Notes for IO

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Chapter 1

Price Competition

1.1 Betrand Paradox

Consider a market demand $q = D(p)$ and two firms simultaneously set their individual price for one product. The rule behind is that the firm with lower price "monopolise" whole market and if prices are equal two firms share market equally. To be specific, the demand of firm i is given by

$$D_i(p_i, p_j) = \begin{cases} D(p_i), & \text{if } p_i < p_j, \\ \dfrac{D(p)}{2}, & \text{if } p_i = p_i = p, \\ 0, & \text{if } p_i > p_j. \end{cases}$$

Now we would like to find the equilibrium of this game and the equilibrium concept we are adapting to is *Nash equilibrium*.

Definition 1.1 (Nash equilibrium)**.** Strategy pair $(s_1^*, \dots, s_n^*)$ is called Nash equilibrium if for all $i = 1, \dots, n$

$$u_i(s_i^*, s_{-i}^*) \geq u_i(s_i, s_{-i}^*) \text{ for all } s_i \in S_i, s_i \neq s_i^*.$$

1.2 Capacity Constraint

$$D_i(p_i, p_j) = \begin{cases} \min\{Q(p_i), k_i\}, & \text{if } p_i < p_j \\ \min\{\dfrac{k_i}{k_i + k_j} Q(p), k_i\}, & \text{if } p_i = p_j = p \\ \max\{0, \min\{Q(p_i) - k_j, k_i\}\}, & \text{if } p_i > p_j. \end{cases}$$

For cases: $p_i < p_j$ and $p_i = p_j = p$, things are clear. We contemplate on $p_i > p_j$ for a moment.

Remark 1.2 (Case: $p_i > p_j$)**.** We consider the residual demand of firm i.

(i) when firm j's capacity can satisfy whole market demand, firm i is facing zero demand: $Q(p_i) \leq Q(p_j) \leq k_j$ implies $Q(p_i) - k_j \leq 0$. Therefore, $D_i(p_i, p_j) = 0$;

(ii) when firm j can not cater for whole market demand and firm i set a relative "high" price, firm i is facing a market demand of $Q(p_i) - k_j$;

(iii) when firm j can not satisfy whole market demand and firm i set a "low" price (of course, it should be greater than p_j), firm i is then supplying a market demand of k_i.

The following example is used to illustrate remark 1.2.

Example 1.3 (Residual demand when $p_i > p_j$)**.** Consider a market demand $Q(p) = 1 - p$ and capacity constraint $k_1 = k_2 = 0.3$. The following three situations correspond to residual demand of 0, $Q(p_i) - k_j$, and k_i.

(i) $p_1 = 0.8$ and $p_2 = 0.7$: residual demand of firm 1 is 0;

(ii) $p_1 = 0.6$ and $p_2 = 0.5$: residual demand of firm 1 is $0.1 = Q(p_1) - k_2$;

(iii) $p_1 = 0.2$ and $p_1 = 0.1$: residual demand of firm 1 is $0.3 = k_1$.

Claim 1.4 (Nash equilibria under capacity constraint)**.** There are three cases of Nash equilibria under capacity constraint.

(i) When $k_i \leq R_i(k_j), i \neq j$, $p_1^* = p_2^* = p(k_1 + k_2)$ is Nash equilibrium: assume firm 2 sticks to $p(k_1 + k_2)$, we now consider firm 1's motivation to deviate.

  (a) If chooses a price $p_1 < p(k_1 + k_2)$, firm 1 is facing a demand:

$$Q_1(p_1) = \min\{Q(p_1), k_1\}.$$

  Since $Q(p_1) \geq Q(p(k_1 + k_2)) = k_1 + k_2$, $Q_1(p_1) = k_1$. If firm 1 insists $p_1 = p(k_1 + k_2)$, it will face a demand:

$$Q_1(p_1) = \min\{\frac{k_1}{k_1 + k_2} Q(p(k_1 + k_2)), k_1\} = k_1.$$

  Therefore, decreasing price from $p(k_1 + k_2)$ to $p_1 < p(k_1 + k_2)$ will not stimulate demand but suffer a loss. Firm 1 does not have incentive to decrease its price.

(b) If chooses a price $p_1 > p(k_1 + k_2)$, firm 1 is facing a demand:

$$\begin{aligned} Q_1(p_1) &= \max\{0, \min\{Q(p_1) - k_2, k_1\}\} \\ &= \min\{Q(p_1) - k_2, k_1\} && (p_2 = p(k_1 + k_2)) \\ &= Q(p_1) - k_2 && (p_1 > p(k_1 + k_2)) \end{aligned}$$

Here, firm 1's best response is to choose a price such that end up facing a demand $R_1(k_2)$, i.e., the optimal quantity chosen by firm 1 when firm 2 produces k_2 units in the Cournot game.

$$\pi_1 = q_1(p(q_1 + k_2)) \implies \frac{\partial^2 \pi}{\partial q_1^2} = \frac{\partial^2 p}{\partial q_1^2} q_1 + 2\frac{\partial p}{\partial q_1}.$$

1.3 Exercise

Exercise 1. Prove aggregate Bertrand profit is less than monopoly profit.