Problem Chosen	2021 MCM/ICM Summary Sheet	Team Control Number
A		2114440

The Study of Fungi in Ground Litter and Wood Fibers Decomposition

Fungi are the main decomposers of plant debris in nature. As fungi play a key role in the chemical decomposition of plants, which is an important part of the carbon cycle. In addition, the carbon cycling has significant impacts to human life. So we built mathematical models to simulate the chemical decomposition process in nature.

Firstly, we develop a preliminary model for the decomposition of the fungus. Our preliminary model has two basic assumptions. 1. We assume that the fungi are in a resource-limited environment. 2. We assume that there are three fungal populations in the environment. The fungal populations conform to the logistic growth as well as the a competitive relationship. Therefore, we choose the Lotka-Volterra Model. we perform an analysis for the original figure in the material to obtain the decomposition rate and the hyphal extension rate as a function. Since we cannot obtain the data for this graph, we use the fitted curve to obtain the optimized function. The result of the model (Fig.3) shows the time requirement of the three fungi to completely decompose the wood.

Secondly, we add moisture to the Lotka-Volterra model as required by the question to obtain a new model (MLV model). With the addition of the moisture factor, the results of the model (Fig.4) yielded the new time required for the complete decomposition of wood by the three fungi.

Thirdly, we create a new competition model (TPMLV model). Based on previous studies, we made three assumptions about the model: 1. We set the main components of wood and their proportions. The components are lignin, cellulose, hemicellulose and polysaccharides. To simplify the model, we assume that the proportion of these four components are 25%. 2. We identify three fungal types and the substances they decompose. The three fungal types were white rot, brown rot, and soft rot. The white rot fungus prefer lignin and polysaccharides, while the brown rot fungus and soft rot fungus both prefer cellulose and hemicellulose. 3. We determine the four enzymes which are required to decompose wood and their optimal temperature and pH. Based on these three assumptions, the model results (Fig.5 6 7) demonstrate the decomposition times of the three fungi.

Finally, we use the TPMLV model to simulate the three fungal populations change in different environmental states to derive the relative merits of the combined diversity and the performance of the fungal populations in different environments by (Fig.8).

Key Words: Fungi; Decomposition Rate; TPMLV Model; Environmental Factors; Diversity of Species;

Contents

1 Introduction

1.1 Problem Background

Fungi are the main decomposers of plant debris in nature[**?**], and can be classified into three types according to their environmental preferences and decay patterns: white rot fungi, brown rot fungi, and soft rot fungi. They always degrade four parts of wood: lignin, cellulose, hemicellulose and polysaccharides[**?**]. According to Hamed, white rot fungi prefer lignin and polysaccharide in wood while brown rot fungi and soft rot fungi prefer cellulose and hemicellulose[**?**]. Fungi degrade the wood through secreting a series of enzymes, including four main enzymes: lignin-degrading cazymes, cellulose-degrading cazymes, hemicellulose-degrading cazymes, and pectin-degrading cazymes[**?**]. The activity of these enzymes are mainly affected by temperature and pH[**?**].

1.2 Problem statement

We are asked to model plant debris decomposition in a given land where multiple types of fungi are present. To achieve this main objective, we will do the following:

- Establish a preliminary mathematical model to describe plant debris decomposition in the presence of multiple fungi.
- Extract the growth rate and moisture tolerance parameters of different fungi given in the question, and establish a new plant debris decomposition model based on the initial model.
- Further add the effects of enzymes, temperature, and pH to establish a new model to describe the short-term and long-term interactions between different types of fungi.
- Arrange and combine multiple fungi to find the relative advantages and disadvantages of the combination. And predict the performance of these combinations in different environmental conditions.
- Based on the conclusions of the above model, describe the relationship between fungal community diversity and decomposition efficiency, and predict the importance of fungal diversity to the local ecosystem.

2 General Assumptions

- Fungus do not inhibit each other, they only compete for resources.
- The composing proportion lignin, cellulose, hemicellulose and polysaccharides in plant debris are 25%, 25%, 25% and 25%. White rot fungus only degrade lignin & polysaccharides, brown rot fungus and soft rot fungus only degrade cellulose & hemicelluloys.
- Only temperature and PH will affect the activity of enzymes.
- The optimum temperature and PH of lignin-degrading cazymes are 30°C and 8.0. (see[**?**]) For cellulose-degrading cazymes are 40°C and 6.5. (see[**?**]) For hemicellulose-degrading cazymes are 65°C and 5.7 on average. (see[**?**]) For pectin-degrading cazymes are 61°C and 4.0. (see[**?**])

- The growth rate of fungus is the hyphal extention rate and ignore the death rate of fungus.
- In normal growth environment, PH is 7, temperature is 22°C, and relative humidity is 50%.
- The moisture trade-off of of fungi 1, fungi 2, fungi 3 are -1, 0, 1, and the corresponding optimum relative humidity are 20%, 50% and 80%.

3 Symbols Description

Table 1: Symbols Table

Symbol	Description	Symbol	Description
$\dot{H}_1(t)$	Population growth rate of fungi1 at time t	T	Current tempreature in the environment
$\dot{H}_2(t)$	Population growth rate of fungi2 at time t	T_1	The optimum tempreature of lignin-degrading cazymes
$\dot{H}_3(t)$	Population growth rate of fungi3 at time t	T_2	The optimum tempreature of cellulose-degrading cazymes
x_1	Current population of fungi1	T_3	The optimum tempreature of hemicellulose-degrading cazymes
x_2	Current population of fungi2	T_4	The optimum tempreature of pectin-degrading cazymes
x_3	Current population of fingi3	k_1	influence coefficient of tempreature and PH on the fungi growth rate
xm_1	Maximum population in the environment of fungi1	k_2	influence coefficient of relative humidity on the fungi growth rate
xm_2	Maximum population in the environment of fungi2	α_1	proportion of lignin-degarding cazymes in fungi1
xm_3	Maxmium population in the environment of fungi3	β_1	proportion of cellulose-degarding cazymes in fungi2 & 3
M	Current relative humidity in the environment	S_1	Competition coefficient of fungi1 when fungi1 and fungi2 grow together in RC & MRC model

Symbol	Description	Symbol	Description
M_1	The optimum relative humidity of fungi1	S_2	Competition coefficient of fungi1 when fungi1 and fungi3 grow together in RC & MRC model
M_2	The optimum relative humidity of fungi2	S_3	Competition coefficient of fungi2 when fungi2 and fungi3 grow together in RC & MRC model
M_3	The optimum relative humidity of fungi3	S_4	Competition coefficient of fungi2 when fungi1 and fungi2 grow together in RC & MRC model
PH	Current PH in the environment	S_5	Competition coefficient of fungi3 when fungi1 and fungi3 grow together in RC & MRC model
PH_1	The optimum PH of lignin-degrading cazymes	S_6	Competition coefficient of fungi3 when fungi3 and fungi2 grow together in RC & MRC model
PH_2	The optimum PH of cellulose-degrading cazymes	S_7	Competition coefficient of fungi2 when fungi2 and fungi3 grow together in TPMRC model
PH_3	The optimum PH of hemicellulose-degrading cazymes	S_8	Competition coefficient of fungi3 when fungi2 and fungi3 grow together in TPMRC mpdel
PH_4	The optimum PH of pectin-degrading cazymes	$D(H)$	Decompensition rate at hyphal extention rate H

4 Presentation of the Model

4.1 Premise of models

According to the relationship between the hyphal extension rate and the resulting wood decomposition rate at virous tempreature in question A, we can obtain the equations of three fitted curve through observations of Figure1 and Figure2. The estimated equations are as follows:

$$D(H) = \begin{cases} 7\sqrt{H} + 5 & ,T = 22°C \\ 5\sqrt{H} + 3 & ,T = 16°C \\ 4\sqrt{H} & ,T = 10°C \end{cases} \tag{1}$$

