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# Optimal fuzzy based economic emission dispatch of combined heat and power units using dynamically controlled Whale Optimization Algorithm

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## HIGHLIGHTS

- Multi-objective combined heat and power economic emission dispatch is studied.
- A novel dynamically controlled whale optimization algorithm is employed.
- Conflicting objectives of fuel cost and emissions are handled using Fuzzy Framework.
- It is tested on the latest CEC test functions and three MO-CHPEED case studies.
- The results obtained validate the new proposed method.

# ARTICLE INFO

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# ABSTRACT

The significance and purpose of this multi-objective Combined Heat and Power Economic Emission Dispatch (MO-CHPEED) problem aims to determine the optimal generator output of the co-generation systems, in which two conflicting objectives of the fuel cost and mass of emissions are to be simultaneously minimized. The nonlinear and nonconvex nature of the objective functions needs a good optimization technique to handle it. This paper proposes a Dynamically Controlled Whale Optimization Algorithm (DCWOA) to solve the multi-objective non-convex MO-CHPEED problem in fuzzy environment. The proposed DCWOA is an improved variant of the traditional WOA method by adding dynamically controlled constriction function. Both the conflicting objectives of fuel cost and mass of emissions are handled using Fuzzy Framework. To highlight the performance of the proposed technique, it is tested on the latest CEC test functions and three different MO-CHPEED case studies. The results obtained by proposed DCWOA after 100 independent trails on latest CEC test functions and compared with latest different published methods show the effectiveness and robustness of the proposed method for getting better average and STD values. Moreover, proposed DCWOA is also tested on different dimensioned MO-CHPEED test functions after 100 independent trails and compared with latest techniques. Again the most compromise

Abbreviations: CHP, Combined Heat and Power; EED, Economic Emission Dispatch; CHPEED, CHP Economic Emission Dispatch; MO-CHPEED, Multi-objective CHPEED; ACO, Ant Colony Optimization; GSA, Gravitational Search Algorithm; KHA, Krill Herd Algorithm; PSO, Particle Swarm Optimization; TVAC-PSO, Time Varying Acceleration Coefficients PSO; GA, Genetic Algorithm; SARGA, Self-Adaptive Real Coded GA; HS, Harmony Search; TLBO, Teaching Learning Based Optimization; DE, Differential Evolution; GSO, Group Search Optimization; CS, Cuckoo Search; BCO, Bee Colony Optimization; IWO, Invasive Weed Optimization; EMA, Exchange Market Algorithm; GWO, Grey Wolf Optimization; NSGA-II, Non dominated Sorting GA-II; SPEA 2, Strength Pareto Evolutionary Algorithm-2; NBIM, Normal Boundary Intersection Method; MOPSO, Multi Objective PSO; IDBEA, Indicator & crowding distance-based Evolutionary Algorithm; FOR, Feasible Operating Region; WOA, Whale Optimization Algorithm; FDO, Fitness Dependent Optimizer; DA, Dragonfly Algorithm; SSA, Salp Swarm Optimization; RVAF, Randomly Varying Acceleration Function; LVAF, Linearly Varying Acceleration Function; EVAF, Exponentially Varying Acceleration Function.

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#### Nomenclature

 $N_{\rm E}$ number of electrical generator units number of CHP generator units  $N_{C}$ number of heat only generator units  $N_H$  $C_r(P_r^E)$ fuel cost of the  $r^{th}$  electrical generator units C<sub>s</sub>(P<sub>s</sub> C<sub>s</sub> H<sub>s</sub> C<sub>s</sub>) fuel cost of the s<sup>th</sup> CHP generator units  $C_t(H_t^{H})$  fuel cost of the  $t^{th}$  heat only generator  $u_r$ ,  $v_r$ ,  $w_r$  fuel cost coefficients of  $r^{th}$  electrical generator unit valve point fuel cost coefficients of  $r^{t\bar{h}}$  electrical generator  $x_r, y_r$  $u_s$ ,  $v_s$ ,  $w_s$ ,  $x_s$ ,  $y_s$  cost coefficients of the  $s^{th}$  CHP unit  $u_C$ ,  $v_C$ ,  $w_{C_3}$  cost coefficients of the  $t^{th}$  heat units total SO<sub>v</sub> and NO<sub>v</sub> emission  $a_p, b_p, c_p, d_p, f_r$  emission coefficients for the electrical generator units  $g_s$ ,  $h_s$ emission coefficients for CHP systems

emission coefficients for heat only generator units

 $E_c$  total CO<sub>2</sub> emission

 $l_r,\,k_s$  and  $m_t$  emission coefficients for the electrical generator, CHP

and heat only generator units respectively

fitness<sub>1</sub> individual fitness of cost objective fitness<sub>2</sub> individual fitness of emission objective

fitness overall compromise fitness

 $B_{rs}$ ,  $B_{0r}$ ,  $B_{00}$  loss coefficients

 ${
m P_r^{Emin}}_{,} {
m P_r^{Emax}}$  minimum & maximum outputs of the  $r^{th}$  electrical generator unit in MW

 $P_s^{Cmin}(H_s^{\ C}), P_s^{\ Cmax}(H_s^{\ C}) \ H_s^{\ Cmin}(P_s^{\ C}), \ H_s^{\ Cmax}(P_s^{\ C})$  boundary limits of feasible operating region of the  $s^{th}$  CHP unit

 $H_t^{Hmin}$ ,  $H_t^{Hmax}$  minimum and maximum outputs of the  $t^{th}$  heat

generator unit.

 $\overrightarrow{\textit{Pos}_{\textit{rand}}}$  a randomly chosen position vector from the current

solution

#### 1. Introduction

 $i_b, j_t$ 

#### 1.1. Introduction to CHP systems

Combined Heat and Power (CHP) units are being popular because of its capability to simultaneously produce electrical and thermal energy, provide economic benefits and reduce environmental emission [1,2]. This CHP system has higher efficiency in the range of 80 to 85% with respect to thermal plants and boilers units [3,4]. Because of this, the overall efficiency is of the system increased by utilizing the waste heat generated [5]. Keeping in view all the benefits of CHP system, it is important to properly schedule these CHP units [6,7]. This is due to the optimal scheduling of CHP system with thermal and boiler units are demanding, called as Combined Heat and Power Economic dispatch (CHPED) and Combined Heat and Power Economic Emission Dispatch (CHPEED) [8 9]. The objective of the CHPED problem is to minimize the cost, whereas the objective of CHPEED problem is to minimize the cost as well as simultaneously minimize the emission, respectively. These two objectives are conflicting in nature and when non convexity, nonlinearity and other practical constraints are considered then this multi-objective problem becomes complex. Therefore, for solving this complex problem with various related constraint, a power optimization technique is needed which is capable to provide best compromise solution.

In last years, many different methods have been implemented to solve CHPEED problem. Some of them are mentioned below: Shaabani et al. [10] solved the stochastic multi-objective CHP units problem by using time-varying acceleration particle swarm optimization (TVAC-PSO). In addition, Monte Carlo method is also considered for solving the stochastic model to understand the risk and uncertainty of the model. Basu [11] studied the non-linear bounded multi-objective problem by applying NSGA-II method. The NSGA-II provided better quality solution with less CPU time. Ahmadi et al. [12] implemented Normal-Boundary Intersection and TOPSIS decision-making method to solve multi-objective combined heat and power Problem. Wang and Singh [13] suggested an IPSO to solve deterministic and stochastic CHP dispatch problem. Sundaram [14] introduced a hybrid NSGA-II and MOPSO method, in which the exploration was done using NSGA-II, while the exploitation phase was done by the MOPSO. In addition, to ensure that

the search space is limited to the linear and nonlinear constraints and the Feasible Operation Region (FOR) of the CHP units, an effective constraint handling strategy is proposed. Sun et al. [15] proposed the IDBEA to provide a solution for the MO-CHPEED conundrum. The objective of the mass of emission, in the form if environmental protection, is given more weightage. The IDBEA is evaluated on standard four, five and seven-unit test systems for validation purposes. A two-stage approach is proposed Li et al. [16], in which multi-objective optimization is combined with integrated decision making. A powerful  $\theta$ -Dominance based Evolutionary Algorithm ( $\theta$ -DEA) forms the first stage of the proposed approach. This is used to find multiple Pareto-optimal solutions of the model. Later in the second stage, Fuzzy clustering separates the obtained solutions into different clusters. From this, by the assessment of the relative projections of the solutions of the same cluster, the best compromised solution is identified by using Grey Relation Projection.

