Incremental learning, game theory, and applications
Lecture 0: a brief panorama of Game Theory

Rida Laraki and Guillaume Vigeral

Master IASD, 2024

Contents

Game Theory ?

Game theory aims to analyze situations involving strategic interaction between several entities called players or agents (human beings, populations, firms, automata, cells,...). The well-being of a player depends on his choices but also those of the others.

Two very different frameworks :

Game Theory ?

Game theory aims to analyze situations involving strategic interaction between several entities called players or agents (human beings, populations, firms, automata, cells,...). The well-being of a player depends on his choices but also those of the others.

Two very different frameworks :

- **Strategic games** : This framework corresponds to the main topic analyzed in this course. The players are autonomous : their *strategies*, are determined (selected, chosen by the players) independently from each other. A *profile* of strategies (one for each player) induces an *outcome* and each player uses a real-valued *utility function* defined on this space of outcomes. The main issue is then to define and analyze rational behavior in this framework.

Game Theory ?

Game theory aims to analyze situations involving strategic interaction between several entities called players or agents (human beings, populations, firms, automata, cells,...). The well-being of a player depends on his choices but also those of the others.

Two very different frameworks :

- **Strategic games** : This framework corresponds to the main topic analyzed in this course. The players are autonomous : their *strategies*, are determined (selected, chosen by the players) independently from each other. A *profile* of strategies (one for each player) induces an *outcome* and each player uses a real-valued *utility function* defined on this space of outcomes. The main issue is then to define and analyze rational behavior in this framework.
- **Coalitional games (not in this course)** : In this approach, the initial data is still given by a set I of players but one takes into account all feasible subsets $C \subset I$, called *coalitions*; and an *effectivity function* associates to each C the subset of outcomes it can achieve. The issue is now to deduce a specific outcome for the set of all players.

Some applications of Game Theory in Computer Science

- Internet search : there is an entire search engine optimization industry, dedicated to influencing search results and an extensive use of machine learning and careful algorithm design to make search results robust to various kinds of attack.

Some applications of Game Theory in Computer Science

- Internet search : there is an entire search engine optimization industry, dedicated to influencing search results and an extensive use of machine learning and careful algorithm design to make search results robust to various kinds of attack.
- Peer-to-peer systems : A fundamental problem in P2P systems is free-riding : users may prefer to consume resources without contributing resources back to the system. Earlier file-sharing protocols failed to successfully align incentives, modern systems (BitTorrent protocol for example) incorporates game theory idea (a kind of tit-for-tat).

Some applications of Game Theory in Computer Science

- Internet search : there is an entire search engine optimization industry, dedicated to influencing search results and an extensive use of machine learning and careful algorithm design to make search results robust to various kinds of attack.
- Peer-to-peer systems : A fundamental problem in P2P systems is free-riding : users may prefer to consume resources without contributing resources back to the system. Earlier file-sharing protocols failed to successfully align incentives, modern systems (BitTorrent protocol for example) incorporates game theory idea (a kind of tit-for-tat).
- Social networks Question and- answer systems or Wikipedia, rely on user participation as well as well-designed algorithms and systems for their success (create incitations, avoid manipulations, fake news and attacks).

Some applications of Game Theory in Computer Science

- Internet search : there is an entire search engine optimization industry, dedicated to influencing search results and an extensive use of machine learning and careful algorithm design to make search results robust to various kinds of attack.
- Peer-to-peer systems : A fundamental problem in P2P systems is free-riding : users may prefer to consume resources without contributing resources back to the system. Earlier file-sharing protocols failed to successfully align incentives, modern systems (BitTorrent protocol for example) incorporates game theory idea (a kind of tit-for-tat).
- Social networks Question and- answer systems or Wikipedia, rely on user participation as well as well-designed algorithms and systems for their success (create incitations, avoid manipulations, fake news and attacks).
- Voting Given a voting procedure, how should one vote ? Does it only depend on your own preferences ?