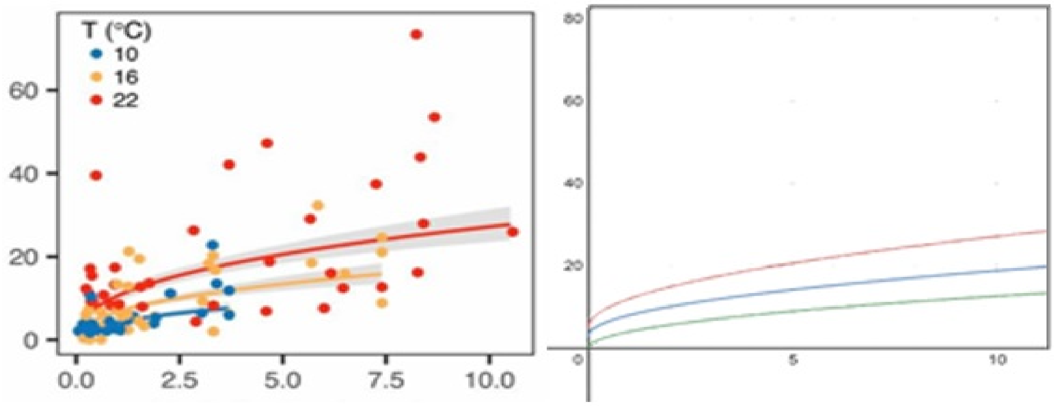


Figure 1: Original figure and the estimation equations

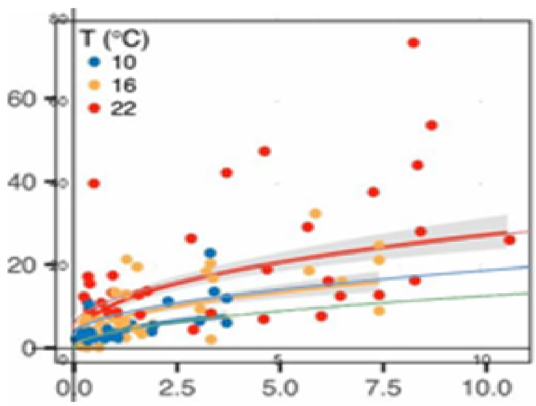


Figure 2: Overlap the original figure and the estimation equations

4.2 Traditional Competition Model

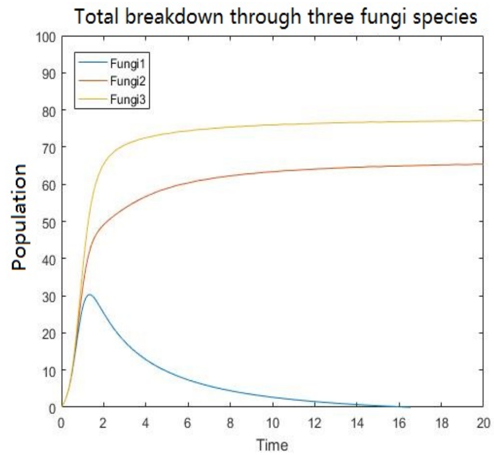
4.2.1 Lotka-Volterra Model

According to the definition of Lotaka-Volterra model, when two or more species live in a same environment and share same resources, they will compete for resources[**?**]. Therefore, the growth of several fungal population in the same environment follows the Lotaka-Volterra Model. Assume that there are three kinds of fungi with the differential equations of their population change can be written as follows:

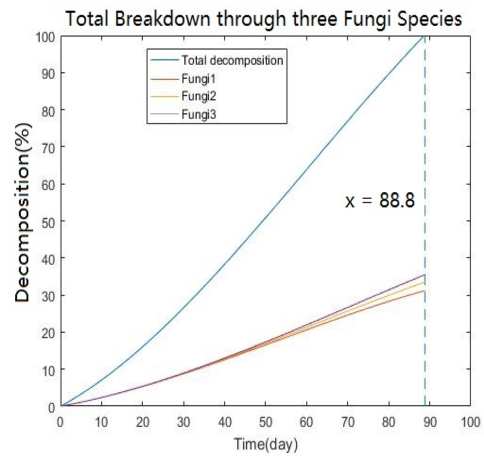
$$\begin{cases} \dot{H}_1(t)=\mathrm{H}_{n1}x_1(1-\frac{x_1}{xm_1}-S_1\frac{x_2}{xm_2}-S_2\frac{x_3}{xm_3}) \\ \dot{H}_2(t)=\mathrm{H}_{n2}x_2(1-\frac{x_2}{xm_2}-S_4\frac{x_1}{xm_1}-S_3\frac{x_3}{xm_3}) \\ \dot{H}_3(t)=\mathrm{H}_{n3}x_3(1-\frac{x_3}{xm_3}-S_6\frac{x_2}{xm_2}-S_5\frac{x_1}{xm_1}) \end{cases} \tag{2}$$

By putting the population growth rate $\dot{H}_1(t)$, $\dot{H}_2(t)$ and $\dot{H}_3(t)$ into equation (1) in 4.1, we get the equations of fungus decomposition rate.

4.2.2 Results of Lotka-Volterra Model



(a)



(b)

Figure 3: Wood decomposition rate and population of three fungal species under Lotka-Volterra model ($H_{n1}, H_{n2}, H_{n3} = 4, xm_1, xm_2, xm_3 = 100, S_1 = 0.8, S_2 = 0.6, S_3 = 0.4, S_4 = 0.7, S_5 = 0.5, S_6 = 0.3$)

The Fig.3(a) above shows the population change of the three fungi when only considering the resource competition among species. The trends of three kinds fungi in the Figure 3 are different from the logistic model because we added competition factors among populations, but the overall trends are consistent, and figure shows that all of them reach a stable state after 16 units of time. Fig.3(a) shows the population of fungus 1 gradually drops to 0 and because the resource competitiveness of fungi 1 is lower than fungi 2 and fungi 3. According to the relationship between fungi growth rate and decomposition rate given in the problem, we can obtain Fig.3(b), where we can see that after 88.8 days, the wood decomposition percentage reaches to 100%.

4.3 Improved Competition Model

4.3.1 MLV Model

Lotka-Volterra model is a basic model to simulate the population change in nature. However, we can add some other influencing factors into consideration. In MLV model, our group add moisture as a influencing factor. According to the relationship between moisture trade-off and decomposition rate, the range of moisture trade-off is [-1, 1]. Therefore, we assume the moisture trade-off of of fungi 1, fungi 2, fungi 3 are -1, 0, 1, and the corresponding optimum relative humidity are 20%, 50% and 80%. In nature, the closer the current humidity is to the optimum humidity, the better the fungi grows. Therefore, the fungi population can be assumed to be inversely proportional to the difference of relative humidity, and we add this factor to the equation (2), as a result, the updated differential equations of their population change can be written as follows:

$$\begin{cases} \dot{H}_1(t)=\mathrm{H}_{n1}x_1(1-k_2(M-M_1)^2)(1-\frac{x_1}{xm_1}-S_1\frac{x_2}{xm_2}-S_2\frac{x_3}{xm_3}) \\ \dot{H}_2(t)=\mathrm{H}_{n2}x_2(1-k_2(M-M_2)^2)(1-\frac{x_2}{xm_2}-S_4\frac{x_1}{xm_1}-S_3\frac{x_3}{xm_3}) \\ \dot{H}_3(t)=\mathrm{H}_{n3}x_3(1-k_2(M-M_3)^2)(1-\frac{x_3}{xm_3}-S_6\frac{x_2}{xm_2}-S_5\frac{x_1}{xm_1}) \end{cases} \tag{3}$$

Similarly, we can put the population growth rate $\dot{H}_1(t)$, $\dot{H}_2(t)$ and $\dot{H}_3(t)$ into equation (1) and we get the equations of fungus decomposition rate of MLV model.