The Whale Optimization Algorithm (WOA) was introduced in 2016 by Mirjalili and Lewis [17], which is a bio-inspired optimization algorithm based on the hunting pattern of humpback whales. The WOA has attracted attention of researcher and used for solving the single objective CHP economic dispatch problem in [18,19]. The basic WOA provides better quality solution for small to medium test system with faster convergence. Moreover, for multi-model large scale complex problem, basic WOA is not proving best quality solution because of premature convergence with slow convergence [20-22]. This is because of a disparity between local exploration and global exploitation in basic WOA [23]. In the literature, different modifications have been done by many researchers to improve the performance of WOA by proper balancing of exploration and exploitation phase. Hongping et al. [24,25] introduced a new inertia weight control parameter same as in PSO and obtained an Improved Whale Optimization Algorithm (IWOA). This control parameter was used to tune the influence of current best solution. Kaveh et al. [26] proposed enhanced whale optimization algorithm (EWOA) to improve convergence speed, reliability, and accuracy of solution. Majdi et al. presented [27] Hybrid Whale Optimization Algorithm for solving feature selection problem. In Hybrid Whale Optimization Algorithm, Simulated Annealing (SA) algorithm was combined with WOA algorithm for improving the best solution in each iteration. The main aim of incorporating Simulated Annealing (SA) algorithm with WOA was to improve the exploitation phase [27]. Ling et al. [20]

proposed Levy Flight Trajectory-Based Whale Optimization Algorithm for solving Global Optimization problem. The proposed method was efficient to maintain a better trade-off between the exploration and exploitation of the WOA. Oliva et al. [28] introduced improved Chaotic Whale Optimization Algorithm (CWOA) for solving parameter estimation of solar photovoltaic cells. In this CWOA method, chaotic maps were used for computation and automatic selection the internal parameters of the method. Mohamed Abd El Aziz et al. [29] proposed a hybrid method consisting of Whale Optimization Algorithm (WOA) and Moth-Flame Optimization (MFO) to obtain the optimal multi-level thresholds for image segmentation. Aljarah et al. [30] employed a new MLP training method based latest developed WOA for optimizing connection weights in neural networks. Abdel-Basset et al. [31] introduced a hybrid whale optimization method with local search strategy for solving permutation flow shop scheduling problem. Kaur et al. [21] proposed Chaotic WOA (CWOA) method by considering various chaotic maps. These chaotic maps were useful for fine tuning the main parameters of WOA, and for maintaining the exploration and exploitation of the proposed CWOA method. Al-Zoubi [32] utilised a hybrid method consisting of Support Vector Machines and WOA for solving the task of identifying spammers in online social networks problem. Xiong et al. [22] proposed an improved whale optimization algorithm (IWOA) for accurately extracting the parameters of different solar photovoltaic model problems. This IWOA method has adopted two prey-searching strategies to effectively maintain the exploration and exploitation balancing for improving the performance of WOA. Bansal et al. employed Grey Wolf Optimizer [33], and Jadhav et al. [34] proposed hybrid method consisting of grey wolf optimizer with whale optimization for solving optimal data clustering problem. Mohamed Abdel-Basset et al. proposed an IWOA for solving 0-1 knapsack problem with different scales.

Dynamically controlled constriction function is used in the proposed DCWOA to fine tune the performance of the WOA, which intend to provide an optimal solution for the economic emission scheduling conundrum of co-generation systems. This constriction function enhances the performance of the basic WOA in terms of convergence, maintaining the balance between exploration and exploitation throughout the search process and improving its efficiency. The proposed DCWOA, when is compared to the basic WOA, has proper balancing of both the exploration and the exploitation phases throughout the search process. First, the performance of DCWOA is tested on the latest CEC benchmark functions. Results obtained by DCWOA after 100 different trials on these test functions are compared with latest published results. The results obtained by the proposed DCWOA technique are compared with basic WOA, and some of the latest published techniques. The comparison results show that proposed DCWOA obtains better quality fitness for conflicting objectives.

The main scientific contributions of this paper are:

- DCWOA is proposed to solve Multi-objectives CHPEED problem with Trapezoidal Fuzzy function, being a new method applied in this problem.
- This DCWOA is improved by adding dynamically controlled constriction function, which has not been found in the literature.
- DCWOA is tested on latest CEC benchmark functions. The results obtained by DCWOA on several standard benchmark functions are compared with latest published results and found to be better.
- DCPSO are tested on different case studies of the Multi-objective CHP Economic Emission Dispatch problem consisting of several related constraints with basic WOA.
- Results obtained by DCWOA after 100 unbiased trials are validated by comparing them with results obtained by basic WOA, and recently

published results by different latest methods. The comparison results show that proposed DCWOA performs much better than other latest methods.

The paper is organized as: the problem formulation is explained in Section 2; The proposed Dynamically Controlled Whale Optimization Algorithm is explained in Section 3; In section 4, the proposed method is applied on several latest CEC functions; Section 5 presents the simulation results and discussion on MOCHPEED case studies, and; The conclusion is presented in Section 6.

#### 2. Problem formulation

This section describes the problem formulation; first for a single objective CHP Economic Dispatch, then a single objective CHP Emission Dispatch problem and, finally, combining them into a MO-CHPEED problem.

# 2.1. CHP economic dispatch problem

The objective of CHP economic dispatch problem is to minimize the total cost of CHP system by optimal allocation of output of electrical and heat units with satisfying all the related constraints [35]. The mathematical equation of total cost, referred from [36], is given by equation (1):

$$Cost_{fuel} = \text{Min} \left[ \sum_{r=1}^{N_E} C_r (P_r^E) + \sum_{s=1}^{N_C} C_s (P_s^C, H_s^C) + \sum_{t=1}^{N_H} C_t (H_t^H) \right] \$ / hour$$
(1)

The non-convex cost function of thermal generators is represented by equation (2) [37]:

$$C_r(P_r) = \left[ u_r (P_r^E)^2 + v_r P_r^E + w_r + |x_r \sin(y_r (P_r^{E_{min}} - P))| \right] \$/hour$$
 (2)

The quadratic cost function of the CHP units is given by equation (3) [38]:

$$C_{s}(P_{s}^{C}, H_{s}^{C}) = \left[u_{s}(P_{s}^{C})^{2} + v_{s}P_{s}^{C} + w_{s} + x_{s}(H_{s}^{C})^{2} + y_{s}H_{s}^{C} + z_{s}H_{s}^{C}P_{s}^{C}\right] \$/hour$$
(3)

where  $u_s$ ,  $v_s$ ,  $w_s$ ,  $x_s$ ,  $y_s$  and  $z_s$  are the cost coefficients of the  $s^{th}$  CHP unit. And the cost function for heat units referred from [37] is written by equation (4):

$$C_{t}(H_{t}^{H}) = \left[u_{t}(H_{t}^{H})^{2} + v_{t}H_{t}^{H} + w_{t}\right] \$/hour$$
 (4)

where  $u_t$ ,  $v_t$  and  $w_t$  are the cost coefficients of the  $t^{th}$  heat only units.