Some applications of Game Theory in Computer Science

- Internet search : there is an entire search engine optimization industry, dedicated to influencing search results and an extensive use of machine learning and careful algorithm design to make search results robust to various kinds of attack.
- Peer-to-peer systems : A fundamental problem in P2P systems is free-riding : users may prefer to consume resources without contributing resources back to the system. Earlier file-sharing protocols failed to successfully align incentives, modern systems (BitTorrent protocol for example) incorporates game theory idea (a kind of tit-for-tat).
- Social networks Question and- answer systems or Wikipedia, rely on user participation as well as well-designed algorithms and systems for their success (create incitations, avoid manipulations, fake news and attacks).
- Voting Given a voting procedure, how should one vote ? Does it only depend on your own preferences ?
- Add Exchanges, Wireless Spectrum Auctions, Prediction Markets, Matching problems, Recommender and Reputation Systems, GAN, etc.

Contents

Stable Matching : Gale-Shapley

- Consider two finite families I and J (men/women, workers/firms,...) with the same cardinality.

Stable Matching : Gale-Shapley

- Consider two finite families I and J (men/women, workers/firms,...) with the same cardinality.
- Each element $i \in I$ (resp. $j \in J$) has a complete strict order on J (resp. I).

Stable Matching : Gale-Shapley

- Consider two finite families I and J (men/women, workers/firms,...) with the same cardinality.
- Each element $i \in I$ (resp. $j \in J$) has a complete strict order on J (resp. I).
- The goal is to find a stable matching namely bijections π from I to J such that there are no couples $(i, \pi(i) = j)$, $(i', \pi(i') = j')$ with both j' preferred by i to j and i preferred to i' by j'.

Stable Matching : Gale-Shapley

- Consider two finite families I and J (men/women, workers/firms,...) with the same cardinality.
- Each element $i \in I$ (resp. $j \in J$) has a complete strict order on J (resp. I).
- The goal is to find a stable matching namely bijections π from I to J such that there are no couples $(i, \pi(i) = j)$, $(i', \pi(i') = j')$ with both j' preferred by i to j and i preferred to i' by j'.
- A simple algorithm allows to do so !

Stable Matching : Gale-Shapley

- Consider two finite families I and J (men/women, workers/firms,...) with the same cardinality.
- Each element $i \in I$ (resp. $j \in J$) has a complete strict order on J (resp. I).
- The goal is to find a stable matching namely bijections π from I to J such that there are no couples $(i, \pi(i) = j)$, $(i', \pi(i') = j')$ with both j' preferred by i to j and i preferred to i' by j'.
- A simple algorithm allows to do so !
- A stable matching may be quite inefficient regarding the average utility of each player ; conversely the matching that maximize the average utility may be unstable. Concept of Price of Anarchy.

Pie-division

- Represent by $[0, 1]$ a set which is to be shared between two players, each of whom have preferences regarding their share of the set.

Pie-division

- Represent by $[0, 1]$ a set which is to be shared between two players, each of whom have preferences regarding their share of the set.
- The game evolves in continuous time between times 0 and 1 and the player who stops first (at time t) wins the share $[0, t]$ of the set, his opponent getting the complement.

Pie-division

- Represent by $[0, 1]$ a set which is to be shared between two players, each of whom have preferences regarding their share of the set.
- The game evolves in continuous time between times 0 and 1 and the player who stops first (at time t) wins the share $[0, t]$ of the set, his opponent getting the complement.
- Assume that $a_1(t)$ (resp. $a_2(t)$), describing the evaluation by player 1, resp. player 2, of the share $[0, t]$, is a continuous function increasing from 0 to 1 ($1 - a_i(t)$ is the evaluation of the complement $(t, 1]$).

Pie-division

- Represent by $[0, 1]$ a set which is to be shared between two players, each of whom have preferences regarding their share of the set.
- The game evolves in continuous time between times 0 and 1 and the player who stops first (at time t) wins the share $[0, t]$ of the set, his opponent getting the complement.
- Assume that $a_1(t)$ (resp. $a_2(t)$), describing the evaluation by player 1, resp. player 2, of the share $[0, t]$, is a continuous function increasing from 0 to 1 ($1 - a_i(t)$ is the evaluation of the complement $(t, 1]$).
- Each player i can obtain $1/2$ by aiming to stop at time t_i with $a_i(t_i) = 1/2$ (if the opponent stops before, even better).