4.3.2 Results of MLV Model

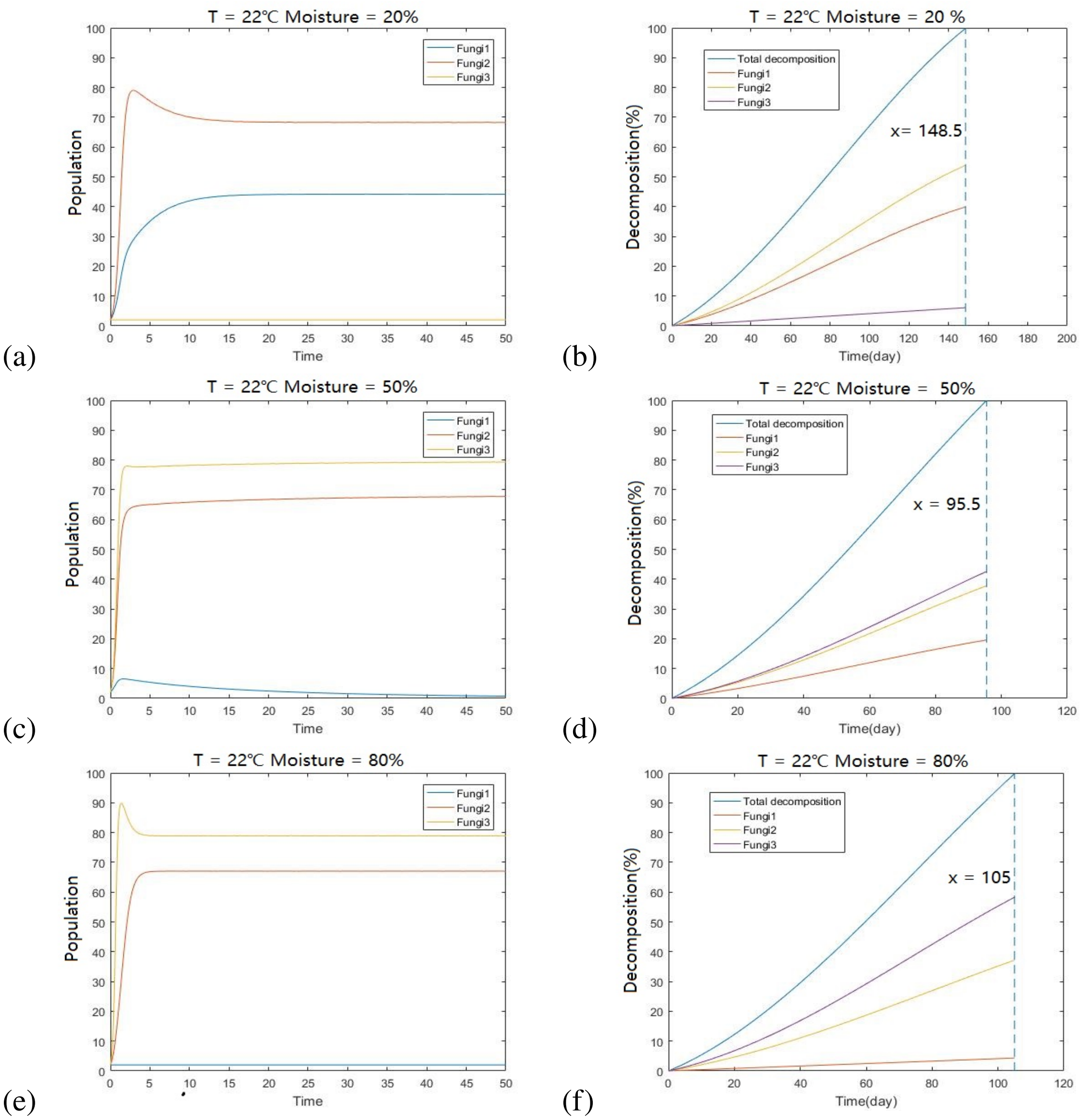


Figure 4: Wood decomposition rate and population of the three fungal species under different humidity and resource competition ($H_{n1} = 2, H_{n2} = 4, H_{n3} = 6, xm_1, xm_2, xm_3 = 100, M_1 = 0.2, M_2 = 0.5, M_3 = 0.8, S_1 = 0.8, S_2 = 0.6, S_3 = 0.4, S_4 = 0.7, S_5 = 0.5, S_6 = 0.3, k_2 = 25/9$)

From the Fig.4(a), 4(b), 4(c), three fungal species are observed to show different population growth rates and different wood decomposition rates under three different relative humidity. In Fig.4(a) the population growth rate of fungi 3 is almost equal to 0, because the current environmental humidity (80%) is much greater than its optimum humidity (20%), which inhibits its growth. The wood decomposition rate of the corresponding fungi 3 in Fig.4(b) is also the lowest. From Fig.4(b), (c), (d), (e), we can see that the population growth rate of fungi 1 is slower than the other two fungal species. This is caused by the low competition coefficient of fungi 1.

Also we observe that the decomposition rate of three fungi in Fig.4(b), (d), (f) are related to their population growth rate. When the environmental humidity is 50%, the three fungal species decompose the wood fastest, reaching 100% in 95.5 days. When the environmental humidity is 20%, the three fungal species used 148.5 days.When the environmental humidity is 80%, the three fungal species used 105 days.

4.3.3 TPMLV Model

Because of the diversity of environmental factors, relative humidity cannot represent all of them. Since the fungi can be classified as white rot fungi, brown rot fungi, and soft rot fungi, we assume that in TPLMV model, fungi1 are white rot fungi, fungi 2 are brown rot fungi, fungi 3 are soft rot fungi. Different from previous models, only fungi 2 and fungi 3 will compete for resources in this model because white rot fungus prefer lignin and polysaccharide in wood while brown rot fungus and soft rot fungus prefer cellulose and hemicellulose [**?**]. In order to make the model more accurate, our group considered two more environmental factors, namely, the temperature and PH. Temperature and PH will firstly change the activity of enzymes which will then affect the population rate of fungi and decomposition rate. The updated differential equations can be expressed as follows:

