Frequent Itemsets and Association Rules

Market Baskets

Frequent Itemsets

A-Priori Algorithm

The Market-Basket Model

- A large set of *items*, e.g., things sold in a supermarket.
- A large set of *baskets*, each of which is a small set of the items, e.g., the things one customer buys on one day.

Market-Baskets – (2)

- A general many-many mapping (association) between two kinds of things.
  - But we ask about connections among "items," not "baskets."
- The technology focuses on common events, not rare events ("long tail").

Support

- Simplest question: find sets of items that appear “frequently” in the baskets.
- *Support* for itemset *I* (s(I)) = the number of baskets containing all items in *I*.
- Given a *support threshold* *s*, sets of items that appear in at least *s* baskets are called *frequent itemsets*.

Example: Frequent Itemsets

- Items={milk, coke, pepsi, beer, juice}.
- Support = 3 baskets.

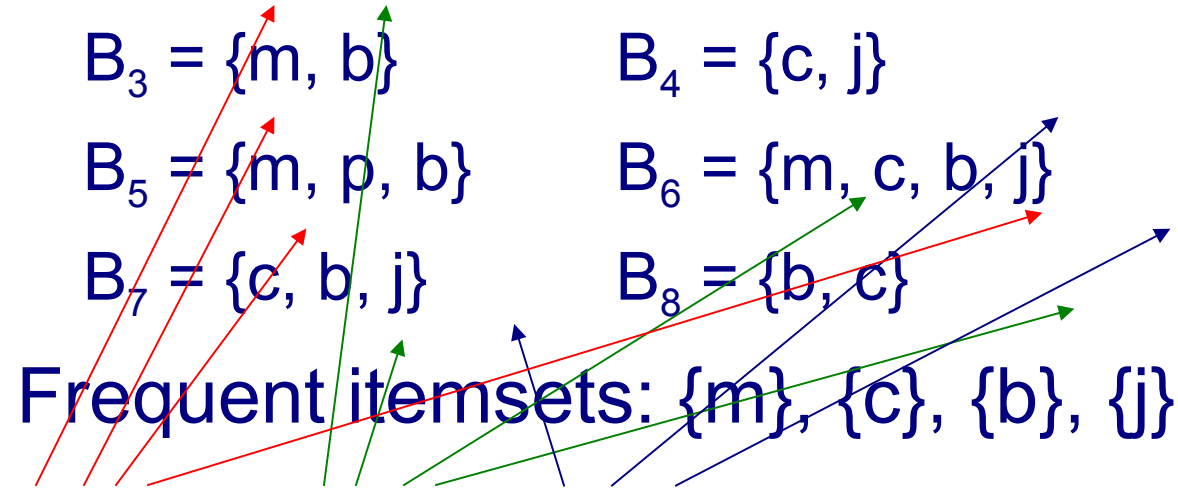
B_1 = {m, c, b} B_2 = {m, p, j}

B_3 = {m, b} B_4 = {c, j}

B_5 = {m, p, b} B_6 = {m, c, b, j}

B_7 = {c, b, j} B_8 = {b, c}

- Frequent itemsets: {m}, {c}, {b}, {j}, {m,b}, {b,c}, {c,j}.



Monotonicity

- For any sets of items I and any set of items J, it holds that:

$$s(I \cup J) \leq s(I)$$

Applications – (1)

- Items = products; baskets = sets of products someone bought in one trip to the store.
- Example application: given that many people buy beer and diapers together:

  Run a sale on diapers; raise price of beer.
- Only useful if many buy diapers & beer.

Applications – (2)

- Items = sets of documents; baskets = for any sentence there is a basket containining all documents with that sentence.
- Items that appear together too often could represent plagiarism.

Applications – (3)

- Baskets = Web pages; items = words.
- Unusual words appearing together in a large number of documents, e.g., “Hollande” and “Merkel”, may indicate an interesting connection.

Scale of the Problem

- WalMart, Carrefour sell more than 100,000 items and can store billions of baskets.
- The Web has billions of words and many billions of pages.

Association Rules

If-then rules I → *j* about the contents of baskets, I is a set of items and j is an item.

- I → *j* means: "if a basket contains all the items in I then it is *likely* to contain *j*."

Confidence of this association rule is the *probability* of *j* given I, i.e.

$$\frac{s(I \cup \{j\})}{s(I)}$$

Example: Confidence

+ B_1 = {m, c, b} B_2 = {m, p, j}

- B_3 = {m, b} B_4 = {c, j}

- B_5 = {m, p, b} +B_6 = {m, c, b, j}

B_7 = {c, b, j} B_8 = {b, c}

- An association rule: {m, b} → c.

  Confidence = ?

Example: Confidence

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- An association rule: {m, b} → c.