Pie-division

- Represent by $[0,1]$ a set which is to be shared between two players, each of whom have preferences regarding their share of the set.
- The game evolves in continuous time between times 0 and 1 and the player who stops first (at time t) wins the share $[0,t]$ of the set, his opponent getting the complement.
- Assume that $a_1(t)$ (resp. $a_2(t)$), describing the evaluation by player 1, resp. player 2, of the share $[0,t]$, is a continuous function increasing from 0 to 1 ($1 - a_i(t)$ is the evaluation of the complement $(t,1]$).
- Each player i can obtain $1/2$ by aiming to stop at time t_i with $a_i(t_i) = 1/2$ (if the opponent stops before, even better).
- However, if $t_i < t_j$ and player i knows it, he can anticipate that player j will not stop before t_j and try to wait until some $t_j - \varepsilon$.

Pie-division

- Represent by $[0, 1]$ a set which is to be shared between two players, each of whom have preferences regarding their share of the set.
- The game evolves in continuous time between times 0 and 1 and the player who stops first (at time t) wins the share $[0, t]$ of the set, his opponent getting the complement.
- Assume that $a_1(t)$ (resp. $a_2(t)$), describing the evaluation by player 1, resp. player 2, of the share $[0, t]$, is a continuous function increasing from 0 to 1 ($1 - a_i(t)$ is the evaluation of the complement $(t, 1]$).
- Each player i can obtain $1/2$ by aiming to stop at time t_i with $a_i(t_i) = 1/2$ (if the opponent stops before, even better).
- However, if $t_i < t_j$ and player i knows it, he can anticipate that player j will not stop before t_j and try to wait until some $t_j - \varepsilon$.
- Interesting questions appear related to information on the characteristics of the opponent, to the anticipation of his behavior (rationality) and to the impact of the procedure on the outcome (if $t_i < t_j$ then player j would like the splitting to move from 1 to 0).

Vickrey Auction

- An indivisible object is proposed for auction and n players have valuations v_i, $i = 1, \ldots, n$, upon it.

Vickrey Auction

- An indivisible object is proposed for auction and n players have valuations v_i, $i = 1, \ldots, n$, upon it.
- One can consider decreasing auctions where the proposed price decreases until acceptance or increasing auctions where the players make successive increasing offers.

Vickrey Auction

- An indivisible object is proposed for auction and n players have valuations v_i, $i = 1, \dots, n$, upon it.
- One can consider decreasing auctions where the proposed price decreases until acceptance or increasing auctions where the players make successive increasing offers.
- Another model corresponds to the case where the players simultaneously provide sealed bids b_i and the item is given to the one with the highest bid.

Vickrey Auction

- An indivisible object is proposed for auction and n players have valuations v_i, $i = 1, \ldots, n$, upon it.
- One can consider decreasing auctions where the proposed price decreases until acceptance or increasing auctions where the players make successive increasing offers.
- Another model corresponds to the case where the players simultaneously provide sealed bids b_i and the item is given to the one with the highest bid.
- If the price to pay corresponds to the second best bid, the strategy $b_i = v_i$ (bidding his true valuation) is "optimal" for all players (e.g. it is called a dominant strategy).

Biology and evolution

- Consider a competition between three types of cells : a produces a virus and an anti-virus, b produces an anti-virus and c produces nothing.

Biology and evolution

- Consider a competition between three types of cells : a produces a virus and an anti-virus, b produces an anti-virus and c produces nothing.
- Individual cells are not rational but in a way the population as a whole is : succesful cells reproduce faster and transmit their genes. So game theory apply even in microbiology !

Biology and evolution

- Consider a competition between three types of cells : a produces a virus and an anti-virus, b produces an anti-virus and c produces nothing.
- Individual cells are not rational but in a way the population as a whole is : succesful cells reproduce faster and transmit their genes. So game theory apply even in microbiology !
- Production has a cost, hence b wins against a (in terms of fitness or reproduction rate) and c wins against b but a kills c. One faces a cycle !

Biology and evolution

- Consider a competition between three types of cells : a produces a virus and an anti-virus, b produces an anti-virus and c produces nothing.
- Individual cells are not rational but in a way the population as a whole is : succesful cells reproduce faster and transmit their genes. So game theory apply even in microbiology !
- Production has a cost, hence b wins against a (in terms of fitness or reproduction rate) and c wins against b but a kills c. One faces a cycle !
- The corresponding dynamics on the simplex of proportions of the types in the population has an interior rest point (where the three types are simultaneously present) but it may be attractive or repulsive.

