Sorting in Linear Time:
fastest (but special purpose) sorting algorithms

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Slides adapted from Dr. Debswapna Bhattacharya's class

- Sorting by comparing pairs of numbers

- Algorithms that sort n numbers in $O(n^2)$ time
  - Insertion, Selection, Bubble

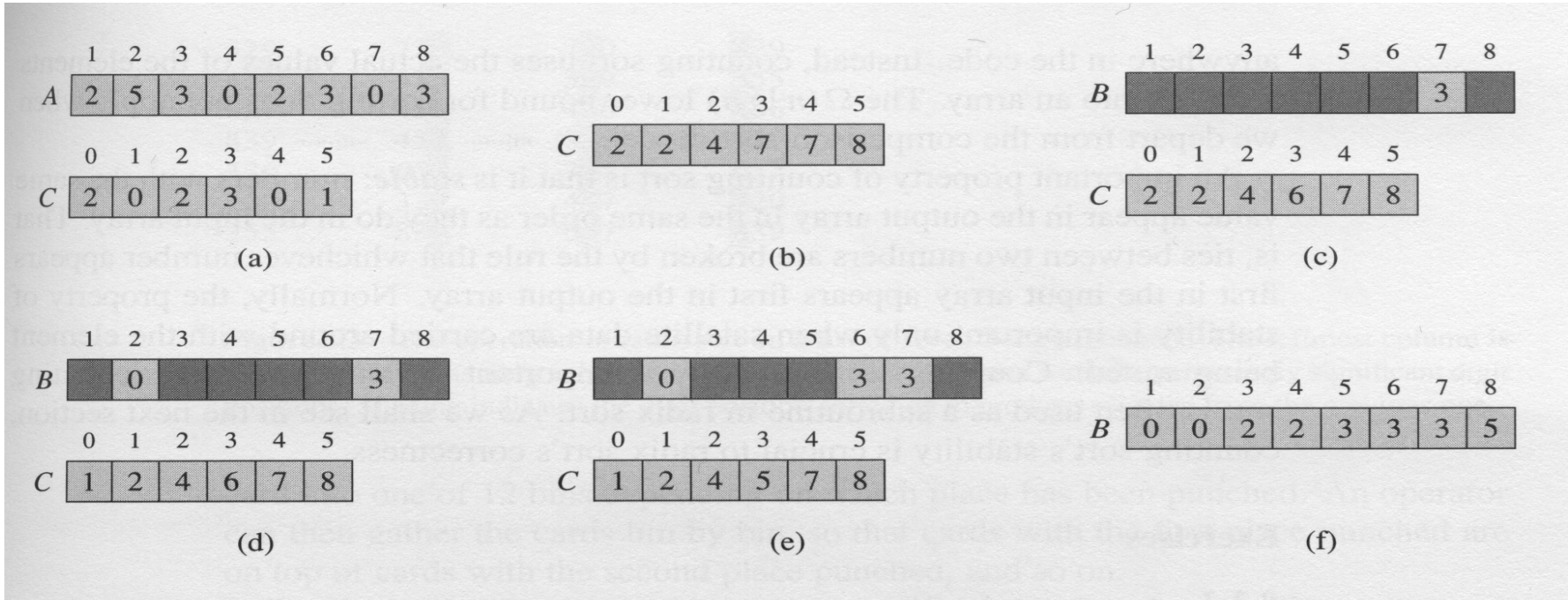
- Algorithms that sort n numbers in O(nlgn) time
  - Merge, Heap and Quick

- Any comparison sort must make **Ω(nlgn)** comparisons.
- So, O(nlgn) is the most efficient they can be!
- Now we will talk about three sorting algorithms – counting, radix and bucket – that are linear
- So, they must use a strategy other than comparing pairs of numbers.