## 2.2. CHP emission dispatch problem

The objective of CHP emission dispatch problem is stated by equation (5) [10]:

 $Min[Mass_{emission}]$ 

where,

$$Mass_{emission} = E_s + E_c (5)$$

The mathematical modelling is given by equations (6) and (7):

$$E_{s} = \left[ \sum_{r=1}^{N_{E}} \left[ a_{r} + b_{r} P_{r} + c_{r} P_{r}^{2} + d_{r} e^{(f_{r} P_{r})} \right] + \sum_{s=1}^{N_{C}} (g_{s} + h_{s}) P_{s} + \sum_{t=1}^{N_{H}} (i_{t} + j_{t}) H_{t} \right] kg/hour$$
(6)

$$E_{c} = \left[ \sum_{r=1}^{N_{E}} l_{r} P_{r} + \sum_{s=1}^{N_{C}} k_{s} P_{s} + \sum_{t=1}^{N_{H}} m_{t} H_{t} \right] kg/hour$$
 (7)

# 2.3. Multi-objective combined heat and power economic emission dispatch (MO-CHPEED)

The objective of MO-CHPEED is to handle two conflicting objectives cost and emission in such a way to get best compromise solution. The expression is given in [10] and expressed by equation (8).

$$Min[Cost_{fuel}, Mass_{emission}]$$
 (8)

In the literature, several methods are available to solve this multiobjective problem, being the objective to obtain the best compromised solution, namely Weighting and price penalty factor [39–41], Analytic Hierarchy Process [42], ε-constraint method [43], TOPSIS [44,45], Fuzzy Framework [46], etc. [47,48]. For solving multi-objective problems using weighting and price penalty factor and Analytic Hierarchy Process, weights for each objective should be defined. For getting best compromise solution between two conflicting objectives, a fitness function as in [46] is given by equation (12):

$$Max fitness = \sqrt{fitness_1 \times fitness_2}$$
 (12)

#### 2.4. Constraints

For solving MOCHPEED problem following constraints are considered as described in [49]:

A. Equality Constraints.

$$\sum_{r=1}^{N_E} P_r^E + \sum_{s=1}^{N_C} P_s^C = P_{demand} + P_{Loss}$$
 (13)

$$\sum_{s=1}^{N_C} H_s^C + \sum_{t=1}^{N_H} H_t^H = H_{demand}$$
 (14)

where

$$P_{Loss} = \sum_{r=1}^{N_E} \sum_{r}^{N_E} P_r^E B_{rs} P_s^E + \sum_{r}^{N_E} \sum_{r}^{N_C} P_r^E B_{rs} P_s^C + \sum_{r=1}^{N_C} \sum_{s=1}^{N_C} P_r^C B_{rs} P_s^C + \sum_{r=1}^{N_E} B_{0r} P_r^E + \sum_{r=1}^{N_C} B_{0r} P_s^C + B_{00}$$

$$(15)$$

However, determination of the value of w for conflicting objectives are the key for handling multi-objective problem [41]. In ε-constraint method [43] single objective is considered and other objective is treated as constraints. Ref. [44,45] solved multi-objective problem using TOP-SIS method to obtain the optimal design scheme. Ref. [44] developed a fuzzy Mahalanobis-Taguchi system method for solving multi-objective problem. Among these methods, Fuzzy Framework is intensively used in various fields [41] and effectively handle two objectives, which are conflicting in nature and, finally, get best optimal compromise solution [46]. Also, there is no need to assign weights for objectives. This paper uses the trapezoidal Fuzzy Framework due to its ease of implementation, where the mathematical equations are referred from Vinay K. Jadoun et. al. [46]. The trapezoidal fuzzy membership function gives a linear and continuous relation between the fuzzy index and fuzzy membership function. After this, it allocates any membership value in the range 0 to 1 to these objectives, respectively as per the equations (9)–(11).

Mathematically, it is stated as [46]:

$$fitness_i = \begin{cases} 1, & v_i \le v_{\min i} \\ Rx_i + S, & v_{\min i} \le v_i \le v_{\max i} \\ 0, & v_{\max i} \le v_i \end{cases}$$
 (9)

 $v_{min\ i}$  and  $v_{max\ i}$  are the minimum and maximum limits of the required objective, respectively and can be changed based on the preference of various operators. The coefficients R and S are computed based upon these minimum and maximum bounds of the fuzzy index  $v_i$  and can be given as in [46]:

$$R = -\frac{1}{v_{\text{total}}} \tag{10}$$

$$S = \frac{v_{\text{max}i}}{v_{\text{max}i} - v_{\text{min}i}} \tag{11}$$

B. Inequality Constraints.

$$P_r^{E_{min}} \le P_r^E \le P_r^{E_{max}} \tag{16}$$

$$P_s^{C_{min}}(H_s^C) \le P_s^C \le P_s^{C_{max}}(H_s^C) \tag{17}$$

$$H_{s}^{C_{min}}(P_{s}^{C}) \le H_{s}^{C} \le H_{s}^{C_{max}}(P_{s}^{C}) \tag{18}$$

$$H_t^{H_{min}} \le H_t^H \le H_t^{H_{max}} \tag{19}$$

where  $P_r^{E_{min}}$  and  $P_r^{E_{max}}$  are the minimum and maximum outputs of the r<sup>th</sup> electrical generator unit in MW,  $P_s^{C_{min}}(H_s^C)$ ,  $P_s^{C_{max}}(H_s^C)$ ,  $H_s^{C_{min}}(P_s^C)$  and  $H_s^{C_{max}}(P_s^C)$  are the linear inequalities which define the feasible operating region of the s<sup>th</sup> CHP unit and  $H_t^{H_{min}}$  and  $H_t^{H_{max}}$  are the minimum and maximum outputs of the t<sup>th</sup> heat generator unit.

# 2.5. Proposed Dynamically Controlled Whale Optimization Algorithm (DCWOA)

In this section, the limitations of the basic WOA are discussed and, based on these, some improvements are also suggested. In basic WOA, the updated position is regulated by multiplication of coefficient vector  $(\overrightarrow{M})$  and distance of the  $\mathbf{r}^{\text{th}}$  whale to the prey  $(\overrightarrow{P})$ , which is subtracted from current position. Whereas  $\overrightarrow{P}$  is dependent upon coefficient vector  $(\overrightarrow{N})$  and from equation (26), it can be said that  $\overrightarrow{N}$  is dependent upon  $\overrightarrow{rand}$ , which is a random vector. Moreover, current best solution is dependent upon coefficient vector  $(\overrightarrow{M})$  It can be observed that  $\overrightarrow{M}$  is dependent upon  $\overrightarrow{m}$ , which is decreasing linearly with respect to iteration from 2 to 0. This may cause uncontrolled updated solutions during the

iterations. It may also cause an unbalance between exploration and exploitation during the search space. Further, it may cause poor convergence. For improving this, an inertia weight  $\xi$  (itr) is introduced into WOA as in references [50,51] as used in PSO algorithm to tune the influence of current best solution. According to references [50,51], the basic WOA equations are modified and rewritten as:

In Encircling prey, the updated method is represented by equations (20) and (21) as in [24,25]:

$$\overrightarrow{P} = \left| \overrightarrow{N} \cdot \xi(itr) \cdot \overrightarrow{Pos}^*(itr) - \overrightarrow{P}(itr) \right|$$
 (20)

$$\overrightarrow{Pos}^{*}(itr+1) = \xi(itr).\overrightarrow{Pos}^{*}(t) - \overrightarrow{M}.\overrightarrow{P}$$
(21)

In exploitation phase, the updated equations are given by (22) and (23) [24,25]:

$$\overrightarrow{Pos}(itr+1) = \overrightarrow{P} \cdot e^{bl} \cdot \cos(2\pi l) + \xi(itr) \cdot \overrightarrow{Pos}^*(itr)$$
 (22)

where  $\overrightarrow{P}$  denotes the distance separating the r<sup>th</sup> whale and the prey (best solution at this stage).