Biology and evolution

- Consider a competition between three types of cells : a produces a virus and an anti-virus, b produces an anti-virus and c produces nothing.
- Individual cells are not rational but in a way the population as a whole is : succesful cells reproduce faster and transmit their genes. So game theory apply even in microbiology !
- Production has a cost, hence b wins against a (in terms of fitness or reproduction rate) and c wins against b but a kills c. One faces a cycle !
- The corresponding dynamics on the simplex of proportions of the types in the population has an interior rest point (where the three types are simultaneously present) but it may be attractive or repulsive.
- One can imagine either that after some time the proportions of the three type will converge to a stable situation with a mix of each type, or that one will face a time-cycle of domination of each type one after the other.

A dynamic game

- Consider a situation where two fishermen fish a species that can be in large quantity (a), or small quantity (b), or in extinction (c).

A dynamic game

- Consider a situation where two fishermen fish a species that can be in large quantity (a), or small quantity (b), or in extinction (c).
- Players have an intensive (I) or reduced (R) activity and the outcome of their activity, which depends on the state of the species (a, b or c), is an amount of fish and a probability distribution on the state for the next period. This defines a *stochastic game*.

A dynamic game

- Consider a situation where two fishermen fish a species that can be in large quantity (a), or small quantity (b), or in extinction (c).
- Players have an intensive (I) or reduced (R) activity and the outcome of their activity, which depends on the state of the species (a, b or c), is an amount of fish and a probability distribution on the state for the next period. This defines a *stochastic game*.
- In state a the description of the strategic interaction is given by :

	I	R
I	$100, 100$	$120, 60$
R	$60, 120$	$80, 80$

and the evolution of the state is defined by :

	I	R
I	$(0.3, 0.5, 0.2)$	$(0.5, 0.4, 0.1)$
R	$(0.5, 0.4, 0.1)$	$(0.6, 0.4, 0)$

In state (b) the data for the utilities is :

	I	R
I	$50, 50$	$60, 30$
R	$30, 60$	$40, 40$

and for the transitions probabilities :

	I	R
I	$(0, 0.5, 0.5)$	$(0.1, 0.6, 0.3)$
R	$(0.1, 0.6, 0.3)$	$(0.8, 0.2, 0)$

and in state c the fishing is 0 and the state is absorbing, i.e. stays in state c at all future stages, whatever the players' choices.

In state (b) the data for the utilities is :

	I	R
I	$50, 50$	$60, 30$
R	$30, 60$	$40, 40$

and for the transitions probabilities :

	I	R
I	$(0, 0.5, 0.5)$	$(0.1, 0.6, 0.3)$
R	$(0.1, 0.6, 0.3)$	$(0.8, 0.2, 0)$

and in state c the fishing is 0 and the state is absorbing, i.e. stays in state c at all future stages, whatever the players' choices.

In state (b) the data for the utilities is :

	I	R
I	$50, 50$	$60, 30$
R	$30, 60$	$40, 40$

and for the transitions probabilities :

	I	R
I	$(0, 0.5, 0.5)$	$(0.1, 0.6, 0.3)$
R	$(0.1, 0.6, 0.3)$	$(0.8, 0.2, 0)$

and in state c the fishing is 0 and the state is absorbing, i.e. stays in state c at all future stages, whatever the players' choices.

In state (b) the data for the utilities is :

	I	R
I	$50, 50$	$60, 30$
R	$30, 60$	$40, 40$

and for the transitions probabilities :

	I	R
I	$(0, 0.5, 0.5)$	$(0.1, 0.6, 0.3)$
R	$(0.1, 0.6, 0.3)$	$(0.8, 0.2, 0)$

and in state c the fishing is 0 and the state is absorbing, i.e. stays in state c at all future stages, whatever the players' choices.

When choosing an activity, there is clearly a conflict between the immediate outcome (today's payoff) and the consequence on the future states (and hence future payoffs). Thus the evaluation of some behavior today depends upon the duration of the interaction.

Contents

Strategic games

A *strategic game in normal form* G is defined by :

- a finite set I of *players* (with cardinality $|I| = n \geq 1$) ;

Strategic games

A *strategic game in normal form* G is defined by :

- a finite set I of *players* (with cardinality $|I| = n \geq 1$) ;
- a non-empty set of *strategies* S^i for each player $i \in I$;

Strategic games

A *strategic game in normal form* G is defined by :

- a finite set I of *players* (with cardinality $|I| = n \geq 1$) ;
- a non-empty set of *strategies* S^i for each player $i \in I$;
- a mapping $g = (g^1, \ldots, g^n)$ from $S = \Pi_{i=1}^n S^i$ to R^n.