- ***Assume that each of the n input elements is an integer in the range 0 to k for some integer k.***

```
COUNTING_SORT(A,B,k)
    let C[0...k] be a new array
1   for i = 0 to k
2       c[i] = 0
3   for j = 1 to A.length
4       c[A[j]] = c[A[j]] + 1
    // c[i] now contains the number of
       elements equal to i
5   for i = 1 to k
6       c[i] = c[i] + c[i − 1]
    // c[i] now contains the number of
       elements less than or equal to i
7   for j = A.length downto 1
8   B[c[A[j]]] = A[j]
9       c[A[j]] = c[A[j]] − 1
```

The operation of Counting-sort on an input array A[1..8]

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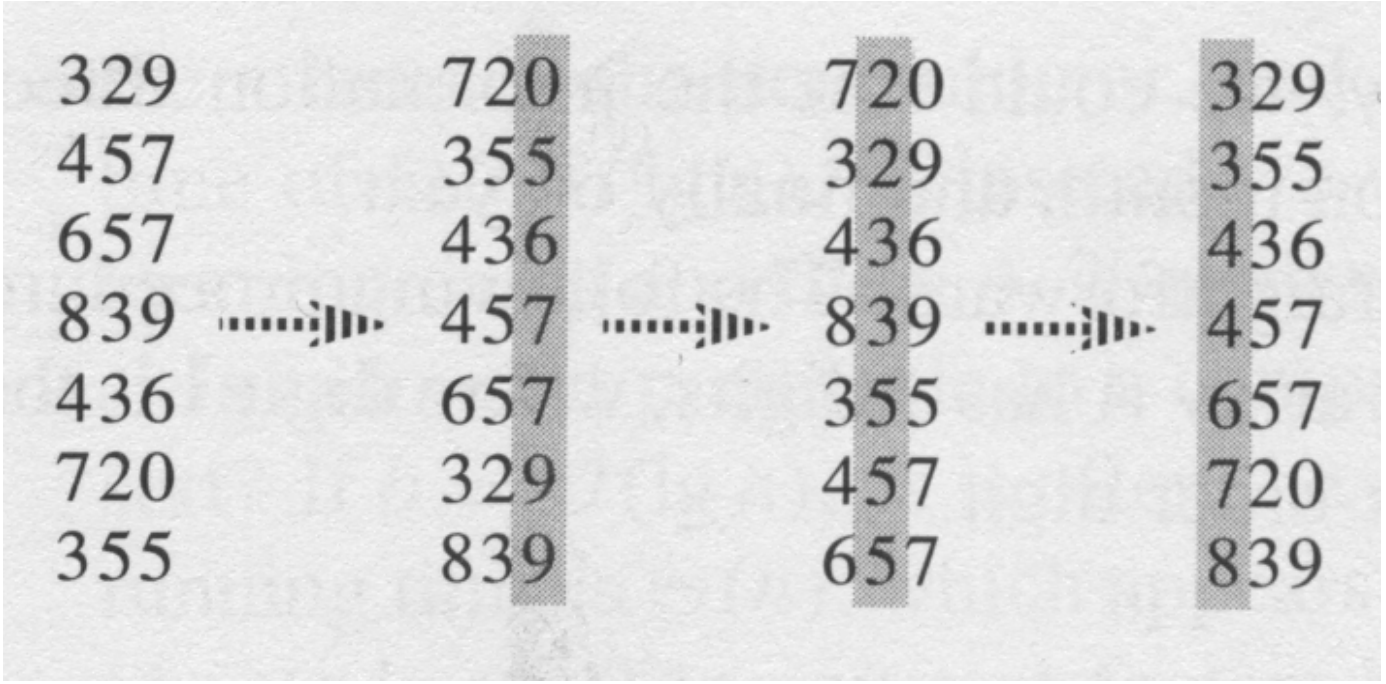
- Approximate Analysis: $\Theta(k+n)$
- $\Theta(n)$ when $k < n$
- $\Theta(k)$ when $k > n$
- What if $k \gg n$?
- Counting sort is *not an in-place* algorithm.
- Counting sort is **stable** (numbers with the same value appear in the output array in the same order as they do in the input array.)