$$\overrightarrow{P} = \left| \xi(itr) \cdot \overrightarrow{Pos}^{*}(t) - \overrightarrow{Pos}(t) \right|$$
 (23)

In the proposed WOA, the 50% probability is considered, which is the same as in basic WOA. The mathematical is modified as equation (24) [24,25]:

$$\overrightarrow{Pos}(itr+1) = \begin{cases} \overrightarrow{\xi(itr)}.\overrightarrow{Pos}^*(itr) - \overrightarrow{M}.\overrightarrow{P}, prob < 0.5\\ \overrightarrow{P}.e^{bl}.cos(2\pi l) + \xi(itr).\overrightarrow{Pos}^*(itr), prob \ge 0.5 \end{cases}$$
(24)

The mathematical model for search for paying (exploration phase) are referred from [14,15], given by equations (25) and (26):

$$\overrightarrow{P} = \left| \xi(itr).\overrightarrow{N}.\overrightarrow{Pos_{rand}} - \overrightarrow{Pos} \right|$$
 (25)

$$\overrightarrow{Pos}(itr+1) = \xi(itr).\overrightarrow{Pos_{rand}} - \overrightarrow{M}.\overrightarrow{P}$$
 (26)

where  $\overline{Pos_{rand}}$  is a random position vector (a random whale) chosen from the current population.

For improving and enhancing the basic WOA performance, dynamically controlled constriction function  $(\xi)$  is considered in the proposed DCWOA. In this proposed variant, dynamically controlled constriction function  $(\xi)$  is varied exponentially as in reference [51], given by equation (27):

$$\xi(itr) = \exp(-\lambda \ln \tau_d) \tag{27}$$

where  $\lambda = \frac{itr}{itr_{max}}$ ,  $itr_{min} \le itr \le itr_{max}$ ; and  $\tau_d$  is the ratio of maximum and minimum bounds of the constriction function.

The flowchart of the proposed DCWOA is presented in Fig. 1.

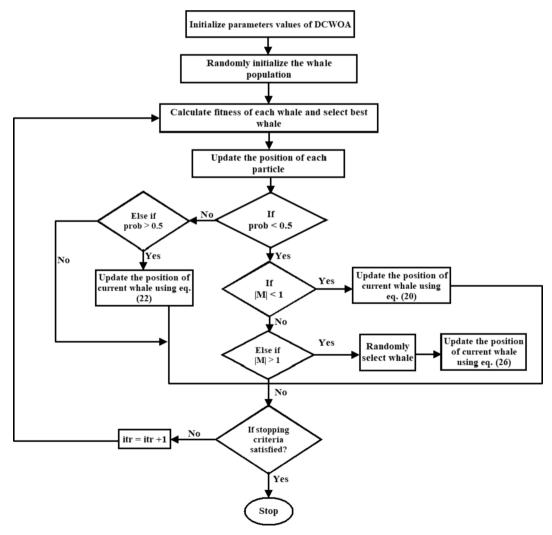


Fig. 1. Flowchart of the proposed DCWOA.

#### 3. Experimental tests and setup

In the existing literature, classical test functions, or IEEE Congress on Evolutionary Computation (CEC) Test Functions, or both, have been used to test the performance of heuristic methods. In the recent past, Black Widow Optimization Algorithm [52], Fitness Dependent Optimizer (FDO) [53], Dragonfly Algorithm (DA) [53], Salp Swarm Optimization (SSA) [53], etc. heuristic methods have been developed. Among these techniques, Black Widow Optimization Algorithm [52], is tested on classical test functions whereas FDO [53], DA [53], SSA [53], etc. are on IEEE Congress on Evolutionary Computation (CEC) Test Functions. In this paper, the proposed DCWOA is tested on latest 10 IEEE CEC Test Functions to determine its performance. These functions are known as "The 100-Digit Challenge", which are to be used in the annual optimization competition taken from references [53,54].

Of these 10 functions, CEC01 to CEC03 are non-rotated functions, while the rest are rotated functions, where the functions are scalable. Note that apart from CEC01 to CEC03, all the other functions are 10-dimensional minimization problems in the range of [-100, 100]. The proposed DCWOA is compared with FDO, DA, SSA and standard WOA. These 4 algorithms are chosen because: they are Swarm-based algorithms, are well cited and popular, have a well-established performance on latest CEC benchmark test functions and in solving actual problems.

The parameters for DCWOA are: Population of whales =100 and Maximum number of iterations (itrmax) =1000. The computer specifications are, Processor – Intel i5 7th Generation @ 2.50 GHz, RAM – 8 GB and Storage Capacity -1 TB. These benchmark functions are simulated on MATLAB 2018b.

The optimization results obtained using the proposed DCWOA and the other four algorithms are compared in Table 1 based on the average value and the standard deviation (STD).

As seen in Table 1, for all the 10 functions, the results, i.e., average and STD given by DCWOA are better or almost equal than FDO [53], DA [53], SSA [53] and standard WOA [53]. This shows the effectiveness of the modifications suggested in the proposed DCWOA. Another point to note is that for CEC02, CEC03, CEC09 and CEC10 have a 0 standard deviation when solved using DCWOA, which implies that in all the trials it has the same result and does not have any chance of further developments and improvement.

#### 3.1. Quantitative measurement metrics

For a further analysis of the proposed DCWOA, the fitness quantitative metric is used as shown in Fig. 2. Here, two sample convergence graphs are shown: one from CEC01 to CEC03 (CEC03 is selected) as they have a different dimension and range compared to CEC04 to CEC10 (CEC09 is selected); the second convergence graph is selected from CEC04 to CEC10. The parameters setting used are: Population of whales (search agents) = 10 and Maximum number of iterations (itrmax) = 200.

Fig. 2 shows the fitness metric, in which the average and best fitness of all the DCWOA whales (search agents) decrease drastically over the

period of iterations. This highlights the point that the proposed algorithm improves the overall whale fitness as well as improves the global best search agent.

# 4. Application of proposed DCWOA in the CHPEED problem

In this paper, two conflicting objectives, cost and emission, are considered. The tendency of these conflictive objectives are: if one objective is minimized, then the other ends up being maximized. Therefore, it is necessary to balance both the objectives. For balancing the objectives, Fuzzy Framework is used, in which equal weightage is given to both the objectives. For evaluating the cost and emission objectives using Fuzzy Framework as a single objective, the bottom and topmost boundary values for cost and emission are required.

There are three steps considered to solve this multi-objective problem: first step is to find the optimal dispatch schedule considering the problem as a single objective economic dispatch problem. This results in obtaining the schedule for the minimized fuel cost. Using this schedule, the lower bound of the fuel cost objective and the upper bound of the mass of emission objective is obtained; The second step is to find the optimal dispatch schedule considering the problem as a single objective emission dispatch problem. These results are obtained for the schedule to minimized mass of emission of the system. Using this schedule, the lower bound of the mass of emission objective and the upper bound of the fuel cost objective are obtained; The third, and the final, step involves using these boundary values of fuel cost and mass of emission objectives to find the compromised solution for the multi-objective CHPEED problem.

Proposed DCWOA, with the basic WOA, is tested on three different case studies. For the single objective economic dispatch and emission dispatch problems, as mentioned above, the schedule obtained using DCWOA for best cost and best emission is presented. Further, the corresponding cost and emission boundary values for applying the Fuzzy Framework, which are obtained using DCWOA, are presented and used for solving the multi-objective CHPEED problem.