  Confidence = # baskets containing {m,b,c} / # baskets containing {m,b} = 2/4 = 0.5

Finding Association Rules

- Question: "find all association rules with support ≥ *s* and confidence ≥ *c* ."

  Note: "support" of an association rule is defined:

  $$s(I \rightarrow j) = s(I)$$

- Hard part: finding the frequent itemsets.

From Frequent Itemsets to AR

Note: if $\{i_1, i_2, \ldots, i_k\} \rightarrow j$ has high support and confidence, then both $\{i_1, i_2, \ldots, i_k\}$ and $\{i_1, i_2, \ldots, i_k, j\}$ are "frequent."

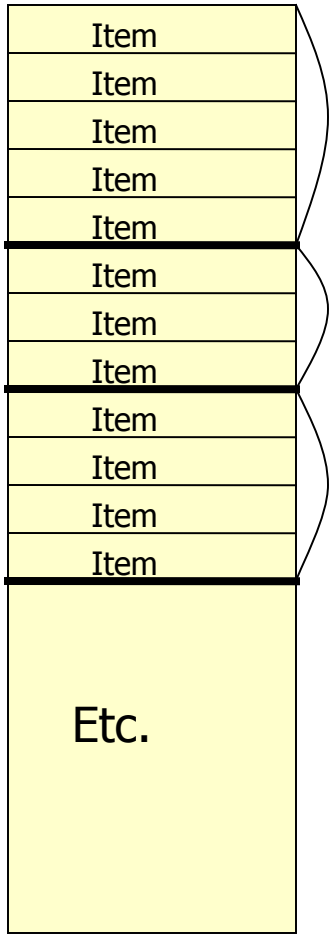
(e.g. conf. = 0.5) then s($\{i_1, i_2, \ldots, i_k, j\}$) >= 0.5$s$.

Then any AR with conf. c and support s can be found among all itemsets with support >= $c \cdot s$.

Computation Model

- Typically, data is kept in raw files rather than in a database system.
  - Stored on disk.
  - Stored basket-by-basket.
  - Expand baskets into pairs, triples, etc. as you read baskets.
    - Use *k* nested loops to generate all sets of size *k*.

File Organization



Basket 1

Basket 2

Basket 3

Example: items are positive integers, and boundaries between baskets are –1.

Computation Model – (2)

- The true cost of mining disk-resident data is usually the number of disk I/O's.
- In practice, association-rule algorithms read data in *passes* – all baskets read in turn.
- Thus, we measure the cost by the number of passes an algorithm takes.

Main-Memory Bottleneck

- For many frequent-itemset algorithms, main memory is the critical resource.
  - As we read baskets, we need to count something, e.g., occurrences of pairs.
  - The number of different things we can count is limited by main memory.

Finding Frequent Pairs

- The hardest problem often turns out to be finding the frequent pairs.
  - Why? Often frequent pairs are common, frequent triples are rare.
    - Why? Probability of being frequent drops exponentially with size; number of sets grows more slowly with size.
- We'll concentrate on pairs, then extend to larger sets.

Naïve Algorithm

- Read file once, counting in main memory the occurrences of each pair.

  From each basket of n items, generate its n (n -1)/2 pairs by two nested loops.

- Fails if (#items)^2 exceeds main memory.

  Remember: #items can be 100K (Wal-Mart) or 10B (Web pages).

Example: Counting Pairs

- Suppose 10^5 items.
- Suppose counts are 4-byte integers.
- Number of pairs of items: $10^5(10^5-1)/2 = 5*10^9$ (approximately).
- Therefore, $2*10^{10}$ (20 gigabytes) of main memory needed.

Details of Main-Memory Counting

Two approaches:

1. Count all pairs, using a triangular matrix.
2. Keep a table of triples [*i*, *j*, *c*] = "the count of the pair of items {*i*, *j* } is *c*."

(1) requires only 4 bytes/pair.

Note: always assume integers are 4 bytes.

(2) requires 12 bytes, but only for those pairs with count > 0.

Triangular-Matrix Approach – (1)

- Number items 1, 2,…

  Requires table of size O(n) to convert item names to consecutive integers.

- Count {i, j } only if $i < j$.
- Keep pairs in the order {1,2}, {1,3},…, {1,n }, {2,3}, {2,4},…,{2,n }, {3,4},…, {3,n }, …{n -1,n }.

Triangular-Matrix Approach – (2)

- Find pair $\{i, j\}$ at the position $(i-1)(n-i/2) + j - i$.
- Total number of pairs $n(n-1)/2$; total bytes about $2n^2$.

Details of Approach #2

- Total bytes used is about 12*p*, where *p* is the number of pairs that actually occur.
  - Beats triangular matrix if at most 1/3 of possible pairs actually occur.
- May require extra space for retrieval structure, e.g., a hash table.