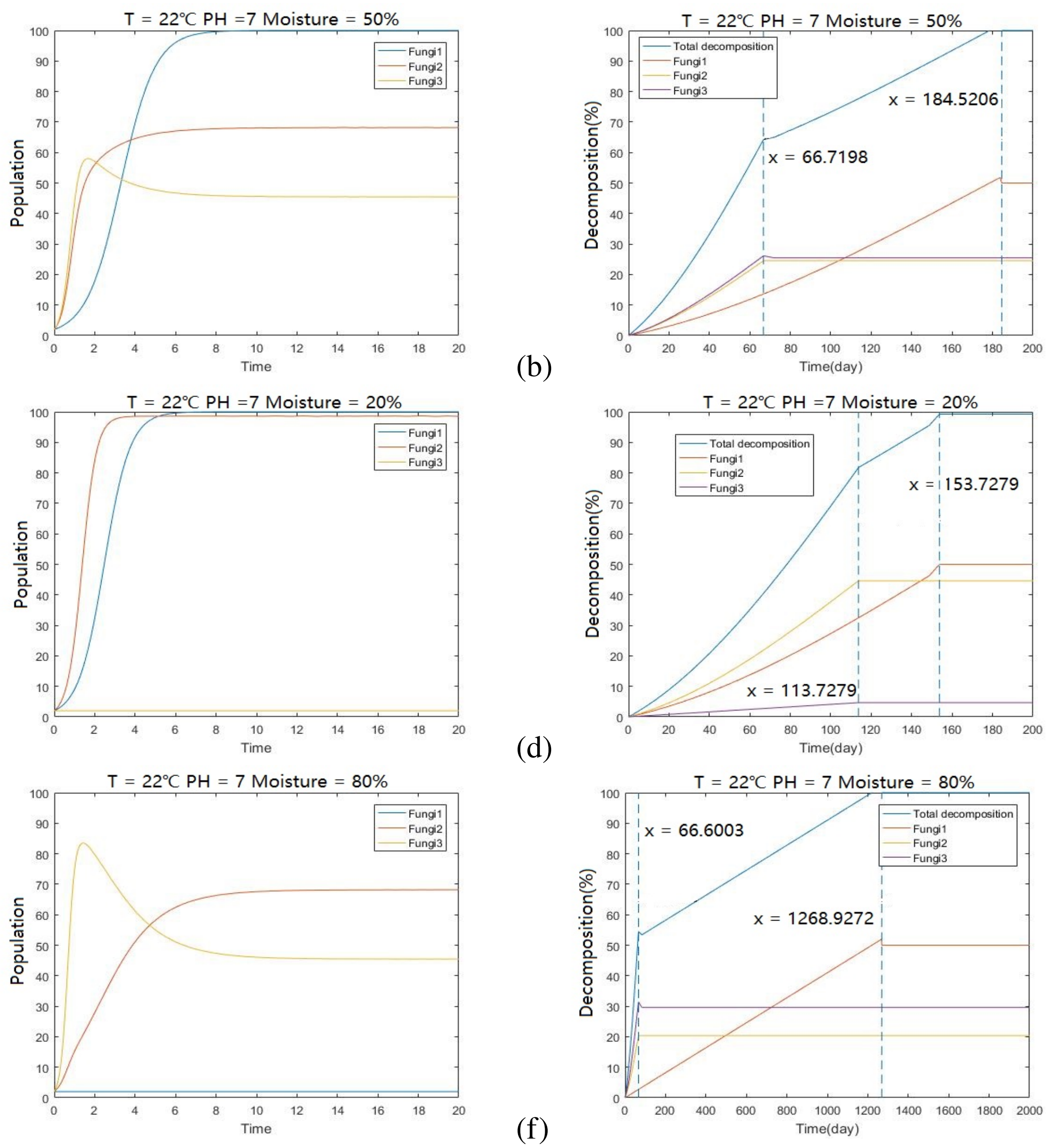
$$\begin{cases} \dot{H}_1(t)=\mathrm{H}_{n1}x_1\{\alpha_1[1-k_1(PH-PH_1)^2(T-T_1)^2]+(1-\alpha_1) \\ \quad \text{- k}_1(PH-PH_4)^2(T-T_4)^2\}(1-k_2(M-M_1)^2)(1-\frac{x_1}{xm_1}) \\ \\ \dot{H}_2(t)=\mathrm{H}_{n2}x_2\{\beta_1[1-k_1(PH-PH_2)^2(T-T_2)^2]+(1-\beta_1) \\ \quad \text{- k}_1(PH-PH_3)^2(T-T_3)^2\}(1-k_2(M-M_2)^2)(1-\frac{x_2}{xm_2}-S_7\frac{x_3}{xm_3}) \\ \\ \dot{H}_3(t)=\mathrm{H}_{n3}x_3\{\beta_1[1-k_1(PH-PH_2)^2(T-T_2)^2]+(1-\beta_1) \\ \quad \text{- k}_1(PH-PH_3)^2(T-T_3)^2\}(1-k_2(M-M_3)^2)(1-\frac{x_3}{xm_3}-S_8\frac{x_2}{xm_2}) \end{cases} \tag{4}$$

Then by putting the population growth rate $\dot{H}_1(t)$, $\dot{H}_2(t)$ and $\dot{H}_3(t)$ into equation (1) in 4.1, we get the equations of fungus decomposition rate of TPMLV model.

4.3.4 Results of TPMLV Model

The fungal population growth lines in Fig.5(a) (c) (e) below are different from the previous two models because we changed the relationship between the three species. Fungus 1 (white-rot fungus) does not compete with the other two fungi for resources, fungus 2 (brown-rot fungus) and fungus 3 (soft-rot fungus) compete with each other. So when in normal growth environment, fungus 1 shows a logistic growth, fungus 2 and fungus 3 fluctuate. However, in Fig.5(c), relative humidity is 20% which is not proper for fungi 3's growth. The yellow line shows that fungi 3 didn't grow. Consequently, fungi 2 do not have the competitor and fungi 1 2 grow in logistic pattern. Similarly in Fig.5(e), fungi 1 do not grow and fungi 2 3 compete for resources.

In model (4), the wood composition will be separated into two parts: composition of lignin, polysaccharide and cellulose, hemicellulose. When relative humidity is 50%, fungi 2 and 3 will use 66.7198 days to finish the decomposition of cellulose & hemicellulose and fungi 1 will use 184.5206 days to finish the decomposition of lignin and polysaccharide. Similarly, when relative humidity is 20%, cellulose and hemicellulose will be decomposed in 113.7279 days and lignin and polysaccharide will be decomposed in 153.7279 days. When relative humidity is 20%, cellulose and hemicellulose will be decomposed in 66.6033 days and lignin & polysaccharide will be decomposed in 1268.9272 days. Compared with the data, our group concluded that relative humidity is an important influence factor. In Fig.5(b) (d) (f), each of the total decomposition lines changes slope for two times. The first transformation means 50% of the total woods (cellulose and hemicellulose) has been decomposed. The second transformation means all the resources has been decomposed.



(a) (b) (c) (d) (e) (f)

Figure 5: Wood decomposition rate and population of the three fungal species in normal environment and 20% and 80% humidity condition environment ($H_{n1} = 2, H_{n2} = 4, H_{n3} = 6, xm_1, xm_2, xm_3 = 100, M_1 = 0.2, M_2 = 0.5, M_3 = 0.8, PH_1 = 8, PH_2 = 6.5, PH_3 = 5.7, PH_4 = 4, T_1 = 30, T_2 = 40, T_3 = 65, T_4 = 61, S_7 = 0.7, S_8 = 0.8, k_1 = 1/32400, k_2 = 25/9, \alpha_1 = 0.5, \beta_1 = 0.5$)

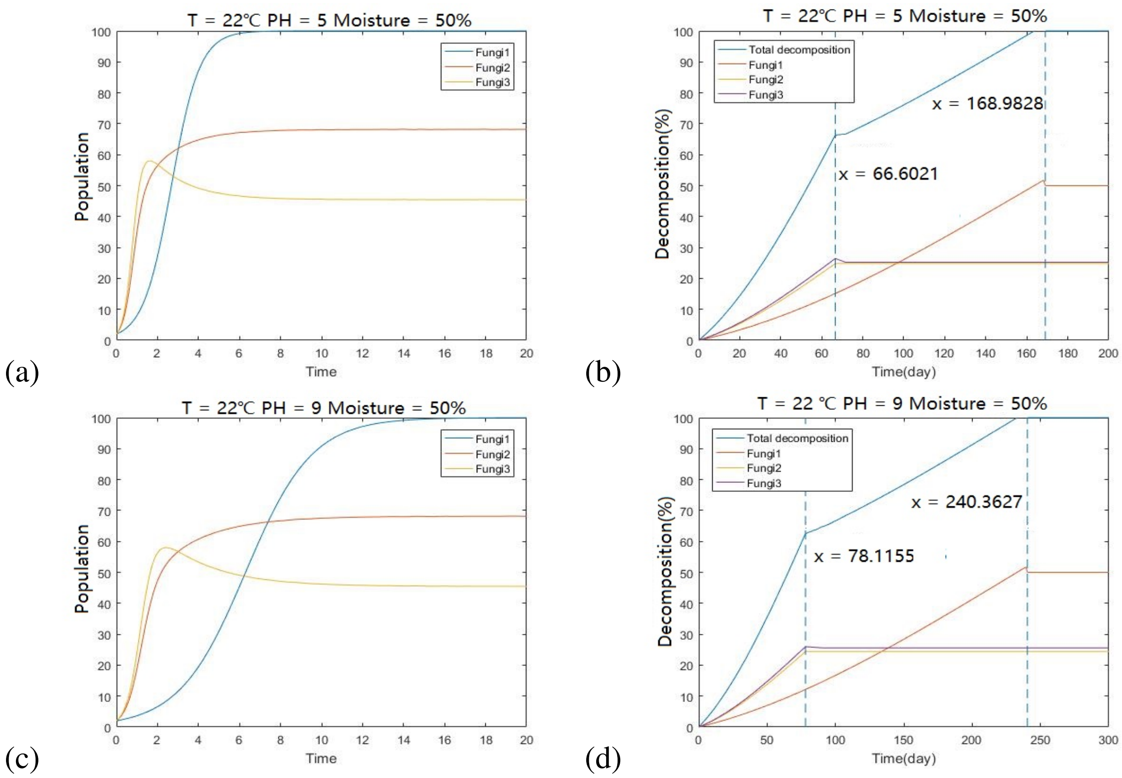
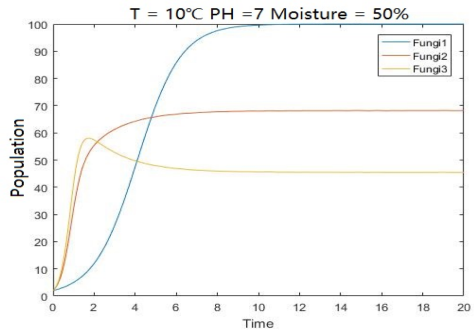
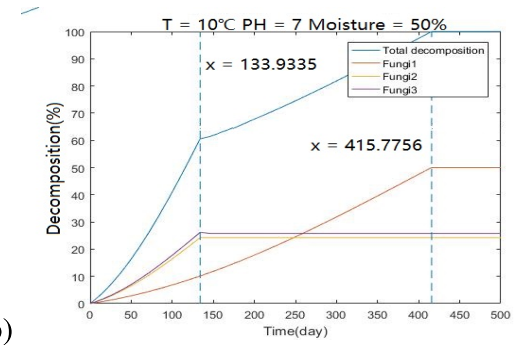