Strategic games

A *strategic game in normal form* G is defined by :

- a finite set I of *players* (with cardinality $|I| = n \geq 1$) ;
- a non-empty set of *strategies* S^i for each player $i \in I$;
- a mapping $g = (g^1, \ldots, g^n)$ from $S = \Pi_{i=1}^n S^i$ to $\mathbb{R}^n$.

This modelizes choices made independently by the players (each $i \in I$ can choose a strategy $s^i \in S^i$) and the global impact on player j when the *profile* $s = (s^1, \ldots, s^n)$ is chosen is measured by $g^j(s) = g^j(s^1, \ldots, s^n)$, called the *payoff* of player j, and g^j is the *payoff function* of j.

Strategic games

A *strategic game in normal form* G is defined by :

- a finite set I of *players* (with cardinality $|I| = n \geq 1$) ;
- a non-empty set of *strategies* S^i for each player $i \in I$;
- a mapping $g = (g^1, \ldots, g^n)$ from $S = \Pi_{i=1}^n S^i$ to R^n.

This modelizes choices made independently by the players (each $i \in I$ can choose a strategy $s^i \in S^i$) and the global impact on player j when the *profile* $s = (s^1, \ldots, s^n)$ is chosen is measured by $g^j(s) = g^j(s^1, \ldots, s^n)$, called the *payoff* of player j, and g^j is the *payoff function* of j.

We will also use the alternative notation $s = (s^i, s^{-i})$, where s^{-i} stands for the vector of strategies of players other than i, and $S^{-i} = \Pi_{j \neq i} S^j$.

Prisoner dilemma

Two members of a criminal gang are arrested and imprisoned. Each prisoner is in solitary confinement with no means of communicating with the other. The prosecutors lack sufficient evidence to convict the pair on the principal charge, but they have enough to convict both on a lesser charge. Simultaneously, the prosecutors offer each prisoner a bargain. Each prisoner is given the opportunity either Defect by testifying that the other committed the crime, or Cooperate by remaining silent. The possible outcomes are :

Prisoner dilemma

Two members of a criminal gang are arrested and imprisoned. Each prisoner is in solitary confinement with no means of communicating with the other. The prosecutors lack sufficient evidence to convict the pair on the principal charge, but they have enough to convict both on a lesser charge. Simultaneously, the prosecutors offer each prisoner a bargain. Each prisoner is given the opportunity either Defect by testifying that the other committed the crime, or Cooperate by remaining silent. The possible outcomes are :

- If A and B each Defect, each of them serves two years in prison

Prisoner dilemma

Two members of a criminal gang are arrested and imprisoned. Each prisoner is in solitary confinement with no means of communicating with the other. The prosecutors lack sufficient evidence to convict the pair on the principal charge, but they have enough to convict both on a lesser charge. Simultaneously, the prosecutors offer each prisoner a bargain. Each prisoner is given the opportunity either Defect by testifying that the other committed the crime, or Cooperate by remaining silent. The possible outcomes are :

- If A and B each Defect, each of them serves two years in prison
- If A Defect B but B Cooperate, A will be set free and B will serve three years in prison (and vice versa)

Prisoner dilemma

Two members of a criminal gang are arrested and imprisoned. Each prisoner is in solitary confinement with no means of communicating with the other. The prosecutors lack sufficient evidence to convict the pair on the principal charge, but they have enough to convict both on a lesser charge. Simultaneously, the prosecutors offer each prisoner a bargain. Each prisoner is given the opportunity either Defect by testifying that the other committed the crime, or Cooperate by remaining silent. The possible outcomes are :

- If A and B each Defect, each of them serves two years in prison
- If A Defect B but B Cooperate, A will be set free and B will serve three years in prison (and vice versa)
- If A and B both Cooperate, both of them will serve only one year in prison (on the lesser charge).