The summary of the three case studies is mentioned in Table 2. The parameters for the basic WOA and proposed DCWOA are: Population of whales = 100 and Maximum number of iterations ( $itr_{max}$ ) = 100;

# 4.1. Case study 1

The different parameters values of CHPEED problem are referred from [55]. The demand of this case study for electrical and thermal energies are 300 MW and 150MWth, respectively. The statistical values calculated by DCWOA after 100 unbiased trails for solving individual objectives are presented in Table 3.

In this paper, for solving multi-objective problem, trapezoidal fuzzy framework is used. This trapezoidal membership function is bounded by minimum and maximum limit (between 0 and 1). For getting the values of these boundaries, limits for both conflicting objectives cost and emission, DCWOA is used to individually solve CHP economic dispatch

 Table 1

 Comparison based on Average and STD obtained using the proposed DCWOA of CEC Test Functions.

Test Function	Proposed Do	CWOA	FDO [53]		DA [53]		WOA [53]		SSA [53]	
	Average	STD	Average	STD	Average	STD	Average	STD	Average	STD
CEC01	4282.443	1480.054	4585.27	20707.627	46835.63	8992.755	411 × 10 <sup>8</sup>	$542 \times 10^8$	$605 \times 10^{8}$	$475\times10^8$
CEC02	3.893	0.0000	4.0	3.224E-9	18.3168	0.0419	17.3495	0.0045	18.3434	0.0005
CEC03	12.703	0.0000	13.702	1.649E-11	12.7024	$1.5  imes 10^{-12}$	13.7024	0.0	13.7025	0.0003
CEC04	33.569	10.206	34.084	16.529	103.3295	20.0041	394.6754	248.5627	41.6936	22.2191
CEC05	1.936	0.0289	2.139	0.0858	1.1773	0.0575	2.7342	0.2917	2.2084	0.1064
CEC06	11.109	0.485	12.133	0.6002	5.5465	$4.3 \times 10^{-8}$	10.7085	1.0325	6.0798	1.4873
CEC07	115.376	12.861	120.486	13.594	898.5188	4.0239	490.6843	194.8318	410.3964	290.5562
CEC08	5.905	0.375	6.102	0.7570	6.2109	0.0017	6.909	0.4269	6.37	0.5862
CEC09	1.911	0.0000	2.0	1.592E-10	2.6011	0.2333	5.9371	1.6566	3.6704	0.2362
CEC10	2.225	0.0000	2.718	8.882E-16	20.0507	0.0709	21.2761	0.1111	21.04	0.078

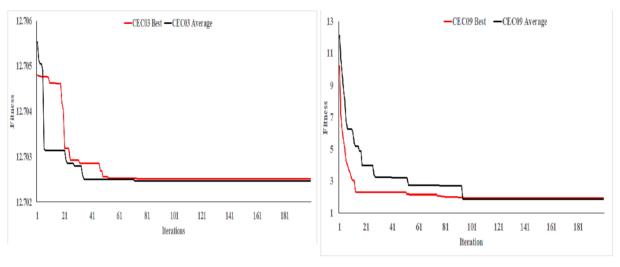


Fig. 2. Average and Best Fitness of DCWOA's whales (search agents) for CEC03 and CEC09 Functions.

 Table 2

 Summary of different case studies consider to solve multi-objective problem.

Case Study	No. of power only units	No. of co-generation units	No. of heat only units	Total no. of units	Power Demand (MW)	Heat Demand (MWth)	Emission Considered
1	1	3	1	5	300	150	SO <sub>2</sub> & NO <sub>x</sub>
2	4	2	1	7	600	150	SO <sub>2</sub> & NO <sub>x</sub>
3	4	2	1	7	600	150	SO <sub>2</sub> , NO <sub>x</sub> & CO <sub>2</sub>

 $\begin{tabular}{ll} \textbf{Table 3} \\ For individual objective Quality Solution Obtained by DCWOA for case study 1. \\ \end{tabular}$ 

Outputs	Optimal CHP Economic Dispatch	Outputs	Optimal CHP Emission Dispatch
P1 (MW)	135.000000	P1 (MW)	34.999867
P2 (MW)	40.759130	P2 (MW)	125.799254
P3 (MW)	19.239881	P3 (MW)	34.200671
P4 (MW)	105.000000	P4 (MW)	104.999333
H2 (MWth)	73.616497	H2 (MWth)	133.864434
H3 (MWth)	36.719346	H3 (MWth)	16.135658
H4 (MWth)	0.000000	H4 (MWth)	0.000000
H5 (MWth)	39.664524	H5 (MWth)	0.000000
Minimum Cost (\$/hour)	13672.7969	Minimum	1.174882
(\$/110ur)		Emission (kg/ hour)	
Average Cost (\$/hour)	13673.5827	Average Emission (kg/hour)	1.174883
Maximum Cost (\$/hour)	13706.4038	Maximum Emission (kg/	1.174887
STD	0.000276	hour) STD	0.000001

**Table 4**The boundaries limits for Trapezoidal membership functions for Optimal CHP Economic and Emission Dispatch using DCWOA for case study 1.

Outputs	Optimal CHP Economic Dispatch	Optimal CHP Emission Dispatch
Cost (\$/hour)	13672.7969	17210
Emission (kg/ hour)	12.0554	1.174882

and CHP emission dispatch, respectively. These boundaries, limits for both conflicting objectives cost and emission are presented in Table 4.

The optimal generators schedule of multi-objective CHPEED for WOA, and proposed DCWOA for case study 1 are mentioned in Table 5.

The best compromise solution of MOCHPEED problem provided by

**Table 5**Optimal Schedule of Multi-objective CHPEED for WOA and proposed DCWOA for case study 1.

Outputs	WOA	DCWOA
P1 (MW)	86.7208	85.86739
P2 (MW)	74.6079	75.05647
P3 (MW)	33.6703	34.07519
P4 (MW)	105	105
H2 (MWth)	75.9014	76.94991
H3 (MWth)	32.5944	32.75026
H4 (MWth)	0.0017	0
H5 (MWth)	41.5025	40.30031
Total Power (MW)	299.999	299.999
Total Heat (MWth)	150	150.000

**Table 6**Multi-objective CHPEED results comparison obtained by proposed DCWOA with WOA and other published methods for case study 1.

Methods	Cost (\$/hour)	Emission (kg/hour)	Fitness
GWO [56]	15243.7	5.4	0.5825
NSGA-II [55]	15008.70	6.0563	0.5857
SPEA 2 [12]	14964.30	6.3667	0.5760
Hybrid of NSGA-II and MOPSO [14]	14,909	5.8794	0.6077
IDBEA [15]	15,182	5.2	0.5981
BCS [16]	15137.3	5.1	0.6127
WOA	15092.97	5.2188	0.6078
DCWOA	15121.40	5.1232	0.6133

DCWOA with WOA, and other published methods are given in Table 6. It can be observed from Table 6 that if cost of some technique is low then its corresponding emission is high. This is due to the conflicting nature of both cost and emission objectives, in which if one objective is minimized, the other objective ends up being maximized. This is reason to choose fuzzy framework in this paper. In this fuzzy environment, fitness of each method is calculated based on their best optimal solutions. In



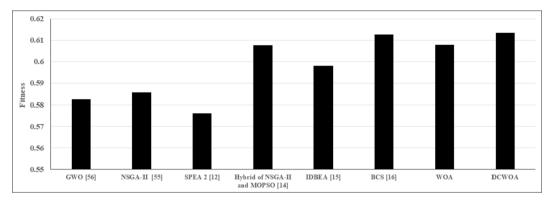


Fig. 3. Fitness comparison for Case Study 1.

Table 6, the fitness of each method is mentioned. It can be observed that the fitness calculated by DCWOA is much better than WOA and other published methods. Based on this, it can be said that DCWOA provide best compromise solution.

