9. SS Lab 9

The 9th Signals and Systems lab covers topics such as signal power, Fourier Series, square and triangular waves in the MATLAB environment.

Suggestions for improvement or correction of the manuscript would be appreciated.

9.1 Lab Objectives

In this lab, the following topics would be addressed:

- Power of continuous and discrete time signals
- Application of Fourier Series
- Synthesis of square wave
- Synthesis of triangular wave

9.2 Signal Power

Average power of continuous time signal can be calculated using the formula:

$$P = \frac{1}{T}\int_{-T/2}^{T/2} |x(t)|^2 dt$$

To carry out the integral, Euler approximation can be used. It simply tells that a definite integral can be approximated using a sum i.e.:

$$\int_a^b x(t)dt \cong \sum_{n=0}^{N-1} x(a+n\Delta t)\Delta t\,, \Delta t = \frac{b-a}{N}$$

In this method, the region over which integral is carried out is divided into N parts or intervals, each of duration t, such that function stays constant over those short intervals. Approximating function in this way is shown below. Note that as the number of intervals N is increased, the approximation gets better.

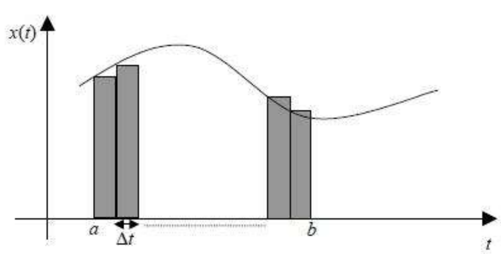

Figure 9.1: Signal power approximation

Approximating integrals using sums is a deep subject of numerical analysis by itself, therefore further details are out of scope and not explored here. It is sufficient to know that Euler's formula is easy to implement and produces good results for almost all the signals that will be studied here as long as N is selected large enough.

Example: Power of Continuous Time Cosine:

```
clc
clear all
close all
t = -1:0.005:0.995;   % time duration of given signal;
xt = cos(2*pi*t/2);   % generate signal
plot(t, xt);          % plot signal
xlabel('time, t');
ylabel('Amplitude, A');
title('Continuous Time Cosine');
abs_xt_2 = abs(xt).^2;    % take absolute square of signal
T = 2; % length of interval
delta_t = 0.005;                  % interval duration
pxt = sum(abs_xt_2)*delta_t/T    % power of given signal
```

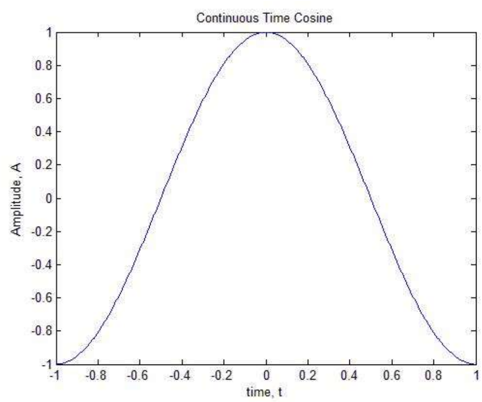

Figure 9.2: Continuous time cosine

```
pxt =
  0.5000
```

9.3 Fourier Series

Fourier series theory states that a periodic wave can be represented as a summation of sinusoidal waves with different frequencies, amplitudes and phase values.

9.3.1 Synthesis of Square wave

The square wave for one cycle can be represented mathematically as:

$$x(t) = \begin{cases} 1 & 0 \le t < T/2 \\ -1 & T/2 \le t < T \end{cases}$$

The complex amplitude is given by:

$$x(t) = \begin{cases} 4/(j * 2 * \pi * k) & \text{for} \quad k = \pm 1, \pm 3, \pm 5, \ldots \\ 0 & \text{for} \quad k = 0, \pm 2, \pm 4, \pm 6, \ldots \end{cases}$$

For $f = 1/T = 25$Hz, only the frequencies ±25, ±50, ±75 etc. are in the spectrum.