Figure 6: Wood decomposition rate and population of the three fungal species under the condition of pH=5 and pH=9 ($H_{n1} = 2, H_{n2} = 4, H_{n3} = 6, xm_1, xm_2, xm_3 = 100, M_1 = 0.2, M_2 = 0.5, M_3 = 0.8, PH_1 = 8, PH_2 = 6.5, PH_3 = 5.7, PH_4 = 4, T_1 = 30, T_2 = 40, T_3 = 65, T_4 = 61, S_7 = 0.7, S_8 = 0.8, k_1 = 1/32400, k_2 = 25/9, \alpha_1 = 0.5, \beta_1 = 0.5$)

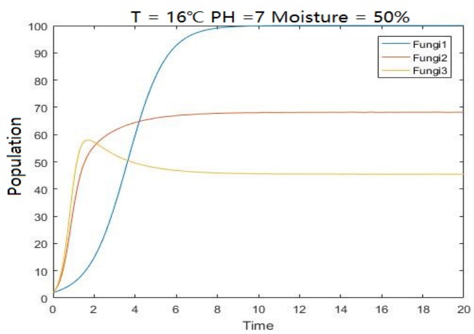
Comparing Fig.6 (a) (c), the variation tendency of lines are very similar, but the time is different. when PH is 5, three fungi species reach to a stable state at 7 time units, and when PH is 9, three fungi species reach to a stable state at 16 time units. Similar in Fig.6 (b) (d) (decomposition graphs), the variation tendency of lines are analogous but when PH is 5, three fungi will use 66.6021 days to finish 50% of the decomposition work and will use 168.9828 to finish all the decomposition work. When PH is 9, they will use 78.1155 days to decompose 50% of the woods and will use 240.3627 days to decompose all the woods. Compared these days, our group concluded that PH is not a important influence factor.



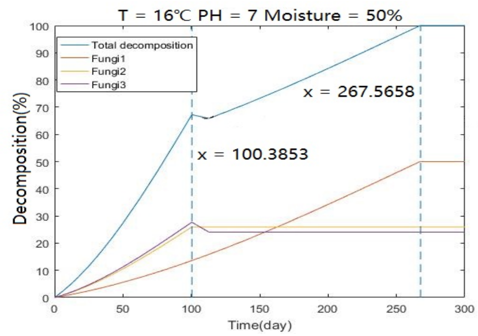
(a)



(b)



(c)



(d)

Figure 7: Wood decomposition rate and population of the three fungal species were under the environment temperature at 10 ℃ and 16 ℃ ($H_{n1} = 2, H_{n2} = 4, H_{n3} = 6, xm_1, xm_2, xm_3 = 100, M_1 = 0.2, M_2 = 0.5, M_3 = 0.8, PH_1 = 8, PH_2 = 6.5, PH_3 = 5.7, PH_4 = 4, T_1 = 30, T_2 = 40, T_3 = 65, T_4 = 61, S_7 = 0.7, S_8 = 0.8, k_1 = 1/32400, k_2 = 25/9, \alpha_1 = 0.5, \beta_1 = 0.5$)

From Fig7. (a) (c), the variation tendency of fungi population are exactly similar. The only difference is when temperature is 16°C, the population of fungi reach the stable state one time unit earlier than 10°C. However, in Fig.7 (d) (f), the decomposition time changes much. According to the equations (1), the temperature is a important factor which will influence the decomposition result. Finally by comparing with decomposition time, our group concluded that temperature is an important influence factor.

4.3.5 Effects of Weather and Environmental Changes on the Decomposition Efficiency of Fungi

In Figure 5 6 7, fungi respond differently to the three environmental factors of relative humidity, pH, and temperature. The decomposition rates of them are mostly sensitive to relative humidity and temperature. Under normal conditions, the humidity and temperature will change over time. Regularly, the humidity is low at noon and high at night. Therefore, fungi decomposes wood more effectively during the day than at night, and is more effective in clear weather than in raining days. It is noted that long-term rainfall will also inhibit the growth of some local fungi and even cause death. This may lead to a decline among local fungal biodiversity. Similarly, temperature is high at noon and low in the morning or at night. So the decomposition rate will be high at noon and also high in summer. But the highest temperature that we analyse is 22°C which is not a very high temperature in nature. From our analysis, low temperature will also inhibit the growth of some local fungi and even cause death.

5 Predictions of Decomposition Time with Different Species and Environment

5.1 Method

Firstly, we use different parameters and combinitions of fungi species and compare with the decomposition time to find the best solution.

Secondly, put the best combination in five environment: arid, semi-arid, temperate, arboreal, and tropical rain forests, as the temperature has been limited as 22°C, 16°C and 10°C

With 3 fungi species, we can obtain 4 combinations:

Table 2: Fungi Combinitions

Combinations	Selection of Fungi
Combination 1	fungi 1, fungi 2
Combination 2	fungi 1, fungi 3
Combination 3	fungi 2, fungi 3
Combination 4	fungi 1, fungi 2, fungi 3

Since combination 3 cannot decompose lignin and polysaccharide, we only compare combinations 1 3 and 4.

For combination 1, the differential equation can be constructed as follows:

$$\begin{cases} \dot{H}_1(t)=\mathrm{H}_{n1}x_1\{\alpha_1[1-k_1(PH-PH_1)^2(T-T_1)^2]+(1-\alpha_1) \\ \quad -\ \mathrm{k}_1(PH-PH_4)^2(T-T_4)^2\}(1-k_2(M-M_1)^2)(1-\frac{x_1}{xm_1}) \\ \\ \dot{H}_2(t)=\mathrm{H}_{n2}x_2\{\beta_1[1-k_1(PH-PH_2)^2(T-T_2)^2]+(1-\beta_1) \\ \quad -\ \mathrm{k}_1(PH-PH_3)^2(T-T_3)^2\}(1-k_2(M-M_2)^2)(1-\frac{x_2}{xm_2}) \\ \end{cases} \tag{5}$$

For combination 3, the differential equation can be constructed as follows:

$$\begin{cases} \dot{H}_1(t)=\mathrm{H}_{n1}x_1\{\alpha_1[1-k_1(PH-PH_1)^2(T-T_1)^2]+(1-\alpha_1) \\ \quad -\ \mathrm{k}_1(PH-PH_4)^2(T-T_4)^2\}(1-k_2(M-M_1)^2)(1-\frac{x_1}{xm_1}) \\ \\ \dot{H}_3(t)=\mathrm{H}_{n3}x_3\{\beta_1[1-k_1(PH-PH_2)^2(T-T_2)^2]+(1-\beta_1) \\ \quad -\ \mathrm{k}_1(PH-PH_3)^2(T-T_3)^2\}(1-k_2(M-M_3)^2)(1-\frac{x_3}{xm_3}) \\ \end{cases} \tag{6}$$