In this case, WOA has a fitness of 0.607766, being slightly low as WOA lacks to balance between exploration and exploitation phases well. Therefore, with the addition of dynamically controlled constriction function, the proposed DCWOA obtained the highest fitness value of 0.613352. It can be observed that DCWOA generates better fitness compared with basic WOA and other methods. It is the one which is able to get balance between the cost and pollutant emission objectives, reactively, and it outperforms the others. The average computational time take by proposed DCWOA is 0.14 s.

The fitness values, as shown in Table 6, are shown in Fig. 3. This visual representation helps to understand the magnitude of improvement achieved by modifying WOA by using the dynamically controlled constriction function.

# 4.2. Case study 2

The different coefficient values and electrical and thermal demands of this case study for handling MOCHPEED are referred from reference [55]. In addition, the B-coefficients values for calculating power loss if given as follows:

 $B = [49 \ 14 \ 15 \ 15 \ 20 \ 25; 14 \ 45 \ 16 \ 20 \ 18 \ 19; 15 \ 16 \ 39 \ 10 \ 12 \ 15; 15 \ 20 \ 10 \ 40 \ 14 \ 11; 20 \ 18 \ 12 \ 14 \ 35 \ 17; 25 \ 19 \ 15 \ 11 \ 17 \ 39] * 10^{-7}.$ 

$$\begin{split} B_0 &= \text{[-0.3908-0.1297 0.7047 0.0591 0.2161-0.6635]} * 10^{-3}, \\ B_{00} &= 0.056. \end{split}$$

**Table 7**For individual objective Quality Solution Obtained by DCWOA for case study 2.

Outputs	Optimal CHP Economic Dispatch	Outputs	Optimal CHP Emission Dispatch
P1 (MW)	59.093608	P1 (MW)	41.525334
P2 (MW)	98.224108	P2 (MW)	47.946629
P3 (MW)	112.688731	P3 (MW)	58.897650
P4 (MW)	209.838039	P4 (MW)	79.641162
P5 (MW)	81.000000	P5 (MW)	247.000000
P6 (MW)	40.000000	P6 (MW)	125.800000
H5 (MWth)	0.000000	H5 (MWth)	150.000798
H6 (MWth)	90.953797	H6 (MWth)	0.000000
H7 (MWth)	59.047166	H7 (MWth)	0.000000
Minimum Cost	9742.286240	Minimum	6.663086
(\$/hour)		Emission (kg/	
		hour)	
Average Cost	9858.342831	Average	6.869391
(\$/hour)		Emission (kg/	
		hour)	
Maximum Cost	10138.845755	Maximum	6.934071
(\$/hour)		Emission (kg/	
		hour)	
STD	0.007567	STD	0.013343

**Table 8**The boundaries limits for Trapezoidal membership functions for Optimal CHP Economic and Emission Dispatch using DCWOA for case study 2.

Outputs	Optimal CHP Economic Dispatch	Optimal CHP Emission Dispatch
Cost (\$/hour)	9742.286240	19,490
Emission (kg/ hour)	28.6073	6.663086

**Table 9**Optimal Schedule of Multi-objective CHPEED for WOA and proposed DCWOA for case study 2.

Control Variables	WOA	DCWOA
P1 (MW)	58.4263	54.286923
P2 (MW)	68.1347	65.847685
P3(MW)	94.5355	88.318419
P4 (MW)	124.9072	124.76644
P5(MW)	214.8449	227.63631
P6 (MW)	40	40
H3 (MWth)	0.0004	0
H4 (MWth)	81.7697	101.41286
H5 (MWth)	68.2299	48.587618
Total Power (MW)	600.8486	600.8558
Total Heat (MWth)	150	150.0005

**Table 10**Multi-objective CHPEED results comparison obtained by proposed DCWOA with WOA and other published methods for case study 2.

Methods	Cost (\$/hour)	Emission (kg/hour)	Fitness
GWO [56]	12974.1	18	0.5571
NSGA-II [55]	13433.19	25.8262	0.2818
SPEA 2 [12]	13448.95	25.7810	0.2837
IDBEA [15]	12957.2	17.3	0.5850
BCS [16]	13282.9	14.6561	0.6363
WOA	13024.54	14.6001	0.6328
DCWOA	13357.03	13.5827	0.6563

The quality solutions for solving individual objective obtained by DCWOA is given in Table 7.

The boundary values of each objective functions for fuzzy membership function are given in Table 8.

The optimal generators schedule of multi-objective CHPEED for WOA and proposed DCWOA are shown in Table 9.

The best compromise solution for MOCHPEED problem provided by basic WOA and proposed DCWOA with the results obtained by GWO [56], NSGA-II [55], SPEA-2 [12], IDBEA [15] and BCS-1 [16] are presented in Table 10.

Table 10 shows that among GWO [56], NSGA-II [55], SPEA-2 [12],



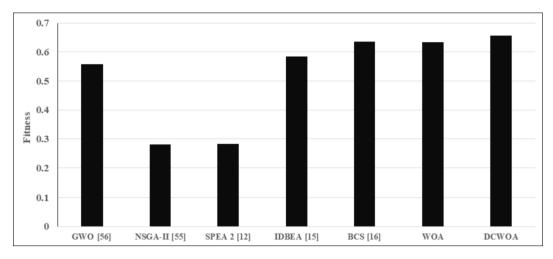


Fig. 4. Fitness comparison for Case Study 2.