Prisoner dilemma

Two members of a criminal gang are arrested and imprisoned. Each prisoner is in solitary confinement with no means of communicating with the other. The prosecutors lack sufficient evidence to convict the pair on the principal charge, but they have enough to convict both on a lesser charge. Simultaneously, the prosecutors offer each prisoner a bargain. Each prisoner is given the opportunity either Defect by testifying that the other committed the crime, or Cooperate by remaining silent. The possible outcomes are :

- If A and B each Defect, each of them serves two years in prison
- If A Defect B but B Cooperate, A will be set free and B will serve three years in prison (and vice versa)
- If A and B both Cooperate, both of them will serve only one year in prison (on the lesser charge).

	C	D
C	-1,-1	-3,0
D	0,-3	-2,-2

Prisoner dilemma

Two members of a criminal gang are arrested and imprisoned. Each prisoner is in solitary confinement with no means of communicating with the other. The prosecutors lack sufficient evidence to convict the pair on the principal charge, but they have enough to convict both on a lesser charge. Simultaneously, the prosecutors offer each prisoner a bargain. Each prisoner is given the opportunity either Defect by testifying that the other committed the crime, or Cooperate by remaining silent. The possible outcomes are :

- If A and B each Defect, each of them serves two years in prison
- If A Defect B but B Cooperate, A will be set free and B will serve three years in prison (and vice versa)
- If A and B both Cooperate, both of them will serve only one year in prison (on the lesser charge).

	C	D
C	-1,-1	-3,0
D	0,-3	-2,-2

The donation game is a form of prisoner's dilemma in which cooperation corresponds to offering the other player a benefit b at a personal cost c with $b > c$. Defection means offering nothing.

Other Games

Coordination : Two friends want to meet in a location (A) or (B).

	A	B
A	1,1	0,0
B	0,0	1,1

Other Games

Coordination : Two friends want to meet in a location (A) or (B).

	A	B
A	1,1	0,0
B	0,0	1,1

Battle of the sexes : A couple hesitate between activity A or B. They want be together, the men prefers A to B and the women B to A.

	A	B
A	2,1	0,0
B	0,0	1,2

“Matching Pennies” :

"Matching Pennies" :

	P	F
P	1,-1	-1,1
F	-1,1	1,-1

- This is a two player game where the sum of payoff is zero : what player 1 wins, player 2 loses.

"Matching Pennies" :

	P	F
P	1,-1	-1,1
F	-1,1	1,-1

- This is a two player game where the sum of payoff is zero : what player 1 wins, player 2 loses.
- This will be the subject of our second part of the course : what is the "solution" to a zero sum game and is there a simple procedure which leads the players to learn the solution ?.

Outline of the course 1/2

- 0) Introduction (these slides).

Outline of the course 1/2

- 0) Introduction (these slides).
- 1) Zero-Sum Games (finite case) : minmax, maxmin, value, mixed strategies, von-Neuman minmax theorem and its link with linear programming. Two learning procedures : fictitious play (follow the leader) / and better-reply (Blackwell approachability).

Outline of the course 1/2

- 0) Introduction (these slides).
- 1) Zero-Sum Games (finite case) : minmax, maxmin, value, mixed strategies, von-Neuman minmax theorem and its link with linear programming. Two learning procedures : fictitious play (follow the leader) / and better-reply (Blackwell approachability).
- 2) Zero-Sum Games (general case) Sion minmax theorem (proofs : by separation, by discretization, by learning -fictitious play-). Extensive Form Games (Zermelo's, Gale Stewart, Kuhn's theorems).

Outline of the course 1/2

- 0) Introduction (these slides).
- 1) Zero-Sum Games (finite case) : minmax, maxmin, value, mixed strategies, von-Neuman minmax theorem and its link with linear programming. Two learning procedures : fictitious play (follow the leader) / and better-reply (Blackwell approachability).
- 2) Zero-Sum Games (general case) Sion minmax theorem (proofs : by separation, by discretization, by learning -fictitious play-). Extensive Form Games (Zermelo's, Gale Stewart, Kuhn's theorems).
- 3) N-player games : Rationalizability, Nash equilibrium, potential games, monotone games. Existence and variational characterisation of NE. Best reply dynamics and fictitious play : convergence in potential/acyclic games, non convergence in general games (Shapley triangle). Fictitious play with finite memory.