Example: Effect of adding fundamental, third, fifth, and seventh harmonics:

```
clc
clear all
close all
t=0:0.0001:8;
ff=0.5;
% WE ARE USING SINE FUNCTION BECAUSE
% FROM EXPONENTIAL FORM OF FOURIER
% SERIES FINALLY WE ARE LEFT WITH SINE TERMS
y = (4/pi)*sin(2*pi*ff*t);
% COMPLEX AMPLITUDE = (4/(j*pi*k)) for k = 3:2:7
fh=k*ff;
x = (4/(k*pi))*sin(2*pi*fh*t);
y=y+x;
end
plot(t,y,'linewidth',1.5);
title('A square wave with harmonics 1st, 3rd, 5th, and 7th');
xlabel('Time');
ylabel('Amplitude');
```

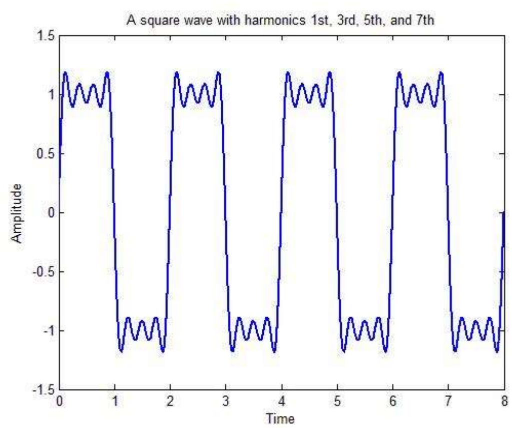

Figure 9.3: Square wave with 1st, 3rd, 5th and 7th harmonics

Example: Effect of adding 1st to 17th harmonics

```
clc
clear all
t=0:0.0001:8;
ff=0.5;
```

```
y = (4/pi)*sin(2*pi*ff*t);
for k = 3:2:17 fh=k*ff;
x = (4/(k*pi))*sin(2*pi*fh*t); y=y+x;
end
plot(t,y,'linewidth',1.5);
title('A square wave with harmonics 1st-17th');
xlabel('Time');
ylabel('Amplitude');
```

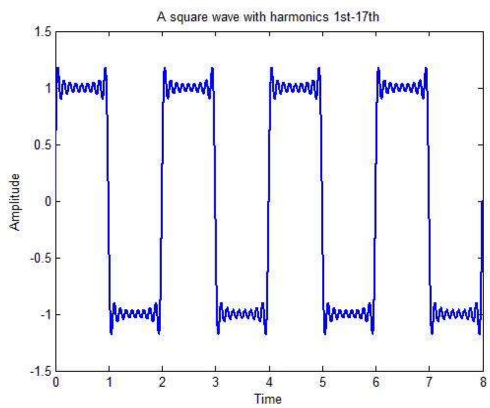

Figure 9.4: Square wave with 1st to 17th (odd) harmonics

Example: Effect of Adding 1st to 27th harmonics

```
clc
clear all
close all
t=0:0.0001:8;
ff=0.5;
y = (4/pi)*sin(2*pi*ff*t);
fh=k*ff;
x = (4/(k*pi))*sin(2*pi*fh*t);
y=y+x;
end
plot(t,y,'linewidth',1.5);
title('A square wave with harmonics 1st to 27th');
xlabel('Time');
ylabel('Amplitude');
```

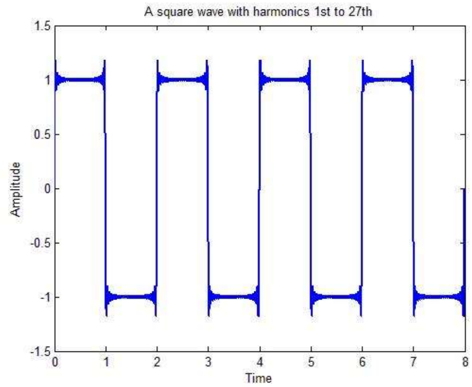

Figure 9.5: Square wave with 1st to 27th (odd) harmonics

9.3.2 Synthesis of Triangular Wave

The triangular wave signal can be represented by complex exponential signal:

$$x(t) = x_k . e^{2*\pi*f*k*t*j}$$

with the complex amplitude given by:

$$x_k = \begin{cases} -8/(\pi^2 * k^2) & \text{for } k \text{ is an odd integer} \\ 0 & \text{for } k \text{ is an even integer} \end{cases}$$

Example: Triangular wave with $N = 3$:

```
clc;
clear all;
close all
t=0:0.001:5;
x=(-8/(pi*pi))*exp(i*(2*pi*0.5*t));
y=(-8/(9*pi*pi))*exp(i*(2*pi*0.5*3*t));
s=x+y;
plot(t,real(s),'linewidth',3);
title('Triangular Wave with N=3');
ylabel('Amplitude');
xlabel('Time');
grid;
```