**Table 11**For individual objective quality solution obtained by DCWOA for case study 3.

Outputs         Optimal CHP Economic Dispatch         Outputs         Optimal CHP Emission Dispatch           P1 (MW)         59.093608         P1 (MW)         74.995836           P2 (MW)         59.299601         P2 (MW)         112.612250           P3 (MW)         98.090388         P3 (MW)         128.501596           P4 (MW)         112.622919         P4 (MW)         163.723436           P5 (MW)         209.831412         P5 (MW)         81.000000           P6 (MW)         81.000000         P6 (MW)         40.00000           H5 (MWth)         40.000000         H5 (MWth)         79.205522           H6 (MWth)         0.000000         H6 (MWth)         70.794894           H7 (MWth)         88.413425         H7 (MWth)         0.000000           Minimum Cost         9794.718722         Minimum         47.718479           (\$/hour)         Emission (ton/hour)           Average Cost         9858.690531         Average Emission         48.070597           (\$/hour)         Maximum         48.531959           (\$/hour)         Emission (ton/hour)           Maximum         48.531959		<i>y</i> 1 <i>y</i>	•	<b>3</b>
P2 (MW) 59.299601 P2 (MW) 112.612250 P3 (MW) 98.090388 P3 (MW) 128.501596 P4 (MW) 112.622919 P4 (MW) 163.723436 P5 (MW) 209.831412 P5 (MW) 81.000000 P6 (MW) 81.000000 P6 (MW) 40.000000 H5 (MWth) 40.000000 H5 (MWth) 79.205522 H6 (MWth) 0.000000 H6 (MWth) 70.794894 H7 (MWth) 88.413425 H7 (MWth) 0.000000 Minimum Cost 9794.718722 Minimum 47.718479 (\$/hour) Emission (ton/hour)  Average Cost 9858.690531 Average Emission 48.070597 (\$/hour) (ton/hour) Maximum Cost 10067.554091 Maximum 48.531959 (\$/hour) Emission (ton/hour)	Outputs		Outputs	
P2 (MW) 59.299601 P2 (MW) 112.612250 P3 (MW) 98.090388 P3 (MW) 128.501596 P4 (MW) 112.622919 P4 (MW) 163.723436 P5 (MW) 209.831412 P5 (MW) 81.000000 P6 (MW) 81.000000 P6 (MW) 40.000000 H5 (MWth) 40.000000 H5 (MWth) 79.205522 H6 (MWth) 0.000000 H6 (MWth) 70.794894 H7 (MWth) 88.413425 H7 (MWth) 0.000000 Minimum Cost 9794.718722 Minimum 47.718479 (\$/hour) Emission (ton/hour)  Average Cost 9858.690531 Average Emission 48.070597 (\$/hour) (ton/hour) Maximum Cost 10067.554091 Maximum 48.531959 (\$/hour) Emission (ton/hour)				
P3 (MW) 98.090388 P3 (MW) 128.501596 P4 (MW) 112.622919 P4 (MW) 163.723436 P5 (MW) 209.831412 P5 (MW) 81.000000 P6 (MW) 81.000000 P6 (MW) 40.000000 H5 (MWth) 40.000000 H5 (MWth) 79.205522 H6 (MWth) 0.000000 H6 (MWth) 70.794894 H7 (MWth) 88.413425 H7 (MWth) 0.000000 Minimum Cost 9794.718722 Minimum 47.718479 (\$/hour) Emission (ton/hour)  Average Cost 9858.690531 Average Emission 48.070597 (\$/hour) (ton/hour) Maximum Cost 10067.554091 Maximum 48.531959 Emission (ton/hour)	P1 (MW)	59.093608	P1 (MW)	74.995836
P4 (MW) 112.622919 P4 (MW) 163.723436 P5 (MW) 209.831412 P5 (MW) 81.000000 P6 (MW) 81.000000 P6 (MW) 40.000000 H5 (MWth) 40.000000 H5 (MWth) 79.205522 H6 (MWth) 0.000000 H6 (MWth) 70.794894 H7 (MWth) 88.413425 H7 (MWth) 0.000000 Minimum Cost 9794.718722 Minimum 47.718479 (\$/hour) Emission (ton/hour)  Average Cost 9858.690531 Average Emission 48.070597 (\$/hour) Maximum Cost 10067.554091 Maximum 48.531959 (\$/hour) Emission (ton/hour)	P2 (MW)	59.299601	P2 (MW)	112.612250
P5 (MW) 209.831412 P5 (MW) 81.000000 P6 (MW) 81.000000 P6 (MW) 40.000000 H5 (MWth) 40.000000 H5 (MWth) 79.205522 H6 (MWth) 0.000000 H6 (MWth) 70.794894 H7 (MWth) 88.413425 H7 (MWth) 0.000000 Minimum Cost 9794.718722 Minimum 47.718479 (\$/hour) Emission (ton/hour)  Average Cost 9858.690531 Average Emission 48.070597 (\$/hour) (ton/hour) Maximum Cost 10067.554091 Maximum 48.531959 (\$/hour) Emission (ton/hour)	P3 (MW)	98.090388	P3 (MW)	128.501596
P6 (MW) 81.000000 P6 (MW) 40.000000 H5 (MWth) 40.000000 H5 (MWth) 79.205522 H6 (MWth) 0.000000 H6 (MWth) 70.794894 H7 (MWth) 88.413425 H7 (MWth) 0.000000 Minimum Cost 9794.718722 Minimum 47.718479 (\$/hour) Emission (ton/hour) Average Cost 9858.690531 Average Emission 48.070597 (\$/hour) (ton/hour) Maximum Cost 10067.554091 Maximum 48.531959 Emission (ton/hour)	P4 (MW)	112.622919	P4 (MW)	163.723436
H5 (MWth) 40.000000 H5 (MWth) 79.205522 H6 (MWth) 0.000000 H6 (MWth) 70.794894 H7 (MWth) 88.413425 H7 (MWth) 0.000000 Minimum Cost 9794.718722 Minimum 47.718479 (\$/hour) Emission (ton/hour)  Average Cost 9858.690531 Average Emission 48.070597 (\$/hour) (ton/hour)  Maximum Cost 10067.554091 Maximum 48.531959 (\$/hour) Emission (ton/hour)	P5 (MW)	209.831412	P5 (MW)	81.000000
H6 (MWth) 0.000000 H6 (MWth) 70.794894 H7 (MWth) 88.413425 H7 (MWth) 0.000000 Minimum Cost 9794.718722 Minimum 47.718479 (\$/hour) Emission (ton/hour)  Average Cost 9858.690531 Average Emission 48.070597 (\$/hour) (ton/hour)  Maximum Cost 10067.554091 Maximum 48.531959 (\$/hour) Emission (ton/hour)	P6 (MW)	81.000000	P6 (MW)	40.000000
H7 (MWth) 88.413425 H7 (MWth) 0.000000  Minimum Cost 9794.718722 Minimum 47.718479  (\$/hour) Emission (ton/hour)  Average Cost 9858.690531 Average Emission 48.070597  (\$/hour) (ton/hour)  Maximum Cost 10067.554091 Maximum 48.531959  (\$/hour) Emission (ton/hour)	H5 (MWth)	40.000000	H5 (MWth)	79.205522
Minimum Cost (\$/hour)         9794.718722         Minimum (\$/hour)         47.718479           Average Cost (\$/hour)         9858.690531         Average Emission (\$/hour)         48.070597           (\$/hour)         (ton/hour)         Maximum (\$/hour)         48.531959           (\$/hour)         Emission (ton/hour)         hour)	H6 (MWth)	0.000000	H6 (MWth)	70.794894
(\$/hour) Emission (ton/hour)  Average Cost 9858.690531 Average Emission 48.070597 (\$/hour) (ton/hour)  Maximum Cost 10067.554091 Maximum 48.531959 (\$/hour) Emission (ton/hour)	H7 (MWth)	88.413425	H7 (MWth)	0.000000
hour   Average Cost   9858.690531   Average Emission   48.070597	Minimum Cost	9794.718722	Minimum	47.718479
Average Cost 9858.690531 Average Emission 48.070597 (\$/hour) (ton/hour)  Maximum Cost 10067.554091 Maximum 48.531959 (\$/hour) Emission (ton/hour)	(\$/hour)		Emission (ton/	
(\$/hour) (ton/hour)  Maximum Cost 10067.554091 Maximum 48.531959 (\$/hour) Emission (ton/hour)			hour)	
Maximum Cost 10067.554091 Maximum 48.531959 (\$/hour) Emission (ton/hour)	Average Cost	9858.690531	Average Emission	48.070597
(\$/hour) Emission (ton/hour)	(\$/hour)		(ton/hour)	
hour)	Maximum Cost	10067.554091	Maximum	48.531959
*	(\$/hour)		Emission (ton/	
STD 0.005096 STD 0.006165			hour)	
	STD	0.005096	STD	0.006165

**Table 12**The boundaries limits for Trapezoidal membership functions for Optimal CHP Economic and Emission Dispatch using DCWOA for case study 3.

Outputs	Optimal CHP Economic Dispatch	Optimal CHP Emission Dispatch
Cost (\$/hour)	9794.7187	10,457
Emission (kg/ hour)	49.8053	47.7185

**Table 13**Optimal Schedule of Multi-objective CHPEED for WOA and proposed DCWOA for case study 3.

Control Variables	WOA	DCWOA
P1 (MW)	74.999935	74.997252
P2 (MW)	110.07812	101.90573
P3 (MW)	113.01986	115.27625
P4 (MW)	181.73069	187.65188
P5 (MW)	80.999937	81
P6 (MW)	39.999937	40
H5 (MWth)	0.280675	0
H6 (MWth)	98.934315	113.66599
H7 (MWth)	50.784952	36.334888
Total Power (MW)	600.8285	600.8311
Total Heat (MWth)	149.9999	150.0009

IDBEA [15], BCS [16] and WOA, DCWOA performs better and obtains higher fitness values. The fitness of NSGA-II [55] and SPEA-2 [12] are very low as the emission values are closer to the upper boundary values set for the fuzzy function as in Table 9. This signifies that DCWOA is able to find the best compromised solution among basic WOA, and other recently published methods for this test system with average computational time of 0.34 s.

