidable): **open**!

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Recursive Queries

Complexity of Query Evaluation

Static Analysis of Queries

Containment and Equivalence

Conjunctive Queries

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Satisfiability in the relational calculus

Definition

A Boolean relational calculus query q is **satisfiable** if there exists a (finite) database D such that $D \models q$.



Satisfiability in the relational calculus

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A Boolean relational calculus query q is **satisfiable** if there exists a (finite) database D such that $D \models q$.

Theorem ([Trakhtenbrot, 1963])

*Satisfiability of the relational calculus (in the finite case) is **undecidable**.*

Proof.

Reduction possible from the POST correspondence problem, technical, see [Abiteboul et al., 1995]. □

Containment and equivalence of the calculus

Theorem

*Containment and equivalence of relational calculus queries are **undecidable** and **co-recursively enumerable**.*

Containment and equivalence of the calculus

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Proof.

Undecidability is by direct reduction from the undecidability of satisfiability.

Co-recursive enumerability is shown directly, by enumerating possible counter-examples. □

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References

References

- **Basics** on what data management research is [Benedikt and Senellart, 2012]
- Classic textbook on the **foundations** of data management, database theory [Abiteboul et al., 1995]
- Modern textbook, being developed, on **database theory** [Arenas et al., 2022]
- Classic textbook on **database systems** [Garcia-Molina et al., 2008]
- **Details of SQL**: standards are not public documents (and not useful for a final user); use the RDBMS documentation instead
- For example, **PostgreSQL** has a comprehensive documentation at <https://www.postgresql.org/docs/> (and using \h in the command line client)

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