A-Priori Algorithm – (1)

- A two-pass approach called *a-priori* limits the need for main memory.
- Key idea: *monotonicity* : if a set of items appears at least *s* times, so does every subset.

  For pairs: if item *i* does not appear in *s* baskets, then no pair including *i* can appear in *s* baskets.

A-Priori Algorithm – (2)

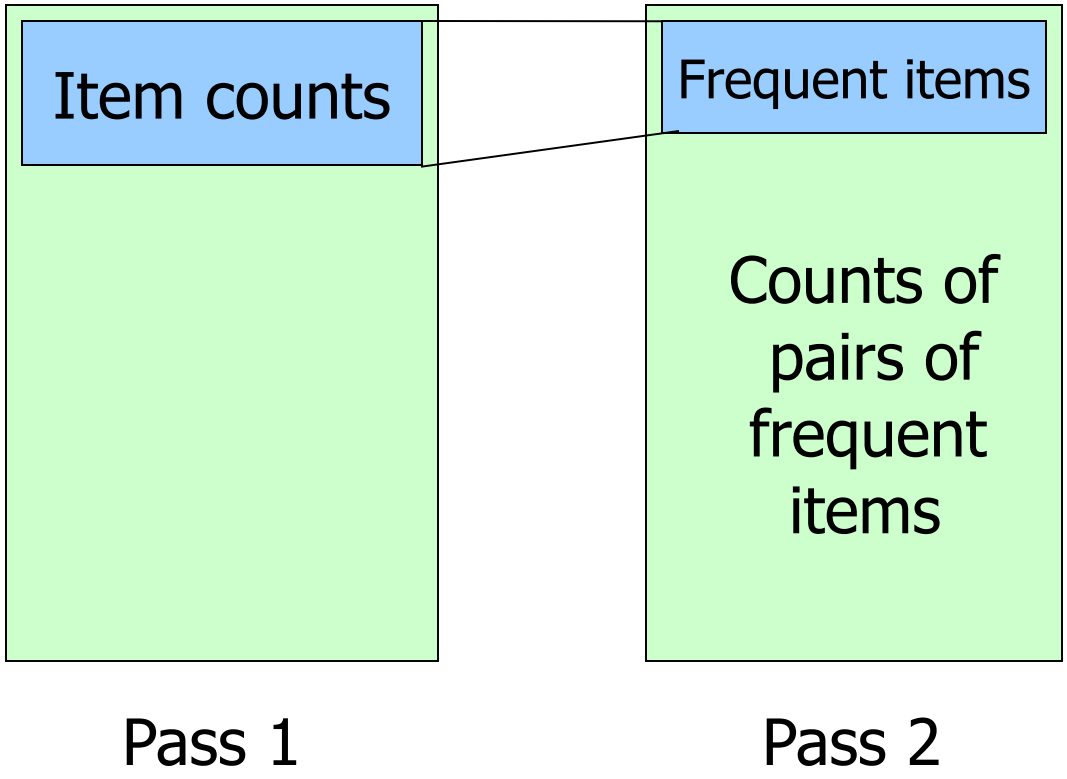
- Pass 1: Read baskets and count in main memory the occurrences of each item.
  - Requires only memory proportional to #items.
- Items that appear at least *s* times are the *frequent items*.

A-Priori Algorithm – (3)

Pass 2: Read baskets again and count in main memory only those pairs both of which were found in pass 1 to be frequent.

- To count number of item pairs use a hash function. Requires memory proportional to square of *frequent* items only, plus a list of the frequent items (so you know what must be counted), plus space for hashing.

Main Memory Data in A-Priori



Detail for A-Priori

- You can use the triangular matrix method with n = number of frequent items.
  - May save space compared with storing triples.
- Trick: number frequent items 1,2,…,n and keep a table relating new numbers to original item numbers.

Frequent Triples, Etc.

- For each k, we construct two sets of *k -sets* (sets of size k):

  C_k = *candidate* k -sets = those that might be frequent sets (support $\geq s$) based on information from the pass for $k-1$.

  L_k = the set of truly frequent k -sets.

A-Priori for All Frequent Itemsets

- One pass for each *k*.
- Needs room in main memory to count each candidate *k* -set.
- For typical market-basket data and reasonable support (e.g., 1%), *k* = 2 requires the most memory.

Frequent Itemsets – (2)

- C_1 = all items
- In general, L_k = members of C_k with support ≥ s.
- C_{k+1} = (k +1) -sets, each k of which is in L_k .
- When do we stop?

Frequent Itemsets – (2)

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- In general, L_k = members of C_k with support ≥ s.
- C_{k+1} = (k +1) -sets, each k of which is in L_k.
- When do we stop? When C_{k+1} is empty.