The fitness values given in Table 10 (Fig. 4) shown the magnitude of improvement achieved by modifying WOA by using the dynamically controlled constriction function.

#### 4.3. Case study 3

The cost and emission coefficients and other demand values of the MO-CHPEED problem are taken form reference [11]. The loss coefficient values for calculating power loss are given by:

 $B = [49 \ 14 \ 15 \ 15 \ 20 \ 25; 14 \ 45 \ 16 \ 20 \ 18 \ 19; 15 \ 16 \ 39 \ 10 \ 12 \ 15; 15 \ 20 \ 10 \ 40 \ 14 \ 11; 20 \ 18 \ 12 \ 14 \ 35 \ 17; 25 \ 19 \ 15 \ 11 \ 17 \ 39] * 10^{-7}.$ 

 $B_0 = [-0.3908 - 0.1297 \ 0.7047 \ 0.0591 \ 0.2161 - 0.6635] \ * \ 10^{-7}.$ 

 $B_{00} = 0.056.$ 

The optimal solutions obtained for individual objective after 100 trials by DCWOA are presented in Table 11.

The boundaries values of each objective for trapezoidal membership function are given in Table 12.

The optimal generators schedule for MOCHPEED problem provided by WOA and proposed DCWOA are given in Table 13.

The best compromise solution for multi-objective CHPEED calculated by proposed DCWOA with basic WOA, and recently published methods TVAC-PSO [11], hybrid of NSGA-II and MOPSO [14], LCA [57] and PSO, are shown in Table 14.

Among rest of the algorithms, it is evident that LCA [57] provides the lowest value of hourly emission, but it has the highest hourly operating cost. Among basic WOA and improved WOA variants, all the values of

**Table 14**Multi-objective CHPEED results comparison obtained by proposed DCWOA with WOA and other published methods for case study 3.

Methods	Cost (\$/hour)	Emission (ton/ hour)	Fitness
TVAC-PSO [11]	10244.0022	50.0453	0.2252
Hybrid of NSGA-II and MOPSO	10102.2	47.9724	0
[14]			
LCA [57]	12451.38	11.1	0
PSO	9957.571337	48.724227	0.6571
WOA	9972.896450	48.375022	0.7125
DCWOA	9940.082644	48.375579	0.7313

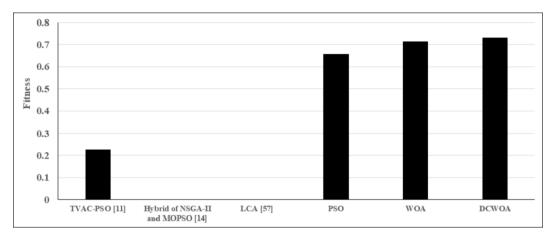


Fig. 5. Fitness comparison for case study 3.

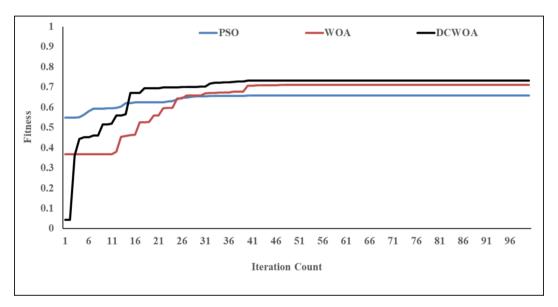


Fig. 6. Convergence Curve of Case Study 3 obtained by PSO, WOA and DCWOA.

cost and emission are close to each other but better than TVAC-PSO [11], hybrid of NSGA-II and MOPSO [14], LCA [57] and PSO. Therefore, fitness is used to evaluate the performance of these techniques. The fitness of TVAC-PSO [11] is the least as the emission and cost values are closer to the boundary conditions set as per Table 12.

The fitness values from Table 14 are shown in Fig. 5. They demonstrate the magnitude of improvement achieved by modifying WOA by using the dynamically controlled constriction function with average computational time of  $0.61\ s.$ 

A convergence characteristic graph for the fitness values of case study 3 with respect to iterations is shown in Fig. 6, comparing the fitness of PSO, basic WOA and proposed DCWOA. It can be concluded from Fig. 6 that basic WOA and DCWOA are performed much better than PSO. It can be observed from this figure that during the latter half, basic WOA stuck in local minima. For improving the efficiency and maintaining the balance between exploration and exploitation, dynamically controlled constriction function is used in proposed DCWOA. By adding dynamically controlled constriction function, proposed fuzzy based DCWOA has sufficient time for proper exploration and exploitation. This can be observed from the results that best compromise solution obtained by fuzzy based DCWOA is much better then basic WOA and other latest published methods.

# 5. Conclusion

The multi-objective economic emission scheduling problem of CHP units is a non-convex, nonlinear, and hard constrained combinatorial problem. This conundrum becomes complex as it deals with two conflicting objectives of fuel cost and the mass of emissions. This paper proposes the use of an improvement of a popular optimization algorithm, the Whale Optimization by adding dynamically controlled constriction function, i.e., DCWOA. The proposed DCWOA is tested on latest CEC benchmark functions and different CHPEED case studies. Furthermore, trapezoidal membership function is used to get the best compromise between the fuel cost and mass of emission conflicting objectives. The results obtained by DCWOA on several standard benchmark functions are compared with latest published results, where the proposed algorithm obtain better average and STD values. In addition, the results provided by proposed DCWOA for the solving MO-CHPEED conundrum are also compared with basic WOA and latest and established optimization algorithms. It can be observed in Table 6, the Multi-objective CHPEED results comparison obtained by proposed DCWOA with WOA and other published methods for case study 1. It can be observed that the proposed DCWOA obtained the highest fitness value of 0.6134 as compared with BCS which provided 0.6127. Table 10 shows the Multi-objective CHPEED results comparison obtained by

proposed DCWOA with WOA and other published methods for case study 2. Table 10 shows that among GWO, NSGA-II, SPEA-2, IDBEA, BCS and WOA, proposed DCWOA performs better and obtains higher fitness values 0.6563. Again, in Table 14, Multi-objective CHPEED results comparison are obtained by proposed DCWOA with WOA and other published methods for case study 3. Proposed DCWOA provide higher fitness of 0.7313 than TVAC-PSO, hybrid of NSGA-II and MOPSO, LCA, PSO and WOA. It can be found that proposed DCWOA outperform better than the mentioned methods to provide good quality of compromise solution and found to be better optimization method for solving hard constraints problem. This study is limited to solve multi-objective Combined Heat and Power Economic Emission Dispatch (MO-CHPEED) problem with fuzzy membership approach consisting of thermal generators, CHP units and heat units with limited constraints. The constrains considered in this work is limited to power & head balance constraints, minimum & maximum limits of electrical generators, CHP units and heat generator units. However, this proposed DCWOA can be extended to solve CHPEED problem in the presence of renewable energy sources, for cost benefit analysis with related constraints.

#### CRediT authorship contribution statement

Vinay Kumar Jadoun: Data curation, Formal analysis, Investigation, Methodology, Software, Supervision, Validation, Visualization, Writing – original draft. G Rahul Prashanth: Data curation, Formal analysis, Investigation, Methodology, Software, Supervision, Validation, Visualization, Writing – original draft. Siddharth Suhas Joshi: Formal analysis, Investigation, Methodology, Software, Supervision, Validation, Visualization. K. Narayanan: Funding acquisition, Investigation, Methodology, Project administration, Resources, Software, Supervision, Validation, Writing – original draft, Writing – review & editing. Hasmat Malik: Data curation, Formal analysis, Investigation, Methodology, Software, Supervision, Validation, Visualization, Writing – original draft. Fausto Pedro García Márquez: Funding acquisition, Investigation, Methodology, Project administration, Resources, Software, Supervision, Validation, Writing – original draft, Writing – review & editing.

# **Declaration of Competing Interest**

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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