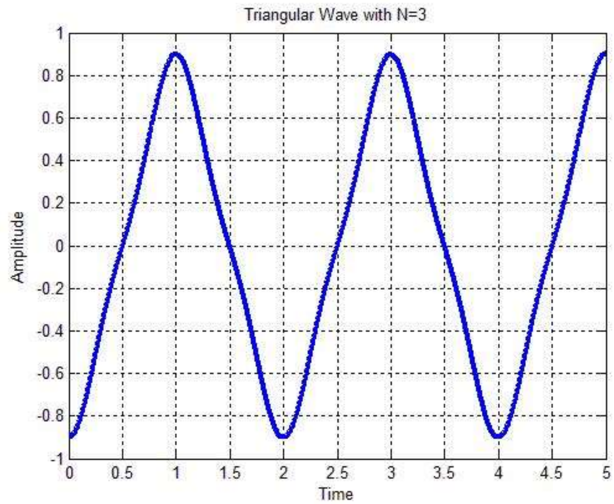

Figure 9.6: Triangular wave with N = 3

Example: Triangular wave with $N = 11$:

```
clc;
clear all;
close all
t=0:0.01:0.25;
ff=25;
x1=(-8/(pi^2))*exp(i*(2*pi*ff*t));
for k=3:2:21
   fh=ff*k;
   x=(-8/(pi^2*k^2))*exp(i*(2*pi*fh*t));
   y=x1+x;
end
plot(t,real(y),'linewidth',3);
title('Triangular Wave with N=11');
ylabel('Amplitude');
xlabel('Time');
grid;
```

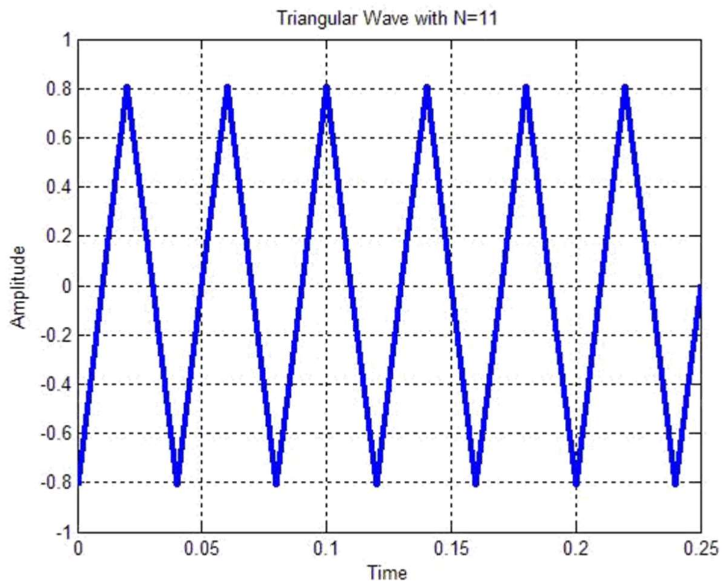

Figure 9.7: Triangular wave with N = 11

9.4 Tasks

Perform the following tasks:

9.4.1 Task 01

Calculate the power of discrete-time cosine signal with period 20, defined over interval 0:19 using the following formula:

$$P = \frac{1}{N}\sum_{0}^{N-1} |x[n]|^2$$

9.4.2 Task 02

Analyze the effect of adding 1st to 17th harmonics and the effect of adding 1st to 27th harmonics in section 9.3.1.

9.4.3 Task 03

Write a program that plots the signal $s(t)$.

$$s(t) = \sum_{n=1}^{N} \frac{\sin(2\pi n t)}{n} \text{ where } n = 1, 3, 5, 7, 9 \text{ and } N = 9$$

OR

$$s(t) = \sin(2\pi * t) + \frac{\sin(6\pi * t)}{3} + \frac{\sin(10\pi * t)}{5} + \frac{\sin(14\pi * t)}{7} + \frac{\sin(18\pi * t)}{9}$$

9.4.4 Task 04

What do you conclude from 9.4.2 and 9.4.3?

9.4.5 Task 05

Write a program that plots the above signal $s(t)$ but with $N = 100$.

9.4.6 Task 06

Generate a triangular wave with $N = 17$.