- Modify the algorithm to sort n integers in the range i-j, i<j, where i>0
- Modify the algorithm to sort n integers in the range i-j, i<j, where i!=0 and either or both of i and j may be negative

RADIX_SORT(A, d)

1 for $i = 1$ **to** d

2 use a stable sort to sort array A on digit i



Given n d-digit numbers in which each digit can take on up to k possible values, Radix sort correctly sorts these number in $\Theta(d(n + k))$ time if Counting Sort is used.

k = 10 so for large n, n+k can be approximated as n, Radix sort runs in $\Theta(dn) = \Theta(n)$ time because d is also a small constant.

- Why is it important that the sorting algorithm used by Radix sort be stable?

- Radix sort is not an in-place algorithm…why?

- Suppose you were to use the Counting Sort algorithm in step 2 of Radix Sort. Will Radix Sort work correctly for all negative integers? Why or why not? If not, can you modify Radix and/or Counting Sort to make this work?

- Suppose you were to use the Counting Sort algorithm in step 2 of Radix Sort. Will Radix Sort work correctly for mixed +ve and –ve integers? Why or why not? If not, can you modify Radix and/or Counting Sort to make this work?

BUCKET_SORT(A)

1 $n = A.length$

2 let B[0…n-1] be a new array

3 **for** $i = 0$ **to** *n-1*

4 make B[i] an empty list

5 **for** $i = 1$ **to** n

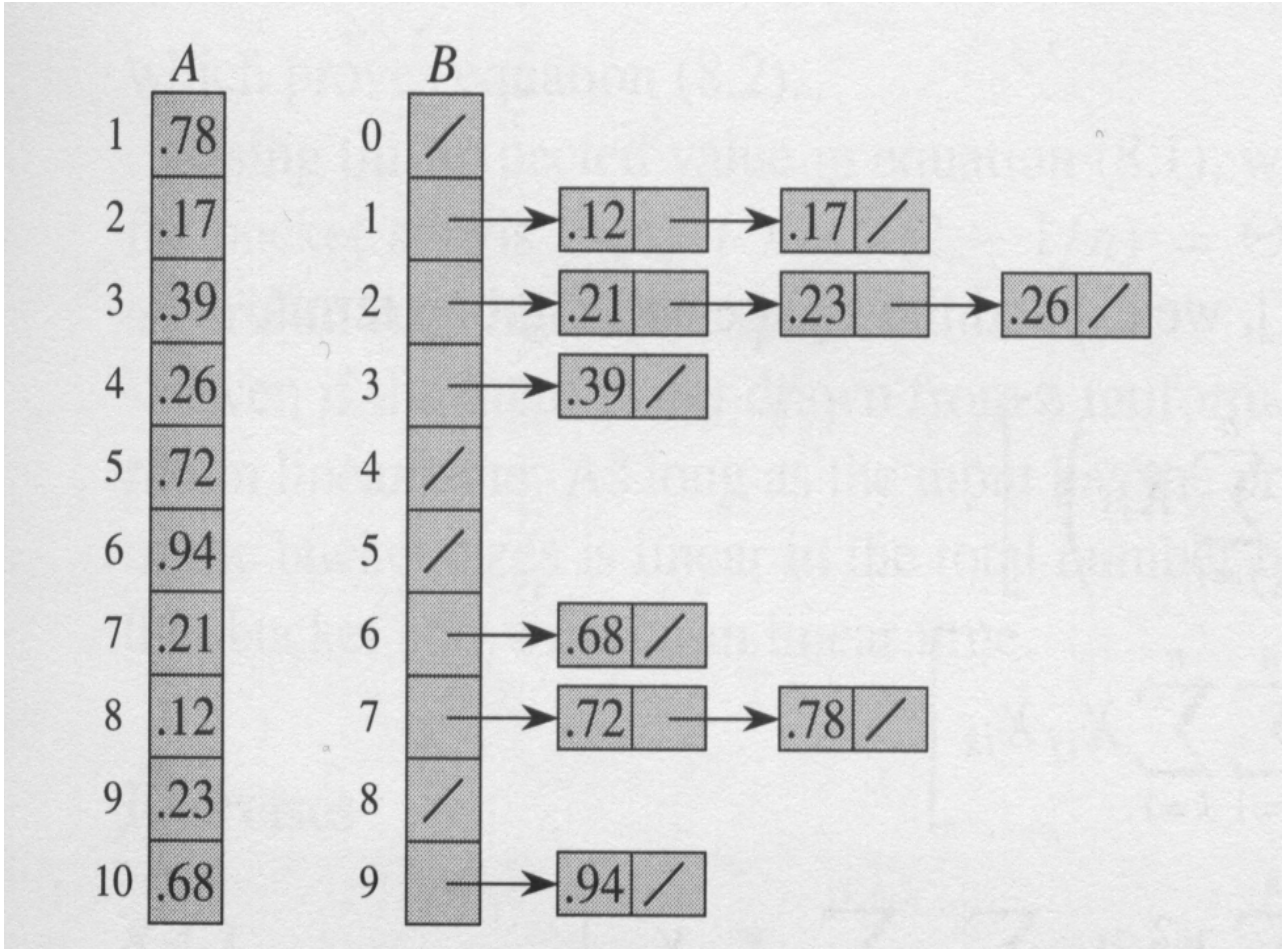
6 insert $A[i]$ into

list $B\lfloor nA[i] \rfloor$

7 **for** $i = 0$ **to** *n-1*

8 sort list $B[i]$ with insertion sort

9 concatenate $B[0], B[1], \ldots, B[n-1]$ in order



Bucket sort is linear, O(n), because if the numbers are distributed uniformly across each of the n buckets, each bucket will only have on average **one** number in it.

- The algorithm requires an input of real numbers uniformly distributed over the interval [0,1). Why is it important that the input to Bucket sort be uniformly distributed over this interval?

