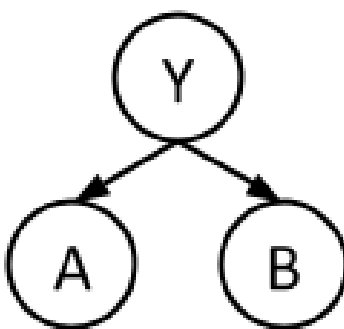


## 1 Naive Bayes

In this question, we will train a Naive Bayes classifier to predict class labels  $Y$  as a function of input features  $A$  and  $B$ .  $Y$ ,  $A$ , and  $B$  are all binary variables, with domains 0 and 1. We are given 10 training points from which we will estimate our distribution.

$A$	1	1	1	1	0	1	0	1	1	1
$B$	1	0	0	1	1	1	1	0	1	1
$Y$	1	1	0	0	0	1	1	0	0	0



- (a) What are the maximum likelihood estimates for the tables  $P(Y)$ ,  $P(A|Y)$ , and  $P(B|Y)$ ?

$Y$	$P(Y)$	$A$	$Y$	$P(A Y)$	$B$	$Y$	$P(B Y)$
0	$3/5$	0	0	$1/6$	0	0	$1/3$
1	$2/5$	1	0	$5/6$	1	0	$2/3$
		0	1	$1/4$	0	1	$1/4$
		1	1	$3/4$	1	1	$3/4$

- (b) Consider a new data point ( $A = 1, B = 1$ ). What label would this classifier assign to this sample?

$$\begin{aligned}
 P(Y = 0, A = 1, B = 1) &= P(Y = 0)P(A = 1|Y = 0)P(B = 1|Y = 0) & (1) \\
 &= (3/5)(5/6)(2/3) & (2) \\
 &= 1/3 & (3) \\
 P(Y = 1, A = 1, B = 1) &= P(Y = 1)P(A = 1|Y = 1)P(B = 1|Y = 1) & (4) \\
 &= (2/5)(3/4)(3/4) & (5) \\
 &= 9/40 & (6) \\
 & & (7)
 \end{aligned}$$

Our classifier will predict label 0.

- (c) Let's use Laplace Smoothing to smooth out our distribution. Compute the new distribution for  $P(A|Y)$  given Laplace Smoothing with  $k = 2$ .

$A$	$Y$	$P(A Y)$
0	0	$3/10$
1	0	$7/10$
0	1	$3/8$
1	1	$5/8$

## 2 Perceptron

You want to predict if movies will be profitable based on their screenplays. You hire two critics A and B to read a script you have and rate it on a scale of 1 to 4. The critics are not perfect; here are five data points including the critics' scores and the performance of the movie:

#	Movie Name	A	B	Profit?
1	Pellet Power	1	1	-
2	Ghosts!	3	2	+
3	Pac is Bac	2	4	+
4	Not a Pizza	3	4	+
5	Endless Maze	2	3	-

$AB$

- (a) First, you would like to examine the linear separability of the data. Plot the data on the 2D plane above; label profitable movies with + and non-profitable movies with - and determine if the data are linearly separable. **The data are linearly separable.**
- (b) Now you decide to use a perceptron to classify your data. Suppose you directly use the scores given above as features, together with a bias feature. That is  $f_0 = 1$ ,  $f_1 = \text{score given by A}$  and  $f_2 = \text{score given by B}$ .

Run one pass through the data with the perceptron algorithm, filling out the table below. Go through the data points in order, e.g. using data point #1 at step 1.

step	Weights	Score	Correct?
1	$[-1, 0, 0]$	$-1 \cdot 1 + 0 \cdot 1 + 0 \cdot 1 = -1$	yes
2	$[-1, 0, 0]$	$-1 \cdot 1 + 0 \cdot 3 + 0 \cdot 2 = -1$	no
3	$[0, 3, 2]$	$0 \cdot 1 + 3 \cdot 2 + 2 \cdot 4 = 14$	yes
4	$[0, 3, 2]$	$0 \cdot 1 + 3 \cdot 3 + 2 \cdot 4 = 17$	yes
5	$[0, 3, 2]$	$0 \cdot 1 + 3 \cdot 2 + 2 \cdot 3 = 12$	no

Final weights:  $[-1, 1, -1]$

- (c) Have weights been learned that separate the data? **With the current weights, points will be classified as positive if  $-1 \cdot 1 + 1 \cdot A + -1 \cdot B \geq 0$ , or  $A - B \geq 1$ . So we will have incorrect predictions for data points 3:**

$$-1 \cdot 1 + 1 \cdot 2 + -1 \cdot 4 = -3 < 0$$

and 4:

$$-1 \cdot 1 + 1 \cdot 3 + -1 \cdot 4 = -2 < 0$$

**Note that although point 2 has  $w \cdot f = 0$ , it will be classified as positive (since we classify as positive if  $w \cdot f \geq 0$ ).**

- (d) More generally, irrespective of the training data, you want to know if your features are powerful enough to allow you to handle a range of scenarios. Circle the scenarios for which a perceptron using the features above can indeed perfectly classify movies which are profitable according to the given rules:

- (a) Your reviewers are awesome: if the total of their scores is more than 8, then the movie will definitely be profitable, and otherwise it won't be. **Can classify (consider weights  $[-8, 1, 1]$ )**
- (b) Your reviewers are art critics. Your movie will be profitable if and only if each reviewer gives either a score of 2 or a score of 3. **Cannot classify**
- (c) Your reviewers have weird but different tastes. Your movie will be profitable if and only if both reviewers agree. **Cannot classify**