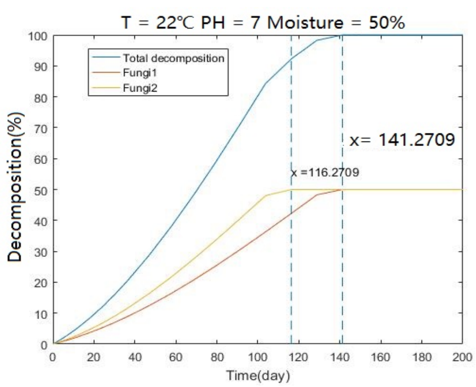
For combination 4, the differential equation can be constructed as follows:

$$
\begin{cases}
\dot{H}_1(t)=\mathrm{H}_{n1}x_1\{\alpha_1[1-k_1(PH-PH_1)^2(T-T_1)^2]+(1-\alpha_1) \\
\qquad \text{- } \mathrm{k}_1(PH-PH_4)^2(T-T_4)^2\}(1-k_2(M-M_1)^2)(1-\frac{x_1}{xm_1}) \\
\\
\dot{H}_2(t)=\mathrm{H}_{n2}x_2\{\beta_1[1-k_1(PH-PH_2)^2(T-T_2)^2]+(1-\beta_1) \\
\qquad \text{- } \mathrm{k}_1(PH-PH_3)^2(T-T_3)^2\}(1-k_2(M-M_2)^2)(1-\frac{x_2}{xm_2}-S_7\frac{x_3}{xm_3}) \\
\\
\dot{H}_3(t)=\mathrm{H}_{n3}x_3\{\beta_1[1-k_1(PH-PH_2)^2(T-T_2)^2]+(1-\beta_1) \\
\qquad \text{- } \mathrm{k}_1(PH-PH_3)^2(T-T_3)^2\}(1-k_2(M-M_3)^2)(1-\frac{x_3}{xm_3}-S_8\frac{x_2}{xm_2})
\end{cases}
\tag{7}
$$

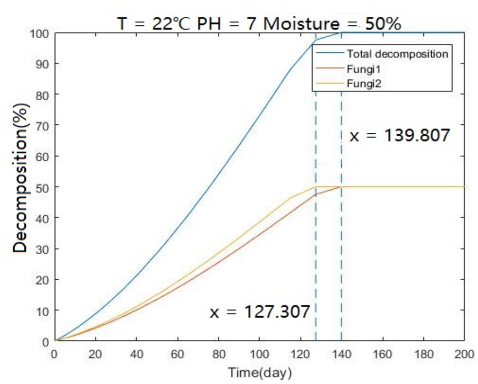
5.2 Prediction Results

From Fig.8(a),it takes 141.2709 days for the combination of fungi 1 and 2 to completely decompose at the same temperature, pH and humidity.From Fig.8(b), the combination of fungi 1 and 3 takes 139.807 days, and from Fig.8(c) the combination of fungi 1, 2 and 3 takes 133.9293 days. Since we use the decomposition rate to measure the pros and cons of the combination, we can draw the following conclusions through comparing their decomposition time. So the hierarchy of three species are:

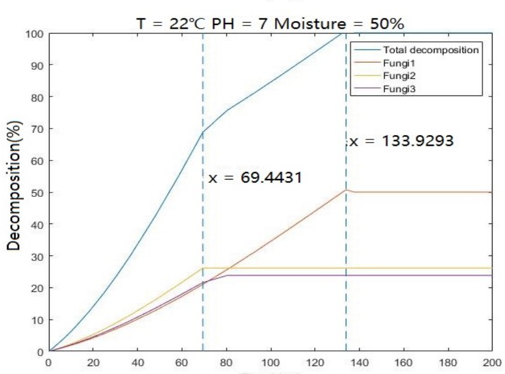
Fungi 1, 2 and 3 > Fungi 1 and 3 > Fungi 1 and 2



(a)



(b)



(c)

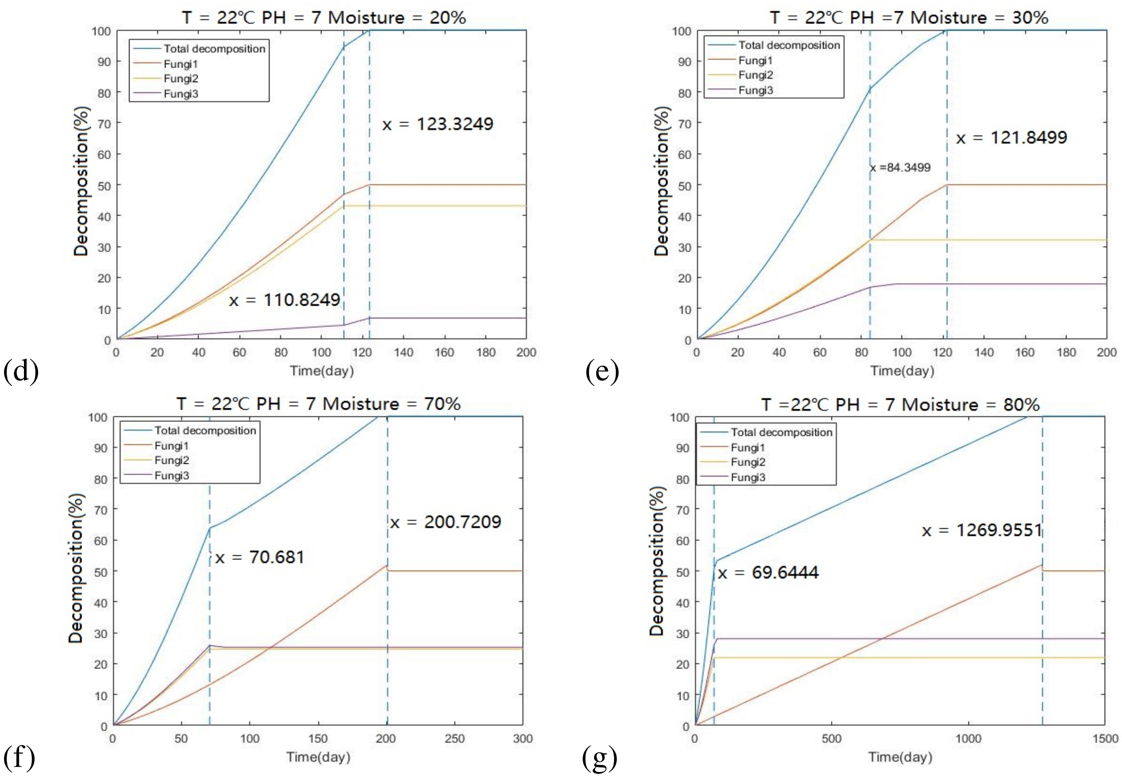


Figure 8: Wood decomposition rate and population size of the three fungal species under different humidity and resource competition($H_{n1} = 4, H_{n2} = 4, H_{n3} = 4, xm_1, xm_2, xm_3 = 100, M_1 = 0.2, M_2 = 0.5, M_3 = 0.8, PH_1 = 8, PH_2 = 6.5, PH_3 = 5.7, PH_4 = 4, T_1 = 30, T_2 = 40, T_3 = 65, T_4 = 61, S_7 = 0.7, S_8 = 0.8, k_1 = 1/32400, k_2 = 25/9, \alpha_1 = 0.5, \beta_1 = 0.5$)

From Fig8.(d) (e) (f) (g), we can see the performance of the combination of fungi 1, 2, and 3 under different environmental conditions. Because we express the different environmental states by keeping the same temperature and PH value and changing the relative humidity. Therefore, we can know that the graphs from 1-5 are arid, semi-arid, temperate, arbor and tropical rain forest environments.