Outline of the course 1/2

- 0) Introduction (these slides).
- 1) Zero-Sum Games (finite case) : minmax, maxmin, value, mixed strategies, von-Neuman minmax theorem and its link with linear programming. Two learning procedures : fictitious play (follow the leader) / and better-reply (Blackwell approachability).
- 2) Zero-Sum Games (general case) Sion minmax theorem (proofs : by separation, by discretization, by learning -fictitious play-). Extensive Form Games (Zermelo's, Gale Stewart, Kuhn's theorems).
- 3) N-player games : Rationalizability, Nash equilibrium, potential games, monotone games. Existence and variational characterisation of NE. Best reply dynamics and fictitious play : convergence in potential/acyclic games, non convergence in general games (Shapley triangle). Fictitious play with finite memory.
- 4) Vector payoff games : Blackwell approachability and the equivalence with no-regret and calibration. Application to zero-sum games. Link with online optimisation (Online Gradient Descent, Follow the leader, Online Mirror Decent).

Outline of the course 2/2

- 5) Smooth fictitious play, link with regret learning (follow the perturbed leader) and convergence to coarse equilibria. Internal regret and prediction : convergence to correlated equilibria.

Outline of the course 2/2

- 5) Smooth fictitious play, link with regret learning (follow the perturbed leader) and convergence to coarse equilibria. Internal regret and prediction : convergence to correlated equilibria.
- 6) Game in extensive form Perfect and imperfect information, behavior and mixed strategies, Kuhn's theorem. Subgame perfect strategies, Bayesian perfect strategies.

Outline of the course 2/2

- 5) Smooth fictitious play, link with regret learning (follow the perturbed leader) and convergence to coarse equilibria. Internal regret and prediction : convergence to correlated equilibria.
- 6) Game in extensive form Perfect and imperfect information, behavior and mixed strategies, Kuhn's theorem. Subgame perfect strategies, Bayesian perfect strategies.
- 7) Repeated Games : cooperation and folk theorems. Bayesian Learning, merging, grain of truth, convergence and impossibilities. Hypothesis Testing and convergence to Nash Equilibria.

Outline of the course 2/2

- 5) Smooth fictitious play, link with regret learning (follow the perturbed leader) and convergence to coarse equilibria. Internal regret and prediction : convergence to correlated equilibria.
- 6) Game in extensive form Perfect and imperfect information, behavior and mixed strategies, Kuhn's theorem. Subgame perfect strategies, Bayesian perfect strategies.
- 7) Repeated Games : cooperation and folk theorems. Bayesian Learning, merging, grain of truth, convergence and impossibilities. Hypothesis Testing and convergence to Nash Equilibria.
- 8) Continuous time learning dynamics, link with discrete time, local stability, ESS, variational stability (if we have time !)

Outline of the course 2/2

- 5) Smooth fictitious play, link with regret learning (follow the perturbed leader) and convergence to coarse equilibria. Internal regret and prediction : convergence to correlated equilibria.
- 6) Game in extensive form Perfect and imperfect information, behavior and mixed strategies, Kuhn's theorem. Subgame perfect strategies, Bayesian perfect strategies.
- 7) Repeated Games : cooperation and folk theorems. Bayesian Learning, merging, grain of truth, convergence and impossibilities. Hypothesis Testing and convergence to Nash Equilibria.
- 8) Continuous time learning dynamics, link with discrete time, local stability, ESS, variational stability (if we have time !)
- 9) Oral examination : presentation by all student of their article (15mn+5mn questions). The final grade is the mean between the oral and the written report grades.

References

- Cesa-Bianchi and Lugosi. Prediction and Learning in Games. Cambridge U Press, 2006.
- Hart and Mas-Colell. *Simple Adaptive Strategies : from Regret Matching to Uncoupled Dynamics*. WSS, 2013.
- Hofbauer and Sigmund. *Evolutionary Games and Population Dynamics*, Cambridge, 1998.
- Laraki, Renault and Sorin. *Mathematical Foundations of Game Theory*. Springer 2019.
- Nizan, Roughgarden, Tardos and Vazirani. *Algorithmic Game Theory*, Cambridge University Press, 2007.
- Sandholm. *Population Games and Evolutionary Dynamics*, M.I.T. Press, 2010.
- Shalev-Shwartz. *Online Learning and Online Convex Optimization*, Foundations and Trends in Machine Learning, 2011.
- Weibull. *Evolutionary Game Theory*, MIT, 1995.
- Young. *Strategic Learning and its Limits*, Oxford University Press, 2004.