# Math Circle (Oct 5, 2024)

Instructor: Hao Zhuang

1. For $y = \ln(x)$, can you graph it on $xy$-plane? Is it concave up or down? Then, on the same picture, given two real numbers $a$ and $b$, can you draw the line through $(a, \ln(a))$ and $(b, \ln(b))$

2. Figure out the equation of this line.

3. Check that for any $0 \leq t \leq 1$, we have $(1-t)a + tb$ between $a$ and $b$.

4. Plug $x = (1-t)a + tb$ into the function $y = \ln(x)$ from Question 1 and the line $y = qx + r$ from Question 2 respectively. Write down these two $y$'s.

5. Which $y$ is bigger? (Hint: Look at the graph. Which function is higher?)

Now that you've shown the inequality

$$\ln\left((1-t)a+tb\right) \geq (1-t)\ln(a)+t\ln(b),$$

we play with $a_1, a_2, \cdots, a_n$:

6. Let $t=\dfrac{1}{2}$, $a=a_1$, $b=a_2$. What does the inequality become?

7. Let $t=\dfrac{1}{3}$, $a=\dfrac{a_1+a_2}{2}$, $b=a_3$. What does the inequality become? Apply the conclusion from Question 6, what do you get?

8. Let $t=\dfrac{1}{4}$, $a=\dfrac{a_1+a_2+a_3}{3}$, $b=a_4$. What does the inequality become? Apply the conclusion from Question 7, what do you get?

9. Take a guess of the result for $a_1, a_2, \cdots, a_n$ and $t=\dfrac{1}{n}$. (If you are interested, you may prove it rigorously using mathematical induction).

10. Apply the exponential "$e$" to each side and then ...