Under drought conditions, we can see that fungi 2 and 3 used 110.8249 days to decompose cellulose and hemicellulose. Since only fungi1 is left for decomposition, the slope of the curve of total decomposition becomes slower. After 123.3249 days, the wood was completely decomposed.

Under semi-arid conditions, we can see fungi 2 and 3 decompose cellulose and hemicellulose in 84.3499 days. Since only fungi1 is left for decomposition, the slope of the curve of total decomposition becomes slower. After 121.8499 days, the wood was completely decomposed

In a temperate environment, we can see fungi2 and 3 decompose cellulose and hemicellulose in 69.4431 days. Since only fungi1 is left for decomposition, the slope of the curve of total decomposition becomes slower. After 133.9293 days, the wood was completely decomposed.

In the arbor environment, we can see fungi2 and 3 decompose cellulose and hemicellulose in 70.681 days. Since only fungi1 is left for decomposition, the slope of the curve of total decomposition becomes slower. At day 200.7209, the wood was completely decomposed

In the tropical rain forest environment, we can see fungi 2 and 3 used 69.6444 days to decompose cellulose and hemicellulose. Since only fungi1 is left for decomposition, the slope of the curve of total decomposition becomes slower. On day 1269.9551, the wood was completely decomposed. Based on the above analysis, we can get the following figure:

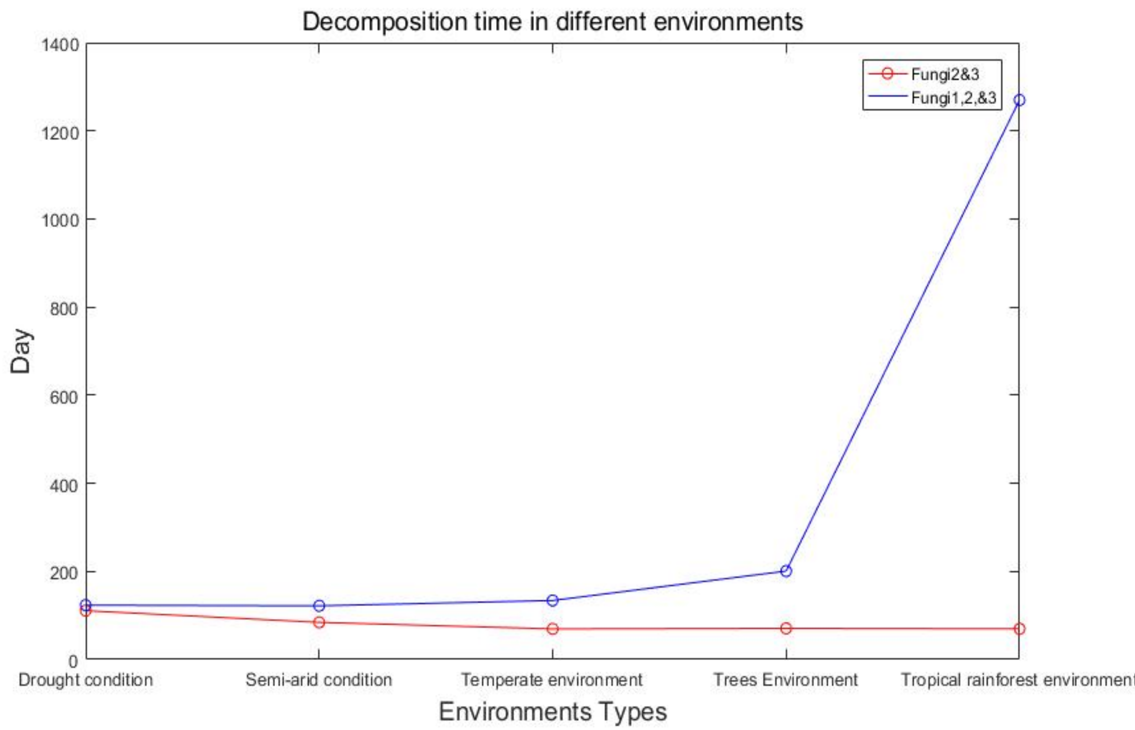


Figure 9: Comparison of decomposition time

The blue line shows the decomposition time of fungi 2 3 (decomposition time of cellulose, hemicellulose, 50% proportion). The red line shows the decomposition time of fungi 1 2 3 (decomposition time of lignin, cellulose, hemicellulose and polysaccharides, 100% proportion).

6 Strengths

- We improved the traditional Lotka-Volterra Model and transform it into TPMLV Model which considers more factors.
- We divided the factors affecting fungal growth into two parts: 1) environmental factors, e.g. temperature, humidity, pH, and 2) species interactions, e.g. competition for resources.
- We analyzed the effect of key factors on the rate of fungal growth and decomposition of wood and predicted the time required to completely decompose the wood.

7 Weaknesses

- Since there are many factors affecting the growth of fungi, we only took three factors, temperature, humidity and PH, for analysis, which may cause some errors.

- Since we lack the provided data between the fungal growth rate and decomposition rate at different temperatures, we perform function estimation, which may lead to some errors.

8 Conclusion

Our work can be summarized as the flow chart below: Based on the above models, we can draw

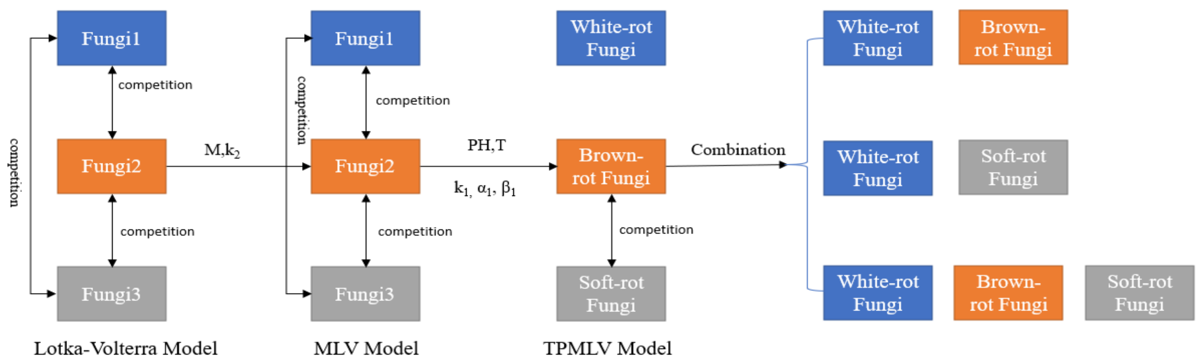


Figure 10: Flowchart

the following conclusions. First, under ideal conditions, the three fungal species will rob each other for resources. Eventually, the populations of the three fungal species will reach a steady state, and the weaker fungal species will face the threat of death. Under these conditions, the efficiency of wood decomposition by the fungi is the fastest. Second, the three fungal species competed for resources considering only humidity. We found that the time required for wood decomposition was shortest at 50% ambient humidity. And the first fungal species was more sensitive to changes in ambient humidity. Third, we set the three fungi as white rot, brown rot and soft rot and considered the effect of ambient temperature and pH on fungal enzymes. We found that the sensitivity of the fungi to changes in temperature, humidity and PH can be ranked from strong to weak: humidity, PH and temperature. The sensitivity of the white rot fungus to PH was greater than the other two fungi, and the brown rot fungus was more adaptable to the change of environmental humidity. And the fungi may face the threat of death when they are in rainy or sunny environment for a long time. Fourth, we predicted different combinations of the three fungi, and we found that wood decomposition efficiency was highest when all three fungi were present together. This suggests that biodiversity is crucial to the carbon cycle in ecosystems.

