5. SS Lab 5

The fifth lab of Signals and Systems is about complex numbers, exponential signals, real exponential signals, and complex exponential signals in the MATLAB environment.

Suggestions for improvement or correction of the manuscript would be appreciated.

5.1 Lab Objectives

In this lab, the following areas would be covered:

- Complex Numbers and plots
- Complex exponential signals
- Real exponential signals

5.2 Complex Numbers

A complex number z is an ordered pair (x, y) of real numbers. Complex numbers can be represented in rectangular form as $z = x + iy$, which is the vector in two-dimensional plane. The horizontal coordinate x is called the real part of z and can be represented as $x = \text{Re}\{z\}$, while the vertical coordinate y is called the imaginary part of z and represented as $y = \text{Imag}\{z\}$. That is:

$$z = (x, y) = x + iy = \text{Re}\{x\} + i\ \text{Imag}\{x\}$$

Another way to represent a complex number is in polar form. In polar form, the vector is defined by its length r or magnitude $|z|$ and its direction θ. A rectangular form can be converted into polar form using formulas:

$$|z| = r = \sqrt{x^2 + y^2}$$

$$\theta\ = \arctan\left(\frac{y}{x}\right) z\ =\ r\,e^{j\theta}$$

where $e^{j\theta} = \cos\theta + i\sin\theta$ and known as the Euler's formula.

5.3 Built-In Matrix Functions

```
Function Description
=====================================================
real        returns the real part x of z
imag        returns the imaginary part y of z
abs         returns the length r of z
angle       returns the direction θ of z
conj        returns the complex conjugate z̲ of z
=====================================================
```

Example: To define the complex number, for instance, $z = (3,4)$ in MATLAB, write the following code in MATLAB editor:

```
>> z = 3 + 4i
```

```
z =
   3.0000 + 4.0000i
```

Example: To find the real and imaginary parts of the complex number, write:

```
>> x = real(z)
x =
   3
>> y = imag(z)
y =
   4
```

Example: To find the length and direction of z, write:

```
>> r = abs(z)
r =
   5
>> θ = angle(z)
θ =
   0.9273
```

Example: To find the conjugate of z, write:

```
>> zx = conj(z)
zx =
   3.0000 - 4.0000i
```

5.4 Complex Exponential Signals

The complex exponential signal is defined as:

$$x'(t) = A\, e^{j(w_0 t + ø)}$$

which is a complex-valued function of t, where the magnitude of $x'(t)$ is:

$|x'(t)| = A$ ⟹ magnitude or length of $x'(t)$

$\arg x'(t) = (w_0 t + ø)$ ⟹ angle or direction of $x'(t)$

Using Euler's formula, it can be expressed in rectangular or Cartesian form, i.e.:

$$x'(t) = A\, ej(w_0 t + ø) = A\, cos(w_0 t + ø) + j\, A\, sin(w_0 t + ø)$$

where

A = amplitude, ø = phase shift, w_0 = frequency in rad/sec

Example:

```
clear, close all, clc
n=0:1/10:10;
k=5;
a=pi/2;
```

```
x=k * exp(a*n*i);
% plot the real part
subplot(2,1,1)
stem(n, real(x), 'filled')

title('Real part of complex exp')
xlabel('sample #')
ylabel('signal amplitude')
grid
% plot the imaginary part
subplot(2,1,2)
stem(n, imag(x), 'filled')
title('Imaginary part of complex exp')
xlabel('sample #')
ylabel('signal amplitude')
grid
```

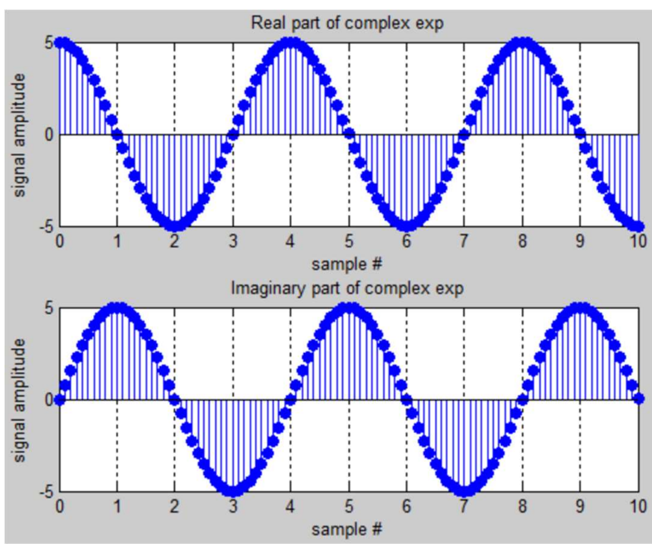

Figure 5.1: Real and imaginary parts of complex exponential signal

5.5 Tasks

Perform the following tasks:

5.5.1 Task 01

Write a MATLAB function `zprint`, which takes a complex number and returns it real part, imaginary part, magnitude, phase in radians, and phase in degrees.

A sample run of program is:

```
>> zprint(z)
Z =    X   +    jY    Magnitude   Phase   Ph(deg)
   3  4  5  0.927    53.13
```

5.5.2 Task 02

Compute the conjugate ź (i.e. `z_conj` [give variable name]) and the inverse $1/z$ (i.e. `z_inv` [give variable name]) for any complex number z. Display the results numerically with `zprint`.

5.5.3 Task 03

Take two complex numbers, compute $z_1 + z_2$ and display the results numerically using `zprint`.

5.5.4 Task 04

Take two complex numbers and compute $z_1 . z_2$ and z_1/z_2. Use `zprint` to display the results numerically.

5.5.5 Task 05

Determine the complex conjugate of the exponential signal given in above example and plot its real and imaginary parts.

5.5.6 Task 06

Generate the complex valued signal

$$y(n) = e^{(-0.2+j0.5)n} \text{ for- } 10 \leq n \leq 10$$

and plot its magnitude, phase, the real part, and the imaginary part in separate subplots.

5.5.7 Task 07

Generate a real-exponential $x = an$ for $a = 0.7$ and n ranging from 0-10. Find the discrete time as well as the continuous time versions of this signal. Plot the two signals on the same graph (by holding both the graphs).

Repeat the same program with value of $a = 1.3$.

5.5.8 Task 08

Multiply the two discrete signals $x_1 = 5\exp\left(i * n * \frac{pi}{4}\right)$ and $x_2 = an$ (use point-by-point multiplication of the two signals). Plot the real as well as the exponential parts for $0 < a < 1$ and $a > 1$.

5.5.9 Task 09

Plot the discrete signal $x = a^{|n|}$ for n ranging from -10 to 10. Draw two subplots for $0 < a < 1$ and $a > 1$.

5.5.10 Task 10

Generate the signal $x(t) = Ae^{j(\omega t + \pi)}$ for A=3, π =- 0.4, and $\omega = 2\pi(1250)$. Take a range for t that will cover 2 or 3 periods.

Plot the real part versus t and the imaginary part versus t. Use `subplot(2,1,i)` to put both plots in the same window.

Verify that the real and imaginary parts are sinusoids and that they have the correct frequency, phase, and amplitude.