- Instead of simply adding new numbers to the head of the linked lists and later sorting the lists using Insertion Sort, you can maintain sorted linked lists by adding a new number in its correct sorted location in step 6 and do away with steps 7 and 8. Will this change the complexity of the algorithm? If so, how? If not, why not?

- The algorithm uses n buckets for n input numbers in the range [0,1) and some buckets may be empty while others contain more than one number and need to be sorted. But if we restrict the input to n integers in a range [0,k] as was the case for Counting Sort, and if you use k+1 buckets, you can avoid having to use Insertion sort. Why?

- How will you modify the Bucket Sort algorithm to accomplish this? Compare this modified algorithm with Counting Sort in terms of space and time efficiency.

- How will you modify the algorithm step 6 to accommodate real number input from some interval [0,p), p>1, not [0,1)?

- How will you modify the algorithm step 6 to accommodate real number input from some interval [i,j), not [0,1)?

- Omit 8.1
- Read 8.2
- Read 8.3 (omit lemmas and proofs)
- Read 8.4 (omit average case analysis p.202-203)

- Do Exercises:
- 8.2-1:8.2-4
- 8.3-1:8.3-3
- 8.4-1:8.4-2

- A universal problem! Many applications incorporate sorting.

- There is a wide variety of sorting algorithms, and they use a rich set of techniques. So sorting provides a good case study of a variety of algorithm design techniques resulting in algorithms of differing complexities.

Popular Sorting Algorithms

- Quadratic Sorts $O(n^2)$
  - Bubble, Insertion, Selection

- A sort that improved on this (of historical interest only)
  - Shell Sort $O(n^{1.5})$

- Efficient Sorts O(nlgn)
  - Merge, Heap, Quick

- Linear (specialized) sorts O(n)
  - Counting, Radix, Bucket

- For a lager list, see https://en.wikipedia.org/wiki/Sorting_algorithm

Four important properties

- Time Efficiency
- Space Efficiency: in-place
- Stability
- Recursive/Non-recursive

- Thinking Assignments
  - Which of the sorting algorithms we discussed are in-place?
  - Which of the sorting algorithms we discussed are stable?

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