9 Further Work

In summary, our model involves a variety of factors and considers a more comprehensive perspective, but due to time constraints, our model cannot consider more indicators. If time is sufficient, we can do the following: 1. Consider more environmental indicators and build a more complete model to get a more accurate model 2. Investigate and find data on fungal growth rate and decomposition rate under different conditions, and get more complete data

CONTENT:

- Fungal Community
- Fungal diversity
- Fungal diversity and ground litter decomposition
- Fungal diversity and carbon cycle

FUNGI DIVERSITY AND ECOSYSTEM CARBON CYCLE

Learning Outcomes

Upon completion of this section, you should be able to:

1. Understand the types of fungi that decompose ground litter

2. Understand the meaning of fungal diversity

3. Understand the role of fungal diversity on the decomposition of ground litter

4. Understand the significance of fungal diversity for the carbon cycle

Fungus, a microorganism with degrading ability. Fungi are of great significance to the decomposition of ground litter, and the decomposition of ground litter has an important impact on the carbon cycle of the ecosystem. Therefore, this chapter will start with fungi and fungal diversity, focusing on the importance of fungi to the ecosystem.

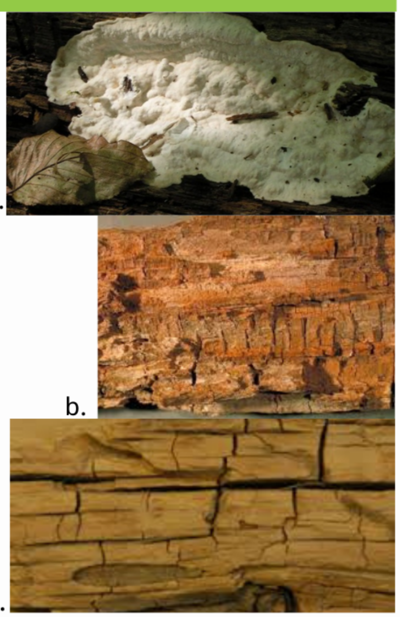
Fungal Community

There are many types and numbers of fungi, but the fungi that degrade ground litter can be divided into three categories according to their growth substrate preference and decay mode: white rot fungi, brown rot fungi, and soft rot fungi. The part that fungi degrade wood is actually lignin, cellulose, hemicellulose and polysaccharides.

White rot fungus (Figure 1a.), a microorganism that can degrade the main components of wood, is named because of the white appearance of decayed wood. White-rot fungi degrade wood as lignin and polysaccharides. However, after white rot, the wood can still remain intact. Therefore, white rot fungi are often used for biological pulping.

Brown rot fungus (Figure 1b.), a microorganism that can degrade cellulose and hemicellulose in wood. And there are various types of brown rot fungi. In North America, there are about 1600-1700 species. In the process of degrading wood by such bacteria, the wood will appear reddish brown in appearance and form fragile pieces. Due to its strong ability to degrade cellulose and hemicellulose, brown rot fungi are of great significance for extracting pulping materials and perfecting biological pulping technology.

Soft rot fungus (Figure 1c.), a microorganism that mainly degrades cellulose and hemicellulose in wood. The erosion pattern of soft rot fungi is unique, and wood that has been soft rot will leave holes. In addition, soft-rot fungi are hygroscopic, so they are more common in aquatic environments.



a.

b.

c.

Figure 1. Three Fungi
a.White rot fungus
b.Brown rot fungus
c.soft rot fungus

Fungal Diversity

Biodiversity is typically a measure of variation at the genetic, species, and ecosystem level. Biodiversity is of great significance to the sustainable development of ecosystem organisms. Here, here, biodiversity measures the ecosystem. Fungal diversity refers to the diversity of fungal species. The more fungal species in the ecological environment, the more diverse the organisms. The ecosystem with the most fungal diversity in this chapter is an ecosystem with three types of fungiat the same time.

Figure 2.Ground Litter

Fungal Diversity and Ground Litter Decomposition

The decomposed components of ground litter (Figure 2.) are mainly lignin, cellulose, hemicellulose and polysaccharides. These ingredients are mainly white rot fungi, brown rot fungi, and soft rot fungi secreting lignin degrading enzymes, cellulose degrading enzymes, hemicellulose degrading enzymes, and pectin degrading enzymes. The decomposition rate is affected by enzyme activity, and the activity of these enzymes is mainly affected by temperature and ph. The impact of fungal diversity on the ecosystem is mainly reflected in the rate of decomposition of ground litter. Under the condition of controlling the activity of enzymes, the more fungal diversity a system has, the more types of fungi the system contains, and the more types of components in the wood are decomposed and faster.

Fungal Diversity and Carbon Cycle

The more diverse the fungus, the faster the fungus decomposes litter on the ground, and the shorter the time it takes to release carbon into the atmosphere. As more carbon is released faster, green plants have enough carbon raw materials to fix, and new organisms can metabolize and grow faster in new cities. Therefore, the carbon cycle (Figure 3.) of the ecosystem can occur healthily. For example, the temperature, pH, and humidity near the equator are the most suitable for the growth of fungi. The more fungi species there are, the faster the fungi decompose litter and accelerate the carbon cycle of the ecosystem. Therefore, the biodiversity near the equator is also richer.

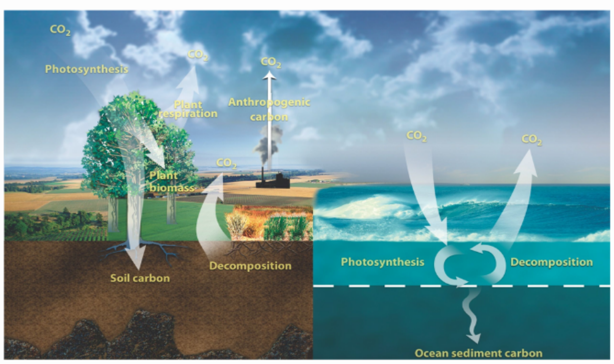


Figure 3. Carbon Cycle

Check your progress:

1. Distinguish different types of fungi that decompose ground litter
2. Explain the meaning of fungal diversity
3. Represent the role of fungal diversity on the decomposition of ground litter
4. State the significance of fungal diversity for the carbon cycle

References

[1] B. Kendrick. Fungi: ecological importance and impact on humans. *eLS*, 2011.

[2] Qin W. Sista Kameshwar, A. K. Comparative study of genome-wide plant biomass-degrading cazymes in white rot, brown rot and soft rot fungi. *Mycology*, 9(2):93–105, 2018.

[3] S. A. M. Hamed. In-vitro studies on wood degradation in soil by soft-rot fungi: Aspergillus niger and penicillium chrysogenum. *International Biodeterioration Biodegradation*, 78:98–102, 2013.

[4] Vazquez J. A. Murado M. A. Prieto, M. A. A new and general model to describe, characterize, quantify and classify the interactive effects of temperature and ph on the activity of enzymes. *Analyst*, 140(10):3587–3602, 2015.

[5] Abd Aziz S. Hassan M. A. Rahman, N. H. A. Production of ligninolytic enzymes by newly isolated bacteria from palm oil plantation soils. *BioResources*, 8(4):6136–6150, 2013.

[6] Bundela P. S. Pandey A. K. Khan J. Awasthi M. K. Sarsaiya S. Gautam, S. P. Optimization for the production of cellulose enzyme from municipal solid waste residue by two novel cellulolytic fungi. *Biotechnology research international*, 2011.

[7] Mechelke M. Baudrexl M. Mennerich D. Hornburg D. Mann M. ... Zverlov V. V. Broeker, J. The hemicellulose-degrading enzyme system of the thermophilic bacterium clostridium stercorarium: comparative characterisation and addition of new hemicellulolytic glycoside hydrolases. . *Biotechnology for biofuels*, 11(1):1–18, 2018.

[8] Desagiacomo C. Polizeli M. D. L. T. D. M. Damasio A. R. D. L. Lima A. M. F. Gomes E. Bonilla-Rodriguez G. O. Trindade, L. V. Biochemical characterization, thermal stability, and partial sequence of a novel exo-polygalacturonase from the thermophilic fungus rhizomucor pusillus a13. 36 obtained by submerged cultivation. *BioMed research international*, 2016.

[9] C Zhu and G Yin. On hybrid competitive lotka–volterra ecosystems. *Nonlinear Analysis: Theory, Methods & Applications*, 71:e1370–e1379, 2009.